



FUNDAÇÃO
GETULIO VARGAS

EPGE

Escola de Pós-Graduação em
Economia

SEMINÁRIOS
DE PESQUISA
ECONÔMICA

“A FRACTIONAL COINTEGRATION ANALYSIS OF PURCHASING POWER PARITY FOR BRAZIL”

DENISARD ALVES

(USP)

LOCAL

Fundação Getúlio Vargas
Praia de Botafogo, 190 - 10º andar - Auditório

DATA

21/11/96 (5ª feira)

HORÁRIO

16:00h

332.413

A 474 F

F

arte

MARCO
ANTONIO

Coordenação: Prof. Pedro Cavalcanti Ferreira
Tel: 536-9353

A Fractional Cointegration Analysis of Purchasing Power Parity for Brazil

Vera Fava¹
Denisard Alves

The idea of equilibrium exchange rate has worried economists in attempting to calculate the correct exchange rate. This idea has been a major building block of models of exchange rate determination and has also been used to provide a rule for exchange rate evaluation in policy discussion.

Although empirical studies report significant deviations from PPP in the short run, the validity of Purchasing Power Parity (PPP) in the long-run has been motive for controversy. Adler and Lehmann (1983), Darby (1983), and Roll (1979) provided evidences suggesting that the real exchange rate can be seen as a random walk process, concluding that shocks have long lived effects. On the other hand, authors such as Abuaf and Jorian (1990), and Roll (1979) presented reported evidence of PPP reversion using multivariate unit root tests. Zini and Cati (1993), using data for Brazil, did not reject the random walk as the model for the real exchange rate in the long run

The apparently mixed findings reflect a major problem associated with the test of long run PPP. A test for the long run PPP entails proper modeling of the low frequency dynamics of economic variables and their equilibrium relationship, while allowing for significant deviations from equilibrium in the short-run. The results depend crucially on the power of the statistical technique used to separate the low frequency from the high-frequency dynamics. The uses of a statistical procedure that can identify a rich class of low frequency dynamics and detect long run relationships from noise data thus appears desirable.

¹ The authors are Professors of the School of Economics, Business Administration and Account of The University of São Paulo.

This study examines the relevance of long run PPP for Brazil using a fractional cointegration framework that integrates the notions of cointegration, suggested by Engle and Granger (1987), and fractional differencing, introduced by Granger and Joyeux (1980) and Hosking (1981) to economics following the study of Cheung and Lai(1993) , using a richer class of statistical model than the Zini and Cati(1993) study.

This article has the following organization Section 1 briefly discuss the PPP relationship. Section 2 discusses the notion of fractional integration and cointegration. Section 3 presents the tests for long run PPP, section 4 reports the results section 5 concludes.

1. THE PPP RELATIONSHIP

The PPP doctrine suggests that currencies are valued for the goods they can buy and, in equilibrium, a given basket of goods should cost the same at home and abroad, in the presence of international arbitrage. This implies a long- run equilibrium relationship between national price levels expressed in common currency units.. For the purpose of empirical testing, the PPP relationship is written as

$$ep_t = \alpha + \beta p_t + \varepsilon_t \quad (1)$$

where α and β are constants, ep_t is the foreign price index converted to domestic currency units, p_t is the domestic price index; and ε_t is an error term capturing deviations from PPP. All variables are in logarithms². A necessary condition for PPP to hold in the long-run is that u_t is a mean reverting process, that is the effect of a chock to the PPP relationship will die out.

This form the basis for a cointegration test. If ep_t and p_t are found to be cointegrated, a linear combination of the variables will be mean-reverting, implying that these variables are moving apart from each other through time. The cointegration approach is useful, since it allows data to determine the underlying long-run relationship and its short-run deviations without imposing the homogeneity condition³.

What price index to use for PPP calculations is an open issue, as discussed by Frenkel (1978). Usually the choice is between wholesale price indexes (WPI's) and consumer

² Under the homogeneity condition, the constant of equation (1) is equal to unity. Taylor(1988) noted that, since observed price index are imperfect proxies at the best for the theoretical price variables, the homogeneity condition does not generally hold empirically. Hence α_t should be estimated rather than imposed *a priori* to be unit. Zini and Catti(1993) imposed the homogeneity condition in their estimation. A fact that could account for the rejection of the cointegration test they used.

³ In general, the cointegration approach can be applied whether the homogeneity condition holds or not , see Cheung and Lai(1993).

price indexes (CPI's). WPI places a heavier weight on tradables than CPI's, and WPI's tend to yield more favorable test results to long run PPP than CPI's (e.g. Kim 1990).

The data used in this study are annual data for Brazil from 1855 to 1990 taken from Zini and Cati (1993). The domestic and foreign price indexes are CPI's measurements. Foreign price levels are expressed in terms of Brazilian currency units by multiplying the price indexes by the exchange rates.

2. FRACTIONAL INTEGRATION AND COINTEGRATION

Engle and Granger (1987) and Granger (1986) suggested the use of the notion of a long-run equilibrium relationship between time series. A series is said to be integrated of order d , denoted by $I(d)$, if it has a stationary, invertible autoregressive-moving average (ARMA) representation after applying the differencing operator $(1-L)^d$. When d is not an integer, the series is said to be fractionally integrated.

Let $X_t = (x_{1t}, x_{2t})'$, where x_{1t} and x_{2t} are supposed to be $I(d)$. In this case the linear combination

$$z_t = \delta X_t \quad (2)$$

will also generally be $I(d)$. If a vector δ exists such that z_t is $I(d-b)$ with $b > 0$, however, x_{1t} and x_{2t} are said to be cointegrated of order (d, b) , and $\delta X_t = 0$ represents an equilibrium constraint operating on the long run component of X_t (Granger 1986).

The typical case considered in empirical work is one in which $d = b = 1$; that is, x_{1t} and x_{2t} are $I(1)$ and z_t is $I(0)$. Thus cointegration requires that the equilibrium error, z_t , is mean-reverting even though x_{1t} and x_{2t} wander widely. The mean reversion behavior of the equilibrium error is of key interest, if economic theory suggests a long-run equilibrium relationship between x_{1t} and x_{2t} . Unless the equilibrium error exhibits mean reversion, a shock to the system will tend to drive x_{1t} and x_{2t} out of equilibrium forever, making the notion of equilibrium of little use even in the long-run.

Engle and Granger (1987) proposed a test procedure for cointegration for which there is an intuitive economic interpretation consistent with the notion of long-run equilibrium. The Engle-Granger procedure consists of two steps: Regress x_{1t} on x_{2t} (or vice-versa) as the equilibrium or the cointegrating regression then check if its residuals are $I(0)$ or not using a unit-root test. If the residuals are found to be $I(0)$, the null hypothesis of no cointegration is rejected.

Analytically, however, the strict $I(1)$ and $I(0)$ distinction is arbitrary. This is an important point because for the equilibrium error to be mean-reverting, it does not have to be $I(0)$

exactly. Fractionally integrated process, as proposed by Granger and Joyeux (1980) and Hosking (1981), also display mean reversion.

A fractionally integrated process can be represented by

$$C(L)(1-L)^d z_t = D(L)v_t \quad (3)$$

$C(L) = 1 - C_1 L - \dots - C_p L^p$, $D(L) = 1 + D_1 L + \dots + D_q L^q$, all roots of $C(L)$ and $D(L)$ lie outside the unit circle, v_t is iid(0, σ^2), and the fractional differencing operator is:

$$(1-L)^d = \frac{\sum_{k=0}^{\infty} \Gamma(k-d) L^k}{\{\Gamma(k+1)\Gamma(-d)\}} \quad (4)$$

where $\Gamma(\cdot)$ is the gamma function. Model (3) is referred to as the autoregressive fractionally integrated moving average (ARFIMA) model, and it extends the standard autoregressive integrated moving average (ARFIMA (p,d,q)) model to real values of d. For $0 < d < .5$, the autocorrelations of z_t show a hyperbolic decay rate proportional to k^{2d-1} , in contrast to a faster, geometric decay of a stationary ARMA process (Hosking 1981). Due to the presence of dependence among distant observations, the ARIMA process is often called a long-memory process.

Model (3) includes $I(1)$, that is, $d=1$, as a special case. The distinction between $d=1$ and $d<1$ is crucial in terms of the mean-reversion property of z_t and so is the cointegration property of x_{1t} and x_{2t} . Although the effect of any shock is known to persist forever for an $I(1)$ process, it dies out, albeit slowly, for an $I(d)$ process with $d<1$. This can be seen by studying the moving average representation for $(1-L)z_t$:

$$(1-L)z_t = A(L)v_t \quad (5)$$

Where $A(L) = 1 + A_1 L + A_2 L + \dots$, derived from

$$A(L) = (1-L)^{1-d} \Phi(L) \quad (6)$$

with $\Phi(L) = C^{-1}(L)D(L)$. The moving average coefficients A_i 's are called the impulse responses (Campbell and Mankiw 1987). The impact of a unit innovation at time t on the value of z at time $t+k$ is equal to $1 + A_1 + A_2 + \dots + A_k$. For a mean-reverting process, the infinite cumulative impulse response $A(1)$ equals zero, implying no long-run impact of the innovation on the value of z . Using equation (4) to find the series representation for $(1-L)^{1-d}$, equation (6) can be written as

$$A(L) = F(d-1, 1, 1; L) \Phi(L), \quad (7)$$

where $F(\cdot)$ is the hypergeometric function defined by

$$F(m, n, p, L) = \frac{\sum_{j=0}^{\infty} \Gamma(m+j)\Gamma(n+j)\Gamma(p)L^j}{\{\Gamma(m)\Gamma(n)\Gamma(p+j)\Gamma(j+1)\}} \quad (8) \text{ (Cheung and Lai 1993)}$$

and from Gradshtein and Ryzhik (1980),

$$F(d-1, 1, 1; L) = 0 \quad (9)$$

for $d < 1$. Hence for $d < 1$,

$$A(1) = F(d-1, 1, 1; 1)\Phi(1) = 0 \quad (10) \text{ (Baillie 1996)}$$

For any process $z \sim I(d)$ and for $d < 1$ it follows from (10) that z will mean reverting. While z will not be covariance stationary for $0.5 < d < 1$ (Hosking 1981), it will nevertheless still be mean reverting, since an innovation has no permanent effect on the value of z . This is in contrast to an $I(1)$ process, which is both covariance non stationary and not mean-reverting. For an $I(1)$ process, the effect of an innovation can persist forever.

To the extent that the equilibrium error can display slow mean reversion, not captured by usual $I(0)$ process, a general test for cointegration should allow for fractional cointegration. Although Engle and Granger (1986) noted that the notion of cointegration can well apply to fractionally integrated process, the only applied work for fractional cointegration is that of Cheung and Lai (1993). Their analysis was for the case in which x_{1t} and x_{2t} are $I(1)$, in this case if the equilibrium error z_t is found to be $I(d - b)$ with $b > 0$, though not necessarily $I(0)$, x_{1t} and x_{2t} are fractionally cointegrated and the effect of a shock to the system will eventually disappear, so that an equilibrium relationship between x_{1t} and x_{2t} will prevail in the long-run.

3. A TEST FOR FRACTIONAL COINTEGRATION

The hypothesis of fractional cointegration raises the problem of testing for fractional integration. Diebold and Rudebush (1991), Sowell (1990) and Fava and Alves (1996) observed that standard unit root tests such as the Dickey-Fuller test may have low power against fractional alternatives. In this article, following Cheung and Lai (1993) and Fava and Alves (1996), a spectral regression based test due to Geweke and Porter-Hudak (1983) is used to detect fractional integration in the error ε_t . The Geweke-Porter-Hudak (GPH) test provide a general test for fractional integration that is not dependent on the nuisance parameter of the underlying process - namely-the parametrization of the ARMA part of the process. The GPH test makes use of the fact that the spectral density of $g_t = (1 - L) \varepsilon_t$ is given by

$$\begin{aligned} f_g &= |1 - \exp(-i\omega)|^{-2(d-1)} f_u(\omega) \\ &= (2 \sin(\omega/2))^{-2(d-1)} f_u(\omega) \end{aligned} \quad (11)$$

where $u_t = \Phi(L)v_t$ is a stationary process and $f_u(\omega)$ is its spectral density. Consider a sample series of g_t of size T . Taking logarithms of (11) and evaluating at harmonic frequencies $\omega_j = 2\pi j/T$ ($j = 0, \dots, T-1$), we have

$$\ln(f_g(\omega_j)) = \ln(f_u(0)) - (d-1)\ln(4 \sin^2(\omega_j/2)) + \ln(f_u(\omega_j)/f_u(0)) \quad (12)$$

For low-frequency ordinates ω_j near 0, say $j \leq n \ll T$, the last term is negligible compared with the other terms. Adding $I(\omega_j)$, the periodogram at ordinate j , to both sides of (12) yields

$$\ln(I(\omega_j)) = \ln(f_u(0)) - (d-1)\ln(4 \sin^2(\omega_j/2)) + \ln(I(\omega_j)/f_g(\omega_j)) \quad (13)$$

This suggests estimating d using a simple linear regression equation

$$\ln(I(\omega_j)) = \beta_0 + \beta_1 \ln(4 \sin^2(\omega_j/2)) + \xi_t \quad j = 1, 2, \dots, n \quad (14)$$

where ξ_t equals $\ln(I(\omega_t)/f_g(\omega_t))$. It is asymptotically iid across harmonic frequencies and $n = g(T)$ is an increasing function of T . The theoretical asymptotic variance of ε_t is known to be equal to $\pi^2/6$. Under some regularity conditions on $g(T)$, satisfied by, for example, T^μ for $0 < \mu < 1$. Geweke and Porter-Hudak (1983) showed that the least squares estimate of β_1 provides a consistent estimate of $1-d$ and hypothesis testing concerning the value of d can be based on the t statistic of the regression coefficient.

4.RESULTS OF COINTEGRATION TESTS

The price series are each first checked for fractional integration using the GPH test⁴. The unit root hypothesis can be tested by determining whether or not the GPH estimate of d from the first-differenced series significantly differs from zero. Under the unit root hypothesis, the first differenced data follow a stationary ARMA process with $d=0$. The results of the GPH estimation are presented in Table1.

By including fractional process as the alternatives, the GPH test provides a different perspective to test the unit root hypothesis. As can be seen in the results, the unit root hypothesis embodied in the first differenced series is accepted by the test, but it also accepts the fractional process as an alternative explanation for the data generating process of the series. The value of d is larger than one but smaller than two, thus suggesting that taking second differences of the series might not be correct, because the correct order of differencing is approximately 1.5, characterizing a fractional alternative for the data generating process of sp_t and p_t series. The results are consistent for different values of $n=T^\mu$, for values of μ from .5 to .6. Note that the estimated values of d varies for different values of μ . This is in accord with GPH's (1983) observation that the estimation result can be "contaminated" as more periodogram ordinates is included in the regression.

⁴ The unit root test, ADF, allowing for a drift and a trend for the individual price series is presented in Zini and Cati (1993). They accept the hypothesis of two unit roots for both series. However, when the series is fractionally integrated the ADF test may have low power against a fractional alternative like the GPH test as noted by Diebold and Rudebush (1991).

TABLE 1
Results of the GPH estimation for the first difference of the sp_t and p_t series

sp_t				p_t			
	$n = T^\mu$	\hat{d}	\hat{t}_{student}		$n = T^\mu$	\hat{d}	\hat{t}_{student}
$\mu = .50$	10	0.5521	4.57	$\mu = .50$	10	0.4793	4.66
$\mu = .55$	15	0.5849	6.84	$\mu = .55$	15	0.5013	6.88
$\mu = .60$	20	0.5417	7.83	$\mu = .60$	20	0.5046	9.33

The test for cointegration is presented in Table 2. The hypothesis of cointegration is rejected by the data. The PPP hypothesis is not sustained as the long run theory for exchange rate change for Brazil. The GPH reports that the residual of the cointegrating regression follows a unit root. Hence the PPP hypothesis can not be seen as the explanation for long-run behavior of exchange rate adjustment for Brazil.

TABLE 2
GPH estimation first difference of the residuals
of the cointegration equation

ε_t			
	$n = T^\mu$	\hat{d}	\hat{t}_{student}
$\mu = .50$	10	0.1199	0.45
$\mu = .55$	15	0.0028	0.02
$\mu = .60$	20	0.1816	0.89

5. CONCLUDING REMARKS

A generalized notion of cointegration, called fractional cointegration, has been used to examine the empirical relevance of long-run PPP based on Brazilian historical data for the 1855-1990 period.

A fractional integration analysis is used due to the fact that we are concerned with the case that the price series are $I(1)$ as reported in Zini and Cati(1993). The conventional unit root test normally used in detecting the order of integration of series is not adequate to

test for fractional alternatives. The results indicate that the $I(1)$ or $I(2)$ and $I(0)$ distinction might not be adequate. The GPH test results indicate that a fractional alternative of 1.5 is more appropriated. The cointegration analysis of the cointegration regression residuals indicates the presence of a unit root, thus showing that shocks do not present a mean reversion behavior which leads like to the rejection of the PPP explanation for long-run exchange rate adjustment for Brazil.

6. REFERENCES

- Abuaf, N. and Jorion, P. (1990), "Purchasing Power Parity in the Long Run." *Journal of Finance*, 45, 157-174.
- Adler, M. and Lehmann, B. (1983), "Deviations From Purchasing Power Parity in the Long Run," *Journal of Finance*, 38, 1471-1487.
- Baillie, R. T., King, M. L. (1996), "Fractional Differencing and Long Memory Processes," *Journal of Econometrics*, 73, 1-59
- Campbell, J. K., and Mankiw, N. G. (1987), "Are Output Fluctuation Transitory ?," *Quarterly Journal of Economics*, 102, 857-880.
- Cheung, Y. W. and Lai, K. S. (1993), "A Fractional Cointegration of Purchasing Power Parity," *Journal of Business and Economics Statistics*, 11, 1, pp.103-112.
- Darby, M. R. (1983), "Movements in Purchasing Power Parity : The Short and Long Runs," in *The International Transmission of Inflation*, eds. M. R. Darby, J. R. Lothian, A. E. Gandolfi, A. J. Schwarz, and A. C. Stockman, Chicago : University of Chicago Press, pp. 462-477.
- Diebold, F. X., Husted, S., and Rush, M. (1991), "Real Exchange Rates Under the Gold Standard," *Journal of Political Economy*, 99, 1252-1271.
- Engle, R. F., and Granger, C. W. J. (1987), "Cointegration and Error Correction : Representation, Estimation, and Testing," *Econometrica*, 55, 251-276.
- Fava, V. L., and Alves, D. C. O. (1996), "Integração Fracionária e Inflação," Economic Department FEA-USP, *Unpublished*.
- Frenkel, J. A. (1978), "Purchasing Power Parity: Doctrinal Perspective and the Evidence From the 1920's," *Journal of International Economics*, 8, 169-191.
- Geweke, J., and Porter-Hudak, S. (1983), "The Estimation and Application of Long Memory Time Series Models," *Journal of Time Series Analysis*, 4, 221-238.
- Gradshteyn, I. R., and Ryzhik, I. M. (1980), *Table of Integrals Series, and Products*, New York: Academic Press.
- Granger, C. W. J., and Joyeux, R. (1980), "An Introduction to Long Memory Time Series Models and Fractional Differencing," *Journal of Time Series Analysis*, 1, 15-39.
- Hosking, J. R. M. (1981), "Fractional Differencing," *Biometrika*, 68, 165-176.
- Kim, Y. (1990), "Purchasing Power Parity: Another Look at the Long-Run Data," *Economics Letters*, 32, 334-339.
- Porter-Hudak, S. (1990), "An Application of the Seasonal Fractionally Differenced Model to the Monetary Aggregates," *Journal of the American Statistical Association*, 85, 338-344.
- Roll, R. (1979), "Violation of Purchasing Power Parity and Their Implications for Efficient International Commodity Markets," in *International Finance and Trade*, eds. M. Sarnat and G. Szego, Cambridge, MA: Ballinger, pp. 133-176.
- Sowell, F. (1990 a.), "The Fractional Unit Root Distribution," *Econometrica*, 58, 495-505.
- _____ (1990 b.) "Maximum Likelihood Estimation of Stationary Univariate

Fractionally Integrated Time Series Models,” working paper, Carnegie Mellon University, GSIA.

Taylor, M. P. (1988), “An Empirical Examination of Long Run Purchasing Power Parity Using Cointegration Techniques,” *Applied Economics*, 20, 1369-1381.

Zini Jr, A. A., and Cati, R. C. (1993), “Co-integração e taxa de câmbio: testes sobre s PPP e os termos de troca do Brasil de 1855 a 1990,” in *Pesquisa e Planejamento Economico*, v.23, 2, pp. 349-374.

N.Cham. P/EPGE SPE A474f

Autor: Alves, Denisard Cneio de Oliveir

Título: A fractional cointegration analysis of purchasing



088372

51509

FGV - BMHS

Nº Pat.:F101/98

000088372

