



FUNDAÇÃO
GETULIO VARGAS

CAIXA POSTAL 9.052 - CEP 22253

RIO DE JANEIRO - RJ - BRASIL

CIRCULAR N° 51

Assunto: Seminários Pesquisa
Econômica I (2ª parte)

Coordenadores: Prof. Fernando de H. Barbosa
e Prof. Gregório Lowe Stukart

Convidamos V.Sa. para participar do Seminário de Pesquisa Econômica I (2ª parte) a realizar-se na próxima 5ª feira:


DATA: 03/06/93

HORÁRIO: 15:30h

LOCAL: Auditório Eugenio Gudin

TEMA: "*Knightian Rational Expectations, Inflationary Inertia and Money Neutrality*".
por James Dow - London Business School-, Mario Henrique Simonsen -EPGE/FGV- e
Sérgio Ribeiro da Costa Werlang -EPGE/ FGV- (conferencista do seminário)

Rio de Janeiro, 02 de junho de 1993.


10- Prof. Fernando de Holanda Barbosa
e Prof. Gregório Lowe Stukart
Coordenadores de Seminários
de Pesquisas/EPGE

338.9
D744b
f

Knightian Rational Expectations, Inflationary Inertia and Money Neutrality

by

James DOW*

Mario Henrique SIMONSEN**

and

Sérgio Ribeiro da Costa WERLANG**

Summary. The connection between rational expectations and Nash equilibrium is already well known in the literature (Townsend(1978), Evans(1983), Simonsen(1986, 1988) and Guesnerie(1992)). It is also known that in order to obtain inflationary inertia in a price setting model, one has to depart from rational expectations, or from Nash equilibrium, except for obvious lags coming from staggered wage-price setting. Simonsen(1986, 1988) obtains inertia by the use of maxmin behaviour. As a result, he also gets a violation of the money neutrality. In this article we use the notion of Nash equilibrium under Knightian uncertainty (Dow and Werlang(1992c, 1993)) to show how inflationary inertia arises in a context of full *Knightian rationality*. Thus, we come to the conclusion that uncertainty (in the sense of Knight(1921)), rather than irrationality, is the driving force behind the existence of inflationary inertia and the non-neutrality of monetary policy.

*London Business School.

**Graduate School of Economics at Getulio Vargas Foundation (EPGE/FGV).

Knightian Rational Expectations, Inflationary Inertia and Money Neutrality

James Dow*

Mario Henrique Simonsen**

Sérgio Ribeiro da Costa Werlang**

1. Introduction: Rational Expectations as Nash Equilibria

The phenomenon of inflationary inertia, here understood as the positive dependence on past inflation of wage-price setting rules, has been observed in countries that suffered chronic high inflation. The majority of Latin American nations during the 70's and the 80's and part of the 90's have experienced persistent and high inflation rates. As a result, there are several contributions by local economists to the understanding of inflationary inertia that are not well known among the profession, albeit being an extremely important fact, mainly because it is an undeniable proof of the lack of realism of the rational expectations hypothesis.

The first reference to introduce a macro model that incorporates inflationary inertia is Simonsen(1970), although the Brazilian stabilization plan of 1964-1966 already made use of related ideas (PAEG(1964)). However, most of these early works aim at modelling the macroeconomic effects of inflationary inertia, not at explaining why it exists. There are various attempts to explain why inflationary inertia arises. The majority of the papers on the area explicitly introduce some sort of adaptive expectations, which yield inertia tautologically. Other authors make use of a maximizing model where one introduces a mass of irrational agents who set their prices based on the past inflation (Lopes(1982, 1986) and Franco(1986, 1989)). The most important contribution to the understanding of inflationary inertia is Simonsen(1986, 1988).

The connection between rational expectations and Nash equilibrium is already well known in the literature (Townsend(1978), Evans(1983), Simonsen(1986, 1988) and Guesnerie(1992)). A rational expectations equilibrium is a Nash equilibrium of a game where all agents of the economy choose optimally an action based on a model of the economy, also chosen by them. In order to have a competitive like behaviour, one has to assume a continuum of nonatomic agents, as in Evans(1983), but this is an unnecessary complication of the main idea. Thus, we simply assume that the two concepts are synonyms. Simonsen(1986, 1988) obtains inertia by the use of maxmin behaviour, instead of Nash equilibrium, on a price setting game.

This paper draws on the ideas of Simonsen(1986, 1988), that prudent behaviour is essential for the existence of inflationary inertia. By making use of Schmeidler-Gilboa's model of Knightian uncertainty (Schmeidler(1982, 1989), Gilboa(1987) and Gilboa and Schmeidler(1989)), Dow and Werlang(1992c, 1993) defined a Nash equilibrium under uncertainty. In this article we use their notion of equilibrium to provide an example where inflationary inertia arises in a context of full *Knightian rationality*. Therefore, we come to the conclusion that uncertainty (in the sense of Knight(1921)), rather than irrationality, is the driving force behind the existence of inflationary inertia, and, more generally, of the non-neutrality of money in the short run.

The paper is organized as follows. Section 2 is essentially taken from Dow and Werlang(1992c, 1993), and describes uncertainty in terms of subadditive probabilities (ie, non-additive probabilities reflecting aversion to uncertainty). Section 3 introduces the definition of Nash equilibrium under uncertainty, due to Dow and Werlang(1992c, 1993), preceded by a heuristic approach to games with uncertainty. Section 4 is the core of the article. It presents the solution under uncertainty to a price setting game discussed by Simonsen(1988). The result is inflationary inertia and non-neutrality of money. Section 5 concludes.

2. Uncertainty

Schmeidler(1982, 1989) and Gilboa(1987) have developed an axiomatic model of rational decision making in which agents' behaviour distinguishes between situations where agents know the probability distributions of random variables and situations where they do not have this information. We refer to the former as risk and the latter as uncertainty, or Knightian uncertainty. Synonyms that are used in the literature include roulette lottery, for risk, and horse lottery and ambiguity, for uncertainty. The traditional model of uncertainty used in economics is that of Savage(1954), which reduces all problems of uncertainty to risk under a subjective probability. The axiomatization of Schmeidler-Gilboa leads to very distinct behaviour: behaviour under uncertainty is inherently different from behaviour under risk. We now give a brief exposition of the main aspects of their model. The reader is referred to the papers by Schmeidler and Gilboa cited above for a complete description and for the underlying axioms, and to Dow and Werlang(1992a) which contains an example and an application to portfolio choice (it also includes a mathematical appendix with all basic material on non-additive probabilities). Dow and Werlang(1992b) have an explanation of the excess volatility puzzle, and Simonsen and Werlang(1991) also describe the implications

for portfolio choice. Also, Wakker(1989) has a model which is very similar to Gilboa(1987).

Bewley(1986) presents a similar model which is also designed to capture Knightian uncertainty. His model predicts that uncertainty leads to inertia, a tendency to favor the "status-quo", while in Schmeidler-Gilboa there is a tendency to choose acts where the agent does not end up bearing uncertainty. In a decision problem, this will lead to different predictions unless the status quo is an act where the agent bears no uncertainty. In Game Theory, it is conventional not to distinguish any particular strategy as the status quo.

The Schmeidler-Gilboa model predicts that agents' behaviour will be represented by a utility function and a (subjective) non-additive probability distribution. A non-additive probability P reflecting aversion to uncertainty satisfies the condition

$$P(A) + P(B) \leq P(A \cup B) + P(A \cap B), \quad (1)$$

rather than the stronger condition satisfied by (additive) probabilities

$$P(A) + P(B) = P(A \cup B) + P(A \cap B).$$

In particular, $P(A) + P(A^c)$ may be less than 1; the difference can be thought of as a measure of the uncertainty aversion attached by the agent to the event A . The *uncertainty aversion of P at event A* is $c(P, A) = 1 - P(A) - P(A^c)$ (Dow and Werlang(1992a)). The uncertainty aversion measures two effects simultaneously: how much an agent dislikes uncertainty (and hence its name), and also the degree of uncertainty. For this reason we will refer to it sometimes as the degree of uncertainty.

All the non-additive probabilities considered in this paper will reflect uncertainty aversion, ie they will satisfy inequality (1). Sometimes we refer to non-additive probabilities of this kind as subadditive probabilities. Also, we will restrict attention to the case of a finite set of states of the world.

The agent maximizes expected utility under a non-additive distribution, where the expectation of a non-negative random variable X is defined as:

$$E[X] = \int_{\mathbb{R}_+} P(X \geq x) dx.$$

Associated with a non-additive probability P is a set Δ of additive probabilities called the core of P , which is defined (analogously to the core in cooperative game theory) as the set of additive probability measures π such that

$\pi(A) \geq P(A)$ for all events A . If the non-additive probability satisfies the inequality (1) (reflecting aversion to uncertainty) the core is non-empty. (A closely related model of behaviour under uncertainty is for the agent to act to maximize the minimum value, over the elements of the core, of expected utility (Gilboa and Schmeidler(1989)).)

The support of a non-additive probability P may be defined analogously to the additive case. One possible definition is the smallest event A such that $P(A) = 1$. Notice that if P reflects positive uncertainty aversion at all events (other than the empty set and the set of all states) then the entire set of all states is the only event with probability one. Intuitively, the interpretation of an event of non-additive probability zero is the same as in the additive case: it is an event which will almost never happen. However, if a set has probability zero, that does not mean its complement is of probability one. But if the complement of a zero-probability set has positive probability (in principle it could be zero too) it has relatively infinitely more chance of happening than the original set. Hence, we are led to an alternative definition based on the idea that the complement of the support has zero probability.

Definition: a *support* of a non-additive probability P is an event A such that $P(A^c) = 0$ and $P(B^c) > 0$ for all events $B \neq A$ with $A \supset B$.

It should be clear that there may be several supports. Also note that any support must be contained in the smallest event with probability one (otherwise a smaller set with the same property would be obtained by taking the intersection of the support with the smallest event of probability one).

Example: this example illustrates the difference between the smallest event of probability one and the support.

$$p_1 = p_2 = p \in]0, 1/3[$$

$$p_3 = 0$$

$$p_{12} = q \in]2p, 1 - p[$$

$$p_{13} = p_{23} = p.$$

The smallest event of probability one is the whole set. The (unique) support is the event of state 1 or state 2.

Example: this example does not have a unique support. Again there are three states.

$$p_1 = 0.5$$

$$p_2 = p_3 = 0$$

$$p_{12} = p_{13} = 0.6$$

$$p_{23} = 0.1.$$

The supports are $\{1, 2\}$ and $\{1, 3\}$.

3. Nash Equilibrium under Uncertainty

A well known debate on the foundations of the theory of noncooperative games is whether the concept of Nash equilibrium provides or not an appropriate description of "rational" behaviour for a player. Since many important theorems can be proven about Nash equilibria, most of the game theoretical literature overlooks the subject taking Nash behaviour as the paradigm of "rationality" in problems of individual choice in an interdependent situation. That is to say, they postulate that an intelligent participant in a game should play Nash. The problem is that this leads to an empirically strange concept of rationality, where smart and thoughtful people are considered "irrational".

In fact, in many noncooperative games smart players do not play a Nash equilibrium, even assuming that such exists and is unique. As an example, taken from Dow and Werlang(1992c, 1993) (after an example in Werlang(1986)), consider the bi-matrix game in Figure 1 below:

		Player II	
		l	r
Player I	u	10, 10	-10, $10-\alpha$
	d	$10-\varepsilon, 10$	$10-\varepsilon, 10-\alpha$

$\alpha > 0$

Figure 1

where 10 stands for 10 million dollars, α and ε for a few thousand dollars. The strategy combination (u, l) is the unique Nash equilibrium. Yet, most people would prefer to choose d, whenever confronted with the role of player I.

The paradox can be easily explained: in a bi-matrix game, as in every game in normal form, players must choose their strategies before knowing other players' choices. The concept of Nash equilibrium assumes that each player blindly trusts that other players will behave rationally. In the example above, the second player chooses i since it is a strictly dominant strategy. The first player guesses the second player's choice and plays her best response to i , namely, u . (This argument shows that the pair (u, i) in the example above is the only one which survives two rounds of elimination of strictly dominated strategies, which is much stronger than a usual Nash equilibrium. See Bernheim(1984), Pearce(1984) and Tan and Werlang(1988).)

The problem of the Nash equilibrium notion is that it overlooks the fact that a cautious player should consider the possibility of irrational behaviour by the other participants of the game. For player II, whether player I behaves rationally or irrationally is immaterial. Now, the converse is highly relevant for player I. Assume that player II slips and plays r , which incidentally is only going to cost her a few thousand dollars. In this case player I will lose a fortune if she plays the Nash strategy u . By playing d , she guarantees a 10-€ million dollar payoff, almost as the Nash equilibrium, independently of how player II behaves.

This type of questioning of the rationality of Nash equilibria arises whenever an equilibrium is not a combination of maxmin strategies. Maxmin means most prudent playing: each participant of the game guarantees his best safe value, independently on how others behave, as does the first player of our numerical example when choosing d . By the way, that is the main reason for the discussion on zero-sum games to take most of the attention of early game theorists: in those games, to behave prudently is the same as doing the best given the prudent choice of the other players. Game theory has not solved the conflict, but only made explicit the existence of two confronting views on how intelligent people behave, namely, Nash X maxmin. Yet, both paradigms are unconvincing. Player I above must be incredibly naive to play Nash. A maxmin strategist, however, takes uncertainty to its extreme point: she actually assumes that she is playing with the devil, whose objective is not to make money, but to harm the other participants. That, of course, means pessimism in its ultimate degree, that hardly can explain how economic agents behave, except perhaps in periods of depression.

The concept of Nash equilibrium under uncertainty proposed by Dow and Werlang(1992c, 1993) provides a solution to the rationality paradox. Broadly speaking, it is a weighted average of Nash and maxmin, that unifies two

apparently conflicting views of rational behaviour in game. Let us develop the heuristics of the idea, and then proceed to a formal definition.

Let $\Gamma = (A_1, A_2, u_1, u_2)$ be a two-person finite game (also known as a bi-matrix game) where the A_i 's are pure strategy sets and u_i 's are utilities (payoffs). This will be called the primitive game, or game without uncertainty. For $i=1$ and 2 , define $\underline{u}_i(a_i) = \min_{a_j \in A_j} u_i(a_i, a_j)$, being $i \neq j$. This amount is what player i will get in case she plays strategy a_i against the devil. The game with constant degree of uncertainty (measured by the uncertainty aversion) c_1 for player I and c_2 for player II ($0 \leq c_1 \leq 1$ and $0 \leq c_2 \leq 1$) is the bi-matrix game $\Gamma' = (A_1, A_2, u'_1, u'_2)$, where $u'_i(a_i, a_j) = (1-c_i) \cdot u_i(a_i, a_j) + c_i \cdot \underline{u}_i(a_i)$, $j \neq i$, $i=1$ and 2 .

A Nash equilibrium in this new game is defined as a Nash equilibrium under uncertainty in the primitive game. An interpretation of this new game is that each player attributes a certain probability that the other player will behave irrationally, acting like the devil. With zero uncertainty one gets the usual definition of Nash equilibrium. With 100% uncertainty for every player, an equilibrium is a combination of maxmin strategies.

It is easy to check that if $(\tilde{a}_1, \tilde{a}_2)$ is at the same time a Nash equilibrium and a combination of maxmin strategies of the primitive game, then it is also a Nash equilibrium of the game with any given constant degree of uncertainty. Games with the property that Nash and maxmin coincide, have been studied by Simonsen(1988).

Moving back to our numerical example, the payoffs of the game under uncertainty, with degrees of uncertainty given by c_1 for player I and c_2 for player II, are described by the bi-matrix of Figure 2.

		Player II	
		l	r
Player I	u	$10(1-2c_1), 10$	$-10, 10-\alpha$
	d	$10-\varepsilon, 10$	$10-\varepsilon, 10-\alpha$

$\alpha > 0$

Figure 2

Without uncertainty, ie, with $c_1 = 0$, the only Nash equilibrium is the implausible (u, l) , as seen above. However, as long as one admits a small degree of uncertainty, that is, any c_1 greater than $\varepsilon/20$, the much more plausible combination (d, l) emerges as the only Nash equilibrium under uncertainty.

Let us now generalize the definition of Nash equilibrium under uncertainty. The point of departure will be a well known definition of mixed strategy in standard theory: an additive probability on the space of pure strategies of the player. As before, we restrict attention to two-person finite games $\Gamma = (A_1, A_2, u_1, u_2)$. In the standard theory, a mixed strategy Nash equilibrium can be defined as follows. Let (μ_1, μ_2) be a pair of (additive) probability measures and let $\text{supp}[\mu_i]$ denote the support of μ_i . In Nash equilibrium, every $a_1 \in \text{supp}[\mu_1]$ is a best response to μ_2 , ie a_1 maximizes the expected utility of player 1 given that player 2 is playing the mixed strategy μ_2 ; conversely, every $a_2 \in \text{supp}[\mu_2]$ is a best response to μ_1 . A subjective interpretation can be given to the Nash equilibrium: the mixed strategy of player 1, μ_1 , may be viewed as the beliefs that player 2 has about the pure strategy play of player 1. Conversely, the mixed strategy of player 2, μ_2 , may be viewed as the beliefs player 1 has about the pure strategy play of player 2.

Now, under uncertainty, what happens is that each player no longer views the strategy of the other player as an additive, but as a subadditive probability on the other player's strategy space. Moreover, we have assumed up to now that the degree of uncertainty $c(P, A)$ is constant for each player. This assumption can be lifted in a general definition.

Definition: We say that a pair (P_1, P_2) of subadditive probabilities (all our non-additive probabilities will satisfy inequality (1) above), P_1 over A_1 and P_2 over A_2 is a *Nash Equilibrium under Uncertainty* if there exists a support of P_1 and a support of P_2 such that:

- (i) for all a_1 in the support of P_1 , a_1 maximizes the expected utility of player 1 given that player 1 beliefs about the strategies of player 2 are P_2 , and conversely;
- (ii) for all a_2 in the support of P_2 , a_2 maximizes the expected utility of player 2 given that player 2 beliefs about the strategies of player 1 are P_1 .

The definition above, reduces to the standard definition of Nash equilibrium, whenever there is no uncertainty (which means that the P 's are additive). One could speculate why we have not used the smallest set of probability one instead of a support in the definition above. The reason is that this set is "too large", and the equilibrium notion thus resulting would be too strong, as well. In fact, the great strength of non-additive models is that an event may be infinitely more likely than its complement, but still have probability less than one. Take, for example, the case of a strategy set with two elements, a and b . Suppose $P(a)=0.8$ and $P(b)=0$. If we want to be "sure" that an event is going to happen, then this event has to be the whole strategy set, because it is the smallest set with probability one. However, the likelihood that the strategy a is going to be used is infinite relative to b (ie the relative likelihood that strategy b is going to be used is zero). Thus, in this case, it would be fair to interpret that strategy b has no chance of happening. Clearly, a standard mixed strategy Nash equilibrium is also a Nash equilibrium under uncertainty.

Also, it is easy to see that the definition above reduces to the heuristic definition given before. In fact, consider the case of a uniform squeeze, ie, $P(A) = (1-c)Q(A)$, for A distinct from the whole set of strategies. From Dow and Werlang(1992a) uniform squeezes have constant uncertainty aversion equal to c and the expected value of a positive random variable X is given by

$$E_P[X] = c \min X + (1-c)E_Q[X].$$

If we find the Nash equilibria under uncertainty where the subadditive probabilities of the players are in this class, we obtain the same equilibria as above. (To see that is simple: one has to check that the (standard) mixed strategy Nash equilibria of the modified game (Γ') correspond to the Q 's of the Nash equilibria under uncertainty of the primitive game.) Hence, we have the following theorem (Dow and Werlang(1993)).

Theorem. Let $\Gamma = (A_1, A_2, u_1, u_2)$ be a two-person finite game, and $(c_1, c_2) \in [0,1] \times [0,1]$. Then, there exists a Nash equilibrium (P_1, P_2) , where both P_1 and P_2 exhibit constant uncertainty aversion, such that c_1 is the uncertainty aversion of P_2 and c_2 is the uncertainty aversion of P_1 . (The reason for the interchange in the subscripts is that P_2 is what player 1 thinks player 2 is going to do, so that the uncertainty aversion of P_2 is a characteristic of player 1, and vice-versa.)

4. Money Neutrality, Inflationary Inertia and Nash Equilibrium under Uncertainty

The following is an adaption of the price setting game presented by Simonsen(1988). Let us assume an economy with a continuum of non-storable goods, each one produced by an individual price-setter, and where the nominal output R is controlled by the government. In line with monetary theory, one may assume that the central Bank controls some monetary aggregate that determines R . The nominal output is pre announced by a credible administration, but prices must be set simultaneously, each agent ignoring how others will decide, except for the fact that no good is expected to be priced above P_{\max} . This means that both R and P_{\max} are common knowledge.

Production starts after prices have been set, according to consumers' orders. Since goods cannot be stored, this rules out the possibility of excess supplies. Supply shortages may indeed occur, but are not anticipated by consumers. Each individual is indicated by a real number $x \in [0,1]$. Accordingly, P_x is the price of the good produced by individual x . Let us assume that the general price level is given by a nonatomic aggregator of the individual prices, which is increasing in the price of the goods, and which is homogeneous of degree one:

$$P = g\left(\int_{[0,1]} z(x, P_x) dx\right) \quad (2).$$

We require that the price index be an increasing function of the prices of the goods: if $P'_x \geq P_x$ for all x , then $P' \geq P$, where P' indicates the price level computed by using expression (2) for the prices P'_x . Homogeneity of degree one means that for any $\lambda \geq 0$:

$$g\left(\int_{[0,1]} z(x, \lambda P_x) dx\right) = \lambda g\left(\int_{[0,1]} z(x, P_x) dx\right).$$

Additionally, assume individual x 's notional utility $U_x(P_x, P, R)$ is homogeneous of degree zero in (P_x, P, R) , and decreasing in P . Also, suppose that for each pair (P, R) there is a unique P_x that maximizes individual x 's notional utility, given by:

$$P_x = f_x(P, R) \quad (3)$$

where $f_x(P, R)$ is a continuous real function, homogeneous of degree one and increasing on both its variables, such that for any $P > 0$ both $f_x(P, 0) = 0$ and $f_x(P, \infty) = \infty$.

We are in face of a standard price setting game, with a continuum of players. Nominal output R is set by government's monetary policies. Each player chooses his price P_x , the aggregate result being the general price level P . Note that the assumption that there are only nonatomic agents implies that the weight of P_x in the general price level is zero. Thus, each individual aims at maximizing his notional utility $U_x(P_x, P, R)$ keeping both P and R constant.

Let us first analyze the game with zero uncertainty. From the preceding assumptions, the function

$$h(P, R) = g\left(\int_{[0,1]} z(x, f_x(P, R)) \, dx\right) \quad (4)$$

is continuous, homogeneous of degree one, increasing in both its variables and that for any $P > 0$ both $h(P, 0) = 0$ and $h(P, \infty) = \infty$. As a result, for any positive P there exists one unique positive R such that:

$$P = h(P, R) \quad (5).$$

Since $h(P, R)$ is homogeneous of degree one, this equation is solved by $R = P/\beta$, where β is a positive constant.

The existence and uniqueness of a Nash equilibrium can now be proven immediately. In a Nash equilibrium all price-setters must correctly locate P . According to the foregoing discussion, a Nash equilibrium is hit if, and only if, all price-setters behave by equation (3), taking P as the solution to equation (5). This solution exists and is unique:

$$P = \beta R \quad (6).$$

The equilibrium presents the usual properties of solutions to rational expectations models: real output $Y=R/P$ depends on money supply, abiding by the neutrality hypothesis. Moreover, P depends of its lagged values, that is to say, there is no price inertia.

Let us consider the game with uncertainty. We will argue by means of an analogy with the finite two-person game. Since $U_x(P_x, P, R)$ is a decreasing function of P , the devil always plays $P = P_{\max}$ (in other words, the minimum utility that player x will get, given the strategy choices by the other players $x' \neq x$, $P_{x'}$, and hence the aggregate price level P , happens exactly for $P = P_{\max}$). Assume that each player has constant uncertainty aversion, and behaves as if he believed that there is a probability $0 \leq c \leq 1$ that all other participants are replaced by the devil. Then, individual x 's expected utility (using the subadditive prior) will be given by:

$$V_x(P_x, P, P_{\max}, R) = (1-c)U_x(P_x, P, R) + cU_x(P_x, P_{\max}, R).$$

It is easy to see that the maxima of this function has to occur for a price P_x in the open interval $]f_x(P, R), f_x(P_{\max}, R)[$. In fact, one can see that by drawing the graphs of the two functions above (Figure 3). Note that the graph of $U_x(P_x, P_{\max}, R)$ lies under the graph of $U_x(P_x, P, R)$, because of the hypothesis that U_x is decreasing in the general price index P . Also, the P_x that maximizes $U_x(P_x, P_{\max}, R)$ is $f_x(P_{\max}, R) > f_x(P, R)$ (which is the P_x that maximizes $U_x(P_x, P, R)$) as was shown above. Therefore, by the graph we see that both $U_x(P_x, P, R)$ and $U_x(P_x, P_{\max}, R)$ are simultaneously increasing for $P_x \leq f_x(P, R)$ and decreasing for $P_x \geq f_x(P_{\max}, R)$. This implies that the maxima of $V_x(P_x, P, P_{\max}, R)$ are attained inside the interval. Suppose further that there is a continuous selection of the (possibly multivalued) function that associate for each (P, P_{\max}, R) the values of P_x that maximize $V_x(P_x, P, P_{\max}, R)$. Call this function $\phi_x(P, P_{\max}, R)$. The solution is homogeneous of degree one in the three variables and increasing in each one of them. Furthermore, for $P < P_{\max}$, and for all x :

$$f_x(P, R) < \phi_x(P, P_{\max}, R) < f_x(P_{\max}, R) \quad (7).$$

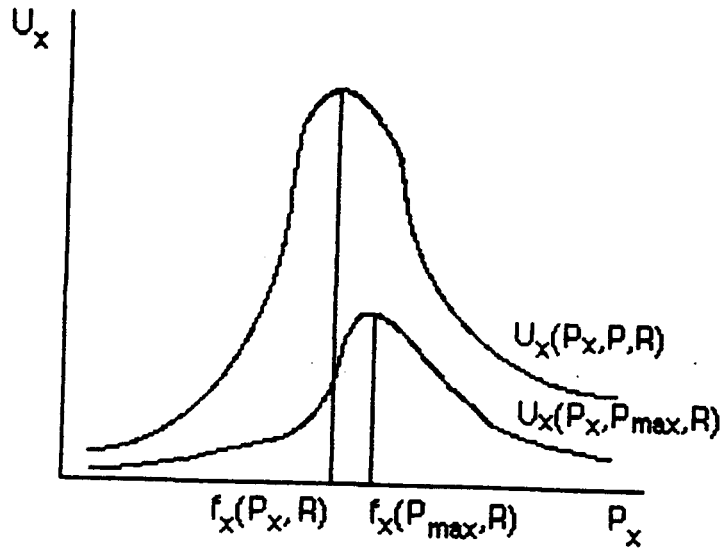


Figure 3

A Nash equilibrium under uncertainty of this game must be a solution of the equation:

$$P = \theta(P, P_{\max}, R) \quad (8), \text{ where}$$

$$\theta(P, P_{\max}, R) = g\left(\int_{[0,1]} z(x, \phi_x(P, P_{\max}, R)) dx\right) \quad (9).$$

The function $\theta(P, P_{\max}, R)$ is homogeneous of degree one in all three variables, continuous, and increasing in each of the variables. Combining (4), (7) and (9), we obtain that for $P < P_{\max}$, and for all x :

$$h(P, R) < \theta(P, P_{\max}, R) < h(P_{\max}, R) \quad (10).$$

We introduce yet another, and by all means very plausible, assumption: $\beta R < P_{\max}$, that is to say, the Nash equilibrium without uncertainty is inferior to the upper bound of the general price level. It follows that the function $Z(P) = \theta(P, P_{\max}, R) - P$ is: (i) strictly positive for $P = \beta R$ [from (10) and the fact that βR is the unique solution to (5)]; and (ii) strictly negative for $P = P_{\max}$ [since we may write $P_{\max} = \gamma \beta R$ with $\gamma > 1$, from the fact that $h(P, R)$ is homogeneous of degree one, $P_{\max} = \gamma \beta R = h(\gamma \beta R, \gamma R)$, which implies that if $\gamma > 1$, as $h(P, R)$ is increasing in R , $P_{\max} = \gamma \beta R > h(\gamma \beta R, R) = h(P_{\max}, R)$]. As $\theta(P, P_{\max}, R)$ is increasing in P and continuous, there exists a unique Nash equilibrium under uncertainty, given by $Z(P) = 0$, so that we may write:

$$P = W(P_{\max}, R)$$

(11),

where

$\beta R < W(P_{\max}, R) < P_{\max}$. As one should expect, the function $W(P_{\max}, R)$ is homogeneous of degree one in its two variables. We are ready to study the consequences of this model.

Consequence 1: Non-Neutrality of Money in the Short Run

As a consequence of the result above, under uncertainty:

$$Y = R/P = R/W(P_{\max}, R) = 1/W(P_{\max}/R, 1),$$

where Y stands for real output. In the short run, P_{\max} is assumed to be fixed. Hence, the real output Y is an increasing function of the nominal R , which is to say that money is not neutral in the short run. One should expect, of course, that money should be neutral in the long run. This can be reconciled with the preceding analysis by assuming that, in the long run, P_{\max} adjusts proportionally to R .

Consequence 2: Inflationary Inertia

Let us now revisit the problem of inflationary inertia. The starting point is a chronic inflation at a constant rate per period. For $t \leq 0$ the government has been expanding the nominal output at a constant rate Π :

$$R_t = R_0 (1+\Pi)^t, \text{ for } t \leq 0.$$

Price-setters have already adjusted for chronic inflation and its moving Nash equilibrium, by using equation (3) with $R=R_t$ and $P=P_t=\beta R_t$, for $t \leq 0$. At the end of period $t=0$, a new administration, whose credibility stays beyond any doubt, announces nominal output stabilization, ie, that it will make $R_t = R_0$ for $t \geq 1$.

In traditional rational expectations models, that correspond to games without uncertainty, inflation would stop immediately, since new Nash equilibria would make $P_t=\beta R_t=\beta R_0=P_0$ for $t \geq 1$. The problem of inertia emerges once uncertainty is brought about to stage. Even if all price setters are convinced that the government will actually stabilize the nominal output at R_0 , they may suspect that other prices will continue to increase. The only plausible assumption that they

can make is that inflation rate should not increase, which makes $P_{1\max} = P_0 (1+\Pi)$. The Nash equilibrium under uncertainty will be P_1 such that $P_0 = \beta R_0 < P_1 < P_{1\max}$. This is to say that the rate of inflation in period 1 will be less than Π , but still positive. Real output will decline, since $P_1 > P_0$, while $R_1 = R_0$. This, of course, means inflationary inertia. It disappears only in the long run, as P_{\max} adjusts to changes in the nominal output.

5. Conclusion

We explore the connection between rational expectations and Nash equilibrium (Townsend(1978), Evans(1983), Simonsen(1986, 1988) and Guesnerie(1992)) to show that in the presence of uncertainty (in the sense of Knight(1921)) there may exist Nash equilibria, and hence Knightian rational expectations equilibria, with indexation on past inflation. Furthermore, in these equilibria money is not neutral in the short run. We use Schmeidler-Gilboa's model of uncertainty and the definition of Nash equilibrium under uncertainty of Dow and Werlang (1992c, 1993). Our main result may be seen as an extension of Simonsen(1986, 1988), that obtained inflationary inertia and non-neutrality of money as a result of maxmin behaviour. The example allows one to understand at least one reason for the failure of rational expectations: the existence of strictly positive uncertainty aversion (as defined by Dow and Werlang(1992a)). It is a completely novel explanation, and does not depend on market incompleteness, wage staggering or asymmetric information.

References

Bernheim, Douglas (1984), "Rationalizable Strategic Behavior", Econometrica 52: 1007-1028.

Bewley, Truman (1986), "Knightian Decision Theory, Part 1", Cowles Foundation Paper No 807, Yale University.

Dow, James and Sérgio R. C. Werlang (1992a), "Uncertainty Aversion, Risk Aversion and the Optimal Choice of Portfolio", Econometrica 60: 197 - 204.

Dow, James and Sérgio R. C. Werlang (1992b), "Excess Volatility of Stock Prices and Knightian Uncertainty", European Economic Review 36: 631 - 638.

Dow, James and Sérgio R. C. Werlang (1992c), "Nash Equilibrium under Knightian Uncertainty: Breaking down Backward Induction", Ensaios Econômicos nº 186 da EPGE/FGV, Graduate School of Economics at Getulio Vargas Foundation.

Dow, James and Sérgio R. C. Werlang (1993), "Nash Equilibrium under Knightian Uncertainty", Graduate School of Economics at Getulio Vargas Foundation and London Business School, mimeo.

Evans, George (1983), "The Stability of Rational Expectations in Macroeconomic Models". In Frydman, Roman and Edmond Phelps eds. Individual Forecasting and Aggregate Outcomes ('Rational Expectations' Examined). Cambridge: Cambridge University Press.

Franco, Gustavo (1986), "Inertia, Coordination and Corporatism", Texto para Discussão 141, Department of Economics, Catholic University (PUC/RJ).

Franco, Gustavo (1989), "Inertia and Coordination: Pacts, Price-Freezes and its Problems" (in Portuguese), Pesquisa e Planejamento Econômico 19: 65 - 84.

Gilboa, Itzhak (1987), "Expected Utility with Purely Subjective Non-Additive Priors", Journal of Mathematical Economics 16: 279 - 304.

Gilboa, Itzhak and David Schmeidler (1989), "Maxmin Expected Utility with a Non-Unique Prior", Journal of Mathematical Economics 18: 141- 153.

Guesnerie, Roger (1992), "How Rational are Rational Expectations?", Invited Lecture at the 11th Latin American Meeting of the Econometric Society, September 1 to 4, El Colegio de México, Mexico.

Knight, Frank (1921), Risk, Uncertainty and Profit. Boston: Houghton.

Lopes, Francisco L. P. (1982), "Inflation and Production Level: An Econometric Study" (in Portuguese) [section 7], Pesquisa e Planejamento Econômico 12: 639-670.

Lopes, Francisco L. P. (1986), "A Note on the Role of Expectations" (in Portuguese), chapter 11 in The Heterodox Shock (in Portuguese). Rio de Janeiro: Campus.

PAEG (1964), Program of Economic Action of the Government 1964-1966 (in Portuguese). Documentos EPEA No. 1, Ministry of Planning and Economic Coordination, Brazilian Government.

Pearce, David (1984), "Rationalizable Strategic Behavior and the Problem of Perfection", Econometrica 52: 1029-1050.

Savage, Leonard J. (1954), The Foundations of Statistics. New York: John Wiley. (Second edition 1972, New York: Dover.)

Schmeidler, David (1982), "Subjective Probability Without Additivity" (Temporary Title), Working Paper, The Foerder Institute for Economic Research, Tel Aviv University.

Schmeidler, David (1989), "Subjective Probability and Expected Utility Without Additivity", Econometrica 57: 571 - 587.

Simonsen, Mario H. (1970), Inflation: Gradualism X Shock Treatment (in Portuguese) [especially chapter 6]. Rio de Janeiro: APEC.

Simonsen, Mario H. (1986), "Rational Expectations, Income Policies and Game Theory", Revista de Econometria 6: 7 - 46.

Simonsen, Mario H. (1988), "Rational Expectations, Game Theory and Inflationary Inertia". In Anderson, Phillip, Kenneth Arrow and D. Pines eds. The Economy as an Evolving Complex System. Addison-Wesley.

Simonsen, Mario H. and Sérgio R. C. Werlang (1991), "Subadditive Probabilities and Portfolio Inertia", Revista de Econometria 11: 1 - 20.

Tan, Tommy and Sérgio R. C. Werlang (1988), "The Bayesian Foundations of Solution Concepts of Games", Journal of Economic Theory 45: 370-391.

Townsend, Robert (1978), Market Anticipations, Rational Expectations and Bayesian Analysis. International Economic Review 19: 481 - 494.

Wakker, Peter (1989), "Continuous Subjective Expected Utility with Non-additive Priors", Journal of Mathematical Economics 18: 1-28.

Werlang, Sérgio R. C. (1986), "Common Knowledge and Game Theory", PhD Thesis, Princeton University.

*Assistant Professor of Finance at the London Business School.

**Professors at the Graduate School of Economics at Getulio Vargas Foundation.

N.Cham. P/EPGE SPE D744k

Autor: Dow, James.

Título: Knightian rational expectations, inflationary inert



087822

51011

FGV - BMHS

Nº Pat.:F47/99

000087822

