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A Monthly Indicator of Brazilian GDP

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A MONTHLY INDICATOR OF BRAZILIAN GDP

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Abstract

This paper constructs an indicator of Brazilian GDP at the monthly frequency. The peculiar instability and abrupt changes of regimes in the dynamic behavior of the Brazilian business cycle were explicitly modeled within nonlinear frameworks. In particular, a Markov switching dynamic factor model was used to combine several macroeconomic variables that display simultaneous comovements with aggregate economic activity. The model generates as output a monthly indicator of the Brazilian GDP and real time probabilities of the current phase of the Brazilian business cycle. The monthly indicator shows a remarkable historical conformity with cyclical movements of GDP. In addition, the estimated filtered probabilities predict all recessions in sample and out-of-sample. The ability of the indicator in linear forecasting growth rates of GDP is also examined. The estimated indicator displays a better in-sample and out-of-sample predictive performance in forecasting growth rates of real GDP, compared to a linear autoregressive model for GDP. These results suggest that the estimated monthly indicator can be used to forecast GDP and to monitor the state of the Brazilian economy in real time.

KEY WORDS: Business Cycle, Dynamic Factor, Markov Switching, Composite Indicators, Kalman filter, Filtered Probabilities, Forecast.

JEL Classification Code: C32, C50, E32

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1. Introduction

In recent years, reduction in capital restrictions in emerging markets substantially increased international financial flows to these countries. In Brazil, the stability of the currency post-Real plan set the basis for reforms, which turned Brazilian markets more competitive. Trade liberalization and a stronger currency led to cheaper imports and propelled national firms to modernize their technology. These changes combined with privatization and deregulation of state monopolies attracted record high foreign investments.

The economic reforms, however, have not decreased the entrenched volatility of the Brazilian economy. Abrupt downturns and upturns have been a regularity affecting both financial markets and the goods producing sector. As international markets become more integrated, there has been a surging interest in the overall economic performance of Brazil. In particular, there has been an increasing international demand for timely and reliable data that would allow real time assessment of the dynamics of the Brazilian economy. The availability of more accurate real time information about the Brazilian economy is not only important for policy or investment decisions. It can also contribute to make it less likely the occurrence of self-fulfilling financial crisis driven by misperceptions regarding fundamentals of the Brazilian economy and the country risk.

A traditional method of monitoring the economic activity is through the use of composite leading and coincident indicators. In the U.S., composite indicators have been used since their first construction by the Department of Commerce in the 1960s,[1] based on the work of Burns and Mitchell (1946). As a result of a throughout statistical analysis of macroeconomic variables, Burns and Mitchell classified over 500 series into lagging, coincident, or leading according to the timing of their cyclical movements with the U.S. economic activity, in a research sponsored by the National Bureau of Economic Research (NBER). The combination of some of these variables resulted in the composite indexes, which are used to signal turning points of business cycles.[2] These indicators are one of the most watched series by the press, businesses, policymakers, and stock market participants.

[1] Since 1995 the Conference Board has been responsible for compiling the U.S. composite indicators.

[2] Turning points refer to peaks and troughs of business cycle phases. Peaks mark the beginning of recession phases (end of expansions) and troughs define the end of recessions (onset of expansions).

The popularity of the U.S. economic indicators led to the construction of these indexes for other countries, such as Australia, Japan, Canada, and other members of the OECD in the 1970s and 1980s. More recently, progressive market integration has induced a worldwide interest in the analysis of cyclical fluctuations through the use of economic indicators. This has placed the traditional method to construct the indicators in the spotlight, which has been under increased scrutiny and criticism in recent years.[3] One of the main criticisms is as old as the method itself. Koopmans (1947) criticized Burns and Mitchell's analysis for being centered on measurement without any explicit model attached to it. This problem is also extended to composite indicators, whic h are constructed as simple weighted averages of statistical transformations of their components. This implies that the dynamic relationships among the variables are not taken into account. For example, the method ignores their cross-correlations or potential long-term relationships. Another criticism is that the method does not include a probabilistic model to determine turning points in the indicators, and the existing ad-hoc rules require the use of substantial ex-post data. That is, turning points can only be identified and predicted a couple of months after their occurrence, which undermines the usefulness of indicators for real time forecasting. Finally, large revisions have been implemented in the indicators over the years to improve their ability in predicting turning points ex-post. These revisions, which include changes in the components, in their weights, and in the construction procedures, have substantially changed the historical record of the indicators' performance. In fact, analysis of their real time forecasting ability can only be evaluated using the unrevised versions of the indicators, as examined in Diebold and Rudebusch (1991).[4]

The disseminated use of economic indicators and awareness of their shortcomings had a corresponding resurgent academic interest in this instrument. Frontier econometric models have been used to formally explore potential dynamic differences across business cycle phases, to generate indicators, and to determine turning points.

[3] Boldin (1998/1997) summarizes the main criticisms to the composite indicators.

[4] Following Diebold and Rudebusch (1991), several other authors have examined the reliability of the unrevised leading indicator in predicting turning points in real time, such as Emerson and Hendry (1996), Lahiri and Wang (1994), Perez-Quiros and Hamilton (1996), Chauvet (1998/1999), among others.

The goal of this paper is to obtain an indicator of Brazilian GDP at the monthly frequency that can be used to forecast business cycles and to monitor the current state of the economy in real time. In particular, a Markov switching dynamic factor model is used to combine several macroeconomic variables that display simultaneous comovements with aggregate economic activity, as in Chauvet (1998). The model generates as output a monthly indicator of the Brazilian GDP and real time probabilities of the current phase of the Brazilian business cycle.

The formal representation of business cycle phases through the dynamic factor model with Markov switching overcomes the drawbacks of the traditional method while maintaining the main points of Burns and Mitchell (1946) and the NBER's definition of business cycles, as suggested by Diebold and Rudebusch (1996). First, since the model includes several variables, it captures pervasive cyclical fluctuations in various sectors of the economic activity. As recessions and expansions are caused by different shocks over time, the inclusion of different variables increases the ability of the model in representing and signaling phases of the business cycles. In addition, the combination of variables reduces measurement errors in the individual series and, consequently, the likelihood of false signaling turning points, which is particularly important for monthly data. For example, one of the findings is that the individual variables that compose the indicator, specially industrial production, do not represent broad changes in the economy and, by itself, give several false turning point signals. Finally, the model allows potential asymmetries across expansions and recessions and analysis of turning points, albeit here their evaluation can be followed in real time.

In order to compile a composite indicator as accurately as possible, a large number of economic time series were evaluated in-depth, with special attention to the peculiar dynamical behavior of the Brazilian business cycle. In particular, the instability and abrupt changes of regimes in the history of Brazilian economic activity were explicitly modeled within nonlinear frameworks.

The primary step in developing composite indicators is the definition of a business cycle chronology to be used as a benchmark. That is, it is necessary to first determine peaks and troughs of the Brazilian business cycle, which are then used to evaluate the predictive performance of the indicators. The NBER has been dating the U.S. business cycle for the

last fifty years. Peaks and troughs are determined from a subjective consensus among the ten members of the Business Cycle Dating Committee. Decisions about business cycle turning points are based on the turning points of several coincident variables, such as manufacturing and trade sales, personal income, industrial production, and non-agricultural employment, among others. However, the NBER dating can not be used to monitor the economy in real time, since the Business Cycle Committee meets only months after a turning point has occurred.

In this paper, the results of the monthly indicator of GDP are compared to the business cycle chronology obtained in Chauvet (2000). Chauvet uses a model-based approach to date the Brazilian business cycle and growth cycle in the last 100 years.[5] The results are compared with several ad-hoc rules, and the dating obtained is very similar. However, the ad-hoc rules require that a couple of months have passed for the turning point dating to be determined. On the other hand, the results of the model-approach method enable analysis of business cycles on a current basis.

The monthly indicator of Brazilian GDP includes variables measuring aggregate economic activity such as employment, industrial production, capacity utilization, compensated hours, and real wages. The estimated indicator shows a remarkable historical conformity with cyclical movements of GDP with respect to volatility, timing of peaks and troughs, and duration of the phases. In addition, the estimated filtered probabilities predict all recessions and expansions in the sample, as dated in Chauvet (2000).

The performance of the indicator is also examined out-of-sample. In order to emulate forecasting in real time, the model is recursively re-estimated out-of-sample. This allows us to reproduce the information content that was available to forecasters at any point in time. The indicator summarizes information in several macroeconomic variables, which is found to be relevant in forecasting economic activity. In particular, the resulting filtered probabilities predict all recessions out-of-sample. The ability of the indicator in linear forecasting growth rates of GDP is also examined within and out-of-sample. In both cases the estimated indicator displays a better predictive performance compared to a linear

[5] More specifically, a Markov switching model is fitted to quarterly and annual real GDP, and the resulting smoothed probabilities are used as filtering rules to determine turning points.

autoregressive model for GDP. In particular, the inclusion of lags of the indicator improves substantially forecasts of the severity of recessions and strength of expansions, as measured by the volatility of changes in GDP.

The results suggest that the indicator and the associated probabilities of business cycle phases could be useful tools to forecast GDP and to monitor the state of the Brazilian business cycle in real time.

The paper is organized as follows. The next section describes the procedure used to analyze and select the variables that compose the model. Section 3 presents the model. In the fourth section, the empirical results are discussed. In particular, the probabilities of business cycle phases and the indicator of the Brazilian GDP are examined within and out-of-sample. The fifth section concludes.

2. Data Analysis

In order to compile the composite indicator, a large number of economic time series were considered. The data were obtained from Getúlio Vargas Foundation (FGV) and the Central Bank of Brazil databases. In order to obtain a historical representative composite indicator of the Brazilian GDP, an important criterion for selecting the series was their sample size. Although there are many candidate variables whose cyclical variation could potentially coincide with the real Brazilian GDP, only a small subset of monthly series have a longer sample going back to the 1970s and 1980s.[6]

As a starting point for the analysis, the variables were seasonally adjusted using the X-11 additive method from the U.S. Bureau of the Census. Variables expressed in nominal terms were deflated using the Global Price Index, IGP-DI, or the Broad Consumer Price Index, IPCA.[7] Finally, all variables were transformed to achieve stationarity.[8]

Several selection criteria were then implemented to find the ones that display simultaneous movements with the Brazilian business cycles. The underlying guideline was

[6] In fact, most series start in the 1990s. Those were not selected, since a decade of observations would not be representative of the historical behavior of recessions and expansions in Brazil.

[7] Since the Broad Consumer Price Index is only available from 1979 on, the IGP-DI was utilized for data with longer sample.

the economic significance of the series, their statistical adequacy, their timing at turning points, and their overall conformity to the business cycle. First, the series were classified as coincident, leading or lagging according to their ability to Granger-cause changes in GDP,[9] their cross-correlation with changes in GDP in time domain, and coherence in frequency domain. Second, they were ranked according to the timing of their turning points with cyclical variation in GDP, which is taken as the reference cycle. Peaks and troughs of GDP mark recession and expansion phases in Brazil, and were obtained as in Chauvet (2000). In particular, two-state Markov switching models were fitted to each of the candidate variables and to GDP, with the states representing economic expansions or contractions.[10] The estimated probabilities of contractions were then used to determine peaks and troughs in the series, and the timing of change of each series was then compared to GDP turning points.

From these procedures, fifteen variables were found to display coincident movements with changes in GDP, which are described in Table 1. These series represent different measurements of industrial production, capacity utilization, real wages, compensated hours, retail sales, employment, unemployment rate, fuel consumption, and production of cement.

[8] During the period analyzed there were several stabilization plans that engendered structural breaks in the Brazilian macroeconomic variables. Thus, both the augmented Dickey-Fuller (1979) and Perron's (1989) tests were used to verify the hypothesis of unit roots.

[9] For comparison with growth rates of GDP, two procedures were undertaken. First, all monthly series were converted to quarterly frequencies as simple averages. Second, changes in GDP were converted into monthly frequency using quadratic-match average for comparison with the monthly series.

[10] The best two-state specification for each series was selected based on the likelihood ratio test.

Table 1. Original Variables at Monthly Frequency

Variables	Source	Measuring unit	Sample	Mnemonic	Transform.
Total Physical Production – General Industry Indicator	IBGE	Index	1975:01	PRODBR	Δlog
Unemployment Rate – Average of 6 Major Metropolitan Regions	IBGE	Percentage	1980:01	URBR	Δ
Total Employment – Brazil	MT	Index	1985:01	EMPLBR	Δlog
Employees in the Industrial Production Sector – Brazil	IBGE	Index	1985:01	EMPLIP	Δlog
Value Added Tax Revenue, ICMS – Brazil	CONFAZ	R$ million	1980:01	ICMS	Δlog
Retail Sales	FCESP	Index	1979:01	SALES	Δlog
Manufacturing Industry – Capacity utilization (*)	BACEN	Percentage	1986:01	CAPBR	Δ
Manufacturing Industry – Real Wages (*)	BACEN	Index	1986:01	WAGEMI	Δlog
Fuel Consumption – Brazil	PETROBRAS	Thousands M3	1980:01	COMB	Δlog
Electricity Consumption – Brazil	ELETROBRAS	GWH	1976:01	ELETBR	Δlog
Total Unemployment, São Paulo	SEADE	Percentage	1984:01	UNEMSP	Δ
Manufacturing Industry – Total Number of Employees, São Paulo	FIESP	Index	1975:01	EMPLOY	Δlog
Manufacturing Industry – Capacity utilization, São Paulo	FIESP	Percentage	1975:01	CAPSP	Δ
Manufacturing Industry – Total of Compensated Hours, São Paulo	FIESP	Index	1975:01	HOURS	Δlog
Manufacturing Industry – Total Wages, São Paulo	FIESP	Index	1975:01	WAGESP	Δlog
Real GDP (quarterly)	IBGE	Index	1975:01	GDP	Δlog

Δ stands for first difference.

IBGE is the Brazilian Institute of Economic Geography. FIESP is the State of São Paulo's Industry Federation. FCESP is the State of São Paulo's Commerce Federation. MT is the Labor Ministry. CONFAZ is the National Treasury Council. BACEN is the Central Bank of Brazil. SEADE is State of São Paulo System of Data Analysis. ELETROBRAS is the Brazilian Electricity Holding Company. PETROBRAS is the Brazilian Petroleum Holding Company.

(*) These data were generated from surveys of the Industry Federation in the following Brazilian States: Amazonas, Ceará, Pernambuco, Bahia, Espírito Santo, Minas Gerais, Rio de Janeiro, São Paulo, Paraná, Santa Catarina, Rio Grande do Sul e Goiás.

3. The Model

The indicator of the Brazilian business cycle is constructed using a Markov switching dynamic factor model, as in Chauvet (1998). Let Y_t be a vector of nx1 observable macroeconomic variables that move simultaneously with the Brazilian Gross Domestic Product:

(1) $$\Delta Y_t = \lambda \Delta F_t + \Delta v_t.$$

Changes in these n macroeconomic variables ΔY_t are modeled as a common unobserved scale factor, ΔF_t, and n individual idiosyncratic terms, Δv_t. The factor loadings, λ, measure the sensitivity of the series to the dynamic factor, ΔF_t.[11] Both the factor and the idiosyncratic terms follow autoregressive processes:

(2) $$\Delta F_t = \mu_{S_t} + \phi \Delta F_{t-1} + \eta_{S_t} \qquad \eta_{S_t} \sim N(0, \sigma^2_{\eta_{S_t}}),$$

(3) $$\Delta v_t = d(L)\, \Delta v_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim \text{i.i.d. } N(0, \Sigma).$$

where η_{S_t} is the common shock that engender changes in the phases of the business cycle and ε_t are the measurement errors. In order to capture potential asymmetries across different states of the business cycle, the intercept and variance of the factor switch regimes according to a Markov variable, S_t, where $\mu_{S_t} = \alpha_1 + \alpha_0 S_t$, and $S_t = 0, 1$. That is, the economy can be either in an expansion state (S_t=1), where μ_{S_t} is positive; or in a contraction phase (S_t=0), with a negative mean growth rate. The volatility of the factor as measured by $\sigma^2_{\eta_{S_t}}$ can also assume different values across states. The switches from one state to another is determined by the transition probabilities of the first-order two-state Markov process, $p_{ij}=\text{Prob}[S_t=j|S_{t-1}=i]$, where $\sum_{j=0}^{1} p_{ij} = 1$, $i, j = 0,1$.

The model separates out common signal underlying the observed variables from individual variations in each sector of the economic activity. The dynamic factor captures widespread simultaneous downturns and upturns movements of several sectors of the

[11] The factor loading for the production series is set equal to one to provide a scale for the latent dynamic factor. This normalization is a necessary condition for identification of the factor and the choice of parameter scale does affect any of the time series properties of the dynamic factor or the correlation with its components.

Brazilian economy, corresponding to Burns and Mitchell's (1946) definition of business cycles. On the other hand, if only one of the variables falls (e.g. industrial production), this would not characterize a recession in the model, and it would be captured by the industrial production idiosyncratic term. A recession (expansion) will occur when all n variables decrease (increase) at about the same time. That is, η_{S_t} and Δv_t are assumed to be mutually independent at all leads and lags, for all n variables, and d(L) is diagonal.

The dynamic factor is the outcome of averaging out the discrete states. Although the n variables measure economic activity in specific sectors, the dynamic factor is a nonlinear combination of them, representing broader movements in the Brazilian economy. The estimation procedure is discussed in the Appendix.

4. Empirical Results

Given that the sample availability differs across variables, the model was estimated using three periods: a) from 1975:01 to 2000:06, b) from 1980:01 to 2000:06, and c) from 1986:01 to 2000:06. Six variables are available from 1975:01 on, which are used to estimate the indicator with the longest period. For the second sample, these variables are combined with the other four that start in 1980:01. Finally, these ten variables are combined with the remaining five to estimate the composite indicator from 1986:01 on.

The dynamic factor summarizes the comovements underlying the macroeconomic series used in the model. Thus, it is crucial that the variables composing the indicator display coincident cyclical movements with each other.[12] Factor analysis and principal components were used to test the variables to be included in the factor. The magnitude of the eigenvalues of the common factor's correlation matrix indicates whether the factor structure is a reasonable representation of the data. In fact, this method can also be used to test for single or multi-factor specifications.

Finally, the selection among those that represent closely related definitions was based on two guidelines. First, it was taken into consideration the economic significance of the specific activity or process as more representative of the Brazilian GDP. For example,

[12] The inclusion of variables with low correlation with each other will generally result in the convergence of the factor to one of the component variables.

total industrial production is chosen instead of production in the manufacturing industry, and so on. Second, the evaluation process described in section 2 was applied to rank and select between closely related variables.

From this analysis, three coincident indicators are found from the combination of a small set of variables. For the sample from 1975:01 on, five variables are combined to yield a monthly indicator of GDP (group 1): total industrial production (PRODBR), total number of employees (EMPLOY), capacity utilization (CAPSP), compensated hours (HOURS), and total wages (WAGESP).[13] For the sample from 1980:01, six variables were combined to obtain the indicator (group 2): total industrial production (PRODBR), unemployment rate (URBR), compensated hours (HOURS), retail sales (SALES), capacity utilization (CAPSP), and total wages (WAGESP). Finally, for the sample from 1985:01 on, six variables enter into the composition of the coincident indicator (group 3): total industrial production (PRODBR), total employment (EMPLBR), compensated hours (HOURS), retail sales (SALES), capacity utilization (CAPBR), and total wages (WAGEMI).[14]

The models using the three sets of variables were estimated by maximizing the likelihood function through a numerical procedure. The nonlinear discrete filter produces two outputs: the dynamic factor and the associated probabilities of the Markov state. The filtered probabilities give at time t the probability of the Markov state using only information available at t, $Pr(S_t=0,1|I_t)$. On the other hand, the smoothing probabilities are obtained through backward recursion using the information in the full sample, $Pr(S_t=0,1|I_T)$.

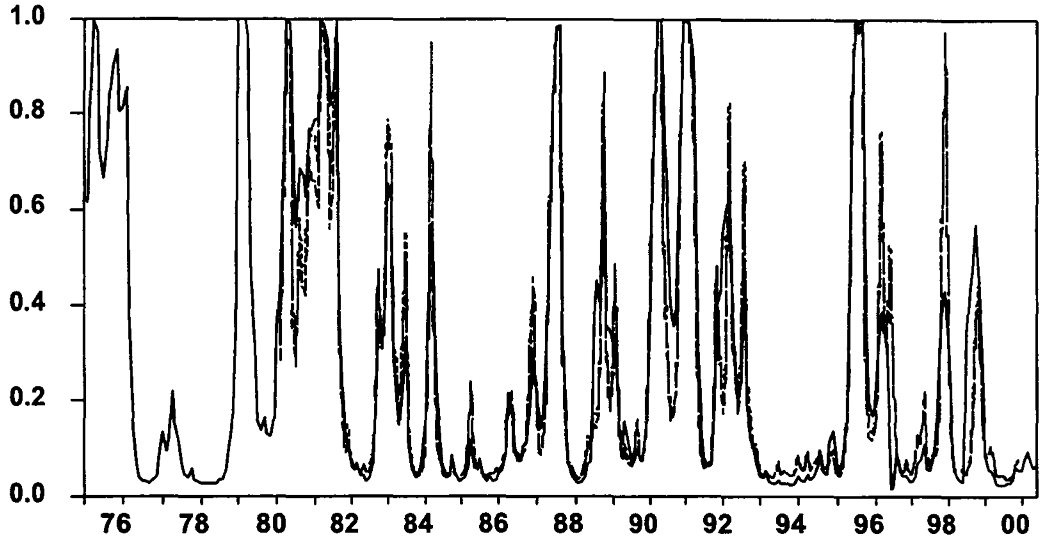
Figure 1 shows the smoothed probabilities of recession for the three sets of variables. The results are striking similar. The probabilities match in terms of timing of changes, duration, and amplitude. This also holds for the dynamic factor from the three models, where the correlation for each pair is around 0.999. This finding is due to the fact

[13] Variables measuring electricity usage were found to lag the economic activity. This result might reflect in part the way some of these series are calculated. For example, observation for time t corresponds to electricity usage from mid-month t-1 to mid-month t. The series on fuel consumption exhibits low correlation with most of the other components and was, therefore, not included in the final composite indicator. The value added tax revenue series (ICMS) did not score very high in the evaluation process – in addition to being a very noisy variable, it also displays low correlation with current economic conditions at the monthly frequency, which may be a consequence of measurement error.

[14] The hypothesis of cointegration for these three group of series was rejected at the 5% level using Johansen's (1991, 1995) test.

that the series used in each subset represent the same sectors. Basically, the variables are broader or narrower measurements of employment, wages, industrial production, capacity utilization, and sales.[15] Thus, the empirical analysis is shown for the monthly indicator composed of the variables with a longer historical sample, group 1.

Figure 1 – Smoothed Probabilities of Recessions for Group 1 (—), Group 2 (—), and Group 3 (- - -):



4.1 Specification Tests

The model assumes that the factor summarizes the common dynamic correlation underlying the observable variables, which implies that the n residuals Δv_t are uncorrelated across variables. This assumption is tested in several ways. First, the one step ahead forecast errors obtained from the Kalman filter are not predictable by lags of the observable variables. Second, the disturbances are regressed on six lags of the observable variables, and the parameters of the equation are not significantly different from zero. These results support

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[15] Notice that these are roughly the same variables used by the NBER, the Conference Board, and the OECD to construct their coincident indicators of the business cycle. The similarity of the results for the three groups also indicate that the series representing the production process in São Paulo are good proxies for the broader production process in the whole country. A further examination of the data explains this finding – São Paulo's GDP has corresponded to around 60% of the total Brazilian GDP in the last twenty years (data for GDP per Sate is obtained from IBGE).

the single factor specification, since the idiosyncratic terms are not capturing common information underlying the observable variables. Finally, this is further tested by examining the eigenvalues of the correlation matrix of the common factor, which also indicates adequacy of the single factor specification.[16]

The i.i.d. assumption of the residuals ε_t is tested using Ljung-Box statistics on their sample autocorrelation, and Brock, Dechert, and Scheinkman's (1996) diagnostic test.[17] Both tests fail to reject the i.i.d. assumption. With respect to the Markov switching process, the number of states is tested using the approach proposed by Garcia (1998), based on Hansen (1993, 1996). The test provides strong evidence for the two-state specification against the assumption of single state. Even though the likelihood ratio test for comparing the Markov switching model with a non-switching model has an unknown sampling distribution (see Hansen 1996), one can evaluate different two-state specifications using standard chi-squared sampling distributions. A likelihood ratio test comparing a model with constant variance and switching mean versus the switching mean and variance model rejects at the 1% level the hypothesis that the additional parameter for the state dependent variance is zero. Finally, the model was estimated allowing either AR(1) or AR(0) processes for the residuals. The likelihood ratio test favors the AR(1) specification at the 1% level.

4.2 Results

Table 2 shows the maximum likelihood estimates of the Markov switching dynamic factor model using the five variables from group 1 for the period from 1975:01-2000:06. The Markov states for the factor are statistically significant. State 1 has a positive mean growth rate, a smaller volatility, and a higher transition probability, while in state 0 the factor has a negative mean growth rate, high variance, and a smaller transition probability. The positive state is associated with the longer and calmer expansion phases of the business cycles. The negative state reflects the more volatile and shorter economic contractions in Brazil. These

[16] The magnitude of the n eigenvalues for each factor reflects how much of the correlation among the observable variables is explained by $k \leq n$ potential factors. For each of the three composite indicators, there is only one eigenvalue greater than one, while the others are much smaller and close to zero.

[17] Leads of 2, 3, 4, 5, and 6 months are used for the residuals and the distance between the two vectors of residuals is set to be equal to their standard deviation.

asymmetries in the phases of business cycles are also found in the U.S., Australia, and several other OECD countries (see Chauvet and Yu 2000).

The factor loadings measure how changes in the dynamic factor affect changes in the observable variables. Industrial production, real wages, and compensated hours are the most sensitive variables to business cycles as measured by the GDP indicator, while employment is relatively less so. That is, some variables are more flexible in adjusting to cyclical economic variation than others. For example, firms facing the prospective of an economic contraction may opt first in reducing the number of compensated hours than actually firing workers.

Table 2
Maximum Likelihood Estimates - Monthly Data: 1975:02-2000:06

Parameters		Parameters		Parameters	
μ_1	1.23	λ_{capac}	0.36	d_{employ}	0.77
	(0.51)		(0.05)		(0.04)
μ_0	-0.79	λ_{wages}	0.57	d_{prod}	-0.38
	(0.44)		(0.10)		(0.05)
σ_1^2	1.73	λ_{hours}	0.76	σ^2_{capac}	2.90
	(0.39)		(0.06)		(0.23)
σ_0^2	9.74	λ_{employ}	0.10	σ^2_{wages}	10.15
	(2.70)		(0.01)		(0.82)
p_{11}	0.93	d_{capac}	-0.20	σ^2_{hours}	0.01
	(0.04)		(0.06)		(0.004)
p_{00}	0.83	d_{wages}	0.10	σ^2_{employ}	0.19
	(0.10)		(0.06)		(0.11)
ϕ	-0.20	d_{hours}	0.98	σ^2_{prod}	6.86
	(0.06)		(0.02)		(0.56)
LogL(θ)	-2526.07				

Asymptotic standard errors in parentheses. The factor loading for production is set to one to normalize the factor.

Probabilities

The variables that enter the dynamic factor model are available in a timely basis. However, they are very volatile, which makes it difficult to use them to discern between broad cyclical movements in the economic activity from individual noise in the different sectors these variables represent. That is, individually, these series give very mixed signals

about the state of the economy. This can be seen from Figure 2, which shows the filtered probabilities of recession obtained from fitting a Markov switching model to each individual component of the dynamic factor model. The probabilities display several extra peaks and troughs that do not correspond to an overall contraction of the Brazilian economic activity. It is particularly interesting to notice that, in the case of Brazil, monthly industrial production by itself is a poor measurement of business cycles – in several periods industrial production decreased or increased substantially while the other sectors of the economy (as well as total GDP) did not show a corresponding movement.

The dynamic factor model combines these variables and extracts the cyclical variation that is common to all of them. In the process of extracting the signal, the noise is isolated into the idiosyncratic terms, ε_t, and the resulting index is a smoother variable that summarizes the common correlation underlying them.

In contrast to the probabilities of the individual series, the probabilities of recession obtained from the dynamic factor capture closely business cycle expansions and contractions, as measured by GDP. Figure 3 plots both the smoothed and filtered probabilities of recessions. The filtered probabilities give at time t the probability of a recession using only data available at t. Thus, these probabilities can be used as a monitoring tool to determine the state of the economy in real time. For example, as a response to the Russian crisis in July and August 1998, the model yields a 54% filtered probability of recession in August 1998, using only information available in August, and a 58% probability in September 1998. On the other hand, the smoothed probabilities indicate that the economy had entered a recession already in July 1998, which lasted until December 1998. Although it is more evident looking backward that the Brazilian economy experienced a recession following the Russian crisis, this assessment was not so clear at the time this event was happening.

Figure 2 – Filtered Probabilities of Recessions Obtained from Fitting a Univariate AR(1) Markov Switching Model to the Factor Components, and Dating of the Brazilian Business Cycle (Shaded Area)

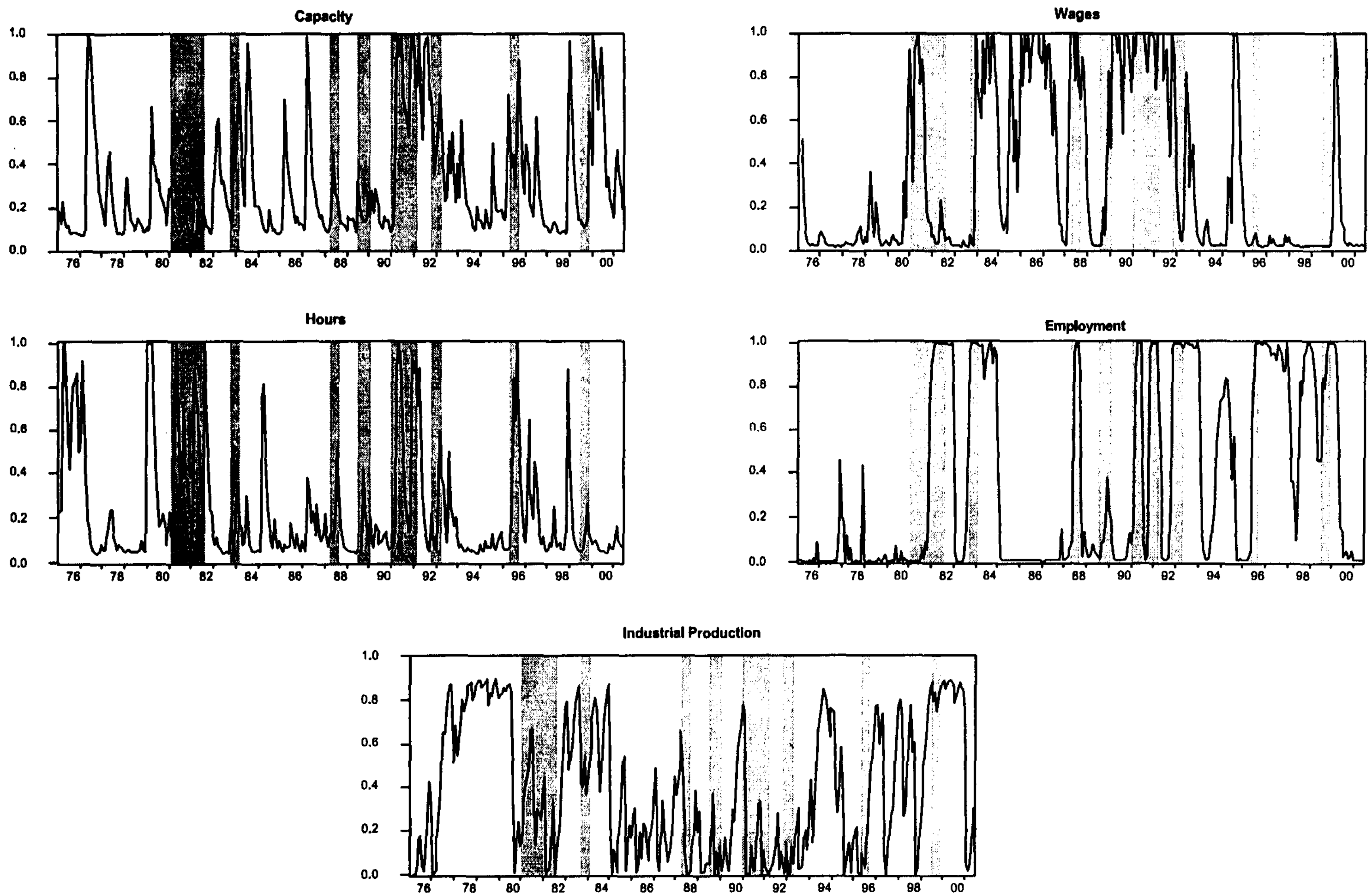


Figure 3 – Smoothed (—) and Filtered (---) Probabilities of Recessions and Dating of the Brazilian Business Cycle (Shaded Area)

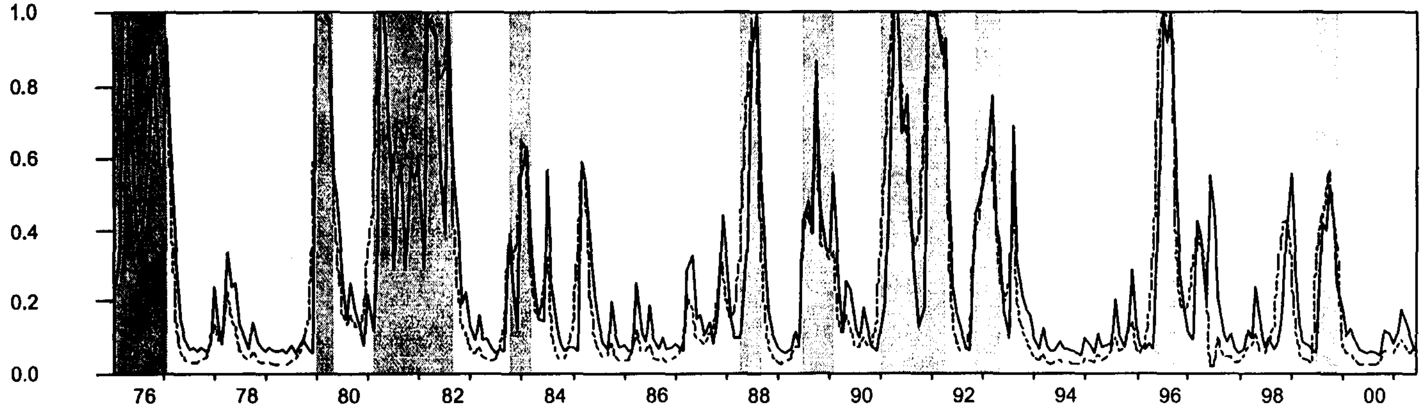


Figure 4 – Smoothed Probabilities of Recessions from GDP (---), and from the Dynamic Factor Model (—), at Monthly Frequency

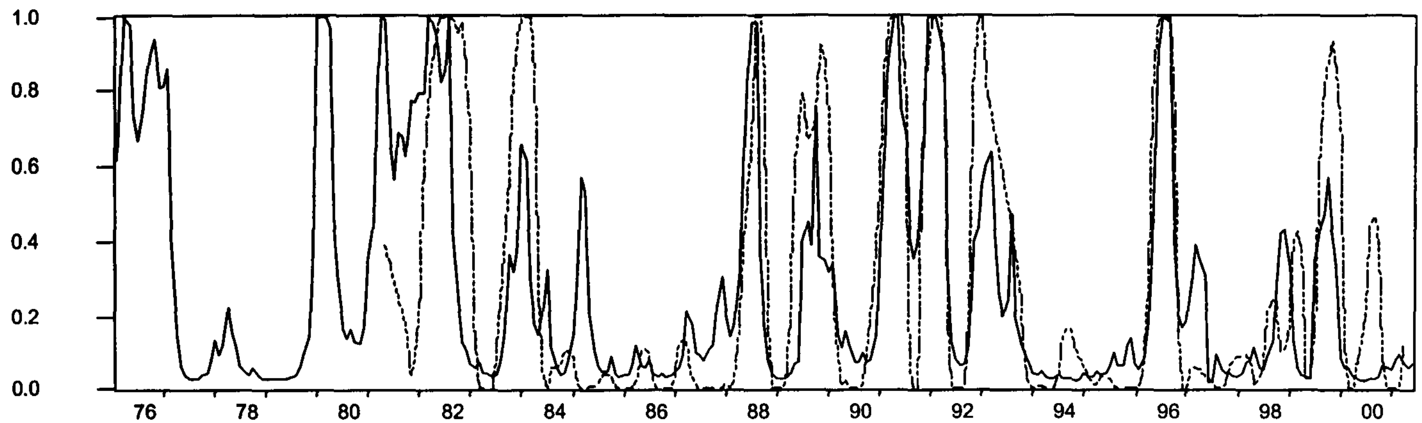


Figure 5 – Dating of the Brazilian Business Cycle for Quarterly (—) and Monthly Frequencies (Shaded Area)

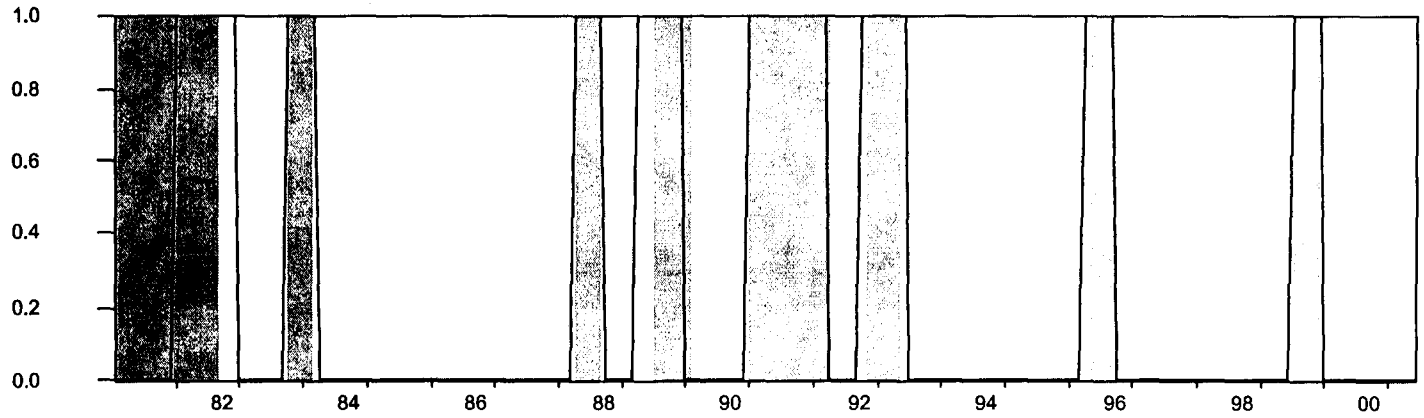


Figure 6 – Smoothed Probabilities of Recessions and Dating of the Brazilian Business Cycle (Shaded Area)

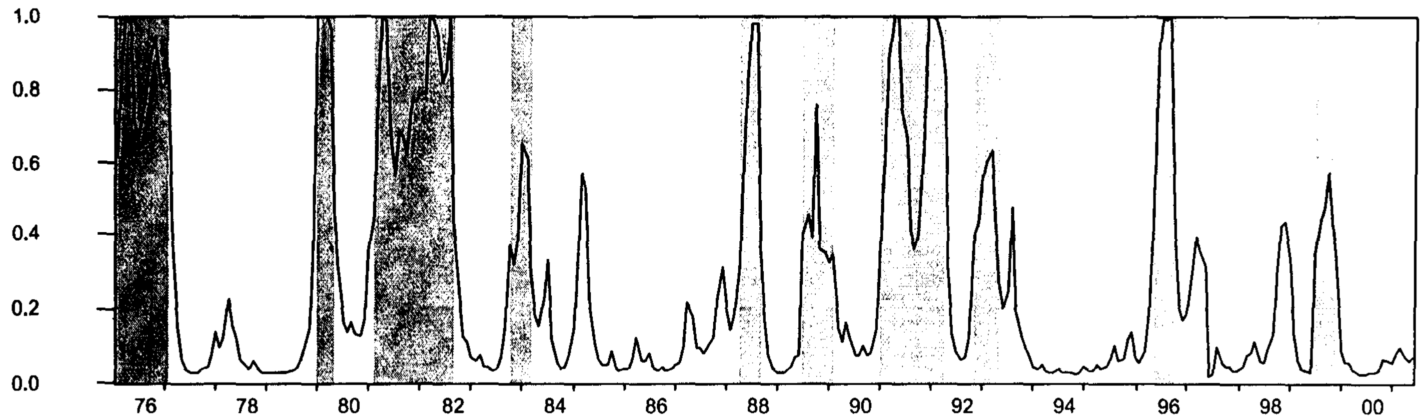


Figure 4 plots the smoothed probabilities of recessions from the monthly series combined into the dynamic factor, and the smoothed probabilities of recessions for quarterly GDP, obtained from Chauvet (2000).[18] The probabilities are very similar. In particular, all recessions are captured by both models, and the timing and duration of the business cycle phases are very close. One difference is the first recession of the quarterly sample, in 1980-82. Since the sample for quarterly GDP starts only in 1980:01, it captures less information about this recession period than the dynamic factor model, whose sample begins in 1975:01.

The smoothed probabilities can be used as a metric to identify historical business cycle turning points in Brazil. One rule is to consider that the economy is in a recession if the smoothed probabilities of recession are above 50%. Alternatively, the frequency distribution of the probabilities can also be used to define a turning point – a rule of thumb to call a peak is when the probabilities of recessions are greater than their mean plus one-half their standard deviation. Figure 5 shows turning points of the monthly dynamic factor and business cycle dating from GDP, obtained from Chauvet (2000). Either of these rules results in very similar dating of expansions and recessions for both models.[19]

Figure 6 shows the smoothed probabilities of recessions and the dating of the Brazilian business cycle at the monthly frequency (shaded area). In the last 25 years Brazil experienced ten recessions and ten expansions. Recessions are generally shorter than expansions. In the sample studied, there were five recessions lasting only six months while the longest recession occurred between 1980 and 1982, lasting 20 months. The longest expansion took place between 1983 and 1987, while the shorter one (6 months) occurred in between the recession induced by the Collor's Plan in 1990-1991 and the subsequent recession in the end of 1991 and beginning of 1992.

Monthly Indicator of GDP

The resulting dynamic factor extracted from the model, $F_{t|t}$, is the monthly indicator of Brazilian GDP. Table 3 presents some statistics for the indicator and its components. All

[18] The quarterly smoothed probabilities were converted to monthly frequency using the quadratic-match average method.

[19] In order to rule out short-term events, such as strikes, tax law changes, etc. from a broad recession in the economy, one of the NBER rules is that a recession corresponds to a general downturn in the economy of at least six months. Here we also consider the minimum duration of business cycle phases to be six months.

components are pro-cyclical, hence displaying a positive correlation with the indicator. The growth rate of the coincident indicator, $\Delta F_{t|t}$, is highly correlated with ΔHours (0.87), followed by ΔProduction (0.61). The other components ΔCapacity and ΔEmployment have a balanced contribution to the indicator, with a correlation around 52%. The least correlated is ΔWages (0.47).

Table 3 - Statistics for the Monthly Indicator and its Components

Statistics	Correlation with $\Delta F_{t\|t}$	Sample Mean	Standard Deviation
$F_{t\|t}$ in level	-	102.062	14.088
$\Delta F_{t\|t\ quarterly}$ (*)	1	0.104	2.041
$\Delta F_{t\|t\ monthly}$	-	0.027	1.070
ΔCapacity	0.522	-0.003	2.243
ΔHours	0.871	-0.127	1.588
ΔWages	0.474	-0.077	2.495
ΔEmployment	0.512	-0.099	0.676
ΔProduction	0.611	0.176	3.998
$\Delta GDP_{quarterly}$	0.705	0.493	2.279
GDP	0.973 (**)	101.81	14.221

(*) Here $\Delta F_{t|t}$ was converted to quarterly frequency for comparison with ΔGDP.
(**) This is the correlation of GDP with the coincident indicator in level, $F_{t|t}$.

In order to compare the results with GDP, two alternative procedures were undertaken. First, monthly variables were converted to quarterly frequency using simple average. Second, GDP was converted to monthly frequency using the local quadratic interpolation method with average matched to the observed data. The growth rate of the indicator displays a 70% correlation with GDP at the quarterly frequency.

Figure 7 plots the growth rate of the dynamic factor indicator and the growth rate of GDP at the monthly frequency. The shaded areas represent recession phases. The estimated indicator is strongly related to movements in GDP. In particular, the volatility of these two series is very close, as well as the timing of changes and amplitude of the oscillations. One exception was the abrupt change in the economy during the Collor Plan. The monthly indicator matches the steep drop in the economic activity upon the introduction of the Plan in the second quarter of 1990, when GDP decreased at a quarterly average rate of –6.7%. The indicator, however, did not go up as much as GDP did in the third quarter

(6.8%). An analysis of the components of the dynamic factor explains this. Neither employment, wages nor hours displayed this strong upswing after the first impact of the Collor plan. In fact, only industrial production rebounded back. Thus, the indicator reflects more accurately the negative impact of the plan in the Brazilian labor market. The series on GDP per sector, obtained from IBGE, corroborates this finding – the responses of GDP from the service and agricultural sectors to the Collor Plan were much smoother than the one from industrial GDP.

Figures 8 and 9 show the estimated indicator in level at the monthly and quarterly frequency, respectively. The indicator in level was obtained directly from the Kalman filter using the identity $F_{t-1} = \Delta F_{t-1} + F_{t-2}$ in the filter.[20] The indicator follows closely cyclical movements in GDP. In particular, the timing of business cycle peaks and troughs coincides for all recessions in the sample. Conformity with the reference cycle in terms of the turning points is one of the most important contributions of the indicator, since it can be used as a monitoring tool to assess the state of the business cycle.

[20] For graphical comparison, the factor is adjusted to have the same trend as GDP.

Figure 7 – Monthly Indicator of Brazilian GDP (—), Growth Rate of Real GDP (---), and Recession Phases (Shaded Area)

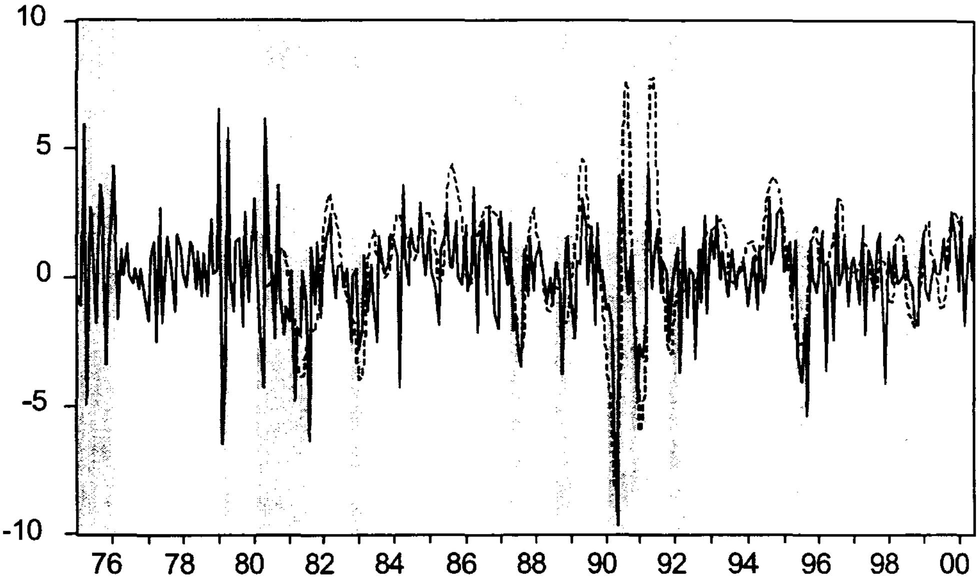


Figure 8 – Indicator of Brazilian GDP in Level (—), Real GDP (---), and Recession Phases (Shaded Area) at Monthly Frequency

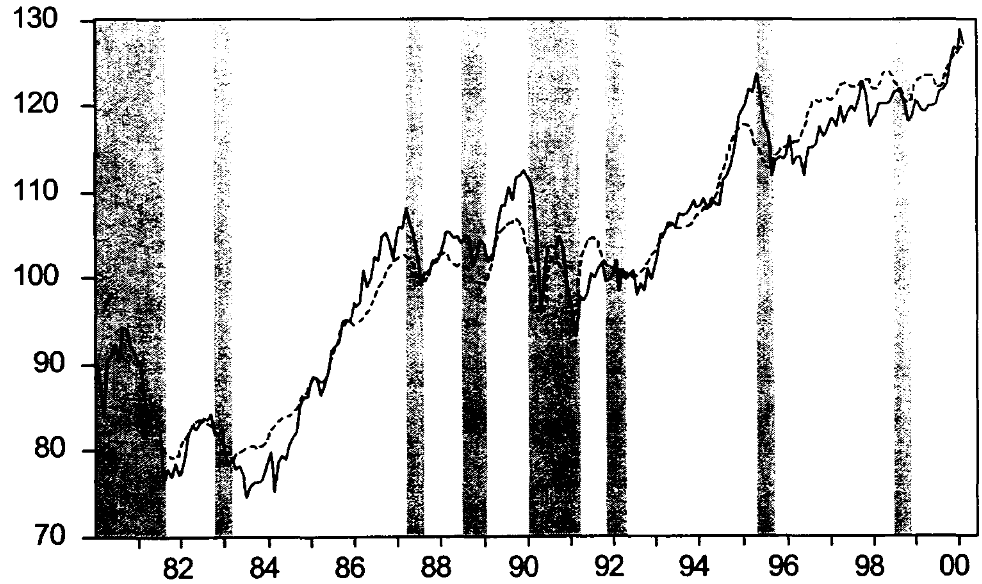


Figure 9 – Indicator of Brazilian GDP (—), Real GDP (—), and Recession Phases (Shaded Area) at Quarterly Frequency

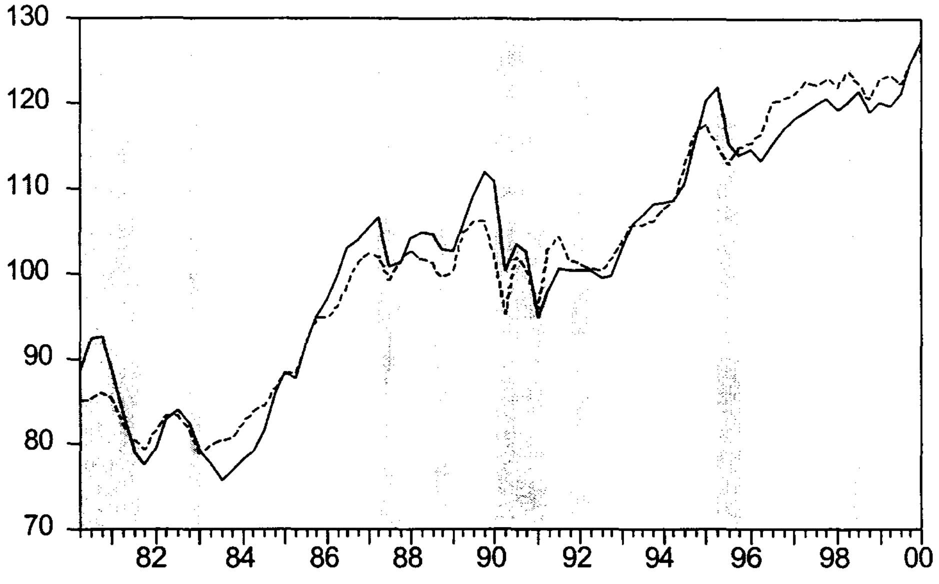
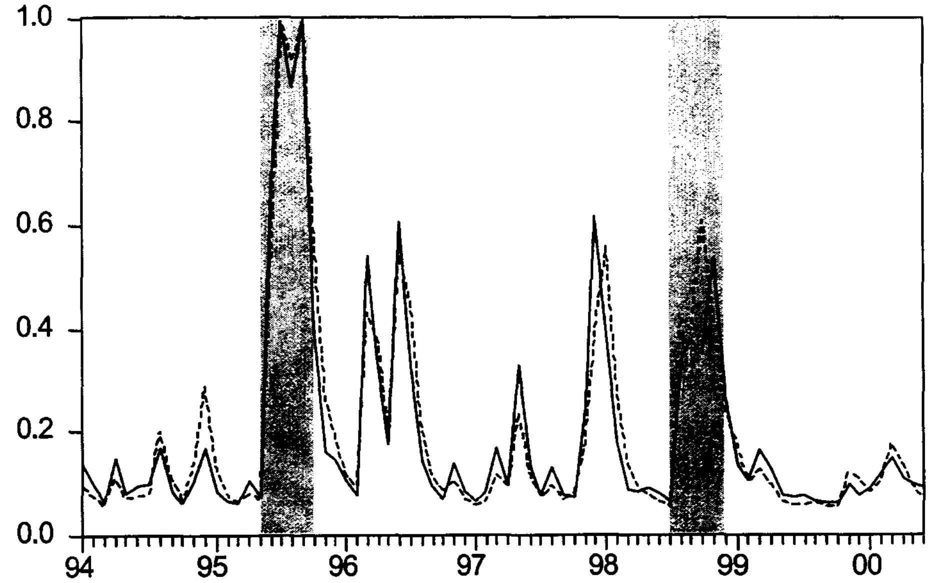


Figure 10 – Out-of-Sample (—) and In-Sample (—) Filtered Probabilities of Recession, and Recession Phases (Shaded Area)



4.3 Out-of-Sample Analysis

The in-sample analysis shows a remarkable historical conformity of the indicator with cyclical movements of GDP. In this section, the conformity of the indicator as well as its linear forecasting ability are verified out-of-sample.

In this exercise, the model is estimated from 1975:02 to 1993:12. The model is then recursively re-estimated for each month from 1994:01 to 2000:06 and out-of-sample recursive forecasts of the filtered coincident indicator and probabilities are computed. This tests the ability of the indicator to predict GDP business cycle phases out-of-sample in real time, and allows us to reproduce the information content that was available to forecasters at any point in time.

Probabilities

Figure 10 compares the in-sample probabilities recessions obtained from estimating the model for the entire sample with the out-of-sample filtered probabilities of recession. Both probabilities are very similar. In particular, the out-of-sample probabilities predict the two recessions in the period, in 1995 and in 1998. The out-of-sample probabilities, however, exhibit more pronounced spikes during the short-lived contractions that occurred in mid-1996 and in the end of 1997. Although these events are not considered recessions given their mildness and short length (only three months), they illustrate the harder task of discerning in real time false peaks from signals of more severe upcoming recessions.

Linear Forecasts

Composite indicators are generally used to predict expansion and recessions phases. The indicators, however, have also been shown to display good linear predictive power to forecast output as well.[21] This section explores the linear forecasting ability of the monthly indicator. Two linear models are used to evaluate the performance of the indicator ($\Delta F_{t|t}$) in predicting growth rates of GDP (ΔGDP).[22] **Model A** is a regression of ΔGDP on six lags of itself, and **Model B** is a vector autoregression of ΔGDP and $\Delta F_{t|t}$ at six lags.[23] The

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[21] See Diebold and Rudebusch (1989, 1991), Hamilton and Perez-Quiros (1996), etc.

[22] Since the two series have different frequencies, the indicator was converted as quarterly using simple average.

[23] Lags were selected using Akaike Information Criteria (AIC) and Schwarz Information Criteria (SIC).

comparison of the two models is a test of the marginal predictive content of the indicator in predicting ΔGDP beyond lags of ΔGDP.

First, both models were estimated for the full sample available for ΔGDP, from 1980:2 to 2000:1, and one-step-ahead forecasts were computed in-sample. Second, both models were estimated from 1980:2 up to 1993:4, and the estimates were then used to calculate recursive one-step-ahead forecasts out-of-sample, from 1994:1 to 2000:1. out-of-sample results are compared to forecasts using the full sample.

Table 4 reports the predictive performance of the models. The model including lags of the composite indicator (Model B) displays a better forecasting performance than using only lags of GDP. The in-sample adjusted coefficient of determination ($\bar{R}^2$) for Model B is 61%, while for Model A it is only 15%. In addition, Model B has a smaller Root Mean Squared Error (RMS), Mean Absolute Error (MAE) and Theil Inequality Coefficient (Theil IC) compared to Model A in-sample. In particular, the Theil Inequality Coefficient is substantially smaller for Model B (0.296). Although both models do well in forecasting the mean of ΔGDP (bias proportion close to zero), Model B has a superior ability in forecasting the variance of ΔGDP, with a variance proportion 3.6 times smaller than Model A's.[24]

Table 4- In-Sample and Out-of-Sample Forecasting Performance of $\Delta F_{t|t}$

Forecasting Performance	In-Sample: 1980.2-2000.1		In-Sample: 1980.2-1993.4 Out-of-Sample: 1994.1 2000.1	
	Model A	Model B	Model A	Model B
$\bar{R}^2$	0.149	0.612	0.169	0.699
RMS	1.981	1.281	1.599	1.436
MAE	1.503	0.998	1.244	1.124
Theil IC	0.548	0.296	0.550	0.424
Bias	0.000	0.000	0.006	0.014
Variance	0.345	0.096	0.317	0.096
Covariance	0.655	0.904	0.677	0.889

Model A: Regression of ΔGDP on 6 lags of ΔGDP.
Model B: VAR(6) of ΔGDP and $\Delta F_{t|t}$.

[24] The covariance proportion of the Theil IC is obtained by residual as the three components add up to one.

Figures 11 and 12 plot in-sample one-step-ahead forecasts, forecast intervals,[25] and actual changes in GDP from Models A and B, respectively. A comparison of the figures illustrates how much closer forecasts from Model B track variation in actual ΔGDP. The one-step-ahead forecast from Model A is much smoother than actual ΔGDP and does not mimic as well its oscillations. In fact, focusing on the recessions phases as represented by the shaded areas, i is noticeable how forecasts from Model B turn down together with ΔGDP around peaks and troughs.

These findings also hold for out-of-sample forecasting. The inclusion of lags of the indicator in the GDP equation improves substantially the model forecasting performance. Again, the RMS, the MAE and the Theil IC are all smaller for Model B compared to Model A (Table 4). Figures 13 and 14 show out-of-sample one-step-ahead forecasts, forecast intervals, and actual changes in GDP from Models A and B, respectively. As within sample, the most striking difference between the two models is how forecasts from Model B (Figure 14) match much more closely the volatility of changes in GDP (smaller variance proportion). This feature is also observed around ΔGDP turning points (shaded area).

Adding up, Model B displays a better forecasting performance for ΔGDP than Model A, both in-sample and out-of-sample. The indicator, which is a parsimonious representation of information in several variables, is useful for both assessing the state of the business cycle as well as to form linear forecast of ΔGDP.

25 Forecast intervals are plus or minus twice the forecast standard errors. The standard error bands give an approximate 95% forecast interval.

Figure 11 – In-Sample One-Step-Ahead Forecasts of ΔGDP from Model A, Actual ΔGDP, and Recession Phases (Shaded Area)

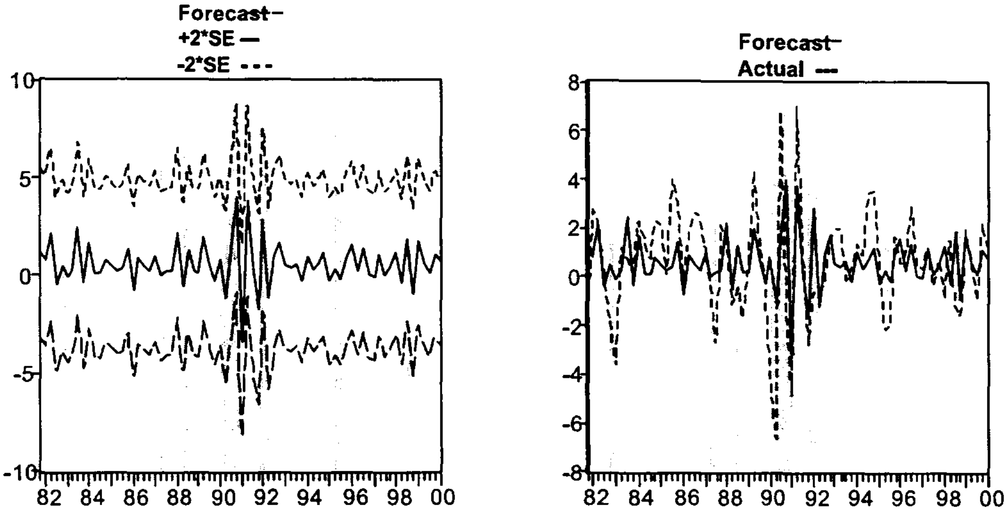


Figure 12 – In-Sample One-Step-Ahead Forecasts of ΔGDP from Model B, Actual ΔGDP, and Recession Phases (Shaded Area)

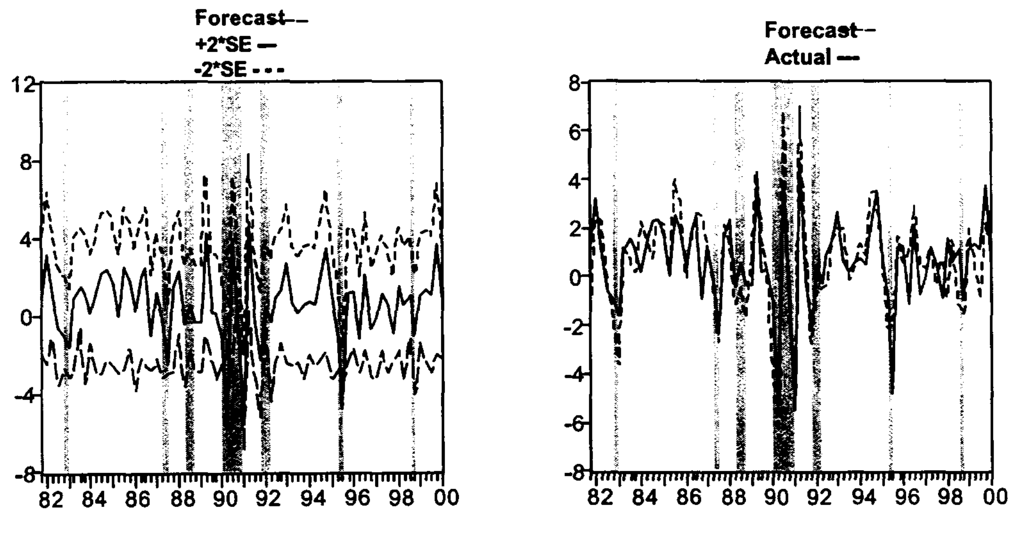
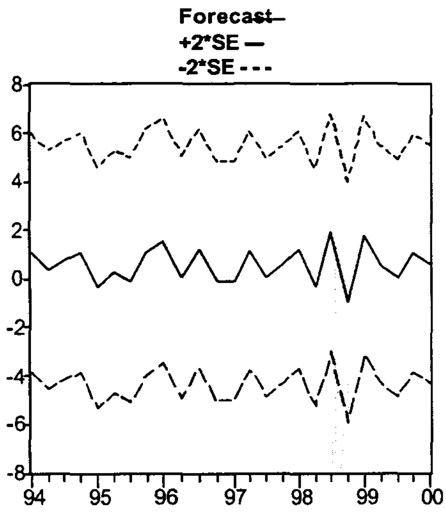


Figure 13 – Out-of-Sample One-Step-Ahead Forecasts of ΔGDP from Model A, Actual ΔGDP, and Recession Phases (Shaded Area)



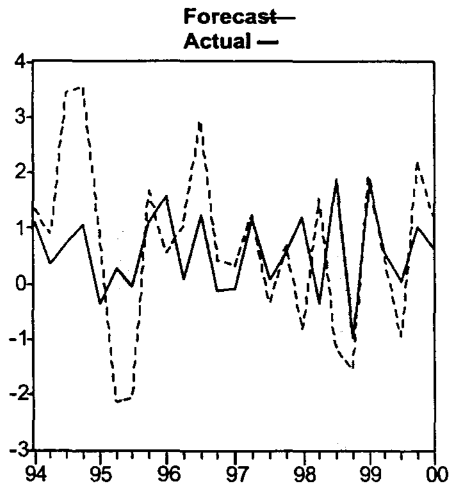
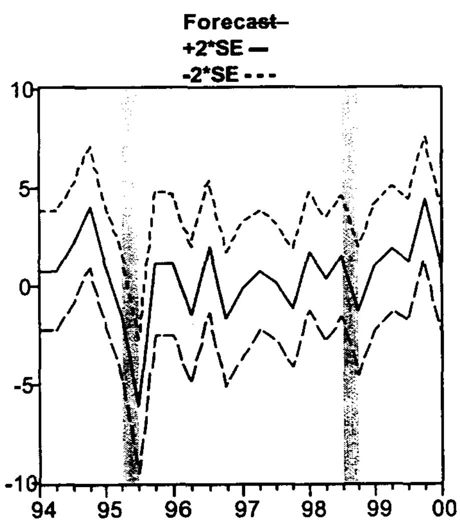
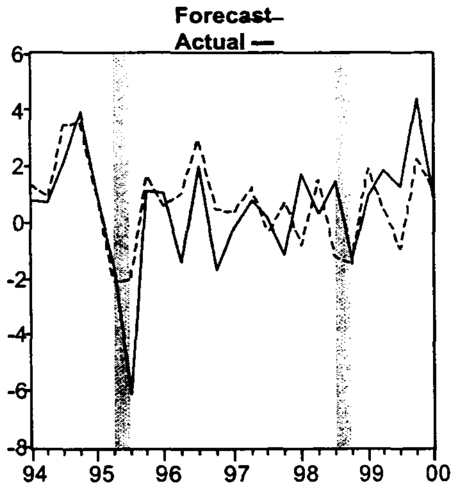


Figure 14 – Out-of-Sample One-Step-Ahead Forecasts of ΔGDP from Model B, Actual ΔGDP, and Recession Phases (Shaded Area)





5. Conclusions

This paper constructs an indicator of Brazilian GDP at the monthly frequency that can be used as a monitoring tool for real time assessment of the current state of the economy. Special attention was given to the peculiar dynamical behavior of the Brazilian business cycle. In particular, the instability and abrupt changes of regimes in the history of Brazilian economic activity were explicitly modeled within nonlinear frameworks. The indicator is the output of a Markov switching dynamic factor model, which combines several macroeconomic variables that display simultaneous comovements with aggregate economic activity. The model generates as outputs a monthly indicator of Brazilian GDP and real time probabilities of the current phase of the Brazilian business cycle.

The formal representation of the indicator through the Markov switching dynamic factor model overcomes the drawbacks of the NBER traditional method. First, the model takes into account the dynamic relationship among the variables and generates an indicator that summarizes common cyclical movements underlying several sectors of the economy. Second, the model yields probabilities that can be used to determine turning points in the indicators in a current basis, and the model can be evaluated in real time.

The monthly indicator of Brazilian GDP shows a remarkable historical conformity with cyclical movements of GDP with respect to volatility, timing of peaks and troughs, and duration of the phases. In addition, the estimated filtered probabilities predict all recessions in-sample and out-of-sample. The indicator, which is a parsimonious representation of information in several variables, can also be used to form linear forecast of GDP. A linear autoregressive model including lags of both the indicator and of the growth rate of GDP displays a better in-sample and out-of-sample forecasting performance than a univariate autoregressive model of GDP. In particular, the inclusion of lags of the indicator improves substantially forecasts of the severity of recessions and strength of expansions, as measured by the volatility of changes in GDP. This feature is particularly accentuated around GDP turning points.

The results suggest that the estimated indicator could be useful in assessing the state of the Brazilian business cycle, through the filtered probabilities, and in forecasting GDP, through the filtered indicator, in real time on a monthly basis.

References

Boldin, M., 1998/1999, "A Critique of the Traditional Composite Index Methodology," *Journal of Economic and Social Measurement*, Vol. 25, No. 3 & 4, 1998/1999, 119-140.

Burns, A. and W. Mitchell, 1946, *Measuring Business Cycles*, (New York: National Bureau of Economic Research).

Brock, W., W. Dechert, B. LeBaron, and J. Scheinkman, 1996, "A Test for Independence Based on Correlation Dimension," *Econometric Reviews*, Vol. 15, No. 3.

Chauvet, M., 2000, "The Brazilian Business Cycle and Growth Cycle," Working Paper, University of California, Riverside.

Chauvet, M., 1998/1999, "Stock Market Fluctuations and the Business Cycle," *Journal of Economic and Social Measurement*, Vol. 25, No. 3 & 4, 235-257.

Chauvet, M., 1998, "An Econometric Characterization of Business Cycle Dynamics with Factor Structure and Regime Switches," *International Economic Review*, Vol. 39, No. 4, 969-96.

Chauvet, M., and C. Yu, 2000, "International Business Cycles in the G-7 Countries," mimeo, University of California, Riverside.

Dickey, D. A., and W. A. Fuller, 1979, "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Society* 74, 427-31.

Diebold, F. X. and G. D. Rudebusch, 1989, "Scoring the Leading Indicators," *Journal of Business*, 62, 369-391.

Diebold, F. X. and G. D. Rudebusch, 1991, "Forecasting Output with the Composite Leading Index: A Real-Time Analysis," *Journal of the American Statistical Association*, 86, 603-610.

Diebold, F. X. and G. D. Rudebusch, 1996, "Measuring Business Cycles: A Modern Perspective," *The Review of Economics and Statistics* 78, 67-77.

Emerson, R. and D. Hendry, 1996, "An Evaluation of Forecasting Using Leading Indicators," *Journal of Forecasting*, 15, 271-296.

Garcia, R., 1998, "Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models," *International Economic Review*, Vol. 39, No. 3, 763-788.

Hamilton, J., 1989, "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica*, 57, 357-384.

Hamilton, J. and G. Perez-Quiros, 1996, "What Do the Leading Indicators Lead?" *Journal of Business* 69, 27-49.

Hansen, B. E., 1993, "The Likelihood Ratio Test Under Non-Standard Conditions: Testing the Markov Trend Model of GNP," in M.H. Pesaran and S. Potter, *Nonlinear Dynamics Chaos and Econometrics*, (John Wiley & Sons), 53-73.

Hansen, B. E., 1996, "Inference When a Nuisance Parameter is not Identified Under the Null Hypothesis," *Econometrica*, 64, 413-430.

Harrison, P. J. and C. F. Stevens, 1976, "Bayesian Forecasting," *Journal of the Royal Statistic Society* Series B, 38, 205-247.

Johansen, S., 1991, "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models," *Econometrica*, 59, 1551–1580.

Johansen, S., 1995, *"Likelihood-based Inference in Cointegrated Vector Autoregressive Models,"* Oxford University Press.

Kim, C. J., 1994, "Dynamic Linear Models with Markov-Switching," *Journal of Econometrics*, 60, 1-22.

Koopmans, T., 1947, "Measurement without Theory," *The Review of Economics and Statistics*, 29, 161-172.

Lahiri, K. and J.G. Wang, 1994, "Predicting Cyclical Turning Points with Leading Index in a Markov Switching Model," *Journal of Forecasting*, vol. 13, pp.245-263.

Perron, P., 1989, "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis," *Econometrica*, 57, 1361-1401.

APPENDIX

Estimation Procedure

The model is estimated using a combination of Hamilton's (1989) Markov switching algorithm and the Kalman filter, as suggested by Kim (1994). The model is cast in state space form, where (1) and (2) are the measurement and transition equations, respectively:

(1) $\Delta Y_t = Z\xi_t$

(2) $\xi_t = \alpha_{\xi_{st}} + T\,\xi_t + u_t,$

The state space representation for the estimated indicator using variables in group 1 is:

$$\Delta Y_t = \begin{vmatrix} \Delta Y_{1t} \\ \Delta Y_{2t} \\ \Delta Y_{3t} \\ \Delta Y_{4t} \\ \Delta Y_{5t} \end{vmatrix}, \quad Z = \begin{vmatrix} \lambda_1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & 0 & 1 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & 0 & 1 & 0 & 0 \\ \lambda_4 & 0 & 0 & 0 & 0 & 1 & 0 \\ \lambda_5 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}, \quad \xi_t = \begin{vmatrix} \Delta F_t \\ \Delta v_{1t} \\ \Delta v_{2t} \\ \Delta v_{3t} \\ \Delta v_{4t} \\ \Delta v_{5t} \\ F_{t-1} \end{vmatrix} \quad \alpha_{\xi_{st}} = \begin{vmatrix} \alpha_1 S_t + \alpha_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix},$$

$$T = \begin{vmatrix} \phi_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & d_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & d_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & d_5 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}, \quad \text{and} \quad u_t = \begin{vmatrix} \Delta\eta_{st} \\ 0 \\ \Delta\varepsilon_{1t} \\ \Delta\varepsilon_{2t} \\ \Delta\varepsilon_{3t} \\ \Delta\varepsilon_{4t} \\ \Delta\varepsilon_{5t} \\ 0 \end{vmatrix}.$$

The term F_{t-1} is included in the state vector to allow estimation of the dynamic factor in levels from the identity $\Delta F_{t-1} = F_{t-1} - F_{t-2}$.

The nonlinear filter forms forecasts of the unobserved dynamic factor, $F^{(i,j)}_{t|t-1}$, and the associated mean squared error matrices, $\theta^{(i,j)}_{t|t-1}$, based on information available up to time t-1, $I_{t-1} \equiv [\Delta Y'_{t-1}, \Delta Y'_{t-2}, \ldots, \Delta Y_1']'$, on the Markov state S_t taking on the value j, and on S_{t-1} taking on the value i, for i, j = 0, 1:

$$F^{(i,j)}_{t|t-1} = E(F_t \mid I_{t-1}, S_t = j, S_{t-1} = i)$$

$$\theta^{(i,j)}_{t|t-1} = E[(F_t - F_{t|t-1})(F_t - F_{t|t-1})' \mid I_{t-1}, S_t = j, S_{t-1} = i)].$$

The filter uses as inputs the joint probability of the Markov-switching states at time t-2 and t-

1 conditional on information up to t-1, $\{\Pr(S_{t-2}=h, S_{t-1}=i|I_{t-1})\}$; an inference about the state vector using information up to t-1, given S_{t-2}=h and S_{t-1}=i, that is, $\{F^{i,j}_{t-1|t-1}\}$; and the mean squared error matrices, $\{\theta^{i,j}_{t-1|t-1}\}$. The outputs are their one-step updated values.

The probability terms are computed using Hamilton's filter. As in the linear Kalman filter, the algorithm calculates recursively one-step-ahead predictions and updating equations of the dynamic factor and the mean squared error matrices, given the parameters of the model and starting values for $F^j_{t|t}$, $\theta^j_{t|t}$ and the probabilities of the Markov states. However, for each date t the nonlinear filter computes 2^k forecasts, where k is the number of states, and at each iteration the number of cases is multiplied by k. This implies that the algorithm would be computationally unfeasible even for the simplest cases. Kim, based on Harrison and Stevens (1976), proposes an approximation introduced through $F^j_{t|t}$ and $\theta^j_{t|t}$ for t>1. This approximation consists of truncating the updating equations into averages weighted by the probabilities of the Markov states.

The conditional likelihood of the observable variables can be obtained as a by-product of the algorithm at each t, which is used to estimate the unknown model parameters. The filter evaluates this lkelihood function, which is then maximized with respect to the model parameters using a nonlinear optimization algorithm. The maximum likelihood estimators and the sample data are then used in a final application of the filter to draw inferences about the dynamic factor and probabilities, based on information available at time t. The final estimated dynamic factor is calculated as:

$$F_{t|t} = \sum_{i=0}^{1} \Pr[S_t = j \mid I_t] F^j_{t|t}.$$

THE BRAZILIAN BUSINESS CYCLE

AND GROWTH CYCLE

MARCELLE CHAUVET[ϒ]

First Draft: July 2000
This Draft: September 2000

Abstract

This paper uses a Markov switching model to date and analyze the Brazilian business cycle and growth cycle. The model is fitted to quarterly and annual real production data. The smoothed probabilities of the Markov states are used as predictive rules to define different phases of cyclical fluctuations of real Brazilian production. At the annual frequency, the model depicts different phases of secular growth: there are periods of slowdowns when the economy grows at an annual rate of 1.15%, and phases of accelerated growth, averaging 7.5% per annum in the last century. At the quarterly frequency, the model identifies periods of expansions, when the economy grows at a more moderate rate of 5% per annum, and recession phases, when the economy reaches an average negative growth of 6%. The model captures asymmetries across the different states of business and growth cycles, in which slowdowns and recessions are short and abrupt, while high growth phases and expansions are longer and less steep. The framework also generates filtered probabilities of the Markov states, which can be used for dating cyclical fluctuations in the Brazilian economy on a timely basis.

KEY WORDS: Business Cycle, Growth Cycle, Markov Switching

JEL Classification Code: C32, C50, E32

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Resumo

Este artigo utiliza modelos the mudança de Markov para datar e analisar ciclos de negócios e ciclos de crescimento no Brasil. O modelo é aplicado a dados de produção trimestrais e anuais. As probabilidades suavizadas dos estados Markovianos são utilizadas como regras de previsão para definir as diferentes fases de flutuações cíclicas na produção real brasileira. Para dados anuais, o modelo identifica diferentes fases de crescimento secular: períodos de baixo crescimento, nos quais a economia cresce a uma taxa annual de 1.15%, e períodos de crescimento acelerado, cuja taxa média é de 7.5% por ano no último século. Para dados trimestrais, a economia apresenta períodos de expansões, nos quais cresce a uma taxa mais moderada de 5% ao ano, e fases de recessões, quando a economia atinge uma média de crescimento negativo de 6%. O modelo capta assimetrias nos diferentes estágios dos ciclos de negócios e de crescimento. Os estados de baixo crescimento e recessões são de curta duração e mais abruptos, enquanto que estados de crescimento acelerado e expansões são mais longos e mais graduais. O modelo também gera probabilidades filtradas dos estados de Markov, que podem ser utilizadas para datar flutuações cíclicas na economia brasileira tempestivamente.

PALAVRAS CHAVES: Ciclos de Negócios, Ciclos de Crescimento, Mudanças de Markov.

1. Introduction

Market economies undergo recurrent fluctuations in aggregate activity. People and firms affected by changes in sales, profits, credit, or employment are very concerned about these swings in the economy. On the other hand, policymakers are also attentive to the differential effects of certain policies, depending on the stage of the business cycle. For example, an increase in interest rates may have a different impact depending on whether business is slow or the economy is booming. Hence, there is a great interest in understanding, measuring, monitoring, and forecasting business cycles.

One of the pioneering studies on business cycles is the work of the National Bureau of Economic Analysis (NBER), founded in the 1920s. The NBER's methodology for empirical analysis of business cycle is summarized in the seminal work of Mitchell (1927) and Burns and Mitchell (1946), which established the following definition:

> "Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions and revivals that merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic. In duration business cycles vary from more than a year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own."

Burns and Mitchell examined the U.S. business cycle based on this definition, and then classified macroeconomic variables as leading, coincident, or lagging according to their conformity to this reference cycle. Based on this comprehensive empirical study, the U.S. Department of Commerce started compiling combinations of these variables as composite indexes, which are used to monitor and forecast business cycle turning points.[1] These indicators have become very popular in the U.S., and have inspired the construction and use of similar ones in several other countries. The private sector and policymakers

[1] Turning points refer to the dates of transition between expansion and recession regimes of business cycles. Peaks are the end of expansions and beginning of recessions, while troughs are the beginning of expansions and end of recessions.

worldwide use these tools to form expectations about the current and future state of the business cycle.

The primary step in constructing composite indicators is the existence of a business cycle chronology that can be used as a common reference point for analysis. The NBER Business Cycle Dating Committee has been dating the U.S. expansions and recessions for the last fifty years. Decisions about business cycle turning points are reached from a subjective consensus among the members of the Committee. The analysis is based on cyclical variation of several variables that move together with business cycles, such as manufacturing and trade sales, personal income, industrial production, and non-agricultural employment, among others. The decision of the Committee is generally accepted as the official dating of the U.S. business cycle and it provides economists with a benchmark for analysis of economic activity.

Although careful deliberations are applied to determine turning points, the NBER procedure can not be used to monitor business cycles. The Business Cycle Committee meets months after a turning point has occurred, and a decision is only released when there is no doubt regarding the dating. This can only be achieved by examining a substantial amount of ex-post revised data. Thus, the NBER dating can not be used in real time. In addition, since the results are based on subjective discussions rather than on formal models, they can not be reproduced.

The NBER methods to date expansions and recessions and to construct composite indicators have remained a standard for the study of business cycle for many decades. However, the widespread use of economic indicators and awareness of their shortcomings has increased academic interest in this type of analysis. Analytic models that formalize the construction of indicators, and probabilistic frameworks to define and evaluate turning points forecasts have gained popularity. For example, Neftci (1982) proposes a method to spot turning points by calculating the likelihood that the regime has changed. A turning point probability signal is called when the estimated probability reaches a pre-determined level of confidence. This approach has been refined by Diebold and Rudebusch (1989) and Hamilton (1989). Diebold and Rudebusch use a Bayesian sequential algorithm to produce ex ante probability forecasts of turning points. Hamilton

suggests modeling sudden changes in the behavior of a time series as the outcome of a Markov switching process, which is governed by an endogenous probability rule. Hamilton (1989) uses a Markov switching univariate model and Chauvet (1998) a Markov switching dynamic factor model to obtain dates for the U.S. business cycle. The chronology found is highly correlated with the ex-post NBER dating. That is, these formal analytic models can be used to monitor turning points and evaluate forecasts in real time, overcoming the drawbacks of the NBER dating.

This paper uses a model-based approach to date turning points of the Brazilian business cycle and growth cycle in the last 100 years.[2] In particular, a Markov switching model is fitted to quarterly and annual real measures of production, and the endogenous probabilities are used as predictive rules to determine the different phases of cyclical aggregate fluctuations. The resulting dating of the Brazilian business cycle and growth cycle can be used as a reference point for construction and evaluation of the predictive performance of coincident, leading, or lagging indicators of economic activity.[3]

The results are compared with several ad-hoc rules, and the dating obtained from these procedures is very similar. However, these ad-hoc rules require a substantial amount of ex-post data, which implies that a turning point dating obtained from them would not be available until at least six months after the turning point in the economy. On the other hand, the modeling approach allows analysis of business cycles on a timely basis, and the dating procedure is reproducible.

A timely recognition of an economic contraction and its severity enables a government policy response that could reduce the amplitude of the downturn. In addition, businesses and investors would be able to reassess projected sales or profits based on knowledge of the transition to a new business cycle phase. In fact, several features of

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[2] Growth cycles correspond to cyclical variation in the deviations from long-term trend of real production and exhibit two phases: slowdowns and high growth states. Business cycles, on the other hand, correspond to a general downturn or upturn in various sectors of the economy and displays two distinct phases: recessions and expansions. Recessions can be interpreted as a more severe slowdown, when the economy grows at negative rates, while expansions are periods of moderate growth. According to Burns and Mitchell's definition, slowdowns precede contractions in the economy, high growth phases correspond to the subsequent revivals, and expansions are phases of normal economic growth.

[3] See, for example, Chauvet (2000a).

the aggregate economy may evolve according to economic cycle stages rather than to calendar time. For example, cyclical changes in inflation can be monitored depending on the state of the economy. A current assessment of the business cycle phase can help identify whether inflationary pressures are arising from tight internal demand or supply markets, which can indicate the adequacy and intensity of policy responses.[4]

The Markov process captures switches and asymmetries across different cycle phases underlying the Brazilian real product. In particular, one state displays a low or negative mean and a shorter average duration, which is associated with economic slowdowns and recessions, respectively. The other state exhibits a positive mean and longer average duration, depicting the features of high growth phases and expansions. These asymmetries regarding duration, amplitude, and deepness across business cycle expansions and recessions are also observed in the OECD countries.[5] At the annual frequency, the Markov switching model depicts different phases of secular growth: during slowdowns the economy grows at an annual rate of 1.15%, while during periods of accelerated growth, it averages 7.5% per annum in the last century. At the quarterly frequency, the model identifies periods of expansions, when the economy grows at a more moderate annual rate of 5%, and periods of recessions, when the economy displays an average negative growth of 6% per annum.

The paper is organized as follows. The next section introduces and interprets the model. In the third section, the empirical results are discussed and the resulting Brazilian growth cycle and business cycle dating is analyzed. The fourth section concludes.

2. The Model

In order to capture cyclical variation in the Brazilian economy, a Markov switching model is fitted to production data:

(1) $$y_t - \mu_{s_t} = \phi(y_{t-1} - \mu_{s_{t-1}}) + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \Omega), \qquad s_t = 0,1,$$

[4] An application of stage analysis is in Chauvet (2000b), which builds leading indicators of inflation for Brazil. One of he methods used in this paper to determine the source of inflationary pressures is examination of the state of the Brazilian business cycle.

[5] See Chauvet and Yu (2000).

where y_t is the log first difference of real production, and s_t is an unobservable first-order two-state Markov chain. μ_{s_t} is the state-dependent mean, which takes the value of μ_0 when the economy is in a low growth phase or in a recession ($s_t = 0$) and μ_1 when the economy is in a high growth state or in an expansion ($s_t = 1$).

The switches between states are governed by the transition probability matrix P with elements $p_{ij} = pr[s_t = j| s_{t-1} = i]$, where i denotes the i^{th} column and j the j^{th} row. Each column of P sums to one, so that $1_2'P = 1_2'$, where 1_2 is a column vector of ones. The transition probability matrix is ergodic and irreducible, and unless each column of the transition matrix is equal to the egordic probabilities π, with $P\pi=\pi$, the Markov chain is serially correlated. The first-order assumption of the Markov chain implies that all relevant information for predicting future states is included in the current state, i.e., $pr[s_{t+1}|I_t, s_t, s_{t-1}, \ldots] = pr[s_{t+1}|s_t]$.

This framework is used to classify business and growth cycle turning points and to capture potential asymmetric behavior across business cycle phases, whose effect could be averaged out in a linear analysis of the whole sample data. That is, within this framework, expansions or high growth phases and recessions or slowdowns can display different duration, amplitude, and steepness.

The model is estimated using a combination of the Dempster, Laird and Rubin (1977) Expectation Maximization (EM) with Hamilton's filter. At each iteration, the algorithm estimates the probabilities of the unobserved states (expectation step), and an estimate of the model parameters is obtained as the solution to the first-order conditions of the likelihood function (maximization step). These parameters are then used to update the filtered probabilities of the states. The estimation procedure and derivation of the likelihood function are described in Hamilton (1989, 1994). The filter yields as output optimal inferences about the probabilities of the Markov states, which are used to date growth cycle and business cycle turning points.

3. Empirical Results

3.1 Data and Specification Tests

The empirical analysis of the Brazilian economic cycles was implemented at both annual and quarterly frequencies. The data were obtained from the Getúlio Vargas Foundation (FGV) and CEPAL database. At the annual frequency, the variable used was real product index from 1900 to 1999. At quarterly frequency, the series used was real GDP from IBGE, from 1980.01 to 2000:01.[6] Both series are compiled by IBGE (Brazilian Institute of Economic Geography).

Augmented Dickey-Fuller (1979) test for unit roots and Perron (1989) test for unit roots in the presence of structural breaks were implemented and they do not reject the null hypothesis of integration against the alternative of stationarity at any significance level. Thus, both series were transformed to achieve stationarity using one hundred times their log first difference.

Tests for the number of states in Markov switching models require non-standard procedures since several of the classical assumptions of asymptotic distribution theory do not hold. The number of states is tested using the approach proposed by Garcia (1998), based on Hansen (1993).[7] The test provides strong evidence for the two-state model.

Several Markov specifications with different autoregressive processes were estimated for the Brazilian GDP.[8] The best specification for both annual and quarterly data is the AR(0) Markov switching model, since autoregressive parameters are not statistically significant. For example, for the AR(1) model, the autoregressive parameter is not statistically significant at the 1% level. However, the log likelihood values are very close and the probabilities obtained from different specifications are very similar. This result is a consequence of the presence of several structural breaks in the Brazilian

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[6] The first observation available for quarterly data is 1980:01. This series was seasonally adjusted using the X-11 method.

[7] Since the transition probabilities are not identified under the null, Hansen (1993, 1996) proposes simulation methods to approximate the asymptotic null distribution of a standardized likelihood test, treating the transition probabilities as nuisance parameters, given that they. The asymptotic one state null distribution is the supremum over all admissible values in the space of transition probabilities.

[8] The likelihood ratio test can be used to choose among alternative specifications of the two-state model.

economy, corresponding to the different stabilization plans in the 80s and 90s. Some of these pulse breaks are very short-lived, resulting in a very small estimated autoregressive process for the period analyzed.

Specification tests are also applied on the assumptions regarding the model residuals. The residuals' sample autocorrelation is close to zero for observations more than one period apart and the autocorrelation functions for the disturbances ε_t are within the limit of two times their asymptotic standard deviation.

3.2 Results

The analysis focuses on both business cycle and growth cycle turning points. That is, we are interested in studying not only recessions and expansions, but also periods of low and high growth in output. This analysis may provide insights on the interrelation between changes in trends and business cycles.

In order to obtain optimal inferences of growth and business cycle turning points, we need first to define procedures to identify these turns. One of the dating guidelines adopted by the NBER is that recessions correspond to a general downturn in various sectors of the economy for a minimum duration of 6 months. The idea is to rule out short-term events, such as strikes, tax law changes, etc., from a broader downturn. In this paper we adopt this criterion for both recessions and expansions, in order to distinguish pervasive and persistent cyclical movements of the economy from brief and fully reversible shocks.[9]

The regime switching model provides probabilities that can be used as prediction rules. In particular, historical turning points are dated using smoothing probabilities, which are obtained by backward recursion based on full sample information, $Prob(s_t = j | I_T)$, $j = 0, 1$.[10] Business and growth cycles chronologies are then determined using two criteria to define a turning point. First, from the frequency distribution of the probabilities, a peak occurs if the probabilities of recessions or slowdowns fall above their mean plus one-half

[9] According to Burns and Mitchell's (1946) definition of business cycle, a full cycle should have a minimum duration of at least one year and a maximum of 10 to 12 years.

[10] For monitoring turning points on a current basis, however, the filtered probabilities should be used instead, which give at time t the probability of the Markov state using only information available at t.

their standard deviation. This criterion tracks turning points according to their specific frequency distribution. Second, a peak occurs if the smoothed probabilities are greater or equal than 50%. That is, the economy is assumed to be in a recession if $P(S_t=0|I_T)\geq P(S_t=1|I_T)$. These two methods yield the same business and growth cycle turning point dating.

The resulting chronology is compared to the ones obtained from two alternative ad-hoc procedures. First, Bry and Boschan (1971) routine is applied to determine turning point dates. Bry and Boschan (B-B) formalized the NBER dating rules into a computer program, which has been refined by Haywood (1973) to include an amplitude criterion.[11] Second, turning points are obtained applying the rule of thumb of two quarters of consecutive decrease in GDP. As examined below, the turning points from these two methods are very similar to the ones obtained from the smoothing probabilities.

Tables 1 and 3 show the maximum likelihood estimated parameters for growth cycles and business cycles, respectively. The coefficients of the Markov states are statistically significant for both models, and capture switches and asymmetries across different cycle phases of the Brazilian economy. In particular, state 0 displays a low or negative mean and a shorter average duration, which is associated with economic slowdowns and recessions, respectively. State 1 exhibits a positive mean and longer average duration, depicting the features of high growth phases and expansions.

3.2.1 The Brazilian Growth Cycle

Figure 1 contrasts the estimated smoothed probabilities of high growth phases and the growth rate of real GDP in the last century. For the annual frequency, the probabilities capture a dichotomous pattern in the series associated with high and low economic growth phases. State 1 is characterized by a high production mean rate (μ_1 = 7.4% per annum), while state 0 displays a low average growth rate (μ_0 = 1.15% per annum), as seen in

[11] The main steps of the B-B routine are: 1) the data are smoothed after outliers are discarded; 2) preliminary turning points are selected and compared with the ones in the original series; 3) duration of the phases is checked and if it is below 6 months the turning points are disregarded; 4) Amplitude criterion is applied, based on a moving standard deviation of the series. In the end, the program selects turning points that would be easily picked simply by visual inspection.

Table 1. The transition probabilities Prob[s_t=i|s_{t-1}= i] = p_{ii} are the probabilities of staying in state i given that the economy is in state i. Their estimates are highly significant and the probability of staying in a high growth phase, p_{11} = 0.77, is higher than the probability of staying in a slowdown, p_{00} = 0.66. That is, on average, high growth phases last 4.4 years in Brazil and are more persistent than slowdowns, which last around 3 years. [12]

Table 1
Maximum Likelihood Estimates
Annual Data: 1901-1999

Parameters		Parameters	
μ_1	7.432	p_{11}	0.774
	(0.548)		(0.082)
μ_0	1.148	p_{00}	0.665
	(0.649)		(0.106)
σ^2	7.886		
	(1.619)		
LogL(θ)	-185.053		

Asymptotic standard errors in parentheses.

Figure 2 plots the smoothed probabilities of slowdowns and the resulting dating of the Brazilian growth cycles. Over the century Brazil experienced eleven growth cycles. Figure 3 contrasts the growth rate of real production with the growth cycle dating, the average annual growth (4.8%) and a band with the standard deviation (4.4%). The smoothed probabilities endogenously define slowdowns as periods in which annual growth reached below 0.4% (mean minus standard deviation).

Table 2 reports dating of the Brazilian growth cycle phases using the smoothed probabilities and Bry-Boschan routine. Both metrics to identify turning points lead to very similar dating of growth cycles. The only difference is that B-B program does not pick up the 1908 slowdown. The reason is that the program discards outliers, and this slowdown was the most severe downturn in the century.

[12] The expected duration of slowdowns and high growth phases can be inferred by the transition probabilities using the formula: $\sum_{k=1}^{\infty} k p_{00}^{k-1}(1-p_{00}) = 1/(1-p_{00})$.

Figure 1 – Growth Rate of the Brazilian Real GDP and (—) and Smoothed Probabilities of High Growth (---) Annual Data: 1900-1999

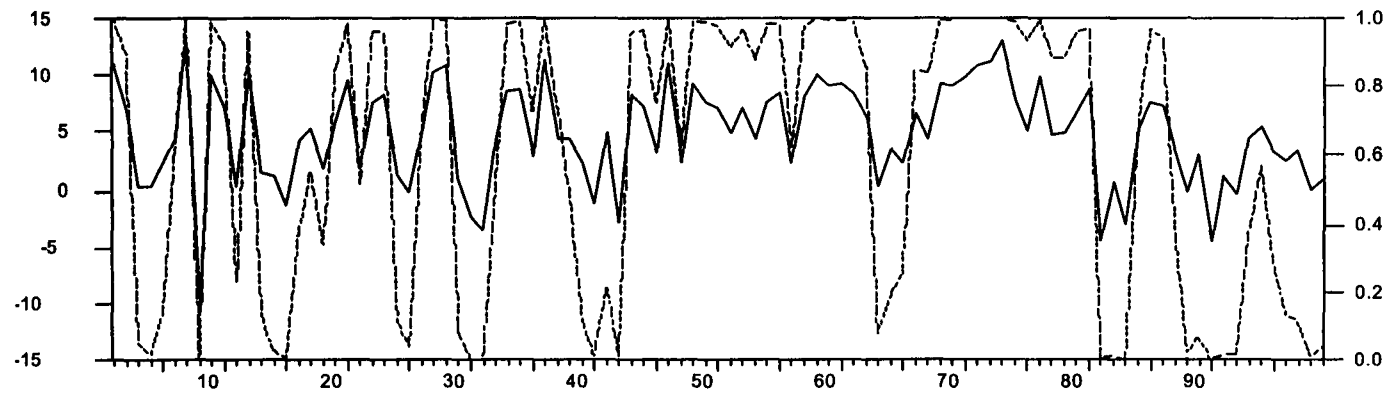


Figure 2 – Smoothed Probabilities of Slowdowns (—) and the Brazilian Growth Cycle Dating. Shaded Area Corresponds to Slowdowns; Annual Data: 1900-1999

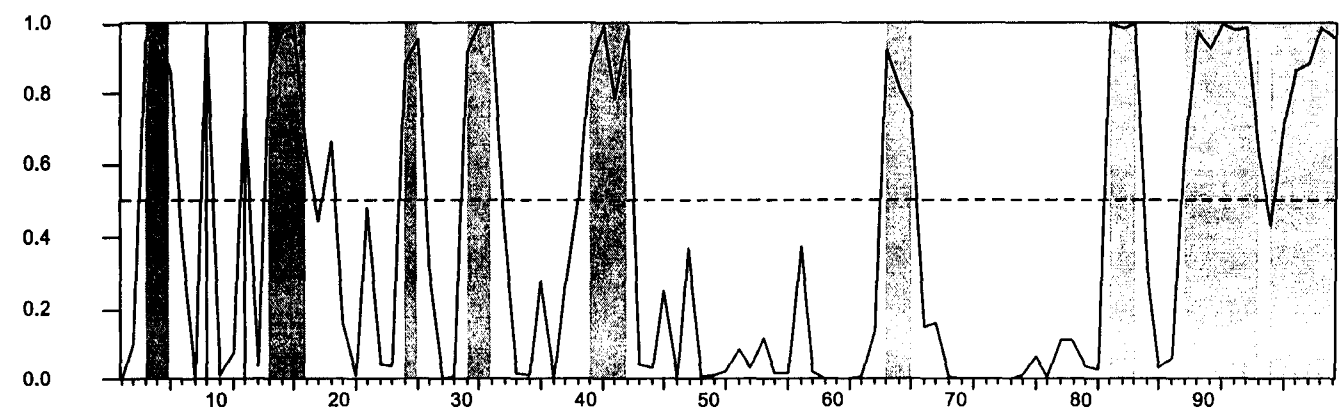


Figure 3 – Growth Rate of Brazilian Real GDP (—) and Low Growth Phases (Shaded Area). Bands are the Mean Plus/Minus the Standard Deviation of the Growth Rate of Real Production.

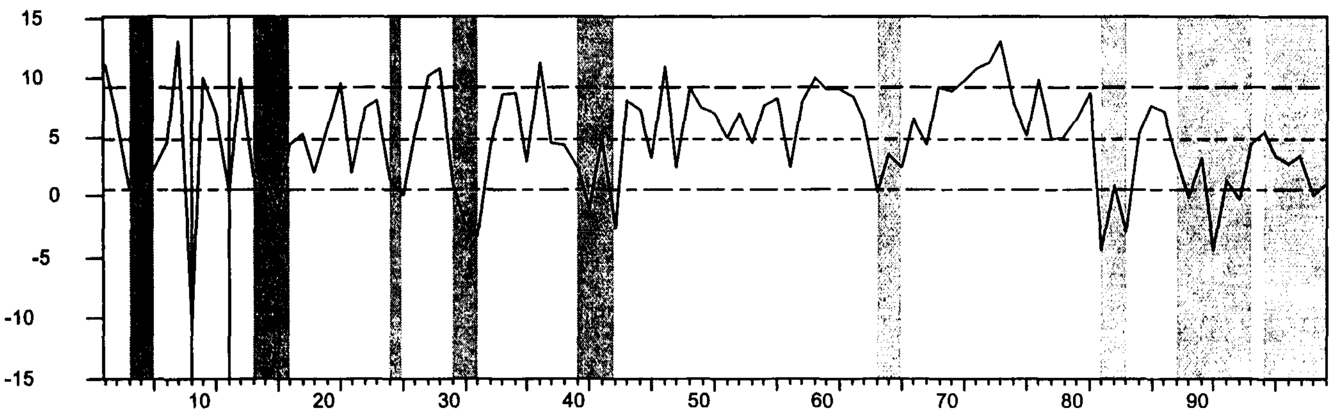


Table 2
Dating of Growth Cycle Turning Points
Annual Frequency: 1901-1999

Smoothed Probabilities		Bry-Boschan	
Peak	**Trough**	**Peak**	**Trough**
1903	1905	1903	1905
1908	1908	-	-
1911	1911	1911	1911
1913	1916	1913	1916
1924	1925	1924	1925
1929	1931	1929	1931
1938	1942	1938	1942
1963	1965	1963	1965
1981	1983	1981	1983
1987	1993	1987	1993
1995	1999	1995	1999

(*) Peaks are the beginning of slowdowns and Troughs mark their end.

There were seven slowdowns in the first half of the sample and four in the second. The most abrupt declines in the real Brazilian production occurred in 1908, 1981, and 1990. Until the 1940s, slowdowns were more frequent and had shorter duration (Figures 2 and 3). In fact, in the beginning of the century the economy was much more volatile, reflecting great uncertainty during the distressed times comprising the two World Wars and the Great Depression.

The first slowdown of the century started in 1903 and lasted 3 years. This was followed by two short slowdowns of 1-year duration in 1908 and in 1911. During the 6 years of the World War I from 1914 to 1918, the Brazilian economy was mostly in a slowdown. The economy was also affected by the Great Depression and the World War II, entering in a low growth phase between 1929 and 1931, and between 1938 and 1942.

In the second half of the century, Brazilian slowdowns exhibited a longer duration, and were much less frequent. In fact, 3 out of the 4 slowdowns during this period occurred in the 1980s and 1990s (Figures 2 and 3). Between 1945 and 1981, the Brazilian economy experienced a long period of economic prosperity. Except for a slowdown between 1963 and 1965, which coincided with times of political instability and the military cup, the economy grew at a high pace for almost 40 years. The high growth phases in the 50s and 60s are concurrent with the Brazilian import substitution program

and heavy public investment on infrastructure. This period also parallels the long expansion in the U.S. and OECD countries during the 60s (see Chauvet and Yu 2000). In the 70s, the oil crisis led many countries to gear back their economies to adjust for the impact of this supply shock. However, Brazil did not follow the tune. Public investment was at all times high, and the economy grew at high annual rates between 7% and 12%, characterizing the period known as the 'Brazilian economic miracle.'

The second oil shock in 1979 and the credit crunch in the early 1980s deeply affected the Brazilian economy and other Latin American countries. One of the reasons of this liquidity constraint was the changes in the procedures of the U.S. Federal Reserve Bank. From October 1979 to the end of 1981, the Fed targeted only the rate of growth of money supply, which caused a substantial increase in the level and volatility of the interest rates. In fact, the U.S. interest rates increased from an average of 4% between 1954 and 1979, to an average of 12% between 1979 and 1981. Increases in oil prices and interest rates led to a severe burden on the highly indebted Latin countries, which was intensified by the Mexican moratorium in 1982. This induced lending banks and international institutions to restrict credit even further, aggravating the liquidity situation of the Latin countries. In this period Brazil entered a low growth phase, between 1981 and 1983.

In the late 80s and first half of the 90s the Brazilian economy experienced very turbulent times, comprising a disrupting hyperinflationary process and six major stabilization plans aiming to control it. These changes in policy regimes engendered structural breaks in the economy and created a very uncertain and unsuitable environment for economic growth. Figure 4 plots inflation as measured by the Broad Consumer Price Index (IPCA) and the stabilization plans in the 80s and 90s. Accordingly, the Brazilian economy entered in two long periods of slowdown: one following the Cruzado Plan from 1987 to 1993, and the other following the Real Plan, from 1995 to 1999. That is, in the last 13 years the Brazilian economy experienced only one year of accelerated growth, in 1994.

3.2.2 The Brazilian Business Cycle

At the quarterly frequency, the Markov model captures business cycle phases. Recessions can be interpreted as more severe slowdowns, while expansions are periods of moderate growth. Figure 5 shows the smoothed probabilities of recessions and slowdowns from 1980:2 to 2000:01.[13] Generally, slowdowns start before the onset of recessions and end after the trough, which implies that recessions are more frequent than slowdowns. While slowdowns correspond to periods of low economic growth, recessions correspond to more severe downturns, in which the economy displays

negative growth. In fact, recessions were so frequent in the last 20 years that in most of this period the economy was in a slowdown phase.

Table 3 shows the maximum likelihood estimates for the quarterly data. State 0 has a negative long run mean rate (μ_0 = -1.5% per quarter or –6% per annum), and a short duration of 2.5 quarters (p_{00}=0.51).[14] In state 1, there is a positive mean rate (μ_1 = 1.4% per quarter or 5.6% per annum) with a longer average duration of 5 quarters (p_{11}= 0.8), which characterizes expansions. That is, the model captures asymmetries in the stages of business cycles, in which recessions are abrupt and shorter, while expansions are more gradual and longer.

Table 3
Maximum Likelihood Estimates
Quarterly Data: 1980:2-2000:1

Parameters		**Parameters**	
μ_1	1.370	p_{11}	0.799
	(0.265)		(0.082)
μ_0	-1.555	p_{00}	0.515
	(0.464)		(0.135)
σ^2	1.747		
	(0.402)		
LogL(θ)	-87.528		

Table 4 reports dating of the Brazilian business cycle using smoothed probabilities, B-B routine, and the rule of two consecutive quarters of decrease in GDP. These dating procedures generate a very similar chronology for business cycles. A minor difference is that the smoothed probabilities identify the beginning of the 1982 and the 1990 recessions as one quarter after what is predicted by the other methods. Although the smoothed probabilities were very high before these recessions, indicating an economic slowdown, the model endogenously identifies the beginning of recessions as quarters in which the economy actually enters a period of negative growth.

[13] For comparison, the annual probabilities were converted to quarterly frequency in this figure using the local quadratic interpolation method with average matched to the observed data.
[14] 'Long run' means that if the state no longer changed.

Figure 4 – IPCA Inflation (—) and Stabilization Plans (Shaded Area)

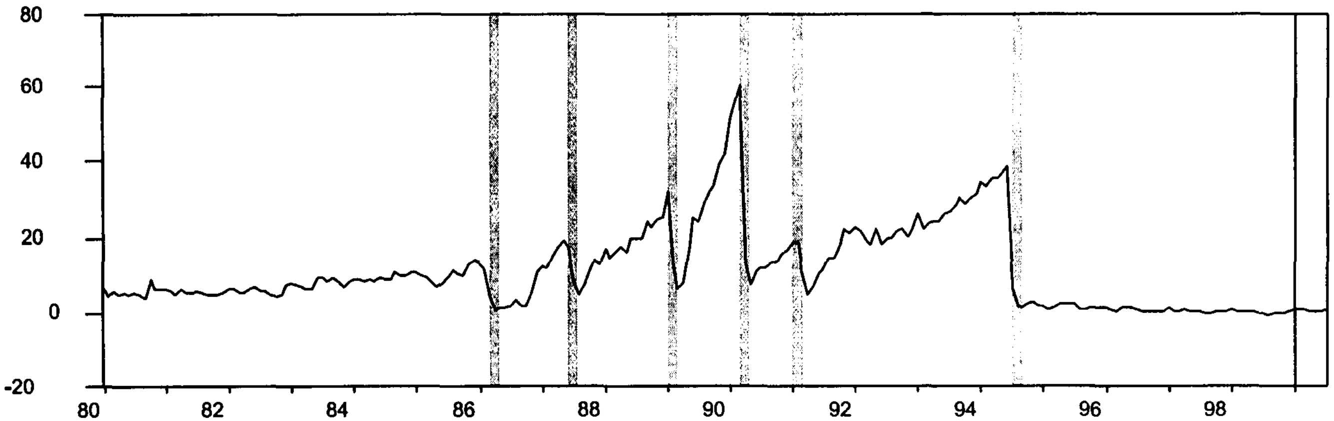
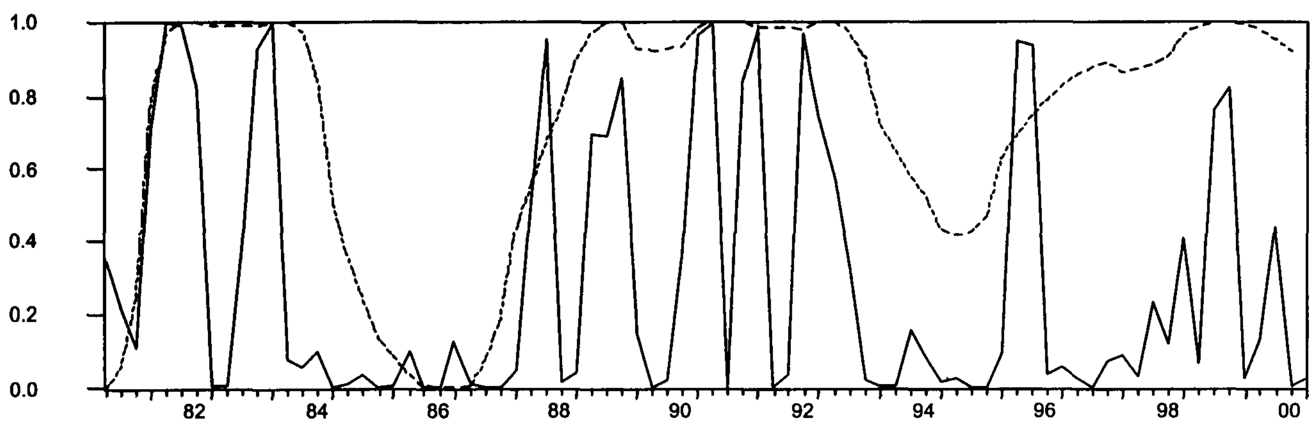


Figure 5 – Smoothed Probabilities of Recessions at Quarterly (—) and Annual Frequencies (—). 1980:2-1999:4



This can be seen in Figures 6 and 7, which plot the estimated smoothed probabilities of recessions, the growth rate of real GDP against expansion and recession phases. The quarterly average growth rate of GDP is 0.49 with a 2.3 standard deviation. The probabilities signal recessions as times in which the economy reaches a growth below the sample average minus the standard deviation (-1.79).

Table 4
Dating of the Brazilian Business Cycle Turning Points at Quarterly Frequency: 1980:I-2000:I

Smoothed Probabilities		Bry-Boschan		Two Consecutive Declines	
Peak	**Trough**	**Peak**	**Trough**	**Peak**	**Trough**
1981:I	1981:IV	1981:I	1981:IV	1981:I	1981:IV
1982:IV	1983:I	**1982:III**	1983:I	1982:III	1983:I
1987:II	1987:III	1987:II	1987:III	1987:II	1987:III
1988:II	1988:IV	1988:II	1988:IV	1988:II	1988:IV
1990:I	1991:I	1990:I	1991:I	**1989:IV**	1991:I
1991:IV	1992:II	1991:IV	1992:II	1991:IV	1992:II
1995:II	1995:III	1995:II	1995:III	1995:II	1995:III
1998:III	1998:IV	1998:III	1998:IV	1998:III	1998:IV

(*) Peaks are the beginning of recessions and troughs are their end.

In the last 20 years, there were 8 recessions and 9 expansions in the Brazilian economy. Several recessions were caused by external shocks. In the early 80s, there were two recessions of short duration that occurred very close to each other. The first one started in the first quarter of 1981 and lasted 4 quarters. The second one went on for only six months, during the fourth quarter of 1982 and first quarter of 1983. These recessions in Brazil correspond to a worldwide contractionary economic period (see Chauvet and Yu 2000). Subsequently, the economy entered in an expansion from 1983 to 1987 – the longest expansion in the last 20 years (16 quarters), when the Brazilian GDP grew at an average rate of 6.5% per annum.

Figure 6 – Smoothed Probabilities of Recessions (—) and Dating of the Brazilian Business Cycle (Shaded Area)

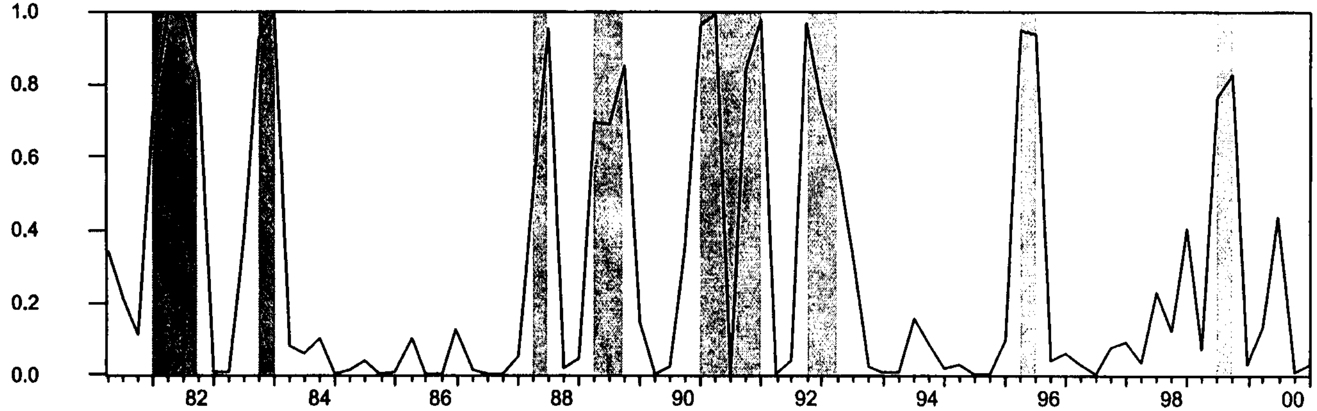
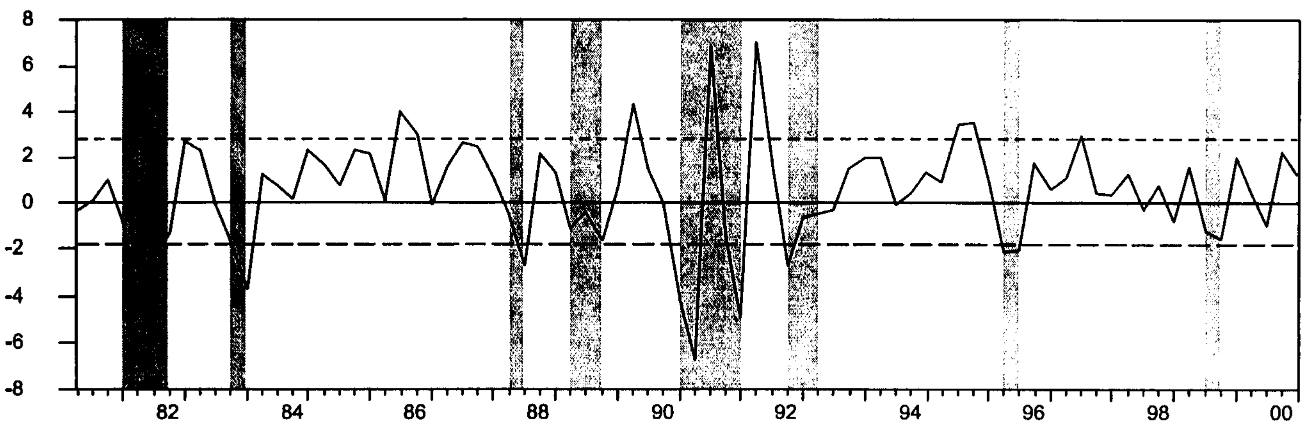


Figure 7 – Growth Rate of Real GDP (—) and the Brazilian Business Cycle– Recessions Phases (Shaded Area). Bands are the Mean Plus/Minus the Standard Deviation of the Growth Rate of Real GDP



As seen above, from 1987 to 2000, the Brazilian economy was in a low growth phase except for 1994. During this same period the economy experienced 6 recessions (negative growth) of relative short duration, generally associated with implementation of stabilization plans. In fact, in the five years between 1987 and 1992, there were five severe economic recessions. The first recession occurred in the second and third quarter of 1987, coinciding with Bresser's Plan. Inflation growth did not subside in spite of the successive plans, and in the second quarter of 1988 until the end of 1988 the economy entered another recession. In the first quarter of 1990, Collor Stabilization Plan gave rise to the most severe recession in the sample (-8.1% annual growth rate) associated with high volatility in the growth rates of real GDP, which lasted for one year. The economy entered a short-lived recovery in 1991, but it lasted only 6 months. In fact, another recession hit the economy in the fourth quarter of 1991, lasting until the second quarter of 1992. At this time the economy shrunk at an annual negative rate of 5.1%. In the third quarter of 1992 the economy finally entered in a period of relative calmness, coinciding with the impeachment of President Collor.

In the last nine years Brazil was in an expansion period 74% of the time. The economy experienced only two short-lasted recessions in this period, which were associated with international shocks. Between the third quarter of 1992 and of 1995, the economy grew at an average annual rate of 5.7% (12 quarters). In fact, in 1994 the economy entered in a period of high growth phase, when GDP grew an annual average growth of 9.25%. In the second and third quarter of 1995 there was a recession associated with the Mexican crisis and a subsequent increase in the domestic interest rates. From the fourth quarter of 1995 until the second quarter of 1998 the economy was in an expansion phase for 11 quarters, with a modest average annual growth rate of 3.4%.

The last four years were marked by high volatility in GDP growth, as the Brazilian economy was hit by several external shocks. This can be seen in the unusual ups and downs of the smoothed probabilities of recessions during this period, in Figure 6. The Asian crisis had a corresponding mild increase in the probabilities of recession in the third quarter of 1997 and in the first quarter of 1998.[15]

The Russian crisis in July 1998 increased the perceived risk of emergent economies, which contributed to a currency crisis in Brazil in the first quarter of 1999. In an attempt to avoid the crisis, the Central Bank increased interest rates in almost 85% in the third and forth quarters of 1998, sustaining a recession during this period. However, contrarily to widespread expectations, a recession did not follow the currency crisis, and the economy entered a modest recovery already in the first quarter of 1999.

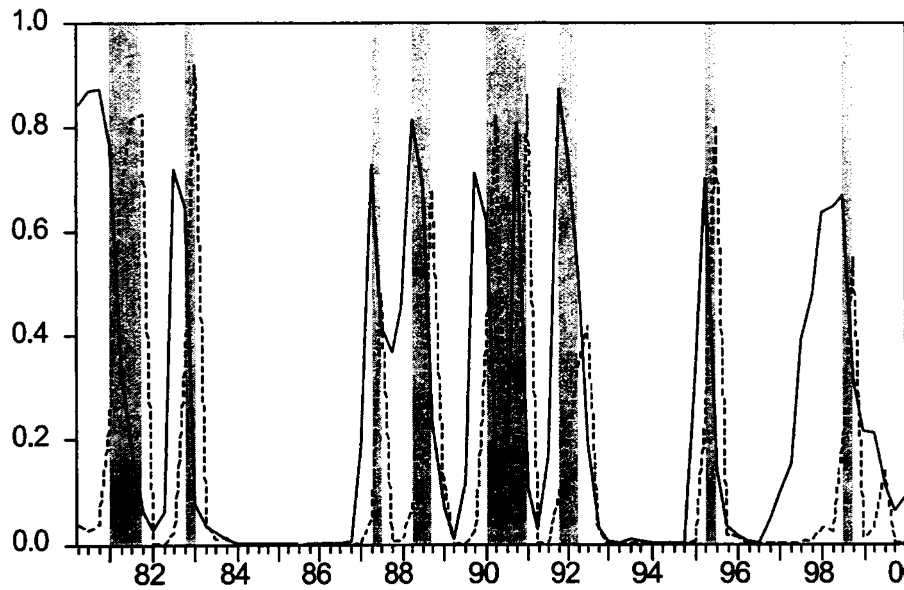
[15] This period is not classified as a recession because of the short duration and small amplitude of the downturn.

More recently, the economy has grown at an average annual rate of 3.9% in the last 5 quarters of the sample. In the fourth quarter of 1999 the growth rate of real GDP was 8.9% per annum while in the first quarter of 2000 the annual growth was 4.9%. The smoothed probabilities shown in Figure 6 indicate a 99% and a 97% probability of the economy being in an expansion in the last quarter of 1999 and first quarter of 2000.

Three Markov States

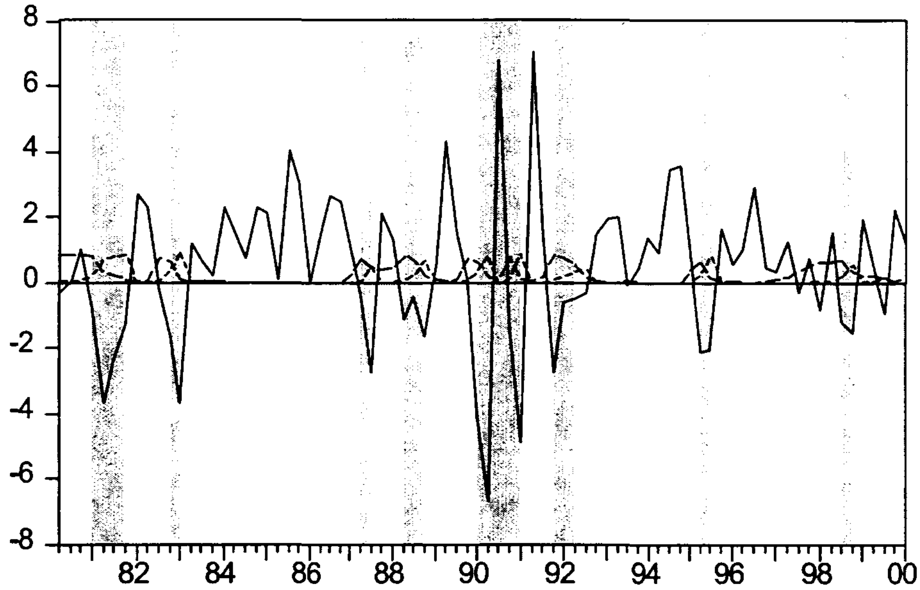
For comparison with the previous results, a Markov switching model with three states was also fitted to quarterly GDP. Figures 8 and 9 show the smoothed probabilities of the contraction states, the growth rate of real GDP, and recession phases obtained from the two-state model. The estimated smoothed probabilities identify three phases: contractions, revivals, and expansions. In fact, downturns are now divided in two phases – the first half corresponds to times in which GDP is contracting, while in the second half GDP is recovering, but it is still below the average growth.[16] The third state roughly coincides with expansions as obtained previously.

Figure 8 – Smoothed Probabilities of Contractions for States 0 and 2, and Recessions Phases (Shaded Area)



[16] In Burns and Mitchell's definition of business cycles, this corresponds to the revival phase.

Figure 9 – Smoothed Probabilities of Contractions for States 0 and 2, Growth Rate of Real Product, and Recessions Phases (Shaded Area)



4. Conclusions

This paper uses a Markov switching model to date and analyze the Brazilian business cycle and growth cycle. The Markov process is a latent variable that is used to define the different phases of cyclical economic fluctuations underlying real Brazilian production. Optimal inferences of turning points are obtained from inferred smoothed probabilities of the Markov states.

The resulting dating of the Brazilian business cycle and growth cycle can be used as a reference point for construction and evaluation of the predictive performance of coincident, leading, or lagging indicators of economic activity. In addition, the filtered probabilities allow assessment of the current state of the Brazilian economy on a timely basis. An early recognit ion of the economic transition to a new business cycle phase can be serve for evaluation of the adequate strength and timing of counter-cyclical policies, reassessment of projected sales or profits by businesses and investors, monitoring of inflation pressures, etc.

The Markov states capture asymmetries across different cycle phases of the Brazilian economy. In particular, the model identifies a state with low or negative mean and a shorter average duration, which is associated with economic slowdowns and recessions. The other state exhibits a positive mean and longer average duration, which characterizes features of high growth phases and

expansions. These asymmetries regarding duration, amplitude, and deepness across business cycle expansions and recessions are also observed in the OECD countries.

More specifically, at the annual frequency the Markov switching model depicts different phases of secular growth: during slowdowns the economy grows at an annual rate of 1.15%, while during periods of accelerated growth, it averages 7.5% per annum in the last century. At the quarterly frequency, the model identifies periods of expansions, when the economy grows at a more moderate annual rate of 5%, and periods of recessions, when the economy displays an average negative growth of 6% per annum.

References

Bry, G., and C. Boschan, 1971, *Cyclical Analysis of Times Series: Selected Procedures and Computer Programs*, (New York: National Bureau of Economic Research).

Burns, A. and W. Mitchell, 1946, *Measuring Business Cycles*, (New York: National Bureau of Economic Research).

Chauvet, M., 2000a, "A Monthly Indicator of Brazilian GDP," Working Paper, University of California, Riverside.

Chauvet, M., 2000b, "Leading Indicators of Inflation for Brazil," Central Bank of Brazil.

Chauvet, M., and C. Yu, 2000, "International Business Cycles in the G-7 Countries," mimeo, University of California, Riverside.

Chauvet, M., 1998, "An Econometric Characterization of Business Cycle Dynamics with Factor Structure and Regime Switches," *International Economic Review*, Vol. 39, No. 4, 969-96.

Dickey, D. A., and W. A. Fuller, 1979, "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Society* 74, 427-31.

Diebold, F. X. and G. D. Rudebusch, 1989, "Scoring the Leading Indicators," *Journal of Business*, Vol. 62, No. 3, 369-391.

Garcia, R., 1998, "Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models," *International Economic Review*, Vol. 39, No. 3, August 1998, 763-788.

Hamilton, J., 1989 "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica* 57, 357-384.

_________, 1994, "*Time Series Analysis*", Princeton University Press.

Hansen, B. E., 1993, "The Likelihood Ratio Test Under Non-Standard Conditions: Testing the Markov Trend Model of GNP," in M.H. Pesaran and S. Potter, *Nonlinear Dynamics Chaos and Econometrics*, (John Wiley & Sons), 53-73.

Haywood, E., 1973, "The Deviation Cycle: A New Index of the Australian Business Cycle, 1950-1973, *Australian Economic Review*, 4, 31-39.

Mitchell, W, 1927, *Business Cycles: The Problem and Its Setting*, (New York: National Bureau of Economic Research).

Neftci, S., 1982, "Optimal Prediction of Cyclical Downturns," *Journal of Economic Dynamics and Control*, No. 4, 225-41.

Perron, P., 1989, "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis," *Econometrica*, 57, 1361-1401.