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RICARDO BUSCARIOLLI PEREIRA

ESSAYS ON ILLIQUIDITY PREMIUM

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Tese apresentada à Escola de Economia de São Paulo da Fundação Getulio Vargas como requisito para a obtenção do título de doutor em economia.

Área do conhecimento: Economia Financeira

Orientador: João Filipe Bernardes Volkmann de Mendonça Mergulhão

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ABSTRACT

This dissertation is composed of three related essays on the relationship between illiquidity and returns. Chapter 1 describes the time-series properties of the relationship between market illiquidity and market return using both yearly and monthly datasets. We find that stationarized versions of the illiquidity measure have a positive, significant, and puzzling high premium. In Chapter 2, we estimate the response of illiquidity to a shock to returns, assuming that causality runs from returns to illiquidity and find that an increase in firms' returns lowers illiquidity. In Chapter 3 we take both effects into account and account for the endogeneity of returns and illiquidity to estimate the liquidity premium. We find evidence that the illiquidity premium is a smaller than the previous evidence suggests. Finally, Chapter 4 shows topics for future research where we describe a return decomposition with illiquidity costs.

JEL classification: G12

Key Words: Liquidity, Empirical Asset Pricing.

RESUMO

Esta tese é composta por três ensaios sobre a relação entre iliquidez e retornos. O Capítulo 1 descreve as propriedades de séries de tempo da relação entre iliquidez e retorno de mercado utilizando dados anuais e mensais. Os resultados mostram que a versão estacionarizada da medida de iliquidez tem um efeito postivo, significante e alto sobre o retorno de mercado. No capítulo 2, estimo a resposta da iliquidez a um choque no retorno de mercado, assumindo que a direção da causalidade se dá do retorno para a iliquidez e mercado. Os resultados indicam que um aumento no retorno reduz a iliquidez de mercado. No capítulo 3 os dois efeitos são considerados mutuamente levando em conta a endogeneidade entre retornos e iliquidez. Os resultados mostram que o prêmio de liquidez é menor do que a evidência prévia sugere. Finalmente, o capítulo 4 exibe tópicos de pesquisa futura e descreve uma decomposição de retornos com custos de iliquidez.

Palavras-chave: Liquidez, Apreçamento Empírico de Ativos.

List of Tables

1.1 Descriptive statistics of yearly data 26

1.2 Tests for unit root - Yearly 29

1.3 Descriptive statistics - Monthly 32

1.4 Tests for unit root - Monthly 35

1.5 Price of the Yearly Amihud (2002) Measure 45

1.6 Measuring Unexpected Amihud (2002) Measure - Yearly 47

1.7 Price of Yearly Amihud's (2002) Measure with the Unexpected Illiquidity given by an AR(1) 49

1.8 Price of Yearly Amihud's (2002) measure with the unexpected illiquidity given by an MA(2) 50

1.9 Pricing of the Yearly market Turnover-Version Amihud (2002) measure 52

1.10 Price of Yearly turnover-version Amihud (2002) measure with the unexpected illiquidity given by an AR(1) 54

1.11 Price of the Monthly Amihud (2002) Measure 56

1.12 Price of the Monthly Amihud (2002) Measure AR(1) 59

1.13 Measuring unexpected Amihud's (2002) measure - Monthly 60

1.14 Price of Monthly Amihud's (2002) measure with the unexpected illiquidity given by an AR(1) 62

1.15 Price of monthly Amihud's (2002) measure with the unexpected illiquidity given by an AR(2) 63

1.16 Akaike (AIC) and Bayesian (BIC) Information Criteria for monthly data 65

1.17 Price of the Monthly Amihud (2002) Measure - Illiquidity lagged twice 67

1.18 Pricing of the Monthly market Turnover-version Amihud (2002) measure 68

1.19 Autoregressive Distributed Lag (ARDL) model with market Turnover measure and size 70

1.20 Quasi-Local Level (QLL) - Yearly 72

1.21 Quasi-Local Level (QLL) - Monthly 72

2.1 Descriptive statistics 80

2.2 Illiquidity response to a change in firms's returns - Yearly Cross-Section 82

2.3 Illiquidity response to a change in firms's returns - Monthly Cross-Section 83

2.4 Response of market illiquidity to changes in market return - Yearly 87

2.5 Response of market illiquidity to changes in market return - AR(1) - Yearly 88

2.6 Response of market illiquidity to changes in market return - Monthly 90

2.7 Response of market illiquidity to changes in market return - AR(1) - Monthly 92

3.1 Reduced form VAR - Monthly data 114

3.2 Structural VAR - Monthly - Coefficients of the matrix of short-run restriction 116

3.3 Structural VAR - Monthly - Structural coefficients estimated with short-run restriction and first-differenced log-transformed illiquidity 117

3.4 Structural VAR - Monthly - Structural coefficients estimated with short-run restriction and first-differenced HP-filtered illiquidity 118

3.5 Structural VAR - Monthly - Structural coefficients estimated with long-run restriction and first-differenced log-transformed illiquidity 119

3.6 Structural VAR - Monthly - Structural coefficients estimated with long-run restriction and HP-filtered log-transformed illiquidity 120

Contents

List of Tables ix

Introduction 1

Chapter 1. An analysis of liquidity over time 4

Abstract 5

- 1.1. Introduction 6
- 1.2. Measures of Illiquidity 9
- 1.3. Methodology 17
- 1.4. Data 24
- 1.5. Results 43
- 1.6. Conclusions 72

Chapter 2. Return Declines and Liquidity 74

Abstract 75

- 2.1. Introduction 76
- 2.2. Measures of Illiquidity 77
- 2.3. Data 78
- 2.4. Cross-sectional evidence 78
- 2.5. Time series evidence 84
- 2.6. Conclusions 91

Chapter 3. Dynamic relationship between illiquidity and return 93

Abstract 94

- 3.1. Introduction 95

3.2. Data and Variable Construction 98

3.3. Estimation strategy 98

3.4. Results 104

3.5. Conclusion 117

Chapter 4. Topics for future research 121

4.1. Introduction 122

4.2. Return decomposition with illiquidity costs 123

4.3. Three-beta model 128

4.4. Volatility, illiquidity, and returns 129

References 131

Appendix . Fama-Macbeth procedure 138

Introduction

This dissertation is composed of three related essays on the relationship between illiquidity and returns. Chapter 1 describes the time-series properties of the relationship between market illiquidity and market return using both yearly and monthly datasets. We update the dataset of Amihud (2002) and find that the illiquidity measure he suggests is non-stationary and, as such, yields a non-significant illiquidity premium. We find, on the other hand, that if we work with stationarized versions of the illiquidity measure we find a premium that is positive and significant, both statistically and economically. Nevertheless, the estimated illiquidity premium is still puzzling high, as previous evidence suggests.

In Chapter 2, we estimate the response of illiquidity to a shock to returns, assuming that causality runs from returns to illiquidity. We analyze and confirm the cross-section evidence of Hameed, Kang and Viswanathan (2010). However, we use the daily Amihud (2002) measure of illiquidity instead of the intraday proxy they construct and find that an increase in firms' returns lowers illiquidity. We also analyze time-series properties and find that a higher than average market return lowers market illiquidity. This implies that the direction of the relationship between return and illiquidity is different from the one in Chapter 1.

In Chapter 3, we relax the causality assumptions of the previous Chapters and take the endogeneity of returns and illiquidity into account to estimate the liquidity premium. We use an structural vector autoregression framework to estimate the illiquidity premium, accounting for the dynamic relation between market return and illiquidity. We find evidence that the illiquidity premium is a smaller than the previous evidence suggests, showing that the puzzle of its large size may be a consequence of model misspecification. We also show

that, using monthly data, the models yield non-significant coefficients at any level. That may suggest that at higher frequencies prices anticipate the information about illiquidity.

Finally, Chapter 4 shows topics for future research where we describe a return decomposition with illiquidity costs.

CHAPTER 1

An analysis of liquidity over time

Abstract

In this Chapter we study the time series properties of Amihud (2002) illiquidity measure. We find that this measure is non-stationary, which has consequences to the estimation of the illiquidity premium over time. We use different stationarized versions of illiquidity and also use an Autoregressive Distributed Lag (ARDL) model after decomposing the original Amihud's (2002) measure into two non-stationary variables: the turnover version of the illiquidity measure and size. To the best of our knowledge such an analysis is novel and the conclusions of it are important for estimating the illiquidity premium in future studies.

1.1. Introduction

In this Chapter we study the time series properties of the illiquidity premium. We start by revisiting the time series evidence on the relation between illiquidity and returns using Amihud (2002) illiquidity measure. Our findings support the notion that illiquidity is priced for both yearly and monthly data. However, this result does not hold if we use the non-stationarized original measure of Amihud (2002).

Both yearly and monthly data are analyzed to check any potential difference in the two frequencies. Results suggest that yearly and monthly illiquidity are priced and that this price is puzzling high, in line with the cross-sectional evidence. These results, however, depend on the strategy we use to convert the illiquidity measure into a stationary variable.

The non-stationarity of Amihud (2002) measure seem to be playing a major role in this analysis. This measure has a negative trend over the years and this was not taken into account in Amihud (2002). His sample ends in 1995, which could have perhaps blurred some of the conclusions about non-stationarity. However, by updating the data sample, it is clear that stationarity has to be addressed. In fact, as market return is a stationary variable, using a non-stationary regressor leads to the conclusion that its coefficient of illiquidity is non-significant at any conventional level.

We first tackle this issue by revisiting Amihud's (2002) approach using updated data, from 1962 to 2011. We make the illiquidity measure stationary by both taking the first difference and filtering it, following the methodology proposed by Hodrick and Prescott (1981). We also estimate coefficients using the same approach of Amihud (2002), i.e., using non-stationary illiquidity as a regressor. We do that to be consistent with his previous work and also to discuss the results and access the gains of adjusting the variables.

Amihud's (2002) measure uses the dollar volume of trading as the proxy of trading activity. Brennan, Huh and Subrahmanyam (2013) (henceforth, BHS) suggest that the estimation of the illiquidity premium should use a measure of illiquidity that relies on turnover as the proxy of trading activity to account separately for firm size effects. Another important reason that also justifies the use of the turnover version is that Amihud

(2002) scales illiquidity with firm size: smaller stocks which have smaller dollar volume for the same turnover (fraction of outstanding shares that are traded) are automatically more illiquid (Cochrane, 2005; Florakis, Gregoriou, and Kostakis, 2011).

Following BHS approach, we decompose the illiquidity measure into turnover and size and re-estimate the illiquidity premium. The results for yearly data show an illiquidity premium that is not as high as the one we find for the original measure. Monthly data yield negative coefficients and their values do not seem to be more "reasonable", as BHS argue.

The BHS illiquidity decomposition allows a different approach for making the illiquidity measure stationary. If the turnover and size components cointegrate, we can re-specify the model to get the estimated illiquidity premium. Although we do not find a cointegration relationship at the yearly level, monthly data evidence suggest that these two components have a long term relationship. Nevertheless, using an Autoregressive Distributed Lag (ARDL) model, the illiquidity premium for the turnover illiquidity version is non-statistical significant at any conventional level.

In Amihud's (2002) time series analysis, the author also studies the effect of "unexpected illiquidity" over returns. He suggests that, although expected illiquidity has a positive impact over the following period's market returns, an increase in unexpected illiquidity decreases contemporaneous market return. This is a consequence of the adjustment mechanism: when market illiquidity goes up, the price level in the contemporaneous period goes down, thus increasing next period's (and decreasing the current period's) return. If the model does not explicitly consider the effect of that unexpected illiquidity, the coefficients may be biased. We also address this issue and, following Amihud (2002), we assume that agents expect illiquidity to be an AR(1) process. The differences between "realized" and expected illiquidity are due to the "unexpected" component.

We add this feature to the specifications that use the turnover versions of illiquidity as well. Results suggest that unexpected illiquidity indeed has a negative impact over returns, which most of the times is significant at a 1% level. However, the magnitude

of the coefficients of the illiquidity variables remain almost unchanged when adding the unexpected component. We test additional models for predicting the expected illiquidity, for robustness, and find no major difference in results.

The final exercise of this Chapter is to check the stability of the estimated models over time. We do so by using the Elliott and Muller's (2006) Quasi-Local Level (QLL) test. Amihud (2002) also addresses this question and finds that his specification is stable. We find evidence that the parameters we estimate using Amihud's (2002) specification are not stable, however, the models that use the stationarized variables are. In other words, the model using Amihud's (2002) measure is not stable but when we transform the variables into some stationary form it is.

The contributions of the Chapter can be summarized as:

1) Using a recent sample, we comprehensively analyze the time series properties of Amihud's (2002) illiquidity measure, and show that it is non-stationary, an issue that was ignored so far. We show that the illiquidity premium is statistical significant on the extended sample only if stationarity is addressed.

2) We analyze the monthly premium of the market Amihud (2002) measure. As far as we know this is the first time the monthly data is considered in a time series study, whereas BHS only study the cross-sectional properties;

3) We use the turnover decomposition of BHS in the time series analysis and measure the premium associated with this variable.

Chapter 1 is organized as follows: section 1.2 presents Amihud's (2002) measure of illiquidity and how it is decomposed into the BHS turnover version; section 1.3 describes the estimation procedures; section 1.4 describes the data we use throughout this dissertation; section 1.5 describes the results for both yearly and monthly data and deals with the stability of the model, and; section 1.6 concludes the Chapter.

1.2. Measures of Illiquidity

What drives the relationship between liquidity and returns is that market makers cannot distinguish between order flows generated by informed traders and by noise traders. Therefore, they set prices that are an increasing function of the imbalance in the order flow (which may indicate informed trading) (Amihud, 2002; Amihud and Mendelson, 1980; Glosten and Milgrom, 1985). This creates a positive relationship between the order flow or transaction volume and price change, commonly called "price impact".

The evidence on the relationship between liquidity and return is quite vast. Many papers argue that liquidity has an impact on the expected return and price of an asset, what Amihud and Mendelson (1991) call "liquidity effect". Under this point of view, illiquid assets and assets with high transaction costs trade at low prices relative to their intrinsic values, in other words, liquidity is priced (Amihud and Medenlson, 1986; Brennan and Subrahmanyam, 1996; Datar, Naik and Radcliffe, 1998; Chordia, Subrahmanyam and Anshuman, 2001).

The liquidity effect is, in broad terms, related to the effect of risk on the returns of assets. The idea is that agents prefer liquid investments that can be traded quickly and at low costs anytime they are in need. Therefore, less liquid investments must offer higher expect returns in order to attract investors in the same way a risk-averse investor would require a higher expected return as a compensation for greater risk (Amihud and Medelson, 1991).

This relationship has been extended in a variety of ways such as in Vayanos (1998), Lo, Mamaysky and Wang (2004); Eisfeldt (2004); Holmstrom and Tirole (2002); Huang (2003), and O'Hara (2003). Acharya and Pedersen (2005), for example, develop a model considering factors related to commonality and risk premia associated with changes in liquidity, finding different risk premia associated with changes in liquidity which turned out to be highly significant in empirical work.

Empirical evidence support this notion that liquidity explains part of the expected returns. Amihud (2002) runs a regression of return on illiquidity and shows that these

two variables are positively related. This suggests that expected stock excess return partly represents an illiquidity premium. Jones (2002) finds that bid-ask spreads and turnover predict U.S. stock returns one period ahead. Another feature found in empirical analysis is that, if liquidity varies systematically, securities whose returns are positively correlated with market liquidity should have higher expected returns (Pastor and Stambaugh (2002); Sadka (2002); Chordia, Roll, and Subrahmanyam (2000); Huberman and Halka (1993)).

There are many liquidity measures and Goyenko, Holden and Trzcinka (2008) show that most of them do a good job on measuring liquidity. They horserace some of these measures, such as Amihud (2002) and Pastor and Stambaugh (2002), considering both price impact and effective spread criteria. They show that among the realized spread measures, Amihud's (2002) is the best overall. They also show that Pastor-Stambaugh Gamma, is dominated by much simpler measures.

Amihud (2002) measure can also be used to uncover the relative impact of turnover and illiquidity over returns. This is the approach taken by BHS, who show that the return premium is "better" captured when turnover (instead of dollar volume) is used to construct the Amihud measure, and firm size effects are accounted for separately. What qualifies it as "better" in BHS is, however, related to what they call a more "reasonable" measure, as the premium estimated using it is not as puzzling high as the one related to Amihud (2002). In the next subsection we show how we calculate the yearly and monthly Amihud (2002) original measure of illiquidity, as well as its turnover version.

1.2.1. Amihud (2002) measure and the turnover version

Amihud (2002) develops an illiquidity measure based on the price impact caused by trading volume that became very popular. We follow the notation of BHS: for security i in each trading day we calculate

$$A^0_{i,d} = \frac{|r_{i,d}|}{DVOL_{i,d}} \tag{1.1}$$

where $r_{i,d}$ is the daily stock return of security i on day d, and; $DVOL_{i,d}$ is daily dollar volume of security i on day d. This measure represents the daily price response associated with one dollar of trading volume. Amihud (2002) argues that this illiquidity measure is strongly related to the liquidity ratio known as the Amivest measure, the ratio of the sum of the daily volume to the sum of the absolute return (Khan and Baker, 1993). It is also positively related to variables that measure illiquidity from microstructure data such as Kyle's (1985) price impact.

The inclusion of this measure in the regression hides, however, the relative importance for asset pricing of turnover and firm size. In order to get these separate effects we decompose the original Amihud (2002) measure (A^0) into its turnover version (A) and a size-related element (S) as in BHS. First, note that the daily dollar volume of security i on day d ($DVOL_{i,d}$) is defined as the multiplication of the total number of shares by the price on day d as

(1.2) $$DVOL_{i,d} = Vol_{i,d} \times p_{i,d}$$

where $Vol_{i,d}$ is the volume traded of security i on day d, and; $p_{i,d}$ is the price of security i on day d. Note, also, that the turnover of security i on day d, $T_{i,d}$, is defined as the daily share volume divided by the total number of shares outstanding

(1.3) $$T_{i,d} = .\frac{Vol_{i,d}}{\#shares_{i,d}}$$

where $\#shares_{i,d}$ is the total number of shares outstanding of security i on day d. We can, therefore, write Amihud's (2002) measure for a given security i in a given day d as

(1.4) $$A^0_{i,d} = \frac{|r_{i,d}|}{DVOL_{i,d}} = \frac{|r_{i,d}|}{T_{i,d}} \frac{T_{i,d}}{DVOL_{i,d}}$$

where $T_{i,d}$ is the turnover of security i on day d. Note that, by definition, the ratio

$$\frac{T_{i,d}}{DVOL_{i,d}} \tag{1.5}$$

can be written as

$$\frac{T_{i,d}}{DVOL_{i,d}} = \frac{\frac{Vol_{i,d}}{\#shares_{i,d}}}{Vol_{i,d} \times p_{i,d}}$$

where $Vol_{i,d}$ is the share volume of security i on day d; $\#shares_{i,d}$ is the total number of shares outstanding of security i on day d, and; $p_{i,d}$ is the price of security i on day d. By simple algebra we have

$$\begin{aligned}
\frac{T_{i,d}}{DVOL_{i,d}} &= \frac{Vol_{i,d}}{\#shares_{i,d}} \times \frac{1}{Vol_{i,d} \times p_{i,d}} \\
&= \frac{1}{\#shares_{i,d} \times p_{i,d}} \\
&= \frac{1}{S_{i,d}}
\end{aligned}$$

where S is firm i's Size, i.e., the market capitalization of firm i. Hence, we can re-write $A^0_{i,d}$ as

$$A^0_{i,d} = \frac{|r_{i,d}|}{T_{i,d}} \left(\frac{1}{S_{i,d}}\right) = A_{i,t} \left(\frac{1}{S_{i,d}}\right) \tag{1.6}$$

By taking natural logarithms on both sides of Equation (1.6) we get the relation between $A^0_{i,d}$, $A_{i,d}$ and firm size, $S_{i,d}$

$$\ln A^0_{i,d} = \ln A_{i,d} - \ln S_{i,d} \tag{1.7}$$

Once we calculate the Amihud (2002) measure for each day we construct the monthly measure for security i by averaging the ratio $\frac{|r_{i,d}|}{DVOL_{i,d}}$ over all the days of the month for each month. We repeat the same process for the yearly measure, averaging it over all the days in each year. Formally, the monthly measure is given by

$$A^0_{i,m} = \frac{1}{D_{i,m}} \sum_{d=1}^{D_{i,m}} \frac{|r_{i,d}|}{DVOL_{i,d}} \tag{1.8}$$

where $D_{i,m}$ is the number of trading days of security i in month m; $|r_{i,d}|$ is the absolute return of security i on day d , and; $DVOL_{i,d}$ is the trading volume (in units of currency) on day d.

The yearly measure for a given firm i over year y, defined as the yearly average of the daily measure, is given by[1]

$$A^0_{i,y} = \frac{1}{D_{i,y}} \sum_{d=1}^{D_{i,y}} \frac{|r_{i,d}|}{DVOL_{i,d}} \tag{1.9}$$

where D_y is the number of trading days in year y; $|r_{i,d}|$ is the absolute return on day d for security i; $DVOL_{i,d}$ is the trading volume (in units of currency) on day d.

BHS do the same for $A_{i,m}$ and $S_{i,m}$, i.e., they define the monthly turnover version of Amihud's (2002) measure, $A_{i,m}$, as

[1]Amihud (2002) has another step after calculating $A^0_{i,y}$. He also calculates the average market illiquidity across securities in each year

$$A^0_{EW,y} = \frac{1}{N_y} \sum_{i=1}^{N_t} A^0_{i,y}$$

In which N_t is the number of securities in year y. This average illiquidity varies considerably over the years, for that reason $A^0_{i,y}$ is replaced by its mean-adjusted value

$$A^{0,MA}_{i,y} = \frac{A^0_{i,y}}{A^0_{EW,y}}$$

Brennan, Huh and Subrahmanyam (2013) do not take this step. This redefinition matters only for cross-sectional and/or panel data analysis as the cross sectional average of $A^{0,MA}_{i,y}$ is equal to 1.

$$A_{i,m} = \frac{1}{D_{i,m}} \sum_{d=1}^{D_{i,m}} \frac{|r_{i,d}|}{T_{i,d}} \tag{1.10}$$

and size, $S_{i,m}$, as the average of daily market values within a month

$$S_{i,m} = \frac{1}{D_{i,m}} \sum_{d=1}^{D_{i,m}} (\#shares_{i,d} \times p_{i,d}) \tag{1.11}$$

Note that the exact equivalence between the $A^0_{i,m}$ and $A_{i,m}$ does not hold taking this approach. That happens because[2]

$$A^0_{i,m} = \frac{1}{D_{i,m}} \sum_{d=1}^{D_{i,m}} \frac{|r_{i,d}|}{DVOL_{i,d}} = \frac{1}{D_{i,m}} \sum_{d=1}^{D_{i,m}} A_{i,t} \left(\frac{1}{S_{i,d}}\right) \neq \frac{\frac{1}{D_{i,m}} \sum_{j=1}^{D_{i,m}} A_{i,t}}{\frac{1}{D_{i,m}} \sum_{j=1}^{D_{i,m}} S_{i,d}} \tag{1.12}$$

Thus, the monthly and yearly turnover versions of Amihud (2002) measure, as calculated by BHS, are not exactly equivalent to the original Amihud's (2002) measure. However, for a matter of consistency, we let those variables as calculated by BHS (2013) in our paper. An alternative would have been to take the logs and then take the average, which is just the geometric mean. However, the logarithm function is not defined for zero returns, which is commonly observed in daily data, and for this reason the measure would suffer some sort of bias. Therefore the implicit assumption in BHS is that

[2] It is actually possible to show how different $E\left[\frac{A_{i,m}}{S_{i,m}}\right]$ is from $\frac{E(A_{i,m})}{E(S_{i,m})}$. Taking the bivariate first order Taylor expansion of $A^0_{i,m} = \frac{A_{i,m}}{S_{i,m}}$ about a given point θ yields

$$E\left[\frac{A_{i,m}}{S_{i,m}}\right] \approx \frac{E\left(A_{i,m}\right)}{E\left(S_{i,m}\right)} - \frac{Cov(A_{i,m}, S_{i,m})}{E\left(S^2_{i,m}\right)} + \frac{Var(S_{i,m})E(A_{i,m})}{E(S_{i,m})^3}$$

Therefore, deviation between the monthly/yearly Amihud's (2002) original measure and the turnover/size measure of Brennan, Huh and Subrahmanyam (2013) is given by

$$\left[\frac{Cov(A_{i,m}, S_{i,m})}{E\left(S^2_{i,m}\right)} + \frac{Var(S_{i,m})E(A_{i,m})}{E(S_{i,m})^3}\right]$$

$$A^0_{i,m} \approx A_{i,m}\left(\frac{1}{S_{i,m}}\right) \tag{1.13}$$

The logarithm version is simply given by

$$\ln A^0_{i,m} = \ln A_{i,m} - \ln S_{i,m} \tag{1.14}$$

The yearly measure is analogous. Even though $A^0_{i,m}$ and $A_{i,m}\left(\frac{1}{S_{i,m}}\right)$ are not exactly equivalent we assume that the values are close enough for the insights of the importance of turnover for analyzing illiquidity. After building the cross section of illiquidity measures, original and turnover versions, the next step is to build the market illiquidity by averaging them over the cross sectional units.

1.2.2. Market Illiquidity

Our interest lies in analyzing the behavior of market illiquidity over market return. In each month/year, we average both Amihud's (2002) original and turnover measures, returns, and size over all the securities admitted in the sample using equally and value weighted criteria in order to get the market measures. Amihud (2002) uses only the Equally Weighted Market Illiquidity (EWMI) while BHS use the Value Weighted Market Illiquidity (VWMI) as an illustration, as they focus on cross sectional data. Even though the researcher is free to choose which one to use, the results may vary (Plyakha, Uppal and Vilkov, 2014). We use the VWMI approach in this Chapter and make comments on the results of EWMI in the text. In the next subsections, we describe how we construct the market illiquidity measures.

1.2.2.1. Value Weighted Market Illiquidity (VWMI). The VWMI of Amihud's (2002) original measure in period t, $A^0_{VW,t}$, is defined as

$$A^0_{VW,t} = \frac{1}{\sum_{i=1}^{N_t} S_{i,t}} \sum_{i=1}^{N_t} A^0_{i,t} S_{i,t} \tag{1.15}$$

where $A^0_{i,t}$ is Amihud's (2002) original measure; $S_{i,t}$ is the market capitalization of firm i in month t; N_t is the number of firms admitted to our sample in period t. We want also to decompose this measure to get the relative importance of turnover using the relation described by equation (1.13). We can re-write equation (1.15) as

$$\begin{aligned}
A^0_{VW,t} &= \frac{1}{\sum_{i=1}^{N_t} S_{i,t}} \sum_{i=1}^{N_t} A_{i,t} \left(\frac{1}{S_{i,t}} \right) S_{i,t} \\
&= \frac{1}{\sum_{i=1}^{N_t} S_{i,t}} \sum_{i=1}^{N_t} A_{i,t} \\
&= \frac{N_t}{\sum_{i=1}^{N_t} S_{i,t}} \frac{1}{N_t} \sum_{i=1}^{N_t} A_{i,t} \\
&= \frac{1}{\underbrace{\frac{1}{N_t} \sum_{i=1}^{N_t} S_{i,t}}_{S_{EW,t}}} \underbrace{\left(\frac{1}{N_t} \sum_{i=1}^{N_t} A_{i,t} \right)}_{A_{EW,t}}
\end{aligned}$$

Therefore

$$A^0_{VW,t} = \frac{1}{S_{EW,t}} \times A_{EW,t} \tag{1.16}$$

where $A_{EW,t}$ is the EWMI of the turnover version of Amihud's (2002) measure, A, in period t; $S_{EW,t}$ is the equally weighted market capitalization of all firms in sample in period t. These two measures are defined as

$$A^0_{EW,t} = \frac{1}{N_t} \sum_{i=1}^{N_t} A_{i,t} \tag{1.17}$$

and

$$S_{EW,m} = \frac{1}{N_m} \sum_{i=1}^{N_m} S_{i,m} \tag{1.18}$$

The relation between the value weighted market original measure and the equally weighted turnover measure is much more direct and easy to access. Taking the natural logarithm on both sides we have

$$\ln(A^0_{VW,t}) = \ln(A_{EW,t}) - \ln(S_{EW,t}) \tag{1.19}$$

Therefore we need the EWMI's of $A_{i,t}$ in order to trace the parallel between $\ln(A^0_{VW,t})$, $\ln(A_{EW,t})$, and $\ln(S_{EW,t})$. For notation purposes we omit the subscript VW and EW henceforth. Hence, $\ln A^0_t$ represents the log transformation of the value weighted average taken across all firms in sample in period t. Similarly, $\ln A_t$ and $\ln S_t$ represent the equally weighted averages of the turnover version and size, respectively, in period t. We do that to avoid confusion in the next sections so we do not have to carry the subscripts in every definition. In the next section we describe the estimation procedure, i.e., the models we estimate using the illiquidity measures we describe in this section.

1.3. Methodology

To determine the relationship between market illiquidity and market return, and also to study the role of each components of the Amihud (2002) measure in asset pricing, we follow Amihud, Mendelson and Wood (1990) and Amihud (2002). In these papers the authors find the expected stock excess return to be an increasing function of expected market illiquidity. They do so by following French, Schwert and Stambaugh (1987), who test the effect of risk on stock excess return. Expected illiquidity is estimated by an autoregressive model and this estimate is employed to test two hypotheses: (i) ex ante

stock excess return is an increasing function of expected illiquidity, and (ii) unexpected illiquidity has a negative effect on contemporaneous unexpected stock return.

In this Chapter, instead of reporting the result with the unexpected illiquidity right away, we take a step-by-step approach: first, we estimate the relationship between market liquidity and market return without explicit considering the role of the unexpected illiquidity; then we show the relative relevance of this unexpected component to the illiquidity premium in both yearly and monthly time series.

It is important to highlight that this is a time-series analysis, a positive time-series relation between market return and illiquidity suggests that when the market experiences a period with an illiquidity change that is higher than its unconditional time-series average, the market return is higher than the average return during that period. Previous studies find a positive cross-sectional return-illiquidty relation, usually applied to individual firms, which suggests that firms with illiquidity changes higher than the cross-sectional average have higher returns than the average market returns (Teets and Wasley, 1996; Sadka and Sadka, 2009). The next subsections describe the procedure for tackling the estimation of the illiquidity premium.

1.3.1. Amihud (2002) original measure

First we focus on estimating the premium for Amihud's original measure. We simplify notation by representing the set of regressions for yearly data by

$$r_t^j = \phi_0 + \phi_1 \ln A_{t-1}^0 + \varepsilon_t \tag{1.20}$$

where $t = m$ or y, $j = M, E$ or $FF3$, the dependent variables are defined as

- r_t^M is the value weighted average of returns (VWMR) for a given period t, and;
- r_t^E is the value weighted average of returns (VWMR) for a given period t in excess of the risk-free rate (r_t^f): $r_t^M - r_t^f$

For monthly data there are still the additional dependent variable, denoted by r_m^{FF3}, which represents the market return adjusted for the Fama-French three factors.

In section 1.5 we see that market return does not seem to have any autoregressive structure. However, as most papers add the AR(1) term in order to control for possible autocorrelation, we also add an AR(1) structure to the models in the following way:

$$r_t^j = \phi_0 + \phi_1 \ln A_{t-1}^0 + \phi_2 r_{t-1}^j + \varepsilon_t \tag{1.21}$$

As we discuss in section 5, A_y^0, A_m^0, and their log-transformations are non-stationary. Even though this question is not taken into account in Amihud (2002), the technology available to estimate time-series models assumes regressors' stationarity. To tackle this issue, and also to show the difference of our approach when compared to Amihud (2002), we estimate models that consider $\ln A_t^0$ as the illiquidity measure, but we also use the first difference and the detrended versions of this variable (applying the Hodrick–Prescott filter, which is a popular empirical technique among researchers to detrend an economic series). Therefore we can write equations (1.20) and (1.21) synthetically as

$$r_t^j = \phi_0 + \phi_1 \ln A_{t-1}^{0,k} + \varepsilon_t \tag{1.22}$$

where $t = m$ or y, $j = M, E$ or $FF3$, and $k = 1,\ 2,\ 3$, which defines each illiquidity variable as:

- $\ln A_t^{0,1}$ is the log transformed Amihud (2002) measure, it is non-stationary for both equally and value weighted market averages.
- $\ln A_t^{0,2} = \Delta \ln A_t^0 = \ln A_t^0 - \ln A_{t-1}^0$, is the first differenced log transformed Amihud (2002) measure, it is stationary for both equally and value weighted market averages.

- $\ln A^{0,3}_{t-1} = \ln A^{0}_{h,t} - trend_{HP}$, is the log transformed Amihud (2002) measure minus the Hodrick–Prescott filter's trend

We estimate these equations by OLS with Newey–West standard errors, as in Amihud (2002). We consider that the error structure can be heteroskedastic and possibly autocorrelated up to some pre defined lag, depending on the structure depicted by the variable's autocorrelation function.

1.3.2. Turnover version

As we do with Amihud's (2002) original measure, A^0, we define the set of models we estimate taking the turnover version of the illiquidity measure as

$$r_t^j = \phi_0 + \phi_1 \ln A_{t-1} + \phi_2 \ln S_{t-1} + \varepsilon_t \tag{1.23}$$

where $t = m$ or y, $j = M, E$ or $FF3$. We regress value weighted market return (VWMR) on the sum of equally weighted market illiquidity (EWMI) and size for the issues we explain in section 3.2.1. We also take the AR(1) specification of returns as

$$r_t^j = \phi_0 + \phi_1 \ln A_{t-1} + \phi_2 \ln S_{t-1} + \phi_3 r_{t-1}^j + \varepsilon_t \tag{1.24}$$

Note that Amihud's (2002) original measure implies that $\phi_1 = \phi_2$, the hypothesis that the coefficient of $\ln A^0_{t-1}$ is not significant is equivalent to testing if $\phi_1 + \phi_2 = 0$.

1.3.3. Measuring the unexpected illiquidity

Amihud (2002) deals with the unexpected component of illiquidity. He states that the unexpected illiquidity has a negative effect on contemporaneous unexpected stock return, and may bias the estimation of the illiquidity premium. In order to understand his

arguments, let us take the ex ante effect of market illiquidity on stock excess return, which is described by

$$r_t^j = f_0 + f_1 \ln A_t^{0,E} + \varepsilon_t \tag{1.25}$$

where $h = EW$ or VW, $t = m$ or y, $j = 1, 2$ or 3. Let $\ln A_t^{0,E}$ be the expected market illiquidity for period t based on information in $t-1$. The hypothesis that expected illiquidity is priced implies that $f_1 > 0$. We also calculate the unexpected illiquidity using $\Delta \ln A_t^0$ and $HP. \ln A_t^0$ when these variables are added to the model, however, we stick to the $\ln A_t^0$ notation in this subsection for clarity.

In order to uncover what $\ln A_t^{0,E}$ means we follow Amihud (2002): investors are assumed to predict illiquidity for period t based on information available in $t-1$ and then use this prediction to set prices that will generate the desired expected return in period t. Market illiquidity is assumed to follow the autoregressive model

$$\ln A_t^0 = c_0 + c_1 \ln A_{t-1}^0 + \upsilon_t \tag{1.26}$$

Therefore, the expected illiquidity is given by

$$E\left[\ln A_t^0\right] = \ln A_t^{0,E} = c_0 + c_1 \ln A_{t-1}^0 \tag{1.27}$$

Under this specification υ_t is the "unexpected" illiquidity, $\ln A_t^{0,U}$, which may be important for the dynamics of the relationship between return and illiquidity. Amihud (2002) claims that it is reasonable to expect $g_1 > 0$ and he uses that expectation to build the important hypothesis that the effect of $\ln A_t^{0,U}$ on contemporaneous stock return should be negative. In order to illustrate this claim assume that $\ln A_t^{0,U} > 0$. As a consequence of $g_1 > 0$ we have $\ln A_{t+1}^{0,E} > \ln A_t^{0,E}$. As we say above, the hypothesis that illiquidity is priced

implies that $f_1 > 0$, therefore a higher expected illiquidity implies a higher return, i.e., $r^j_{t-1} > r^j_t$. For that to be true we must have a drop in current *price* in order to reflect the increase next period's *return* (the "liquidity premium"). In this way the current *return,* r^j_t, will be negatively affected as the drop occurs in period t's *price*. One assumption made in Amihud (2002) is that corporate cash flows are unaffected by market illiquidity.

This drop in current price as a consequence of the rise in next period's return can be understood with a simple return decomposition. Assume that in period t one asset can be purchased today for price P_t and this asset yields a dividend D_t. In the next period, $t + 1$, this asset is sold for price P_{t+1}. The return on this investment is given by

$$r_{t+1} = \frac{D_t + (P_{t+1} - P_t)}{P_t} \tag{1.28}$$

We can re-write this in terms of "gross return"

$$1 + r_{t+1} = \frac{D_t + P_{t+1}}{P_t} \tag{1.29}$$

Rearranging this equation we have

$$P_t = \frac{D_t}{1 + r_{t+1}} + \frac{P_{t+1}}{1 + r_{t+1}} \tag{1.30}$$

To ease notation, we define

$$R_t = 1 + r_t \tag{1.31}$$

Thus, the equation for prices can be just written as

$$P_t = \frac{D_t}{R_{t+1}} + \frac{P_{t+1}}{R_{t+1}} \tag{1.32}$$

Therefore Amihud's (2002) claim makes sense. A positive shock in the next period's expect return decreases the current price thus there should be a negative relationship between unexpected illiquidity and contemporaneous stock return. Or, in other terms, $cov(\varepsilon_t, \upsilon_t) < 0$. Stambaugh's (1999) shows that the estimated coefficient f_1 of equation (1.25) is biased upward. Amihud (2002) adds that this bias can be eliminated by including in model (1.25) the unexpected illiquidity, or the residual υ_t of model (1.26). Therefore we estimate the following model

$$r_t^j = f_0 + g_1 \ln A_{t-1}^0 + g_2 \ln A_t^{0,U} + \varepsilon_t \tag{1.33}$$

where $g_0 = f_0 + f_1 c_0$, $g_1 = f_1 c_1$ and $\ln A_t^{0,U} = \upsilon_t$. The hypothesis made in Amihud (200) implies that $g_1 > 0$ and $g_2 < 0$.

Amihud (2002) does one final adjust in estimating models (1.26) and (1.33). The estimated coefficients are supposed to be biased downward due to finite samples, therefore he uses Kendall's (1954) bias correction approximation procedure. We report in the tables the unadjusted coefficients and but in the text we make comments on the values augmented by the term $\frac{1+3\widehat{c}_i}{T}$ where T is the sample size and $\widehat{c}_i$ represents the estimated coefficients.

We repeat the exercise for the turnover version of illiquidity measure, the model for the expected illiquidity, considering this turnover measure, is given by

$$\ln\left[\frac{A_t}{S_t}\right] = c_0 + c_1 \ln\left[\frac{A_{t-1}}{S_{t-1}}\right] + \upsilon_t \tag{1.34}$$

where, υ_t is the stationary residual representing the unexpected illiquidity. We also consider the models with AR(1) specification for returns, i.e., we add r_{t-1}^j to model (1.25) and for the analogous model using the turnover version of the illiquidity measure.

As we discuss in the next section, the series $\ln A_t^0$ is non-stationary. Therefore, we need to take that into account to estimate the equation of expected illiquidity. We replace the non-stationary series by the stationarized versions: first difference and HP-filtered series.

We perform an additional exercise to get a better measure of the unexpected illiquidity. The Autocorrelation Function (ACF) and the Partial Autocorrelation Function (PACF) of yearly data show no indication that illiquidity and its transformations follow an AR(1). Therefore we assume different model specifications as alternatives for the expected yearly and monthly illiquidity. We discuss this issue and the dataset, as well as the properties of the variables we use in this dissertation, in the next section.

1.4. Data

We collect daily data from stocks traded in the NYSE/AMEX (henceforth, NYAM) from January 1962 to December 2011. Daily stock returns and the number of shares outstanding are obtained from the CRSP daily file. The data on risk-free and market index returns are drawn from Kenneth French's website. We restrict our analysis to NYAM-traded stocks in order to avoid the effects of differences in market microstructures (Reinganum, 1990).

We include all stocks that satisfy the criteria below for monthly and yearly data in order to be consistent with Amihud (2002), which uses yearly data, and BHS which use monthly data. The criteria for yearly data are[3]:

[3]We had access to daily and monthly adjusted returns in the CRSP database but we had to calculate yearly returns for each firm. In order to do so we must adjust the quoted daily prices and volumes. We divide each daily price of each firm by the cumulative factor to adjust prices (cfacpr) and we multiply each daily number of shares by the cumulative factor to adjust shares (cfacshr) so we get the adjusted variables. By doing that we can get the adjusted yearly return, given by

$$TotalReturn = \left[\frac{adjprc + \frac{divamt}{\frac{cumfacpr}{facpr}}}{prev\_adjprc} \right] - 1$$

where adjprc is the adjusted price at the end of the period and prev_adjprc is the adjusted price at the end of previous period.

- **The security must have at least 200 days of valid observations during year** $y-1$.This makes the estimated parameters more reliable. Also, the stock must be listed at the end of year $y-1$;
- **Price at the end of the year must be higher than US$**5 because returns on low-price stocks are greatly affected by the minimum tick of $1/8, which adds noise to the estimations;
- **Every observation with missing values for** our **size** variable (market capitalization) is **dropped**;
- Only observations with **no zero monthly volume** are considered;
- **Outliers are winsorized**, i.e., stocks whose estimated returns and A_y^0 in year $y-1$ are at the highest or lowest 1% tails of the distribution are replaced by the value right before the 1%-tile[4].

The surviving criteria for monthly data are

- Stocks must have **five zero-volume days or fewer** within a month;
- Stocks must have monthly returns for the **past 24 months;**
- **Only common stocks** (share code 10 or 11 in CRSP) are used;
- **Outliers are winsorized**, i.e., stocks whose estimated return and A_m^0 in month $m-1$ is at the highest or lowest 1% tails of the distribution are replaced by the value right before the 1%-tile.

Data series for firms are winsorized in each period to alleviate the influence of outliers. Following Amihud (2002) and BHS, we apply a logarithmic transformation to all of the illiquidity variables reported to reduce the influence of extreme observations and to allow a simple additive decomposition of the Amihud measure. Statistics for the transformed variables are shown in Table 1.

[4]We test other criteria to trimm outliers in yearly and monthly. We run regressions without trimming any observartion, trimming the 0.5% and 1% extremes and winsorizing the 0.5% and 1% extremes. The only difference between each one of them is that the more data are trimmed the more higher the magnitude of the coefficients but the economic significance of them change by a very small figure.

All descriptive statistics are presented below. We divide the description of market illiquidity measures and market returns to ease the exposition of them. To be consistent with Amihud (2002) we multiply the illiquidity variables by 10^6. Yearly and monthly data are very unique so we explain each one of them separately.

1.4.1. Yearly variables

Table 1.1 report values of means, medians, standard deviations, and other descriptive statistics for the original Amihud (2002) measure, $\ln A^0$, and its stationary transformations.

Table 1.1. Descriptive statistics of yearly data

Variable	Obs	Mean	Std. Dev.	Min	Max
lnA0	50	-4.18	1.72	-7.35	-1.89
ΔlnA^0	49	-0.11	0.29	-0.70	0.63
HP.lnA^0	50	0.00	0.19	-0.39	0.53
lnA	50	2.23	0.10	2.02	2.39
ΔlnA	49	-0.07	0.19	-0.52	0.31
HP.lnA	50	0.00	0.01	-0.02	0.04
lnS	50	14.16	1.23	12.61	16.00
ΔlnS	49	0.07	0.12	-0.31	0.24
HP.lnS	50	0.00	0.07	-0.18	0.15
Market Return	50	14.97	17.08	-26.43	42.29
Market Return - Risk Free Rate	50	9.70	17.08	-32.98	35.51

This table reports descriptive statistics of key yearly variables for NYSE-AMEX (hereafter NYAM) stocks from January 1961 to December 2011 (50 years). The table shows statistics for the log-transformed values [indicated by ln(.)] of Amihud (2002) measure. It also shows statistics for both the log-transformed turnover version of the measure and for the log-transformed size. For each measure we perform two tranformations in order to make them stationary: we take the first difference [indicated by Δ(.)] and then apply the filtering procedure described by Hodrick and Prescott (1981) [denoted by HP.(.)].

After averaging, there is a total of 50 periods ranging from 1962 to 2011. The mean of the log transformed time series of yearly VWMI, $\ln A^0$, is -4.18 over the sample period, and its the standard deviation is 1.72. Figure 1 plots its path and shows that this measures follows a generally decreasing trend, reflecting improvement in market liquidity since the early 1970's. During the 1990's illiquidity decreases steeply, however illiquidity starts to increase in 2008 and in 2009 there is a peak, probably a consequence of the financial

turmoil that was seen in these years. In 2011 the level of illiquidity is close to the level of 2007.

[INSERT FIGURE 1 ABOUT HERE]

There seem to be a negative trend, which suggests that this variable is non-stationary. Table 1.2 shows the statistics for both Augmented Dickey-Fuller (ADF) and Phillips-Perron tests. We report the statistics for testing a random walk against a stationary autoregressive process of order one, AR(1), and for a random walk against a stationary AR(1) with drift and a time trend. The null hypothesis of a unit root is not rejected at any conventional level for both tests: the p-values of Dickey-Fuller and Phillips-Perron tests for unit root are, respectively, 0.97 and 0.99 for the test assuming that the underlying model has neither drift nor trend, and 0.19 and 0.27 assuming that the underlying model has both drift and trend. These results suggest that illiquidity is a non-stationary process that contains an unit root and a stochastic trend. Re-running the test using more lags to control for serial correlation yields statistics that also fail to reject the null hypothesis of unit root.

Amihud (2002) uses equally weighted criterion to build the market illiquidity in a sample that ends in 1996. When we use a dataset that mimics Amihud (2002), the illiquidity also have a trend. The DF test-statistic for the equally weighted market illiquidity using this mimicking dataset (trimming the 1%-outliers and using the 1962-1996 time span) yields an statistic of -0.62. The p-value relative to this statistic is 0.86, therefore, it fails to reject the null hypothesis of unit root. Adding lags to the ADF test lowers the p-value of the test-statistic. We find similar evidence for Phillips-Perron's test and for the specification with the underlying model with trend and drift.

The technology commonly adopted to estimate parameters in a time series regression assumes that variables in the model are stationary, the model estimated in Amihud (2002) is, therefore, inaccurate. Thus, we need to transform them into some stationary form. As we discuss previously, the log of Amihud (2002) measure seem to have a stochastic trend and trying to detrend such series with simpler approaches does not remove the

non-stationarity. Indeed, for illustration, we fit a linear trend to the VWMI by running a regression assuming that it follows a process such as

$$\ln A_{t-1}^{0} = c_0 + c_1 \ln A_{t-1}^{0} + \rho t + \upsilon_t \tag{1.35}$$

so that we can find the detrended illiquidity by

$$\ln A_t^{0,D} = \ln A_t^{0} - \widehat{\rho} t \tag{1.36}$$

where $\ln A_{t-1}^{0,D}$ represents the detrended illiquidity. The estimated $\widehat{\rho}$ is -0.033 and the ADF statistic (p-value) calculated with this new series is -0.015 (0.96), which suggests non-stationarity. We choose two transformation strategies: (1) we take the first difference of the illiquidity measure (which we denote by $\Delta \ln A^0$) and (2) we use de-trended Amihud's (2002) original measure using the filter described by Hodrick and Prescott (1981), referred simply as HP-filter (which we denote as $HP. \ln A^0$). This filter depends on the choice of the value of a smoothing parameter that penalizes variability in the growth component. We follow the conventional value of 6.25 for yearly data. We add the HP-filtered illiquidity for illustration and compare results with the first differenced illiquidity.

The tests for unit root for first differenced and HP-filtered versions of $\ln A^0$ do reject the null hypothesis of unit root. Table 1.2 shows that all of the p-values of both Augmented Dickey-Fuller and Phillips-Perron tests for both of these transformations are 0.00. Figure 2 and Figure 3 show the path of the transformed variables, the graphical analysis also seem to reveal stationary variables. This suggests that $\ln A^0$ is integrated of order 1.

[INSERT FIGURE 2 ABOUT HERE]

[INSERT FIGURE 3 ABOUT HERE]

Table 1.1 shows the averages of $\Delta \ln A^0$ and $HP. \ln A^0$, which are respectively -0.07 and 0.00. The dispersion of these variables is also high considering their means of 0.19

Table 1.2. Tests for unit root - Yearly

	Dickey-Fuller test for unit root		Phillips-Perron test for unit root	
H0	RW without drift	RW with trend and drift	RW without drift	RW with trend and drift
lnA^0	0.25	-2.81	0.75	-2.63
	(0.97)	(0.19)	(0.99)	(0.27)
ΔlnA^0	-6.81	-6.85	-8.99	-7.02
	(0.00)	(0.00)	(0.00)	(0.00)
$HP.lnA^0$	-6.23	-6.15	-6.21	-6.17
	(0.00)	(0.00)	(0.00)	(0.00)
lnA	-0.45	-2.52	-0.31	-2.35
	(0.90)	(0.32)	(0.92)	(0.40)
ΔlnA	-6.80	-6.86	-6.90	-7.20
	(0.00)	(0.00)	(0.00)	(0.00)
HP.lnA	-6.39	-6.14	-6.46	-6.27
	(0.00)	(0.00)	(0.00)	(0.00)
lnS	0.23	-2.38	0.22	-2.42
	(0.97)	(0.39)	(0.97)	(0.37)
ΔlnS	-6.52	-6.46	-6.52	-6.45
	(0.00)	(0.00)	(0.00)	(0.00)
HP.lnS	-6.22	-6.15	-6.21	-6.12
	(0.00)	(0.00)	(0.00)	(0.00)
Market Return	-7.84	-7.75	-7.92	-7.85
	(0.00)	(0.00)	(0.00)	(0.00)
Market Return - Risk Free Rate	-8.07	-8.07	-8.23	-8.35
	(0.00)	(0.00)	(0.00)	(0.00)

This table reports the Dickey-Fuller and Phillips-Perron test statistics (and MacKinnon approximate p-values in parenthesis) of key variables for NYAM stocks constructed with yearly data from January 1961 to December 2011 (50 observations). The table shows test statistics and p-values of the log-transformed Amihud (2002) measure and its stationary transformations. It also shows the test statistics and respective p-values of the turnover version of Amihud (2002) measure and the size variable, including also their starionary versions.

for $\Delta \ln A^0$ and 0.01 for $HP. \ln A^0$. Figures 4, 5, 6 and 7 show the autocorrelation (ACF) and partial autocorrelation (PACF) functions of $\Delta \ln A^0$ and $HP. \ln A^0$. They show no clear indication of the number of AR or MA terms[5].

[INSERT FIGURE 4 ABOUT HERE]

[INSERT FIGURE 5 ABOUT HERE]

[INSERT FIGURE 6 ABOUT HERE]

[INSERT FIGURE 7 ABOUT HERE]

The first PACF is negative and it cuts off after lag 2 (however, it is 5% significant at lag 22) and the first ACF is negative and the only 5% significant point is lag-2. This suggests that $\Delta \ln A^0$ may follow an MA(2) process. The autocorrelation and partial autocorrelation functions of the HP-filtered $\ln A^0$ also suggest an MA(2) process.

We observe a similar behavior in the equally weighted series of $\ln A$ and $\ln S$. In Table 1.1 we see that the mean (standard deviation) of $\ln A$ and $\ln S$ are given, respectively, by 2.23 (0.10) and 14.16 (1.23). These variables are also non-stationary, the p-values of the ADF (Phillips-Perron) tests for unit root are 0.90 (0.92) and 0.97 (0.97) respectively for $\ln A$ and $\ln S$, when we test for a random walk against a stationary AR(1). When we change the underlying model assumption and test for a random walk against a stationary AR(1) with drift and a time trend, we have p-values of 0.32 (0.40) for the calculated ADF (Phillips-Perron) statistic of the $\ln A$ series and 0.39 (0.37) of the $\ln S$ series. We see a decreasing trend on the path of $\ln A$, which is shown in Figure 8, and an upward trend in size, shown in Figure 9.

[INSERT FIGURE 8 ABOUT HERE]

[INSERT FIGURE 9 ABOUT HERE]

It is interesting to highlight that, while the peak of illiquidity in the recent years is seen in 2009 for the original measure, the turnover measure has its peak in 2008. This reflects

[5]We can use the rule of thumb to tackle the number of AR or MA terms: if the PACF of the differenced series displays a sharp cutoff then we may add an AR term to the model. The lag at which the PACF cuts off is the indicated order of the AR term. If the ACF of the differenced series displays a sharp cutoff then we may add an MA term to the model. The lag at which the ACF cuts off is the indicated order of the MA term.

that both measures may have different sensitivities for what happens in the markets. Cochrane's (2005) argument that smaller stocks, which have smaller dollar volume for the same turnover, are automatically more illiquid for Amihud's (2002) original measure. Maybe a decrease of value of large companies lowered the $\ln A^0$. However, as $\ln A$ takes turnover into account, this measure could capture the drop in illiquidity without the noise created by firms' loss of value.

We also perform the same transformations we use in $\ln A^0$ to construct stationary versions of $\ln A$ and $\ln S$. The transformed measures are stationary, Table 2 shows that the p-values relative to the ADF and Phillips-Perron tests for the first difference and HP-filtered variables are all 0.00, rejecting the null hypothesis of unit root.

The nonstationarity of the illiquidity measure is an important point for the estimation of illiquidity premium. In previous studies this issue is not considered. Amihud's (2002) sample ends in 1996 and he does not report any test for unit root/stationarity of illiquidity in his time-series analysis. Any updated study that measures illiquidity premium over time should take that into account.

1.4.1.1. Yearly market return. We construct Equally and Value Weighted Market Returns in a way analogous to market illiquidity, however, we just report tables with the results of the Value Weighted Market Returns, henceforth VWMR, which we denote by r_y^M. We also construct the VWMR in excess of the risk-free rate, given by the annualized one-month T-bill rate, which we denote by r_y^E.

Table 1.1 shows that the values of the means of r_y^M and r_y^E, respectively given by 14.97 and 9.70. Both ADF and Phillips-Perron tests for unit root reject the null hypothesis, at 1% level, that yearly market returns have a unit root. As expected, yearly returns are stationary. Figures 10, 11, and 12 show, respectively, the path, the ACF, and the PACF of r_y^M (the figures relative to r_y^E are analogous).

[INSERT FIGURE 10 ABOUT HERE]

[INSERT FIGURE 11 ABOUT HERE]

[INSERT FIGURE 12 ABOUT HERE]

Yearly market returns do not show any sign of having a structure, all the points in both of these functions do not seem to be different from zero (taking Bartlett's formula for 95% confidence bands). In the next subsection we analyze the monthly data in a way analogous to what we do with yearly data.

1.4.2. Monthly variables

In this subsection we discuss descriptive statistics for monthly data. Table 1.3, analogous to Table 1.1, reports values of means, medians, standard deviations, and other descriptive statistics for the monthly original Amihud (2002) measure, $\ln A^0$, and its stationary transformations.

Table 1.3. Descriptive statistics - Monthly

Variable	Obs	Mean	Std. Dev.	Min	Max
$\ln A^0$	600	-3.24	1.51	-6.31	0.17
$\Delta \ln A^0$	599	-0.01	0.22	-0.76	1.81
$HP.\ln A^0$	600	0.00	0.40	-1.31	1.47
lnA	600	24.28	0.51	23.05	25.69
ΔlnA	599	0.00	0.17	-0.54	0.69
$HP.\ln A^0$	600	0.00	0.30	-0.70	1.03
Market Return	600	1.40	4.39	-20.70	18.23
Market Return - Rf	600	0.98	4.39	-21.30	17.72
Market Return - FF3F	600	0.52	4.07	-17.19	16.70

This table reports descriptive statistics of key yearly variables for NYSE-AMEX (hereafter NYAM) stocks from January 1961 to December 2011 (600 months). The table shows statistics for the log-transformed values [indicated by ln(.)] of Amihud (2002) measure and the measures of Return. It also shows statistics for both the log-transformed turnover version of the measure and for the log-transformed size. For each measure we perform two transformations in order to make them stationary: we take the first difference [indicated by Δ(.)] and then apply the filtering procedure described by Hodrick and Prescott (1981) [denoted by HP.(.)].

There is a total of 600 periods ranging from January 1962 to December 2011. The mean of the log-transformed VWMI, $\ln A^0$, is -3.24 over the sample period and its the standard deviation is 1.55. Figure 13 plots the path of the monthly $\ln A^0$ and it shows that this measure follows a generally decreasing trend.

[INSERT FIGURE 13 ABOUT HERE]

During the 1990's illiquidity decreases steeply. In March and July of 2008 there are two spikes in illiquidity and after the second one the level of illiquidity remains higher until August 2009, when it returns to a level close to the one of December of 2007. In August of 2009 the log of the Amihud measure is -5.33, and in December 2007 it is -5.31. In between these two periods the mean of the log illiquidity is -4.65.

As in the yearly case, Figure 13 shows a path that can suggest a non-stationary process. The first and third columns of Table 1.4 report the statistics of both the Augmented Dickey-Fuller (ADF) and Phillips-Perron tests for unit root against a stationary autoregressive process of order one. The ADF statistic (MacKinnon approximate p-values) of $\ln A^0$ is -3.20 (0.02), therefore the null hypothesis of unit root is not rejected at a 1% level, however it is rejected at a 5% level. The Phillips-Perron test yields an statistic of -1.92 (0.325), which means that the null hypothesis of a unit root is not rejected at any conventional level.

Columns 2 and 4 of Table 1.4 show the statistics of both the Augmented Dickey-Fuller (ADF) and Phillips-Perron tests for unit root against a stationary AR(1) with drift and a time trend. The ADF statistic (MacKinnon approximate p-values) of $\ln A^0$ is -8.75 (0.00), therefore the null hypothesis of a unit root is rejected at a 1% level. The Phillips-Perron test yields an statistic of -8.19 (0.00), which means that the null hypothesis of a unit root is also rejected at a 1%-significance level.

Taken together, these tests suggest that the $\ln A^0$ series is trend-stationary. If this is the case, we should remove the non-stationarity by detrending. Taking the first-difference also "removes" the non-stationarity but that adds an MA(1) structure into the errors. We make the same variable transformations we use in the yearly section: (1) we take the first difference of the illiquidity measure ($\Delta \ln A^0$) and (2) we use de-trended Amihud's (2002) original measure using the filter described by Hodrick and Prescott (1981) ($HP. \ln A^0$) with the smoothing parameter set to 129,600, which is the conventional choice.

There are other ways of detrending a series, including simply fitting a linear trend and subtracting it from the original series. We fit a linear trend to the VWMI, the estimated

coefficient of the trend is -0.0018, and subtract it from the original series. This procedure yields a stationary series, the Dickey-Fuller test statistic for the detrended series is -4.008, the related p-value is 0.0014.

The tests for unit root for both the first-differenced and the HP-filtered versions of $\ln A^0$ reject the null hypothesis of unit root, as we report in Table 1.4. All the p-values of both Augmented Dickey-Fuller and Phillips-Perron tests for all these transformations are 0.00.

Figures 14 and 15 show the path of these transformed variables, the graphical analysis also seem to reveal stationary variables. This suggests that $\ln A^0$ calculated with monthly data is integrated of order 1.

[INSERT FIGURE 14 ABOUT HERE]

[INSERT FIGURE 15 ABOUT HERE]

The averages of $\Delta \ln A^0$ and $HP. \ln A^0$ are respectively -0.01 and 0.00. As happens with yearly data, their dispersion is high considering their means, the standard deviation of $\Delta \ln A^0$ is 0.43 and of $HP. \ln A^0$ is 0.46.

Figures 16 and 17 show the autocorrelation and partial autocorrelation functions of $\Delta \ln A^0$, figures 18 and 19 show the same functions of $HP. \ln A^0$. The first two PAC of $\Delta \ln A^0$ are 5% significant and negative. The first ACF is negative 5% significant. The rule of thumb would suggest to estimate an ARMA (2,1) specification or at least to consider some MA structure. However when we use 2 lags in the model, i.e., an AR(2) structure, the MA term becomes non significant at any conventional level. We test several model versions and use AIC and BIC statistics as indicators and their lowest values (AIC of 519.24 and BIC of 536.82) are related to the AR(2) model.

[INSERT FIGURE 16 ABOUT HERE]

[INSERT FIGURE 17 ABOUT HERE]

[INSERT FIGURE 18 ABOUT HERE]

[INSERT FIGURE 19 ABOUT HERE]

Table 1.4. Tests for unit root - Monthly

	Dickey-Fuller test for unit root		Phillips-Perron test for unit root	
H0	RW without drift	RW with trend and drift	RW without drift	RW with trend and drift
$\ln A^0$	-3.20	-8.75	-1.92	-8.19
	(0.02)	(0.00)	(0.33)	(0.00)
$\Delta \ln A^0$	-38.11	-38.08	-43.87	-43.86
	(0.00)	(0.00)	(0.00)	(0.00)
$HP.\ln A^0$	-12.91	-12.9	-13.35	-13.34
	(0.00)	(0.00)	(0.00)	(0.00)
lnA	-3.92	-4.55	-3.72	-5.4
	(0.00)	(0.00)	(0.00)	(0.00)
ΔlnA	-25.75	-25.73	-25.92	-28.68
	(0.00)	(0.00)	(0.00)	(0.00)
HP.lnA	-7.08	-7.07	-7.18	-8.84
	(0.00)	(0.00)	(0.00)	(0.00)
lnS	0.24	-3.52	0.12	-2.05
	(0.97)	(0.04)	(0.97)	(0.03)
ΔlnS	-22.22	-22.24	-22.30	-22.43
	(0.00)	(0.00)	(0.00)	(0.00)
HP.lnS	-6.12	-6.11	-6.76	-5.74
	(0.00)	(0.00)	(0.00)	(0.00)
Market Return	-22.99	-22.97	-22.97	-22.97
	(0.00)	(0.00)	(0.00)	(0.00)
Market Return - Risk Free Rate	-22.95	-22.96	-22.93	-22.94
	(0.00)	(0.00)	(0.00)	(0.00)
Market Return - FF3 adjusted	-24.12	-24.19	-24.15	-24.21
	(0.00)	(0.00)	(0.00)	(0.00)

This table reports the Dickey-Fuller and Phillips-Perron test statistics (and MacKinnon approximate p-values in parenthesis) of key variables for NYAM stocks constructed with monthly data from January 1961 to December 2011 (600 observations). The table shows test statistics and p-values of Returns and the log-transformed Amihud (2002) measure, including its stationary transformations. It also shows the test statistics and respective p-values of the turnover version of Amihud (2002) measure and the size variable, as well as their starionary versions.

The ACF of $HP. \ln A^0$ decrease exponentially until the 9th lag, and the PACF cuts off after the 2nd lag. It seems that and ARMA with enough lags can capture the dynamic of this variable, we test several lags combinations using AIC and BIC as comparison criteria. The lowest AIC and BIC, 466.60 and 488.58 respectively, are achieved when we estimate an ARMA (2,2).

The Equally Weighted Market Illiquidity using the turnover version of Amihud (2002) measure calculated with monthly data is harder to analyze. Figures 20 show the path of this variable, it seems to have a decreasing trend but it is not as clear as the yearly counterpart.

[INSERT FIGURE 20 ABOUT HERE]

As we show in Table 4, both ADF and Phillips-Perron tests reject the hypothesis of unit root at a 1%-level when we do not control for serial correlation of this series. Figure 21 shows the autocorrelation function of $\ln A$, we see that it dumps out close to the 20th lag (which looks like a non-stationary process). We run the ADF test allowing the autocorrelation up to the 20th lag and it yields a calculated statistic (p-value) of -2.71 (0.07). It rejects the null hypothesis of the existence of a unit root at a 10%-level.

[INSERT FIGURE 21 ABOUT HERE]

The results in Table 1.4 also show that the first-difference of $\ln A$ and the HP-filtered transformation are stationary as all of the calculated statistics of both ADF and Phillips-Perron tests reject the null hypothesis of existence of unit root at a 1%-level. We must also access the properties of the log-transformed size, as this variable is one of the components of the turnover version of Amihud (2002). The log-transformed size seem to be non-stationary, the p-values of both the Augmented Dickey-Fuller and Phillips-Perron tests for unit root are 0.97. Figure 22 shows the path of $\ln S$ which has an upward trend.

[INSERT FIGURE 22 ABOUT HERE]

Apart from looking for strategies to stationarize $\ln A$ and $\ln S$, we analyze in the next subsection if these series cointegrate. We show that for yearly data they do not seem to

cointegrate and for monthly data they do seem to have a cointegration relation, therefore, we can use an Autoregressive Distributed Lag (ARDL) specification to estimate the model. So far we have focused on the monthly illiquidity, in the next subsection we focus on the description of monthly returns.

1.4.2.1. Monthly market return. We construct both equally and value weighted market returns but throughout the text we just show the results for the Value Weighted Market Return (VWMR). Once the average of cross-sectional returns is calculated for each period, we make two adjusts: (1) we subtract the risk-free rate, given by the one-month T-bill rate of each market return getting the return in excess of the risk-free rate, and; (2) we use Fama and French's (1993) Three Factors (FF3F) to adjust returns. These adjusts yields three market return variables for monthly data.

We denote the value VWMR in month m by r_m^M and the return in excess of the monthly risk-free rate by r_m^E. We define the additional measure, representing the market return in excess of the Fama and French (1993) three factors. The construction of this variable requires the estimation of the factor loadings, what we do using the entire sample period. Let $r_{i,m}$ denote the return of firm i's stock on month m; and r_m^f denote the risk-free rate of return for month m, given by the return on the one month T-Bill. We estimate the following model

$$\left(r_{i,m} - r_m^f\right) = a_i + b_{i,1}MKT + b_{i,2}SMB + b_{i,3}HML + \varepsilon_t \tag{1.37}$$

where MKT can be defined as the VWMR in month m; SMB stands for "Small Minus Big" and HML stands for "High Minus Low" which are calculated after forming 6 portfolios using market capitalization and book-to-market ratio. The resulting FF3F-adjusted excess return, denoted by $r_{i,m}^{FF3}$ for stock i in each month m, is calculated as the sum of the intercept and the residual from time-series regressions in which the dependent variable is the stock return in excess of the one-month T-bill rate. Then the FF3-adjusted returns can be written as

$$r_{i,m}^{FF3} \equiv \left(r_{i,m} - r_m^f\right) - \left(\widehat{b}_{i,1}MKT + \widehat{b}_{i,2}SMB + \widehat{b}_{i,3}HML\right) \tag{1.38}$$

and the VWMR adjusted for the three Fama-French Factors, r_m^{FF3}, is given by

$$r_m^{FF3} = \frac{1}{\sum_{i=1}^{N_m} S_{i,m}} \sum_{i=1}^{N_m} S_{i,m} r_{i,m} \tag{1.39}$$

All the market return measures are multiplied by 100 for expositional convenience. In Table 3 we report statistics for regressions with three different dependent variables: (i) the unadjusted market returns (r_m^M), (ii) adjusted excess return using the one-month T-bill rate (r_m^E); and (iii) the FF3-adjusted excess return (r_m^{FF3}).

The mean (standard deviation) values of the VWMR, VWMR in excess of the risk-free rate, and VWMR adjusted by the Fama-French three factors are given, respectively, by 1.40 (4.39), 0.98 (4.39), and 0.52 (4.07). The dispersion seem to be high. Some of these definitions of returns exhibit some structure but only after the 5th lag. Figures 23, 24 and 25 graph the autocorrelation of, respectively, r_m^M, r_m^E, and r_m^{FF3} and we see that the market return's only significant (considering the 95% Bartlett's band) autocorrelation is located on the 14th lag (the same can be seen for the VWMR in excess of the risk-free rate). The r_m^{FF3} has a significant autocorrelation on the 5th lag.

[INSERT FIGURE 23 ABOUT HERE]

[INSERT FIGURE 24 ABOUT HERE]

[INSERT FIGURE 25 ABOUT HERE]

We are not seeking to model any structure for market return but we may have to take it into account in the model, hence improving the estimation of the illiquidity premium. Most papers assume that returns follow an AR(1) structure as they add the lagged market return as a regressor. As depicted in the figures above the addition of an AR(1) term do not add much to results. Just to illustrate this point we run AR(1) regressions for

the three return variables, the estimated coefficients (p-values) for the lagged VWMR, VWMR in excess of risk-free rate and VWMR adjusted to the Fama-French three-factors are 0.062 (0.091), 0.063 (0.083), and 0.014 (0.682). None are 1% or 5% significant, the first two are, however, 10% significant. All of them are positive, so for a 10% level, a stock which shows a positive return over the past month may be expected maintain the positive trend. We do add an AR(1) term in the regressions we run in the next sections.

1.4.3. Cointegration relation

We show in section 1.5 that $\ln A_{t-1}$ and $\ln S_{t-1}$ seem to be non-stationary. One strategy to deal with that issue is to check if they cointegrate. In the next subsection we rewrite the model we describe in equation (1.23) and present the results of some standard cointegration tests. If the variables do not cointegrate we can simply take the first difference or filter them. This subsection shows the reparametrization we use to write equation (1.23) in the Autoregressive Distributed Lag (ARDL) formulation. We also tests if there is a cointegration relation between $\ln A$ and $\ln S$. Replacing $\ln(A_{t-2}) + [\ln(A_{t-1}) - \ln(A_{t-2})]$ for $\ln(A_{t-1})$, i.e., $\ln(A_{t-1}) = \ln(A_{t-2}) + \Delta\ln(A_{t-1})$ and $\ln(S_{t-1}) = \ln(S_{t-2}) + \Delta\ln(S_{t-1})$ in equation (1.23) we have the following model

$$(1.40)\quad r_t^j = \phi_0 + \phi_1 [\ln(A_{t-2}) + \Delta\ln(A_{t-1})] + \phi_2 [\ln(S_{t-2}) + \Delta\ln(S_{t-1})] + \varepsilon_t,\ j = 1,2$$

which is a reparametrization to get the ARDL. By simple algebra, it can be written as

$$(1.41)\quad r_t^j = \phi_0 + \phi_1\Delta\ln(A_{t-1}) + \phi_2\Delta\ln(S_{t-1}) + [\phi_1\ln(A_{t-2}) + \phi_2\ln(S_{t-2})] + \varepsilon_t,\ j = 1,2$$

or, normalizing the term in brackets by ϕ_1, we have

$$(1.42)\quad r_t^j = \phi_0 + \phi_1\Delta\ln(A_{t-1}) + \phi_2\Delta\ln(S_{t-1}) + \phi_1\left[\ln(A_{t-2}) + \frac{\phi_2}{\phi_1}\ln(S_{t-2})\right] + \varepsilon_t,\ j = 1,2$$

If there is a cointegration relation between $\ln(A_{t-2})$ and $\ln(S_{t-2})$ all of the independent variables are stationary. In order to check if that cointegration relation exists we perform 3 tests: Engle and Granger (1987), Johansen (1995), and Pesaran, Shin and Smith's (2001) (PSS, henceforth) bound tests.

Engle and Granger's (1987) approach is straightforward: we run the following regression

$$\ln A_t = a + b \ln S_t + v_t \tag{1.43}$$

If the cointegration relation exists the residuals v_t must be stationary. For yearly data, the statistic of the Augmented Dickey-Fuller test for unit root is -2.34, the related p-value is 0.16, which means that the test fails to reject the null hypothesis of unit root. Therefore, we can conclude that for yearly data the turnover illiquidity variable and size data do not seem to cointegrate. This result holds when we add a drift and a trend to the underlying mode and also when we add more lags to control for autocorrelation in the series.

For monthly data the statistic of the Augmented Dickey-Fuller test of the residuals of the estimated equation (1.43) for unit root is -4.58, using the MacKinnon critical values, the p-value is 0.00. This result holds when we control for more lags of autocorrelation and test against an underlying model that includes a trend. It other words, the test rejects the null hypothesis of unit root in a 1%-level, suggesting that $\ln A$ and $\ln S$ cointegrate. Alternative unit-root tests, like Phillips-Perron's, yield a similar result.

Johansen (1995) suggests a VAR based test to cointegration. Differently from Engle Granger, that is OLS based, this test is a Maximum Likelihood procedure and, for this reason, it relies to a large extent on asymptotic properties. If applied to small samples the results might be difficult to understand. The test uses the relationship between the rank of a matrix and its eigenvalues or characteristic roots and it is possible to obtain more then a single cointegrating relationship. For yearly data the rank 0 hypothesis has

a trace statistic of 7.61, the 5% critical value is 15.41. This shows signs that the variables do not cointegrate. For monthly data we have a different scenario, the trace statistic for the rank 0 hypothesis is 27.75, which is higher than the 5%-critical value of 15.41. The trace statistic for the rank 1 hypothesis is 0.07 and the 5%-critical value is 3.76. This result suggests that there is a cointegration relation between $\ln A$ and $\ln S$.

Engle-Granger and Johansen approaches have some disadvantages. First, they require that all the underlying variables to have the same order of integration. Second, the ML-based Johansen cointegration test relies on asymptotic properties that do not hold in small sample. The most popular alternative cointegration approach that is more adequate to small sample size is PSS bound test, which can be done when underlying variables are I(0) or I(1) and is adequate for small sample size estimation.

PSS builds two sets of critical values for both situations where all the underlying variables are I(0), and all underlying variables are either I(0), or I(1). Then we have values of F and t test statistics that indicate the possible existence of a long-term relationships between the underlying variables.

In order to perform the PSS test all variables must not be I(2). As we show in section 5.2, they all seem to be I(1). The test is run by estimating model (1.41) and testing if $\phi_1 = \phi_2 = 0$. The issue is that we cannot use an usual F-test because the distribution of the test statistic is non-standard. To tackle that issue, PSS calculate bounds on the critical values for the asymptotic distribution of the F-statistic. The assumption to get the lower bound is that all of the variables are I(0). The assumption for the upper bound is that all of the variables are I(1). However, it can be also the case in which we are somewhere in between these two extremes.

Then we can just the computed F-statistic, if it lies below the lower bound then the variables are I(0) and there is no possible cointegration. If the F-statistic lies above the upper bound, there is cointegration. If the F-statistic lies between the bounds, the test is inconclusive.

The value of the F-statistic is 5.827 and there are 2 variables in our model. The lower and upper bounds for the F-test statistic at the 10%, 5%, and 1% significance levels are [4.04, 4.78], [4.94, 5.73], and [6.84, 7.84] respectively. The value of the calculated F-statistic exceeds the upper bound of both 10% and 5% significance levels, therefore there is evidence of a long-run relationship between the two time-series at these levels.

We use three tests for monthly data that indicate that there is a long-run relationship between the turnover version of Amihud (2002) measure and Size. For yearly data we use two of these tests and get the conclusion that this relation does not hold. This means that for yearly data we can use the same transformations we use for Amihud's (2002) original measure, i.e., we use first differenced and HP-filtered illiquidity variables. For monthly data we can reparametrize to get the long term relation between $\ln A$ and $\ln S$ and then reestimate the model (we do use the same methodology we adopt for yearly data. i.e., first difference and HP filter, for monthly data too for comparison). We can estimate the coefficients of the cointegrating ARDL model in two ways that are asymptotically equivalent:

$$r_t^j = \phi_0 + \phi_1 \ln(A_{t-2}) + \phi_2 \ln(S_{t-2}) + \phi_3 \Delta \ln(A_{t-1}) + \phi_4 \Delta \ln(S_{t-1}) + \varepsilon_t, \;\; j = 1, 2 \tag{1.44}$$

and

$$r_t^j = \phi_0 + \phi_1 \Delta \ln(A_{t-1}) + \phi_2 \Delta \ln(S_{t-1}) + \phi_3 \upsilon_t + \varepsilon_t \tag{1.45}$$

We also run the same models assuming that returns follow an AR(1) process, i.e., we add $r_{VW,t-1}^j$ as a regressor. The inclusion of this variable does not represent any econometric problem as it is stationary.

1.5. Results

This section analyses the return premium associated with the log transformed original Amihud (2002) measure, $\ln A^0$, the turnover-version, $\ln A$, and size, $\ln S$. We show the results in two separate subsections. The first focus on yearly data, the second on monthly. We do so for clarity, in each subsection we discuss the results and their interpretations.

1.5.1. Yearly

Yearly data shows that Amihud (2002) measure is priced. However, to access the premium we need to make the illiquidity series stationary. By doing that, we find positive and significant coefficients, which represent the yearly illiquidity premium. We take into account the presence of the unexpected illiquidity but it does not change the values of the estimated coefficients. This subsection shows the estimated coefficients and their respective statistical and economic significance (the impact on returns of a one-standard deviation of the illiquidity measure).

The last exercise of this subsection is the decomposition of the original measure of Amihud (2002) into the turnover version plus the size component. The annual premium implied by the estimated coefficients using this decomposition is lower, therefore less puzzling high than the ones we find using the original measure. We also estimate the unexpected illiquidity and add it to the model to eliminate potential bias. All coefficients, results and interpretations follow below.

1.5.1.1. Amihud (2002) original measure. We verify that there is a return premium associated with the stationary log-transformed original Amihud (2002) measure. We estimate the set of equations described in models (1.20) and (1.21) using data for market return and illiquidity in the period from January 1962 to December 2011 for NYAM-listed stocks.

Table 1.5 shows that illiquidity is priced. In the table there are results of time series regressions, the direction and values of the coefficients of all the illiquidity measures we use in the paper. Amihud (2002) measure, taken as it is, i.e., without any transformation to

make it stationary, is not significant at any conventional level. As the variables of market returns are stationary, this result should not be surprising. The value of the coefficient of $\ln A^0$ is 0.594, when the dependent variable is the VWMR, and -0.294 when the dependent variable is the VWMR in excess of the risk-free rate.

Therefore, the results of Amihud's (2002) approach to a databank ending in 2011 is supposed to lead to the conclusion that illiquidity effect is not priced. In terms of goodness of fit, the model with $\ln A^0$ yields an adjusted R-square of -0.0177 (-0.0204) for VWMR (VWMR in excess of the risk-free rate) as the dependent variable.

When we turn to the stationary versions of the measure, we see that illiquidity is indeed priced in yearly data. The coefficients of the Amihud (2002) measure for the stationary versions of Amihud's (2002) original measure, $\Delta \ln A^0_{t-1}$ and $HP. \ln A^0_{t-1}$, are positive and significant (even though at only a 10%-level for the first differenced log-measure, $\Delta \ln A^0_{t-1}$). The adjusted R-squares increase as well, for VWMR they are 0.036 ($\Delta \ln A^0$) and 0.199 ($HP. \ln A^0$) and for the VWMR in excess of the risk-free rate they are 0.0446 ($\Delta \ln A^0$) and 0.237 ($HP. \ln A^0$).

The interpretation in terms of the economic impacts of the reported coefficients shows a puzzling high illiquidity premium, which is also pointed out by BHS. The estimated coefficients imply that a one-standard deviation change in $\Delta \ln A^0_{t-1}$ changes expected yearly VWMR by 4.16 basis points. A one standard deviation increase in the HP-filtered Amihud's original measure accounts for an increase of 8.01 points in VWMR. For the VWMR in excess of the risk-free rate these results are slightly higher, of 4.46 for $\Delta \ln A^0_{t-1}$ and 8.71 for $HP. \ln A^0_{t-1}$. As we discuss in the previous sections, the $\ln A^0$ series seem to have an stochastic trend, therefore $\Delta \ln A^0_{t-1}$ should be the variable to focus. We maintain the HP-filtered transformation throughout this subsection as a benchmark.

Even though we do not report the results for the equally weighted market illiquidity (EWMI) and return (EWMR), we do run models using this criterion to make a comparison with Amihud's (2002) methodology. The coefficients of the EWMI regressions are similar to the ones of the VWMI. However, it is interesting to notice that under the EWMI

Table 1.5. Price of the Yearly Amihud (2002) Measure

VARIABLES	Return			Return - Rf		
$\ln A^0$ (-1)	0.594			-0.291		
	(0.486)			(-0.263)		
$\Delta \ln A^0$ (-1)		14.317*			15.346*	
		(1.807)			(1.814)	
$HP.\ln A^0$ (-1)			41.660***			45.330***
			(4.714)			(5.802)
Constant	17.907***	16.722***	15.187***	8.934*	11.447***	9.833***
	(3.382)	(6.139)	(6.785)	(1.899)	(4.452)	(4.750)
Observations	49	48	49	49	48	49
F	0.236	3.267	22.22	0.0689	3.289	33.67

This table reports the results of yearly time-series regressions that include log-transformed Amihud (2002) measure. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variable is Return and Return-Rf. The definitions of the variables are as follows. Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). The returns are multiplied by 100. The illiquidity measures are $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta \ln A^0$: the first difference of $\ln A^0$. $HP.\ln A^0$: the HP-filtered $\ln A^0$. All the explanatory variables are lagged in one period For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must have The security must have at least 200 days of valid observations during year y-1.The stock must be listed at the end of year y-1; Price at the end of the year must be higher than 5 dollars; Every observation with missing values for our size variable (market capitalization) is dropped; Only observations with no zero monthly volume are considered; Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

criterion a one standard deviation increase in $\Delta \ln A^0_{t-1}$ increases the expected VWMR by 5.37 points, and increases the VWMR in excess of the risk-free rate in 5.59 points, which represents an even higher effect of illiquidity over returns. Not only is the effect more meaningful but it is also 5%-significant, in contrast to the 10%-significance level we find by using the VWMI. It is also interesting to note that under the EWMI rationale the coefficients of the non-stationary illiquidity $\ln A^0_{t-1}$ remain non significant at any conventional level, but they are also higher and more economically relevant than its VWMI counterpart.

As we show in the previous sections, there is no indication that yearly market return has any AR or MA structure. Therefore adding a lagged return on this regression is not supposed to change much. We add this lagged term in any case and the coefficient relative to it is non significant at any conventional level and the illiquidity premium increases a little. Take the economic impact of $\Delta \ln A^0_{t-1}$ for instance, it represents an additional increase of 0.03 basis points in VWMR and a decrease in the adjusted R-square, that falls to 0.0149.

The overall result of this subsection implies that, when market illiquidity is higher than its unconditional time-series average, market return during that period is higher-than-average.

The Kendall's (1954) bias correction has a small impact over the results. The effect of a one-standard deviation increase in return on $\Delta \ln A^0_{t-1}$ [$HP. \ln A^0_{t-1}$] is 4.42 [8.50] and on return in excess of the risk-free rate is 4.74 [13.97]. This represents an increase of 0.27 [0.49] of yearly market return points for a shock in $\Delta \ln A^0_{t-1}$ [$HP. \ln A^0_{t-1}$].

The next step is to include the unexpected illiquidity. Table 1.6 shows the estimated coefficients of the *expected* illiquidity. The first thing to note is that for the non-stationary measure, $\ln A^0_{t-1}$, the coefficient is 1%-significant and its value is almost 1, a direct consequence of the unit root process. When we turn to the stationary versions, the first differenced illiquidity, $\Delta \ln A^0_{t-1}$, has an AR(1) term of -0.037, which is non significant at any conventional level. The estimated coefficient of the AR(1) term of the HP-filtered

illiquidity is 0.066, which is also non-significant. That result is expected, as we say in this section, the ACFs and PACFs show no indication that the illiquidity processes follow an AR(1).

Table 1.6. Measuring Unexpected Amihud (2002) Measure - Yearly

VARIABLES	$\ln A^0$	$\Delta \ln A^0$	HP.$\ln A^0$
$\ln A^0$ (-1)	1.006***		
	(39.880)		
$\Delta \ln A^0$ (-1)		-0.037	
		(-0.240)	
HP. $\ln A^0$ (-1)			0.066
			(0.438)
Constant	-0.082	-0.111**	-0.001
	(-0.730)	(-2.462)	(-0.034)
Observations	49	48	49
R^2	0.971	0.001	0.004
F	1590	0.0576	0.191

This table reports the results of the estimated coefficients to measure the expected illiquidity. It assumes that illiquidity follows an AR(1). The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The illiquidity measures are the value weighted averages of the following variables $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta \ln A^0$: the first difference of $\ln A^0 . HP.lnA^0 : theHP - filteredlnA^0$. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics To survive in the sample, stocks must have The security must have at least 200 days of valid observations during year y-1.The stock must be listed at the end of year y-1; Price at the end of the year must be higher than 5 dollars; Every observation with missing values for our size variable (market capitalization) is dropped; Only observations with no zero monthly volume are considered; Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

Using this estimated coefficients we find the unexpected illiquidity and estimate the model we describe in equation (1.33). The results in Table 1.7 show that the addition of this unexpected component does not change any of the estimated illiquidity premia. They remain with the same values and directions. We also see that when the illiquidity variables we use are $\ln A^0_{t-1}$ and $\Delta \ln A^0_{t-1}$ the unexpected illiquidity is 1%-significant and negative, as Amihud (2002) predicts. The economic impact of the unexpected illiquidity variables are, however, quite high. The standard deviation of the unexpected $\ln A^0_{t-1}$ and $\Delta \ln A^0_{t-1}$ is 0.29 for both, a increase of one-standard deviation on these two variables represent a decrease of 7.44 and 7.34 points of VWMR, respectively, and a decrease of 7.66 and 7.61 of VWMR in excess of risk-free rate.

The adjusted R-square increases its value when we add the unexpected illiquidity when $\Delta \ln A^0$ is used. For VWMR, the measure when the unexpected component is included is 0.2084, quite higher than the 0.036 we find previously. For the $HP. \ln A^0$, on the other hand, the R-square is reduced slightly, its value is 0.1904, while in the previous case it is 0.199. The evidence is similar for when we use the VWMR in excess of the risk-free rate.

By using the MA(2) model as an alternative to obtain the expected $\Delta \ln A^0_{t-1}$ and $HP. \ln A^0_{t-1}$ we have the estimated coefficients reported on Table 1.8. The coefficients of the MA(2) component are -0.286 for $\Delta \ln A^0_{t-1}$ and -0.520 for the HP-filtered $\ln A^0_{t-1}$ and they are, respectively, 10% and 1% significant.

The coefficients of $\Delta \ln A^0_{t-1}$ are non significant at any conventional level, unlike the coefficients we find by using the AR(1)-expected illiquidity (which we show above to be 10%-significant). The coefficients representing the illiquidity premium of the lagged HP-filtered $\ln A^0_{t-1}$ are 1%-significant. Their values are a little higher than the ones of the previous models we estimate. The range from 41.660 in the AR(1) approach to 42.405 when the dependent variable is VWMR and from 45.330 to 46.030 when the dependent variable is VWMR in excess of the risk-free rate.

The unexpected HP-filtered $\ln A^0$, on the other hand, is only 10%-significant for VWMR and non significant for VWMR in excess of the risk-free rate. A one-standard

Table 1.7. Price of Yearly Amihud's (2002) Measure with the Unexpected Illiquidity given by an AR(1)

VARIABLES	Return			Return - Rf		
$\ln A^0$ (-1)	0.594			-0.291		
	(0.586)			(-0.319)		
Unexpected $\ln A^0$	-25.622***			-26.394***		
	(-3.474)			(-3.483)		
$\Delta \ln A^0$		14.317*			15.346*	
		(1.908)			(1.884)	
Udamihud0		-25.033***			-25.963***	
		(-3.228)			(-3.214)	
$HP.\ln A^0$ (-1)			41.660***			45.330***
			(4.713)			(5.785)
Unexpected $HP.\ln A^0$			-8.020			-7.975
			(-0.871)			(-0.792)
Constant	17.907***	16.722***	15.187***	8.934**	11.447***	9.833***
	(4.380)	(7.601)	(6.812)	(2.413)	(5.456)	(4.782)
Observations	49	48	49	49	48	49
F	10.35	13.31	11.65	6.741	16.40	17.57
t-statistics in parentheses						

This table reports the results of yearly time-series regressions that include log-transformed Amihud (2002) measure and its stationary transformations as well as the unexpected illiquidity, given by the residuals of an AR(1) regression. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variable is Return and Return-Rf. The definitions of the variables are as follows. Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). The returns are multiplied by 100. The illiquidity measures are $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta \ln A^0$: the first difference of $\ln A^0$. $HP.\ln A^0$: the HP-filtered $\ln A^0$. All the explanatory variables are lagged in one period, except by the unexpected illiquidity, we use the "contemporaneous" variable, given by the residuals of AR(1) regressions on the illiquidity variables. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must have The security must have at least 200 days of valid observations during year y-1.The stock must be listed at the end of year y-1; Price at the end of the year must be higher than 5 dollars; Every observation with missing values for our size variable (market capitalization) is dropped; Only observations with no zero monthly volume are considered; Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

deviation increase in the unexpected $HP. \ln A^0$ decreases yearly VWMR by 2.27 points and VWMR in excess of the risk-free rate in 2.13 points.

It seems that the addition of the unexpected illiquidity does not represent such an improvement in the estimated illiquidity premium, specially if we assume an AR(1) process for getting the expected value of illiquidity. Changing the specification of the expected

Table 1.8. Price of Yearly Amihud's (2002) measure with the unexpected illiquidity given by an MA(2)

VARIABLES	Return			Return-Rf	
ΔlnA^0 (-1)	12.577			13.581	
	(1.630)			(1.609)	
Unexpected ΔlnA^0	-28.398***			-28.810***	
	(-3.879)			(-3.671)	
HP.lnA^0 (-1)		42.405***			46.030***
		(4.709)			(5.692)
Unexpected HP.lnA^0 (-1)		-13.820*			-12.982
		(-1.714)			(-1.515)
Constant	16.569***	15.175***	8.934**	11.291***	9.822***
	(8.278)	(6.800)	(2.413)	(5.795)	(4.779)
Observations	48	49	49	48	49
F	14.45	11.25	6.741	16.09	17.16

This table reports the results of yearly time-series regressions that include log-transformed Amihud (2002) measure and its stationary transformations as well as the unexpected illiquidity, given by the residuals of an MA(2) regression. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variable is Return and Return-Rf. The definitions of the variables are as follows. Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). The returns are multiplied by 100. The illiquidity measures are lnA^0: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. ΔlnA^0: the first difference of lnA^0. HP.lnA^0: the HP-filtered lnA^0. All the explanatory variables are lagged in one period, except by the unexpected illiquidity, we use the "contemporaneous" variable, given by the residuals of MA(2) regressions on the illiquidity variables. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must have The security must have at least 200 days of valid observations during year y-1.The stock must be listed at the end of year y-1; Price at the end of the year must be higher than 5 dollars; Every observation with missing values for our size variable (market capitalization) is dropped; Only observations with no zero monthly volume are considered; Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively..

illiquidity to an MA(2), following the evidence we drawn from data, changes the results slightly. However, it is worth noting that the impact of unexpected illiquidity seem to be relevant, it lowers market returns by a large amount in terms of return basis points.

1.5.1.2. Turnover version. This section analyses the Turnover version of Amihud's (2002) measure, as described in equations (1.23) and (1.24), as well as their unexpected illiquidity counterparts. The estimates of Models (1.23) are reported in Table 1.9. The

rationale for the interpretation of the coefficients follow BHS. We decompose the Amihud (2002) measure, $\ln A^0$, into its turnover version, $\ln A$, and firm size, $\ln S$, including them as separate regressors.

The effect of the turnover-version Amihud measure, $\ln A$, is highly dependent on the strategy we use to make the variable stationary. Another important feature is that the original Amihud (2002) measure constrains the coefficients of $\ln A$ and $\ln S$ to be equal and of the opposite sign, therefore we test if the sum of these two coefficients is equal to 0.

The effects of the HP-filtered $\ln A$ are positive and 5% and 1% significant for, respectively, VWMR and VWMR in excess of the risk-free rate. For the HP-filtered illiquidity, the coefficients of the size component, $\ln S$, are negative and 1% statistically significant. Moreover, the hypothesis that the sum of the coefficients of $\ln A$ and $\ln S$ is equal to 0 is rejected at a 1% level.

The effects $\Delta \ln A$ are non significant at any conventional level. The log-transformation of size is negative but also non significant, therefore the test fails to reject the hypothesis that the sum of both coefficients are equal to 0. The coefficient of the non-stationary illiquidity, $\ln A$, is 5% significant but the impact of $\ln S$ is positive, which is not what was expected.

The economic significance of the turnover-version measure, $\ln A$, show a more reasonable magnitude for the illiquidity effect. A one-standard-deviation change in $\ln A$ is associated with incremental expected market return (return in excess of risk-free rate) of about 1.83% (1.77) per year. A one-standard-deviation change in $\Delta \ln A$ and in $HP.\ln A$ represent an incremental expected VWMR (VWMR in excess of risk-free rate) of about 4.24% (4.19%) and 4% (4.65%) per year. This figures, specially for the HP-filtered case, make more sense than the almost 8% premium that is estimated using Amihud's original measure, even though some of these coefficients are non-significant at any conventional level.

Table 1.9. Pricing of the Yearly market Turnover-Version Amihud (2002) measure

VARIABLES	Return			Return - Rf		
lnA (-1)	17.479**			16.880**		
	(2.505)			(2.484)		
lnS (-1)	6.835*			7.823**		
	(1.847)			(2.178)		
ΔlnA (-1)		14.689			11.164	
		(0.957)			(0.727)	
ΔlnS (-1)		-2.815			-12.276	
		(-0.181)			(-0.708)	
HP.lnA (-1)			28.171**			30.603***
			(2.593)			(2.774)
HP.lnS (-1)			-123.657***			-127.640***
			(-5.786)			(-5.951)
Constant	-253.088**	16.117***	15.321***	-266.468**	11.233***	9.984***
	(-2.161)	(6.914)	(7.193)	(-2.336)	(5.048)	(5.105)
Observations	49	48	49	49	48	49
F	3.573	0.666	28.74	3.098	0.825	38.22
F stat - Illiq+size=0	5.43	0.67	12.40	5.91	0.21	12.40
p-value - Illiq+size=0	0.02	0.52	0.00	0.02	0.65	0.00

This table reports the results of yearly time-series regressions that include log-transformed turnover-version Amihud (2002) measure and its stationary transformations. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variable is Return and Return-Rf. The definitions of the variables are as follows. Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). The returns are multiplied by 100. The illiquidity measures are lnA: the log-transformation turnover-version Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to turnover multiplied by 1,000,000. ΔlnA: the first difference of lnA. HP.lnA: the HP-filtered lnA. All the explanatory variables are lagged in one period. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must have The security must have at least 200 days of valid observations during year y-1.The stock must be listed at the end of year y-1; Price at the end of the year must be higher than 5 dollars; Every observation with missing values for our size variable (market capitalization) is dropped; Only observations with no zero monthly volume are considered; Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

For comparison, we re-estimate the model with Amihud's (2002) original measure and add log-transformation of size. The illiquidity premia is remarkably closer to the premia of the turnover-version. Size presents the same behavior we describe in this section: is non-significant at any conventional level when the illiquidity variable is $\Delta \ln A^0$ and is negative and 1%-significative when taken with $HP. \ln A^0$.

The addition of the unexpected illiquidity given by the residuals of an AR(1) regression, as in Amihud (2002), changes the values of the coefficients, as we show in Table 1.10. The only illiquidity variable with 1%-significant coefficients is the HP-filtered version. A one-standard deviation of $HP. \ln A$ accounts for an increase of 0.71 points in yearly VWMR and 0.74 points in yearly VWMR in excess of the risk-free rate. That is considerably lower than the figures we describe in the previous subsection. The effect of the unexpected illiquidity is negative and 1%-significant for every regression.

It is not clear, therefore, if the decomposition of illiquidity brings gains to understanding yearly illiquidity premium as many coefficients are non-significant at any conventional level. It is clear that, in terms of the magnitude of the premium, the decomposition brings more reasonable results, i.e., not as high as the ones we find by using Amihud (2002) original measure. However, the statistical significance of the illiquidity premium shows no robust results. If we look exclusively at the HP-filtered variables we do have 1%-significant coefficients but this result does not hold for the first-differenced $\ln A$. It is unclear which measure dominates, as it seems to be clear in monthly cross-sectional data, as Brennan, Huh and Subrahmanyam (2013) point out.

1.5.2. Monthly

As we do with yearly data, this section shows the coefficients estimated with monthly data and their respective statistical and economic significance (the impact on returns of a one-standard deviation illiquidity measure). We show that in most of the models illiquidity is non-significant at any conventional level and the estimated premium has a negative direction, which implies that a higher illiquidity predicts a lower return in the following

Table 1.10. Price of Yearly turnover-version Amihud (2002) measure with the unexpected illiquidity given by an AR(1)

VARIABLES	Return			Return - Rf		
lnA (-1)	4.024			3.612		
	(0.623)			(0.520)		
lnS (-1)	-0.221			0.843		
	(-0.061)			(0.210)		
Unexpected lnA (-1)	-63.363***			-64.300***		
	(-7.221)			(-7.079)		
ΔlnA (-1)		-10.471			-13.858	
		(-1.070)			(-1.462)	
ΔlnS (-1)		-57.204***			-66.150***	
		(-2.789)			(-3.157)	
Unexpected ΔlnA (-1)		-70.510***			-71.704***	
		(-8.614)			(-8.455)	
HP.lnA (-1)			53.449***			55.800***
			(4.439)			(4.489)
HP.lnS (-1)			-125.139***			-129.623***
			(-4.799)			(-4.707)
Unexpected HP.lnA			-58.561***			-58.282***
			(-4.142)			(-3.968)
Constant	-19.556	20.882***	17.039***	-35.825	15.984***	11.657***
	(-0.172)	(8.809)	(7.207)	(-0.288)	(6.482)	(4.997)
Observations	49	47	48	49	47	48
F	41.86	31.17	40.30	42.40	31.95	40.33

This table reports the results of yearly time-series regressions that include log-transformed turnover-version Amihud (2002) measure and its stationary transformations. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variable is Return and Return-Rf. The definitions of the variables are as follows. Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). The returns are multiplied by 100. The illiquidity measures are lnA: the log-transformation turnover-version Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to turnover multiplied by 1,000,000. ΔlnA: the first difference of lnA. HP.lnA: the HP-filtered lnA. All the explanatory variables are lagged in one period. For each explanatory variable there are the coefficient estimates, and the values in parenthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must have The security must have at least 200 days of valid observations during year y-1.The stock must be listed at the end of year y-1; Price at the end of the year must be higher than 5 dollars; Every observation with missing values for our size variable (market capitalization) is dropped; Only observations with no zero monthly volume are considered; Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

period. However, this result depends on the strategy we use to make data stationary. HP-filtered illiquidity measures are 1%-significant and positive.

We also estimate the unexpected component of illiquidity and add it to the model in order to alleviate potential bias. We decompose the original measure of Amihud (2002) into the turnover version plus size and find results that are similar in magnitude to the ones we find using the original measure. Therefore it is harder to make statements about which measure makes "more sense", like BHS do.

1.5.2.1. The Amihud (2002) original measure. Table 1.11 shows the results of the estimated coefficients of models (1.20) and (1.21) for monthly data as well as their t-statistics and respective statistical significances. Illiquidity is only 1%-significant when the HP-filtered transformation is used. Both the first-differenced illiquidity and non-stationary illiquidity yield coefficients that are non significant at any conventional level. We find mixed results about the direction of the effects. The first differenced $\ln A^0$ has a negative direction, i.e., an increase in this variable is supposed to lower next period's return, which was not expected, even though the coefficient of this variable is non significant at any conventional level. The additional Kendall's bias correction adds nothing to the analysis as we have many (600) observations.

The coefficients of $\ln A^0$ and $\Delta \ln A^0$ are non-significant at any conventional level. Apart from the lack of statistical significance, they are also not very meaningful. A one-standard deviation shock in $\ln A^0$ increases the VWMR by 0.096 points (the resulting product of the standard deviation 1.55 by the coefficient 0.062), which represents an annualized increase of 1.16 points in VWMR. A one-standard deviation shock in $\Delta \ln A^0$, on the other hand, decreases monthly VWMR by -0.017 points, which represents -0.20 points in terms of the annualized VWMR. The value of the coefficient of $HP. \ln A^0$ is 1.199, which is 1%-significant. A one-standard deviation shock in it increases monthly VWMR by 0.554 points, which represents an increase of 6.85 return points in annualized terms. If we assume that $\ln A^0$ is trend-stationary the detrended HP-filtered series should be the one that makes more sense.

When the dependent variable is VWMR in excess of the risk-free rate the coefficients show a similar behavior. As in the previous case, $\ln A^0$ and $\Delta \ln A^0$ are non significant at any conventional level, only the $HP. \ln A^0$ is 1% significant. A one-standard deviation shock in $\ln A^0$, $\Delta \ln A^0$, and $HP. \ln A^0$ increase this dependent variable, respectively by 0.003, -0.022, and 0.559, which represents effects of 0.04, -0.27, and 6.92 in annualized terms.

Table 1.11. Price of the Monthly Amihud (2002) Measure

VARIABLES	Return			Return-Rf			Return - FF3F		
$\ln A^0$ (-1)	0.062			0.002			-0.012		
	(0.504)			(0.017)			(-0.681)		
$\Delta \ln A^0$ (-1)		-0.039			-0.052			-0.047	
		(-0.132)			(-0.175)			(-0.890)	
HP.$\ln A^0$ (-1)			1.199***			1.211***			0.070*
			(2.709)			(2.766)			(1.912)
Constant	1.611***	1.409***	1.410***	0.990**	0.982***	0.984***	-0.369***	-0.381***	-0.381***
	(3.582)	(7.654)	(7.518)	(2.176)	(5.320)	(5.232)	(-13.357)	(-17.783)	(-17.899)
Observations	599	598	599	599	598	599	599	598	599
F	0.254	0.0174	7.336	0.000	0.0305	7.653	0.464	0.792	3.656

This table reports the results of monthly time-series regressions that include log-transformed Amihud (2002) measure. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variables are: Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. The illiquidity measures are: $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta \ln A^0$: the first difference of $\ln A^0$. HP.$\ln A^0$: the HP-filtered $\ln A^0$. All the explanatory variables variables are lagged in one period For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

The final dependent variable is return adjusted by the VWMR adjusted for Fama-French three factors. The main differences in this case are that (i) the coefficient on $\ln A^0$ is negative, however, it is still non significant and (ii) the economic and statistical effects of $HP.\ln A^0$ are reduced. If a one-standard deviation shock takes place on $\ln A^0$, $\Delta \ln A^0$ and $HP.\ln A^0$, the effects on the measure of return are -0.019, -0.020, and 0.032, which is translated in -0.22, -0.24, and 0.39 per year. It means that the response of return adjusted to the FF3F to an increase in the HP-filtered variable is 6.53 points lower than the response of VWMR in excess of the risk-free rate. Furthermore, $HP.\ln A^0$ is only 10%-significant when this measure of return is used.

The difference between the annualized effect of a one-standard deviation in the illiquidity variables and the annual effect we estimate with yearly data is not so large for the HP-filtered series. The yearly VWMR (VWMR in excess of the risk-free rate) premium relative to the $HP.\ln A^0$ is 8.01 (8.71). The annualized monthly impact of a one standard deviation on the HP-filtered variable is 1.16 (1.79) points lower than its yearly counterpart[6].

We add a lagged return variable to these models in order to check any effect of an autorregressive component and report the estimated coefficients in Table 1.12. The coefficients of $\ln A^0$ and $\Delta \ln A^0$ remain statistically non-significant at any conventional level, but in this specification the coefficient of $\Delta \ln A^0$ is positive. The adjusted R-square increases when we add the lagged return. The value of this measure is -0.0017 for the model with only $\Delta \ln A^0$ as covariate. The addition of the lagged return increases it to 0.0006.

The coefficients relative to $HP.\ln A^0$ remain 1% and 5% significant, depending on the return variable we use. For VWMR, the annualized monthly premium of $HP.\ln A^0$ increases in 0.20 annual return points comparing to the model estimated without the AR(1) term. The adjusted R-square increases a little, from 0.0143 to 0.0174.

Taking VWMR in excess of the free rate as the dependent variable, the impact of the HP-filtered illiquidity is increased by 0.21 annual return points, and the adjusted R-square

[6]We also detrend the $\ln A^0$ series by fitting a linear trend. The estimated coefficients of the lagged detrended illiquidity variable on all the return variables are non-significant at any conventional level.

increases from 0.0146 to 0.0179. When the dependent variable is VWMR adjusted for the Fama-French three factors the results are a little different. The HP-filtered coefficient becomes 5%-significant and the annualized impacts of a one-standard deviation shock is lowered by 0.12 points. The adjusted R-square decreases from 0.0094 to 0.0082.

Just for comparison, if the variables were calculated using EWMI, the results would not change their directions but they would indeed change their statistical significances. By using the EWMI criterion, all the coefficients relative to the premium of $\ln A^0$ are 10% significant when return is used as dependent variable. The coefficients relative to $\Delta \ln A^0$ are 5%-significant with any of the dependent variables. The economic impacts of the coefficients, however, are quite different. Take the specification without the AR(1) term for illustration. A one-standard deviation increase in $\ln A^0$, $\Delta \ln A^0$ and $HP. \ln A^0$ accounts for a respectively annualized increase of 8.76, a decrease of 11.41 and an increase of 7.28 points in annual market returns. This represents a difference of 7.601, -11.208, and 0.431 return points comparing to the VWMI. This highlights how important the strategy adopted to average out cross-sectional data is.

As we do in the yearly analysis, we add the unexpected illiquidity and check the results in order to eliminate some potential bias. If Amihud's (2002) rationale holds, by including the unexpected illiquidity the coefficients should reduce their values. Table 1.13 shows the estimated coefficients of the AR(1) regression of Amihud (2002) illiquidity measure and its first differenced and HP-filtered versions.

All the coefficients are 1% significant. The value of the coefficient of the lagged $\ln A^0$ is almost 1, as expected after all the tests showing signs that this variable follows an unit root process. The coefficient of the lagged first differenced $\ln A^0$ is negative and of the lagged HP-filtered $\ln A^0$ is positive, in other words, the first coefficient is saying that if illiquidity decreased from period $t-1$ to period t it is expected that it will increase in period $t+1$, in some mean-reversion-like way. The positive coefficient of the HP-filtered says that if the filtered variable is high in one period it will be likely high in the next one.

Table 1.12. Price of the Monthly Amihud (2002) Measure AR(1)

VARIABLES	Return			Return-Rf			Return - FF3F		
$\ln A^0$ (-1)	0.068			0.012			-0.005		
	(0.579)			(0.102)			(-0.420)		
$\Delta \ln A^0$ (-1)		0.093			0.083			-0.001	
		(0.325)			(0.288)			(-0.023)	
HP.$\ln A^0$ (-1)			1.233***			1.246***			0.048**
			(2.959)			(3.021)			(1.974)
Return (-1)	0.063	0.064	0.069	0.064	0.065	0.070	0.430***	0.431***	0.423***
	(1.388)	(1.393)	(1.492)	(1.418)	(1.433)	(1.544)	(6.468)	(6.431)	(6.217)
Constant	1.542***	1.319***	1.314***	0.961**	0.918***	0.915***	-0.213***	-0.217***	-0.220***
	(3.540)	(6.403)	(6.261)	(2.206)	(4.731)	(4.634)	(-7.678)	(-8.712)	(-8.573)
Observations	599	598	599	599	598	599	599	598	599
F	1.081	0.972	6.109	1.006	1.027	6.227	20.93	20.70	24.49

This table reports the results of monthly time-series regressions that include log-transformed Amihud (2002) measure and the lagged return. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variables are: Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. The illiquidity measures are: $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta \ln A^0$: the first difference of $\ln A^0$. HP.$\ln A^0$: the HP-filtered $\ln A^0$. All the explanatory variables are lagged in one period For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

Table 1.13. Measuring unexpected Amihud's (2002) measure - Monthly

VARIABLES	amihud0	damihud0	hp1amihud0
$\ln A^0$ (-1)	0.964***		
	(86.347)		
$\Delta \ln A^0$ (-1)		-0.124***	
		(-5.221)	
HP.$\ln A^0$ (-1)			0.073*
			(1.669)
Constant	-0.122***	-0.006	0.004
	(-3.042)	(-0.355)	(0.207)
Observations	599	597	598
R-squared	0.926	0.044	0.005

This table reports the results of the estimated coefficients to measure the monthly expected illiquidity. It assumes that illiquidity follows an AR(1). The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The illiquidity measures are the value weighted averages of the following variables $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta \ln A^0$: the first difference of $\ln A^0 . HP.lnA^0$: $theHP - filteredlnA^0$. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

Table 1.14 shows results of the estimated coefficients of the illiquidity premium considering the unexpected illiquidity in the model. All of the coefficients of unexpected illiquidity are negative, as expected, and 1%-significant. The coefficients of the lagged illiquidity variables, however, are non-statistical significant a any conventional level, with the exception of the HP-filtered version, which is 1%-significant. A one-standard deviation increase in $HP. \ln A^0$ has an annualized impact over VWMR 4.88 times higher than the one estimated without the unexpected illiquidity. If we add an AR(1) term to the

model the coefficients remain with the same statistical significance. Only $HP.\ln A^0$ is 1%-significant, and the value of its coefficient is increased.

The magnitude of the adjusted R-square suggests that the models with the unexpected illiquidity have a better fit. The value of the R-square of the model that includes the VWMR and $\Delta \ln A^0$ increases to 0.0439. We verify the same behavior to all of the models we estimate.

The last exercise related to the unexpected illiquidity is to change the model specification according to the model selection criteria we show in Section 4.2. As we discuss in that section the first differenced illiquidity seem to follow an AR(2) model, if we adjust it to data we get a new measure of the unexpected illiquidity.

The model we run using this new measure yields an estimated illiquidity premia relative to the VWMR and VWMR in excess of the risk free rate that are the same as the ones we show in Table 1.15. The main difference is when the return measure is the VWMR adjusted to the Fama-French three factors. The coefficient of $HP.\ln A^0$ decreases to 0.926. It means that a one-standard deviation increase in the HP-filtered variable increases the monthly VWMR adjusted to the FF3F in 0.43 points, which represents 5.25 annualized basis points. This is lower than the previous estimated effects, that were always very high.

We also adjust an ARMA(2,2) model to the HP-filtered illiquidity and get the residuals representing the unexpected illiquidity. The results are very close to the ones we have to the AR(2) case. The illiquidity premium for VWMR, for instance, is given by the coefficient of the lagged $HP.\ln A^0$ of 1.163. This value is not so different from the 1.199 we find by estimating the model with the AR(2) specification for the unexpected illiquidity. The result is also 1%-significant and both the value of the coefficient and the economic impact of the unexpected illiquidity are quite similar to the previous case. Therefore, the attempt to improve the model of the expected illiquidity may reveal a less puzzling high illiquidity premium.

Table 1.14. Price of Monthly Amihud's (2002) measure with the unexpected illiquidity given by an AR(1)

VARIABLES	Return			Return-Rf			Return - FF3F		
$\ln A^0$ (-1)	0.062			0.002			-0.073		
	(0.545)			(0.018)			(-0.669)		
Unexpected $\ln A^0$	-2.049***			-2.096***			-1.798***		
	(-3.401)			(-3.435)			(-3.948)		
$\Delta \ln A^0$ (-1)		-0.608*			-0.627*			-0.200	
		(-1.879)			(-1.923)			(-0.686)	
Unexpected $\Delta \ln A^0$		-2.345***			-2.370***			-1.890***	
		(-3.742)			(-3.722)			(-3.923)	
HP.$\ln A^0$ (-1)			2.113***			2.133***			1.760***
			(3.425)			(3.458)			(3.679)
Unexpected HP.$\ln A^0$ (-1)			-1.678***			-1.689***			-1.518***
			(-2.654)			(-2.637)			(-2.904)
Constant	1.611***	1.408***	1.405***	0.990**	0.981***	0.977***	0.295	0.529***	0.524***
	(3.855)	(8.300)	(7.941)	(2.339)	(5.775)	(5.516)	(0.786)	(3.238)	(3.149)
Observations	599	597	598	599	597	598	599	597	598
F	5.922	7.009	6.248	5.898	6.939	6.280	7.898	8.258	7.844

This table reports the results of monthly time-series regressions that include log-transformed Amihud (2002) measure and the lagged return. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variables are: Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. The illiquidity measures are: $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta \ln A^0$: the first difference of $\ln A^0$. HP.$\ln A^0$: the HP-filtered $\ln A^0$. All the explanatory variables are lagged in one period, except by the unexpected illiquidity, we use the "contemporaneous" variable, given by the residuals of AR(1) regressions on the illiquidity variables. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

Table 1.15. Price of monthly Amihud's (2002) measure with the unexpected illiquidity given by an AR(2)

VARIABLES	Return			Return - Rf			Return - FF3F		
$\ln A^0$ (-1)	0.062			0.002			-0.073		
	(0.545)			(0.018)			(-0.669)		
Unexpected $\ln A^0$ (-1)	-2.049***			-2.096***			-1.798***		
	(-3.401)			(-3.435)			(-3.948)		
$\Delta \ln A^0$ (-1)		-0.039			-0.052			0.259	
		(-0.143)			(-0.192)			(0.986)	
Unexpected $\Delta \ln A^0$		-2.496***			-2.524***			-1.967***	
		(-3.714)			(-3.690)			(-3.861)	
HP.$\ln A^0$ (-1)			1.199**			1.211**			0.926**
			(2.393)			(2.450)			(2.162)
Unexpected HP.$\ln A^0$			-1.819***			-1.834***			-1.617***
			(-2.754)			(-2.737)			(-2.996)
Constant	1.611***	1.409***	1.410***	0.990**	0.982***	0.984***	0.295	0.529***	0.530***
	(3.855)	(8.400)	(8.042)	(2.339)	(5.853)	(5.601)	(0.786)	(3.253)	(3.210)
Observations	599	598	599	599	598	599	599	598	599
F	5.922	6.901	6.315	5.898	6.812	6.336	7.898	7.914	7.884

This table reports the results of monthly time-series regressions that include log-transformed Amihud (2002) measure and the lagged return. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variables are: Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. The illiquidity measures are: $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta \ln A^0$: the first difference of $\ln A^0$. HP.$\ln A^0$: the HP-filtered $\ln A^0$. All the explanatory variables are lagged in one period, except by the unexpected illiquidity, we use the "contemporaneous" variable, given by the residuals of AR(2) regressions on the illiquidity variables. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

BHS use the second lag of the illiquidity measure in their analysis of the cross-sectional premium of Amihud's (2002) measure, even though they do not mention why they choose such specification. One way to see the different impacts of each lag of the market illiquidity to the market return is to plot the cross-correlation between these variables. Figure 26 plots the cross-correlation between the VWMR and VWMI. Figure 27 and 28 plot, respectively, the cross-correlations between VWMR and the first-differenced VWMI, and between the VWMR and the HP-filtered VWMI.

[INSERT FIGURE 25 ABOUT HERE]

[INSERT FIGURE 26 ABOUT HERE]

[INSERT FIGURE 27 ABOUT HERE]

All the lags of the VWMI have a positive correlation with the VWMR, even though they seem to be very close to zero. The first-differenced VWMI alternates between having positive and negative correlations with the VWMR. The correlation between the first lag of the first-differenced VWMI and VWMR is negative, the second and third are positive, but they are all close to zero. Therefore may expect that adding the second lag to the regressions we estimate yields a positive but non-significant coefficient. The lags of the HP-filtered VWMI have all positive correlations with the VWMR. It is interesting to note that all the contemporaneous illiquidity measures have a negative impact on the VWMR.

In Chapter 3 we use a VAR specification to estimate the illiquidity premium which may allow a better assessment of the number of lags we add to the model. However, in this Chapter, we use AIC and BIC criteria to access the difference between three model specifications depending on the number of lags we use. We compare models that include only the first-lagged, only the second-lagged, and both first and second lagged illiquidity measures. The results are reported in Table 1.16 and show that adding the second lag may be relevant.

Table 1.17 shows the estimated coefficients of the second lag of illiquidity As we expect, the second lags of $\ln A^0$ and $\Delta \ln A^0$ are non-significant at any conventional level. However, the value of the coefficient of $\Delta \ln A^0$ is positive. The coefficient of $HP.\ln A^0$

Table 1.16. Akaike (AIC) and Bayesian (BIC) Information Criteria for monthly data

	AIC			BIC		
	Return	Return-Rf	Return - FF3 adjusted	Return	Return-Rf	Return - FF3 adjusted
$\ln A^0$ (-1)	3473.50	3475.64	3382.40	3482.29	3484.44	3391.19
$\ln A^0$ (-2) and $\ln A^0$ (-2)	3470.65	3472.77	3378.96	3483.83	3485.95	3392.15
$\ln A^0$ (-2)	3468.65	3470.79	3377.27	3477.44	3479.57	3386.06
$\Delta \ln A^0$ (-1)	3468.95	3470.77	3377.60	3477.74	3479.56	3386.38
$\Delta \ln A^0$ (-1) and $\Delta \ln A^0$ (-2)	3465.07	3466.99	3372.03	3478.24	3480.17	3385.21
$\Delta \ln A^0$ (-2)	3463.17	3465.07	3371.71*	3471.95	3473.85	3380.49*
HP.$\ln A^0$ (-2)	3464.14	3465.83	3376.18	3472.93	3474.62	3384.97
HP.$\ln A^0$ (-2) and HP.$\ln A^0$ (-2)	3458.27*	3459.72*	3372.71	3471.45*	3472.90	3385.89
HP.$\ln A^0$ (-2)	3458.92	3460.36	3374.04	3467.71	3469.15*	3382.83

This table reports the Akainke and Bayesian information criteria of different mode specifications. The models are different according to their dependent cariable and number of lags. The models are estimated with NYAM-listed stocks from January 1962 to December 2011. The dependent variables are: Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. The illiquidity measures are: $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta \ln A^0$: the first difference of $\ln A^0$. HP.$\ln A^0$: the HP-filtered $\ln A^0$. All the explanatory variables are lagged in one or two periods. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

is 1%-significant and its value of 1.224 is not so different from the coefficient of the first lagged one, 1.233.

This subsection shows that the premium for the HP-filtered VWMI measure is positive and 1%-significant. A one-standard deviation increase in this variable increases the next period's VWMR by annualized figures that fluctuate between 7 and 8 return points, as the previous BHS evidence shows, it is puzzling high. The VWMR in excess of the Fama and French's three factors increases by 0.39 yearly return points when faced by the same increase in the HP-filtered illiquidity. Changes in the model specification and the addition of the unexpected illiquidity does not change this figures by any relevant amount. The effects of the VWMI and the first-differenced VWMI have no statistically-significant impact on the return variables. The next section shows the results of the turnover version of the illiquidity measure.

1.5.2.2. Turnover version. This section analyses the monthly turnover version of Amihud's (2002) measure, as described in equations (1.23) and (1.24), as well as their unexpected illiquidity counterparts. We decompose the Amihud (2002) measure, $\ln A^0$, into its turnover version, $\ln A$, and firm size, $\ln S$, and include them separately in the regression. The estimated coefficients as well as their t-stats are reported in Table 1.18.

The only measure that is 5%-significant is the HP-filtered variable, $HP.\ln A$, all the others are non-significant at any conventional level. The puzzling part is that the values of the coefficients of $HP.\ln A$ are negative for all the definitions of return. The coefficients of the lagged size are also negative for all the models we estimate and 1%-significant when taken together with the lagged $HP.\ln A$. The addition of the lagged return to the model or using the second lag of the illiquidity measure do change the value of the estimated coefficients but do not change their statistical significances and directions.

The coefficient of $HP.\ln A$ for VWMR as the dependent variable is -2.23. The effect of one-standard deviation on $HP.\ln A$ implies a reduction of 1.14 points of the following month's VWMR, which means a 12.88 reduction in annualized VWMR points. The result for the other variables is similar in magnitude. Adding the AR(1) term changes the value

Table 1.17. Price of the Monthly Amihud (2002) Measure - Illiquidity lagged twice

VARIABLES	Return			Return - Rf			Return - FF3F		
$\ln A^0$ (-2)	0.064			0.005			-0.094		
	(0.518)			(0.042)			(-0.797)		
$\Delta \ln A^0$ (-2)		0.386			0.379			0.478	
		(0.962)			(0.946)			(1.422)	
HP.$\ln A^0$ (-2)			1.224***			1.249***			0.718*
			(2.641)			(2.722)			(1.689)
Constant	1.617***	1.414***	1.409***	0.999**	0.986***	0.982***	0.224	0.533***	0.528***
	(3.567)	(7.645)	(7.681)	(2.174)	(5.318)	(5.345)	(0.553)	(3.105)	(3.073)
Observations	598	597	598	598	597	598	598	597	598
F	0.269	0.926	6.974	0.00175	0.896	7.410	0.635	2.022	2.851

This table reports the results of monthly time-series regressions that include log-transformed Amihud (2002) measure and the lagged return. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variables are: Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. The illiquidity measures are: $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta \ln A^0$: the first difference of $\ln A^0$. HP.$\ln A^0$: the HP-filtered $\ln A^0$. All the explanatory variables are lagged in two periods. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

Table 1.18. Pricing of the Monthly market Turnover-version Amihud (2002) measure

VARIABLES	Return			Return - Rf			Return - FF3Ft		
lnA (-1)	0.352			0.358			0.365		
	(0.775)			(0.787)			(0.896)		
lnS (-1)	0.044			0.131			0.231		
	(0.231)			(0.693)			(1.263)		
ΔlnA (-1)		-0.925			-1.022			-0.247	
		(-0.850)			(-0.942)			(-0.243)	
ΔlnS		3.908			3.828			-1.540	
		(0.869)			(0.857)			(-0.350)	
HP.lnA (-1)			-2.233**			-2.323**			-2.296**
			(-2.370)			(-2.532)			(-2.546)
Hp.lnS			-13.764***			-14.066***			-12.171***
			(-6.965)			(-7.253)			(-6.248)
Constant	-7.734	1.389***	1.409***	-9.518	0.962***	0.983***	-11.540	0.535***	0.529***
	(-0.613)	(7.580)	(7.829)	(-0.755)	(5.253)	(5.478)	(-1.006)	(2.941)	(3.117)
Observations	599	598	599	599	598	599	599	598	599
F	0.329	1.461	24.92	0.359	1.586	26.80	0.820	0.0657	20.14

This table reports the results of monthly time-series regressions that include log-transformed turnover-version Amihud (2002) measure and its stationary transformations. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variables are: Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. The illiquidity measures are: lnA: the log-transformation of the turnover version of Amihud (2002) measure. ΔlnA: the first difference of lnA. HP.lnA: the HP-filtered lnA. lnS: is the log-transformed market capitalization. All the explanatory variables are lagged in one period. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

of the coefficients but for such a small amount that we do not even report them. The coefficient of $HP.\ln A$ using VWMR as the dependent variable becomes -2.17, for instance.

If we use the cointegration relation between $\ln A$ and $\ln S$ to get the coefficients representing the illiquidity premium we get the same puzzling results. We estimate the following models:

$$r_t^j = \phi_0 + \phi_1 \ln (A_{t-2}) + \phi_2 \ln (S_{t-2}) + \phi_3 \Delta \ln (A_{t-1}) + \phi_4 \Delta \ln (S_{t-1}) + \varepsilon_t,\ j = 1, 2 \tag{1.46}$$

and

$$\ln (A_t) = a + b \ln (S_t) + \upsilon_t \tag{1.47}$$

$$r_t^j = \phi_0 + \phi_1 \Delta \ln (A_{t-1}) + \phi_2 \Delta \ln (S_{t-1}) + \phi_3 \upsilon_t + \varepsilon_t \tag{1.48}$$

The results reported in Table 1.19 show no significant results at any conventional level. The lack of significance could be understood as an evidence that the path of illiquidity is predictable, and as such it is priced. The first lag of the illiquidity variable would add no new information to the market return and therefore its coefficient is non-significant (Sadka and Sadka, 2008). However, as the HP-filtered coefficients of both original and turnover measures are significant, even though with different directions. It is hard to make such a claim without further investigation.

To illustrate more results, we run the same regression considering the cointegration relation using the EWMR as the dependent variable. The results are once again different as the estimated premia are negative. The estimated coefficients relative to the lag of the turnover illiquidity measure are 5%-significant (for market return and market return in excess of the risk-free rate) and 10%-significant for the market return adjusted for

Table 1.19. Autoregressive Distributed Lag (ARDL) model with market Turnover measure and size

VARIABLES	Return		Return - Rf		Return - FF3F	
ΔlnA (-1)	-0.655	-0.777	-0.754	-0.874	-0.054	-0.148
	(-0.591)	(-0.704)	(-0.682)	(-0.795)	(-0.052)	(-0.144)
ΔlnS (-1)	4.218	2.760	4.112	2.678	-1.371	-2.308
	(0.941)	(0.624)	(0.919)	(0.608)	(-0.311)	(-0.530)
lnA (-2)	0.483		0.495		0.381	
	(1.095)		(1.120)		(0.912)	
lnS (-2)	0.063		0.152		0.240	
	(0.349)		(0.845)		(1.293)	
Residual1		-0.641				
		(-1.353)				
Residual2				-0.642		
				(-1.365)		
Residual3						-0.429
						(-0.958)
Constant	-11.207	1.395***	-13.171	0.968***	-12.040	0.540***
	(-0.918)	(7.685)	(-1.078)	(5.336)	(-1.028)	(2.991)
Observations	598	598	598	598	598	598
F	1.142	1.608	1.296	1.718	0.449	0.391

This table reports the results of monthly time-series regressions that include log-transformed turnover-version Amihud (2002) measure and its stationary transformations. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variables are: Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. The illiquidity measures are: $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta \ln A^0$: the first difference of $\ln A^0$. HP.$\ln A^0$: the HP-filtered $\ln A^0$. All theexplanatory variables are lagged in one period, except by the unexpected illiquidity, we use the "contemporaneous" variable, given by the residuals of AR(2) regressions on the illiquidity variables. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

the Fama-French three factors. All of the coefficients are negative, what is not what we expected in the first place. These results show the limitation of using the turnover version to monthly data and highlight that there is still a long way to go in understanding the relation of illiquidity and returns. In the next section we study the stability of the parameters on the models we estimate so far.

1.5.3. Persistence in time variation

The last exercise of the Chapter is to analyze the stability of the parameters in the time series. Amihud (2002) states that they are stable and we get similar results by using the Elliott and Muller's (2006) Quasi-Local Level (QLL) test. This approach is based on the idea that seemingly different approaches of structural breaks and random coefficients are in fact equivalent. The null hypothesis is that the regression model is stable, i.e.

$$y_t = X_t'\overline{\beta} + Z_t'\delta + \varepsilon_t, t = 1, ..., T \tag{1.49}$$

which is supposed to be distinguished by using this test from the alternative hypothesis of the unstable model

$$y_t = X_t'\beta_t + Z_t'\delta + \varepsilon_t, t = 1, ..., T \tag{1.50}$$

with non-constant $\{\beta_t\}$, where y_t is a scalar, X_t, β_t are $k \times 1$ vectors, Z_t and δ are $d \times 1$, $\{y_t, X_t, Z_t\}$ are observed, β_t, $\{\beta_t\}$, and δ are unknown and ε_t is a mean-zero disturbance. Elliott and Muller (2006) add that this tests whether the coefficient vector that links the observables X_t to y_t remains stable over time, while allowing for other stable links between y_t and the observables through Z_t. The estimated statistics of the test of yearly data are reported in Table 1.20 and the ones for monthly data are reported in Table 1.21.

Amihud (2002) measure yields models that are not stable for both yearly and monthly data. For yearly data, the calculated statistics of the model using $\ln A^0$ is -10.24, at a 5%-level, the null hypothesis of stability is rejected. All the other measures seem to be stable up to a 10%-level.

The same conclusion is drawn from monthly data; The calculated test statistic for the model that uses $\ln A^0$ as the illiquidity measure is -13.38, therefore the test rejects the null hypothesis of stability at a 1%-level. The remaining versions of illiquidity seem to be

Table 1.20. Quasi-Local Level (QLL) - Yearly

		Critical Values		
	Test stat	1%	5%	10%
$\ln A^0$	-10.24	-11.05	-8.36	-7.14
$\Delta \ln A^0$	-5.52	-11.05	-8.36	-7.14
$HP.\ln A^0$	-3.51	-11.05	-8.36	-7.14
lnA and lnS	-8.62	-17.57	-14.32	-12.80
ΔlnA and ΔlnS	-11.08	-17.57	-14.32	-12.80
HP.lnA HP.lnS	-7.98	-17.57	-14.32	-12.80

This table reports the results of yearly Quasi-Local Level (QLL) test that include both log-transformed and turnover-version Amihud (2002) measure and its stationary transformations.

Table 1.21. Quasi-Local Level (QLL) - Monthly

		Critical Values		
	Test stat	1%	5%	10%
$\ln A^0$	-13.383	-11.05	-8.36	-7.14
$\Delta \ln A^0$	-2.033	-11.05	-8.36	-7.14
$HP.\ln A^0$	-5.884	-11.05	-8.36	-7.14
lnA and lnS	-4.8	-11.05	-8.36	-7.14
ΔlnA and ΔlnS	-2.824	-11.05	-8.36	-7.14
HP.lnA and HP.lnS	-3.21	-11.05	-8.36	-7.14

This table reports the results of monthly Quasi-Local Level (QLL) test that include both log-transformed and turnover-version Amihud (2002) measure and its stationary transformations.

stable, according to the test. Thus, the only model that is not stable is precisely the one Amihud (2002) argues to be stable, and we have evidence of this for both monthly and yearly data.

1.6. Conclusions

In this Chapter we analyze the time series properties of Amihud's (2002) illiquidity measure as comprehensively as possible. We find that the measures of illiquidity are non-stationary, and that is important to get estimates of the illiquidity premium.

We find that yearly illiquidity is priced as the coefficients related to the lagged illiquidity are positive, statistically significant and economically meaningful. The size of this

effect, however, seem to be puzzling high. We also find that the yearly unexpected illiquidity lowers contemporaneous market return but the addition of this variable to the model adds a small effect on the value of the estimated illiquidity premium. If we use the measure suggested by Amihud (2002), without any transformation to make it stationary, we get coefficients that are non-significant at any conventional level. This point should be noted for future research that considers the effect of illiquidity over time.

The monthly Amihud (2002) measure is also non-stationary, however it seems to have a deterministic trend. If we use the HP-filtering as the detrending strategy, the coefficients reveal a positive, significant, and economically meaningful illiquidity premium. We use three versions of market return in the monthly analysis, and for two of them (VWMR and VWMR in excess of the risk-free rate) the premium is as puzzling high as the yearly evidence shows. The premium relative to the VWMR adjusted for the Fama and French's three factors is considerably lower.

We decompose Amihud (2002) measure into its turnover version, as suggested by BHS. Most of the coefficients are non-significant at any conventional level. For monthly data we find a negative illiquidity premium, a result on the opposite direction of the ones we find using Amihud (2002).

These results show how much there is still to be uncovered about the effect of illiquidity over returns. The size and even the direction of the illiquidity premia seem to be puzzling and the way we use to deal with data in order to make them stationary has a great impact over the results.

CHAPTER 2

Return Declines and Liquidity

Abstract

In this Chapter we analyze the relationship of illiquidity and past returns. Using Amihud's (2002) measure of illiquidity, we test this relationship both at cross-sectional and time series level. Furthermore, we assume that causality runs from return to illiquidity. In Chapter 1, we provide evidence that returns increase with past illiquidity, implying a positive relationship between these two variables. Here we show that illiquidity may have a negative relationship with past returns.

2.1. Introduction

The rationale we describe in the previous Chapter suggests that causality runs from illiquidity to return, supported by evidence that returns increase with illiquidity, implying a positive relation between these variables. In this Chapter, we assume that causality runs from return to illiquidity. We analyze the response of illiquidity to shocks in returns, for both market and firm levels. We use Amihud's (2002) measure to examine the cross-sectional and time-series properties of this relation. We find that illiquidity decreases with return, implying a negative relation.

The impact of return over illiquidity is reported by Hameed, Kang and Viswanathan (2010). They find that negative return decrease liquidity, specially during times of tightness in the funding market. This argument is supported by a number of theoretical models, that have as a common point the prediction that large market declines increase the demand for liquidity (as agents liquidate their positions across assets) and reduce the supply of liquidity (as liquidity providers hit their wealth or funding constraints).

These models can be grouped in three broad categories: (1) Collateral-based models, which state that market makers face funding constraints and are forced to liquidate reaching a low liquidity, high margin equilibrium (Brunnermeier and Pedersen, 2009; Anshuman and Viswanathan, 2005; Garleanu and Pedersen, 2007; Gromb and Vayanos, 2002); (2) Limits-to-arbitrage models, which suggest that shocks to noise traders make price move away from fundamentals and make liquidity providers take advantage of arbitrage opportunities becoming liquidity demanders as they liquidate their positions (Kyle and Xiong, 2001; Xiong, 2001; Mitchel, Pedersen and Pulvino, 2007) and (3) Coordination failure models, which state that traders have different trading limits to sell, what may push the price down (Bernardo and Welch, 2003; Morris and Shin, 2004; Vayanos, 2004).

Hameed, Kang and Viswanathan (2010) provide empirical evidence of the impact of illiquidity over return. They regress a measure of asset illiquidity taken from intraday data on firms' returns and find a negative relation between these variables. This, according to the authors, supports the notion that a drop (increase) in returns is followed by a decrease

(increase) of asset's liquidity. In this Chapter we use Amihud's (2002) popular measure of illiquidity instead of the bid-ask spread, as Hameed, Kang and Viswanathan (2010) do, as Amihud's depends on daily data, which is simpler and more available.

First, we measure the cross-sectional response of illiquidity to a shock in return using Fama-Macbeth's (1973) methodology. Second, we estimate time series models and get the time-variation properties of the impact of returns on illiquidity.

Once again, we highlight the there is a difference in the interpretation of time-series and cross-sectional results. A negative time-series relation between illiquidity and returns suggests that when the market experiences a period with a return change higher than its unconditional time-series average, there will be a lower-than-average illiquidity during that period. A negative cross-sectional illiquidty-return relation suggests that firms with returns whose changes are higher than the cross-sectional average have lower illiquidity than the average market illiquidity (Sadka and Sadka, 2009;Teets and Wasley, 1996).

To our knowledge analyzing the impact of returns on Amihud (2002) measure of illiquidity is novel, as well as the time series properties of it. This may be considered an intermediate step to the dynamic model we develop in Chapter 3. The Chapter proceeds as follows: section 3 describes the measure of illiquidity, which is described in Chapter 1; section 4 describes the data, which is also the same throughout the whole dissertation; section 5 explains the estimation procedure and results for cross-sectional data; section 6 describes the methodology and results for time-series data; section 7 concludes.

2.2. Measures of Illiquidity

We use the log transformed Amihud's (2002) original measure for each firm and for the market by taking the value weighted averages of the individual measures, as we do in Chapter 1, however we do make comments relative to the equally weighted average. We also do not use BHS decomposition into "turnover plus size" as we are interested in taking liquidity as the dependent variable.

2.3. Data

Data on stock return, volume, share outstanding are taken from CRSP and the risk free data are taken from Kenneth french's website. Our sample starts in 1962 and ends in 2011. Returns and illiquidity variables are 1% winsorized. Similarly to Chapter 1, we use yearly and monthly data to account for the differences in the premia according to the periodicity. The measures of illiquidity are the same of the previous Chapter and we use the stationary versions of (log-transformed) Amihud's (2002) measure, $\ln A^0$, in the time-series analysis. For more details, please refer to section 1.4 of the previous Chapter.

2.4. Cross-sectional evidence

In this section, we analyze the impact of returns over illiquidity in cross-sectional data using Amihud'(2002) measure. We describe the estimation procedure and, in the next subsection, we report results.

The goal of this section may be summarized by the following equation

$$ILLIQ_{i,t} = \beta_0 + \beta_1 R_{i,t-1} + \varepsilon_{1,i,t} \tag{2.1}$$

where $R_{i,t}$ is the yearly (monthly) return of stock i in year (month) t; $ILLIQ_{i,t}$ represents the illiquidity measure; $\varepsilon_{1,i,t}$ is the residual. We also include the following controls:

- **Dividend Yield** ($DIVYLD$): sum of the dividends during a given period divided by the end-of-period price. Dividend yields attract different types of investor due to tax reasons (higher tax rate on dividends compared to the tax on capital gains, Brennan et al, 1998). Redding (1997) suggests that large investors prefer companies with high liquidity and also prefer receiving dividends.
- **Standard deviation of returns** ($SDRET$): standard deviation of daily returns of each stock and of the market index for the previous year/month. Vayanos (2004) suggests demand for liquidity depends on volatility. Risk averse investors

will rebalance their portfolios with less risky assets (flight for quality), increasing illiquidity for volatile stocks.

- **Size** ($SIZE$): firm's market capitalization (stock price times shares outstanding). Size is negatively related to illiquidity since a larger firms attract more investors, either through index tracking funds either through more analyst/media coverage.

Table 2.1 shows descriptive statistics for the cross sectional variables used in this Chapter.

The estimation strategy we use in this section follows Fama and Macbeth (1973) method (FM, henceforth) for each year, as in Amihud (2002), Hameed, Kang and Viswanathan (2010), and Brennan, Huh and Subrahmanyam (2013). The traditional approach to this method essentially consists of two steps. First, for each time period, cross-sectional regressions are used to obtain estimates of the parameters of interest. Then, in the second step, the time series of these estimates are used to obtain final estimates for the parameters and standard errors so that t-statistics can be computed. Accurate computation of t-statistics, however, depends on accurate computation of standard errors.

The most important aspect for the computation of standard errors is the cross-sectional and serial correlation of the residuals and the independent variables in the panel regressions. Skoulakis (2006) shows that the properties of the FM estimator can be affected by the length of N (the total number of firms in our case) or T (the number of periods). We use his approach and run regressions using the "traditional" two steps. In the first step, for each single time period, a cross-sectional regression is performed. In the second step, the final coefficient estimates are obtained as the average of the first step coefficient estimates.

We also follow the Brennan, Chordia, and Subrahmanyam (1998) approach and use individual stock data instead of building portfolios. This procedure avoids the data-snooping bias inherent in portfolio-based approaches (Roll 1977; Lo and MacKinlay 1990). Ang, Liu, and Schwarz (2008) argue that using individual stocks provides more efficient

Table 2.1. Descriptive statistics

Variable	Obs	Mean	Std. Dev.
YEARLY			
$\ln A^0$	80,034	-3.15	2.80
Return	70,395	17.97	50.38
Dividend Yield	70,395	0.03	0.03
Std. Deviation	70,395	0.02	0.01
Size	70,384	2,719,328	12,700,000
ln Size	70,384	12.76	1.97
MONTHLY			
$\ln A^0$	1,273,329	-2.27	3.27
Return	1,264,522	1.25	13.04
Dividend Yield	1,264,522	0.00	0.01
Std. Deviation	1,263,453	0.03	0.02
Size	1,262,690	1,949,937	10,600,000
ln Size	1,262,690	11.96	2.24

This table reports descriptive statistics of key yearly and monthly variables for NYSE-AMEX (hereafter NYAM) stocks from January 1961 to December 2011 (50 years/600 months). The table shows statistics for the log-transformed values [indicated by ln(.)] of Amihud (2002) measure. It also shows the average firm return in the sample. It also shows statistics of Size (market capitalization), Dividend Yield and the Standard Deviation of daily returns in one year or month. The returns are multiplied by 100. The yearly criteria for an firm to survive in the sample is: Stocks must have the security must have at least 200 days of valid observations during year y-1.The stock must be listed at the end of year y-1; Price at the end of the year must be higher than 5 dollars; Every observation with missing values for our size variable (market capitalization) is dropped; Only observations with no zero volume are considered. The monthly criteria for a firm to survive in the sample is: Stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months; Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized.

tests of whether factors are priced. In Appendix 1, we detail the FM estimation procedure. Next subsection presents and discusses the results.

2.4.1. Results

This section analyses the response of the log transformed original Amihud (2002) measure, $\ln A^0$, to returns in a cross-sectional dataset. We present results for yearly and monthly

data separately. We find evidence confirming Hameed, Kang and Viswanathan's (2010) claim that return declines increase illiquidity. We estimate model (2.1) using yearly and monthly data, using data for firms' returns and illiquidity from January 1962 to December 2011 for NYAM-listed stocks.

Table 2.2 shows results of the estimated coefficients and t-statistics for yearly data. The first column of the table shows the estimated coefficients of the regression that does not include any control. The coefficient of the lagged yearly return is negative and significant at 1%. Although we find the same negative relationship that Hameed, Kang and Viswanathan's (2010) report, we cannot compare the coefficients directly as both the measures of illiquidity and the frequencies are different. The value of -0.003 implies that one-standard deviation increase in returns reduces illiquidity by 0.6 points.

The second column of the table shows the results of the estimated coefficients when we include the additional control variables. The magnitude of the coefficient of lagged returns decreases, but the relation is still negative and significant, with a decrease of 0.35 illiquidity points when returns increase by one-standard deviation. The relationship of illiquidity with dividend yield is also significantly negative, which supports Redding's view that large investors prefer high dividend yield firms increasing liquidity. The coefficient for standard deviation is negative, which is different from the expected and also what Hameed et al. (2010) found. We believe that the argument of volatility feedback does not apply with yearly frequency. Hong and Sraer (2013) suggest that in recent years, speculative investors have increased their exposition to riskier stocks, which could explain the positive sign of the standard deviation coefficients. Regarding size, the coefficient is significant and with the expected negative relationship.

These last three variable are also used as controls in Amihud's regressions where, similarly to Chapter 1, returns are the dependent variable but the rationale to include them is different. The third column of table 2.2 only uses the controls to explain cross-sectional illiquidity. Excluding returns from the regression does not impact greatly on the fit of the model used in the first stage. This evidence supports the view we explore

in the third Chapter, where we consider the possibility that returns and illiquidity are endogenously determined.

Table 2.2. Illiquidity response to a change in firms's returns - Yearly Cross-Section

VARIABLES	$\ln A^0$		
	(1)	(2)	(3)
Return (-1)	-0.00339**	-0.00154***	
	(-3.12)	(-6.34)	
Dividend Yield (-1)		-0.881*	-0.729
		(-2.28)	(-1.91)
Std. Dev (-1)		-19.62***	-20.80***
		(-13.70)	(-14.35)
ln Size (-1)		-1.197***	-1.202***
		(-55.14)	(-57.09)
Constant	-3.537***	12.46***	12.51***
	(-13.11)	(54.62)	(56.82)
Observations	70395	70384	70384
R^2	0.0212	0.864	0.862
Number of groups	49	49	49
F	9.739	1135.4	1282.7

This table reports the results of yearly cross-sectional regressions that include firms' returns and the log-transformed Amihud (2002) measure. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variable is the log-transformed Amihud (2002) measure. The covariates are: firms' returns; dividend yield; and the standard deviation of daily returns over each year. We also include the lagged Amihud (2002) measure. All the explanatory variables variables are lagged in one period. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must have the security must have at least 200 days of valid observations during year y-1.The stock must be listed at the end of year y-1; Price at the end of the year must be higher than 5 dollars; Every observation with missing values for our size variable (market capitalization) is dropped; Only observations with no zero monthly volume are considered; Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

Table 2.3 shows the estimated coefficients and t-statistics estimated with monthly data. As we do in the previous table, the first column of Table 2.3 shows the results of the model controlling only for the lagged monthly return. The estimated coefficient

is -0.020, which is significant at 1% level. If returns are increased by a one-standard deviation, illiquidity is reduced by 0.26 points. Alternatively, we can say that a return decrease increases illiquidity, which is the initial claim of Hameed, Kang and Viswanathan (2010).

Table 2.3. Illiquidity response to a change in firms's returns - Monthly Cross-Section

VARIABLES	$\ln A^0$		
	(1)	(2)	(3)
Return (-1)	-0.020***	-0.006***	
	(-15.62)	(-16.32)	
Dividend Yield (-1)		1.075***	1.042**
		(3.91)	(2.33)
Std. Dev (-1)		1.163***	0.271
		(3.66)	(0.41)
ln Size (-1)		-1.264***	-1.269***
		(-178.55)	(-82.46)
Constant	-2.426***	12.94***	13.00***
	(-33.53)	(171.27)	(80.08)
Observations	1264522	1262289	1262289
R^2	0.0238	0.864	0.862
Number of groups	599	599	599
F	243.9	10115.0	2268.2

This table reports the results of monthly cross-sectional regressions that include firms' returns and the log-transformed Amihud (2002) measure. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variable is the log-transformed Amihud (2002) measure. The covariates are: firms' returns; dividend yield; and the standard deviation of daily returns over each year. We also include the lagged Amihud (2002) measure. All the explanatory variables variables are lagged in one period. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized.

The second column of Table 2.3 shows the results of the model that includes the remaining control variables. The coefficients for dividend yield and standard deviation are now positive and significant, while negative in the yearly sample. Our intepretation is

that portfolios changes due to dividend tax arbitrage are mitigated in the yearly variables. The monthly frequency is also capturing the volatility feedback effect which is not present in the yearly sample. The coefficient of the lagged return decreases, in absolute terms, to -0.006. Nevertheless, the average fit of the first stage regressions is higher indicating, once again, that these controls are important in defining both returns and illiquidity.

Overall, the results of both samples reinforce the hypothesis that illiquidity responds to shocks in returns. Firms with higher return changes than the cross-sectional average have lower illiquidity than the average market returns. In other words, firms facing a return decrease that is higher than the cross-sectional average have higher illiquidity than the average market illiquidity..

We also run the regressions using the demeaned variables, which seems to be the procedure adopted by Amihud (2002) and Hameed, Kang and Viswanathan (2010), however the difference between the results estimated using both strategies is not significant. Results are available upon request.

2.5. Time series evidence

The estimation procedure is this section analogous to section 1.3, however the dependent variable is market return, and the independent is market illiquidity. We do not reproduce the exercise of expected/unexpected return and illiquidity in this section as we do not have any theoretical evidence that illiquidity anticipates market return in some predictable way. We somehow relax that in Chapter 3 by using a structural VAR specification.

First, we estimate the market illiquidity response to a variation in returns focusing on Amihud's original measure. We simplify notation by writing the set of regressions by

$$\Delta \ln A_t^0 = \phi_0 + \phi_1 r_{t-1}^j + \varepsilon_t \tag{2.2}$$

and

$$HP.\ln A_t^0 = \phi_0 + \phi_1 r_{t-1}^j + \varepsilon_t \tag{2.3}$$

When $t = m$ we have monthly data, i.e., the illiquidity measure is calculated for a given month m. When $t = y$ we are dealing with yearly data, in other words, it is the illiquidity measure for a given year y.

The independent variables are the log-transformed market illiquidity, the same of Chapter 1, and are defined as:

- $\Delta \ln A_{t-1}^0$ is the first difference of the log of the value weighted average of Amihud's (2002) original measure;
- $HP.\ln A_{t-1}^0$ is the HP-filtered version of the log of the value weighted average of Amihud's (2002) original measure;

As we do in Chapter 1, the dependent variables are defined as

- r_t^M is the value weighted average of returns (VWMR) for a given period t, and;
- r_t^E is the value weighted average of returns (VWMR) for a given period t in excess of the risk-free rate (r_t^f): $r_t^M - r_t^f$

For monthly data we have an additional return variable

- r_m^{FF3} is value weighted average of returns (VWMI) for a given period t adjusted for Fama and French (1993) three factors (we estimate the FF3 factor loadings using the entire sample period.):

$$r_m^{FF3} = \sum_{i=1}^{N_t} \left\{ \frac{S_{i,t}}{\sum_{i=1}^{N_t} S_{i,t}} \left[\left(r_{i,t} - r_t^f \right) - \left(\widehat{b}_{i,1} MKT_t + \widehat{b}_{i,2} SMB_t + \widehat{b}_{i,3} HML_t \right) \right] \right\}$$

In Chapter 1 we show that monthly illiquidity seem to have some structure. In order to account for that, we add an AR(1) term to the models we estimate, i.e., we add the lagged market illiquidity to models

$$\Delta \ln A_t^0 = \phi_0 + \phi_1 r_{t-1}^j + \phi_2 \Delta \ln A_{t-1}^0 + \varepsilon_t \tag{2.4}$$

and

$$HP. \ln A_t^0 = \phi_0 + \phi_1 r_{t-1}^j + \phi_2 HP. \ln A_{t-1}^0 + \varepsilon_t \tag{2.5}$$

We estimate these equations by OLS with Newey–West standard errors, as in Amihud (2002), considering that the error structure can be heteroskedastic and possibly autocorrelated. In the next section we present the results of the estimation of these models. Next subsection presents the results of the estimation for yearly and monthly data.

2.5.1. Results

This section analyses the illiquidity response associated with the log transformed original Amihud (2002) measure and its stationary-transformed versions in time series data. The first subsection shows the results obtained with yearly data and the second the results with monthly data.

2.5.1.1. Yearly. Table 2.4 shows the estimated coefficients of equations (2.2) and (2.3). The effect of market return (VWMR) on the non-stationary illiquidity ($\ln A^0$) is non significant at any conventional level, however the impact of the market return in excess of the risk free rate over $\ln A^0$ is significant. A one-standard deviation increase in market return in excess of the risk-free rate represents a decrease of 0.38 points of $\ln A^0$. Alternatively, one can interpret this as a one standard deviation negative shock on VWMR in excess of the risk-free rate increases market illiquidity by 0.38 basis points.

The coefficients relative to the effects of both VWMR and VWMR in excess of the risk-free rate over the stationary illiquidity measure ($\Delta \ln A^0$) are 1%-significant. A one standard deviation increase in these measures of return have similar impacts on $\Delta \ln A^0$, lowering it by 0.12 points.

Table 2.4. Response of market illiquidity to changes in market return - Yearly

VARIABLES	$\ln A^0$	$\Delta\ln A^0$	$HP.\ln A^0$	$\ln A^0$	$\Delta\ln A^0$	$HP.\ln A^0$
Return (-1)	-0.015	-0.007***	-0.006***			
	(-1.242)	(-4.236)	(-5.477)			
Return - Rf (-1)				-0.022**	-0.007***	-0.007***
				(-2.272)	(-4.037)	(-5.770)
Constant	-4.001***	-0.002	0.098***	-4.010***	-0.038	0.064***
	(-6.433)	(-0.065)	(4.467)	(-7.393)	(-1.178)	(3.509)
Observations	49	49	49	49	49	49
F	1.543	17.94	30.00	5.163	16.30	33.30

This table reports the results of yearly time-series regressions that include firms' returns and the log-transformed Amihud (2002) measure. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variable is the log-transformed Amihud (2002) measure. The covariates are: firms' returns; dividend yield; and the standard deviation of daily returns over each year. All the explanatory variables variables are lagged in one period. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must have the security must have at least 200 days of valid observations during year y-1.The stock must be listed at the end of year y-1; Price at the end of the year must be higher than 5 dollars; Every observation with missing values for our size variable (market capitalization) is dropped; Only observations with no zero monthly volume are considered; Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

The same analysis for the HP-filtered illiquidity ($HP.\ln A^0$) yield 1%-significant coefficients. A one-standard deviation increase in market returns (excess of market returns) represents a decrease of 0.10 (0.12) points in market illiquidity. If we use the equally weighted criterion for generating the yearly averages, we get very similar coefficients to the ones under the value weighted criterion and with the same statistical significances.

Table 2.5 shows the results of the estimation of models (2.4) and (2.5). The main difference is that all of the coefficients are 1%-significant. The impact of a one-standard deviation increase in both VWMR and VWMR in excess of the risk-free rate, lowers $\ln A^0$ in 0.12 points. The shock on both of these two variables lower $\Delta \ln A^0$ by 0.17 points. The $HP.\ln A^0$ is lowered by 0.10 (VWMR) and 0.12 (VWMR in excess of the risk-free rate) points.

Table 2.5. Response of market illiquidity to changes in market return - AR(1) - Yearly

VARIABLES	$\ln A^0$	$\Delta \ln A^0$	$HP.\ln A^0$	$\ln A^0$	$\Delta \ln A^0$	$HP.\ln A^0$
Return (-1)	-0.007***	-0.010***	-0.006***			
	(-4.162)	(-4.823)	(-5.429)			
Return - Rf (-1)				-0.007***	-0.010***	-0.007***
				(-4.027)	(-5.452)	(-5.703)
Illiq (-1)	1.001***	-0.318**	0.012	0.995***	-0.325**	0.019
	(56.381)	(-2.145)	(0.084)	(55.688)	(-2.309)	(0.129)
Constant	-0.000	0.014	0.097***	-0.058	-0.038	0.064***
	(-0.004)	(0.326)	(4.345)	(-0.773)	(-0.964)	(3.482)
Observations	49	48	49	49	48	49
F	1633	11.64	14.77	1651	15.10	16.32

This table reports the results of yearly time-series regressions that include firms' returns and the log-transformed Amihud (2002) measure. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variable is the log-transformed Amihud (2002) measure. The covariates are: firms' returns; dividend yield; and the standard deviation of daily returns over each year. All the explanatory variables variables are lagged in one period. We also include the lagged Amihud (2002) measure. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must have the security must have at least 200 days of valid observations during year y-1.The stock must be listed at the end of year y-1; Price at the end of the year must be higher than 5 dollars; Every observation with missing values for our size variable (market capitalization) is dropped; Only observations with no zero monthly volume are considered; Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

We test alternative criteria for dealing with outliers and they do not change much the results. The value of coefficients become slightly larger the higher the percentile we trim/winsorize but the statistical significance and direction of the effects are the same for all criteria.

The analysis of yearly time series data confirms the notion that market return declines increase market illiquidity. Therefore, the results suggest that when the market experiences a period with a return change higher than its unconditional time-series average, illiquidity exhibits a lower-than-average illiquidity during that period.

2.5.1.2. Monthly. Table 2.6 shows the estimated coefficients of equations (2.2) and (2.3) with monthly data. The effect of the lagged VWMR is 10%-significant for both $\ln A^0$, and 1% significant for both $\Delta \ln A^0$ and $HP. \ln A^0$. The values of the estimated coefficients for $\ln A^0$, $\Delta \ln A^0$, and $HP. \ln A^0$ are, respectively, -0.030, -0.018, and -0.023. This implies that a one-standard deviation shock in VWMR decreases $\ln A^0$ in 0.04 points, $\Delta \ln A^0$ in 0.03, and $HP. \ln A^0$ in 0.03 points.

The coefficient of the VWMR in excess of the risk-free rate impact over $\ln A^0$ is 5%-significant. Its value is -0.038, which imply a decrease of 0.04 points of illiquidity when the return variable increases in one-standard deviation, 0.44 in annualized points. The coefficients relative to the impact over $\Delta \ln A^0$ and $HP. \ln A^0$ are 1%-significant and their values are -0.018 and -0.023. A one-standard deviation increase in VWMR in excess of the risk-free rate represent a decrease of 0.02 points for both$\Delta \ln A^0$ and $HP. \ln A^0$, in annualized terms, the decrease is respectively 0.21 and 0.27 points, respectively, a little lower than the ones related to the VWMR.

The coefficients of VWMR adjusted to Fama and French's three factors are all 1%-significant and their values are -0.046 ($\ln A^0$), -0.016 ($\Delta \ln A^0$), and -0.022 ($HP. \ln A^0$). These coefficients imply that a one-standard deviation increase on the VWMR adjusted to Fama and French's three factors decreases illiquidity in -0.02, -0.01, and -0.01 for $\ln A^0$, $\Delta \ln A^0$, and $HP. \ln A^0$, which are translated, respectively in decreases of 0.29, 0.10, and 0.14 of annualized points.

Table 2.6. Response of market illiquidity to changes in market return - Monthly

VARIABLES	$\ln A^0$	$\Delta\ln A^0$	$HP.\ln A^0$	$\ln A^0$	$\Delta\ln A^0$	$HP.\ln A^0$	$\ln A^0$	$\Delta\ln A^0$	$HP.\ln A^0$
Return (-1)	-0.030*	-0.018***	-0.023***						
	(-1.941)	(-6.121)	(-4.868)						
Return - Rf (-1)				-0.038**	-0.018***	-0.023***			
				(-2.396)	(-6.209)	(-4.944)			
Return - FF3F (-1)							-0.046***	-0.016***	-0.022***
							(-2.699)	(-4.578)	(-4.454)
Constant	-3.203***	0.019*	0.035	-3.208***	0.012	0.025	-3.221***	0.002	0.013
	(-22.667)	(1.790)	(1.051)	(-23.084)	(1.155)	(0.767)	(-23.307)	(0.190)	(0.416)
Observations	599	599	599	599	599	599	599	599	599
F	3.769	37.47	23.70	5.743	38.55	24.45	7.282	20.96	19.84

This table reports the results of monthly time-series regressions that include return and log-transformed Amihud (2002) measure. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variables are: $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta\ln A^0$: the first difference of $\ln A^0$. $HP.\ln A^0$: the HP-filtered $\ln A^0$. The covariates are: Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. All the explanatory variables variables are lagged in one period For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

Table 2.7 shows the results of the estimated coefficients of models (2.4) and (2.5). Due to the non-stationarity of the illiquidity variable the model that has as the independent variable $\ln A^0$ is not well estimated considering the technology we use. All of the estimated coefficients are 1%-significant. The coefficients of both measures VWMR and VWMR in excess of the risk-free rate relative to $\Delta \ln A^0$ $(HP. \ln A^0)$ are -0.27 (-0.020) 0.09 points higher (0.03 points lower), in absolute terms, than the version without the AR(1) term. The impact of VWMR adjusted to the Fama-French three factors change slightly as well, it is -0.024 for $\Delta \ln A^0$ and -0.018 for $HP. \ln A^0$.

As happens with yearly time series, the analysis of monthly data confirms the notion that market return declines increase market illiquidity. In other words, a firm that experiences a period with a return change higher than its unconditional time-series average exhibits a lower-than-average illiquidity during that period.

2.6. Conclusions

In the first part of this Chapter we show evidence that the cross-section of asset returns affect the Amihud (2002) measure of illiquidity. We use a broad and long data sample starting from the early 1960s. This reinforces Hameed, Kang and Viswanathan's (2010) findings that, for high frequency measures, return decreases increase illiquidity.

The second part of the Chapter analyses the time-series of the response of illiquidity to return shocks. We also verify that return drops decrease illiquidity. Moreover, causality may run from returns to illiquidity as lagged market returns have a negative relation with the market illiquidity. We analyze causality in the next Chapter.

Table 2.7. Response of market illiquidity to changes in market return - AR(1) - Monthly

VARIABLES	$\ln A^0$	$\Delta \ln A^0$	$HP.\ln A^0$	$\ln A^0$	$\Delta \ln A^0$	$HP.\ln A^0$	$\ln A^0$	$\Delta \ln A^0$	$HP.\ln A^0$
Return (-1)	-0.019***	-0.027***	-0.020***						
	(-6.188)	(-8.239)	(-7.296)						
Return - Rf (-1)				-0.019***	-0.027***	-0.020***			
				(-6.392)	(-8.360)	(-7.398)			
Return - FF3F (-1)							-0.017***	-0.024***	-0.018***
							(-4.972)	(-6.624)	(-6.180)
$\ln A^0$ (-1)	0.962***			0.961***			0.961***		
	(62.074)			(62.025)			(61.472)		
$\Delta \ln A^0$ (-1)		-0.474***			-0.475***			-0.461***	
		(-6.243)			(-6.274)			(-5.937)	
$HP.\ln A^0$ (-1)			0.559***			0.559***			0.560***
			(5.367)			(5.379)			(5.377)
Constant	-0.102**	0.030**	0.030*	-0.113**	0.018	0.022	-0.124**	0.004	0.011
	(-1.965)	(2.428)	(1.926)	(-2.203)	(1.589)	(1.421)	(-2.408)	(0.360)	(0.739)
Observations	599	598	599	599	598	599	599	598	599
F	1927	35.18	52.32	1928	36.12	52.44	1890	25.83	41.34

This table reports the results of monthly time-series regressions that include return and log-transformed Amihud (2002) measure. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The dependent variables are: $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta \ln A^0$: the first difference of $\ln A^0$. $HP.\ln A^0$: the HP-filtered $\ln A^0$. The covariates are: Return (the value weighted average of returns across all firms included in the sample); Return-Rf (the value weighted yearly average of excess return in excess of the T-bill rate); and Return - FF3F (the value weighted market return adjusted for the Fama and French's three factors). The returns are multiplied by 100. All the explanatory variables variables are one period lagged. We also add the lagged illiquidity variable as a covariate. For each explanatory variable there are the coefficient estimates, and the values in paranthesis in the second row of each variable are t-statistics computed based on Newey-West. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

CHAPTER 3

Dynamic relationship between illiquidity and return

Abstract

This Chapter studies this illiquidity premium allowing for a joint determination between market return and illiquidity. We use the structural vector autoregression methodology that nests Amihud (2002) and Hameed, Kang and Viswanathan (2010). By imposing short and long-run restrictions, we recover the structural coefficients of the model. We find that this methodology estimates a lower illiquidity premium for yearly data. The results for monthly data still show a puzzling high premium.

We also access the effect of the unexpected illiquidity without imposing the AR(1) structure Amihud (2002) adopts. We find that it lowers contemporaneous return and this reduction is reverted, going to zero after one period in yearly data, and about 5 periods for monthly data.

3.1. Introduction

In this Chapter we deal with the joint determination of returns and liquidity we study in Chapters 1 and 2. We use a structural vector autorregression, employing short and long-run restriction as our identification assumptions. We can, therefore, re-analyze Amihud's (2002) model and get the impact of the unexpected illiquidity, i.e., the residuals of the illiquidity equation, over returns.

A number of papers studies the relationship between the liquidity and return of financial assets (Amihud and Medenlson, 1986; Brennan and Subrahmanyam, 1996; Datar, Naik and Radcliffe, 1998; Chordia, Subrahmanyam and Anshuman, 2001). Theoretical models and empirical evidence, however, diverge on the direction and the causal relationship between these two variables.

Causality runs from illiquidity to return in Amihud (2002), who provides evidence for the negative impact liquidity has over the return of an asset. Hameed, Kang and Viswanathan (2010), on the other hand, show that stock illiquidity is decreasing on its return, implying that causality runs from return to illiquidity. In other words, the former evidence shows that if a stock becomes less liquid, its return is supposed to increase (a negative relation), while the latter suggests that if a stock's return increase it is supposed to get more liquid (a positive relation).

These two approaches, taken together, suggest that both variables are determined simultaneously which, for a start, may imply that the magnitude of the estimated effect may be inaccurate (i.e., the estimated coefficients are biased).

Another aspect related to the joint determination are the effects of the relation between returns and illiquidity through time. Amihud (2002) suggests that only expected stock excess return reflects a compensation for expected market illiquidity. Unexpected market illiquidity lowers contemporaneous stock prices, increasing future returns. In order to get evidence on this claim, he isolates the effect of "expected" and "unexpected" illiquidity on returns assuming that liquidity follows an AR(1) process. This isolation procedure depends on the model used to describe what "expected" liquidity means. Taking the

simultaneous approach allows us to (i) relax the restriction imposed on the behavior of the expected liquidity, and (ii) get insights on their impact in a dynamic setting.

The evidence on the relationship between liquidity and return is large. Many papers argue that liquidity has an impact on the expected return and price of an asset, what Amihud and Mendelson (1991) call "liquidity effect". Under this point of view illiquid assets (or assets with high transaction costs) trade at low prices relative to their fundamental values, in other words, liquidity is priced (Amihud and Medelson, 1986; Brennan and Subrahmanyam, 1996; Datar, Naik and Radcliffe, 1998; Chordia, Subrahmanyam and Anshuman, 2001).

The liquidity effect is, in broad terms, related to the effect of risk on the returns of assets. The idea is that agents prefer liquid investments that can be traded quickly and at low costs anytime they are in need. Therefore, less liquid investments must offer higher expect returns in order to attract investors in the same way a risk-averse investor would require a higher expected return as a compensation for greater risk (Amihud and Medelson, 1991).

This relationship has been extended in a variety of ways, see for instance Vayanos (1998); Lo, Mamaysky and Wang (2004); Eisfeldt (2004); Holmstrom and Tirole (2002); Huang (2003); and O'Hara (2003). Acharya and Pedersen (2005) develop a model considering factors related to commonality and risk premia, associated with changes in liquidity. They find different risk premia associated with changes in liquidity. These risk premia turned out to be highly significant in empirical work.

Empirical evidence support this notion that liquidity explains part of the expected returns. Amihud (2002) regresses returns on illiquidity and uncovers a positive relation. This suggests that expected stock excess return partly represents an illiquidity premium (Jones, 2002).

Another feature found in empirical analysis is that if liquidity varies systematically, securities with returns positively correlated with market liquidity should have high expected returns (Huberman and Halka, 1993; Chordia, Roll, and Subrahmanyam, 2000; Pastor and Stambaugh, 2002; Sadka, 2002).

In the papers aforementioned, the relationship runs from liquidity to returns. However, reverse causal relationship has also been suggested in some other recent papers. Hameed, Kang and Viswanathan (2010) find that negative returns decrease liquidity, specially during times of tightness in the funding market. This argument is supported in a number of theoretical models, that have as a common point, the prediction that large market declines increase the demand for liquidity (as agents liquidate their positions across assets) and reduce the supply of liquidity (as liquidity providers hit their wealth or funding constraints).

The empirical issues brought by this potential joint determination motivates this essay (analogous to what happens between volatility and returns studied by Bollerslev, Litvinov and Tauchen (2006)).

The estimation strategy we use is the structural vector autoregression (SVAR), or identified VAR. This approach focus on the errors of the system,which are linear combinations of the structural shocks. For this reason, if the equations are valid for the system of variables, then they also apply to the unexpected part of the variables (Lutkepohl and Kratzig, 2004). This is particularly interesting to understand the role of the unexpected illiquidity without imposing an AR(1) structure, as in Amihud (2002). The SVAR has also the advantage of using just enough restrictions to identify the system.

The Chapter proceeds as follows:.Section 3 discusses the measures of illiquidity (the same ones we use in Chapters 1 and 2), section 4 describes Data (also the same of Chapters 1 and 2). Section 5 describes the estimation strategy, which is the Structural VAR (SVAR). Section 6 reports results and section 7 concludes.

3.2. Data and Variable Construction

The measures of illiquidity are the stationary versions of (log-transformed) Amihud (2002) original measure: $\Delta \ln A^0$ and $HP.\Delta \ln A^0$. Data on stock return, volume, share outstanding are taken from the CRSP, just like in Chapter 1. The risk free rate is taken from Kenneth French's website. The construction of variables also follows the same steps we describe in Chapter 1. We do not take the turnover version into account so we can focus exclusively on the gains os using the dynamic model without other concerns, such as how to add the size effect to the model.

3.3. Estimation strategy

The models described in Chapters 1 and 2 can be nested in the more flexible VAR specification. Let r_t be a measure of market return (the VWMR, VWMR in excess of the risk-free rate, or the VWMR adjusted to the Fama-French three factors, for the monthly data case), and $illiq_t$ the log-transformed stationary (first-differenced or detrended) version of Amihud (2002) original illiquidity measure. The models we describe in the previous two Chapters can be written as restricted versions of the following system

$$\left\{ \begin{aligned} r_t &= b_{10} - b_{12} illiq_t + \gamma_{11} r_{t-1} + \gamma_{12} illiq_{t-1} + \varepsilon_{r,t} \\ illiq_t &= b_{20} - b_{21} r_t + \gamma_{21} r_{t-1} + \gamma_{22} illiq_{t-1} + \varepsilon_{illiq,t} \end{aligned} \right. \tag{3.1}$$

$\varepsilon_{r,t}$ and $\varepsilon_{illiq,t}$ are the structural errors, so that

$$\begin{pmatrix} \varepsilon_{r,t} \\ \varepsilon_{illiq,t} \end{pmatrix} \sim \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_{illiq}^2 \end{pmatrix} \right] \tag{3.2}$$

Rewriting this system in matrix form

$$
\underbrace{\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} r_t \\ illiq_t \end{bmatrix}}_{\mathbf{y}_t} = \underbrace{\begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}}_{\mathbf{\Gamma}_0} + \underbrace{\begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}}_{\mathbf{\Gamma}_1} \underbrace{\begin{bmatrix} r_{t-1} \\ illiq_{t-1} \end{bmatrix}}_{\mathbf{y}_{t-1}} + \underbrace{\begin{bmatrix} \varepsilon_{r,t} \\ \varepsilon_{illiq,t} \end{bmatrix}}_{\boldsymbol{\varepsilon}_t} \tag{3.3}
$$

or,

$$
\mathbf{B}\mathbf{y}_t = \mathbf{\Gamma}_0 + \mathbf{\Gamma}_1 \mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t \tag{3.4}
$$

Pre-multiplying this equation by $\mathbf{B}^{-1}$ yields

$$
\begin{aligned}
\mathbf{y}_t &= \underbrace{\mathbf{B}^{-1}\mathbf{\Gamma}_0}_{a_0} + \underbrace{\mathbf{B}^{-1}\mathbf{\Gamma}_1}_{R_1}\mathbf{y}_{t-1} + \underbrace{\mathbf{B}^{-1}\boldsymbol{\varepsilon}_t}_{u_t} \\
&= \mathbf{R}_0 + \mathbf{R}_1\mathbf{y}_{t-1} + \mathbf{u}_t
\end{aligned} \tag{3.5}
$$

This is called the reduced form. $\mathbf{u}_t$ are called the reduced form errors, which are linear combinations of the structural errors. The covariance matrix of the reduced form errors is given by

$$
E\left[\mathbf{u}_t\mathbf{u}_t'\right] = \mathbf{B}^{-1}E\left[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\right]\mathbf{B}'^{-1} = \mathbf{\Omega} \tag{3.6}
$$

We cannot uniquely solve for the structural parameter $\mathbf{B}$, $\mathbf{\Gamma}_0$, $\mathbf{\Gamma}_1$, and $\mathbf{\Omega}$ with the estimated coefficients of the reduced form. There are less parameters in the reduced form (9) than in the structural representation (10). Therefore, we need to impose some restrictions if we want to recover the structural coefficients.

The evidence we revisit in Chapters 1 and 2 are, therefore, restrictions imposed to this more general formulation. Amihud's (2002) specification imposes that $b_{12} = b_{21} = \gamma_{21} = 0$. Hameed, Kang and Viswanathan (2010) restrict the model such that $b_{10} = b_{12} = \gamma_{11} = \gamma_{12} = 0$. The VAR formulation is less restrictive and more general in the sense that it

nests the models of Chapters 1 and 2. We test two different sets of restrictions: a short term restriction, imposed on matrix **B**, and a long term restriction. We explain each group in the next subsections.

One last issue is the order of the VAR. For each model we use several information criteria to choose the order with the best fit. The criteria we use are: final prediction error (FPE), Akaike's information criterion (AIC), Schwarz's Bayesian information criterion (SBIC), and the Hannan and Quinn information criterion (HQIC).

3.3.1. Short-run restrictions

This kind of restriction is imposed over matrix **B**. In this Chapter we assume that illiquidity does not affect contemporaneous returns. In order to understand the impact of the unexpected illiquidity ($\varepsilon_{illiq,t}$) over returns we must assume that market illiquidity has a contemporaneous effect over market return. We test that by assuming that $b_{21} = 0$ as our restriction. We can get the structural coefficients as well as the impulse-response function to understand the role played by the unexpected illiquidity.

By assuming that $b_{21} = 0$ we have the following system

$$\begin{bmatrix} 1 & b_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_t \\ illiq_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} r_{t-1} \\ illiq_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{r,t} \\ \varepsilon_{illiq,t} \end{bmatrix} \tag{3.7}$$

The reduced for is then given by

$$\begin{bmatrix} r_t \\ illiq_t \end{bmatrix} = \begin{bmatrix} 1 & -b_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} 1 & -b_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} r_{t-1} \\ illiq_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & -b_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{r,t} \\ \varepsilon_{illiq,t} \end{bmatrix} \tag{3.8}$$

or

$$
(3.9) \qquad \begin{bmatrix} r_t \\ illiq_t \end{bmatrix} = \begin{bmatrix} \phi_{10} \\ \phi_{20} \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} r_{t-1} \\ illiq_{t-1} \end{bmatrix} + \begin{bmatrix} u_{r,t} \\ u_{illiq,t} \end{bmatrix}
$$

The parameters of the VAR can now be identified by the 9 equations

$\phi_{10} = b_{10} - b_{12}b_{20}$

$\phi_{11} = \gamma_{11} - b_{12}\gamma_{21}$

$\phi_{12} = \gamma_{12} - b_{12}\gamma_{22}$

$\phi_{20} = b_{20}$

$\phi_{21} = \gamma_{21}$

$\phi_{22} = \gamma_{22}$

$var(u_{r,t}) = \sigma_r^2 + b_{12}^2\sigma_{illiq}^2$

$var(u_{illiq,t}) = \sigma_{illiq}^2$

$cov(u_{r,t}, u_{illiq,t}) = -b_{12}\sigma_{illiq}^2$

In matrix notation,

$$
(3.10) \qquad \mathbf{\Gamma}_0 = \mathbf{B}\mathbf{R}_0
$$

and

$$
(3.11) \qquad \mathbf{\Gamma}_1 = \mathbf{B}\mathbf{R}_1
$$

The element of the restricted matrix **B** is estimated in an equivalent way as estimating a simultaneous equation model with covariance restrictions, by maximum-likelihood for details see Lutkepohl and Kratzig (2004). We have estimates for **B**, from the $\mathbf{R}_0$, and $\mathbf{R}_1$, therefore we can recover the structural parameters straightforwardly. In the next subsection we present an alternative approach to identify the system, the well known long-term restriction (Blanchard and Quah, 1989).

3.3.2. Long-run restrictions

This kind of restriction can be adopted when we assume that one of the structural errors has only a temporary effect over the other variable in the system. In our VAR specification we assume that an illiquidity shock has no long-run effect on market return.

We assume that for two reasons: (i) the first one is more "statistic", return is a stationary, I(0), variable, and as such has no long term trend and any variable that moves its path cannot have any long term predictable effect; (ii) the second part of the rationale is related to non-arbitrage, if some event can affect the long-term path of a return variable, investors would demand higher amounts of the underlying security, which would increase prices and bring returns down. This restriction allows the identification of the system and the recovery of the structural parameters.

This restriction implies that in the long run the cumulative effect of $\varepsilon_{illiq,t}$ in $\varepsilon_{r,t}$ is zero. To understand this restriction we re-write the VAR specification using Wold representation as an infinite MA. Take the reduced form of VAR

$$\mathbf{y}_t = \mathbf{R}_0 + \mathbf{R}_1\mathbf{y}_{t-1} + \mathbf{u}_t \tag{3.12}$$

We can write it as

$$\mathbf{R}(L)\mathbf{y}_t = \mathbf{R}_0 + \mathbf{u}_t \tag{3.13}$$

where $\mathbf{R}(L) = (\mathbf{I} - \mathbf{R}_1 L)$. Thus, we can re-write the equation above as

$$\mathbf{y}_t = \underbrace{(\mathbf{I} - \mathbf{R}_1 L)^{-1}\mathbf{R}_0}_{\boldsymbol{\mu}} + \underbrace{(\mathbf{I} - \mathbf{R}_1 L)^{-1}}_{\psi(L)}\mathbf{u}_t \tag{3.14}$$

where $\psi(L) = (\mathbf{I} - \mathbf{R}_1 L)^{-1} = \sum_{k=o}^{\infty} \psi_k L^k$

From equation (3.5) we have that

$$\mathbf{u}_t = \mathbf{B}^{-1}\boldsymbol{\varepsilon}_t \tag{3.15}$$

We can then re-write equation (3.14) as

$$\mathbf{y}_t = \boldsymbol{\mu} + \underbrace{\psi(L)\mathbf{B}^{-1}}_{\boldsymbol{\Theta}(L)}\boldsymbol{\varepsilon}_t \tag{3.16}$$

where $\boldsymbol{\Theta}(L) = \sum_{k=o}^{\infty} \boldsymbol{\Theta}_k L^k$

Hence, the VAR specification can be written as

$$\begin{bmatrix} r_t \\ illiq_t \end{bmatrix} = \begin{bmatrix} \mu_{10} \\ \mu_{20} \end{bmatrix} + \begin{bmatrix} \theta_{11}^{(0)} & \theta_{12}^{(0)} \\ \theta_{21}^{(0)} & \theta_{22}^{(0)} \end{bmatrix} \begin{bmatrix} \varepsilon_{r,t} \\ \varepsilon_{illiq,t} \end{bmatrix} + \begin{bmatrix} \theta_{11}^{(1)} & \theta_{12}^{(1)} \\ \theta_{21}^{(1)} & \theta_{22}^{(1)} \end{bmatrix} \begin{bmatrix} \varepsilon_{r,t-1} \\ \varepsilon_{illiq,t-1} \end{bmatrix} + \ldots \tag{3.17}$$

The long-run cumulative impact of the structural shocks is captured by

$$\boldsymbol{\Theta}(1) = \begin{bmatrix} \sum\limits_{s=o}^{\infty} \theta_{11}^{(s)} & \sum\limits_{s=o}^{\infty} \theta_{12}^{(s)} \\ \sum\limits_{s=o}^{\infty} \theta_{21}^{(s)} & \sum\limits_{s=o}^{\infty} \theta_{22}^{(s)} \end{bmatrix} \tag{3.18}$$

Therefore, the imposition of a long-run restriction is reflected in a restriction on the parameters of $\boldsymbol{\Theta}(1)$. Here we assume that $\varepsilon_{illiq,t}$ has no long-run cumulative impact on r_t. It implies that

$$\theta_{21}^{(1)} = \sum_{s=o}^{\infty} \theta_{21}^{(s)} = 0 \tag{3.19}$$

and we can recover the structural parameters as

$$\Theta(1) = \psi(1)\mathbf{B}^{-1} \tag{3.20}$$

$$= (\mathbf{I} - \mathbf{R}_1)^{-1}\mathbf{B}^{-1} \tag{3.21}$$

thus

$$\mathbf{B} = [(\mathbf{I} - \mathbf{R}_1)\Theta(1)]^{-1} . \tag{3.22}$$

$\Theta(1)$ is also estimated by maximum likelihood, once again we refer to Lutkepohl and Kratzig (2004) for details. With the estimates of $\Theta(1)$ we can calculate $\mathbf{B}$ and, as we can always find the coefficients of the reduced-form VAR and the relations described by equations (3.10) and (3.11), we can recover the structural coefficients.

3.4. Results

This section describes the results of the estimated Structural VAR models (SVAR). We divide the evidence for yearly and monthly data in two different subsections. We report the estimated coefficients, impulse-response functions (IRF) and interpretations os each model in each subsection

3.4.1. Yearly

This subsection reports the results of the SVAR models we estimate using yearly data. We find evidence that liquidity is priced and that unexpected illiquidity lowers contemporaneous return. However, our results show that the illiquidity premium may be lower than the one we estimate in Chapter 1. This highlights the importance of considering the mutual effect of return and illiquidity in the specification of the model. The next two subsections report results for the two different sets of restrictions we use: short and long-run.

3.4.1.1. Short-run restriction. In this subsection we recover the structural parameters using the short-run restriction and analyze the impulse-response functions to get the graphical path of the response of each variable to a structural shock. To make the mechanics of the model clear we show all the steps to recover the structural elements in the first model, using the VWMR as the dependent variable and $\Delta \ln A^0$ as the measure of illiquidity. Then, we summarize the results of the other measures of return and illiquidity. The reduced form is a VAR(1), we choose that order after comparing the information criteria of several specifications with several lags. The calculated statistics are 17.82 (FPE), 8.55 (AIC) 8.64 (HQIC) 8.80 (SBIC), all of them indicate that the VAR(1) is preferred.

The equation system below reports the estimated matrix described by model (3.9). We show the coefficients, the t-statistics in parenthesis, and the significance at the 1%, 5% and 10% levels is indicated by ***, ** and *, respectively.

(3.23)

$$\begin{bmatrix} r_t^M \\ \Delta \ln A_t^0 \end{bmatrix} = \begin{bmatrix} \underset{(5.13)}{16.668^{***}} \\ \underset{(0.29)}{0.014} \end{bmatrix} + \begin{bmatrix} \underset{(0.03)}{0.004} & \underset{(1.50)}{14.438} \\ \underset{(-3.94)}{-0.010^{***}} & \underset{(-2.15)}{-0.318^{**}} \end{bmatrix} \begin{bmatrix} r_{t-1}^M \\ \Delta \ln A_{t-1}^0 \end{bmatrix} + \begin{bmatrix} u_{r^M,t} \\ u_{\Delta \ln A^0,t} \end{bmatrix}$$

The coefficients of the reduced form are the same ones we find with the AR(1) specification in Chapters 1 and 2, which should be obvious as each equation of the reduced VAR is estimated by OLS. The t-statistics are not exactly the same as in the first Chapter we use Newey-West methodology for calculating the standard errors. The coefficient of the lagged illiquidity, for instance, is non-significant at a 10% level, however, using Newey-West standard error we get a 10%-significant coefficient (that increases in significance according to the number of lags we add to get the robust standard deviations). We define the estimated matrix $\mathbf{B}$, applying the short-run restriction, with the return variable given by r_t^M, and the illiquidity by $\Delta \ln A_{t-1}^0$, as $\mathbf{B}^{SR}_{r^M,\Delta \ln A^0}$, and it is given by

$$\mathbf{B}^{SR}_{r^M,\Delta \ln A^0} = \begin{bmatrix} 1 & \underset{(228.60)}{32.996^{***}} \\ 0 & 1 \end{bmatrix} \tag{3.24}$$

By plugging this matrix and the matrix of reduced-form coefficients in equations (3.10) and (3.11), we recover matrices $\mathbf{\Gamma}_0$ and $\mathbf{\Gamma}_1$ straighforwardly by

$$\mathbf{\Gamma}_0 = \begin{bmatrix} 1 & 32.996 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 16.668 \\ 0.014 \end{bmatrix} = \begin{bmatrix} 17.144 \\ 0.014 \end{bmatrix} \tag{3.25}$$

and

$$\mathbf{\Gamma}_1 = \begin{bmatrix} 1 & 32.996 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.004 & 14.438 \\ -0.010 & -0.318 \end{bmatrix} = \begin{bmatrix} -0.317 & 3.955 \\ -0.0097 & -0.318 \end{bmatrix} \tag{3.26}$$

Therefore, the structural model can be written as

$$\begin{bmatrix} r^M_t \\ \Delta \ln A^0_t \end{bmatrix} = \begin{bmatrix} 17.144 \\ 0.014 \end{bmatrix} + \begin{bmatrix} -0.317 & 3.955 \\ -0.0097 & -0.318 \end{bmatrix} \begin{bmatrix} r^M_{t-1} \\ \Delta \ln A^0_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{r^M,t} \\ \varepsilon_{\Delta \ln A^0,t} \end{bmatrix} \tag{3.27}$$

Under this formulation, the coefficient of the lagged illiquidity is 3.955, this is a large decrease in its value when compared to the case of Chapter 1 (the reduced form). When $\Delta \ln A^0$ is increased in one-standard deviation, the expected annual VWMR increases in 1.15 points. This figure is a lot less puzzling high than the 4.16 basis points we report in Chapter 1.

Figure 28 traces out the effect of a one-unit shock to $\varepsilon_{\Delta \ln A^0,t}$ on the time path of r^M_t (the VWMR). This shock causes r^M_t to fall by 33 points. In the following period, r^M_{t+1} becomes positive, however, it is not different from zero taking the 95% confidence interval. The impulse response function with the structural VAR is a way of accessing the impact of

the unexpected illiquidity. As Amihud (2002) suggests, it indeed lowers contemporaneous return, and using this IRF we see that this effect lasts for only one period. Furthermore, the SVAR methodology is more flexible than the Amihud (2002) specification as we are not imposing an AR(1) structure for the expected illiquidity.

[INSERT FIGURE 28 ABOUT HERE]

We repeat the same process we describe for the VWMR in excess of the risk-free rate (r_t^E). The estimated matrix of $\mathbf{B}_{r^E,\Delta \ln A^0}$ is

$$\mathbf{B}^{SR}_{r^E,\Delta \ln A^0} = \begin{bmatrix} 1 & \underset{(246.60)}{35.59^{***}} \\ 0 & 1 \end{bmatrix} \tag{3.28}$$

The structural coefficients associated with this matrix yield the following model

$$\begin{bmatrix} r_t^E \\ \Delta \ln A_t^0 \end{bmatrix} = \begin{bmatrix} 10.25 \\ -0.038 \end{bmatrix} + \begin{bmatrix} -0.37 & 3.20 \\ -0.0099 & -0.325 \end{bmatrix} \begin{bmatrix} r_{t-1}^E \\ \Delta \ln A_{t-1}^0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{r^E,t} \\ \varepsilon_{\Delta \ln A^0,t} \end{bmatrix} \tag{3.29}$$

The difference to the previous model with the VWMR is small, the coefficient of $\Delta \ln A_{t-1}^0$ is 3.20, 0.755 lower than the previous case. This estimated coefficient is also quite lower than 14.438 we estimate in the reduced form. Figure 29 plots the response of a one-unit shock to $\varepsilon_{\Delta \ln A^0,t}$ on the time path of r_t^E. The results and the interpretation are virtually the same of the previous case as the coefficient of the contemporaneous impact is so similar.

[INSERT FIGURE 29 ABOUT HERE]

The coefficients we estimate using the reduced-form specification, given by (3.9), using $HP.\ln A_t^0$ as the measure of illiquidity and , are reported in the matrix below. We also report the t-statistics in parenthesis, and the significance at the 1%, 5% and 10% levels, indicated by ***, **, and *, respectively.

(3.30)

$$\begin{bmatrix} r_t^M \\ \\ HP.\ln A_t^0 \end{bmatrix} = \begin{bmatrix} \underset{(5.76)}{16.361^{***}} \\ \\ \underset{(3.20)}{0.097^{***}} \end{bmatrix} + \begin{bmatrix} \underset{(-0.62)}{-0.077} & \underset{(3.61)}{41.014^{***}} \\ \\ \underset{(-4.83)}{-0.006^{***}} & \underset{(0.10)}{0.012} \end{bmatrix} \begin{bmatrix} r_{t-1}^M \\ \\ HP.\ln A_{t-1}^0 \end{bmatrix} + \begin{bmatrix} u_{r^M,t} \\ \\ u_{HP.\ln A^0,t} \end{bmatrix}$$

The estimated $\mathbf{B}^{SR}_{r^M,HP.\ln A^0}$ matrix using these variables is

$$\mathbf{B}_{r^M,HP.\ln A^0} = \begin{bmatrix} 1 & \underset{(122.63)}{17.52^{***}} \\ \\ 0 & 1 \end{bmatrix} \tag{3.31}$$

Therefore, the structural model is given by

(3.32)

$$\begin{bmatrix} r_t^M \\ \\ HP.\ln A_t^0 \end{bmatrix} = \begin{bmatrix} 18.066 \\ \\ 0.0973 \end{bmatrix} + \begin{bmatrix} -0.190 & 41.219 \\ \\ -0.0064 & 0.0117 \end{bmatrix} \begin{bmatrix} r_{t-1}^M \\ \\ HP.\ln A_{t-1}^0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{r^M,t} \\ \\ \varepsilon_{HP.\ln A^0,t} \end{bmatrix}$$

The magnitude of the coefficient of $HP.\ln A^0$ increases by using the SVAR specification. If the HP-filtered variable increases in one-standard deviation, return increases in 7.92 basis points. The coefficients of the reduced-form imply an increase of 7.88 points of return, 0.04 points lower than the one related to the SVAR-structural coefficient.

Figure 30 traces out the effect of a one-unit shock to $\varepsilon_{HP.\ln A^0,t}$ on the time path of r_t^M. This shock causes r_t^M to fall by 18 points. In the following period, r_{t+1}^E becomes positive, being different from zero (in the 95% confidence interval) until the second year after the shock.

[INSERT FIGURE 30 ABOUT HERE]

Applying the same procedure but using the VWMR in excess of the risk-free rate yields the following matrix $\mathbf{B}^{SR}_{r^E,HP.\ln A^0}$

$$\mathbf{B}_{r^E, HP.\ln A^0} = \begin{bmatrix} 1 & \underset{(144.41)}{20.63^{***}} \\ 0 & 1 \end{bmatrix} \tag{3.33}$$

and the structural model is given by

$$\begin{bmatrix} r_t^E \\ HP.\ln A_t^0 \end{bmatrix} = \begin{bmatrix} 10.91 \\ 0.064 \end{bmatrix} + \begin{bmatrix} -0.245 & 44.96 \\ -0.006 & 0.0193 \end{bmatrix} \begin{bmatrix} r_{t-1}^E \\ HP.\ln A_{t-1}^0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{r^E,t} \\ \varepsilon_{HP.\ln A^0,t} \end{bmatrix} \tag{3.34}$$

The effect of a one-unit shock to $\varepsilon_{HP.\ln A^0,t}$ on the time path of r_t^E is analogous to the one in figure 30. These results show that the impact of illiquidity over returns can be affected by the simultaneous relation between these two variables. We impose a short term restriction and assume that there is a contemporaneous relation that flows from illiquidity to return. In the next subsection we study the estimated coefficients by assuming a long-run restriction.

3.4.1.2. Long-run restriction. In this subsection we add the long-run restriction we describe in the previous section. We assume here that the illiquidity shock has no long-run effect on market return. This is translated on a restriction on the estimated matrix $\mathbf{\Theta}(1)$. The first model we analyze is the relation between VWMR $\Delta \ln A_{t-1}^0$. The estimated $\mathbf{\Theta}_{r_t^M, \Delta \ln A_{t-1}^0}$ is given by

$$\mathbf{\Theta}_{r_t^M, \Delta \ln A_{t-1}^0} = \begin{bmatrix} \underset{(9.80)}{13.76^{***}} & 0 \\ \underset{(-5.52)}{-0.172^{***}} & \underset{(9.80)}{0.178^{***}} \end{bmatrix} \tag{3.35}$$

As we have the matrix of the reduced-form coefficients, we can easily find matrix $\mathbf{B}$, which is given by

$$(3.36) \qquad \mathbf{B}^{LR}_{r^M_t,\Delta \ln A^0_{t-1}} = \begin{bmatrix} 0.0659 & 0.722 \\ 0.025 & 4.54 \end{bmatrix}$$

The estimated structural parameters are, therefore

$$(3.37) \qquad \mathbf{\Gamma}_0 = \begin{bmatrix} 1.11 \\ 0.498 \end{bmatrix}$$

and

$$(3.38) \qquad \mathbf{\Gamma}_1 = \begin{bmatrix} -0.006 & 0.722 \\ -0.044 & -1.07 \end{bmatrix}$$

The coefficient of the lagged illiquidity is 0.722. A one-standard deviation in this variable represents an increase of 0.20 points of VWMR. In this case, as in the previous, the contemporaneous relation between return and illiquidity yield a premium that is a lot less puzzling than the one the literature commonly estimates, and that we show in Chapter 1. It is also quite lower than the coefficient of 3.95 we find by using the short-run restriction.

The effect of a one-unit shock to $\varepsilon_{\Delta \ln A^0,t}$ on the time path of r^M_t (the VWMR) is show in Figure 31. The shock on $\varepsilon_{\Delta \ln A^0,t}$ causes r^M_t to fall by more than 2 points and in the following period, r^M_{t+1} becomes positive. The confidence interval is not plotted when long-run restrictions are imposed, however, we see that the path of VWMR after shock goes back to zero after 2 periods. This confirms the notion that the unexpected illiquidity decreases contemporaneous return and that the decrease can be significant and meaningful.

[INSERT FIGURE 31 ABOUT HERE]

By definition, the short-run restriction does not change the value of the coefficient of the lagged return over the illiquidity variable. The long-run restriction, however, allows a change in the value of this impact. The reduced form yields a coefficient of -0.318, the structural coefficient is -1.07. Both are very high, they imply that a one standard deviation increase in VWMR reduces illiquidity in 5.43 and 17.07 points, respectively.

If we use the VWMR in excess of the risk-free rate as the measure of market return we can estimate the coefficients of the matrix of restrictions $\boldsymbol{\Theta}_{r_t^E,\Delta \ln A_{t-1}^0}$

$$\boldsymbol{\Theta}_{r_t^E,\Delta \ln A_{t-1}^0} = \begin{bmatrix} \underset{(9.80)}{13.30^{***}} & 0 \\ \underset{(-5.74)}{-0.176^{***}} & \underset{(9.80)}{0.172^{***}} \end{bmatrix} \tag{3.39}$$

The respective matrix $\mathbf{B}$ is, therefore, given by

$$\mathbf{B}^{LR}_{r_t^E,\Delta \ln A_{t-1}^0} = \begin{bmatrix} 0.065 & 0.741 \\ 0.03 & 4.70 \end{bmatrix} \tag{3.40}$$

The estimated structural parameters are, therefore

$$\boldsymbol{\Gamma}_0 = \begin{bmatrix} 0.743 \\ 0.17 \end{bmatrix} \tag{3.41}$$

and

$$\boldsymbol{\Gamma}_1 = \begin{bmatrix} -0.009 & 0.741 \\ -0.47 & -1.09 \end{bmatrix} \tag{3.42}$$

The magnitude of the coefficients are very close to the previous case.

We also use the HP-filtered version of the illiquidity measure. The matrices of restrictions for the VWMR and the VWMR in excess of the risk-free rate are given by

$$\Theta_{r_t^M, HP.\ln A^0_{t-1}} = \begin{bmatrix} \underset{(9.90)}{11.15^{***}} & 0 \\ \underset{(-1.44)}{-0.32} & \underset{(9.90)}{0.154^{***}} \end{bmatrix} \tag{3.43}$$

and

$$\Theta_{r_t^E, HP.\ln A^0_{t-1}} = \begin{bmatrix} \underset{(9.90)}{10.34^{***}} & 0 \\ \underset{(-1.24)}{-0.027^{***}} & \underset{(9.90)}{0.153^{***}} \end{bmatrix} \tag{3.44}$$

These restrictions imply the following structural models

(3.45)
$$\begin{bmatrix} r_t^M \\ HP.\ln A_t^0 \end{bmatrix} = \begin{bmatrix} 1.36 \\ 0.29 \end{bmatrix} + \begin{bmatrix} -0.023 & 2.77 \\ -0.036 & -0.646 \end{bmatrix} \begin{bmatrix} r_{t-1}^M \\ HP.\ln A_{t-1}^0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{r^M,t} \\ \varepsilon_{HP.\ln A^0,t} \end{bmatrix}$$

and

(3.46)
$$\begin{bmatrix} r_t^E \\ HP.\ln A_t^0 \end{bmatrix} = \begin{bmatrix} 0.95 \\ 0.16 \end{bmatrix} + \begin{bmatrix} -0.03 & 3.12 \\ -0.036 & -0.73 \end{bmatrix} \begin{bmatrix} r_{t-1}^E \\ HP.\ln A_{t-1}^0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{r^E,t} \\ \varepsilon_{HP.\ln A^0,t} \end{bmatrix}$$

These coefficients are a lot lower than the ones we find imposing the short-run restriction. A one-standard deviation in the HP-filtered imply an increase of 0.53 points in VWMR and 0.6 points in VWMR in excess of the risk-free rate

This subsection shows that the restrictions we impose reduce the estimated illiquidity premium. This is convenient as the previously reported coefficients are considered puzzling high, as BHS state. The next subsection analyzes the estimated coefficients for monthly data

3.4.2. Monthly

This subsection presents results of the SVAR model using monthly data. We find that most of the coefficients of the lagged illiquidity measures are non-significant, what raises doubts about the pricing of monthly illiquidity. The short-run restriction applied to the first-lagged illiquidity yields results that imply that illiquidity lowers the future market return, what is not expected. The direction of the structural coefficients of the HP-filtered variables are positive, according to what we expect. Long run restrictions yield positive coefficients whose magnitudes are lower than the reduced form VAR implies for both illiquidity transformations.

In Chapters 1 and 2 we estimate models with only the first-lagged illiquidity variable. In this Chapter we can relax this assumption and estimate the model that shows "better" features. The information criteria we describe previously in the Chapter indicate that the model with the best fit would be either a VAR(3) or a VAR (4). Taking the model with the VWMR and the first-differenced illiquidity measure as an example, the FPE (1.92) and AIC (6.33) criteria indicate a VAR (3). The HQIC (6.37) and SBIC (6.43) indicate a VAR (4). We run a structural VAR considering a 4-lag specification and the coefficients of the fourth lag of return on the equation of the market illiquidity, are non-significant at any level. The same is seen for the models with alternative return and illiquidity measures. Therefore, the next subsections show results of models we estimate with 3 lags and imposing short and long-run restrictions.

3.4.2.1. Short-run restriction. This section reports the results of the estimated coefficients of the reduced and structural VAR (1) specifications we estimate with monthly data. We, one more time. recover the structural parameters using the short-run restriction.

We start our analysis for the first-differenced illiquidity measure. Table 3.1 reports the coefficients of the reduced-form VAR we estimate for all the return and illiquidity variables we use. These coefficients are not exactly the same we estimate in the previous Chapters as we use a 3-lags structure. The t-statistics are also different as in this Chapter we do not use Newey-West methodology.

Table 3.1. Reduced form VAR - Monthly data

	$\Delta \ln A^0$		$\Delta \ln A^0$ - Return-Rf		$\Delta \ln A^0$ - Return - FF3		HP.$\ln A^0$		HP.$\ln A^0$ - Return - Rf		HP.$\ln A^0$ - Return - FF3	
VARIABLES	$\Delta \ln A^0$	Return	$\Delta \ln A^0$	Return-Rf	$\Delta \ln A^0$	Return - FF3	HP.$\ln A^0$	Return	HP.$\ln A^0$	Return-Rf	HP.$\ln A^0$	Return - FF3
$\Delta \ln A^0$ (-1)	-0.743***	0.327	-0.747***	0.301	-0.665***	0.939*						
	(-17.866)	(0.604)	(-17.952)	(0.553)	(-16.012)	(1.957)						
$\Delta \ln A^0$ (-2)	-0.496***	0.653	-0.501***	0.617	-0.404***	1.319**						
	(-10.476)	(1.059)	(-10.565)	(0.997)	(-8.631)	(2.439)						
$\Delta \ln A^0$ (-3)	-0.131***	0.366	-0.134***	0.335	-0.069*	0.777						
	(-3.215)	(0.691)	(-3.293)	(0.630)	(-1.676)	(1.638)						
Return (-1)	-0.031***	0.071*					-0.029***	0.063				
	(-9.750)	(1.701)					(-9.433)	(1.514)				
Return (-2)	-0.022***	-0.043					-0.022***	-0.038				
	(-6.522)	(-0.967)					(-6.766)	(-0.881)				
Return (-3)	-0.009***	0.036					-0.010***	0.058				
	(-2.638)	(0.793)					(-3.189)	(1.373)				
Return - Rf (-1)			-0.032***	0.073*					-0.029***	0.064		
			(-9.836)	(1.731)					(-9.491)	(1.534)		
Return - Rf (-2)			-0.023***	-0.043					-0.022***	-0.038		
			(-6.591)	(-0.965)					(-6.809)	(-0.866)		
Return - Rf (-3)			-0.009***	0.033					-0.010***	0.057		
			(-2.701)	(0.738)					(-3.225)	(1.349)		
Return - FF3 (-1)					-0.027***	0.025					-0.026***	0.023
					(-7.417)	(0.610)					(-7.526)	(0.549)
Return - FF3 (-2)					-0.019***	0.019					-0.020***	0.015
					(-5.000)	(0.441)					(-5.682)	(0.360)
Return - FF3 (-3)					-0.004	0.073*					-0.007**	0.073*
					(-1.035)	(1.663)					(-2.074)	(1.731)
HP.$\ln A^0$ (-1)							0.195***	0.867	0.193***	0.865	0.242***	1.101**
							(5.001)	(1.643)	(4.957)	(1.635)	(6.118)	(2.285)
HP.$\ln A^0$ (-2)							0.192***	0.762	0.192***	0.773	0.190***	0.512
							(4.981)	(1.462)	(4.986)	(1.481)	(4.767)	(1.058)
HP.$\ln A^0$ (-3)							0.331***	-0.002	0.333***	0.024	0.285***	-0.538
							(8.750)	(-0.004)	(8.808)	(0.046)	(7.404)	(-1.150)
Constant	0.075***	1.342***	0.048***	0.942***	0.013	0.498***	0.090***	1.291***	0.065***	0.899***	0.033**	0.468***
	(4.633)	(6.392)	(3.245)	(4.849)	(0.904)	(2.935)	(5.917)	(6.244)	(4.570)	(4.675)	(2.341)	(2.756)
Observations	596	596	596	596	596	596	597	597	597	597	597	597
R-square	0.387	0.387	0.389	0.389	0.333	0.333	0.506	0.506	0.507	0.507	0.469	0.469

This table reports the results of monthly time-series regressions that include return and log-transformed Amihud (2002) measure. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The variables included in the model are: $\ln A^0$: the log-transformation of original Amihud (2002) measure defined as the yearly average of daily ratios of absolute return to dollar volume in multiplied by 1,000,000. $\Delta \ln A^0$: the first difference of $\ln A^0$. HP.$\ln A^0$: the HP-filtered $\ln A^0$. Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. For each explanatory variable there are the coefficient estimates, and paranthesis in the second row of each variable there are t-statistics computed. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

Most of the coefficients related to the first lag of the illiquidity variable are non-significant at any level. The only exception is the effect of the first-differenced lagged illiquidity over the market return adjusted to the Fama-French three factors, which is 10%-significant. This may indicate that for a higher frequency the prices incorporate the information about illiquidity. If market return incorporates all the information about illiquidity this variable fails to predict the market return. All of the coefficients of the impact of the lagged market return over illiquidity are, on the other hand, 1%-significant.

We use many specifications with different lags and many definitions of return in this section, thus, instead of reporting each equation in the structural form as we do with yearly data, we report in Table 3.2 the coefficient of matrix **B** and in Table 3.3 the structural coefficients we estimate using the short-run restrictions.

All of the coefficients representing contemporaneous restrictions are positive and 1%-significant. This implies that the impact of the unexpected shock on illiquidity is negative and close to -2 to the models we estimate in this subsection.

Table 3.3 reports that most of the structural coefficients are negative, except the ones for the second and third lags using the VWMR adjusted for the Fama-French three factors. The impact of returns over illiquidity behaves according to what we expected, they are all negative, implying that return decrease makes the market less liquid. The coefficients of the lagged return over illiquidity, which are significant in the reduced form, have magnitudes really similar to the reduced form estimates.

Table 3.4 reports the structural coefficients of the models we estimate using the HP-filtered transformation to measure illiquidity. Using this variable all of the lagged market illiquidity measures have a positive impact over the market return and the lagged market return has a negative effect over market illiquidity. The magnitude of these coefficients is higher than the ones we estimate in the reduced form. The coefficients corresponding to the response of illiquidity to shocks in returns remain almost unchanged.

As the reduced-form coefficients corresponding to the illiquidity premium are non-significant, the structural coefficients are probably also non-significant. We do not attempt

Table 3.2. Structural VAR - Monthly - Coefficients of the matrix of short-run restriction

	Return	Return - Rf	Return - FF3
ΔlnA^0	2.91***	2.96***	2***
	(71.02)	(72.23)	(48.93)
HP.lnA^0	2.55***	2.59***	1.88***
	(62.49)	(63.31)	(46.04)

This table reports the coefficients of the matrix of short term restrictions, B, of monthly time-series. It includes return and stationarized log-transformed Amihud (2002) measure in the Structural VAR. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The variables included in the model are: ΔlnA^0: the first difference of lnA^0. HP.lnA^0: the HP-filtered lnA^0. Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. For each explanatory variable there are the coefficient estimates, and paranthesis in the second row of each variable there are t-statistics computed. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

to calculate their standard deviations to access their significance in this Chapter for practical matters: we use OLS to estimate the reduced-form variable and we use maximum-likelihood to estimate the elements of the restriction matrix. For future research we can estimate all the parameters simultaneously in order to access the properties of the estimated coefficients. In the next subsection we describe the results of the structural coefficients we find using the long-run restriction.

3.4.2.2. Long-run restriction. In this subsection we add the long-run restriction on the effects of market illiquidity over market return. Table 3.5 reports the structural coefficients we find using the first-differenced log of market illiquidity ($\Delta \ln A^0$). Most of the coefficients related to the illiquidity premium are positive, except the ones related

Table 3.3. Structural VAR - Monthly - Structural coefficients estimated with short-run restriction and first-differenced log-transformed illiquidity

	Return		Return-Rf		Return-FF3	
	Return	ΔlnA^0	Return-Rf	ΔlnA^0	Return-FF3	ΔlnA^0
ΔlnA^0 (-1)	-1.84	-0.74	-1.91	-0.75	-0.39	-0.66
ΔlnA^0 (-2)	-0.79	-0.50	-0.86	-0.50	0.51	-0.40
ΔlnA^0 (-3)	-0.01	-0.13	-0.06	-0.13	0.64	-0.07
Return (-1)	-0.02	-0.03	-0.02	-0.03	-0.03	-0.03
Return (-2)	-0.11	-0.02	-0.11	-0.02	-0.02	-0.02
Return (-3)	0.01	-0.01	0.01	-0.01	0.06	0.00
Constant	1.56	0.07	1.08	0.05	0.52	0.01

This table reports the structural coefficients after applying short-run restrictions in the monthly dataset. It includes return and first-differenced log-transformed Amihud (2002) measure in the Structural VAR. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The variables included in the model are: ΔlnA^0: the first difference of lnA^0. Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. For each explanatory variable there are the coefficient estimates, and paranthesis in the second row of each variable there are t-statistics computed. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

to the first lag of market illiquidity when we use VWMR and VWMR in excess of the risk-free rate. The magnitude of the coefficients is a lot smaller than the reduced-form and the structural ones we find with short-run restrictions.

Table When we use the HP-filtered log of market illiquidity ($HP. \ln A^0$) we find only positive coefficients relative to the lag illiquidity variables and negative coefficients for the impact of market returns on market illiquidity. This behaves exactly like we expect. The magnitude of the coefficients is also a lot smaller than the reduced form specification and are also smaller than the ones we find using the short-run restrictions.

3.5. Conclusion

In this Chapter we estimate the illiquidity premium allowing for the contemporaneous relation between market return and illiquidity. We find that for yearly data this strategy

Table 3.4. Structural VAR - Monthly - Structural coefficients estimated with short-run restriction and first-differenced HP-filtered illiquidity

	Return		Return-Rf		Return-FF3	
	Return	HP.lnA^0	Return-Rf	HP.lnA^0	Return-FF3	HP.lnA^0
HP.lnA^0 (-1)	1.37	0.20	1.37	0.19	1.56	0.24
HP.lnA^0 (-2)	1.25	0.19	1.27	0.19	0.87	0.19
HP.lnA^0 (-3)	0.84	0.33	0.89	0.33	0.00	0.28
Return (-1)	-0.01	-0.03	-0.01	-0.03	-0.03	-0.03
Return (-2)	-0.09	-0.02	-0.09	-0.02	-0.02	-0.02
Return (-3)	0.03	-0.01	0.03	-0.01	0.06	-0.01
Constant	1.52	0.09	1.07	0.06	0.53	0.03

This table reports the structural coefficients after applying short-run restrictions in the monthly dataset. It includes return and HP-filtered log-transformed Amihud (2002) measure in the Structural VAR. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The variables included in the model are: HP.lnA^0: the HP-filtered lnA^0. Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. For each explanatory variable there are the coefficient estimates, and paranthesis in the second row of each variable there are t-statistics computed. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

lowers the illiquidity premium to a more reasonable coefficient, i.e., less puzzling high than the one we find in Chapter 1, which is the standard in the literature.

For monthly data, however, the coefficients of the lagged illiquidity on the reduced form VAR are non-significant. It may be the case that for higher frequencies, such as monthly, all of the information is already incorporated in the prices leading to the non-significance of the coefficients. The structural coefficients relative to the first-differenced log of market illiquidity in monthly data are negative when we use the short-term restrictions. The same restriction applied for the HP-filtered variable yields positive coefficients that have higher magnitudes than the reduced-form counterparts. The long-run restrictions yield structural coefficients relative to the illiquidity premium that are positive and a lot smaller than the reduced form ones, for both first-differenced and HP-filtered versions of the illiquidity variable

Table 3.5. Structural VAR - Monthly - Structural coefficients estimated with long-run restriction and first-differenced log-transformed illiquidity

	Return		Return-Rf		Return-FF3	
	Return	$\Delta\ln A^0$	Return-Rf	$\Delta\ln A^0$	Return-FF3	$\Delta\ln A^0$
$\Delta\ln A^0$ (-1)	-0.02	-2.26	-0.02	-2.28	0.00	-1.91
$\Delta\ln A^0$ (-3)	0.09	-1.49	0.08	-1.51	0.19	-1.15
$\Delta\ln A^0$ (-3)	0.07	-0.39	0.06	-0.40	0.17	-0.19
Return (-1)	0.01	-0.09	0.01	-0.09	0.00	-0.08
Return (-2)	-0.01	-0.07	-0.01	-0.07	0.00	-0.05
Return (-3)	0.01	-0.03	0.01	-0.03	0.02	-0.01
Constant	0.32	0.29	0.22	0.19	0.13	0.04

This table reports the structural coefficients after applying longt-run restrictions in the monthly dataset. It includes return and first-differenced log-transformed Amihud (2002) measure in the Structural VAR. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The variables included in the model are: $\Delta\ln A^0$: the first difference of $\ln A^0$. Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. For each explanatory variable there are the coefficient estimates, and paranthesis in the second row of each variable there are t-statistics computed. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

This Chapter highlights that the illiquidity premium may be a lot smaller than previously thought, not because of some issue with the illiquidity measure, as suggest BHS, but because of the effect of simultaneous effect between market return and illiquidity. These results rely on the identification hypothesis we adopt and for that reason it is hard to make comparisons between the results using long and short-run restrictions. Another point is the statistical properties of the structural coefficients. As we use the conventional methodology of structural VAR, we combine the OLS estimates of the reduced-form VAR with the maximum likelihood estimation of the structural VAR. For future research the simultaneous approach may be developed to take more properties into account.

Table 3.6. Structural VAR - Monthly - Structural coefficients estimated with long-run restriction and HP-filtered log-transformed illiquidity

	Return		Return-Rf		Return-FF3	
	Return	HP.lnA^0	Return-Rf	HP.lnA^0	Return-FF3	HP.lnA^0
HP.lnA^0 (-1)	0.46	0.52	0.46	0.51	0.50	0.66
HP.lnA^0 (-3)	0.43	0.51	0.43	0.51	0.31	0.53
HP.lnA^0 (-3)	0.44	0.96	0.45	0.96	0.13	0.85
Return (-1)	-0.02	-0.09	-0.02	-0.09	-0.02	-0.08
Return (-2)	-0.04	-0.06	-0.04	-0.06	-0.02	-0.06
Return (-3)	0.00	-0.03	0.00	-0.03	0.01	-0.02
Constant	0.41	0.19	0.29	0.14	0.15	0.08

This table reports the structural coefficients after applying long-run restrictions in the monthly dataset. It includes return and HP-filtered log-transformed Amihud (2002) measure in the Structural VAR. The table contains the result for NYAM-listed stocks from January 1962 to December 2011. The variables included in the model are: HP.lnA^0: the HP-filtered lnA^0. Return: the value weighted average of returns across all firms included in the sample. Return-Rf: the value weighted yearly average of excess return (in excess of the T-bill rate). Return - FF3F: the value weighted market return adjusted for the Fama and French's three factors. The returns are multiplied by 100. For each explanatory variable there are the coefficient estimates, and paranthesis in the second row of each variable there are t-statistics computed. To survive in the sample, stocks must five zero-volume days or fewer within a month, and monthly returns for the past 24 months. Only common stocks (share code 10 or 11 in CRSP) are used. Outliers are 1-percent winsorized. Significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

CHAPTER 4

Topics for future research

4.1. Introduction

In this dissertation we study the positive effect that market illiquidity has over market returns. We also take into account the negative effect that market return has over illiquidity. In the first Chapter we show that the illiquidity is priced, if the variable that proxies illiquidity is stationary. In the second Chapter we find evidence that market return declines lowers market liquidity. In the third Chapter we estimate the illiquidity premium taking this dynamic between return and illiquidity.

A consequence of this exercise is that we could notice the effect of the contemporaneous illiquidity over returns. The full effect of return can be better understood with a decomposition such as Campbell and Shiller (1988) in which we consider explicitly the effects of illiquidity. We do that in this section: we present a return decomposition that takes into account the illiquidity costs.

There are two interesting consequences if this decomposition holds. First, we can use it to find a three-beta model in a way analogous to Campbell and Vuolteenaho (2004). One beta would reflect news about market's future cash flows, one reflecting news about the market's discount rate, and an extra one reflecting news about illiquidity. Small companies seem to be more illiquid, and this illiquidity is more persistent than the one for larger and more liquid companies. It may help explaining why some firms have higher than average returns.

The second consequence of this decomposition is that we can use it to understand the role of the contemporaneous illiquidity. Sadka and Sadka (2009) use a similar reasoning to access the effect of earnings on contemporaneous return and make interesting statements concerning the effect of predictability and the relation between earnings and return. We do not dig into that rationale for now but it may be also a topic of further research.

Third, in Chapter 2 we show evidence of a significant relation between standard deviation and illiquidity. Also, the main claim this dissertation implies is that illiquidity

affects return, and returns affect illiquidity. We also have evidence of the interaction between volatility and returns (Bollerslev, Litvinov and Tauchen, 2006). It is possible that there is an interaction between these three variables that may help explaining the overall effect we observe. It may be the case that the a stock's volatility increases illiquidity, and this variable increases the expected return. Modelling this relation may be important to understand both illiquidity premium and volatility feedback. The next section presents the return decomposition with illiquidity costs and the last sections explains what we can do with that.

4.2. Return decomposition with illiquidity costs

In this section we introduce a log-linear approximation to the present-value identity following Campbell and Shiller (1988) and Campbell (1991). We model the return in one period: an investor who holds a given security and wants to sell it has to incurs into an illiquidity cost C_i, modeled simply as the per-share cost of selling a given security i.

The different trading horizons of investors are not a major concern, as our objective is to model the return an investor has by holding a given security for a given period. The relation we are modelling is an accounting identity instead of a behavioral model, but it can also be thought in the context of an overlapping generations economy in which each agent lives for two periods, such as Amihud and Mendelson (1986).

In this section we show all the steps to get to the final equations of the return decomposition, which are (4.8) and (4.9). Let R_{t+1} be the return an investor has for holding a security for one period. Let D_{t+1} be the dividend paid by the security for this given period, and C_{t+1} be the cost of selling a given security. From the definition of return we have that

$$R_{t+1} = \frac{P_{t+1} + D_{t+1} - C_{t+1}}{P_t}$$

This is simply an accounting identity. Dividing both sides by R_{t+1} and multiplying $\frac{P_t}{D_t}$ we get to

$$\begin{aligned}
\frac{P_t}{D_t} &= \frac{P_t}{D_t}\frac{1}{R_{t+1}}\left(\frac{P_{t+1}+D_{t+1}-C_{t+1}}{P_t}\right) \\
&= \frac{1}{R_{t+1}}\left(\frac{P_{t+1}-C_{t+1}}{P_t}\frac{P_t}{D_t}+\frac{D_{t+1}}{P_t}\frac{P_t}{D_t}\right) \\
&= \frac{1}{R_{t+1}}\left(\frac{P_{t+1}-C_{t+1}}{D_t}+\frac{D_{t+1}}{D_t}\right) \\
&= \frac{1}{R_{t+1}}\left(\frac{P_{t+1}-C_{t+1}}{D_t}\frac{D_t}{D_{t+1}}+1\right)\frac{D_{t+1}}{D_t} \\
&= \frac{1}{R_{t+1}}\left(\frac{P_{t+1}-C_{t+1}}{D_{t+1}}+1\right)\frac{D_{t+1}}{D_t} \qquad (4.1)
\end{aligned}$$

taking logs and with lowercase letters denoting logs of uppercase letters we have

$$\begin{aligned}
\ln\left(\frac{P_t}{D_t}\right) &= \ln\left[\frac{1}{R_{t+1}}\left(\frac{P_{t+1}-C_{t+1}}{D_{t+1}}+1\right)\frac{D_{t+1}}{D_t}\right] \\
\ln(P_t)-\ln(D_t) &= \ln\left(\frac{1}{R_{t+1}}\right)+\ln\left(\frac{P_{t+1}-C_{t+1}}{D_{t+1}}+1\right)+\ln\left(\frac{D_{t+1}}{D_t}\right) \\
\ln(P_t)-\ln(D_t) &= -\ln(R_{t+1})+\ln\left(\frac{P_{t+1}-C_{t+1}}{D_{t+1}}+1\right)+\ln(D_{t+1})-\ln(D_t) \\
p_t-d_t &= -r_{t+1}+\ln\left(1+e^{p_{t+1}-d_{t+1}}\right)+\Delta d_{t+1} \qquad (4.2)
\end{aligned}$$

Defining $P_{t+1}-C_{t+1}\equiv P^*_{t+1}$ and assuming that the points P_{t+1} and P^*_{t+1} are sufficiently close to each other, we have

$$\begin{aligned}
\ln(P_t)-\ln(D_t) &= -\ln(R_{t+1})+\ln\left(\frac{P^*_{t+1}}{D_{t+1}}+1\right)+\ln(D_{t+1})-\ln(D_t) \\
p_t-d_t &= -r_{t+1}+\ln\left(1+e^{p^*_{t+1}-d_{t+1}}\right)+\Delta d_{t+1} \qquad (4.3)
\end{aligned}$$

We need now to solve the term in parenthesis. Lets take a Taylor expansion of $\ln\left(1+e^{p^*_{t+1}-d_{t+1}}\right)$ about a point $\frac{P^*}{D}=\overline{p}-\overline{d}$

$$\ln\left(1+e^{p^*_{t+1}-d_{t+1}}\right)\approx\ln\left(1+e^{\overline{p}^*-\overline{d}}\right)+\frac{e^{\overline{p}^*-\overline{d}}}{1+e^{\overline{p}-\overline{d}}}\left[p^*_{t+1}-d_{t+1}-(\overline{p}^*-\overline{d})\right]$$

Lets define

$$\rho=\frac{e^{\overline{p}^*-\overline{d}}}{1+e^{\overline{p}^*-\overline{d}}} \tag{4.4}$$

We know that $\rho<1$ and as $\ln\left(1+e^{\overline{p}^*-\overline{d}}\right)$ and $\frac{e^{\overline{p}^*-\overline{d}}}{1+e^{\overline{p}^*-\overline{d}}}(\overline{p}^*-\overline{d})$ are constants. Therefore we have that

$$\ln\left(1+e^{p_{t+1}-d_{t+1}}\right)\approx c+\rho\left(p^*_{t+1}-d_{t+1}\right)$$

and we can write

$$\begin{aligned}p_t-d_t&\approx-r_{t+1}+\left[c+\rho\left(p^*_{t+1}-d_{t+1}\right)\right]+\Delta d_{t+1}\\&=c-r_{t+1}+\Delta d_{t+1}+\rho\left(p^*_{t+1}-d_{t+1}\right)\end{aligned}$$

As we assume that P_{t+1} and P^*_{t+1} are sufficiently close, we can focus on p^*_{t+1}

$$p^*_{t+1}=\ln(P_{t+1}-C_{t+1})$$

Taking a Taylor expansion of this term yields

$$\ln(P_{t+1}-C_{t+1})=\ln\left(e^{p_{t+1}}-e^{c_{t+1}}\right)$$

$$\ln\left(e^{p_{t+1}}-e^{c_{t+1}}\right)\approx\frac{e^{\overline{p}}}{e^{e^{\overline{p}}}-e^{\overline{d}}}\left(p_{t+1}-\overline{p}\right)-\frac{e^{\overline{c}}}{e^{e^{\overline{p}}}-e^{\overline{c}}}\left(c_{t+1}-\overline{c}\right)$$

note that $\frac{e^{\overline{p}}}{e^{e^{\overline{p}}}-e^{\overline{d}}}, \frac{e^{\overline{d}}}{e^{e^{\overline{p}}}-e^{\overline{d}}} < 1$. Lets define

$$\eta = \frac{e^{\overline{p}}}{e^{e^{\overline{p}}} - e^{\overline{c}}}$$

and

$$\upsilon = \frac{e^{\overline{c}}}{e^{e^{\overline{p}}} - e^{\overline{c}}}$$

in this way

$$p_t - d_t \approx k - r_{t+1} + \Delta d_{t+1} + \rho\left(\eta p_{t+1} - \upsilon c_{t+1} - d_{t+1}\right)$$

Re-writing the equation above as

$$r_{t+1} = k + (1-\rho)d_{t+1} + (\alpha p_{t+1} - p_t) - \beta c_{t+1}$$

and noting that we can write r_{t+1} as

$$r_{t+1} = k + (1-\rho)d_{t+1} + (\alpha p_{t+1} - p_t) - \beta\left[E_t\left(c_{t+1}\right) + \left(E_{t+1} - E_t\right)c_{t+1}\right]$$

which is equal to

$$r_{t+1} = k + (1-\rho)d_{t+1} + (\alpha p_{t+1} - p_t) - \beta E_t\left(c_{t+1}\right) - \beta UE_c$$

where $\alpha = \rho\eta < 1$ and $\beta = \rho\upsilon < 1$. Taking the difference of the expectation operators $E_{t+1} - E_t$ we find

$$r_{t+1} - E_t\left(r_{t+1}\right) = (1-\rho)\left[d_{t+1} - E_t\left(d_{t+1}\right)\right] + \alpha\left[p_{t+1} - E_t\left(p_{t+1}\right)\right] - \beta\left[c_{t+1} - E_t\left(c_{t+1}\right)\right] \tag{4.5}$$

It means that the an unexpectedly good stock return occurs either because either dividends or prices were above the expected. Furthermore, if the unexpected cost (illiquidity)

goes up it reduces return. We can find the term $p_{t+1} - E_t(p_{t+1})$. Take the equation for $p_t - d_t$. By forward recursive substitution and assuming no rational bubbles, such that

$$\lim_{m\to\infty} k\sum_{j=1}^{m}\alpha^{j-1} = \frac{k}{1-\alpha} \tag{4.6}$$

and

$$\lim_{\alpha\to\infty} \alpha^{m}\sum_{j=1}^{m} d_{t+m} = 0 \tag{4.7}$$

as d_{t+m} is a constant and $\alpha < 1$ we have

$$p_t = \frac{k}{1-\alpha} - \sum_{j=0}^{m}\alpha^j r_{t+j+1} + (1-\rho)\sum_{j=0}^{m-1}\alpha^j d_{t+j+1} - \beta\sum_{j=0}^{m}\alpha^j c_{t+j+1}$$

Applying the difference $E_{t+1} - E_t$ to the equation above, plugging it to equation (4.5) and after some algebra, we have

$$\begin{aligned} r_{t+1} - E_t(r_{t+1}) &= (E_{t+1} - E_t)\left((1-\rho)\sum_{j=0}^{m-1}\alpha^{j-1}d_{t+j+1}\right) - \\ &\quad - (E_{t+1} - E_t)\left(\sum_{j=0}^{m}\alpha^j r_{t+j+1}\right) - (E_{t+1} - E_t)\left(\beta\sum_{j=0}^{m}\alpha^{j+1}c_{t+j+1}\right) \end{aligned} \tag{4.8}$$

or, we can re-write it as

$$r_{t+1} = E_t(r_{t+1}) + N_{CF} - N_R - N_C \tag{4.9}$$

Re-writing equation (4.8) in terms of ρ we find

$$r_{t+1} - E_t\left(r_{t+1}\right) = \left(E_{t+1} - E_t\right)\left((1-a)\sum_{j=0}^{\infty}\rho^j d_{t+j+1}\right) - \left(E_{t+1} - E_t\right)\left(\sum_{j=0}^{\infty}\rho^j r_{t+j+1}\right) - \left(E_{t+1} - E_t\right)\left(b\sum_{j=0}^{\infty}\rho^{j+1} c_{t+j+1}\right) \tag{4.10}$$

where ρ is a measure given by $\frac{P}{P+D}$. Campbell and Shiller assume that $\rho = 0.95$. a is a consequence of the illiquidity cost, and it is given by $\frac{P-C}{P+D}$. c is also a consequence of this cost, and it is given by $\frac{C}{P+D}$.

This return decomposition allows a discussion about the effect of contemporaneous illiquidity and also the relative impact that these three components for a risk-averse, long-term investor who holds a market portfolio. In the next two sections we describe a little of the work that can be done in those areas.

4.3. Three-beta model

We rely on an argument similar to the one Campbell and Vuolteenaho use. It may be the case that returns on the market portfolio have three components, and the recognition of them may change something in the valuation of equities. They argue that the value of the market portfolio may fall because investors receive bad news about future cash flows (wealth decreases and investment opportunities are unchanged) or because investors increase the discount rate they apply to these cash flows (wealth decreases but future investment opportunities improve). It may also be the case that the value of the market portfolio falls because investors receive bad news about liquidity, or the cost they have to pay when cashing an asset in. Investors may demand higher premium to hold assets that covary with the market's cash-flow and illiquidity news.

The issue related to finding the beta is related to the model we use to find the expected illiquidity. Campbell and Vuolteenaho (2004) use a VAR specification to get the expected

log excess market return ($r^e_{M,t-1}$) and then they use this measure to get the news about future discount rates as

$$N_R = (E_{t+1} - E_t) \left(\sum_{j=0}^{\infty} \rho^{j-1} r_{t+j} \right)$$

where ρ is set to be equal to 0.95. The news about future cash flows is given by the residual after we find N_R. Our approach poses an issue at this point. We cannot find N_{CF} by the residual as we now have an additional component, Actually, c_t is not even observable. Our suggestion is to use of a model of illiquidity, such as the one we report in Chapters 2 and 3, to get the expected illiquidity and use it as a proxy for c_t multiplied by the constant given by b. From equation (4.10) we would have

$$N_C = \sum_{j=0}^{\infty} \rho^{j+1} \left(bc_{t+j+1} \right) \approx \sum_{j=0}^{\infty} \rho^{j+1} \left(Illiq_{t+j+1} \right)$$

where $Illiq$ is an illiquidity measure that proxies for the cost of illiquidity. Then, the news about future cash flows can still be found by the residual and we can proceed following Campbell and Vuolteenaho (2002) to get a three-beta model. However, it all relies on the return decomposition and on how well the news about illiquidity actually proxies the real cost of illiquidity.

4.4. Volatility, illiquidity, and returns

One final topic of future research is a consequence of the relationship between standard deviation and illiquidity we study in Chapter 2, and the simultaneous relation between returns and illiquidity we deal in Chapter 3. The idea is that volatility is supposed to increase illiquidity, which is supposed to increase return. If we take this and add the mutual relationship between returns and volatility we may have a jointly endogenous determined system. We can deal with that using the rationale of Klein and Vella (2010) or some methodology similar to it.

The rationale behind it comes from an empirical regularity in the literature: there is an asymmetry in the relationship between equity market returns and volatility. The causes behind this asymmetry are debated but one of the leading explanation for the volatility asymmetry is related to the time-varying risk premium, or volatility feedback effect (Campbell and Hentschel ,1992). The idea is that volatility is priced, therefore an anticipated increase in volatility raises the required rate of return, what lowers the contemporaneous stock-price. Therefore, according to this rationale, causality runs from volatility to prices.

In this dissertation we find that increases in illiquidity also seem to imply higher future returns and that volatility seem to increase illiquidity. We want to capture this effect to understand the relative importance of each of these components on channelling the effects to return.

Klein and Vella (2010) develop a methodology to adjust for endogeneity in the triangular simultaneous equations model where there are no available exclusion restrictions to generate suitable instruments. If we use that we can model both effects and get evidence of their relative importance. We could also try to model volatility and add liquidity as an explanatory variable, which is similar to the approach of Bollerslev, Litvinova and Tauchen (2006). This exercise would help understanding the mutual effects between illiquidity and volatility and how they affect return, what may contribute to both the volatility feedback and illiquidity-effects literature.

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APPENDIX

Fama-Macbeth procedure

Fama and Macbeth's (1973) methodology is described in detail by Skoulakis (2006). The first step is to demean data using the cross sectional average. The standard linear regression model in the context of panel data can be expressed as

$$A_{it}^{0} = X\prime_{it} b + \varepsilon_{it}, i = 1, ..., N, t = 1, ..., T$$

where A_{it}^{0} is the Amihud's (2002) original measure; X_{it} is the vector of explanatory variables ($SDRET_{i,t-1}$, $SIZE_{i,t-1}$, $DIVYLD_{i,t-1}$, $R_{i,t-1}$, we also add $A_{i,t-1}^{0}$ to one of the models we estimate). The following decomposition of residuals hold

$$\varepsilon_{it} = \xi_t + \mu_i + w_{it}$$

The firm-invariant time effect must be eliminated, this is done by demeaning. For each time period t we subtract the corresponding cross-sectional mean from each observation

$$A_{it}^{0} - \overline{A}_{N}^{0} = \left(X_{it} - \overline{X}_{N}\right) \prime b + \left(\varepsilon_{it} - \overline{\varepsilon}_{N}\right)$$

or

$$\widetilde{A}_{it}^{0} = \widetilde{X}_{it}\prime b + \widetilde{\varepsilon}_{it}$$

where $\widetilde{A}_{it}^{0} = y_{it} - \overline{A}_{N}^{0}$, $\widetilde{X}_{it} = X_{it} - \overline{X}_{N}$, and $\widetilde{\varepsilon}_{it} = \varepsilon_{it} - \overline{\varepsilon}_{N}$.

If there are no firm invariant time effects, the demeaning step is redundant. However, in the presence of firm-invariant time effects, Skoulakis (2006) show that omitting the demeaning step might result in severe deterioration in the performance of FM estimators.

The usual FM procedure consists of running cross-sectional regressions as the first step and then getting the final coefficient as the average of the first step coefficient estimates. The description of the model starts with the equation of y_{it} after demeaning $(\widetilde{A}_{it}^0)$

$$\widetilde{A}_{it}^0 = \widetilde{X}_{it}\prime b + \widetilde{\varepsilon}_{it}$$

An estimate of b based only on observations from time period t can be obtained as follows

$$\widehat{b}_t = (\widetilde{X}_t \widetilde{X}_t \prime)^{-1} \widetilde{X}_t \widetilde{A}_t^0$$

The FM estimator is then obtained by averaging over t the individual estimates of $\widehat{b}_t$.

$$b_T^{FM} = \frac{1}{T}\sum_{t=1}^{T} \widehat{b}_t = \frac{1}{T}\sum_{t=1}^{T} (\widetilde{X}_t \widetilde{X}_t \prime)^{-1} \widetilde{X}_t \widetilde{A}_t^0$$

Adapting the original idea of FM to a time-series context, an estimate of the covariance matrix $Avar[b_T^{FM}]$ is obtained by treating the individual estimates $\widehat{b}_t$ as a sample from a stationary time series. This gives rise to the heteroskedasticity and autocorrelation consistent covariance estimator using linearly decaying weights, suggested by Newey and West (1987)

$$V_{b,T}^{FM} = \Omega_{0,T}^{FM} + \sum_{j=1}^{m_T} \left(1 - \frac{j}{m_T + 1}\right) \left[\Omega_{j,T}^{FM} - \left(\Omega_{j,T}^{FM}\right)\prime\right]$$

where

$\Omega_{j,T}^{FM} = \frac{1}{T} \sum_{j=t+1}^{T} \left(\widehat{b}_t - b_T^{FM}\right) \left(\widehat{b}_{t-j} - b_T^{FM}\right)\prime$

and the bandwidh m_T increases with T. In this paper we use autocorrelation consistent covariance estimator assuming autocorrelation of 2 periods. By using this procedures to equations (2.1) we get the results described in Section 4.

Figure 1: Path of the Yearly log-transformed annual Amihud (2002) - original measure (lnA0).

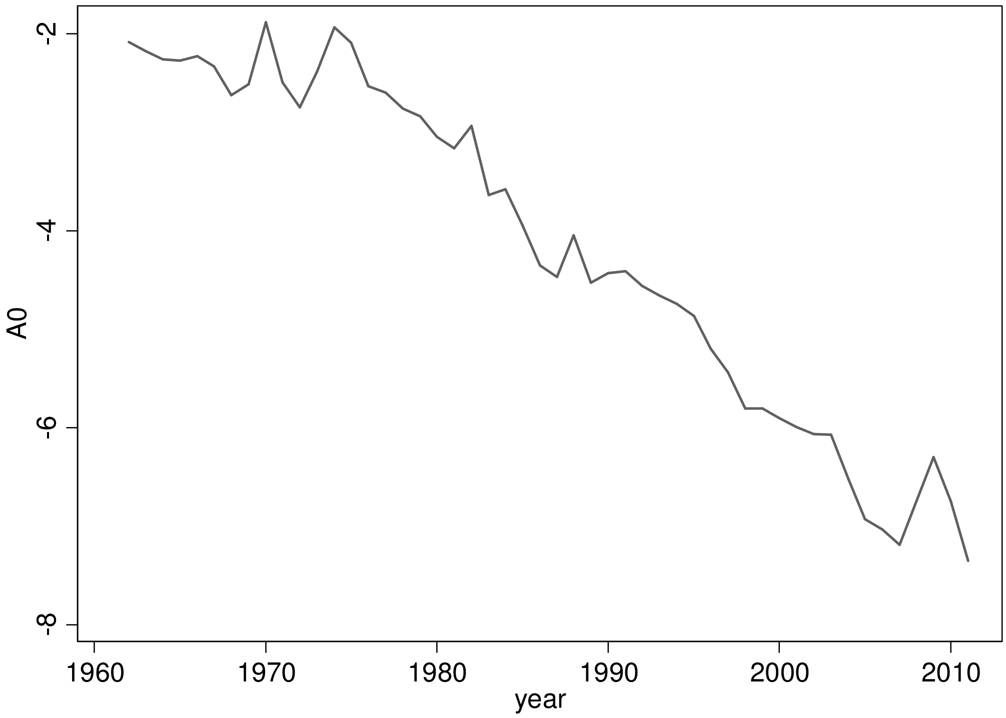

Figure 2: Path of Yearly first-differenced log-transformed annual Amihud (2002) - original measure (DELTAlnA0).

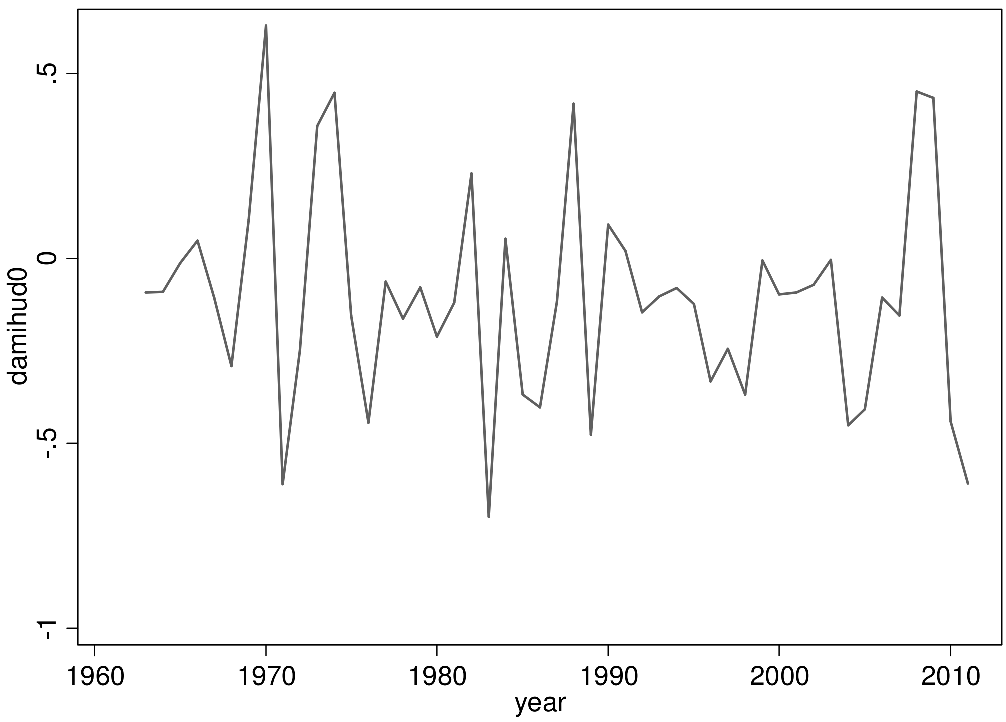

Figure 3: Path of the Yearly HP-filtered log-transformed annual Amihud (2002) - original measure (HP.lnA0).

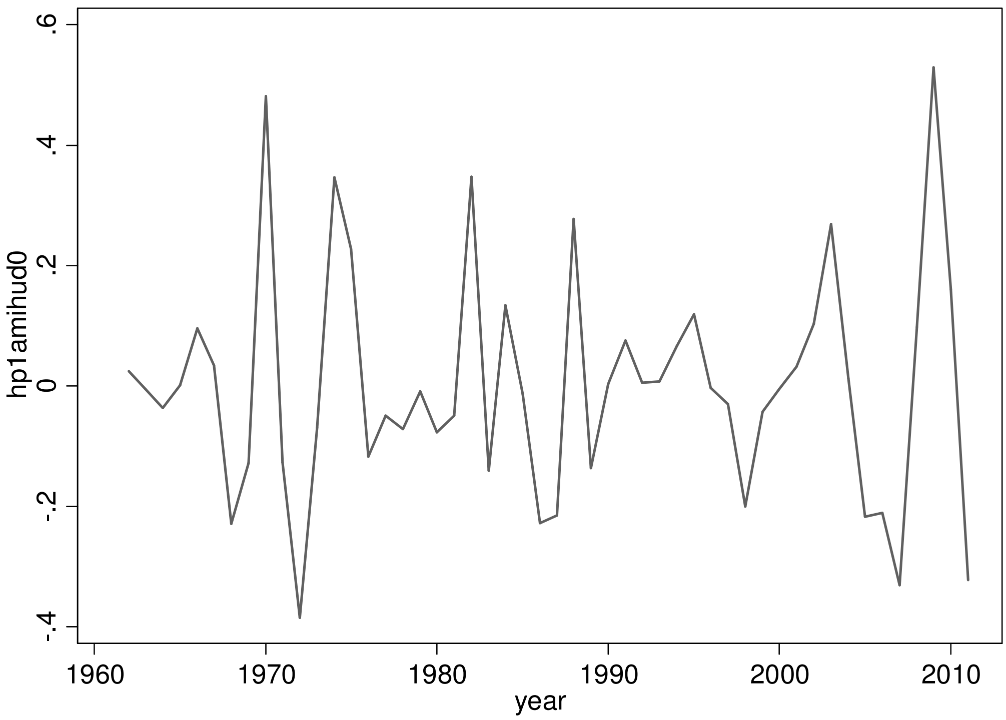

Figure 4: ACF of the Yearly first-differenced log-transformed annual Amihud (2002) - original measure (DELTAlnA0).

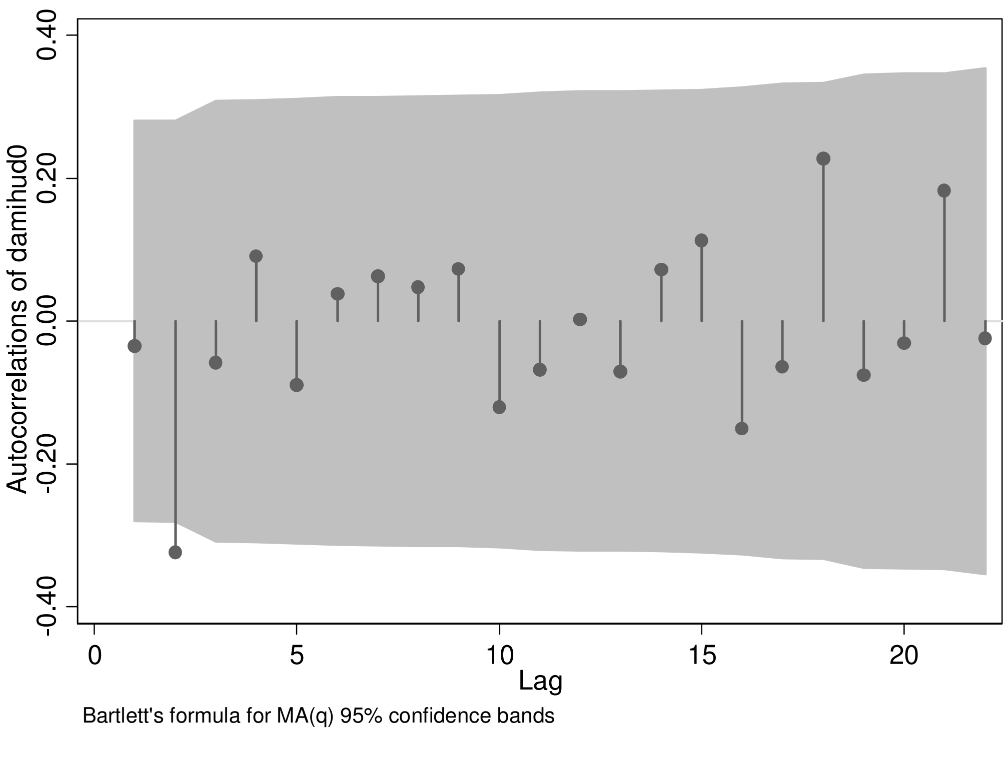

Figure 5: PACF of the Yearly first-differenced log-transformed annual Amihud (2002) - original measure (DELTAlnA0).

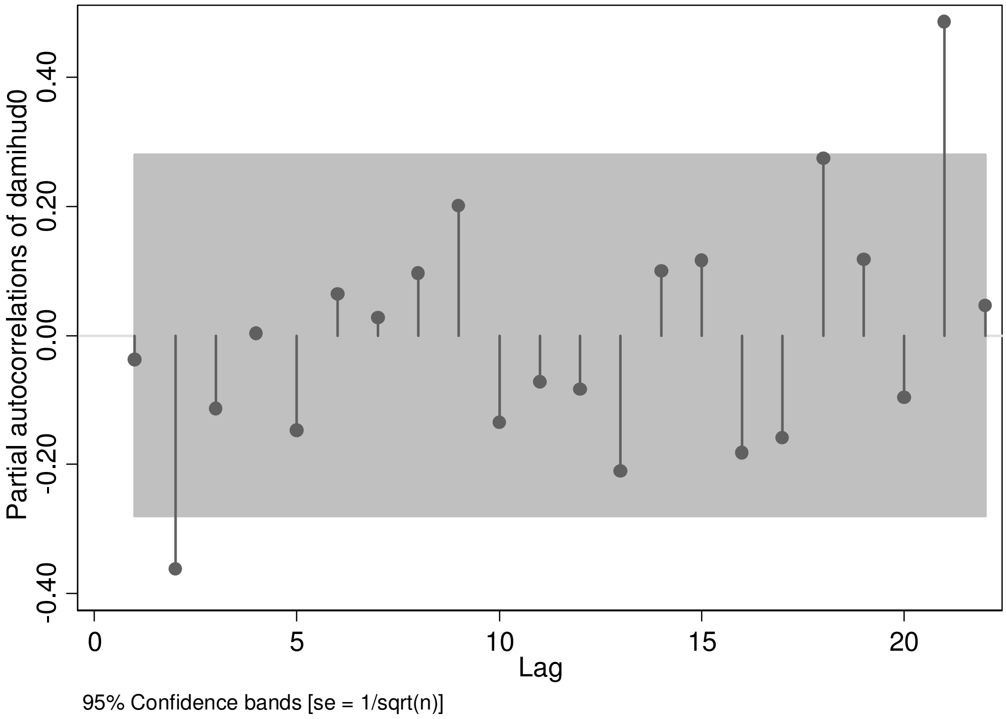

Figure 6: ACF of the Yearly HP-filtered log-transformed annual Amihud (2002) - original measure (HP.lnA0).

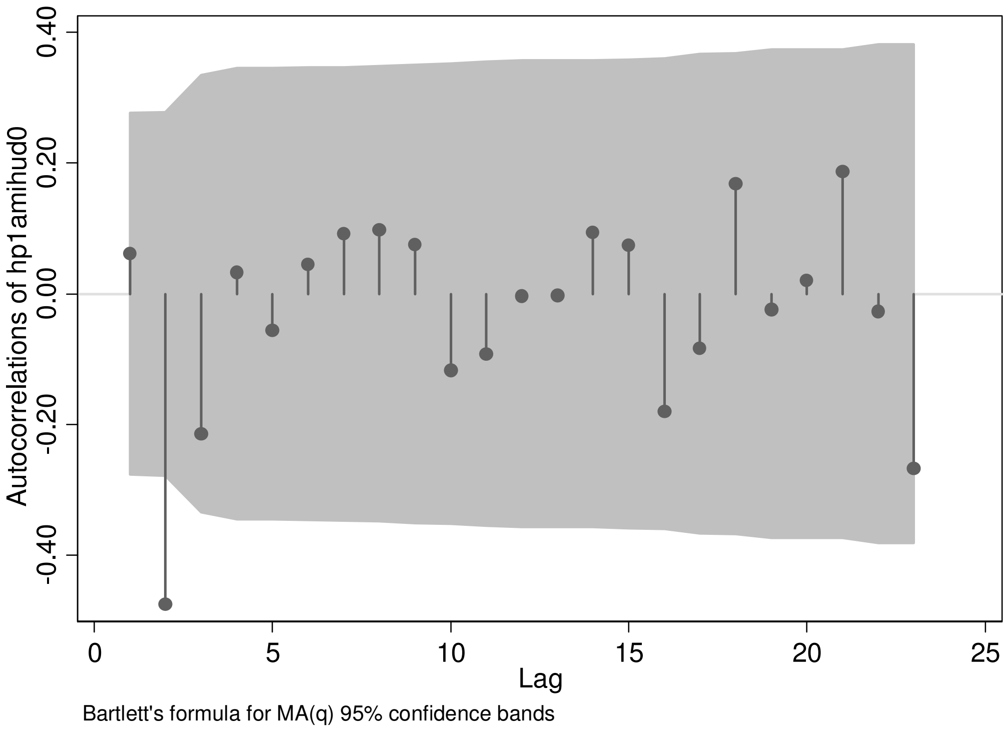

Figure 7: PACF of the Yearly HP-filtered log-transformed annual Amihud (2002) - original measure (HP.lnA0).

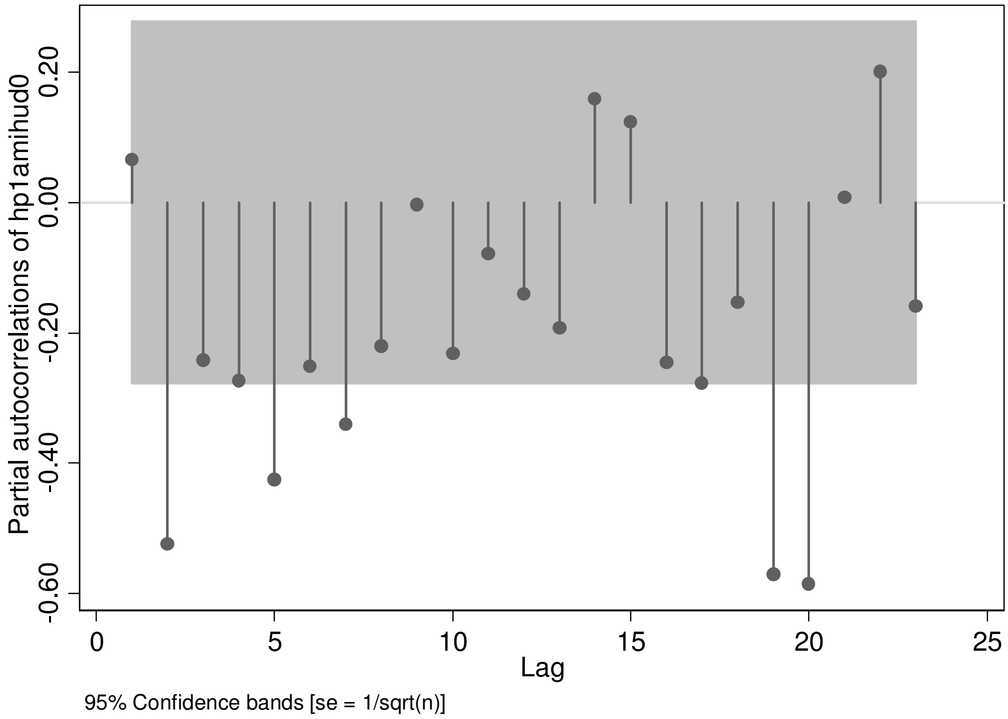

Figure 8: Path of the Yearly log-transformed annual Amihud (2002) – turnover version (lnA).

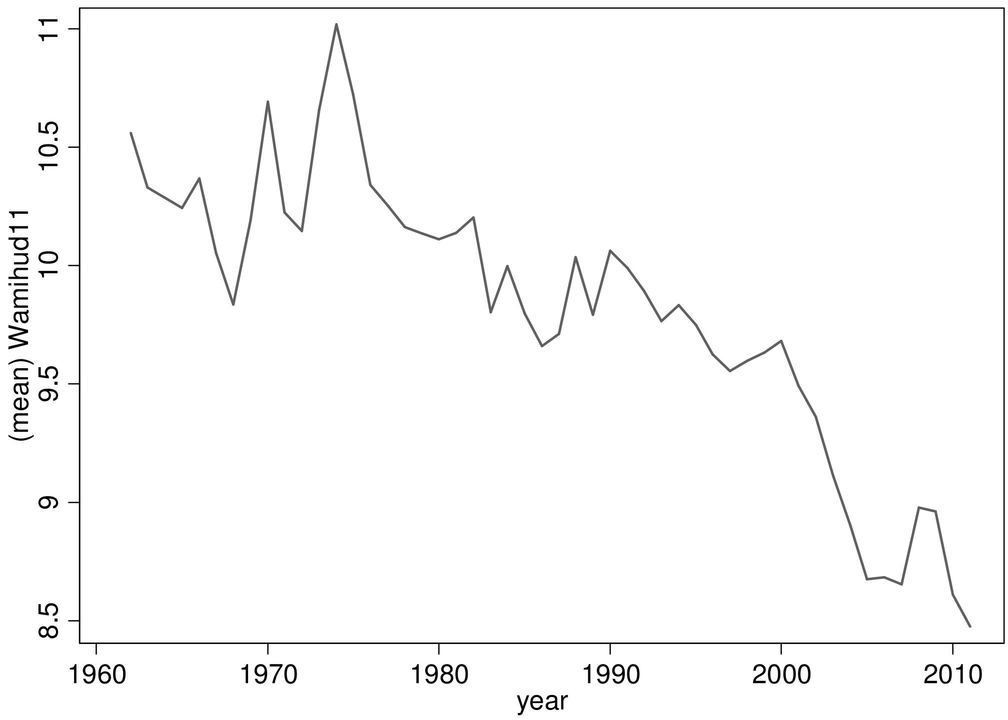

Figure 9: Path of the Yearly log-transformed annual Size (lnS).

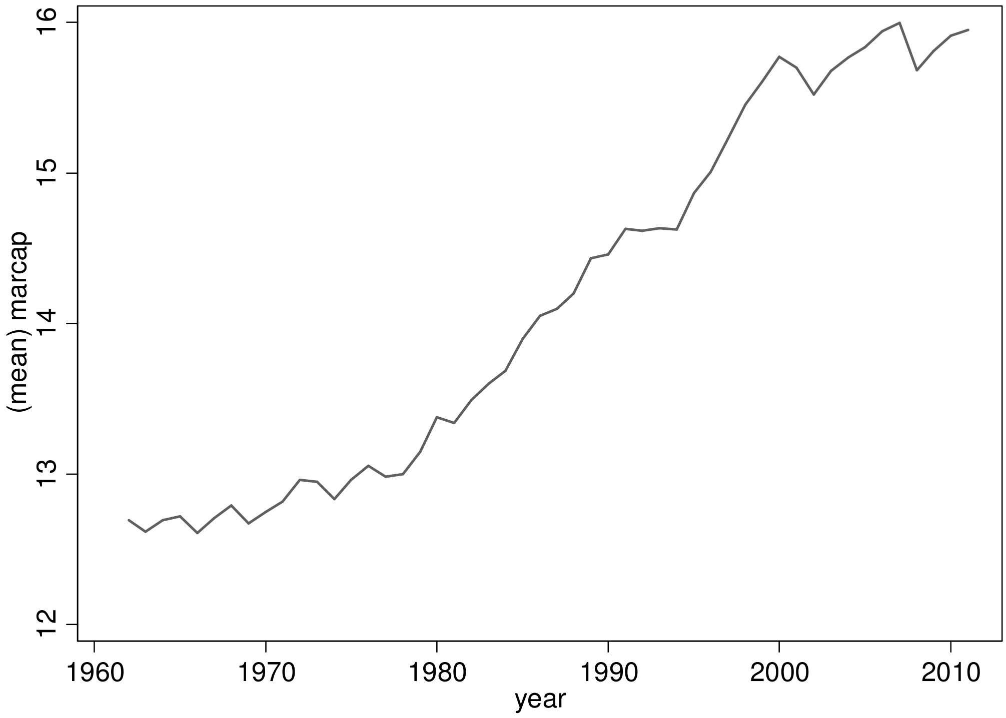

Figure 10: Path of the Yearly VWMR.

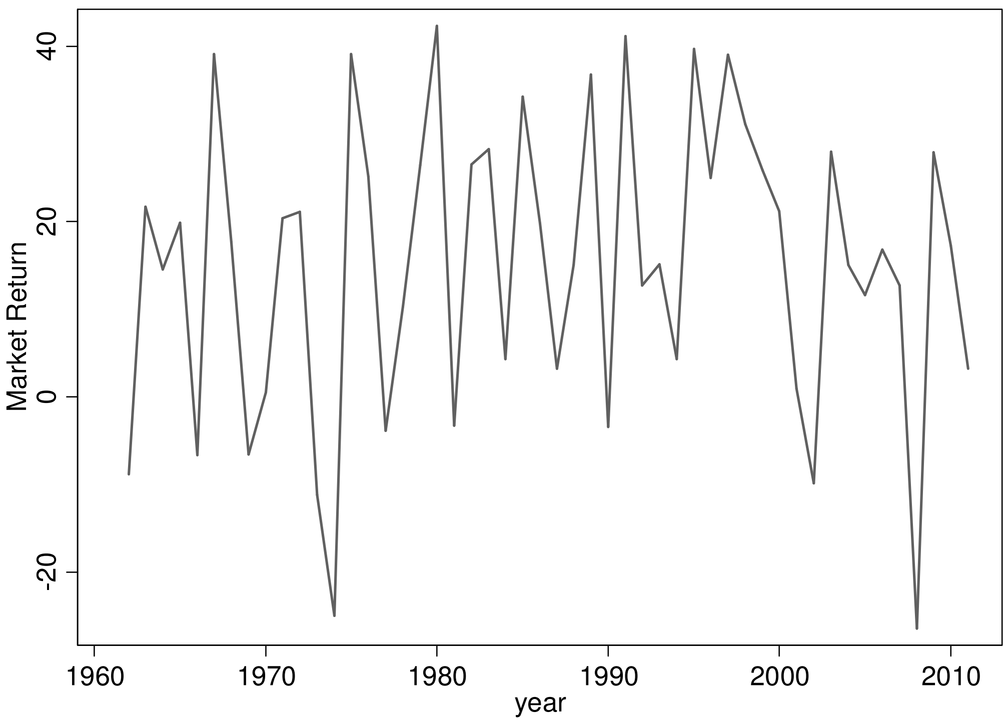

Figure 11: AC of the Yearly VWMR.

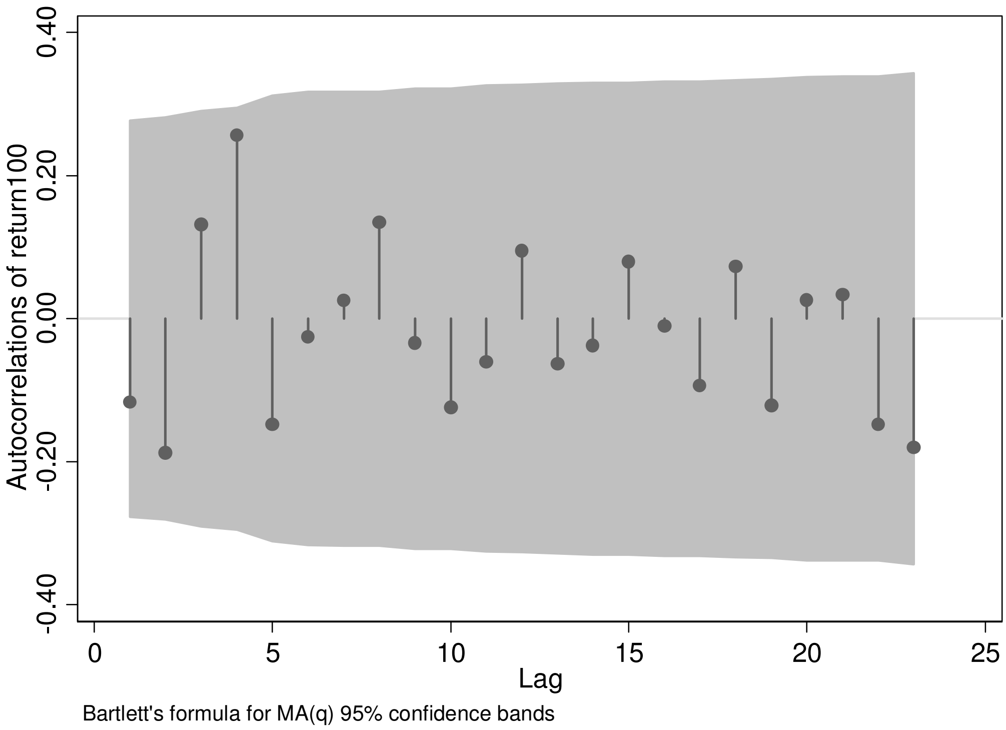

Figure 12: PAC of the Yearly VWMR.

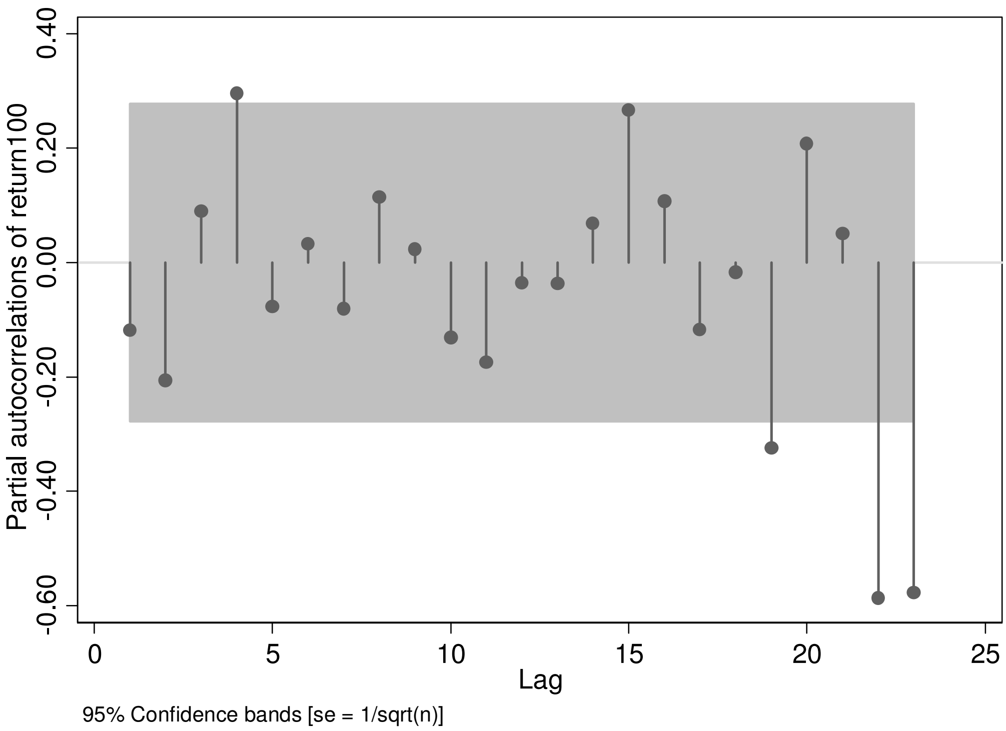

Figure 13: Path of the Monthly log-transformed Amihud (2002) - original measure (lnA0).

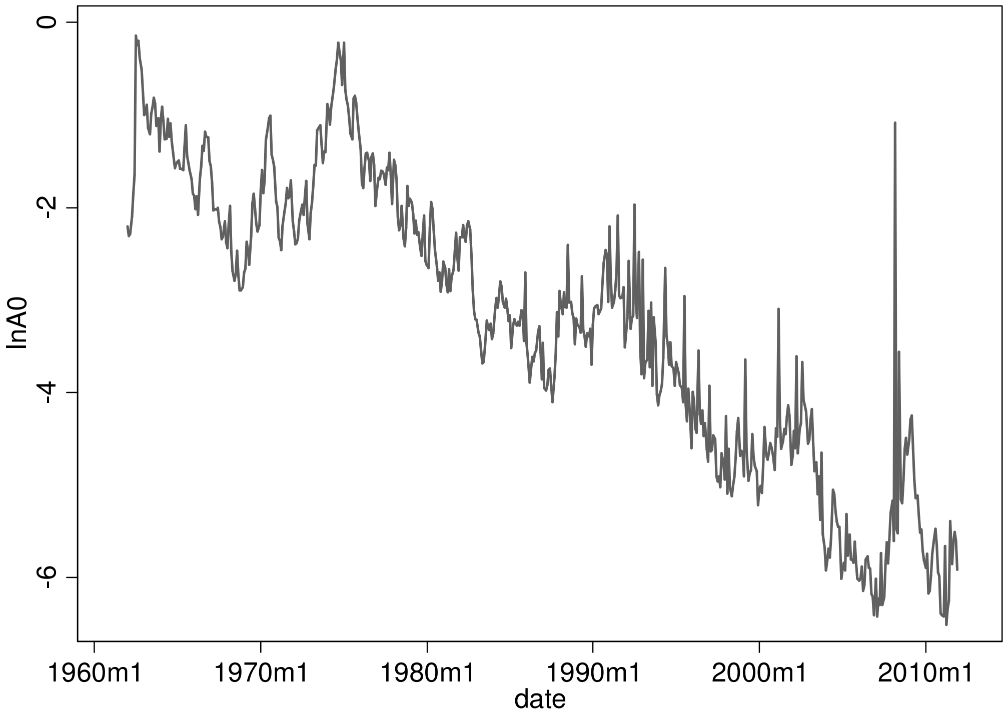

Figure 14: Path of the Monthly first-differenced log-transformed Amihud (2002) - original measure (lnA0).

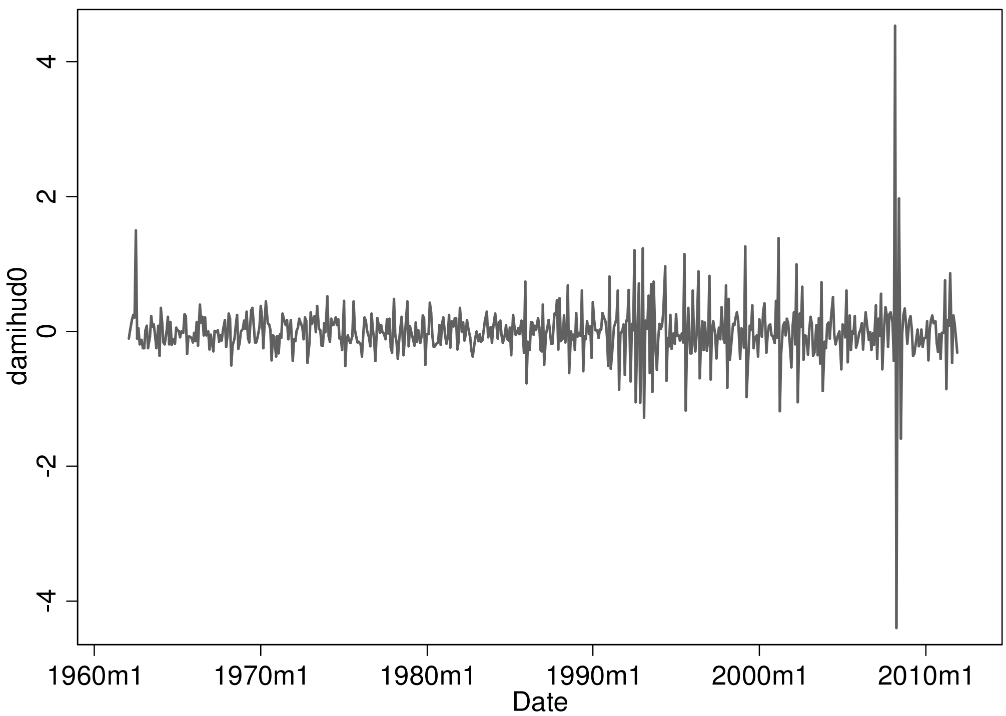

Figure 15: Path of the Monthly HP-filtered log-transformed Amihud (2002) - original measure (lnA0).

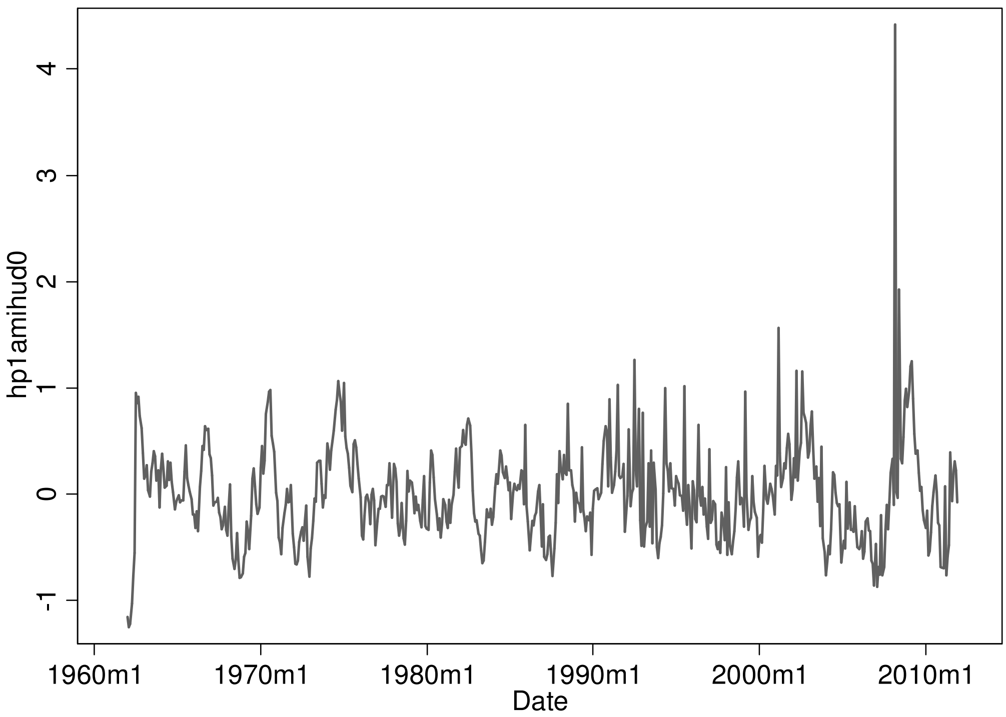

Figure 16: ACF of the Monthly first-differenced log-transformed Amihud (2002) - original measure (DELTAlnA0).

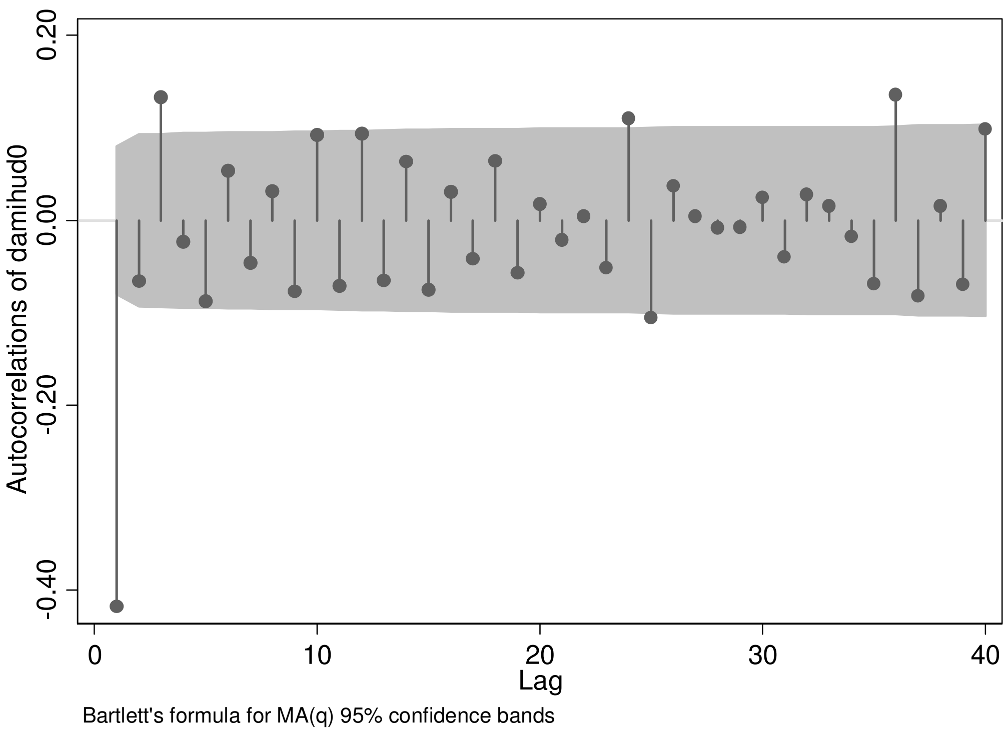

Figure 17: PACF of the Monthly first-differenced log-transformed Amihud (2002) - original measure (HPlnA0).

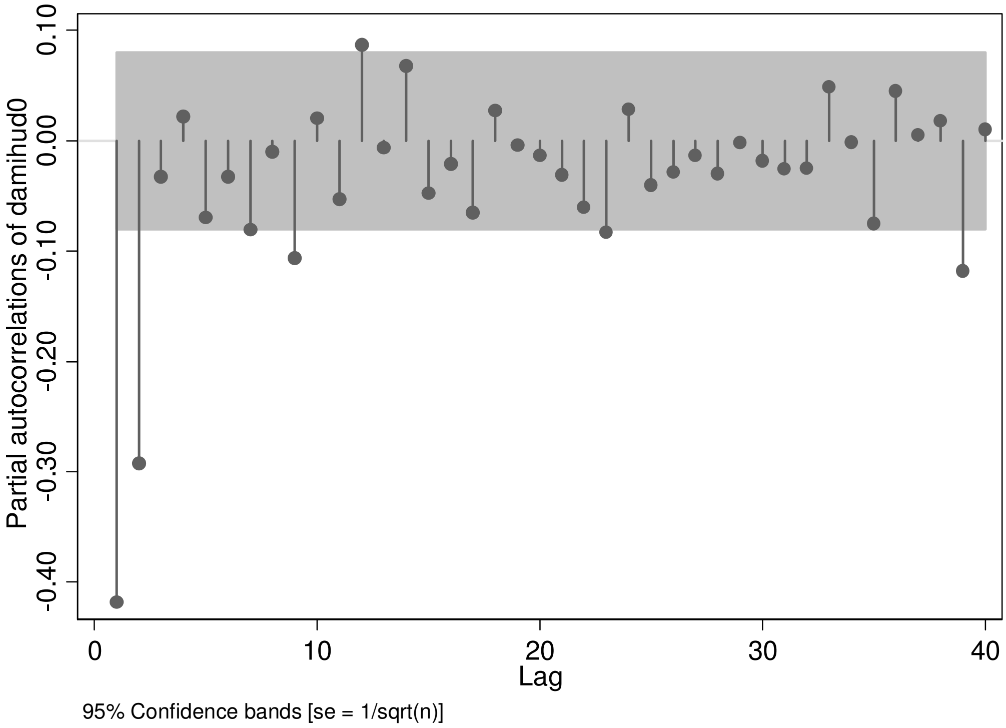

Figure 18: ACF of the Monthly HP-filtered log-transformed Amihud (2002) - original measure (HPlnA0).

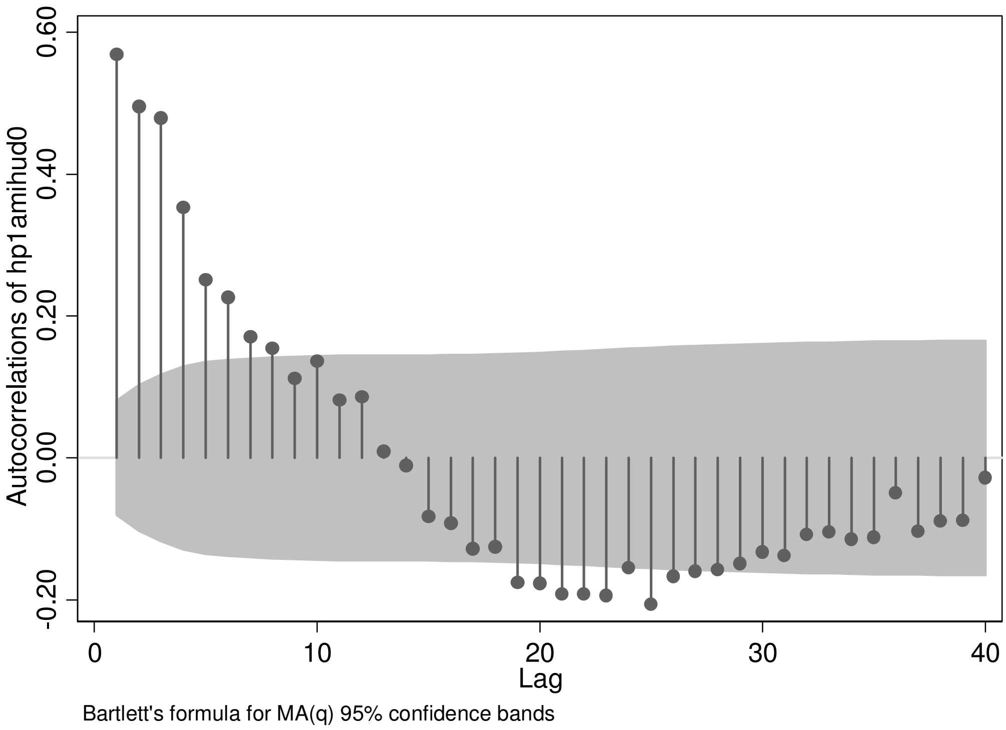

Figure 19: PACF of the Monthly HP-filtered log-transformed Amihud (2002) - original measure (HPlnA0).

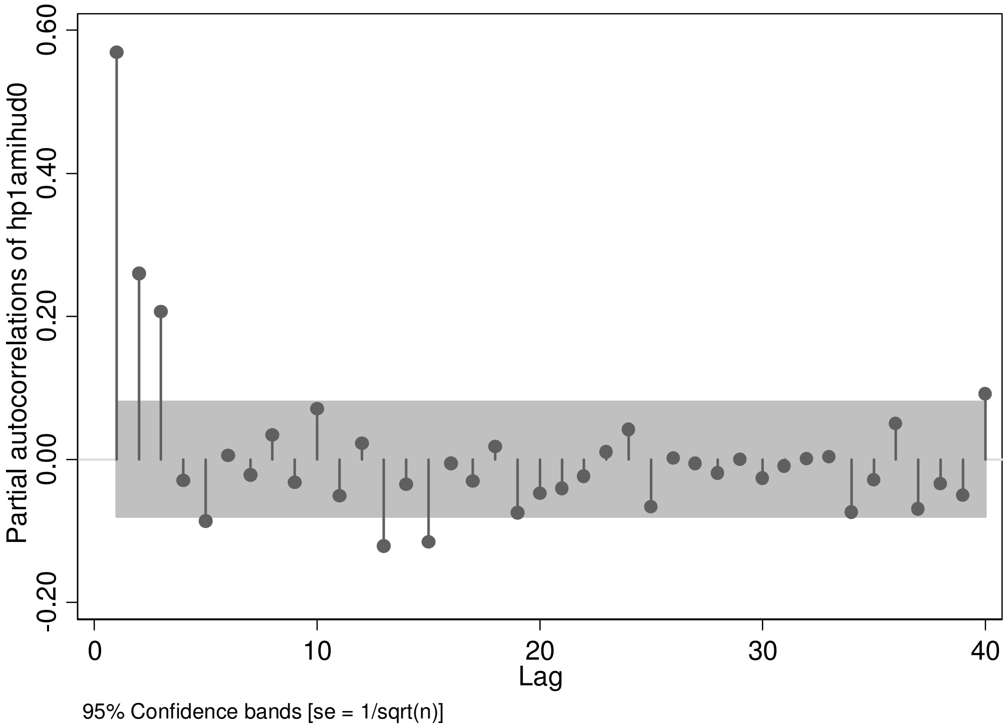

Figure 20: Path of the Monthly log-transformed turnover version of Amihud (2002) – turnover version (lnA).

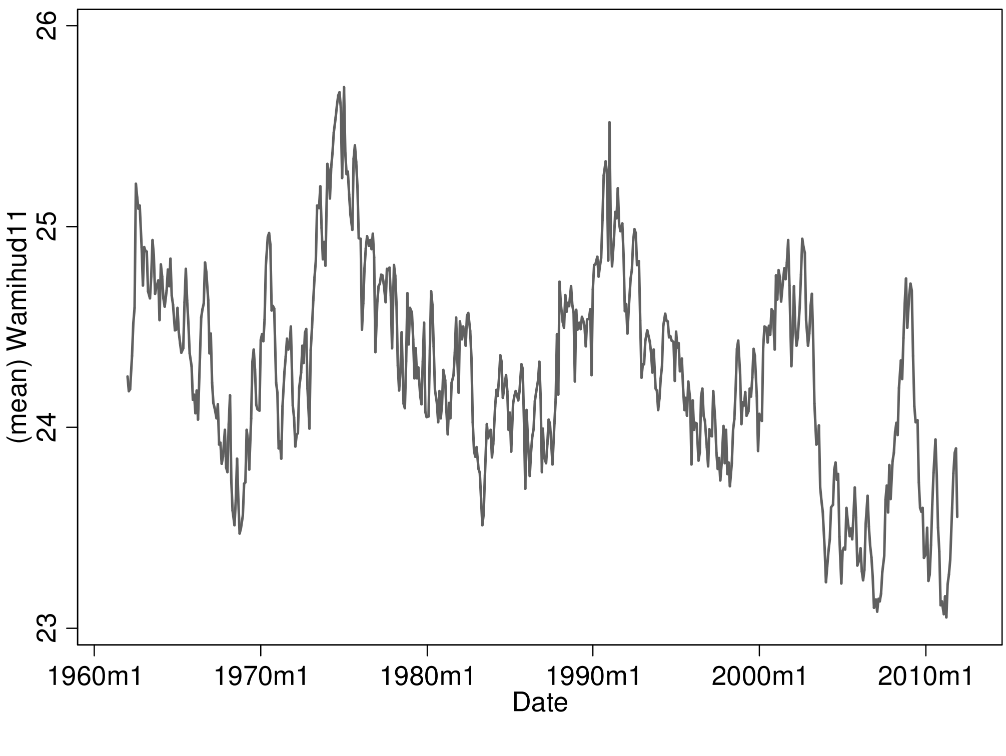

Figure 21: ACF of the Monthly log-transformed Amihud (2002) – turnover version (lnA).

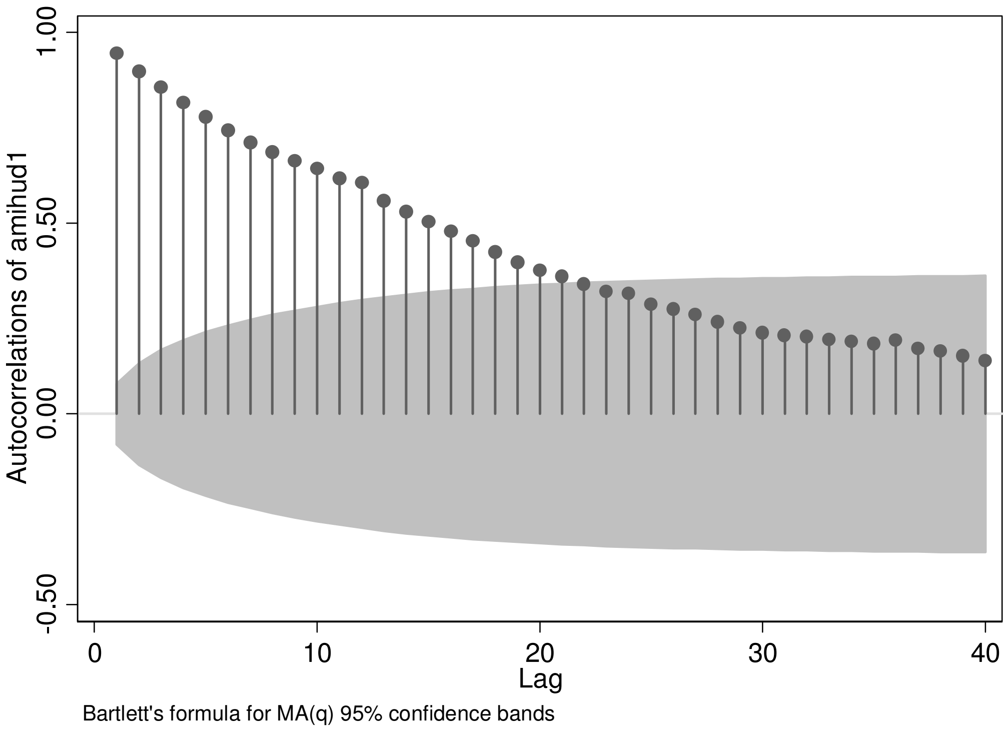

Figure 22: Path of the Monthly log-transformed Size

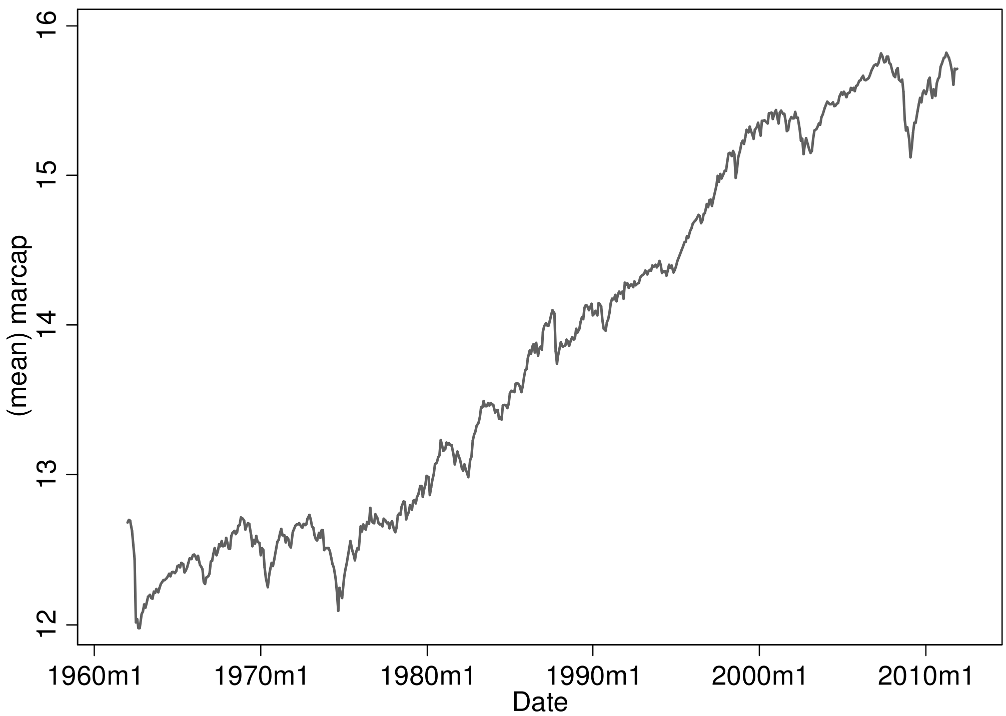

Figure 23: AC of the Monthly log-transformed monthly market return

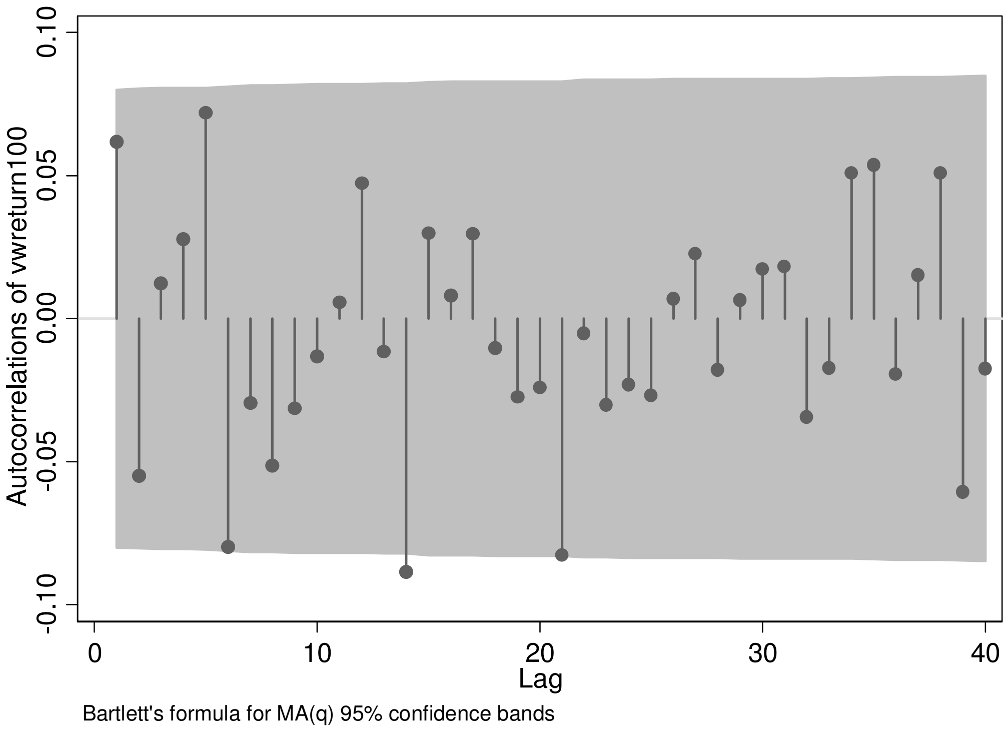

Figure 24: AC of the Monthly log-transformed market return in excess of the risk free rate

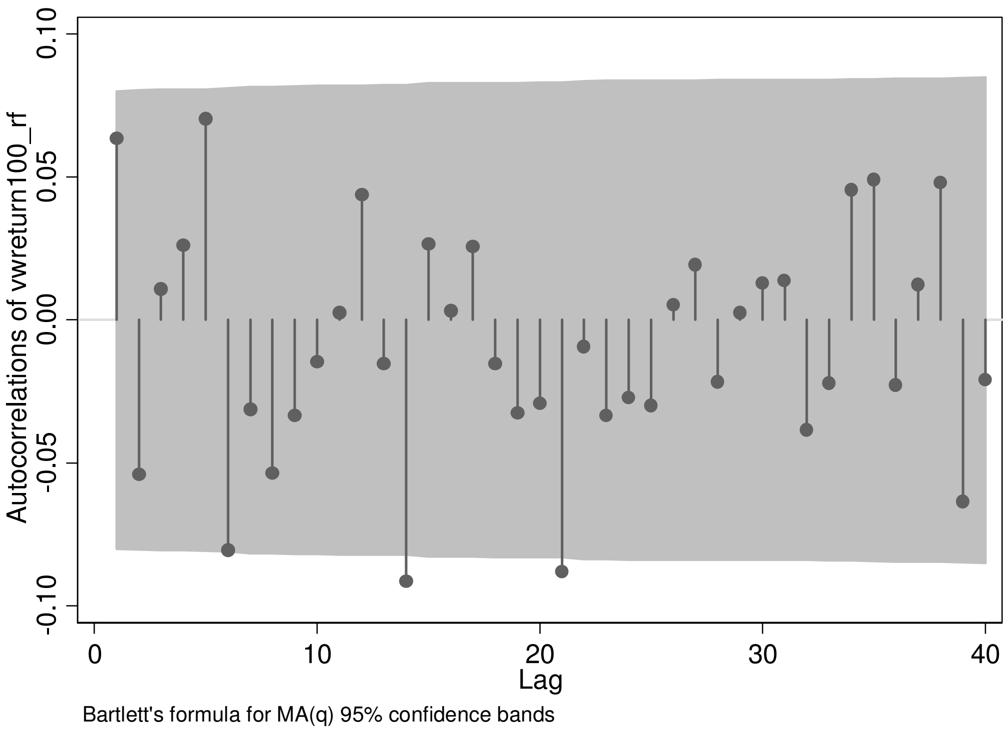

Figure 25: AC of the Monthly log-transformed market return adjusted for the FF3

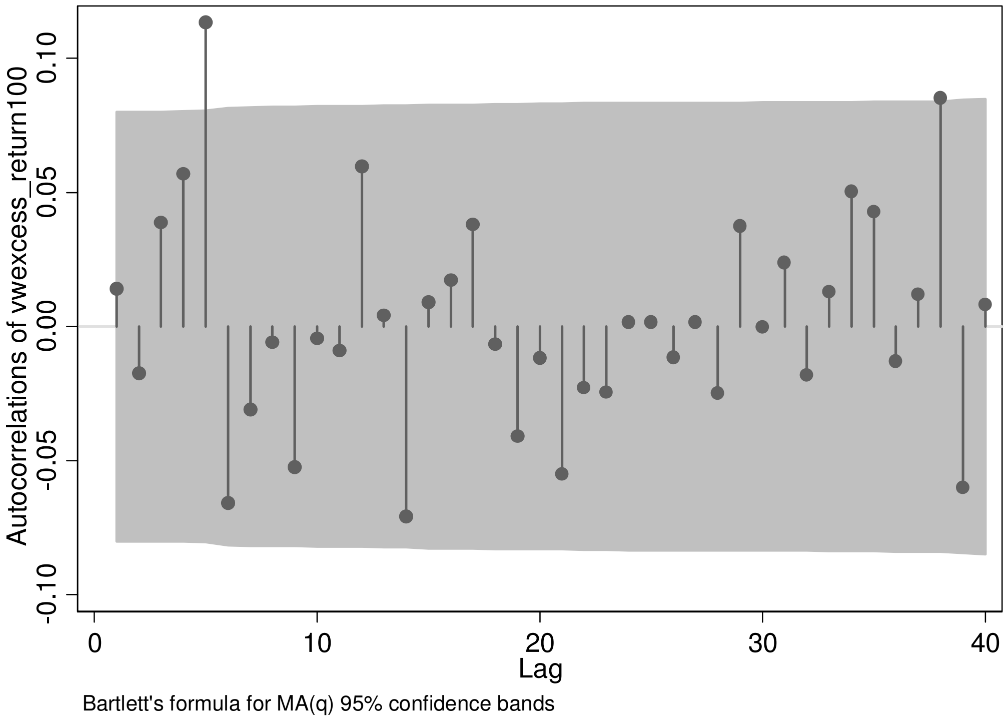

Figure 22: Path of the Monthly log-transformed Amihud (2002) – turnover version (lnA) – and the trend calculated using HP-procedure.

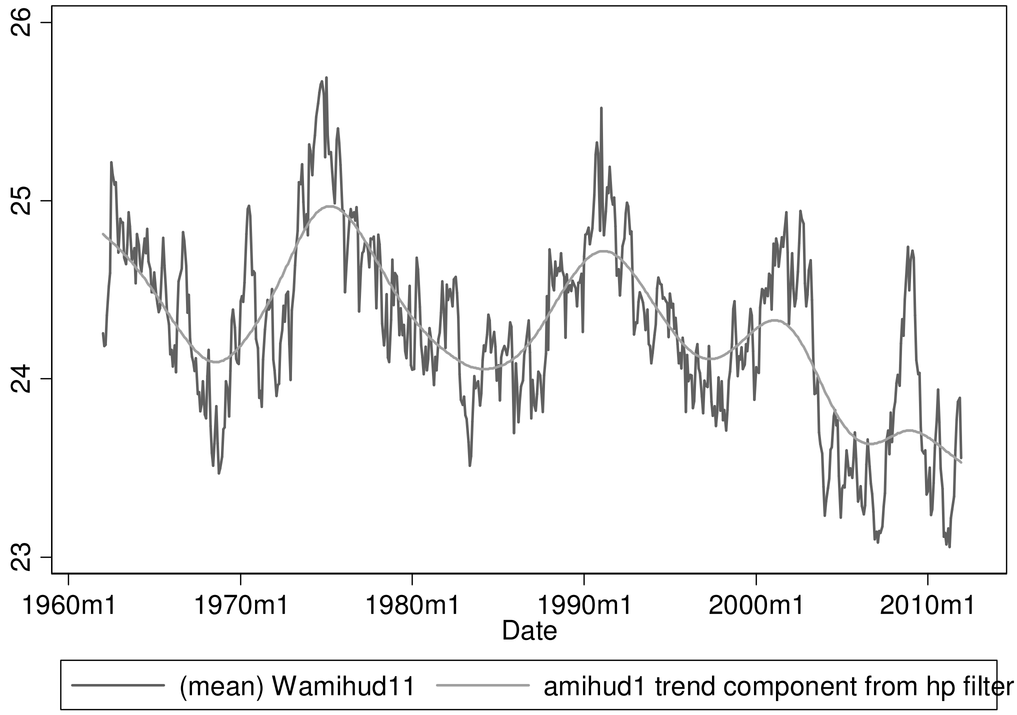

Figure 23: Path of the Monthly value weighted log-transformed monthly Amihud (2002) – turnover version (lnA)

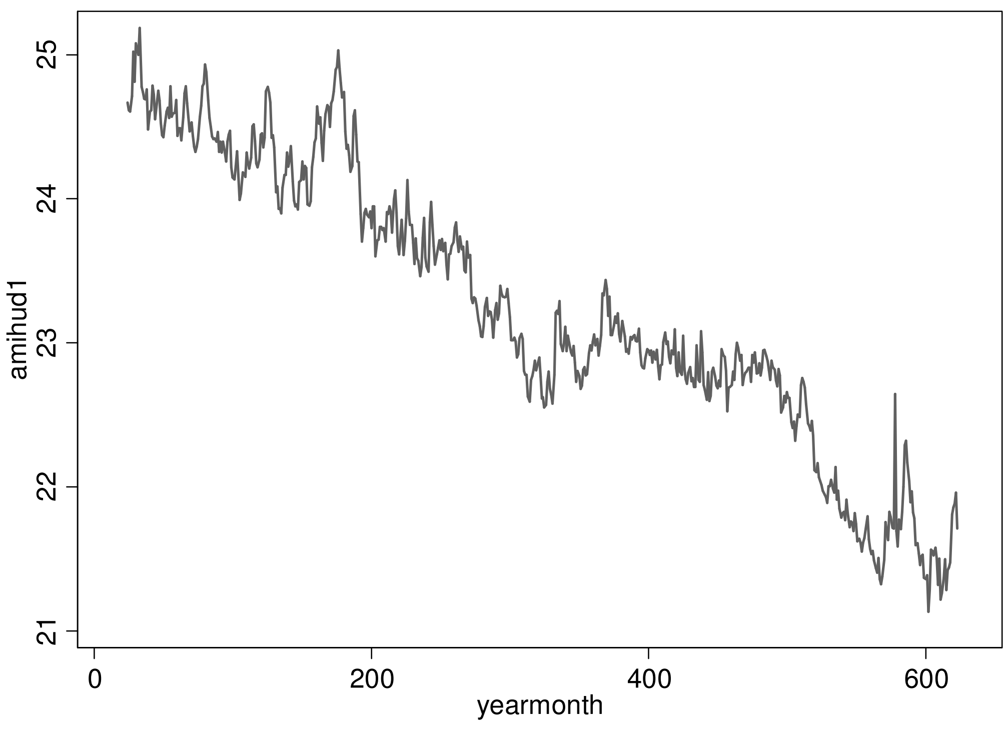

Figure 24: Path of the Monthly first differenced lnA

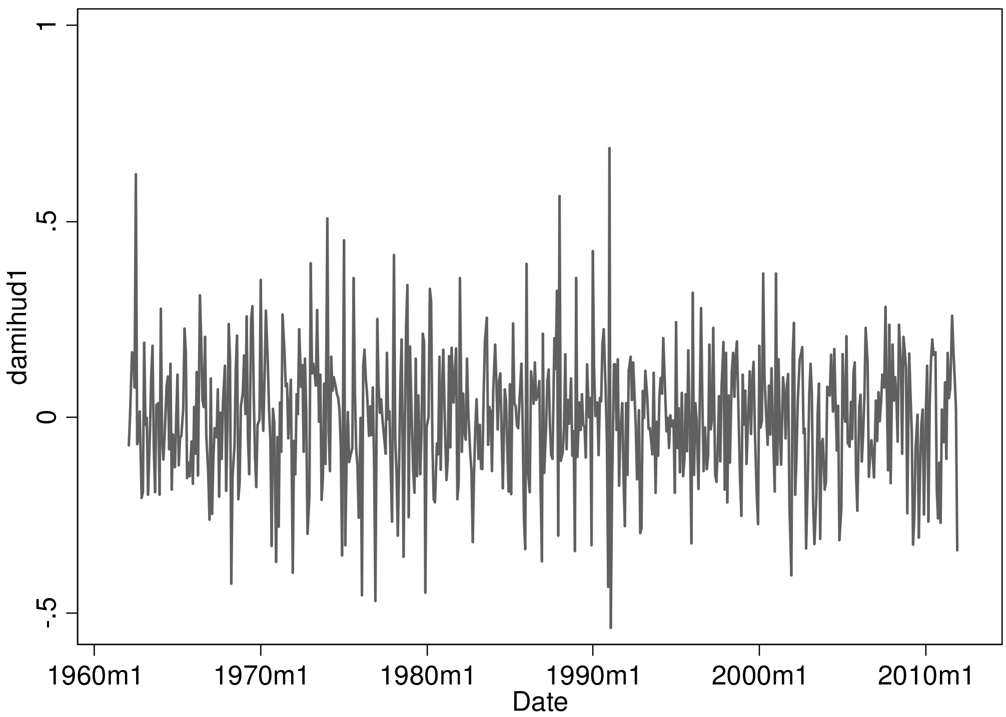

Figure 25: Path of the Monthly HP-filtered lnA

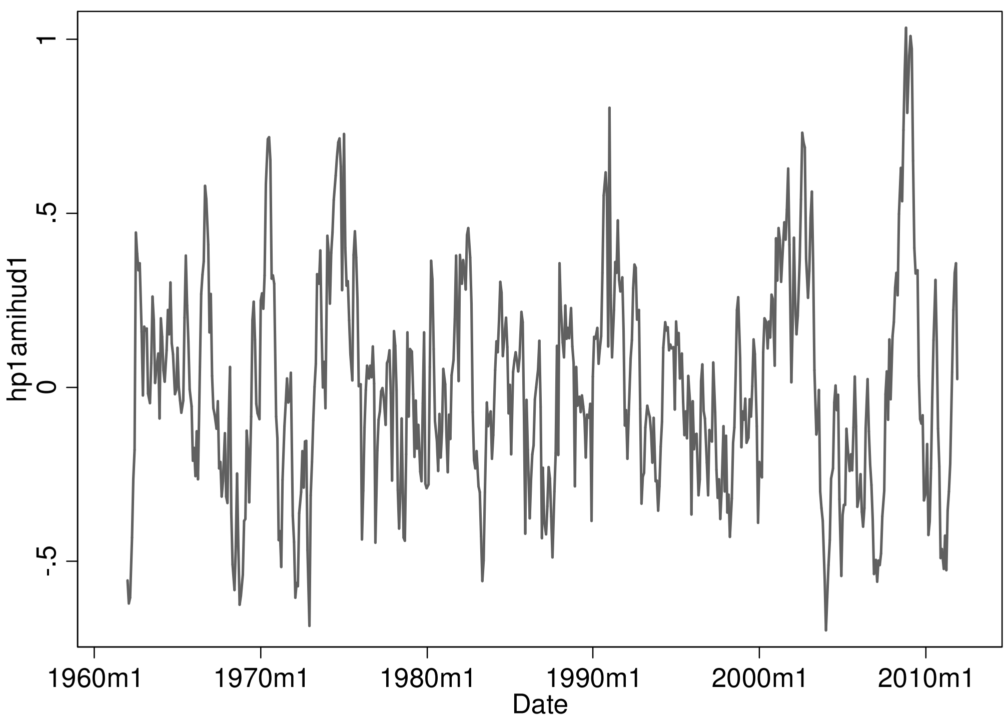

Figure 26: Cross-correlogram of Monthly VWMR and VWMI

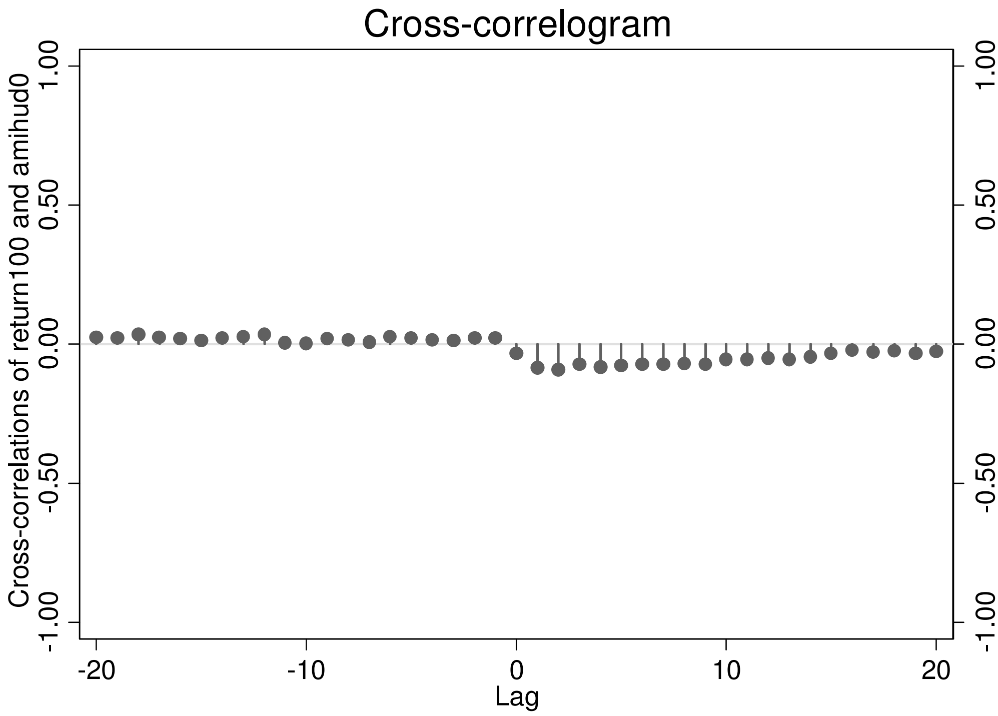

Figure 26: Cross-correlogram of Monthly VWMR and first-differenced VWMI

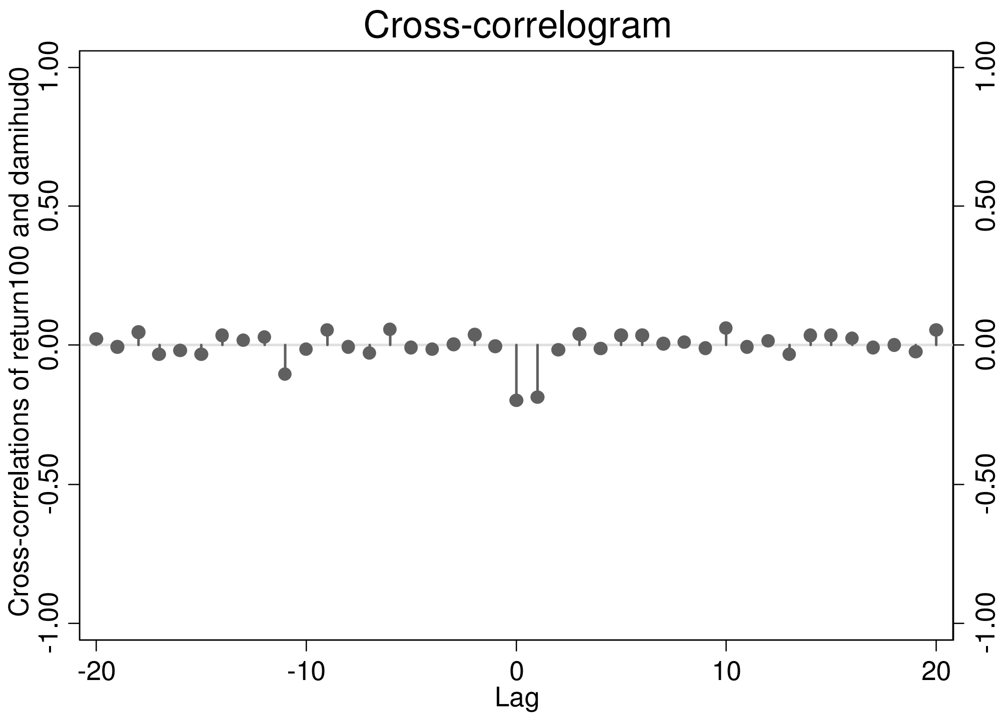

Figure 27: Cross-correlogram of Monthly VWMR and first-differenced VWMI

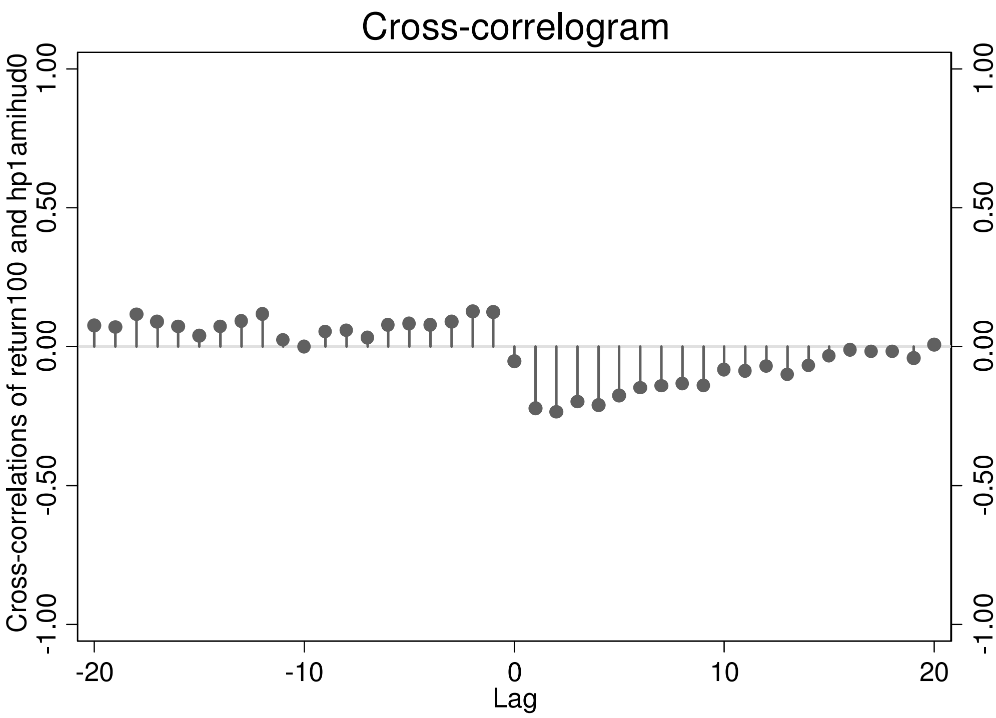

Figure 28: Yearly - Impulse-response function – response of VWMR to an impulse in lnA0 – Short-run restriction

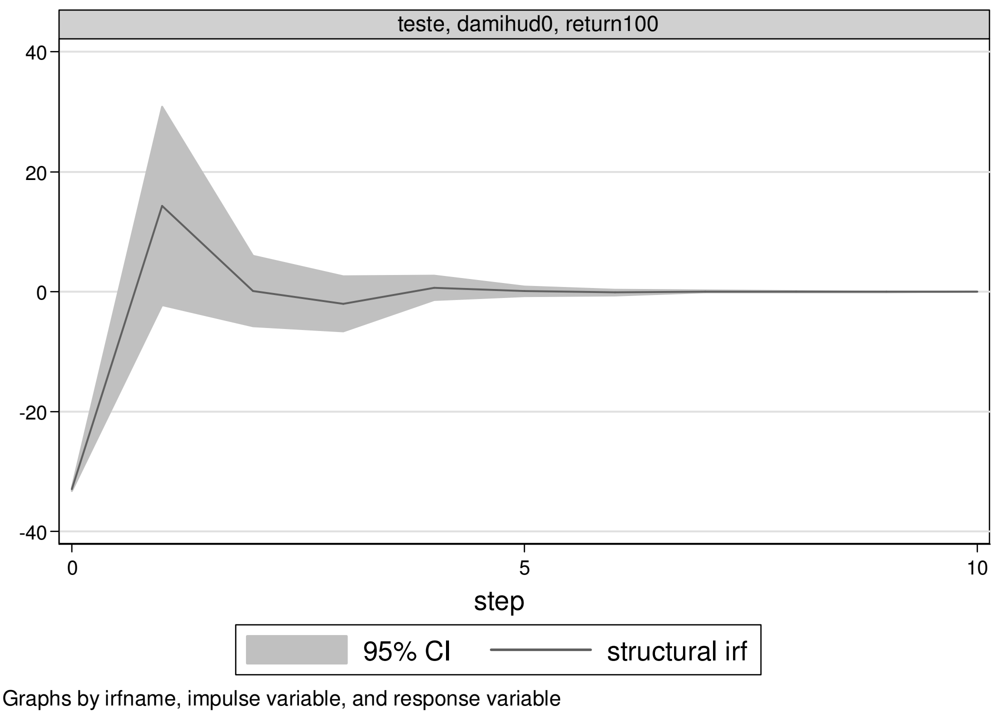

Figure 29: Yearly - Impulse-response function – response of VWMR inj excess of the risk-free rate to an impulse in lnA0 – Short-run restriction

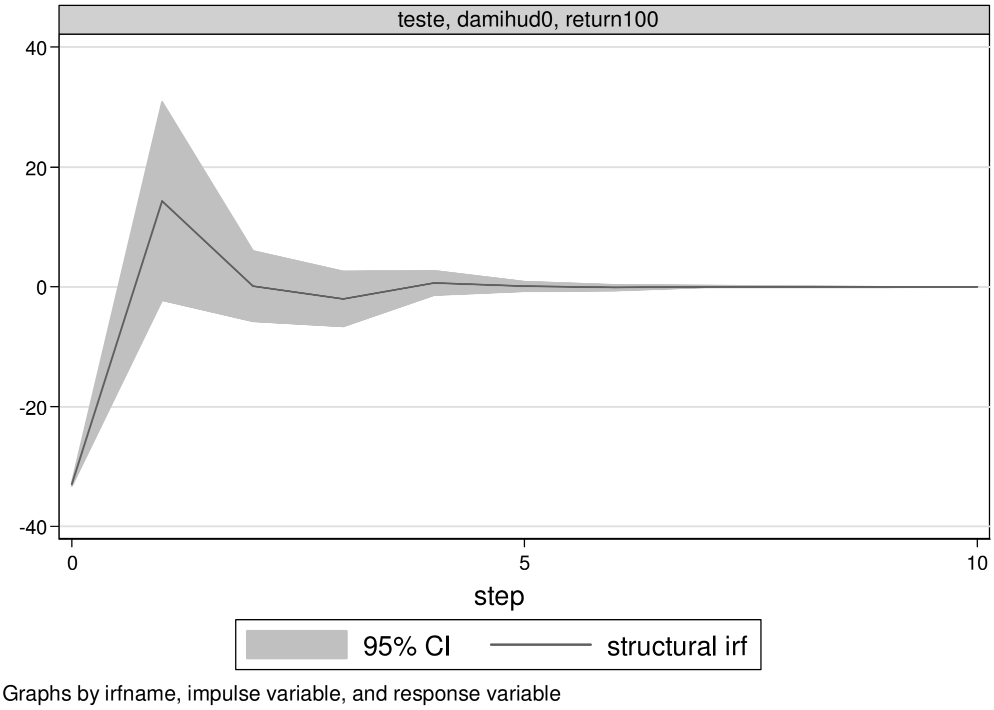

Figure 30: Yearly - Impulse-response function – response of VWMR to an impulse in lnA0 – Long-run restriction

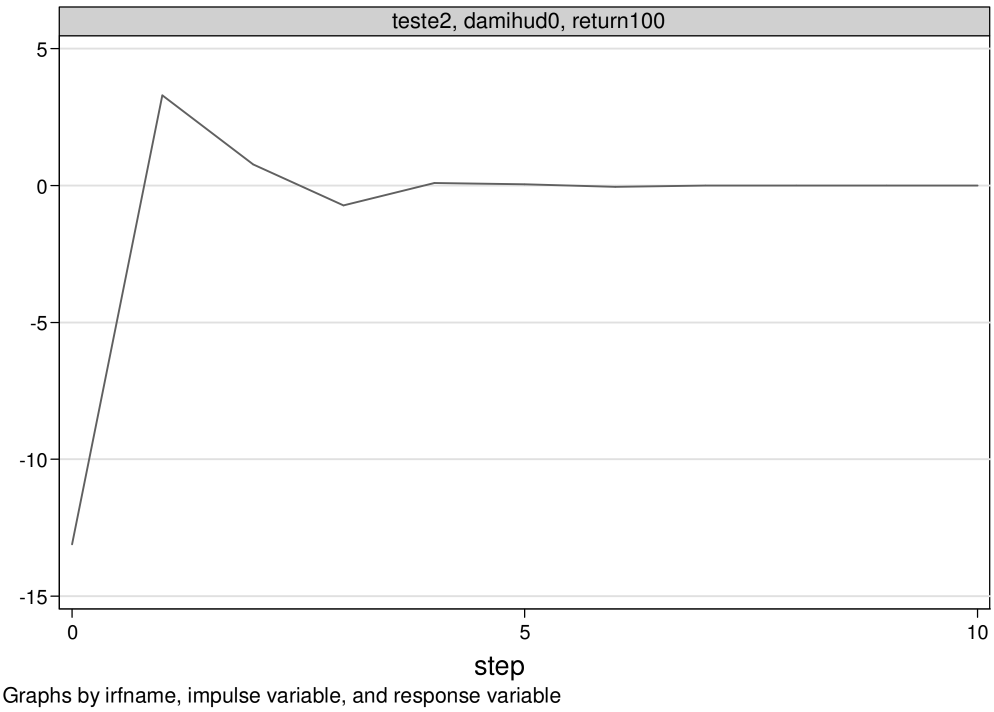

Figure 31: Monthly - Impulse-response function – response of VWMR to an impulse in lnA0 – Short-run restriction

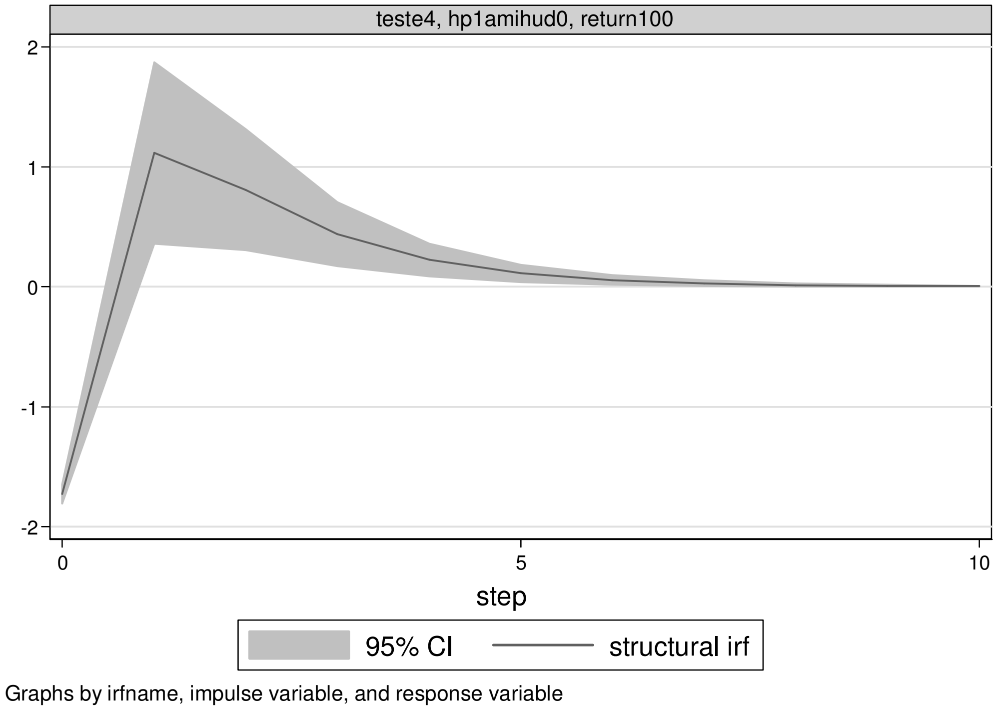

Figure 32: Monthly - Impulse-response function – response of VWMR to an impulse in lnA0 – Long-run restriction