

# **“Rationing by Waiting as a Means of Second-Degree Price Discrimination in a Vertically Differentiated Market.”**

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EDUARDO PEDRAL SAMPAIO FIUZA

( Doutorando EPGE/FGV e Pesquisador do IPEA/DIPES )

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**Coordenação: Prof. Pedro Cavalcanti Gomes Ferreira**  
**Email: ferreira@fgv.br - ☎ (021) 536-9250**

# RATIONING BY WAITING AS A MEANS OF SECOND-DEGREE PRICE DISCRIMINATION IN A VERTICALLY DIFFERENTIATED MARKET

Eduardo Pedral Sampaio Fiuza<sup>1</sup>

## INTRODUCTION

In the classical models it is an auctioning mechanism that adjusts prices so as to balance supply and demand in a Walrasian equilibrium. Whenever a binding price control is enforced in a perfectly competitive market some kind of shortage arises. In monopolistic, collusive or imperfectly competitive cases, though, supply and total surplus of a market can always be increased by means of a price control, as long as it is not "too tight" and the good is not modifiable.

However, in case the government's enforcement power is feeble, the monopolist (or a cartel of dominant firms) may try to circumvent the price control in many ways. One of them is simply to deviate the output to a black market and charge there the monopolistic price, while undersupplying the formal market. That was an ordinary practice in the former planned economies. Another resource is to "make up" the good in order to evade the inspection, and charge the same monopolistic price, either by stating that the good was another one not subject to the price control, or by overreporting the actual quality or quantity content. That practice was widespread in Brazil during long periods of price freezes, especially at the time of the stabilization plans (starting in the Cruzado Plan, in 1986): the quality or quantity content of several goods was reduced, so as to compensate for the difference between desired and allowed price. Last, the monopolist could still bundle the controlled good to a complementary non-controlled (or easier to be "made up") good<sup>2</sup>.

When goods are differentiated, a monopolistic multiproduct firm can still combine the strategies above and circumvent the price control for one of its goods in another fashion: by inducing shortage or quality downgrade of the controlled good, it may "dissuade" part of the consumers from their original consumption plan and replace it with another one free from control, or, more generally, with one that yields a higher mark-up.

In this paper we will analyze this phenomenon within a second-degree price discrimination framework: the monopolist offers two packages to the consumers, and each of them in turn chooses the one that best fits their tastes. In a sequentially rational way, then, the monopolist manipulates the price-quality or price-quantity binomial so as to extract the maximum surplus from the consumers, subject to two types of constraint: a) the **individual rationality** (or participation) constraint, which means that the consumer will only participate in the market if the package allows her a utility at least as great as the outside alternative (in

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<sup>1</sup> Doutorando da EPGE / FGV e pesquisador do IPEA / DIPES.

<sup>2</sup> Ability to make up is greater when the good is highly differentiated and has multiple characteristic dimensions.

general normalized to zero); b) the **incentive compatibility** constraint, i.e., that the package designed for consumer  $j$  is not chosen (provides a lower utility) by consumer  $i$ , and vice-versa. Tirole (1988, ch. 3) used this framework to show that an unregulated monopolist who can charge two-part tariffs but cannot distinguish between the two types of consumers will offer two packages: the first one, with a lower quantity (quality), a lower fixed fee and a higher marginal price, provides a suboptimal quantity (quality); the second one is meant to the higher taste consumers, and charges a higher fixed fee, a lower marginal price (equal to marginal cost) and higher (optimal) quantity (quality). Tirole's reasoning is quite similar to Mussa and Rosen's (1978)<sup>3</sup>.

The introduction of a quality-dependent price control can change this result. In fact, we can show that a well-designed price-control can increase total surplus in some situations, even when the monopolist responds to this kind of control by creating a queue. In the limit, a very well-designed price-control may reach the first-best: optimal quality (i.e., maximal total surplus) for both goods is obtained, and no queue arises. Despite the allocative efficiency, however, the distribution of benefits is not the one envisioned by policymakers, as we shall see below.

The present article is organized as follows. Section 1 reviews the literature on queuing. Section 2 introduces the model we utilize. The final section summarizes the conclusions and the possible extensions from this analysis.

## 1. RATIONING BY WAITING

Not many papers on rationing by waiting can be found in the economic literature. A classical article on that subject was published by Barzel (1974). In his pioneer work, he introduced the role of waiting time as a total price. Consequently, a supply and demand of waiting time fills the "rationing" gap that arises when the market for the good is in disequilibrium. But he does not explore the possibility of discrimination: "In the absence of discrimination among consumers only one time-price will prevail in the market." (p.75), and the equilibrium length of time spent is determined by the value, net of price, that consumers place on marginal units of the controlled good. The mechanics of distribution of the good (e.g. how fast the customer is served, what size of batches, etc.) do not matter. But the "time-price" varies negatively in response to variations of the size of the allotment served or of the number of consumers, and positively to variations of income or prices of substitutes (just like a scarcity price). His insight that time-price will eventually consume all the consumer's surplus is particularly useful for our following analysis.

Deacon and Sonstelie (1989) surveys the literature on price control and rent-seeking behaviors. They list several (costly) actions that the consumers

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<sup>3</sup> The basic difference between them is that the former assumes constant marginal cost and concave indifference curves, while the latter assumes increasing marginal cost and linear indifference curves instead.

undertake to minimize the cost of waiting, such as buying larger batches at longer intervals -- incurring in costs of storage and/or spoilage --, alternating the standing on the queue with neighbors, hiring someone to stay in the queue, etc. Even a well-designed and well-managed coupon system will suffer from misallocation problems, due to insufficient knowledge of the demand. If coupons are oversupplied, rent-seeking behavior is again induced, adding to the deadweight loss as well. In another paper (Deacon and Sonstelie, 1985), they report an experiment run with participants of a queue at a gas station: the motorists faced a choice between waiting in line for low-priced gasoline at a Chevron station, subject to price control, and purchasing at a higher price without waiting, at stations serving other brands which were not subject to that control. Their choices were input into a revealed preference model to estimate value of time, which was found quite similar to individuals' after-tax wages.

Kornai (1980) discussed the producers' and consumers' behaviors when their input supplies are subject to permanent scarcity. The consumer, for instance, searches for the good, and if she does not find it, she can search again, substitute another good for it, wait, try again, and so forth. He did not really formalize the process in a complete mathematical model, though. Also unfortunately for our purposes, his concern was describing a planned economy<sup>4</sup>, where the prices have practically no allocative role -- only the production plans do.

Our model relies heavily on Tirole's simple model. By introducing a price control and the possibility of creating a queue, we extract very powerful results. The model's description follows.

## 2. THE MODEL

Following Tirole (1988), we assume a market with two products, vertically differentiated by a one-dimensional characteristic: quality, which can be  $q_1$  or  $q_2$  (necessarily  $q_2 > q_1$ ). There are two types of consumers, with unit demands and utility functions differentiated only by a multiplicative coefficient: consumer  $i$ 's utility (or surplus) is  $U_i(q, T) = \theta_i V(q) - T$ ,  $i = 1, 2$ , where  $\theta_2 > \theta_1$ ,  $T_i$  is a two-part tariff paid for package  $i$  and  $V'(q) \geq 0$  and  $V''(q) \leq 0$ <sup>5</sup>. Tariffs and quality are observed by all the agents: monopolist, consumers and government. Fixed cost is assumed to be zero, and variable and marginal cost is assumed constant. We allow the marginal cost for good 1 to be less than or equal to the marginal cost for good 2:  $c_2 \geq c_1$  (may be lower because of a lower tax rate).

The taste parameters  $\theta_i$  are only observed by the own consumers; monopolist and government only know the proportion each taste appears in the market; consumers 1 occur with probability  $\lambda$ , and consumers 2 with probability  $1 - \lambda$ .

<sup>4</sup> As a matter of fact, he is Hungarian and the book was written before the downfall of communism in Eastern Europe.

<sup>5</sup> The  $V(\cdot)$  function can be interpreted as the total surplus associated with quality, and its derivative as the negative of the inverse demand as a function of quality.

Further, it is assumed that any kind of binding price control enforced only for good 1 can be circumvented by the monopolist by means of creating a queue. Following the literature, we assume that the waiting time subtracts utility from the consumers in a linear fashion:

$$U_i(q, T, t) = \theta_i V(q) - T - \delta_i t,$$

where  $t$  is the waiting time and  $\delta_i$  is the consumer  $i$ 's "impatience rate", and the coefficient of disutility for the queue is also greater for consumer 2 than for consumer 1,  $\delta_2 > \delta_1$ . Moreover, we assume that  $\delta_2 \theta_2 > \delta_1 \theta_1$ , that is, the ratio between impatience and taste is greater for consumer 2 than for consumer 1.<sup>6</sup>

The price control we assume is a very general one, and may be non-binding:  $T_1 \leq M(q_1)$ . The  $M(\cdot)$  function can be a constant, otherwise we assume it is "sufficiently" convex; that means that controlling the price of good 1 does not bind the price of good 2, even if it binds the price for good 1. In other words, the price ceiling rises much faster than total surplus for good 2, so the price control is indeed exclusively targeting the "plain good".

Since the price control is assumed never-binding for good 2, there is no point for the monopolist to create a queue for it, as it can appropriate the "waste" of time through the fixed fee. Thus, we simply set  $t_2 = 0$  at once. On the other hand, the market for good 1 can display this kind of "waste" of time, because this waiting time can prevent the luxury consumer from buying good 1. If there existed a continuum of consumers' tastes and impatience rates, this would mean that creating a queue would refrain some of the luxury consumers from buying plain goods. In our simplified model, the queue enables the monopolist to charge a higher tariff  $T_2$  from **all** the type-2 consumers.

The objective function that the firm maximizes is, therefore:

$$\Pi = \max \{ \lambda [T_1 - c_1 q_1] + (1-\lambda) [T_2 - c_2 q_2] \}$$

s.t.

$$\text{QD: } q_2 \geq q_1$$

$$\text{WT: } t_1 \geq 0$$

$$\text{PC: } M(q_1) \geq T_1$$

$$\text{IR}_1: \theta_1 V(q_1) - T_1 - \delta_1 t_1 = 0$$

$$\text{IR}_2: \theta_2 V(q_2) - T_2 \geq 0$$

$$\text{IC}_1: \theta_1 V(q_1) - T_1 - \delta_1 t_1 \geq \theta_1 V(q_2) - T_2$$

$$\text{IC}_2: \theta_2 V(q_2) - T_2 \geq \theta_2 V(q_1) - T_1 - \delta_2 t_1$$

<sup>6</sup> This is very sensible: a queue is feasible if the threaten of a waiting time "scares" the "luxury consumer", as compared to the "plain consumer", more than they differ in their preferences for quality (just read the inequality above as  $\delta_2 \theta_1 > \theta_2 \delta_1$ ).

where QD stands for Quality Difference;

WT for Waiting Time;

PC for Price Control;

IR for Individual Rationality;

IC for Incentive Compatibility.

Substituting IR<sub>1</sub> into IC<sub>1</sub>, IC<sub>2</sub> and WT, the Lagrangian becomes:

$$\begin{aligned}\mathcal{L} = & \lambda[T_1 - c_1 q_1] + (1-\lambda)[T_2 - c_2 q_2] + \eta\left[\frac{\theta_1 V(q_1) - T_1}{\delta_1}\right] + \mu[M(q_1) - T_1] + \\ & + v_2\{\theta_2[V(q_2) - V(q_1)] - (T_2 - T_1) + \delta_2\left[\frac{\theta_1 V(q_1) - T_1}{\delta_1}\right]\} + \\ & + v_1\{T_2 - \theta_1 V(q_1)\} + \phi[\theta_2 V(q_2) - T_2] + \gamma(q_2 - q_1)\end{aligned}$$

and the First-Order Conditions are:

- (1)  $1 - \frac{\mu}{\lambda} - \frac{\eta}{\lambda \delta_1} + \frac{v_2}{\lambda} \left\{1 - \frac{\delta_2}{\delta_1}\right\} = 0$
- (2)  $\theta_1 V'(q_1) \left\{ \frac{\eta}{\lambda \delta_1} + \frac{v_2}{\lambda} \left[ \frac{\delta_2}{\delta_1} - \frac{\theta_2}{\theta_1} \right] \right\} + \frac{\mu}{\lambda} M'(q_1) = c_1 + \gamma$
- (3)  $(1-\lambda) - v_2 + v_1 - \phi = 0$
- (4)  $\theta_2 V'(q_2) \left[ v_2 + \phi - v_1 \frac{\theta_2}{\theta_1} \right] = (1-\lambda)c_2 - \gamma$
- (5)  $q_2 - q_1 \geq 0$
- (6)  $\frac{\theta_1 V(q_1) - T_1}{\delta_1} \geq 0$
- (7)  $M(q_1) - T_1 \geq 0$
- (8)  $\theta_2[V(q_2) - V(q_1)] - (T_2 - T_1) + \delta_2\left[\frac{\theta_1 V(q_1) - T_1}{\delta_1}\right] \geq 0$
- (9)  $T_2 - \theta_1 V(q_1) \geq 0$
- (10)  $\theta_2 V(q_2) - T_2 \geq 0$
- (11)  $\gamma(q_2 - q_1) = 0$
- (12)  $\eta\left[\frac{\theta_1 V(q_1) - T_1}{\delta_1}\right] = 0$
- (13)  $\mu[M(q_1) - T_1] = 0$

$$(14) v_2 \{ \theta_2 [V(q_2) - V(q_1)] - (T_2 - T_1) + \delta_2 \left[ \frac{\theta_1 V(q_1) - T_1}{\delta_1} \right] \} = 0$$

$$(15) v_1 \{ T_2 - \theta_1 V(q_1) \} = 0$$

$$(16) \phi [\theta_2 V(q_2) - T_2] = 0$$

The Kuhn-Tucker cases to be examined are displayed on Table 1. They are numbered from 1 to 32 and cover all the possible combinations between binding and non-binding constraints, except the cases where  $q_2 = q_1$ , which we did not take the time to examine, as we are not interested in this kind of situation. Each constraint corresponds to a column, and each case to a line. A cell's content may be either a zero or a plus. A zero means that the constraint is binding (e.g. IR<sub>2</sub>:  $\theta_2 V(q_2) - T_2 = 0$ ) for that particular case, and a plus indicates that it is slack (in the same example, IR<sub>2</sub>:  $\theta_2 V(q_2) - T_2 > 0$ ). Note that some cases are readily ruled out with one or more "X's" because of incompatibilities concerning the constraints. The incompatibilities we found are named "problems" and are carefully described in the Appendix. The cases without any "X" are possible solutions to be chosen from by the firm..

As Table 1 shows, only four cases are not ruled out: 13, 14, 21 and 29. Case 21, in particular, is exactly Tirole's result, but within our present framework its occurrence depends on the parameters of impatience and distribution of consumers. We display below the FOC's for the four selected cases, and the particular conditions for their occurrence.

**Case 13:** (5) > 0; and (11) =>  $\gamma = 0$ ; (QD slack)  
 (6) > 0; and (12) =>  $\eta \geq 0$ ; (WT slack)  
 (7) = 0; and (13) =>  $\mu = 0$ ; (PC binding)  
 (8) = 0; and (14) =>  $v_2 \geq 0$ ; (IC<sub>2</sub> binding)  
 (9) > 0; and (15) =>  $v_1 = 0$ ; (IC<sub>1</sub> slack)  
 (10) > 0; and (16) =>  $\phi = 0$ . (IR<sub>2</sub> slack)

(8), (6) and (10) =>  $\theta_1 V(q_1) > T_1 > k \theta_1 V(q_1)$ ,  
 (19)

where  $k = \frac{(\frac{\delta_2}{\delta_1} - \frac{\theta_2}{\theta_1})}{\frac{\delta_2}{\delta_1} - 1} < 1$ ;

TABLE 1

Case	QD	WT	PC	IC <sub>2</sub>	IC <sub>1</sub>	IR <sub>2</sub>	Problems											
							1	2	3	4	5	6	7	8	9	10	11	12
1	+	+	+	+	+	+	X											
2	+	+	+	+	+	0	X											
3	+	+	+	+	0	+		X										
4	+	+	+	+	0	0			X									
5	+	+	+	0	+	+	X											
6	+	+	+	0	+	0	X											
7	+	+	+	0	0	+				X								
8	+	+	+	0	0	0			X	X								
9	+	+	0	+	+	+					X							
10	+	+	0	+	+	0					X							
11	+	+	0	+	0	+						X						
12	+	+	0	+	0	0			X									
13	+	+	0	0	+	+												
14	+	+	0	0	+	0												
15	+	+	0	0	0	+							X					
16	+	+	0	0	0	0			X									
17	+	0	+	+	+	+								X				
18	+	0	+	+	+	0									X			
19	+	0	+	+	0	+								X				
20	+	0	+	+	0	0			X									
21	+	0	+	0	+	+												
22	+	0	+	0	+	0										X		
23	+	0	+	0	0	+											X	
24	+	0	+	0	0	0			X									
25	+	0	0	+	+	+								X				
26	+	0	0	+	+	0												X
27	+	0	0	+	0	+								X				
28	+	0	0	+	0	0			X									
29	+	0	0	0	+	+												
30	+	0	0	0	+	0										X		
31	+	0	0	0	0	+											X	
32	+	0	0	0	0	0			X									



$$(1) \text{ into } (2): (1 - \frac{\mu}{\lambda}) k \theta_1 V'(q_1) + \frac{\mu}{\lambda} M'(q_1) = c_1 \quad (18)$$

$$(3) \text{ into } (1): \mu = \lambda - (1-\lambda)(\frac{\delta_2}{\delta_1} - 1) = 1 - (1-\lambda)\frac{\delta_2}{\delta_1} < \lambda \quad (19)$$

$$(19) \text{ into } (13): \frac{\delta_2}{\delta_1} \leq \frac{1}{1-\lambda} \quad (20)$$

$$(3) \text{ into } (4): \theta_2 V'(q_2) = c_2 \quad (\text{Optimal quality for package 2}) \quad (21)$$

From this case we learn that, as long as condition (20) is satisfied, the monopolist can choose to create a queue in order to evade the price control and continue to charge the “luxury” consumers a high tariff. If the firm did not compensate for the price control with a waiting time, it would have to lower tariff  $T_2$  accordingly, so as not to violate  $IC_2$ . We will be able to check that on graph 2 further below.

**Case 14:** (5) > 0; and (11) =>  $\gamma = 0$ ; (QD slack)  
 (6) > 0; and (12) =>  $\eta = 0$ ; (WT slack)  
 (7) = 0; and (13) =>  $\mu \geq 0$ ; (PC binding)  
 (8) = 0; and (14) =>  $v_2 \geq 0$ ; ( $IC_2$  binding)  
 (9) > 0; and (15) =>  $v_1 = 0$ ; ( $IC_1$  slack)  
 (10) = 0; and (16) =>  $\phi \geq 0$ . ( $IR_2$  binding)

$$\text{From (10): } T_2 = \theta_2 V(q_2) \quad (22)$$

$$(8), (6) \text{ and } (10) \Rightarrow \theta_1 V(q_1) > T_1 = k \theta_1 V(q_1), \quad (23)$$

$$(1) \text{ into } (2): (1 - \frac{\mu}{\lambda}) k \theta_1 V'(q_1) + \frac{\mu}{\lambda} M'(q_1) = c_1 \quad (18)$$

$$(3) \text{ into } (1): \phi = \{ \mu - [1 - (1 - \lambda) \frac{\delta_2}{\delta_1}] \} / (\frac{\delta_2}{\delta_1} - 1) \quad (24)$$

From (7), (13), (10) and (16), we can note that the expression inside brackets in condition (24) is not necessarily non-negative, as it was required in case 13. This

means that  $\frac{\delta_2}{\delta_1}$  may actually be greater than  $\frac{1}{1-\lambda}$ . As a matter of fact, a relatively

high impatience rate for consumer 2 is a possible explanation for how the monopolist is able to extract **all the surplus** from them, something impossible in ordinary self-selection models. The mere threat of waiting in a queue can scare the luxury consumers so much that they pay for the right not to wait, adding to the fixed fee, to the extent that the consumer surplus is entirely extracted<sup>7</sup>.

<sup>7</sup> Actually the firm could resort to this action even if it were not subject to a price control. Also note that this threat is analogous to the threat of a punishment in a tacit collusion equilibrium of a repeated game

$$(3) \text{ into } (4): \theta_2 V'(q_2) = c_2 \quad (\text{Optimal quality for package 2}) \quad (21)$$

**Case 21:** (5) > 0; and (11) =>  $\gamma = 0$  (QD slack)  
(6) = 0; and (12) =>  $\mu \geq 0$ ; (PC binding)  
(7) > 0; and (13) =>  $\eta = 0$ ; (WT slack)  
(8) = 0; and (14) =>  $v_2 \geq 0$ ; (IC<sub>2</sub> binding)  
(9) > 0; and (15) =>  $v_1 = 0$ ; (IC<sub>1</sub> slack)  
(10) > 0; and (16) =>  $\phi = 0$ . (IR<sub>2</sub> slack)

$$(6): T_1 = \theta_1 V(q_1) \quad (25)$$

$$(1) \text{ and } (3) \text{ into } (2): \theta_1 V'(q_1) \left\{ 1 - \frac{(1-\lambda)}{\lambda} \left[ \frac{\theta_2}{\theta_1} - 1 \right] \right\} = c_1 \quad (\text{Suboptimal quality}) \quad (26)$$

$$(3) \text{ into } (1): \frac{\eta}{\delta_1} = 1 - (1-\lambda) \frac{\delta_2}{\delta_1} < \lambda \quad (27)$$

$$(27) \text{ into } (13): \frac{\delta_2}{\delta_1} \leq \frac{1}{1-\lambda} \quad (20)$$

$$(3) \text{ into } (4): \theta_2 V'(q_2) = c_2 \quad (\text{Optimal quality for package 2}) \quad (21)$$

As we remarked above, this is Tirole's benchmark case, which is quite the same result one encounters in any self-selection model. What we introduce here is the necessary -- but not sufficient -- condition (20) for this result.

**Case 29:** (5) > 0; and (11) =>  $\gamma = 0$  (QD slack)  
(6) = 0; and (12) =>  $\mu \geq 0$ ; (PC binding)  
(7) = 0; and (13) =>  $\eta \geq 0$ ; (WT binding)  
(8) = 0; and (14) =>  $v_2 \geq 0$ ; (IC<sub>2</sub> binding)  
(9) > 0; and (15) =>  $v_1 = 0$ ; (IC<sub>1</sub> slack)  
(10) > 0; and (16) =>  $\phi = 0$ . (IR<sub>2</sub> slack).

$$(6) \text{ and } (7): T_1 = \theta_1 V(q_1) = M(q_1) \quad (25)$$

(1), (3) into (2):

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with perfect monitoring; on the equilibrium path punishment never actually takes place. Last, but not least, note that  $T_2$  has a "pay-not-to-wait" component in case 13 as well, but in that case it is not enough to extract the entire surplus.

$$\theta_1 V'(q_1) \left\{ 1 - \frac{\mu}{\lambda} - \frac{(1-\lambda)}{\lambda} \left[ \frac{\theta_2}{\theta_1} - 1 \right] \right\} + \frac{\mu}{\lambda} M'(q_1) = c_1 \quad (28)$$

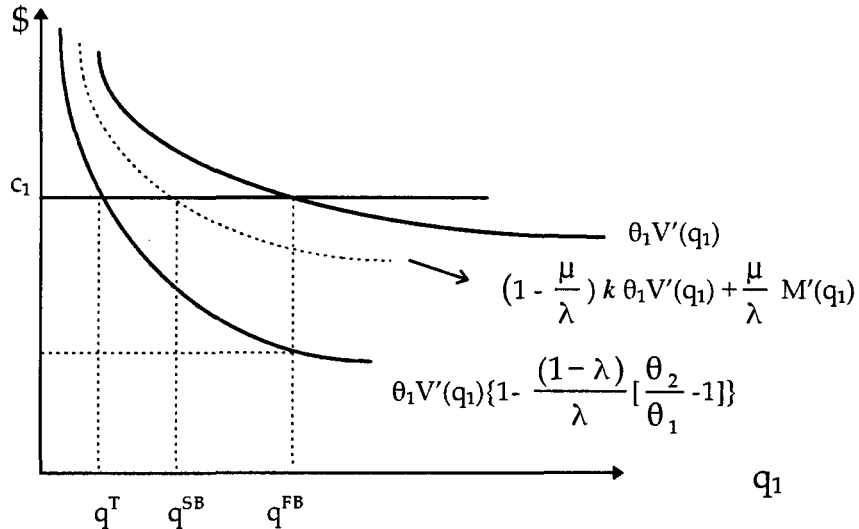
$$(3) \text{ into } (1): \mu = 1 - (1-\lambda) \frac{\delta_2}{\delta_1} - \frac{\eta}{\delta_1} < \lambda \quad (29)$$

$$(3) \text{ into } (4): \theta_2 V'(q_2) = c_2 \quad (\text{Optimal quality for package 2}) \quad (21)$$

Here the way the price control was designed prevents waiting time and can enhance quality as compared to case 21, as we show below.

Note that the four results assure optimal quality in market 2. It so happens in case 14 that the consumers do not have any share in this maximal total surplus, since the firm appropriates all of it. It is also worth noting that, because of our assumption that the firm can always adopt a rationing by waiting scheme, the type- $\theta_1$  consumers never have any surplus: in cases 21 and 29 they are totally appropriated by the firm, and in cases 13 and 14 part of it is dissipated due to the (social) waste of waiting time. Note further from condition 18 that the (binding) price control allows quality to be greater than in the unregulated equilibrium (case 21), as the marginal cost is now a linear combination of the inverse demand for quality and the price control quality-schedule. Now, remember that  $V'(q)$  is non-increasing, while  $M'(q)$  is non-decreasing. From graph 1, one can see that the government can always find a price schedule that improves quality for consumer 2, despite the fact that a queue is obtained.

**GRAPH 1**



Algebraically, it is easy to show in case 13 that we can always find such a price schedule. Substituting (19) into (18), we obtain:

$$\theta_1 V'(q_1) \frac{(1-\lambda)}{\lambda} \left( \frac{\delta_2}{\delta_1} - \frac{\theta_2}{\theta_1} \right) + \left[ 1 - \frac{(1-\lambda)}{\lambda} \left( \frac{\delta_2}{\delta_1} - 1 \right) \right] M'(q_1) = c_1 \quad (30)$$

Now, let  $q_1^{SB} = \text{argsolve}(30)$ . One can see on graph 1 that, as long as  $M'(q_1)$  is non-negative everywhere, the condition

$$\theta_1 V'(q_1^{SB}) \frac{(1-\lambda)}{\lambda} \left( \frac{\delta_2}{\delta_1} - \frac{\theta_2}{\theta_1} \right) + \left[ 1 - \frac{(1-\lambda)}{\lambda} \left( \frac{\delta_2}{\delta_1} - 1 \right) \right] M'(q_1^{SB}) > \theta_1 V'(q_1^{SB}) \left\{ 1 - \frac{(1-\lambda)}{\lambda} \left[ \frac{\theta_2}{\theta_1} - 1 \right] \right\} \quad (31)$$

guarantees that the price control schedule increases quality (and total surplus) in market 1, as compared to the unregulated monopolist. By simplifying the inequality above, we obtain a sufficient condition for such improvement:

$$M'(q_1^{SB}) > \theta_1 V'(q_1^{SB}) \quad (32)$$

Condition 32 also rules out the possibility that a quality-independent price control (i.e., a mere price ceiling, or  $M'(q_1) = 0$  for all  $q_1$ ) raises total surplus in market 1.

The limit of case 13 when waiting time tends to zero is case 29. Therefore we can also verify what price control schedule will achieve the first-best optimum. To do so, we look for the  $M'(q_1)$  which equals condition (28) -- from case 14 -- to the welfare maximization condition:

$$\theta_1 V'(q_1^{FB}) = c_1, \quad (33)$$

that is,

$$\theta_1 V'(q_1^{FB}) \left\{ 1 - \frac{\mu}{\lambda} - \frac{(1-\lambda)}{\lambda} \left[ \frac{\theta_2}{\theta_1} - 1 \right] \right\} + \frac{\mu}{\lambda} M'(q_1^{FB}) = \theta_1 V'(q_1^{FB}), \quad (34)$$

whose solution is:

$$M'(q_1^{FB}) = \theta_1 V'(q_1^{FB}) \left\{ 1 + \frac{(1-\lambda)}{\mu} \left[ \frac{\theta_2}{\theta_1} - 1 \right] \right\} > \theta_1 V'(q_1^{FB}), \quad (35)$$

where  $\mu$  is given by condition (26).

Let us now give a numerical example. Let:

---

$\theta_1 = 2$	$\delta_1 = 1$	$c_1 = 0.5$	$V(q) = \frac{1 - (1-q)^2}{2}$	$\lambda = 0.6$
$\theta_2 = 3$	$\delta_2 = 2$	$c_2 = 0.6$	$M(q) = \frac{1}{2} q^2 + \frac{3}{16} q$	

---

### **Case 13:**

$$(19): \mu = 0.2;$$

$$(18): q_1^{SB} = 11/16;$$

$$\text{Total Surplus 1: } W_1^{SB} = [\theta_1 V(q_1^{SB}) - \delta_1 t_1] - c_1 q_1^{SB} = T_1 - c_1 q_1^{SB} = \frac{66}{256}$$

### **Case 21:** (Tirole)

$$(26): q_1^T = 5/8 < q_1^{SB}$$

$$\text{Total Surplus 1: } W_1^T = \theta_1 V(q_1^T) - c_1 q_1^T = \frac{140}{256} < W_1^T$$

### **First Best:**

$$(32): \theta_1 V'(q_1^{FB}) = c_1 \Rightarrow q_1^{FB} = 0.75 > q_1^{SB} > q_1^T$$

$$W_1^{FB} = \theta_1 V(q_1^{FB}) - c_1 q_1^{FB} = \frac{144}{256} > W_1^{SB} > W_1^T.$$

### **Market 2:**

$$(21) q_2 = 0.8;$$

$$W_2 = \frac{66}{50} \text{ (for all cases).}$$

One can also check that the price schedule  $M(q) = \frac{2q^2 + q}{4}$  achieves the first-best optimum (only possible in case 29), supposing  $\eta = 0$ . Hint: substitute  $M'(q)$  into (28).

As an illustration, Graph 2 displays equilibrium in case 13. The indifference curves are displayed below.  $I_1'$  and  $I_2'$  are “virtual” indifference curves, in the sense that they are only perceived by the consumer, while the firm collects as little money as indicated by  $I_1$  and  $I_2$  (the former are the latter respectively shifted by  $\delta_1 t_1$  and  $\delta_2 t_1$ ). The formula for the indifference curve  $I_i$  is given by the implicit function theorem:

$$\frac{dT}{dq_i} = - \frac{\partial U_i / \partial q_i}{\partial U_i / \partial T} = \theta_i V'(q_i)$$

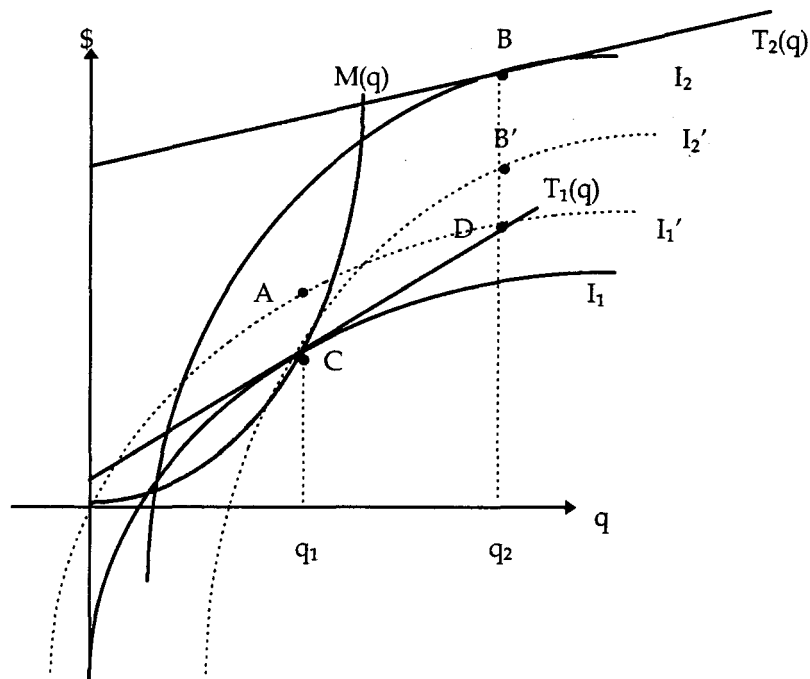
Thus, indifference curves that cross the y-axis at the origin stand for a zero-utility level. Indifference curves crossing the y-axis below the origin allow for higher utility levels. Indifference curves crossing the y-axis above the origin are not achievable, because of the IR constraints. Tangency of the indifference curve provides the quality level that maximizes revenue for a given tariff schedule<sup>8</sup>.

Binding  $IC_2$  and PC constraints are viewed at point C. Slack WT is seen on graph 2 by noting that  $I_1$  is below  $I_1'$ , and  $I_2$  is above  $I_2'$ . The consumer 1's utility dissipation due to waiting time,  $\delta_1 t_1$ , is measured by the vertical distance between points A and C, while  $\delta_2 t_1$  is given by the distance between B and B'. It is because of this and because of condition (8) that  $I_2'$  is the relevant curve for  $IC_2$ . Analogously (because of (9))  $I_2$  and  $I_1'$  are the relevant curves for  $IC_1$ . Slack  $IC_1$  is represented by point B above D (binding  $IC_1$  would require B and D coinciding). By raising quality along  $M(q)$  curve, the firm would be able to raise  $T_1$  and consequently appropriate part of the waiting time cost, but this would be a second-order effect: profit would increase  $(p-c)q_1$ , where  $p$  is the marginal price. On the

<sup>8</sup> Tangency is located at  $q = \text{argsolve}\{T'(q) = \theta V'(q)\}$ , which means that marginal price equals marginal total surplus. In turn,  $\theta V'(q) = c$  ensures socially optimal quality, which maximizes total surplus.

other hand,  $I_2'$  would also go up, and that would violate  $IC_2$ , so the firm would need to reduce  $T_2$ 's fixed fee, a first-order effect.

**GRAPH 2**



### 3. CONCLUSIONS

We investigated in this paper the effects that a quality-dependent price control may have in a vertically differentiated market with two-part tariffs, if it targets a “popular” or “plain” (low-quality) good. If the price control is slack, the firm will remain in the unregulated separating equilibrium where the “plain” consumer is served a suboptimal-quality good and has no surplus, while the “luxury” consumer enjoys the optimal quality and a positive surplus (case 21). When the price control is binding, three scenarios may arise, depending on the parameters of distribution of consumers and of relative preferences for quality and for time. In the first one (case 13), the firm creates a queue to scare luxury consumers and prevent them from buying the plain good. Nonetheless, by forcing the firm to raise quality, the price control may raise total surplus in the plain good market (market 1). The caveat is that this arrangement may ultimately benefit the luxury good consumers by forcing the firm to decrease  $T_2$ 's fixed fee; quality 2 is not affected, so total surplus in market 2 is only redistributed from the firm to consumers. In other words, a price control that was meant to benefit poor consumers will ultimately not affect their utility, while it will increase rich consumers' surplus.

In the second scenario (case 14), waiting time is such a burden for consumers 2 that actually the luxury consumers pay not to wait in a queue to bthe

extent that the firm is able to extract all their surplus. In market 1, the firm extracts all surplus, except the amount dissipated by waiting time. Thus, the price control may also increase the sum of total surpluses from markets 1 and 2, but this improvement does not benefit the consumers at all; on the contrary, it burdens the luxury consumers without helping the plain consumers.

In the third scenario (case 29), the price control's improvement as compared to case 21 range from nothing -- the control is irrelevant and results the same quality 1 -- to a first-best social optimum -- achieved through a very well-designed price-quality schedule that enables the firm to appropriate the whole surplus in market 1 with a tariff which charges marginal price equal to marginal cost and a fixed fee equal to consumer surplus. Meanwhile, total surplus in market 2 remains the same, but the fixed fee is redistributed to luxury consumers: quality 1's increase forces the firm to do so because of  $IC_2$  constraint.

In sum, a quality-dependent price control is only partially effective if the firm can circumvent the control by making consumers wait. This type of evasion prevents the targeted consumers from enjoying an increase in utility, and may allow the control to benefit the "wrong" public (the monopolist and the luxury consumers).

The analysis may be enriched by allowing a competing firm to enter the market. Another possibility is the introduction of some other kind of information asymmetry -- we already have a revelation problem of the consumer's types. These possible extensions are currently being considered by the author.

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## APPENDIX

Here we show why one rules out the other 28 cases.

**Problem 1:**  $QD > 0$ ;  $WT > 0$ ;  $PC > 0$ ;  $IC_1 > 0$ , and

**Problem 2:**  $QD > 0$ ;  $WT > 0$ ;  $PC > 0$ ;  $IC_2 > 0$ ;  $IC_1 = 0$ ;  $IR_2 > 0$ .

One can raise the fixed fee of  $T_1$  (and thus decrease  $t_1$ ) up to the point where:

- $IC_2$  becomes binding (remember that  $I_2'$  moves up "faster" than  $I_1$  for the same decrease of time, because  $\delta_2 > \delta_1$ ), or
- $PC$  becomes binding (beyond that, you can only raise  $T_1$  if you increase quality), or
- $WT$  becomes binding (limit of firm's response to  $PC$  binding),

or still raise  $T_2$  until  $IC_2$  is binding.

**Problem 3:**  $IC_1 = 0$ ;  $IR_2 = 0$ .

(9):  $T_2 = \theta_2 V(q_2)$ ;

(10):  $T_1 = \theta_2 V(q_2)$ .

As it is assumed that  $\theta_2 > \theta_1$ , the conditions above are compatible only if  $V(q_2) = V(q_1) = 0 = T_2$ . Clearly the firm will not pursue this goal.

**Problem 4:**  $QD > 0$ ;  $WT > 0$ ;  $PC > 0$ ;  $IC_2 = 0$ ;  $IC_1 = 0$ .

The firm can raise  $T_1$  and  $T_2$  jointly, as long as  $PC$ ,  $IC_2$  and  $WT$  are not binding (moving the two tariffs simultaneously may relax  $IC_2$  momentarily).

**Problem 5:**  $QD > 0$ ;  $WT > 0$ ;  $PC = 0$ ;  $IC_2 > 0$ ;  $IC_1 > 0$ .

The firm can raise  $T_1$  (and decrease  $t_1$ ) along  $PC$  curve until  $WT=0$  or  $IC_2=0$  obtains.

**Problem 6:**  $QD > 0$ ;  $WT > 0$ ;  $PC = 0$ ;  $IC_2 > 0$ ;  $IC_1 = 0$ ;  $IR_2 > 0$ .

The firm can raise  $T_2$  (as quality 2 is suboptimal) and  $T_1$  (along  $PC$  curve) until  $WT = 0$  or  $IC_2 = 0$ .

**Problem 7:**  $QD > 0$ ;  $WT > 0$ ;  $PC = 0$ ;  $IC_2 = 0$ ;  $IC_1 = 0$ ;  $IR_2 > 0$ .

This result requires  $\delta_1 > \delta_2$ . Moreover, it originates a superoptimal quality for good 2.

**Problem 8:**  $QD > 0$ ;  $WT = 0$ ;  $PC \geq 0$ ;  $IC_2 > 0$ ;  $IC_1 \geq 0$ ;  $IR_2 > 0$ .

The firm can raise  $T_2$ , because  $IC_2$  and  $IR_2$  (and in cases 17 and 19, also  $PC$ ) are slack.



**Problem 9:**  $QD > 0$ ;  $WT = 0$ ;  $PC > 0$ ;  $IC_2 > 0$ ;  $IC_1 > 0$ ;  $IR_2 = 0$

Impossible. From (10):  $\theta_2 V(q_2) - T_2 = 0$ .

But from (8) and (6):  $\theta_2 V(q_2) - T_2 > \theta_2 V(q_1) - T_1 - \delta_2 t_1 = (\theta_2 - \theta_1) V(q_1) \geq 0$   
(Contradiction).

**Problem 10:**  $QD > 0$ ;  $WT = 0$ ;  $PC > 0$ ;  $IC_2 = 0$ ;  $IC_1 > 0$ ;  $IR_2 = 0$ .

From (10) and (8):  $T_2 = \theta_2 V(q_2) - (\theta_2 - \theta_1) V(q_1) = \theta_2 V(q_2)$  (35)

From (35) and (6):  $T_1 = \theta_1 V(q_1) = 0$  (unless  $\theta_2 = \theta_1$ ).

Clearly the firm will not pursue this goal.

**Problem 11:**  $QD > 0$ ;  $WT = 0$ ;  $PC > 0$ ;  $IC_2 = 0$ ;  $IC_1 = 0$ ;  $IR_2 > 0$ .

From (9) and (8):  $T_2 = \theta_2 V(q_2) - (\theta_2 - \theta_1) V(q_1) = \theta_1 V(q_2)$  (36)

This is true iff  $q_1 = q_2$  (violates  $QD > 0$ ) or  $\theta_2 = \theta_1$  (consumers would be equal, so what is the point in differentiating goods?).

**Problem 12:**  $QD > 0$ ;  $WT = 0$ ;  $IC_2 > 0$ ;  $IR_2 = 0$

From (6):  $T_1 = \theta_1 V(q_1)$  (37)

From (10):  $T_2 = \theta_2 V(q_2)$  (38)

Now, (37) and (38) contradict  $IC_2 > 0$ .



N.Cham. P/EPGE SPE F565r

Autor: Fiuza, Eduardo Pedral Sampaio.

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