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Neighborhood Feedbacks, Endogenous Stratification, and Income Inequality

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Abstract

This paper explores the evolution of the cross-section income distribution in economies where endogenous neighborhood formation interacts with positive within-neighborhood feedback effects. We study an economy in which the economic success of adults is determined by the characteristics of the families in the neighborhood in which a person grows up. These feedbacks take two forms. First, the tax base of a neighborhood affects the level of education investment in offspring. Second, the effectiveness of education investment is affected by a neighborhood's income distribution, reflecting factors such as role model or labor market connection effects. Conditions are developed under which endogenous stratification, defined as the tendency for families with similar incomes to choose to form common communities, will occur. When families are allowed to choose their neighborhoods, wealthy families will have an incentive to segregate themselves from the rest of the population. This resulting stratification is supported by house price differences between rich and poor communities. Endogenous stratification can lead to pronounced intertemporal inequality as different families provide very different interaction environments for offspring. When the transformation of human capital into income exhibits constant returns to scale, cross-section income differences may also grow across time. As a result, endogenous stratification and neighborhood feedbacks can interact to produce long run inequality.

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1. Introduction

This paper illustrates how income inequality can emerge in economies through the interaction of a positive feedback structure between members of a common neighborhood with the tendency of families to endogenously stratify themselves into economically homogeneous neighborhoods. The underlying logic of the model we study is straightforward. The presence of positive feedbacks means that the income distribution of a neighborhood will strongly affect the future economic status of children within the community. Endogenous stratification, by allowing the sorting of families with like attributes into separate neighborhoods, allows children from different neighborhoods to experience very different feedback effects, so relative economic status is transmitted across generations. By focusing on the neighborhood as the primary intergenerational transmission mechanism, the model exhibits very different income distribution dynamics from those which emerge in the private education economies originally studied by Becker and Tomes [1979] and Loury [1981].

The class of models we study represents a generalization of the incomplete market/coordination failure models which have formed the basis for much recent work in macroeconomics. As illustrated by Cooper and John [1988], the various models in this literature possess a common positive feedback structure which creates the potential for multiple, Pareto ranked equilibria to exist for a given microeconomic specification. This occurs because high and low levels of activity on the part of all agents render these activity levels optimal for each individual. The macroeconomic literature on positive feedbacks and multiple equilibria has generally taken the interaction structure between agents to be exogenous. Typically, this literature has focused on economies where the spillover effects are symmetric between all agents. As such, this literature is useful in explaining how all agents in an economy can simultaneously achieve high or low incomes, but

is less equipped to explain persistent income disparities within an economy.

By endogenizing the interaction sets of agents, in the sense that agents can affect the set of other agents with whom they interact, the ideas underlying incomplete markets macroeconomics can provide a general framework for understanding how inequality can emerge within a given economy. When agents segregate themselves according to some class of attributes, we say that the economy exhibits stratification. When these attributes are also the source of feedback effects between members of a given interaction set, then stratification can lead to inequality. By analogy to the coordination failure literature, high incomes and low incomes can both emerge as equilibria among subsets of families within the economy when families are stratified by income between neighborhoods. One implication of the current paper is that by endogenizing the interaction structure of agents, the macroeconomic models of positive feedbacks and multiple equilibria also provide a theory of inequality.

A body of recent theoretical research has studied the role of neighborhood influences on the distribution of income in ways related to the current paper. In Bénabou [1991], the productivity of family-specific investment in human capital is affected by the level of human capital investment among one's classmates. Rich neighborhoods are those characterized by high human capital investment by families, whereas poor neighborhoods are those with low human capital investment. Durlauf [1992] demonstrates how local financing of public education and the dependence of the productivity of human capital investment on the empirical income distribution of a neighborhood can combine to allow permanently poor neighborhoods to emerge and coexist with permanently wealthy ones. Montgomery [1990,1991] shows that if one's success in the labor market depends on referrals by neighborhood members, then multiple equilibria may exist in the distribution of occupations within neighborhoods. Streufert [1991] studies an economy in which the absence of role models in poor

communities prevents children from accurately assessing the marginal product of effort in school. Much of this new work has been stimulated by work in sociology by Wilson [1987] which has documented the persistence of poverty among families trapped in inner city ghettos. An important common feature of the role model, tax base and labor market connection effects which underlie the models in this literature is that the economic status of adults in a community will affect the economic success of the neighborhood's offspring.

Our analysis extends previous work in several respects. First, we present an analysis of how the cross-section income distribution evolves over time. The bulk of the work on neighborhood effects models has been static. Second, we provide an analysis of how relative inequality evolves over time. The focus of models such as Bénabou [1991], Durlauf [1992] and Montgomery [1990,1991] has been on the existence of absolute low income or poverty traps. Our analysis complements this work by showing how permanent relative inequality can emerge in an economy where the incomes of all families are growing. Third, we provide a relatively general framework which allows the study of the roles of both local financing of education as well as sociological feedbacks in perpetuating inequality. Fourth, the model provides a simple way of showing how house prices can support the stratification of neighborhoods over time even when the number of communities is endogenously determined, thereby generalizing the stratification mechanism in a number of previous models.

This paper is related in a number of ways to Bénabou [1992], which studies the effect of stratification on economic growth.[1] That paper focuses on the differences between aggregate growth dynamics in economies when the rich and poor live in separate communities in all periods versus when all

[1]See also Lundberg and Startz [1992] who study racial income inequality in the context of a private education model where blacks and whites are exogenously segregated.

families share a common community, and shows how the long run growth rate in an economy may be augmented by economic integration even if the short run growth rate is reduced. Our paper focuses on the role of different spillovers in sustaining relative inequality over different horizons. In addition, we show how to endogenize the neighborhood formation process. At the same time, both papers perform their analysis in the context of a constant returns to scale technology and together with Durlauf [1992] provide an overview of the dynamic properties of multicommunity economies for a range of microeconomic structures.

In developing the analysis, we work with a relatively tightly parameterized model in the sense that specific forms are chosen for the utility and production functions. These functional forms allow us to explicitly analyze the time series properties of the income distribution. In the absence of such a tight parameterization, it is difficult to describe the relationship between the model's primitives and dynamics due to the complex interaction structure which links families over time. At the same time, one goal of the analysis is to show in what sense our results on persistent income inequality are robust to alternative functional forms.

The empirical importance of neighborhood effects in determining economic success, even after controlling for family background, has been confirmed in a number of studies.[2] Datcher [1982] found that the mean income of a community helps to predict the years of schooling among urban males; Corcoran, Gordon, Laren and Solon [1989] uncovered a similar link between the percentage of families on welfare in a community and future offspring wages. Crane [1991] demonstrated how the percentage of professional workers in a community correlates negatively with the high school dropout rate among teenagers. Case and Katz [1991] similarly find that neighborhood characteristics help predict the incidence of many social

[2]See Jencks and Mayer [1991] for a survey of the empirical work on neighborhood effects.

problems among poor youths. Further, studies such as Card and Krueger [1992] which establish a relationship between school quality and eventual economic success of students, in combination with local financing of education, suggest a relationship between a neighborhood's income distribution and the economic status of offspring.[3]

Section 2 of the paper describes a basic model of neighborhood feedback effects. Section 3 characterizes the cross-section equilibrium in the economy, with an emphasis on the relationship between the cross-section income distribution and the allocation of families across neighborhoods. Section 4 analyses the dynamics of the income distribution and provides conditions under which persistent inequality will emerge.Section 5 discusses the role of the various functional form assumptions on the results. Section 6 concludes. An appendix follows which contains proofs of all propositions.

2. A model of family income and neighborhood spillovers

i. Population structure

The population consists of a continuum of families, indexed by i. Family i,t is composed of agent $i,t-1$, who is born at $t-1$, and his offspring. Agents live two periods. Each agent receives education when young and has one child and works when old.

Families live in neighborhoods indexed by n. The set of families occupying neighborhood n at t is $N_{n,t}$; $\mu(N_{n,t})$ denotes the Lebesgue measure of the set of families in the neighborhood and therefore measures a community's population size. Within a neighborhood, $\widehat{F}_{Y^{\cdot},n,t}$ is the

[3] Approximately 50% of all expenditures on public education between kindergarten and 12th grade in the United States are funded through locally raised revenues.

empirical probability distribution for family incomes; $\widehat{F}_{Y,n,t}(s)$ denotes the percentage of families in neighborhood n at t with incomes less than or equal to s. Finally, the average income among families in n at t is $MY_{n,t}$.

ii. Preferences

Each agent $i, t-1$ derives utility from his own consumption as an adult, $C_{i,t}$, and the expected income of his offspring as an adult, $Y_{i,t+1}$.

$$U_{i,t-1} = \pi_1 log(C_{i,t}) + \pi_2 \mathrm{E}(log(Y_{i,t+1}) \mid \mathfrak{F}_t). \tag{1}$$

The assumption that an adult's utility depends on the income rather than the utility of his offspring is used by Becker and Tomes [1979], among others, and is common in the literature on income distribution dynamics. The assumption is designed to proxy for the idea that parent's care about the budget constraints of their offspring.[4] The importance of the Cobb-Douglas specification for the utility function will be discussed in Section 5.

iii. Income Determination

The realized income of each offspring in a neighborhood is determined by a constant returns to scale production function which combines two factors. First, education endows each child in neighborhood n at t with a common level of human capital $H_{n,t}$. Second, each offspring experiences a white noise productivity shock $\xi_{i,t+1}$ as an adult.

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[4]This assumption is also important in ensuring the existence of an equilibrium. For example, we are unaware of any proof for the existence of an equilibrium in this economy for the utility function $U_{i,t-1} = u(C_{i,t}) + \beta U_{i,t}$, given the nonconvexities in the model which derive from the neighborhood formation process.

$$Y_{i,t+1} = \phi H_{n,t}\xi_{i,t+1} \tag{2}$$

The shocks $\xi_{i,t+1}$ are identically distributed across individuals and neighborhoods at all dates and are independent across neighborhoods. Each shock has mean 1 and lies in a closed, uniformly bounded positive interval of $\mathbb{R}$. Without loss of generality, we assume that $\xi_{i,t+1}$ consists of a neighborhood-wide component $\nu_{n,t+1}$ and an idiosyncratic component, $\gamma_{i,t+1}$,

$$\xi_{i,t+1} = \nu_{n,t+1}\gamma_{i,t+1}, \tag{3}$$

where the idiosyncratic component obeys the law of large numbers in the sense that the empirical and population probability distributions of the idiosyncratic shocks are always equal, i.e. $\hat{F}_{\gamma,n,t+1}(s) = F_{\gamma,n,t+1}(s)\ \forall\ s$.[5]

iv. Budget constraint

Agent $i, t-1$ divides his income between consumption and taxes $T_{i,t}$.

$$Y_{i,t} = C_{i,t} + T_{i,t} \tag{4}$$

v. Tax determination

Given realized family incomes at t, each neighborhood chooses a proportional income tax rate $\tau_{n,t} \in [0,1]$ whose proceeds are used to finance education.[6] Taxes are determined by majority voting. A given $\tau_{n,t}$ is an

[5]All results in the paper will hold so long as the empirical probability distribution of the shocks within a neighborhood is well defined each period. We choose this specification to simplify the statement of some of the proofs.

[6]The use of income taxes rather than property taxes simplifies the analysis with no substantive effect on the results.

equilibrium rate for $N_{n,t}$ if at least one half of the members of $N_{n,t}$ prefer the rate to any fixed alternative. Individual taxes therefore obey

$$T_{i,t} = \tau_{n,t} Y_{i,t} \;\forall\; i \in N_{n,t}. \tag{5}$$

Following Loury [1981], neighborhoods cannot borrow in order to finance education, since there exists no mechanism to enforce repayment of such loans by offspring.

vi. Education investment

All children in a neighborhood receive the same level of education investment, $ED_{n,t}$, which is determined by the total tax revenues raised in the community. The level of human capital induced by this investment is characterized below. The level of education expenditures within a neighborhood, $TE_{n,t}$, is assumed to have two parts: a component that increases in the level of education investment but which must be spent regardless of the number of students receiving the investment and a component that requires spending in proportion to the total number of students. The fixed cost component in spending is assumed to be a constant fraction of the total, so that

$$TE_{n,t} = \lambda_1 ED_{n,t} + \lambda_2 \mu(N_{n,t}) ED_{n,t}. \tag{6}$$

Since per capita education investment costs are decreasing in neighborhood size, there exists an incentive for wealthier families to live with poorer ones.

vii. Sociological feedbacks and human capital formation[7]

The actual level of human capital formation induced by a given level of education investment is assumed to depend on the empirical income distribution within the neighborhood, $\widehat{F}_{Y,n,t}$. This dependence is designed to reflect the influence of role model and labor market connection effects on the returns to schooling and is consistent with the empirical work on neighborhood effects and economic status. We assume that this feedback takes the form

$$H_{n,t} = \Theta(\widehat{F}_{Y,n,t})ED_{n,t}. \tag{7}$$

Three restrictions are placed on $\Theta(\cdot)$. First, rightward shifts in a neighborhood's income distribution increase the amount of human capital produced by a given level of education investment. If $\widehat{F}_{Y,n,t}$ and $\widehat{F}'_{Y,n,t}$ denote two income distributions such that $\widehat{F}_{Y,n,t}(s) < \widehat{F}'_{Y,n,t}(s)\ \forall\ s$, then

$$\Theta(\widehat{F}'_{Y,n,t}) < \Theta(\widehat{F}_{Y,n,t}). \tag{8}$$

Second, the percentage change in the marginal productivity of education investment induced by a proportional increase in the incomes of all members of a neighborhood is assumed to become nonincreasing as the minimum income in a neighborhood becomes large. This condition limits the degree of fragmentation of neighborhoods over time by ensuring that along growing income paths, the incentive for wealthy families to segregate themselves from proportionately poorer neighborhoods is not continually growing as well. For every pair of income distributions $\widehat{F}_{Y,n,t}(\overline{Y})$ and

[7]We refer to these feedbacks as sociological not only because their causes and magnitudes have been largely studied by sociologists rather than economists but also because some aspects of these feedbacks, such as the "culture of poverty" cannot be readily derived from conventional microeconomic reasoning. Montgomery [1990,1991] and Streufert [1991] , however have shown how these neighborhood feedbacks can emerge due to information imperfections.

$\widehat{F}'_{Y,n,t}(\overline{Y})$, there exists a number $\overline{Y} < \infty$ such that if $\widehat{F}_{Y,n,t}(\overline{Y}) = 0$ and $\widehat{F}'_{Y,n,t}(\overline{Y}) = 0$, then for any $K > 1$,

$$| \Theta(\widehat{F}_{Y,n,t}) - \Theta(\widehat{F}'_{Y,n,t}) | \geq | \Theta(\widehat{F}_{Y,K,n,t}) - \Theta(\widehat{F}'_{Y,K,n,t}) | . \tag{9}$$

where $\widehat{F}_{Y,K,n,t}(Ks) = \widehat{F}_{Y,n,t}(s)\ \forall s$ and $\widehat{F}'_{Y,K,n,t}(Ks) = \widehat{F}'_{Y,n,t}(s)\ \forall s$.

Third, $\Theta(\cdot)$ obeys a continuity restriction. For any income distribution F, let F_ϵ denote the same income distribution when the poorest $\epsilon \times 100$ percent of families have been omitted. $\Theta(F_\epsilon)$ is assumed continuous in $\epsilon \in [0,1)$. This ensures that each family can determine its most desired set of neighbors for any economy-wide income distribution by ruling out any effects from moving a measure zero set of families between neighborhoods.

viii. Neighborhood formation rules

At the beginning of each period t, all incomes $Y_{i,t}$ are realized. Once incomes are realized, families are allowed to form neighborhoods costlessly. This means that there is no stock of housing which exists for more than one generation. Any set of families is free to form an initial neighborhood and build houses. There is no limit to the number of neighborhoods which may be formed. Once families build houses, the units may be resold at a common neighborhood housing price $P_{n,t}$.[8] Any family wishing to join an already established community must therefore pay the prevailing house price in order to enter. At the same time, any family can move costlessly to any empty neighborhood. In an equilibrium configuration of families across

[8]Metaphorically, each neighborhood passes a zoning law which precludes additional construction once the initial set of houses is built. Under the assumption that the neighborhoods are voluntarily formed, one may show that in equilibrium, families in a given neighborhood will unanimously support such a restriction if all neighborhoods with higher incomes have such restrictions.

neighborhoods, the set of house prices which prevails must be such that no family will wish to move.

3. Characterization of equilibrium

This section describes the existence and cross-section properties of the equilibrium in the economy.

i. Existence of equilibrium

In order to establish the existence of an equilibrium it is necessary to prove first that contingent on an allocation of families across neighborhoods, there exists an equilibrium tax rate in each community and second, that there is a way to allocate families across neighborhoods such that at available housing prices, no one wishes to move.

The Cobb-Douglas preference specification not only ensures the existence of an equilibrium tax rate within a neighborhood, but also means that the equilibrium tax rate will always be unanimously preferred regardless of the neighborhood's empirical income distribution. To see this, observe that each agent's preferred tax rate over all possible τ will maximize

$$\pi_1 log((1-\tau)Y_{i,t}) + \pi_2 \mathrm{E}(log(\phi H_{n,t}(\tau)\xi_{i,t+1}) \mid \mathfrak{F}_t), \tag{10}$$

where $H_{n,t}(\tau)$ expresses the level of human capital as a function of the tax rate. By substituting in eqs. (6) and (7) to solve for $H_{n,t}(\tau)$, the preferred tax rate will maximize

$$\pi_1 log((1-\tau)) + \pi_1 log(Y_{i,t}) + \pi_2 log(\phi\Theta(\widehat{F}_{Y,n,t})\frac{\mu(N_{n,t})}{(\lambda_1 + \lambda_2\mu(N_{n,t}))})$$

$$+\pi_2 log(\tau)+\pi_2 log(MY_{n,t})+\pi_2 \mathrm{E}(log(\xi_{i,t+1})\,|\,\mathfrak{F}_t) \tag{11}$$

Simple algebra reveals that the most preferred tax rate for agent $i,t-1$ is $\pi_2/(\pi_1+\pi_2)$, which is independent of either the agent's income or the composition of his neighborhood. This argument leads to Proposition 1.

Proposition 1. Agreement on most preferred tax rate within a neighborhood

For any neighborhood configuration $N_{n,t}$ the tax rate $\tau = \dfrac{\pi_2}{\pi_1+\pi_2}$ is

i. unanimously preferred to any alternative,

ii. independent of the composition of the neighborhood.

The Cobb-Douglas specification also implies that the economy will exhibit a positive feedback structure between families within a neighborhood, as increases in the income of one's neighbors always improve the welfare of a given family. Since all families agree on the utility maximizing tax rate within a community, rightward shifts in a neighborhood's income distribution will both increase the size of the tax base and raise the marginal product of education investment. Similarly, these effects induce a positive feedback from a neighborhood's income distribution to the expected income of offspring.

Proposition 2. Relationship between adult expected utility, offspring expected income and the empirical income distribution in a neighborhood

For a given population size $\mu(N_{n,t})$,

i. the expected utility of any agent $i,t-1$ who lives in $N_{n,t}$ is increasing in

monotonic rightward shifts of the empirical income distribution over all other families in the neighborhood.

ii. the expected income of any agent i, t *who lives in* $N_{n,t}$ *is increasing in monotonic rightward shifts of the empirical income distribution over all other families in the neighborhood.*

The presence of neighborhood-level positive spillovers is sufficient to prove the existence of an equilibrium allocation of families across neighborhoods under voluntary neighborhood formation which can be supported by house price differentials. Intuitively, the positive spillover structure within neighborhoods means that communities with high incomes will be more desirable than those with low incomes. Under voluntary community formation, communities with nonoverlapping income ranges will emerge. When the lowest income in one neighborhood is at least as great as the highest income in another, we say the neighborhoods are *stratified.* Once these communities form, a set of house prices will emerge which deter members of the different communities from moving. Proposition 3 states not only that there exists a stratified allocation of families across neighborhoods in all periods such that no family wishes to move given the prevailing level of house prices, but also that all equilibria must be stratified.

Proposition 3. Existence of equilibrium under endogenous neighborhood formation

i. There exists a joint stochastic process over income, consumption, human capital, neighborhood housing prices and neighborhood membership for all families at all dates such that each adult maximizes expected utility subject to majority voting over neighborhood taxes.

ii. Either all families prefer to live in a common neighborhood or any equilibrium neighborhood configuration is stratified.

Given the local dependence of each individual's human capital on the empirical income distribution of the neighborhood in which he grows up as well as the white noise assumption on the productivity shocks, the equilibrium law of motion for individual income will have a neighborhood-based structure, as stated in Proposition 4.

Proposition 4. Probability law for individual income

Each family's income obeys the transition rule

$$Prob(Y_{i,t+1} \mid \mathfrak{F}_t) = Prob(Y_{i,t+1} \mid \widehat{F}_{Y,n,t}, \mu(N_{n,t})).^9 \tag{12}$$

ii. Stratification and the cross-section income distribution

We now characterize the range of possible equilibrium allocations of families across neighborhoods. The positive feedback structure as described in Proposition 2 implies that all families will wish to move to neighborhoods with as affluent an income distribution as possible. Proposition 3 establishes the possibility of a stratified equilibrium, but does not show that one will ever actually occur. High income families benefit from large communities due to the decreasing average cost of human capital. On the other hand, poorer neighbors erode the tax base and reduce the marginal product of education investment through the human capital formation function. The consequences of this tradeoff for the realized distribution of families across

[9] Throughout, $Prob(x \mid y)$ denotes the conditional probability of x given y.

neighborhoods will depend on the empirical income distribution across the entire economy. Proposition 5 shows that stratification will always occur when the income gap between the wealthiest and poorest families in the economy is wide enough. The proposition implies that a large enough degree of cross-section inequality can have strong intertemporal consequences by ensuring that the offspring of rich and poor families experience different neighborhood feedbacks.

Proposition 5. Endogenous stratification as a consequence of the cross-section income distribution

For any income level $\overline{Y}^{low}$ and population values μ_1 and μ_2, there exists an income level $\overline{Y}^{high}$ such that no neighborhood will form which contains both μ_1 families with incomes above $\overline{Y}^{high}$ and μ_2 families with incomes below $\overline{Y}^{low}$.

At the same time, the decreasing average cost structure for human capital formation provides incentives for heterogeneous neighborhoods. Further, these incentives will become more pronounced the smaller the population of families. This means that sufficiently small neighborhoods will replicate themselves over time. Specifically, one can demonstrate that once the populations of individual neighborhoods lie below a threshold $\overline{\mu}^{thresh}$, which will generally depend on $\widehat{F}_{Y,n,t}$, the neighborhood population will either stabilize or offspring from different neighborhoods will begin to form joint neighborhoods as adults. Further, there will exist a uniform lower bound on this minimum size as a neighborhood becomes wealthier, so the size of wealthy neighborhoods will eventually stabilize. These features are summarized in Proposition 6.

Proposition 6. Self-replication or recombination of small neighborhoods

i. There exists a maximum value μ^{thresh}, *which may depend on* $\widehat{F}_{Y,n,t}$, *such that if* $0 < \mu(N_{n,t}) \leq \mu^{thresh}$ *then either*

a. all offspring in $N_{n,t}$ *will form a common neighborhood at* $t+1$, *or*

b. some offspring in $N_{n,t}$ *will combine with offspring from other neighborhoods at* $t+1$.

ii. There exists a number $\overline{Y}$ *such that if* $\widehat{F}_{Y,n,t}(\overline{Y}) = 0$, *then there exists a number* $\mu^* > 0$ *such that* $\mu^{thresh} > \mu^*$.

Finally, we observe that Proposition 6 implies that this economy does not exhibit economy-wide positive feedback effects, even though, as stated in Proposition 3, the economy does exhibit positive feedback effects conditional on a neighborhood's population size. This follows from the possibility that a monotonic rightward shift in the economy-wide empirical income distribution can result in a reallocation of families across neighborhoods which acts to the detriment of poorer families. In this respect, an endogenous neighborhoods model will possess a different structure from the positive feedback models conventionally studied in macroeconomics. Once agents can choose the set of other agents with whom they interact, the welfare consequences of increases in the income of other agents are very different from an economy where the interaction environment is exogenously given.

Proposition 7. Effect of changes in economy-wide income distribution on expected utility of parents and expected income of offspring

For any economy-wide cross-section income distribution at t, there exist

some monotonic rightward shifts of the distribution, under which no family's income declines, such that the expected utility of some adults and the expected income of their offspring are both reduced.

4. Income distribution dynamics

In this section, we consider some predictions of the model for the evolution of the cross-section income distribution. Our goal is to characterize how neighborhood feedbacks and endogenous stratification can combine to generate intertemporal inequality.

i. One generation-ahead dynamics

The role of stratification in transmitting inequality stems from the effects of higher neighborhood income on both the tax base and marginal product of educational investment. These effects increase both the total and average product of tax revenues raised within a community, which by Proposition 2.ii means that high and low income communities are more likely to produce high and low income offspring respectively. The dependence of the expected income of offspring on a neighborhood's income distribution holds regardless of the neighborhood formation mechanism.

Endogenous stratification of communities has an important effect on the intergenerational transmission of economic status by allowing the rich to separate themselves from the poor. The equilibrium formation process in this economy turns out to maximize the degree of inequality between rich and poor families. Proposition 8 states that among all possible neighborhood configurations, endogenous stratification maximizes the expected income of the offspring of the highest income family in the economy. Similarly, endogenous stratification minimizes the expected income of the offspring of

the lowest income family among all configurations which leave the size of the poorest family's neighborhood unaffected. This latter caveat is required since, if endogenous stratification leaves the poorest family living with a positive measure of wealthier neighbors, the expected income of that family's offspring could be further reduced by isolating the poorest family from those neighbors.

Proposition 8. Effect of endogenous stratification on the expected income of offspring of highest and lowest income families

Under endogenous stratification,

i. the expected offspring income of the wealthiest family in the economy is maximized relative to any other configuration of families across neighborhoods.

ii. the expected offspring income of the poorest family in the economy is minimized relative to any other configuration of families across neighborhoods which does not reduce the size of that family's neighborhood.

By implication, frictions in the neighborhood formation process will reduce intergenerational inequality.

Endogenous stratification not only maximizes inequality among offspring with respect to possible neighborhood configurations, but also has implications for the relationship between the income gap across parents in different neighborhoods and the income gap across their offspring. In order to see this, we consider the behavior of the average income growth rate for families in neighborhood n at t, $g_{n,t}$.

$$g_{n,t} = \frac{\mathrm{E}(Y_{i,t+1} \mid \widehat{F}_{Y,n,t}, \mu(N_{n,t})) - MY_{n,t}}{MY_{n,t}} \tag{13}$$

When wealthy neighborhoods and poor neighborhoods invest at a common tax rate, the feedbacks which affect the marginal product of education investment imply that the wealthy neighborhood's investment will be at least as productive as a poor neighborhood's investment. As a result, the mean income gap between equal-sized rich and poor neighborhoods will grow across time, as stated in Proposition 9. The equal-size restriction is important because for some cross-section income distributions the higher marginal product of education investment in a high income neighborhood is offset by a higher per capita cost of education investment due to a small population size induced by the desire of the wealthy to isolate themselves from the poor. Conversely, the proposition indicates how the gap between a rich majority and poor minority will increase over time.

Proposition 9. Growth in mean income differential between parents and offspring across stratified communities

For any two neighborhoods n and n' such that $\widehat{F}_{Y,n,t}(s) < \widehat{F}_{Y,n',t}(s)$ $\forall s$, and $\mu(N_{n,t}) \geq \mu(N_{n',t})$, average output growth among families in n will exceed growth in n'.

$$g_{n,t} - g_{n',t} > 0. \tag{14}$$

Therefore, positive feedbacks and endogenous stratification can lead to growing inequality.

ii. Long run dynamics

The model can exhibit interesting long run relative income dynamics

when family incomes are growing across time.[10] We therefore make an assumption which ensures that the model can exhibit income growth in some neighborhoods.

Assumption 1. Conditions for growth in family incomes across neighborhoods.

There exist numbers $\overline{Y}$ *and* $\overline{\mu} \leq \mu^*$ *such that if* $\widehat{F}_{Y,n,t}(\overline{Y}) = 0$ *and* $\overline{\mu} \leq \mu(N_{n,t})$, *then*

$$\frac{\pi_2}{\pi_1 + \pi_2} \cdot \frac{\mu(N_{n,t})}{\lambda_1 + \lambda_2 \mu(N_{n,t})} \cdot \phi\Theta(\widehat{F}_{Y,n,t}) > 1.^{11} \tag{15}$$

This cumbersome looking expression has a straightforward interpretation. The first term, $\pi_2/(\pi_1 + \pi_2)$ is the equilibrium tax rate and hence equals the marginal change in per capita spending with respect to an increase in the mean income of the neighborhood. The second term $\mu(N_{n,t})/(\lambda_1 + \lambda_2)$ is the marginal change in per capita education investment with respect to a change in the per capita spending in a neighborhood of size $\mu(N_{n,t})$. The third term, $\phi\Theta(\widehat{F}_{Y,n,t})$, is the marginal product in terms of offspring income of a change in education investment. The product of the

[10]In general, long run inequality can occur in a economy without growth only if one assumes both that the offspring of the sufficiently wealthy cannot have lower incomes than their parents and that the offspring of the sufficiently poor cannot have higher incomes than their parents. This requires a strong set of assumptions on the joint properties of preferences, technologies and productivity shocks. In the current model there is the additional problem that the presence of constant tax rates and constant returns to scale means that all family incomes may converge to zero in absence of growth. This problem can be corrected, though, by assuming that each adult receives a minimum income that is linearly augmented by his level of human capital.

[11]Proposition 6 defines μ^* as the lower bound on the population size at which all neighborhoods above some income threshold will replicate themselves. Notice that this condition will always hold for large enough ϕ.

three terms therefore equals the marginal change in the mean income of offspring with respect to an increase in the mean income of parents. The proof of Proposition 6 implies that any neighborhood whose population is less than μ^* and whose income distribution is shifted far enough to the right will replicate itself over time. Assumption 1 in conjunction with Proposition 6 therefore means that sufficiently wealthy neighborhoods can stabilize in size and experience positive average income growth.

In growing economies, endogenous stratification can combine with positive feedbacks to induce long run inequality across families in two senses. First, relative income rankings can become fixed along some sample paths of the economy. Second, the gap between the richest and poorest families along these paths can grow without bound. These features occur because first, once families are separated into distinct communities, wealthier communities can, by Proposition 9 grow faster than poorer ones, and second, once communities separate, these growth rate differentials can preserve the separation of families across time. As a result, if the cross-section income distribution induces a sufficiently large gap between the average incomes of rich and poor neighborhoods, then the economy can exhibit permanent inequality with positive probability. This relationship between the economy-wide income distribution at t and the long run dynamics of the economy is formalized in Proposition 10.

Proposition 10. Permanent inequality between population subgroups under some initial conditions

There exists a set of economy-wide income distributions at time t and associated neighborhoods $N_{n,t}$ and $N_{n',t}$ such that with positive probability,

i. relative income rankings are preserved over all future generations,

$$Prob(MY_{i,t+s}/MY_{i',t+s} \geq MY_{i,t}/MY_{i',t}\ \forall\ s \mid i \in N_{n,t},\ i' \in N_{n',t}) > 0.$$

ii. the income ratio between rich and poor becomes arbitrarily large. For any $K > 1$, there exists a k such that

$$Prob(MY_{i,t+l}/MY_{i',t+l} \geq K\ \forall\ l \geq k \mid i \in N_{n,t},\ i' \in N_{n',t}) > 0.$$

The proof of Proposition 10 requires that a small population of poor families becomes permanently isolated from a larger population of wealthy families, so that the lower per capita education costs induced by fixed costs permit the income gap to be perpetuated. While it seems reasonable that the depopulation and subsequent deterioration of the tax base of poor communities such as East Saint Louis has contributed to the breakdown of public services such as education, population differences do not in general seem to be that plausible a cause of long run inequality. Observe, however, that if $\Theta(\cdot)$ can grow without bound, then it is straightforward to show that the gap between a wealthy and a poor neighborhood can, depending on the properties of $\Theta(\cdot)$, still obey Proposition 10, even if the poor neighborhood is larger than the wealthy one. For example, if the productivity gap between initially rich and poor neighborhoods is preserved whenever the absolute gap between the two is preserved, then one can show that the initially higher income of the wealthy neighborhood can place it on a sample path where its human capital is always more productive than that of its poor counterpart.[12]

Can the required economy-wide income distribution for permanent inequality occur over the sample path realization of the economy? The

[12]For more general specifications of $\Theta(\cdot)$ than the one we study, differences between the human capital productivity of rich and poor can be perpetuated even if Θ is bounded. For example, if the marginal product of capital depends not only on a community's absolute income distribution but also on its income distribution relative to that of the rest of society, then productivity differences can become permanent.

answer is yes, in two senses. First, by setting $t = 0$, Proposition 10 implies that some initial conditions for the economy will induce long run inequality. This interpretation seems appropriate for analyzing an economy such as that of the United States, where the economic status of blacks has been so strongly affected by historical factors. Second, by choosing the distribution of $\xi_{i,t}$, one can ensure that the necessary economy-wide income distribution can occur with positive probability even if all families start off in the same neighborhood.

A second issue relates to whether the model is path dependent. Can two distinct sample path realizations of the economy exhibit different long run dynamics given the same initial conditions? Again, the answer is yes. The neighborhood-specific shocks imply that different sample realizations can exhibit different forms of limiting behavior, as can be seen through the following argument. Suppose there is a probability distribution G such that a neighborhood containing the entire population will never break up if $\hat{F}(s) \leq G(s)\ \forall\ s$. This economy will exhibit long run equality, if the economy-wide income distribution ever exceeds G and if all families ever occupy the same neighborhood. However, the economy-wide income distribution depends on the realization of the individual productivity shocks, which will not obey the law of large numbers due to the neighborhood-wide components. One could therefore have one sample path realization of the economy where after an initial split, all neighborhoods grow over time and eventually recombine into a common community whereas along another sample path, the separation is preserved as an initial income gap grows as some neighborhoods prosper and others decline. What is required for a permanent split is that the gap between rich and poor is large enough by the time the poorest community's income distribution exceeds G that the rich wish to preserve separation. This is most likely to occur when the effect on $\Theta(\cdot)$ of including very poor families is large, so that very poor families are segregated into neighborhoods with low marginal product of education

investment early in the history of the economy.

5. Role of assumptions

i. Preference restrictions

The Cobb-Douglas assumption for preferences is a strong one and plays a critical role in our results.[13] To see the importance of the assumption, we consider the more general utility function

$$U_{i,t} = \mathrm{E}(u(C_{i,t}) + v(Y_{i,t+1}) \mid \mathfrak{F}_t), \tag{16}$$

where $u(\cdot)$ and $v(\cdot)$ are both increasing and concave.

The first important feature of the Cobb-Douglas assumption is that it ensures that an equilibrium configuration of families across neighborhoods is achievable based upon the combination of voluntary neighborhood formation and house price differences. This holds in two respects. First, in the case where all members of a rich community prefer that community to a poor one, the Cobb-Douglas assumption, through the equality of tax rates across neighborhoods, ensures that this self segregation is supportable by house prices for any distribution of income. To see this, suppose that family i lives in a wealthy community w, and family i' lives in a poor community p. In a stratified equilibrium, it must be the case that if

$$\begin{aligned} u((1-\tau_w)Y_{i,t}) \;+\; \mathrm{E}(v(H_{w,t}, \xi_{i,t}) \mid \mathfrak{F}_t) = \\ u((1-\tau_p)(Y_{i,t} + P_{w,t} - P_{p,t})) \;+\; \mathrm{E}(v(H_{p,t}, \xi_{i,t}) \mid \mathfrak{F}_t) \end{aligned} \tag{17}$$

[13] In fact, the Cobb-Douglas assumption has been used by many authors studying multi-community/income distribution models in order to avoid some of problems we describe. See for example Bénabou [1992] and Glomm and Ravikumar [1992].

then

$$u((1-\tau_w)(Y_{i',t}-P_{w,t}+P_{p,t})) \;+\; \mathrm{E}(v(H_{w,t},\xi_{i',t})) \leq \\ u((1-\tau_p)(Y_{i',t})) \;+\; \mathrm{E}(v(H_{p,t},\xi_{i',t}) \mid \mathfrak{F}_t), \tag{18}$$

which jointly require that

$$u((1-\tau_p)(Y_{i',t}))-u((1-\tau_w)(Y_{i',t}-P_{w,t}+P_{p,t})) \geq \\ u((1-\tau_p)(Y_{i,t}+P_{w,t}-P_{p,t}))-u((1-\tau_w)Y_{i,t}). \tag{19}$$

For this condition to hold for all possible income distributions, it must hold if an arbitrarily small percentage of families in neighborhood p have incomes equal to $Y_{i,t}-\epsilon$. In this case however, eq. (19) can only hold for all values of τ_p, τ_w, and $P_{w,t}-P_{p,t}>0$ if $\tau_w \geq \tau_p$. Otherwise, there may not be a set of prices which support stratification. Intuitively, when taxes are lower in rich neighborhoods, the cost of entry on the part of members of poorer neighborhoods is partially defrayed by the lower tax burden. Cobb-Douglas preferences ensure the existence of supporting prices since they imply that taxes are constant across neighborhoods. Otherwise, either tax rates will need to be increasing in neighborhood size, or the differences in school quality between neighborhoods will need to be large enough to ensure that the $P_{w,t}-P_{p,t}$ which makes a rich family indifferent to moving is so large that no one in the poor neighborhood will be willing to pay it.

Second, the Cobb-Douglas assumption, because of the within-neighborhood unanimity in voting over tax rates, ensures that the desired configuration of families is well defined in the sense that all members of a neighborhood agree on the set of families whom they prefer as neighbors. This agreement stems from the fact that each member of a neighborhood is

always the median voter. When agents disagree on the first best tax rate, then they can also disagree on the preferred population of a community as different populations will induce different tax rate equilibria. Consequently, the model will not have a well defined neighborhood configuration without additional restrictions on the way in which neighborhoods form.

At the same time, it is possible to add restrictions on the neighborhood formation rules in order to ensure that an equilibrium exists for more general preference structures, in ways discussed in Durlauf [1992]. If neighborhoods can implement minimum income requirements for all families through majority voting, then the ability of the rich to segregate themselves from the poor will be enhanced. When tax rates are nonincreasing as neighborhoods become more affluent, and when housing consumption and nondurables consumption are perfect substitutes in the utility function, these minimum income requirements will be equivalent to zoning restrictions which require members of a neighborhood to consume some minimum level of housing. Authors such as Hamilton [1975] have shown in other contexts how zoning restrictions are required to augment house price differences in order to achieve neighborhood stratification. If minimum income requirements are further augmented by a requirement that families move sequentially between neighborhoods, then disagreements over the desired composition of neighborhoods can be resolved by the order in which families form communities.

Finally, observe that the constancy of the marginal tax rate across neighborhoods is also important for the intertemporal inequality results in Section 4. If rich neighborhoods were to save a smaller fraction of their incomes than poor ones, then the relatively greater productivity of education investment in the rich community would no longer be sufficient to ensure that the relative income between the communities is growing across time. In this case, Propositions 9 and 10 can still hold, although it would be necessary to place joint restrictions on the utility function and $\Theta(\cdot)$ to

ensure that the sample path of each neighborhoods's tax rate was nondecreasing in mean income.

ii. Production function restrictions

The assumptions of multiplicative productivity shocks and the linear transformation of education expenditures into human capital are important factors both in the determination of neighborhood tax rates as well as the dynamic properties of the model. The general production function

$$Y_{i,t+1} = f(ED_{n,t}, \widehat{F}_{Y,n,t}, \xi_{i,t+1}) \tag{20}$$

can always be rewritten as

$$Y_{i,t+1} = \phi ED_{n,t}\epsilon_{i,t+1}(ED_{n,t}, \widehat{F}_{Y,n,t}) \tag{21}$$

where $\epsilon_{i,t+1}(ED_{n,t}, \widehat{F}_{Y,n,t})$ is a shock whose conditional distribution (at t) is a function of $ED_{n,t}$ and $\widehat{F}_{Y,n,t}$. This modified production function has several important implications for the analysis.

First, under this more general functional form, the tax rate in a neighborhood will generally depend on the empirical income distribution. To see this, consider the new first order condition for an agent's preferred tax rate.

$$\frac{\pi_1}{(1-\tau)} = \frac{\pi_2}{\tau} + \pi_2 \mathrm{E}\left(\frac{\frac{\partial \epsilon_{i,t+1}}{\partial H_{i,t}} \frac{\partial H_{i,t}}{\partial ED_{n,t}} \frac{\partial ED_{n,t}}{\partial \tau}}{\epsilon_{i,t+1}(H_{i,t}, \widehat{F}_{Y,n,t})} \mid \mathfrak{F}_t \right) = 0. \tag{22}$$

In the basic model studied above, the third term in this expression is zero since $\partial \epsilon_{i,t+1}/\partial H_{i,t}$ is zero. When this term is nonzero, differences in the third term can induce different tax rates across neighborhoods. For

example, suppose that $Y_{i,t+1} - \mathrm{E}(f(ED_{n,t}, \widehat{F}_{Y,n,t}, \xi_{i,t+1}) \mid \mathfrak{F}_t)$ is uniformly bounded. This would imply that the support of $\xi_{i,t+1}$ shrinks towards 1 as $H_{i,t}$ increases. In this case, higher human capital formation would provide insurance against adverse productivity shocks, so that poor neighborhoods may wish to choose higher tax rates than rich ones. The possibility that endogenous stratification may not be supportable by house price differences may arise, even under the Cobb-Douglas preference structure. Again, minimum income requirements may be necessary to support a stratified equilibrium. A second effect of tax rate differences is that poor communities may invest a greater percentage of their income in education than rich ones, which would mean that Proposition 9 no longer necessarily holds.

Second, under a more general production function, the model can exhibit some form of decreasing returns to scale, which would affect the relative growth rates of rich and poor communities. This would occur if

$$\frac{\partial \mathrm{E}\epsilon_{i,t+1}(ED_{n,t}, \widehat{F}_{Y,n,t})}{\partial ED_{n,t}} < 0. \tag{23}$$

In this case, it would be possible for equal sized poorer communities to grow faster than richer communities if the concavity of the income function offsets the higher human capital formation associated with a given level of education investment, which would again violate Proposition 9.

However, observe that tax rate differences and production function concavity will not necessarily imply that the model exhibits no permanent inequality in growing economies. Specifically, if the production function becomes asymptotically linear, then the tendency for poorer communities to grow faster than richer ones due to higher savings rates and marginal product of education investment may not be sufficient to cause the expected income difference between any two families to converge to zero over time – Proposition 10 can still hold. The reason for this is that once a

poorer community's income grows, it will experience the same range of the production function experienced previously by the wealthier community. Concavity of the production function and tax rate differences add a transition component to the growth rates of different neighborhoods which will diminish as the poorer neighborhood's income grows. This transition component has important empirical implications in the sense that it means that the observation that poorer families have higher growth rates than richer ones has no necessary implication that contemporaneous inequality in an economy will disappear over time.[14]

Finally, observe that nonlinearity of the aggregate production function will permit the marginal product of education investment in a poor neighborhood to exceed that in an equal size rich neighborhood, which cannot occur under the constant returns to scale assumption. In this case, it could be efficient for rich neighborhoods to lend to poor ones, if such contracts were enforceable. In the current model, the marginal product of a given level of education investment in a poor neighborhood can exceed that in a rich neighborhood only if the poor neighborhood is larger.

Similarly, if the average product of education investment is increasing over some range, a transfer of tax revenues from a rich to poor neighborhood may be output maximizing. For example, suppose there exists a number $\overline{E}$ and a probability distribution $\overline{F}$ such that $Y_{i,t+1} = f(\alpha ED_{n,t}, \xi_{i,t+1})$, $\alpha < 1$, if $ED_{n,t} \leq \overline{E}$ and $\widehat{F}_{Y,n,t}(s) < \overline{F}(s)$, $Y_{i,t+1} = f(ED_{n,t}, \xi_{i,t+1})$ otherwise. In other words, if a poor neighborhood chooses a per capita investment of at least $\overline{E}$, its production function will exhibit a jump. In this case, it is easy to construct an income distribution at t such that under internal neighborhood finance a

[14] A formal demonstration of this argument can be found in Bernard and Durlauf [1992]. Notice that the argument implies that the negative correlation between intergenerational family income growth and initial family income found by Solon [1992] and Zimmerman [1992] is compatible with long run inequality in relative economic status.

neighborhood may not achieve this level of education investment, whereas total output at $t+1$ would be maximized by transferring tax revenues to this neighborhood from a rich one. This type of nonlinearity will also allow the model to exhibit poverty traps, where some communities becomes permanently fixed below some absolute income level whereas others experience sustained income growth. This type of model is analysed in Durlauf [1992].

Overall, the assumptions on preferences and technology clearly play an important role in establishing the existence of an equilibrium. This sensitivity is not surprising, as the existence of multicommunity equilibria has been established in the urban economics literature only for very specialized economic environments (see for example Westhoff [1977]). Further, some aspects of the dynamic properties of the economy are strongly affected by these assumptions. At the same time, these assumptions are clearly sufficient rather than necessary, so they provide a useful benchmark for understanding what requirements exist for persistent inequality to emerge.

6. Conclusions

This paper has developed a model of the evolution of the cross-section income distribution in an economy characterized by feedbacks from a neighborhood income distribution to the economic prospects of its offspring. These feedback effects create incentives for wealthier families to isolate themselves from poorer ones. The model we develop is shown to support this stratification of the economy through a sequence of neighborhood specific house prices. When stratification occurs, the economy will exhibit persistent inequality in two senses. First, the differences in average income between parents across neighborhoods are shown to be

transmitted to offspring. Second, for some cross-section income distribution, permanent inequality is shown to emerge with positive probability. These results collectively suggest that when agents react to positive feedbacks by choosing with whom they interact, these feedbacks can produce sustained inequality.

One important extension of our model concerns a formal analysis of the properties of the productivity shocks which augment human capital in the determination of the cross-section income distribution among offspring within a neighborhood. The current model takes this distribution as exogenous. Given the implication of the analysis that a sufficiently unequal cross-section distribution will induce intertemporal inequality, it is clearly important to understand what factors determine this distribution. One likely candidate is the way in which labor markets channel workers with different skills into jobs. This process, in turn will depend both on the way in which agents with different skills interact in the aggregate production function and on the mechanism by which information about job availability diffuses through the population. Bénabou [1991] and Montgomery [1990] have studied aspects of these issues in a static context. Development of a dynamic analogue of this work would not only permit a better understanding of the connections between the cross-section and intertemporal spillovers, but would also be important in allowing the basic model to capture higher frequency fluctuations in the cross-section income distribution.

Appendix

Proof of Proposition 1

Proposition 1 follows immediately from eqs. (10) and (11) and the subsequent discussion in the text.

Proof of Proposition 2

At an initial equilibrium tax rate, agent $i, t-1$'s utility is increasing in rightward shifts of the income distribution of his neighbors, given eqs. (6), (7) and (8). Further, by Proposition 1, the equilibrium tax rate is unaffected by a shift in the neighborhood income distribution. This proves part i. Further, since the equilibrium tax rate in a neighborhood is independent of its composition, the increases in both the tax base and human capital shift factor induced by a rightward shift in the income distribution of a given family's neighbors immediately implies part ii.

Proof of Proposition 3

We prove the existence of an equilibrium by describing an algorithm by which the neighborhoods are constructed each period. Given the realized income distribution at t, place the highest income family in neighborhood 1. Add to that neighborhood the largest collection of families which maximizes the highest income agent's utility. This set is well defined because $\Theta(F_\epsilon)$ is continuous in ϵ. Place the highest income family among those not assigned to neighborhood 1 into neighborhood 2. Add to that neighborhood the collection of families not assigned to neighborhood 1 which will maximize the utility of the highest income family in neighborhood 2. Repeat this procedure until all families are assigned to neighborhoods 1 to D. We claim

that this algorithm produces an equilibrium in the sense that these neighborhoods are supportable by a set of house prices. Since Proposition 1 states that an equilibrium tax rate exists for any allocation of families within a neighborhood, the proof is complete once it is shown that there are house prices $P_{d,t}$ such that at this allocation, no family can improve utility by moving.

Consider first the existence of a house price sequence which separates families in neighborhoods 1 and 2. In order for $P_{1,t}$ and $P_{2,t}$ to support stratification, it must be the case for any agent $i, t-1 \in N_{1,t}$ and agent $i', t-1 \in N_{2,t}$ that if

$$\begin{aligned} &\pi_1 log((1-\tau^*)Y_{i,t}) + \pi_2 log(\phi H_{i,t}) = \\ &\pi_1 log((1-\tau^*)(Y_{i,t} + P_{1,t} - P_{2,t})) + \pi_2 log(\phi H_{i',t}) \end{aligned} \tag{A.1}$$

then

$$\begin{aligned} &\pi_1 log((1-\tau^*)(Y_{i',t} - P_{1,t} + P_{2,t})) + \pi_2 log(\phi H_{i,t}) \leq \\ &\pi_1 log((1-\tau^*)(Y_{i',t})) + \pi_2 log(\phi H_{i',t}). \end{aligned} \tag{A.2}$$

Since the required price differential $P_{1,t} - P_{2,t}$ for (A.1) to hold is increasing in $Y_{i,t}$ whereas the required range of price differentials such that (A.2) holds is decreasing in $Y_{i',t}$, the existence of a stratifying price sequence will hold so long as (A.1) implies (A.2) when $Y_{i,t} = Y_{i',t}$. However, this requires,

$$\frac{\pi_1 Y_{i,t}}{Y_{i,t} + P_{1,t} - P_{2,t}} \leq \frac{\pi_1 Y_{i,t}}{Y_{i,t} - P_{1,t} + P_{2,t}} \tag{A.3}$$

which will always hold if $P_{1,t} - P_{2,t} > 0$. However, since all families agree on the relative ranking of neighborhoods, this last inequality must hold since the wealthiest family in neighborhood 1 prefers that neighborhood to

neighborhood 2. This establishes that there exists a house price differential between the first two neighborhoods which supports the stratification. One can proceed sequentially to construct the house price differential between neighborhoods 2 and 3 to support stratification, and so on to show how the stratified equilibrium is supported across all communities. This proves part i. of Proposition 3.

Part ii. follows from two arguments. First, by Proposition 2, all members of a neighborhood wish to have the highest income neighbors for a neighborhood of fixed size. Second, under Cobb-Douglas preferences, all families agree on the relative rankings of different neighborhoods. Hence the relative neighborhood ranking by the richest family in the population is agreed upon by all families. Suppose that two neighborhoods have nonstratified income distributions. In this case, there exists a reallocation of families across neighborhoods so that the richest member across the two neighborhoods lives in a community of equal size with a preferred income distribution; under this reallocation, the population will now be stratified by income. All members of the now wealthier neighborhood are also better off. Hence under the assumption of initial voluntary neighborhood formation, the original configuration could not have been an equilibrium.

Proof of Proposition 4

Proposition 4 is an immediate consequence of eq. (8), the assumption that all financing of education is determined within a neighborhood subject to a no borrowing constraint, and the white noise assumption on $\xi_{i,t+1}$.

Proof of Proposition 5

The proposition will hold for all neighborhoods if it holds for a neighborhood consisting of μ_1 families with income Y^{high} and μ_2 families

with incomes Y^{low}. In this case, the high income families will only agree to live with the low income families if

$$\frac{\pi_2}{\pi_1+\pi_2}\mu_2 Y^{low} - \lambda_2\mu_2 ED_{n,t} > 0 \tag{A.4}$$

where $ED_{n,t}$ is the level of per capita education expenditure when the two groups live together. However, since the tax rate is independent of neighborhood composition, $ED_{n,t}$ is increasing in Y^{high} in such a way that for a sufficiently large value of Y^{high}, (A.4) will be violated.

Proof of Proposition 6

Define the collection of offspring of $N_{n,t}$ as O_{t+1} with the empirical income distribution $\widehat{F}_{Y,O,t+1}$ and the associated mean income $MY_{O,t+1}$. We consider the desired neighborhood configuration of the wealthiest offspring among members of O_{t+1}, designated as agent $i,t+1$, who also resides in neighborhood n as an adult.

First, observe that if we define $\kappa_{i,t+1}$ as

$$\kappa_{i,t+1} = \frac{\phi\Theta(\widehat{F}_{Y,n,t+1})\mu(N_{n,t+1})MY_{n,t+1}}{(\lambda_1+\lambda_2\mu(N_{n,t+1}))Y_{i,t+1}}, \tag{A.5}$$

then agent $i,t+1$'s optimal tax choice will implicitly maximize

$$\pi_1 log(Y_{i,t+1} - T_{i,t+1}) + \pi_2 \mathrm{E}(log(\kappa_{i,t+1}T_{i,t+1}\xi_{i,t+2} \mid \mathfrak{F}_{t+1})), \tag{A.6}$$

implying that the agent will choose his neighborhood to maximize $\kappa_{i,t+1}$.

In order for agent i,t to choose $N_{n,t+1} \subset O_{t+1}$, it must be the case that dropping all families below some income threshold will make i,t better off when compared to living with all offspring in $N_{n,t}$. Let $O_{\epsilon,t+1}$ denote the population of families in O_{t+1} that remains after dropping the poorest

$\epsilon \times 100$ percent of families; $\widehat{F}_{Y,O,\epsilon,t+1}$ denotes the income distribution and $MY_{O,\epsilon,t+1}$ denotes the mean income after dropping the ϵ families. Without loss of generality, assume that $\Theta(\widehat{F}_{O,\epsilon,t+1})$ and $MY_{O,\epsilon,t+1}$ are differentiable with respect to ϵ.

In order for $N_{n,t+1} \subset O_{t+1}$ to denote an equilibrium neighborhood configuration, it must be the case that the derivative of $\kappa_{i,t+1}$ with respect to ϵ is zero or positive at some $\epsilon > 0$. This derivative is

$$\left(\frac{\phi}{Y_{i,t+1}}\right)\left(\left(\frac{\partial(\Theta(\widehat{F}_{O,\epsilon,t+1})MY_{O,\epsilon,t+1})}{\partial\epsilon}\right)\left(\frac{\mu(O_{\epsilon,t+1})}{(\lambda_1+\lambda_2\mu(O_{\epsilon,t+1}))}\right)+\right.$$
$$\left.\Theta(\widehat{F}_{O,\epsilon,t+1})MY_{O,\epsilon,t+1}\cdot\left(\frac{\lambda_1}{(\lambda_1+\lambda_2\mu(O_{\epsilon,t+1}))^2}\right)\right)\cdot\left(\frac{\partial\mu(O_{\epsilon,t+1})}{\partial\epsilon}\right) \quad \text{(A.7)}$$

The variation in $\phi\Theta(\widehat{F}_{O,\epsilon,t+1})MY_{O,\epsilon,t+1}/Y_{i,t+1}$ is bounded for all ϵ by eqs. (3) and (9) and the uniform boundedness of $\xi_{i,t+1}$ (which bounds the ratio of richest to poorest offspring within a neighborhood). Further, $\partial\mu(O_{\epsilon,t+1})/\partial\epsilon = -1$. Therefore, when $\mu(O_{t+1})$ is small enough, a desired reduction in population from O_{t+1} by agent i,t requires that

$$-\frac{\phi\Theta(\widehat{F}_{Y,O,\epsilon,t+1})MY_{O,\epsilon,t+1}}{Y_{i,t+1}}\cdot\frac{\lambda_1}{(\lambda_1+\lambda_2\mu(O_{\epsilon,t+1}))^2} \geq 0. \quad \text{(A.8)}$$

However, for $\mu(N_{n,t})$ small enough, (A.8) can never hold, since $\mu(O_{\epsilon,t+1}) < \mu(N_{n,t})$ for $\epsilon > 0$, proving part i.

To prove part ii. observe that if $\overline{F}$ is shifted far enough to the right, it must be the case that $\Theta(\widehat{F}_{O,\epsilon,t+1}) - \Theta(\widehat{F}_{O,t+1})$ is nonincreasing in the lower bound on incomes in O_{t+1} by eq. (9). This means that changes in the largest admissible value of μ^{thresh} as the income distribution shifts to the right must eventually become negligible, which implies part ii.

Proof of Proposition 7

Suppose that all families start with any initial income $\bar{Y}^{low}$. Under decreasing average costs, all families will inhabit the same neighborhood in this case. Now suppose that the incomes of a positive measure of families increase to $\bar{Y}^{high}$, where $\bar{Y}^{high}$ is chosen, (as can always be done by Proposition 6) so that those families with incomes greater than or equal to $\bar{Y}^{high}$ refuse to form neighborhoods with those with incomes equal to $\bar{Y}^{low}$. Clearly, those families whose incomes did not change are worse off. By extension, for any initial income distribution, there must exist some rightward shifts of the economy-wide income distribution, under which no family's income declines, such that a positive measure of those families who reside in the poorest neighborhood worse off because they lose their wealthier neighbors. This verifies the proposition.

Proof of Proposition 8

As discussed in the proof of Proposition 6, the wealthiest family will live in the neighborhood which maximizes $\kappa_{i,t}$ among all possible neighborhood configurations. Since $T_{i,t}$ is independent of which neighborhood the richest family resides in, this means, given eqs. (A.5) and (A.6), that the human capital investment of the family's offspring is maximized, which proves part i. For the lowest income family, endogenous stratification means that the poorest family lives in the neighborhood with the lowest income distribution among all those of a given size. Hence by Proposition 2.ii, this must be the neighborhood which minimizes the expected income of its offspring.

Proof of Proposition 9

If two neighborhoods n and n' are of equal size at t, then

$$\frac{ED_{n,t}}{MY_{n,t}} = \frac{ED_{n',t}}{MY_{n',t}} \tag{A.9}$$

since each will choose the same tax rate under Cobb-Douglas preferences. Further, $\Theta(\widehat{F}_{Y,n,t}) > \Theta(\widehat{F}_{Y,n',t})$ by assumption of the proposition. Therefore, substituting (A.9) and eq. (2) into eq. (13), one finds that

$$g_{n,t} = \frac{\phi\Theta(\widehat{F}_{Y,n,t})ED_{n,t} - MY_{n,t}}{MY_{n,t}} >$$
$$\frac{\phi\Theta(\widehat{F}_{Y,n',t})ED_{n',t} - MY_{n',t}}{MY_{n',t}} = g_{n',t}. \tag{A.10}$$

Further,

$$\frac{\partial \frac{ED_{n,t}}{MY_{n,t}}}{\partial\mu(N_{n,t})} = \frac{\tau\lambda_1}{(\lambda_1 + \lambda_2\mu(N_{n,t}))^2} > 0 \tag{A.11}$$

so that (A.10) still holds if $\mu(N_{n,t}) \geq \mu(N_{n',t})$, proving the proposition.

Proof of Proposition 10

This proposition is verified by constructing an economy-wide distribution of income at t and showing that it fulfills the proposition.

1. Assign a population of μ_1 families to neighborhood n and μ_2 to neighborhood n', where $\mu_1 > \mu_2$ and $\overline{\mu} \leq \mu_1 \leq \mu^*$. By choosing a large enough minimum income for these neighborhoods at t, one can ensure that these neighborhoods are in the sense of Proposition 6 and that the expected average growth rate in income is positive among families in $N_{n,t}$.

2. Assume that at time t, all families have the same incomes within each of the neighborhoods that have been formed. Further, choose the incomes within each neighborhood to be far enough apart that not only will these neighborhoods be equilibrium configurations at t, but that offspring across the neighborhoods will not recombine at $t+1$ if the wealthier

neighborhood's mean income grows at a faster rate than the poorer one's. This can always be done, since the proof of Proposition 5 establishes that if the minimum income among offspring in one neighborhood is sufficiently greater than the maximum in another these offspring will not combine to form a common neighborhood as adults when the mean income is constant or increasing even when the marginal product of capital is unaffected by the neighborhood income distribution.

3. If at $t+s$, $MY_{n,t+s}/MY_{n',t+s} \geq MY_{n,t}/MY_{n',t}$, then given the construction of the neighborhoods at t, one can always write a law of motion for $log(MY_{n,t+s+1}/MY_{n',t+s+1})$ of the form

$$
\begin{aligned}
& log(MY_{n,t+s+1}/MY_{n',t+s+1}) = \\
& \mathrm{E}(log(MY_{n,t+s+1}/MY_{n,t+s}) - log(MY_{n',t+s+1}/MY_{n',t+s})) + \\
& log(MY_{n,t+s}/MY_{n',t+s}) + \zeta_{t+s+1} \qquad \text{(A.12)}
\end{aligned}
$$

where for some $\nu > 0$,

$$\mathrm{E}(log(MY_{n,t+s+1}/MY_{n,t+s}) - log(MY_{n',t+s+1}/MY_{n',t+s})) \geq \nu \quad \text{(A.13)}$$

and ζ_{t+s+1} is a martingale difference. This follows because the expected growth rate of the wealthier community must exceed that of the poorer one by some minimum amount due to the difference in population size. Further, ζ_{t+s+1} has uniformly bounded support by eq. (3).

Given (A.12), if

$$Prob(MY_{n,t+s}/MY_{n',t+s} \geq MY_{n,t}/MY_{n',t} \ \forall \ s) > 0, \qquad \text{(A.14)}$$

then the *log* of the income gap between families i and i' will, with positive probability, behave as a random walk with drift. A random walk with a drift parameter which is bounded from below by any positive ν with

uniformly bounded innovations has two important features. First, the process will diverge with probability 1. Second, following Durlauf [1992], one can show that by the law of large numbers for uniformly bounded martingale differences (see Stout [1974] for a statement), the set of values less than or equal to the initial value of such a process is a transient set of states for the process. This means that if (A.14) holds, the mean incomes of the rich and poor neighborhoods will obey both parts of the proposition.

4. The probability that the income ratio between the two communities is bounded from below by 1 can determined by studying the auxiliary process Z_t, where

$$Z_{t+1} = \nu + Z_t + \zeta^*_{t+1} \tag{A.15}$$

where ζ^*_{t+1} is a uniformly bounded martingale difference sequence such that $\zeta^*_{t+s} = \zeta_{t+s}$ if $MY_{n,t+r}/MY_{n',t+r} \geq MY_{n,t}/MY_{n',t}$ $\forall$ $r = 1 \ldots s-1$, and $Z_t = log(MY_{n,t}/MY_{n',t})$.

By construction,

$$\begin{aligned} Prob(MY_{n,t+s}/MY_{n',t+s} \geq MY_{n,t}/MY_{n',t} \ \forall \ s) \geq \\ Prob(Z_{t+s} \geq Z_t \ \forall \ s). \end{aligned} \tag{A.16}$$

However, the process Z_t is a random walk with constant drift and uniformly bounded increments, which means that the set of values of the support of the process which are less than or equal to the initial state of the process is transient,

$$Prob(Z_{t+s} \geq Z_t \ \forall \ s) > 0, \tag{A.17}$$

which completes the proof.

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