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A Theory of Weekly Specials

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Abstract

Multiproduct retailers facing similar costs and serving the same public commonly announce different weekly specials. These promotional prices also seem to evolve randomly over the weeks. Here, weekly specials are viewed as the strategic outcome of an oligopolistic price competition among multiproduct retail stores facing nonconvex costs. Existence of an equilibrium in mixed strategies is proven. Identical stores serving the same public will never charge the same price vector with probability one (cross-store price dispersion). Mixed strategies can generate random price dispersion over time in the repeated version of the model.

Keywords: Bertrand competition, weekly specials, sales, price promotion, loss leading, retail stores, supermarkets. JEL Classification: D43, L13, M21, M37.

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1 Introduction

All over the world, families receive weekly information about special prices practiced by different stores (such as competing supermarkets, drugstores, and electronics and hardware stores). Typically, the weekly specials are announced by multiproduct retailers through catalogs mailed to surrounding addresses.[1] Two key features distinguish weekly specials from other price promotions. First, the announced prices differ across similar stores serving the same public. Second, in many cases, the special prices seem to evolve randomly over the weeks.

The existing literature raises different motives for spatial and intertemporal price dispersion. None of these theories is able to generate these two particular features of the weekly specials. This paper develops a game theoretical model to explain the pricing policies of similar stores competing for well-informed consumers. Weekly specials are viewed as the strategic outcome of an oligopolistic price competition in an environment where consumers buy multiple products in the same place and stores face a nonconvex technology. Consumers shopping for different products in a single place and technological nonconvexities associated with the retailing service are the basic elements behind the existence of multiproduct stores, as argued next. It is shown here that these two elements are also behind the existence of weekly specials.

Price Dispersion, Sales, and Loss Leading

The existing explanations for cross-store price dispersion argue that consumers pay different prices for the same basket of goods because: (i) they face differentiated transportation costs for visiting each store (see Shilony, 1977); (ii) they are not perfectly informed about the prices practiced by competing stores (see Salop and Stiglitz, 1977, Varian, 1980, and Burdett and Judd, 1983); (iii) they face search costs for acquiring information about the prices practiced by each store (see Robb, 1985). None of these motives seem appealing to explain price dispersion across similar stores serving well informed consumers who face identical transportation costs.

Price dispersion has also been studied in the sales literature. Sales are motivated by different reasons, such as: (i) to attract customers during seasons with low store traffic; (ii) to promote stock clearing in order to avoid losses from products whose values decrease with the length of time on the shelf (see Lazear, 1986, and Pashigian, 1988); (iii) to advertise a new product (see Bass, 1980); and (iv) to discriminate

[1] TV and radio advertisements are also used sometimes, and the special promotions have a periodicity different from one week in some sectors.

prices over time in order to extract surplus from impatient and inelastic consumers (see Stokey, 1981). Unlike weekly specials, sales do not generate systematic price dispersion across neighboring stores. Moreover, the intertemporal price dispersion generated by sales is not random over the weeks (in fact, they tend to follow well defined rules depending on the particular motivation for the sale).

A third related problem studied in the literature is loss leading—the practice of charging less than the marginal costs for some products in order to increase the sales of other products either due to complementarity effects or via attracting more customers. Holton (1957) first pointed out to this pricing policy practiced by monopolistic multiproduct retailers. However, weekly specials are frequently practiced by competing stores and, as shown by Bliss (1988) and Walsh and Whelan (1999), loss leading is not an equilibrium outcome in a scenario where two or more stores compete via prices in a monopolistic competition setting. In the oligopolistic model studied here, loss leading might emerge in equilibrium but not as cause of weekly specials.

A Model of Oligopolistic Retail Pricing

Retailers add services to the original goods, leading consumers to purchase these goods from them rather than directly from the producers. For instance, retailers deal with different producers and transport all products to the same place, reducing the shopping time expended by consumers. They also provide quality assurance by searching for good products among different producers. Moreover, they bargain for better prices in sectors where the wholesale market is not competitive—acting on behalf of a large number of individuals who separately would have low bargaining power against the manufacturers. All such activities are subject to fixed costs, such as the investments needed for transporting different goods and the time expended on searching and bargaining. These technological nonconvexities restrict competition and drive the pricing behavior in this sector.

This paper models price competition among $J > 1$ multiproduct retailers facing fixed transaction costs. The stores choose which products to offer and a retail price for each of them. The retail and wholesale prices are linear (i.e., the price per unit does not depend on the number of units traded). The consumers are perfectly informed about prices and choose freely one store to shop from. Attracting customers and pricing product complementarities are the two key elements driving the retailers' pricing strategies.

Models of multiproduct price competition with nonconvex costs have not yet been explored in the economic literature. This is probably due to the difficulty in

characterizing an equilibrium for them. The presence of fixed costs rule out the standard Bertrand equilibrium (in which prices equal marginal costs). Moreover, the usual arguments for existence of a Nash equilibrium do not apply in this setting because the payoff functions need not be continuous.

Here, existence of a Nash equilibrium in mixed strategies is proven by means of the techniques developed in the literature of games with discontinuous payoffs (specifically, in Simon and Zame, 1990). The equilibrium outcome replicates the basic features of weekly specials. First, identical retailers facing positive fixed costs will never charge the same prices with probability one (i.e., there is price dispersion across identical stores). This result follows from the fact that such an equilibrium would imply two or more stores charging prices above marginal costs (since the stores face fixed costs) and, then, there would be a profitable deviation for one of them. Besides, the mixed-strategy equilibrium can generate random price dispersion over the weeks in the repeated version of the game. This is, in fact, the only possible outcome when consumers are homogeneous—as degenerated strategies are not played in equilibrium when fixed costs are positive and consumers are identical.

The remainder of the paper is organized as follows. Section 2 describes the model; Section 3 develops an equilibrium concept and states the existence theorem; Section 4 discusses the equilibrium properties; and Section 5 concludes. Simple proofs are sketched in the text and the existence proof is left for the appendix.

2 The Model

Consider an economy with $L+1$ goods, $I > 1$ consumers, and $J > 1$ retail stores. Good zero is the numeraire good, whose price is normalized to one. The other $L > 1$ goods are the consumption goods, which are produced by a competitive sector and sold for the retailers at a fixed per unit cost, $c_l > 0$, $\forall l = 1, ..., L$.[2]

The consumers are endowed with $w^i > 0$ units of the numeraire good. Their preferences are represented by the demand functions, $x^i : \mathbb{R}^L_+ \rightarrow \mathbb{R}^{L+1}_+$, and the indirect utility functions, $v^i : \mathbb{R}^L_+ \rightarrow \mathbb{R}$—where $\mathbb{R}^L_+$ is the space of prices for the L consumption goods. These functions are assumed to display the following properties: (i) v^i is continuous and decreasing in prices; (ii) x^i is continuous in prices and there exists a threshold level $p_{\max} > 0$ such that $p_l \geq p_{\max}$ implies $x^i_l(\cdot) = 0$—where p_l is the price of good l faced by consumer i.

The lack of market expertise and the need for special transportation for each

[2] As usual, L, I, and J refer to positive integer numbers.

type of good preclude the consumers from interacting directly with the wholesale sector. Furthermore, shopping is assumed to be unpleasant and consumers must buy all products in the same place. In the real world, these are the two main forces behind the existence of multiproduct retail stores.

The retailers are profit-maximizing enterprises which face the competitive wholesale prices $c = (c_1, ..., c_L)$ and choose the retail prices $p^j = (p_1^j, ..., p_L^j)$ to be charged from their customers. Once the price vector p^j is announced, the store j must deliver all units demanded by its customers.[3]

Transacting with the wholesalers is costly. A trip to wholesaler l costs $r_l \geq 0$, regardless of the volume traded. These fixed costs account for market expertise and special transportation costs inherent to the retailing activity. They are assumed to be small, such that a monopolistic store would be able to make positive profits (otherwise the game becomes uninteresting as stores would have no motive to operate).

Retail prices lie in the compact set $\Gamma = [0, p_{\max}]^L$. For the sake of notational simplicity, the choice $p_l^j = p_{\max}$ is interpreted as if retailer j did not transact with wholesaler l. The transaction cost r_l is avoided whenever $p_l^j = p_{\max}$, and the choice $p^j = \vec{p}_{\max} = (p_{\max}, ..., p_{\max})$ represents the non-entry decision.

This economy is modeled as a two-stage game. In the initial node, the retailers choose (simultaneously) the retail prices to be charged from their customers. In the second stage, after observing the retail offers, the consumers choose a retail store to shop from.

2.1 The Second Stage

After observing the retail prices from the first stage, $p = \left(p^1, ..., p^J\right)$, the consumers go shopping in one of the retail stores. They compare the welfare that would be obtained by shopping from different stores and, then, decide which store to go. Consumer i's favorite retail store, $s^i(p) \in \mathbb{J} = \{1, ..., J\}$, must be such that:

$$v^i\left(p^{s^i}\right) \geq v^i\left(p^j\right), \forall j \in \mathbb{J}. \tag{1}$$

Note that some consumers might be indifferent between two (or more) stores to shop from. Hence, there might be multiple second-stage solutions associated with

[3]For notational simplicity, stores are not allowed to offer limited quantities of the promotional products. This could be incorporated into the model, with additional notation, without changing the main forces driving the results.

each p. No exogenous rule is imposed here to define what consumers do in case of indifference. As defined in Section 3, the stores acting in the first stage will hold endogenous and consistent beliefs about the likelihood of each second-stage solution being played.

2.2 The First Stage

In the initial node, the J profit-maximizing retailers move simultaneously and choose their offers, $p^j \in \Gamma$. For each possible price vector chosen in the first stage, $p \in \Gamma^J$, there will be a second-stage solution, $\left\{s^i(p), \forall i \in \mathbb{I}\right\}$, which yields the following payoff for retailer j:

$$\text{Profit}\,(j,p) = \sum_{\{i \in \mathbb{I}: s^i(p)=j\}} \sum_{l=1}^{L} \left(\left(p_l^j - c_l\right) x_l^i(j,p^j) - \chi\left(p_l^j\right) r_l\right), \tag{2}$$

where $\chi\left(p_l^j\right)$ is an indicator function that equals 0 when $p_l^j = p_{\max}$.

3 Equilibrium

All subgame perfect Nash equilibria for this game can be computed backwards. In the second stage, the consumers' actions are not strategic. For each node p, a second-stage solution is a list of s^i satisfying condition (1), $\forall i \in \mathbb{I} = \{1, ..., I\}$. Anticipating the possible second-stage solutions associated with each p, the retailers make their first-stage decisions.

In order to precisely define a subgame perfect Nash equilibrium, a few technical details will be needed.[4] Note that there might be multiple second-stage solutions associated with each $p \in \Gamma^J$. In these cases, the retailers acting in the initial node will have to hold beliefs about the likelihood of each solution occurring. Here, these beliefs are modeled as endogenous sharing rules, in the spirit of Simon and Zame (1990).

First, define $\Xi(p)$ as the set of all second-stage equilibria associated with the price schedule $p \in \Gamma^J$. Formally, let $\Xi : \Gamma^J \to \mathbb{J}^I$ be a correspondence such that $\left(s^1, ...s^I\right) \in \Xi(p)$ if and only if s^i satisfies (1), $\forall i \in \mathbb{I}$. Second, let $\Pi(p) = \left(\Pi^1(p), ..., \Pi^J(p)\right)$ be the set of all profit payoffs that can be achieved in the second-

[4] Braido (2005) uses a similar approach in a financial innovation context.

stage solutions associated with p, that is: $\Pi(p) = \{\theta \in \mathbb{R}^J : \exists \left(s^1, ...s^I\right) \in \Xi(p)$ s.t. $\theta^j = \text{Profit}(j,p)$ as in (2), $\forall j \in \mathbb{J}\}$.

Finally, let $co\Pi$ be the convex hull of Π and $\pi = (\pi^1, ..., \pi^J)$ be a Borel measurable function selected from $co\Pi$.[5] Since $\Pi\left(p\right)$ represents the set of all profit profiles associated with p, $co\Pi(p)$ defines the universe of all possible expected profit payoffs associated with p and $\pi(p)$ is a particular expected profit function reflecting the retailers' beliefs about the second stage.

Definition 1. The retailers' expected profit is a Borel measurable function, $\pi : \Gamma^J \rightarrow \mathbb{R}^J$, selected from the correspondence $co\Pi : \Gamma^J \rightarrow \mathbb{R}^J$.

Therefore, the reduced game in the initial node is given by $\{\mathbb{J}, \Gamma, \pi\}$. Each retailer chooses a strategy (potentially mixed) to maximize π^j. Define $\mathcal{B}$ as the Borel sigma-algebra for Γ and Λ_Γ as the set of probability measures on $(\Gamma, \mathcal{B})$. Thus, retailer j's strategy is a probability measure $\sigma^j \in \Lambda_\Gamma$ that is chosen as a best response to the other retailers' strategies (σ^{-j}). Formally, the problem faced by each retailer $j \in \mathbb{J}$ in the initial node is:

$$\max_{\sigma^j \in \Lambda_\Gamma} \int \pi^j(p) d\sigma^j \times d\sigma^{-j}. \tag{3}$$

Definition 2. A subgame perfect Nash equilibrium consists of a Borel measurable function selected from $co\Pi$ and pricing mixed strategies, namely $\pi = \left(\pi^1, ..., \pi^J\right)$ and $\sigma = (\sigma^1, ..., \sigma^J)$, such that σ^j solves (3), given σ^{-j}, for all $j \in \mathbb{J}$.

Proposition 1. There exists a subgame perfect Nash equilibrium for this economy.

Proof. See appendix. ■

4 Equilibrium Properties

Proposition 1 proves the existence of an equilibrium in mixed strategies for the model of price competition among multiproduct retailers. Here, I show two key properties of the equilibrium.

[5]The convex hull of a given set Ω is the smallest convex set containing Ω.

4.1 Cross-Store Price Dispersion

One must first note that, in equilibrium, there will always be at least two entrant stores (i.e., stores whose strategies are not degenerated in $\vec{p}_{\max}$). This is proven in Proposition 2.

Proposition 2. In equilibrium, there will always be at least two entrant stores.

Proof Sketch. If all stores chose $p^j = \vec{p}_{\max}$, then any of them would profit by deviating and choosing a price vector that yields the monopoly profit (which is positive by assumption—see Section 2). Moreover, if all but one store chose $p^j = \vec{p}_{\max}$, then the entrant should be making the monopoly profit and there would be a profitable deviation for any of the nonentrant stores. By charging slightly less than the monopoly price, a nonentrant store could attract all customers from the entrant (thanks to v^i being decreasing in prices) and, then, make positive profits (thanks to continuity of individual demand functions). ■

Propositions 1 and 2 do not rule out the possibility of an equilibrium where all mixed strategies are degenerated in the same price vector. In this case, there would be no price dispersion across stores (a key feature of weekly specials). The next proposition shows that this possibility is never an equilibrium when the fixed transaction costs are positive.

Proposition 3. If $r_l > 0$ for all l, then there is no equilibrium strategy in which two or more entrant stores choose the same price vector with probability one.

Proof Sketch. Note first that there is no pure strategy equilibrium in which a store charges less than the wholesale prices for all products offered—otherwise, the store would make a loss since the fixed costs are positive. But if two or more stores chose the same price vector different from $\vec{p}_{\max}$, with at least one retail price above its wholesale cost, then there would be a profitable deviation for any of them. By making an infinitesimal reduction in one of the prices charged above the marginal cost, a store can attract all customers from the other stores and keep its own customers (thanks to v^i being decreasing in prices). Since the consumers' demand functions are continuous, this deviation would increase profits. ■

Obviously, Proposition 3 does not hold when $r_l = 0$ for all l, in which case the standard Bertrand solution (where prices equal marginal costs) would be an equilibrium. This stresses the importance of technological nonconvexities in the

determination of pricing policies of competing retailers. Surprisingly, this result shows that competing retailers facing fixed costs will never charge the same prices even though they access the same technology and serve the same consumers who are perfectly informed and free to choose which store to attend. This was the main puzzling characteristic of weekly specials.

4.2 Intertemporal Price Dispersion

The model presented here is static but, in order to study intertemporal price dispersion, one could imagine this game being repeated for a finite or infinite number of weeks. Stores playing a static Nash equilibrium in each week is a possible equilibrium outcome. (In fact, it is the only subgame perfect equilibrium outcome if the number of repetitions is finite.)

Prices could then differ over time due to two reasons. First, if the static game had multiple equilibria, the stores may play a different pricing equilibrium in each week. Furthermore, if there is one static equilibrium in which stores use nondegenerated mixed strategies, then the equilibrium where this strategy profile is repeated every week will generate a random price dispersion over the weeks.

So far, one cannot rule out the possibility of a game having only one equilibrium in pure strategies, in which case the finitely repeated game will not display intertemporal price dispersion. (Even though one would still have price dispersion across neighboring stores, as shown in Section 4.1.) The next proposition identifies preference heterogeneity as a key element behind this possibility. It shows that no pure strategy equilibrium exists when consumers have homogeneous preferences. As a consequence, weekly specials evolving randomly over time are the only possibility in neighborhoods and sectors where consumers have identical tastes for the products being sold.

Proposition 4. If $u^i = u$ and $r_l > 0$, for all i and l, then there is no equilibrium in pure strategies.

Proof Sketch. Suppose that $(\pi, p) \in co\Pi \times \Gamma^J$ was a pure strategy equilibrium and notice that there would exist a profitable deviation for at least one of the stores. To see this, note first that there is no pure strategy equilibrium in which a store charges less than the wholesale prices for all products offered. From Proposition 2, there must be at least two entrant stores. From Proposition 3, these two entrant stores (say, stores 1 and 2) must be charging different prices, i.e., $p_1 \neq p_2$. Moreover, they must be selling strictly positive quantities, since their revenues must at least pay

the fixed and variable costs. Hence, it must be the case that the identical consumers are indifferent between the two stores, i.e., $v^i(p^1) = v^i(p^2)$, for all i. But then each store could increase its profit by promoting an infinitesimal reduction in prices. By doing so, it will attract the customers from the other store (since v^i is decreasing in prices) and increase profits (since the demand functions are continuous). ■

5 Conclusion

This paper models price competition among multiproduct retail stores facing fixed transaction costs (multiproduct Bertrand competition with fixed costs). Consumers are perfectly informed about prices and they must choose only one store to buy their bundles from. An equilibrium exists and is able to replicate the key features of weekly specials. When fixed costs are positive, the retailers will never charge the same prices, even though they are identical and act in the same neighborhood. Moreover, if consumers are homogeneous, the equilibrium strategies will be totally mixed (i.e., nondegenerated distributions). Hence, the model is able to predict not only price dispersion across stores but also random price dispersion over time.

A Proof of Proposition 1

First, notice that $\Xi(p)$ is nonempty—that is, there is a second-stage solution associated with each $p \in \Gamma^J$. This is straightforward since the consumers' utility functions are continuous, their budget sets are compact for positive prices, and the number of retail stores is finite.

One needs to prove the existence of a solution for the reduced game described in Section (3). Since Γ is compact, one needs only to prove that $co\Pi$ is bounded and upper hemicontinuous with nonempty, convex, and compact values. The existence result would then follow from the main theorem in Simon and Zame (1990), pp. 865.

Lemma 1. The correspondence $co\Pi : \Gamma^J \to \mathbb{R}^J$ is bounded and upper hemicontinuous with nonempty, convex, and compact values.

Proof. First, note that $\Pi : \Gamma^J \to \mathbb{R}^J$ is bounded—gains and losses are bounded since the demand functions are bounded for p in Γ^J. Moreover, Π has a closed graph and is therefore upper hemicontinuous and compact valued. To see the closed-graph

property, take any two sequences $\{p_n\} \rightarrow \bar{p}$ and $\{\theta_n\} \rightarrow \bar{\theta}$ s.t. $p_n \in \Gamma^J$ and $\theta_n \in \Pi(p_n)$, $\forall n \in \{1, 2, ..., \infty\}$. From the definition of $\Pi(p_n)$, there exists $\left(s_n^1, ..., s_n^I\right) \in \Xi\left(p_n\right)$ such that $\theta_n^j = \text{Profit}(j, p_n^j)$ as in (2), $\forall j, n$. The sequence $\left\{s_n^1, ..., s_n^I\right\}_{n=1}^{\infty}$ is bounded and, then, there exists a subsequence converging to $\left(\bar{s}^1, ..., \bar{s}^I\right)$. Since $s_n^i \in \mathbb{J} = \{1, ..., J\}$, this subsequence must be constant for sufficiently large n. Hence, for sufficiently large n, one has $v^i\left(p_n^{\bar{s}^i}\right) \geq v^i\left(p_n^j\right)$, $\forall j$. Since v^i is continuous in prices, this implies $v^i\left(\bar{p}^{\bar{s}^i}\right) \geq v^i\left(\bar{p}^j\right)$, $\forall j$. Thus, $\left(\bar{s}^1, ..., \bar{s}^I\right) \in \Xi\left(\bar{p}\right)$ and $\bar{\theta} = \lim_{n \rightarrow \infty} \theta_n \in \Pi(\bar{p})$.

Therefore, $co\Pi$ is bounded and compact valued since Π is bounded and compact valued; and it is upper hemicontinuous since Π is compact valued and upper hemicontinuous. Finally, $co\Pi$ is nonempty since $\Xi\left(p\right)$ is nonempty and convex from the definition of convex hull. ■

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