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# **“EXTINCTION, POLLUTION, AND THE SUSTAINABLE USE OF EXHAUSTIBLE AND RENEWABLE RESOURCES”**

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by

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## Abstract

### Extinction, Pollution, and the Sustainable Use of Exhaustible and Renewable Resources

by

Dan Biller

Single ownership of natural resources is common in many developing countries and socialist economies. The sole owner is usually the state or society at large, and governments are responsible for either distributing exploitation rights or engaging in exploitation through their own corporations. Under this circumstance, the notion of externality may not fully explain pollution problems existent in these nations.

This paper studies the case where a single agent owns both exhaustible and renewable resources, and attempts to maximize its welfare. The resources are either perfect or imperfect substitutes. Initially, exhaustible resource extraction does not affect the renewable resource, and sustainable growth is attainable. A factor of pollution flowing from the extraction of the non-renewable resource into the growth of the renewable resource is introduced. The continuous exploitation of the exhaustible resource leads to the "optimal" extinction of the renewable resource, and sustainable growth is no longer reached. Regulation from a supra governmental agency such as an multinational institution may prove to be of utmost importance, if sustainability is to be achieved.

The paper is divided into five sections. Section two provides a brief survey of the relevant literature. Section three presents the model without pollution. This factor is introduced in section four. The final section discusses some possible approaches for attaining sustainable growth, and contains the concluding remarks.

Extinction, Pollution, and the Sustainable Use of Exhaustible and  
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Introduction

Natural Resources can be divided into two categories: Exhaustible and renewable. Exhaustible resources may in geological time be replaced, yet under an economic time frame they are doomed to total depletion. Alternatively, there is no steady state for the growth of an exhaustible resource; therefore, a country that bases its growth on exhaustible resource extraction - given the absences of both technological progress and reinvestment in renewable activities - will at a point in time notice the collapse of its economic strategy.

Renewable resources may be harvested under a steady state; that is, as long as the growth characteristic of a given renewable resource is respected, it may be harvested indefinitely. A country that bases its growth on renewable resource harvesting may sustain its economic strategy. Yet, renewable resources also have a point of collapse in which they cannot be regenerated. Mismanagement, or economic factors such as discount rates may ultimately lead to the extinction of a renewable resource.

This paper analyses the case in which the extraction of an exhaustible resource generates pollution affecting the growth of a renewable resource. It compares the solution of the pollution problem with the case where pollution is absent, and shows that with pollution the renewable resource may disappear.

The paper is divided into five sections. The next section presents a brief survey of relevant literature. The third section introduces the model without pollution, and considers the cases of perfect and imperfect substitutes. Pollution is inserted in the fourth segment. The last section examines the main findings of the paper, and contains the concluding remarks.

## Literature Review

An important occurrence studied in this paper is the extinction of renewable resources. This has been extensively discussed in Clark (1976). The standard result on renewable resource management links extinction to the discount rate; that is, it is usually optimal to exploit the renewable resource more than the maximum sustainable yield<sup>1</sup> when the rate of discount exceed its maximum growth<sup>2</sup>. Extinction may also come about related to problems of open access and common property (the "tragedy of the commons"). For the sole owner's case, as studied by Lewis and Schmalensee (1977), and Cropper (1988), extinction may be further associated with the initial size of the renewable resource stock and with the choice for the resource's growth function. Finally, Swallow (1990) studies the trade-offs between renewable resource production and environmental development motivated by the irreversible impacts of coastal zone development. In this case, extinction only occurs if development proceeds too far.

Substitutability as translated in the use of natural resources is also discussed in this paper. In the economic literature, the output side of substitution - i.e. the joint production of natural resources - was addressed in detail by Pindyck (1982), Hoel (1983), Kemp and Long (1984), and Wirl (1987 and 1988). Kemp and Long (1984) presents a comprehensive study on the economics of exhaustible resources. Pindyck (1982) explores market functioning in the presence of joint production with storage costs and uncertain future resource demand. Hoel (1983) combines an exhaustible resource with a backstop technology<sup>3</sup>, where there is a gradual transition from the former to the latter.

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<sup>1</sup> The maximum sustainable yield (MSY) is the point where the growth function of a renewable resource is maximized. This point is often considered optimal in the biological framework but not in economic analysis.

<sup>2</sup> As Clark (1976) points out, the term "optimal" refers solely to the idea of maximizing present values. It does not consider aesthetic or moral issues pertaining the decision of extinction. These are undoubtedly very important particularly when dealing with irreversible actions.

<sup>3</sup> A backstop technology can be defined as a resource which is available at infinite supply associated with very high costs.

Finally, Wirl (1987 and 1988) analyze in a dynamic context the joint production of exhaustible resources and their optimal extraction policies.

The works mentioned in the paragraph above address primarily the cases of exhaustible resources or exhaustible resource and a backstop technology. The mixed case, i.e. the simultaneous use of exhaustible and renewable resources, was studied in Biller (1990) and Aarrestad (1990). Biller (1990) models the behavior of a monopoly that produces both types of resources for the cases of perfect and imperfect substitutes. Aarrestad (1990) investigates an economy for which the welfare function is dependent on the production (and consumption) of perfect substitute renewable and exhaustible resources.

Finally, an additional topic of relevance to the models presented below is the issue of sustainability. Although this issue serves as a link between natural resource and environmental economics, finding an unique definition for sustainability has been a difficult task. Pezzey (March 1989), for example, gives more than forty-five definitions of sustainability coming from the literature. As pointed out by Barbier and Markandya (1990), at least two categories of definitions may be identified: a broader concept involving economic, ecological and social development and a narrower notion focused on intertemporal optimal resource and environmental management. Like Barbier and Markandya, our model will center on the latter. That is, we will consider sustainable any growth for which a steady state is possible.

#### Economic Use of Mixed Resources Without Pollution

Substitutability as emphasized by Wirl (1987 and 1988) is a concept that can be largely defined by consumers. In this sense, at the firm level the substitutability of jointly produced resources would appear in the demand faced by producers<sup>4</sup>. In the welfare case studied by Aarrestad (1990), substitutability emerges in the utility function of the economic agents. Both cases, though different in approach, hold primarily the same idea. For the case of perfect

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<sup>4</sup>This idea was largely used in Biller (1990).

substitutes, price or utility is basically a function of the sum of both resources. Imperfect substitutes would involve different price and utility systems.

#### a. The Perfect Substitute Case

Consider an economy that derives its welfare from the extraction of an exhaustible resource and the production of a renewable one. The instantaneous social welfare can thus be written as:

$$U = U[q(t) + y(t)] \quad (1)$$

with

$$U_q > 0, U_y > 0, U_{qq} < 0, U_{yy} < 0.$$

The planner's problem would be:

$$W = \text{Max} \int_0^{\infty} e^{-rt} U[q(t) + y(t)] dt$$

s.t.:

$$q(t) \geq 0, y(t) \geq 0$$

$$\dot{R} = -q$$

$$\dot{H} = G(H) - y$$

state variables:  $R(t), H(t)$

control variables:  $q(t), y(t)$

where the variable  $t$  in parenthesis represents time dependency and will be omitted throughout most of this paper. The superscript "dot" depicts the time derivative of a given variable, and the subscripts represent the partial derivatives of the function with respect to its arguments.  $R$  is the remaining reserves of the exhaustible resource,  $H$  is the stock of the renewable resource with a natural growth function of  $G(H)$ ,  $q$  is the extraction rate,  $y$  is the production of the renewable resource, and  $r$  is the discount rate.

As mentioned previously, from the point of view of society both resources are assumed to be perfect substitutes with no costs attached to their extraction or production. It should also be mentioned that there exists a finite and positive level of utility for which the non-renewable resource is exhausted (a "choke" level of utility)<sup>5</sup>.

The Hamiltonian of this problem is given by

$$\tilde{H} = e^{-rt} [U(q+y)] - \sigma q + \mu (G(H) - y) \quad (2)$$

which has the following first order conditions

$$\tilde{H}_q = e^{-rt} U_q - \sigma = 0 \quad (3)$$

$$\tilde{H}_y = e^{-rt} U_y - \mu = 0 \quad (4)$$

$$-\dot{\tilde{H}}_R = \dot{\sigma} = 0 \quad (5)$$

$$-\dot{\tilde{H}}_H = \dot{\mu} = -\mu G_H \quad (6)$$

$\sigma$  and  $\mu$  are the shadow prices of the exhaustible and the renewable source, respectively. By taking the time derivatives of equations (3) and (4), and incorporating equations (5) and (6), it is possible to find the extraction rate and production path of the resources. More importantly, we can find the relationship between the two resources as given in the proposition below:

**Proposition 1** (Biller, 1990): In the case of jointly used perfect substitute exhaustible and renewable resources - when the exhaustible resource

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<sup>5</sup> The assumptions of no costs and the existence of a "choke utility" are kept throughout the paper. Extraction and production costs would not change the problem significantly, but it would create additional algebraic complications. This cost less assumption permits us to choose which resource is used first. We consider the non-renewable as the first comer avoiding the possibility of a steady state in the renewable side without the extraction of the exhaustible resource. The "choke utility" assumption allows for the total economic exhaustion of the non-renewable resource.



is extracted first - the switching point occurs when the marginal growth of the renewable resource is zero.

**Proof:** The time derivatives of equations (3) and (4) yield the Hotelling rule for the exhaustible resource and a "variation" for the renewable resource. These equations will only hold simultaneously when  $G_H = 0$ .<sup>6</sup>

$$\frac{\dot{U}_q}{U_q} = r, \quad \frac{\dot{U}_y}{U_y} = r - G_H$$

The condition above is also related to the steady state of the renewable resource. Steady state requires that the derivatives with respect to time of the renewable resource's shadow price, stock and production are equal to zero. This requisite implies that the marginal growth is zero as above, and this point is commonly known as the maximum sustainable yield. Figure 1 depicts the case in which the production path is constant over time.

[Place figure 1 here]

#### **b. The Imperfect Substitute Case**

In the imperfect substitute case, the marginal utility derived from each resource may be different; therefore, the utility function cannot be written as a function of the sum of both resources. The remaining conditions are kept the same. The problems, thus, becomes:

$$W = \text{Max} \int_0^{\infty} e^{-rt} U[q(t), y(t)] dt$$

s.t.:

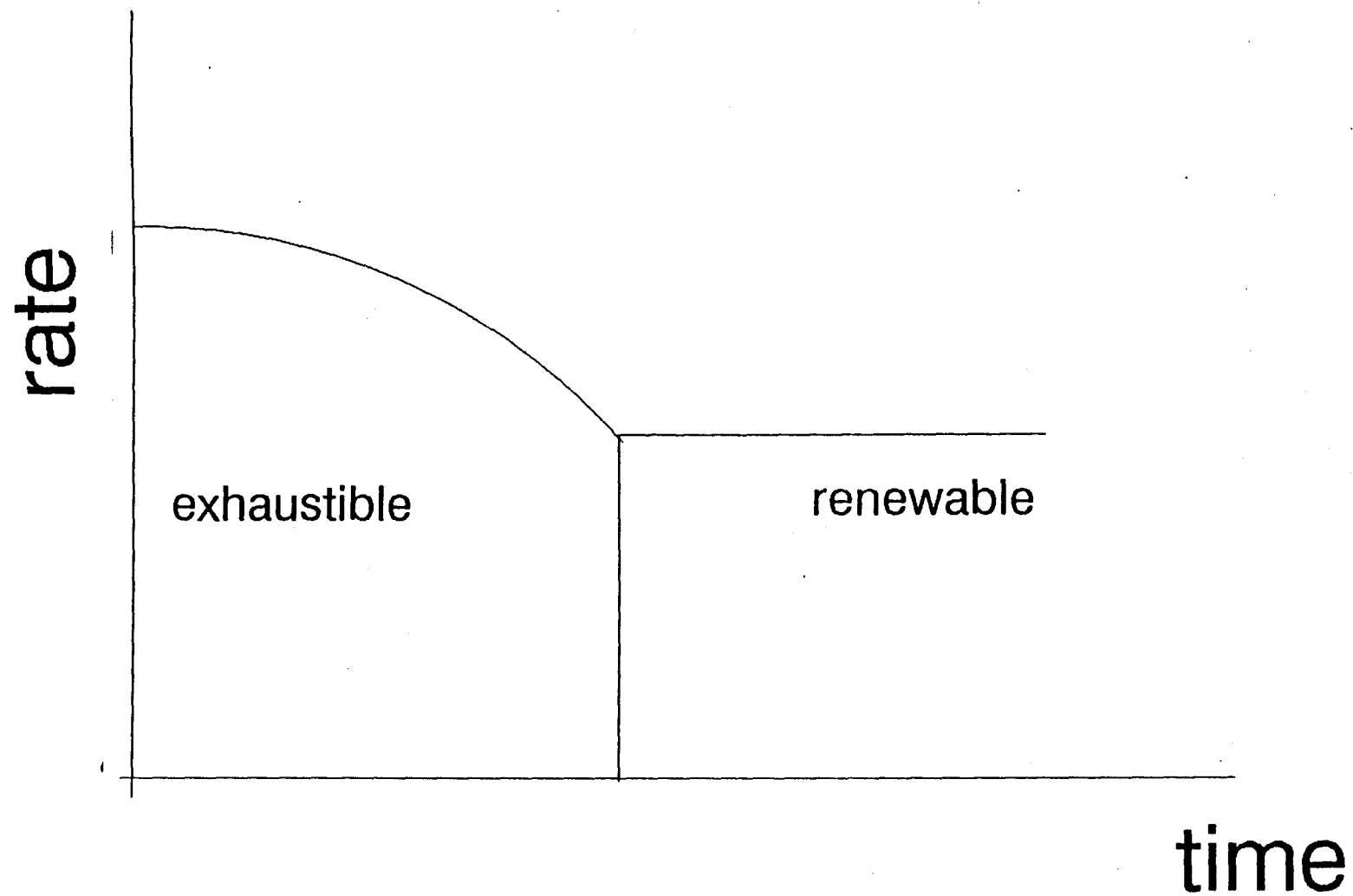
$$q(t) \geq 0, y(t) \geq 0$$

$$\dot{R} = -q$$

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<sup>6</sup>Note that  $U_q = U_y$ .

Figure 1: JOINT USE OF EXHAUSTIBLE AND RENEWABLE RESOURCES  
The Perfect Substitute Case



$$\dot{H} = G(H) - y$$

state variables:  $R(t), H(t)$

control variables:  $q(t), y(t)$

The Hamiltonian of this problem is given by

$$\tilde{H} = e^{-rt} [U(q, y) - \sigma q + \mu (G(H) - y)] \quad (7)$$

which has the following first order conditions

$$\tilde{H}_q = e^{-rt} U_q - \sigma = 0 \quad (8)$$

$$\tilde{H}_y = e^{-rt} U_y - \mu = 0 \quad (9)$$

$$-\tilde{H}_R = \dot{\sigma} = 0 \quad (10)$$

$$-\tilde{H}_H = \dot{\mu} = -\mu G_H \quad (11)$$

Taking the time derivatives of equations 8 and 9 yields the time path of the exhaustible resource extraction and the renewable resource production:

$$\dot{q} = [r(U_q U_{yy} - U_y U_{qy}) + G_H U_y U_{qy}] / \Delta \quad (12)$$

$$\dot{y} = [r(U_y U_{qq} - U_q U_{qy}) - G_H U_y U_{qq}] / \Delta \quad (13)$$

where

$$\Delta = U_{qq} U_{yy} - (U_{qy})^2$$

Since the resources have different marginal utilities, simultaneous extraction and production are possible. Yet, the results of a steady state for the renewable resource are less trivial, and the sign of the stock's natural growth will depend on the relation  $1 - (U_q U_{qy} / U_y U_{qq})$ . Figure 2 below depicts a

possible solution for this system.

[Place figure 2 here]

### Economic Use of Mixed Resources With Pollution

In this section, we will introduce an additional factor to the model presented above. Consider now the situation where the extraction of the exhaustible resource causes pollution affecting the renewable resource. The pollution alters the renewable resource growth function, but its flow depends on the extraction rate and on the ability of the environment to "clean-up" itself. Once again, we consider the cases of perfect and imperfect substitutability.

#### a. The Perfect Substitute Case

The planner's problem would be:

$$W = \text{Max} \int_0^{\infty} e^{-rt} U[q(t) + y(t)] dt$$

$$\text{s.t.:} \\ q(t) \geq 0, y(t) \geq 0$$

$$\dot{R} = -q$$

$$\dot{H} = G(H, S) - y$$

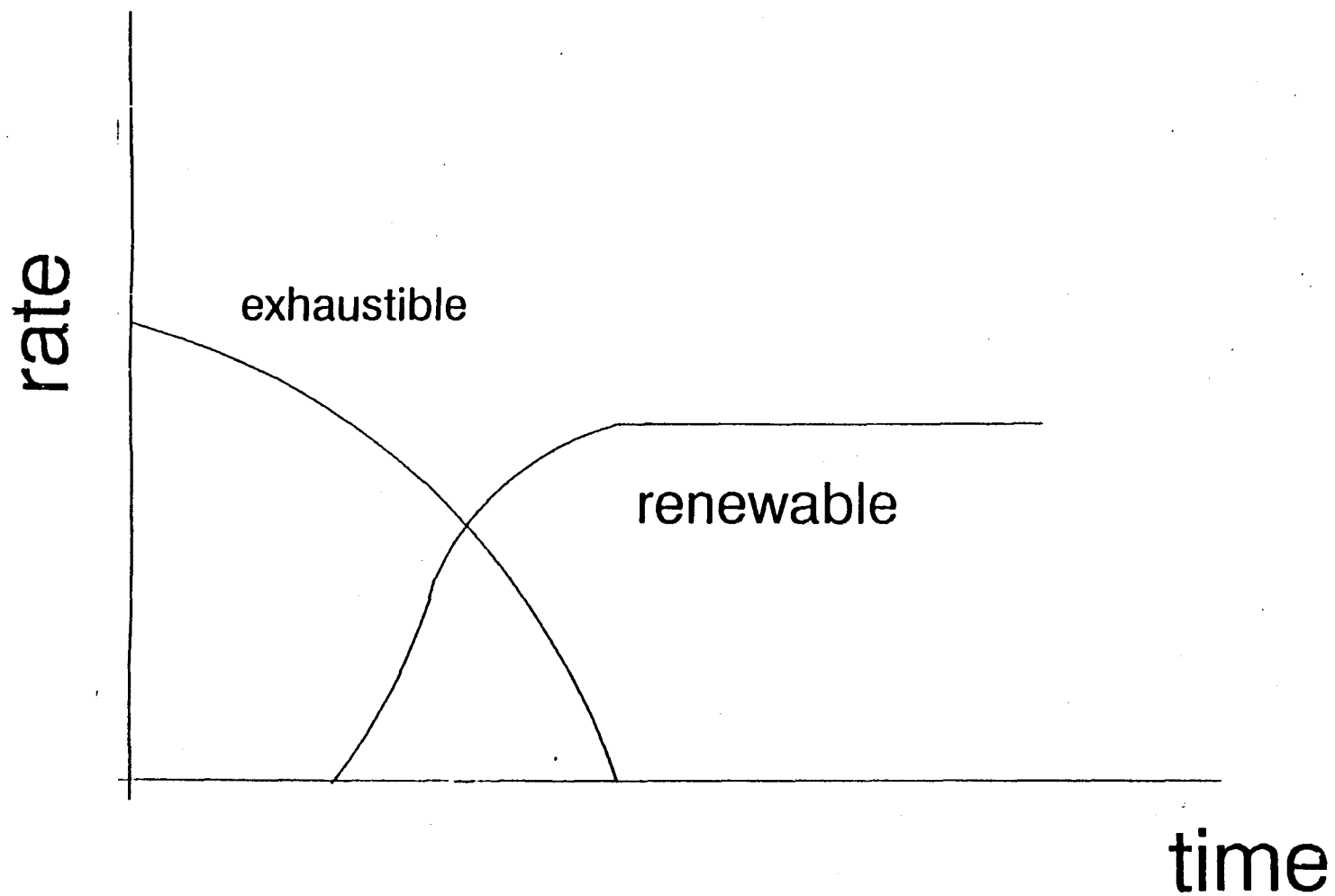
$$\dot{S} = aq - bS; a, b > 0$$

state variables:  $R(t), H(t), S(t)$

control variables:  $q(t), y(t)$

where besides the variables already discussed in the previous sections, a new state equation is introduced. The time derivative of  $S$  is the flow of pollution as suggested in Dasgupta (1982), which is a function of the emissions 'aq' and the stock of pollution itself. The parameters  $a$  and  $b$  are the emission-output ratio and the "cleaning - up" ratio of the environment, respectively. The planner is thus faced with a problem of two control and three state variables.

Figure 2: JOINT USE OF EXHAUSTIBLE AND RENEWABLE RESOURCES  
The Imperfect Substitute Case



The Hamiltonian of this problem is given by

$$\tilde{H} = e^{-rt} [U(q+y)] - \sigma q + \mu (G(H, S) - y) + \tau (aq - bS) \quad (14)$$

which has the following first order conditions:

$$\tilde{H}_q = e^{-rt} U_q - \sigma + \tau a = 0 \quad (15)$$

$$\tilde{H}_y = e^{-rt} U_y - \mu = 0 \quad (16)$$

$$-\tilde{H}_R = \dot{\sigma} = 0 \quad (17)$$

$$-\tilde{H}_H = \dot{\mu} = -\mu G_H \quad (18)$$

$$-\tilde{H}_S = \dot{\tau} = \tau b - \mu G_S \quad (19)$$

$\tau$  is the pollution's shadow price. By taking the time derivatives of (15) and (16) and equating them with (17)-(19), the natural growth rate at the switching point can be determined and is given by:

$$G_H = \frac{b\sigma e^{rt}}{U_q} - b - aG_S \quad (20)$$

It should be noted that a positive and ever increasing  $G_H$  means the extinction of the renewable resource. The exponential factor in equation (20) should dominate over all other factors, and the only negative factor in the equation is likely to be small<sup>7</sup>. The renewable resource could thus be extinct prior to the beginning of its exploitation. Alternatively, as depicted in Figure 3, under the presence of pollution from the extraction of an exhaustible resource

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<sup>7</sup> Note that  $G_S < 0$ . In other words, when the stock of pollution increases the growth of the renewable resource diminishes, thus the only negative factor in the equation (20) is '- b'.

a renewable resource becomes in fact optimally exhaustible. This is a rather gloomy scenario if one considers the sustainable option of the previous section.

[Place figure 3 here]

#### b. Imperfect Substitute Case

The imperfect substitute case yields a similar result to the case above; that is, in both cases it will be optimal in present value terms to render extinction to the renewable resource.

Under imperfect substitutes with pollution, the planner faces the following problem:

$$W = \text{Max} \int_0^{\infty} e^{-rt} U[q(t), y(t)] dt$$

s.t.:

$$q(t) \geq 0, y(t) \geq 0$$

$$\dot{R} = -q$$

$$\dot{H} = G(H, S) - y$$

$$\dot{S} = aq - bS; a, b > 0$$

state variables:  $R(t), H(t), S(t)$

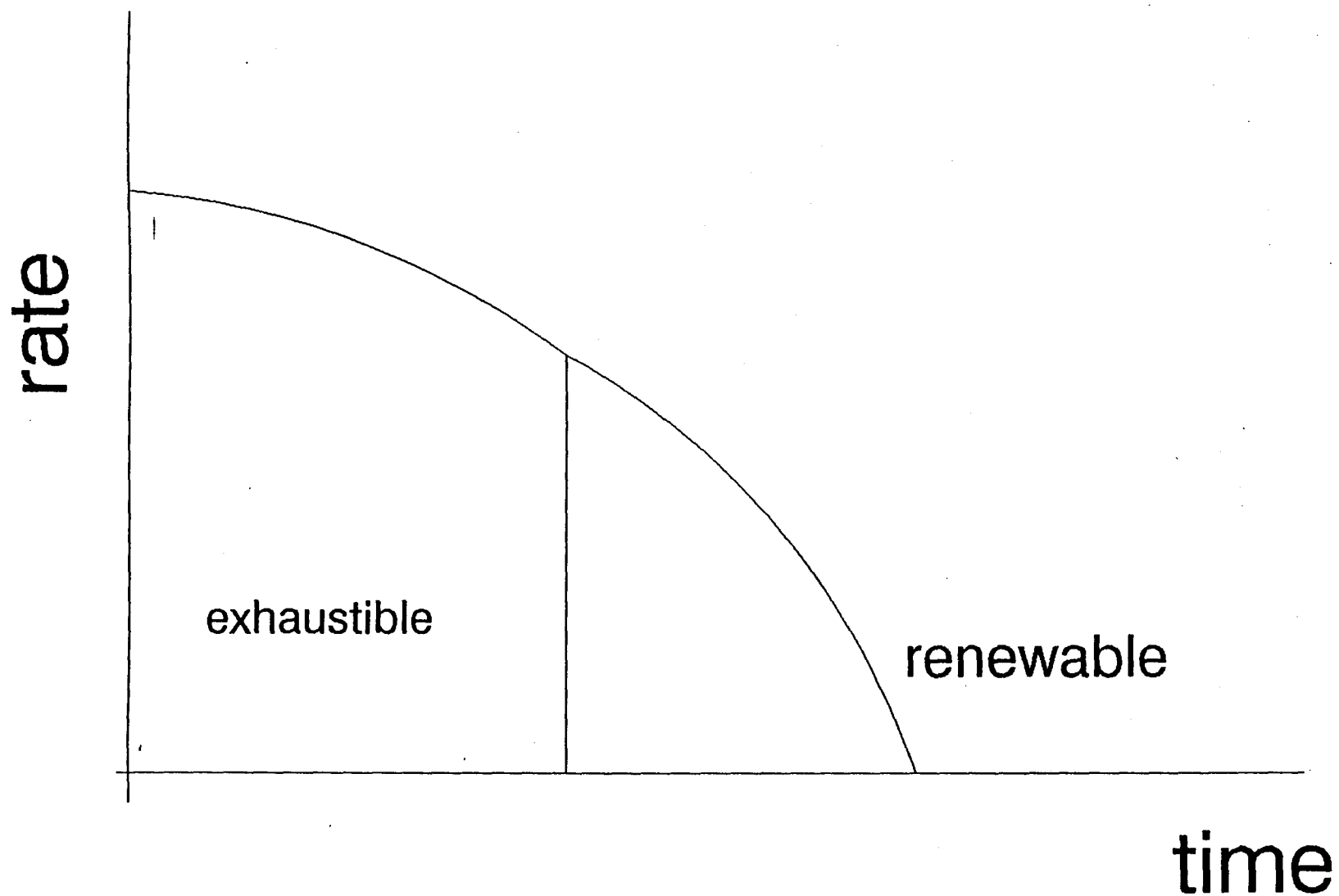
control variables:  $q(t), y(t)$

The hamiltonian and the first order conditions of this problem are the same as in the perfect substitute case, but it should be noted that now  $U_q$  is different than  $U_y$ .

Taking the time derivatives of (15) and (16) and equating them with (17)-(19) yields the time path of the exhaustible resource extraction and the renewable resource production:

$$\dot{q} = [U_{yy}(U_q(r+b) + U_y G_S a - e^{-rt} b \sigma) - U_{qy}(r - G_H) U_y] / \Delta \quad (21)$$

Figure 3: JOINT USE OF EXHAUSTIBLE AND RENEWABLE RESOURCES WITH POLLUTION  
The Perfect Substitute Case





$$\dot{y} = [U_{qq}(r - G_H) U_y - U_{yq}(U_q(r + b) + U_y G_S a - e^{rt} b \sigma)] / \Delta \quad (22)$$

where

$$\Delta = U_{qq} U_{yy} - (U_{qy})^2$$

Due to the exponential factor, the production of the renewable resource cannot have a steady state. In addition, its time derivative is negative, implying that at one point the renewable resource will be exhausted. Figure 4 depicts a possibility for the extraction/production paths.

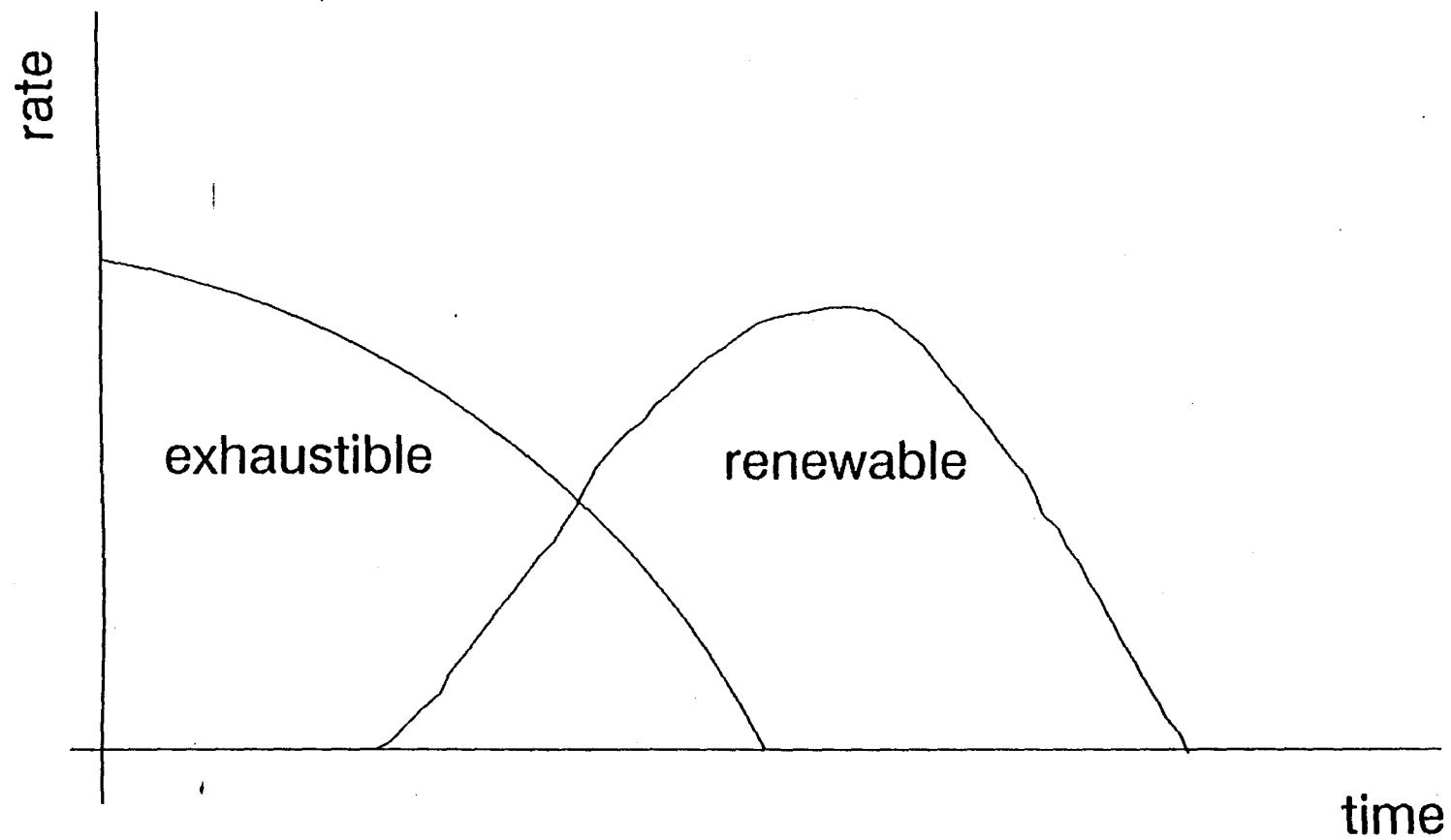
[place figure 4 here]

### Final Considerations

In the sections above, we have discussed an economy that derives its welfare from the exploitation of two resources, one exhaustible and the other renewable. In the first instance, the exhaustible resource has no negative effect on the renewable side, and sustainable growth as defined in the existence of a steady state is attainable. When a pollution factor flowing from the exhaustible to the renewable resource is introduced, steady state and thus sustainability is no longer reached. The renewable resource sooner or later becomes extinct, and the economy is doomed to collapse some time after the total economic depletion of the exhaustible resource.

Besides comparing the case of pollution with the case without pollution, at least three results arise from our model. Pollution at any level becomes a factor for extinction of the renewable resource, unless the environment's "cleaning rate" is sufficiently large to compensate the negative factor. Therefore, since steady state for the renewable resource is not possible with positive rates of marginal growth, the definition that we are using for sustainability is not satisfied. This result contradicts the findings of Berck (1981), in which a steady state is attainable. This likely stems from the fact that Berck treats this "alien" factor as a component of the welfare function, and

Figure 4: JOINT USE OF EXHAUSTIBLE AND RENEWABLE RESOURCES WITH POLLUTION  
The Imperfect Substitute Case



does not consider an exhaustible source.

We have been avoiding the use of the term "externality", for in our case a single agent controls both renewable and exhaustible resources. In fact, in the cases studied here, the externality notion is absent, because no other party is affected by the pollution of the exhaustible resource exploitation. The exercise becomes a trade-off between the production of a renewable resource, the extraction of an exhaustible resource, and its pollution.

In addition, contrary to most of the available literature on sustainability where extinction is either not optimal or suboptimal, in our case unsustainable production of the renewable resource in present value terms is indeed optimal. As previously discussed, this result arises from the maximization of a standard welfare function, and under different assumptions may no longer hold. One such assumption is the introduction of irreversibility factors as analyzed, for example, in Bishop (1978) and Fisher and Hanemann (1990).

Nonetheless, this doomsday scenario may closely explain the behavior of several developing nations and previously socialist economies. The basis of the problem rests in the idea of property rights. In many countries, exhaustible resources such as minerals and renewable resources such as fish and forests belong to the state or to society at large. Governments are generally responsible for distributing exploitation rights or exploiting the resources through their own corporations. This is in essence a single ownership case. This does not imply, however, that externality considerations are irrelevant; rather, it suggests that focusing on externalities only may not fully explain the issue of pollution and unsustainable growth.

A third interesting outcome of the model lies on the possibility for correcting of the pollution "problem". Since under the described cases with pollution, the social planner is actually behaving optimally, what would it take to change his / her behavior and reach the sustainability of the two initial examples?

A concerned multinational institution, that has as objective sustainable

growth, could attempt to influence the social planner by subsidizing conservation. This goal may be attained at lower costs than initially expected. If the multinational institution acknowledges a direct benefit to the rest of the world of not polluting a renewable resource, it can suggest a change in the objective function of the resources' owner and compensate the country for the change. In a similar instance, Berck (1981) has found the possibility of steady growth.

Yet, if the multinational institution attempts to deal directly with the problem described above, it will face substantial costs. As previously shown, the "pollution factor" grows at an exponential rate. Unless the environment's "cleaning rate" is sufficiently large (usually not the case), any attempt to change this situation would have to be considerable so as to compensate for the exponential growth. If a partial payment approach is taken, the only effect that the policy might have is delaying the extinction of the renewable resource.

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