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**Arthur Galego Mendes**

# Central Bank Balance Sheet Concerns and Credible Optimal Escape from Liquidity Trap

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GETULIO VARGAS

**ARTHUR GALEGO MENDES**

**C ENTRAL BANK BALANCE SHEET CONCERNS AND CREDIBLE ESCAPE FROM THE LIQUIDIT TRAP .**

Dissertação apresentada ao Curso de Mestrado em Economia da Escola de Pós-Graduação em Economia para obtenção do grau de Mestre em Economia.

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**ASSINATURA DOS MEMBROS DA BANCA EXAMINADORA**

**Tiago Couto Berriel**
Orientador (a)

**Marco Antonio Cesar Bonomo**

**Eduardo Zilberman**

*Dedicatória...*

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**Abstract**

We show that when a central bank is financially independent from the treasury and has balance sheet concerns, an increase in the size or a change in the composition in the composition of the central bank's balance sheet (quantitative easing) can serve as a commitment device in a liquidity trap scenario. In particular, when the short-term interest rate is up against the zero lower bound, an open market operation by the central bank that involves purchases of long-term bonds can help mitigate the deflation and a large negative output gap under a discretionary equilibrium. This is because such an open market operation provides an incentive to the central bank to keep interest rates low in future in order to avoid losses in its balance sheet.

# 1 Introduction

Arbitrage between money and bonds prevents the nominal interest rate from becoming negative, an economic concept often refered to as the "zero lower bound" of nominal interest rates. When nominal rates have reached this bound, the quantity of money becomes irrelevant because money and bonds are essentially perfect substitutes and conventional monetary/interest rate policy is ineffective. Such a situation, also known as the "liquidity trap," for long was simply a matter of theoretical concern. After Japan‘s late 1980s economic slump and the outbreak of the 2008 and 2009 financial crisis however, this topic has been brought to the center of the economic research and debate.

In particular, when an economy is in a liquidity trap, in models with sticky prices, a large deflation and negative output gap can occur as real interest rates are much higher than their flexible-price counterparts. Since convenational monetary policy runs out of ammunition during a liquidity trap, it is therefore of major interest to investigate what other policy options remain for the central bank. Several important paper, such as Krugam (1998) and Eggertsson and Woodford (2003) have shown that if central bankers can manage private-sector expectations, they can generate as much expected future inflation as necessary to bring real interest rates down, even though nominal rates are bounded to non-negative values and hence provide the necessary incentive for further spending. In this sense, mamanging a liquidity trap fundamentally involves a credibility problem: if central bankers can commit to being irresponsible - that is, convince the market that it will in fact allow prices to rise sufficiently in the future - it will succeed to fight current deflation and negative output gap. Nonetheless, because of the time-inconsistent nature of this policy, achieving such credibility is not easy, since central bankers normally see themselves as defenders against rather than promoters of inflation, and might reasonably be expected to renege on its promise at the first opportunity. Krugman (1998) and Eggertsson (2006) model these issues in standard monetary models.

In this paper we show how shifts in the portfolio of the central bank, so called "quantitatice easing," could be of value in making credible to the private sector the central bank's own commitment to a particular kind of future policy. The central bank in our model is financially independent from the treasury and has concerns about its capital. In this environment, a purchase of long-term nominal bonds by the central bank acts a commitment device as it provides it with an incentive to keep the short-term interest rate low in future as failure to do so would result in losses in its balance sheet.

Our paper is directly related to the literature that assumes balance sheet concerns on the part of the central bank, such as Sims (2005), Berriel and Bhattarai (2009), and Park (2012).

The most closely related work is Jeanne and Svensson (2007), who showed that if the central bank suffer losses when its net worth falls under a fixed level, thet it is possible to create a commitment mechanism that allows an independent central bank to commit to a higher future price level through a current currency depreciation. This mechanism, however, crucially relies on the assumption of a small open economy, and is perhaps not so relevant to the economy of the United States.

# 2 Model

We consider a closed economy. Households consume and save buying riskless claims on government debt. The central bank conducts monetary policy by minimizing a standard quadratic loss function of inflation and the output gap. We show that the economy fall into a liquidity trap with excessively low output and inflation as a result of an unanticipated fall in expected productivity growth and a related fall in the natural interest rate as in Jeanne and Svensson (AER, 2007).

## 2.1 Structure of the Economy

Time is separated into discrete periods, $t = ..., -1, 0, 1, ....$ The economy has a private sector, consisting of a household and firms, and a public sector, consisting of a central bank and a government. The household consumes and saves according to the utility function:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + U\left(\frac{M_{t+i}}{P_{t+i}}\right) - \frac{N_{t+i}^{1+\varphi}}{1+\varphi} \right] \qquad \sigma, \varphi > 0$$

where $\mathbb{E}_t$ denotes expectation conditional on information available in period $t$, $\beta$ is the discount factor, $C_t$ denotes consumption of the consumption good in period $t$, $M_t$ denotes the household's holding of currency, $P_t$ denotes the aggregate price level, $N_t$ denotes supply of labor and $\sigma$ is the coefficient of risk aversion. The function $U(M_t/P_t)$ represents the liquidity services of real money, which consist of time saved in the transactions of the consumption good.

The liquidity services function is continuous and continuously differentiable for $M_t/P_t > 0$ and has additional properties $U(M_t/P_t) < U_0$; $U'(M_t/P_t) > 0$; $U"(M_t/P_t) < 0$ for $M_t/P_t < e^{\delta}$; $U(M_t/P_t) = U_0$ for $M_t/P_t \geq e^{\delta}$ and $U'(M_t/P_t) \to 0$, where $e^{\delta} > 0$ is the satiation level os real money. That is, $U(M_t/P_t)$ is increasing at a decreasing rate for $M_t/P_t < e^{\delta}$ and has a maximum equal to $U_0$ for $M_t/P_t \geq e^{\delta}$. There is a positive demand for real balances regardless of how high the interest rate is.

The consumption good, $C_t$, is a Dixit-Stiglitz composite of a infinity of varieties of mass one, each of them produced by a monopolist firm.

$$C_t \equiv \left[ \int_0^1 c_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\varepsilon$ is the the consumer's elasticity of substitution over the varieties.

The corresponding price index satisfies:

$$P_t = \left[ \int_0^1 p_{jt}^{1-\xi} dj \right]^{\frac{1}{1-\xi}}$$

In the stage, each intermediate $j$ is produced by a single firm $j$ with a technology that is linear in labor input with a exogenous stochastic process $A_t$:

$$Y_t(j) = A_t N_t(j)$$

where $N_t(j)$ denotes labor input in the production of intermediate good $j$. There is hence a continuum of firms producing intermediate goods. Aggregate labor supply and demand will be given by,

$$N_t \equiv \int_0^1 N_t(j) dj$$

Prices are set as in Calvo (1983). Every period a firm is able to revise its price with probability $1 - \alpha$. The lottery that assigns rights to change prices is $i.i.d$ over time and across firms. Firm j's price, $p(j)$, is chosen so as to maximize the expected utility value of profits.

The budget constraint in period $t$ for the home household is

$$T_t^h + P_t C_t + B_t^{hh,s} + Q_t B_t^{hh} + M_t \leq (1 + i_{i_{t-1}}) B_{t-1}^{hh,s} + (1 + Q_t) B_{t-1}^{hh} + W_t N_t + M_{t-1} + \Pi_t$$

where $B_t^{hh,s}$ and $B_t^{hh}$ are riskless claims on the government debt held by the household. The first, the short-term bond, costs $\frac{1}{1+i_t}$ in period $t$ and pays a dollar in period $t+1$. The second, the long-term bond, costs $Q_t$ dollars in period $t$ and pays a dollar in perpetuity. Here, $T_t^h$ are nominal lump-sum taxes.

We assume a Non-Ponzi condition, where the real value of net wealth of private agent, $NW_t = \frac{M_t}{P_t} + \frac{B_t^{hh,s}}{P_t} + \frac{Q_t B_t^{hh}}{P_t}$ does not become arbitrarily negative. Adding a tranversality condition, we get the boundary condition that the rate of growth of private wealth cannot exceed $\beta^{-1}$:

$$\lim_{i \to \infty} \mathbb{E}_t \beta^i \left[ C_{t+i}^{-\sigma} NW_{t+i} \right] = 0$$

If the public sector model reflects the fact that the central bank's balance sheet is isolated from the balance sheet of fiscal authority, then the budget constraint for the central bank is:

$$\underbrace{(1 + i_{t-1}) B_{t-1}^{cb,s} + (Q_t + 1) B_{t-1}^{cb} - M_{t-1}}_{\equiv K_t} = B_t^{cb,s} + Q_t B_t^{cb} - M_t \tag{1}$$

where $B_t^{cb,s}$ and $B_t^{cb}$ are respectively the total of short and long term bonds held by the central bank. We denote the central bank capital level, $K_t$, as the difference between the current value of its assets and its monetary liabilities as in (1). We omit various accounts such as risky loans to the financial sector in order to keep the model simple.

The fiscal side of the public sector is described by the budget constraint the treasury faces:

$$T_t^h + B_t^s + Q_t B_t = (1 + i_{t-1})B_{t-1}^s + (1 + Q_t)B_{t-1}^l + G_t \tag{2}$$

where $B_t^s$ and $B_t$ are total outstanding treasury debt respectively of short and long maturities, $B_t^s = B_t^{hh,s} + B_t^{cb,s}$ and $B_t = B_t^{hh} + B_t^{cb}$.

Whenever we allow transfers between the two authorities to be perfectly state contingent equations (1) and (2) can be consolidated in a unique public sector budget constraint. In this work we assume the existence of a central bank that is financially independent from the treasury and we attain it as we rule out this transfers. That is, we assume $T_t^h = 0$ for all $t$ [1].

## 2.2 Equilibrium

Imposing market clearing and log-linearizing the first-order-conditions of the firm's and household's problems around the zero-inflation steady state results in the usual system[2]:

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right) \tag{3}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t \tag{4}$$

where $\tilde{y}_t$ is the output gap, defined as $y_t \equiv y_t - y_t^n$, $y_t^n$ is the natural level of output, the log of the flexible-prices output, and $r_t^n$ is the natural rate of interest. Moreover, $\pi_t = log(P_t/P_{t-1})$ is inflation in period t.

Because we are interested in analysing monetary policy around the zero lower bound we add to (3), (4) restrictions (5), (6):

$$m_t - p_t \begin{cases} = h(y_t, i_t) & \text{if } i_t > 0 \\ \geq \delta & \text{if } i_t = 0 \\ \end{cases} \tag{5}$$

$$i_t \geq 0 \tag{6}$$

[1] Any exogenous process for $T_t^h$ could be assumed without any significant change in our findings

[2] $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\alpha\epsilon}\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$

### 2.3 Productivity and Natural Rate of Interest

The dynamics of the natural rate is driven by the exogenous stochastic process for productivity,

$$
\begin{aligned}
r_t^n &= \rho + \sigma \mathbb{E}_t \Delta y_{t+1}^n \\
&= \rho + \sigma \tau_a \mathbb{E}_t \Delta a_{t+1}
\end{aligned}
$$

Jeanne and Svensson (2007) design a simple framework to analyse monetary policy in a liquidity trap. In this framework, the technological process implies that the natural rate of interest is equal to its positive steady state value $\rho$ for all periods except for period 1 when it becomes unexpectedly negative. We keep the framework, with a small change, we allow the natural rate of interest to remain negative one more period, in $t = 2$, with some probability $p$. This simplifies the analysis without substantial restriction of generality, since policies that succeed in extracting the economy from a liquidity trap in periods 1 and 2 can be implemented in others periods, too.

## 3 Monetary Policy

We assume that the central bank has an objective function corresponding to inflation and output targeting augmented by concerns about the central bank balance sheet. The central bank's intertemporal loss function can be written

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j L_{t+j}$$

with the period loss function

$$L_t = \frac{1}{2}\left[\pi_t^2 + \lambda \tilde{y}_t^2\right] + \phi \chi_t \tag{7}$$

The first term in the loss function captures the concerns of the central bank in a zero inflation targeting regime when $\lambda > 0$ is the relative weight on output-gap stabilization.

Strong evidence of central bank balance sheet concerns is reported in Jeanne and Svensson (2007) and Berriel and Bhattarai (2009). Here we follow Jeanne and Svensson (2007) in a simple way of capturing its balance sheet concerns, we assume that the central bank suffers an additional loss $\phi \geq 0$ if its capital suffers a negative variation greater than a given nonnegative level, $\Delta$. We represent this additional loss with the term $\phi \chi_t$ in the loss function (7), where the

indicator variable $\chi_t$ takes values 0 or 1 according to how the central bank capital varies:

$$\chi_t = \begin{cases} 1 & \text{if} \quad k_t - k_{t-1} < -\Delta \\ 0 & \text{otherwise} \end{cases}$$

This innovation in the loss function reflects the conjecture that mismanagement of the central bank capital may require a capital injection of the treasure. The treasure may take advantage of this situation to influence the central bank actions undermining its independence. We assume, furthermore, that the private sector is perfectly aware of the central bank's concern's about the level of their capital.

# 4 Liquidity Trap

In this section, we temporarily assume that the central bank does not have any balance sheet concerns, so $\phi = 0$. We show that, as a result, the economy may fall in a liquidity trap in which the monetary authorities are impotent because their announcements are not credible.

## 4.1 Equilibrium without Liquidity Trap

We will see below that a liquidity trap will arise in each period when the natural interest rate turns out to be too low. This situation occurs in period 1 and perhaps in period 2, in all other periods there is no liquidity trap.

When the economy is out of the liquidity trap, restriction (6) is never binding, and the Phillips curve is the only relevant constraint. In those periods, the central bank optimization problem under discretion is to minimize the period loss function (7) subject to (4) only. This implies the targeting rule:

$$\pi_t = -\frac{\lambda}{\kappa}\tilde{y}_t$$

In these cases, the solution is,

$$\pi_t = \left(\frac{\lambda}{\lambda + \kappa^2}\right)\pi_{t+1|t}$$
$$\tilde{y}_t = \left(\frac{\kappa}{\lambda + \kappa^2}\right)\pi_{t+1|t}$$

The actual level of output and inflation in these periods depends only on the private sector's next period inflation expectations. Expectations are formed according to the state of the econ-

omy that are dictated solely by the process of the natural interest rate. Hence, for $t > 2$ and $i \geq 0$, it must be that $\pi_{t+i|t} = \pi_t$. Iterating forward and using the law of iterated expectations:

$$\pi_t = \left(\frac{\lambda}{\lambda + \kappa^2}\right)^i \pi_{t+i|t} \to 0 \qquad \text{as } i \to \infty$$
$$\tilde{y}_t = \left(\frac{\lambda}{\lambda + \kappa^2}\right)^i \pi_{t+i|t} \to 0 \qquad \text{as } i \to \infty$$

**Proposition 1.** *Assume that the central bank has no balance sheet concerns* ($\phi = 0$)*. If policy is conducted under discretion,* $\tilde{y}_t = \pi_t = 0$ *and* $i_t = \rho$ *for* $t \leq 0$ *and* $t \geq 3$*.*

## 4.2 Equilibrium with Liquidity Trap

Now we analyse monetary policy when the natural interest rate becomes negative. We can summarize the model as equations (4) and (8),

$$\begin{aligned} \tilde{y}_t &= \tilde{y}_{t+1|t} - \sigma^{-1}\left[i_t - \pi_{t+1|t} - r_t^n\right] \\ &\leq \tilde{y}_{t+1|t} + \sigma^{-1}\left[\pi_{t+1|t} + r_t^n\right] \end{aligned} \tag{8}$$

The inequality (8) follows from equation (3) combined with (6) and the equation (4) is the Phillips curve. The relation between the nominal and ex ante real interest rate, $r_t$, is:

$$i_t = \pi_{t+1|t} + r_t \geq 0 \tag{9}$$

The economy is in a liquidity trap in period $t$ if the constraint (9) prevents the central bank from setting output at its potential level, that is, when $\pi_{t+1|t} + r_t^n < 0$.

Proposition (1) implies that $\pi_{3|2} = 0$. Hence, if the natural rate of interest is negative in period 2 the economy falls in a liquidity trap and the best the central bank can do is to set the nominal interest rate to zero, though flex-price employment requires this rate to be negative.

**Proposition 2.** *Assume that the central bank has no balance sheet concerns* ($\phi = 0$)*. The economy falls in a liquidity trap in period 2 if and only if the natural rate of interest is negative,* $r_2^n < 0$*. Moreover, assuming* $r_2^n < 0$ *and that policy is conducted under discretion,* $i_2 = 0$*,* $\pi_2 = \kappa\sigma^{-1}r_2^n < 0$ *and* $\tilde{y}_2 = \sigma^{-1}r_2^n < 0$*.*

We analyse period 1, in this case,

$$\pi_{2|1} = p\pi_2(s^l) + (1-p)\pi_2(s^h) \tag{10}$$

Where $s^i$ indicates the state of the economy in period 2: $i = l$ for negative natural rate of interest and $i = h$ for positive. The private sector forms its expectation about next period's inflation as an average of each state contingent inflation weighted by the likelihood of realization of each of state. Proposition (1) assures that $\pi_2(s^h) = 0$ and Proposition (2) that $\pi_2(s^l) = \pi_1$.

Combine (10) with (4) and (8) to rewrite the central bank problem in period 1 as one of direct choosing $\pi_1$ so as to minimize the period loss function (7) subject to,

$$\pi_1 = \kappa\tilde{y}_1 \leq \left[\frac{\sigma^{-1}\kappa}{(1-\beta p)(1-p) - \sigma^{-1}p\kappa}\right] r_1^n \equiv \psi\kappa r_1^n$$

**Proposition 3.** *Assume that the central bank has no balance sheet concerns ($\phi = 0$) and $(1-p)(1-\beta p) - \sigma^{-1}p\kappa > 0$. The economy falls in a liquidity trap in period 1 if and only if $r_1^n < 0$. Moreover, if policy is conducted under discretion, $i_1 = 0$, $\pi_1 = \psi\kappa r_1^n < 0$ and $\tilde{y}_1 = (1-\beta p)\psi r_1^n < 0$.*

Hence, if policy is conducted under discretion, the economy falls in a liquidity trap in period 1 and in the bad state of nature in period 2. In this situation deflation and inefficient low output turn up because the the zero lower bound prevents the central bank to stimulate aggregate spending appropriately. When it occurs, the dynamics of the economy reverses, deflation expectations drive the real interest rate positive when it should be negative leading to possibly quite bad outcomes, as in period 1.

### 4.3 Optimal Policy under Commitment

The discretion equilibrium is not optimal, with a negative output gap that is unnecessary large, because private-sector inflation expectations are too low. If possible, it would be better for the central bank to credibly commit to generate inflation when the economy finally recovers and create private-sector expectations in period 1 of higher period 2 and 3 inflation. This would lower the real interest rate and reduce the magnitude of the output gaps.

In order to specify this policy we consider the optimal policy in a liquidity trap under commitment. The relevant loss function in period 1 for the central bank is the intertemporal loss function (**??**) that is minimized subjected to (4), (3) and (6) for all $t \geq 1$. To simplify this problem we eliminate all uncertainty in the economy by assuming in this section that $p = 0$.

Hence, the economy falls in a liquidity trap in period 1 and gets out of it with certainty in period 2. This simplification does not change the nature of the problem and allows us to solve it analytically. [3]

The relevant loss function reduces is,

$$\frac{1}{2}\left[\pi_1^2 + \lambda \tilde{y}_1^2 + \beta\left(\pi_2^2 + \lambda \tilde{y}_2^2\right)\right] \tag{11}$$

In period 1, the central bank minimizes (11) subject equation (8) for period 1 in addition to (4) for periods 1 and 2. Because we allowed the central bank to commit to future policy, actual period 2 inflation and output gap, $\tilde{y}_2$ and $\pi_2$, rather than expected appear in (4). That is, the central bank can manage expectations - as if control variables - but in equilibrium these expectations must be fulfilled. The Lagrangian is,

$$L = \frac{1}{2}\left\{(\beta\pi_2 + \pi_2)^2 + \lambda\tilde{y}_1^2 + \beta\left[\pi_2^2 + \lambda(\kappa^{-1}\pi_2)^2\right]\right\} + \varphi\left[\tilde{y}_1 - \kappa^{-1}\pi_2 - \sigma^{-1}(\pi_2 + r_1^n)\right]$$

Where $\varphi \geq 0$ is the lagrange multiplier for constraint (8). We let "c" denote the values of variables in the commitment equilibrium. The first-order condition with respect to $\pi_2$ and $\tilde{y}_1$ and the complementary slackness condition are then, respectively,

$$\begin{aligned}
&(1+\beta)^2\pi_2^c + \beta\pi_2^c + \beta\lambda\kappa^{-2}\pi_2^c - \varphi(\kappa^{-1} + \sigma^{-1}) = 0 \\
&\lambda\tilde{y}_1^c + \varphi = 0 \\
&\varphi\left[\tilde{y}_1^c - \kappa^{-1}\pi_2^c - \sigma^{-1}(\pi_2^c + r_1^n)\right] = 0
\end{aligned} \tag{12}$$

In a liquidity trap, $\varphi > 0$. Eliminating $\varphi$ from (12) results in a determined linear system that is satisfied uniquely by,[4]

[3] Eggertsson and Woodford (2003) design the optimal monetary policy under commitment with stochastic natural rate of interest. This policy involves committing to the the creation of an output boom once the natural rate again becomes positive, and hence to the creation of future inflation. In their numerical experiment, the state contingent nominal interest rate falls to zero immediately after the natural rate turns negative and keeps low a few periods after the natural rate turned back positive. Hence the central bank delivers the inflation and output boom it had compromised - so as to push real interest rates negative - when the economy entered the liquidity trap.

[4] $\eta_2^\pi \equiv \frac{\mu}{(\sigma^{-1}+\kappa^{-1})(\lambda^{-1}+\mu)}$, $\eta_1^\pi \equiv \frac{(1+\beta)\mu}{(\sigma^{-1}+\kappa^{-1})(\lambda^{-1}+\mu)}$, $\eta_1^y \equiv \left(1 - \frac{\mu}{\lambda^{-1}+\mu}\right)$, $\eta_2^y \equiv \kappa^{-1}\frac{\mu}{(\sigma^{-1}+\kappa^{-1})(\lambda^{-1}+\mu)}$

$$0 > \tilde{y}_1^c = \eta_1^y \sigma^{-1} r_1^n > \tilde{y}_1$$

$$\pi_1^c = -\eta_1^\pi \sigma^{-1} r_1^n > 0 > \pi_1$$

$$\pi_2^c = -\eta_2^\pi \sigma^{-1} r_1^n > 0 = \pi_2$$

$$\tilde{y}_2^c = -\eta_2^y \sigma^{-1} r_1^n > 0 = \tilde{y}_2$$

Hence, if commitment is implementable, the central bank is able to avoid deflation and to reduce the strength of the negative output gap in period 1. The commitment equilibrium entails a welfare improvement, the reason is that it can take advantage of an additional trade off: when the central bank can manage expectations, it can stimulate the economy out of the trap in period 1 by generating inflation in period 2. The result is a higher output level in period 1 that is paid with inflation above the target in period 2.

This policy involves committing to the creation of an output boom once the natural rate again becomes positive in period 2. Such a commitment stimulates aggregate demand and reduces deflationary pressures while the economy remains in the liquidity trap through substitution and wealth effects: expectations of inflation lowers real interest rates dropping the current cost of consumption with relation to the future. And finally, the expectation of higher future income should stimulate current spending in accordance with the permanent income hypothesis.

It very useful to find the nominal interest rates required to implement each equilibrium. Plug discretion and commitment equilibrium variables in equation (3) to find that $i_1^c = i_1 = 0$ and $i_2^c < i_2 = \rho$. Then we stress the following result,

**Proposition 4.** *Assume that the central bank has no balance sheet concerns* $(\phi = 0)$ *and there is no uncertain in period 2* $(p = 0)$. *If policy is conducted under commitment, the central bank can avoid deflation in period 1,* $\pi_1^c > 0$, *and increase output,* $\tilde{y}_1^c > \tilde{y}_1$, *by committing to keep low nominal interest rates in period 2, when the natural rate is positive,* $i_2^c < \rho$.

### 4.4 Problems in implementing the Commitment Equilibrium

The commitment equilibrium is time-inconsistent. Despite the central bank's desire in period 1 to commit to next period's policy, it will have no incentives to keep this commitment in period 2. If expectations are rational, the central bank can no longer manage the private sector's expectation to stimulate the economy out of the trap in period 1.

The reason for that is very simple. Assume the central bank has no balance sheet concerns

($\phi = 0$) and there is no uncertainty in period 2 ($p = 0$). Denote by "c" and "d" the values of the period loss function (7) at the commitment and discretion equilibriums, respectively. Evidently, $L_1^d > L_1^c$ and $L_2^c > L_2^d = 0$. As long as the model is purely forward looking, in period 2 the central bank faces the same restrictions regardless its actions in period 1. Hence, the central bank will be tempted to implement the commitment outcome in period 1 and the discretion in period 2. Expectations are rational, though. Then, the private sector anticipates the central bank lack of incentives to keep its commitment, expect a zero inflation target for period 2 and unemployment rises in period 1. That is the reason why the literature address a liquidity trap as a credibility problem.

## 5 Balance Sheet Policies

Since December 2008, along with a zero to 25-basis-point target band for the federal funds rate the Federal Reserve bank has engaged in a number of Targeted Asset Purchase Programs (TAPP). These include the Fed's first and second Large Scale Asset Purchase Programs (LSAPP) 1 and 2 and the Fed's Maturity Extension Program (MEP). These programs have resulted in a substantial increase in the size of the Fed's balance sheet, but even more notably, they have resulted in a great increase in the degree to which the Fed holds longer-term securities, rather than relatively short-dated Treasury bills, on its balance sheet.

We turn now to the question of what effect should one expect these Balance Sheet Policies to have on the private sector's expectation in a liquidity trap under the assumption that the central bank has capital concerns ($\phi > 0$).

We model a TAPP in a simple way assuming the following set up: in the beginning of period t, when the economy falls in a liquidity trap, along with the choice for the policy rate, $i_t$, the central bank announces that it will pursue a target for its holdings of long-term bonds, $B^{tg}$, and the private sector forms its expectations based on all information it has so far[5]. To capture some uncertainty about this additional monetary policy, we assume that the private sector is not certain that the target will be accomplished, rather, they attribute a distribution of probability on $B_t$, described by some density function $f(\cdot)$. In the sequence, markets are opened, assets are priced and the central bank acquires the required quantity of assets necessary to carry out both policies.[6]

[5] We assume that the central bank accomplishes this program by a proportionate expansion in its monetary liabilities - in this case equation (5) will not be a restriction to the program if the zero lower bound has been reached

[6] Note that, if the zero lower bound is reached in period 1, we can use Wallace's irrelevance theorem (1981) to assure that the equilibrium price of the long term bond is unaffected by the central bank's additional demand for it. In this case, we can assume that first the the asset is priced (for its state contingent pecuniary returns) and then, in the end of the period, the central bank purchases any quantity of it.

We add a final assumption that the central bank quits the TAPP as soon as the economy recovers and that the private sector fully understands it. The only reason we added this assumption is to keep the model easy to solve and hence to maintain the intuition behind it.

### 5.1 Central Bank Balance Sheet

In this section we develop the side of the central bank balance sheet. We use the fact that the central bank capital plus its monetary liabilities must equal the value of its assets $K_t + M_t = B_t^{cb} + Q_t B_t^{cb,l}$ to rewrite equation (1) in the recursive form:

$$
\begin{aligned}
K_t &= (1+i_{t-1})B_{t-1}^{cb,s} + (Q_t+1)B_{t-1}^{cb} - M_{t-1} \\
&= (1+i_{t-1})B_{t-1}^{cb,s} + (Q_t+1)B_{t-1}^{cb} - \left(B_{t-1}^{cb,s} + Q_{t-1}B_{t-1}^{cb} - K_{t-1}\right) \\
&= K_{t-1} + i_{t-1}B_{t-1}^{cb,s} + \left(1+Q_t-Q_{t-1}\right)B_{t-1}^{cb}
\end{aligned}
$$

Then we take a first order Taylor expansion around the zero inflation steady state to yield:

$$
k_t = k_{t-1} + \left(\frac{b^{cb,s}i}{k}\right) i_{t-1} + \left(\frac{b^{cb,s}i}{k}\right) b_{t-1}^{cb,s} + b^{cb}b_{t-1}^{cb} + b^{cb}\left(\frac{\beta}{(1-\beta)k}\right)(q_t - q_{t-1}) \tag{13}
$$

Where lowercase letters denote the log deviation of each variable from its steady state, which is denoted by lowercase letter without the time index. From $b^{cb,s}$ and $b^{cb}$ we drop the upper index $cb$ since, from here after, we only refer to short and long term bonds held by central bank.

The asset pricing relation for the long term government bonds $1 = \beta\mathbb{E}_t\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{1+Q_{t+1}}{Q_t}\right)\frac{P_t}{P_{t+1}}$ can be linearized by a first order Taylor expansion around the zero inflation steady state to yield[7]:

$$
q_t = -\sum_{i=0}^{\infty}\beta^i\left(i_{t+i|t} - r_{t+i|t}^n\right) \tag{14}
$$

Plugging equation (14) into (13) yields[8],

$$
k_t = \Lambda_{t-1} + b\left(\frac{\beta}{(1-\beta)k}\right)\sum_{i=0}^{\infty}\beta^i\left[\left(i_{t+i|t-1} - i_{t+i|t}\right) - \left(r_{t+i|t}^n - r_{t-1+i|t-1}^n\right)\right] \tag{15}
$$

Where we gather all predetermined terms of equation (15) in $\Lambda_{t-1}$, since they play no role in optimal choice of policy under discretion.

Equation (15) expresses how the short run interest rate path and the private sector's expectations about it affect the central bank current level of capital. As we see, whenever the central

[7] see details in the appendix

[8] $\Lambda_t = k_{t-1} + \frac{b^{cb,s}i}{k}i_{t-1} + \frac{b^{cb,s}i}{k}b_{t-1}^{cb} + b^{cb}b_{t-1}^{cb} + b^{cb}\left(\frac{\beta}{(1-\beta)k}\right) - b\left(\frac{\beta}{(1-\beta)k}\right)(i_{t-1} - r_{t-1}^n)$

bank holds long run bonds in its balance sheet and it fixes the interest rates above what was expected by the private sector in the previous periods, it suffers a capital loss from the negative variation of the bond's price. We see next how we can use this as a mechanism that the central bank can use to commit itself to low interest rates in the future.

# 6 Central Bank's Credible Commitment to Low Interest Rates in the Future

This section shows that the central bank can implement the commitment equilibrium under discretion if the balance sheet concerns are sufficiently strong, represented by being sufficiently large and exceeding a lower bound $\bar{\phi} > 0$ to be determined. Consider a central bank that finds itself in a liquidity trap in period 1. The central bank cannot commit to a particular state contingent interest rate in the future. We show that it can, nevertheless, use a Targeted Asset Purchase Program to lower expectations of period 2 interest rate.

To analyze monetary policy with a independent central bank we add equation (15) to system (3), (4) and (6). The additional equation introduces a endogenous state variables to the system, $i_{t+1|t}$. The basic idea now is to define a minimum set of state variables that directly affect market conditions and assume that the strategies of the central bank and the private sector expectations depend only on this minimum state. In the model, this minimum state is given by $\left(i_{t+1|t}, b, s_{t+1}^{i}\right)$. Rational expectations hypothesis states that private sector's predictions of the future value of any economically relevant variables equals its true statistical expected value, then,

$$i_{t+1|t}\left(i_{t|t-1}, b, s_t^l\right) = \sum_{j\in h,l} p_{lj}\mathbb{E}^f\left[\bar{i}_{t+1}\left(i_{t+1|t}, b', s_{t+1}^j\right)\right] \tag{16}$$

Each state of nature in period $t+1$ is captured by the realization of variables $i_{t+1|t}$, $b'$ and $s_{t+1}^j$, and $\bar{i}_{t+1}(\cdot)$ is the argument that minimizes equation (7) subject to (3), (4), (6) and (15) in each one of these states. Let $p_{ll} = p$, $p_{lh} = 1-p$ and $\mathbb{E}^f$ be the expectation operator under the density $f(\cdot)$. Then, expression (16) means that next period's expected interest rate is an average of the optimal interest rate in each possible state of nature weighted by the probability of realization of each state.

Note that $i_{t+1|t}$ is itself a state variable for period 2 since it is a relevant information in restriction (15). If expectations are supposed to be fulfilled in equilibrium, private sector's expectations must be consistent with this fact. Hence, the set of values of $i_{t+1|t}$ that are consistent with equilibrium is the set of fixed points of expression (16).

Note also that, considering the timing of the game, the full composition of the central bank's balance sheet in period $t$ is unknown when expectations are formed. Due to the uncertainty of the monetary policy, although the central bank has announced its intention to pursue the target $b^{tg}$, the private sector is uncertain of the quantity of long term bonds the central bank will hold in the end of the period $t$, that will affect market conditions in period 2, so they take an expectation according to $f$.

It very is useful to note that exists a $\bar{b} > 0$ such that for all $b > \bar{b}$:

$$\chi_t(i_{t|t-1}, b, s_t^l) = \begin{cases} 1 & \text{if} \quad i_t > i_{t|t-1} \\ 0 & \text{if} \quad i_t \leq i_{t|t-1} \end{cases}$$

It follows directly from equation (15). The reason for this result is simple, high duration assets are very sensitive to interest rate changes. Then, for high enough $b$ any difference between $i_t$ and $i_{t|t-1}$ causes a large impact in the central bank capital. This considerations lead us to the following result,

**Lemma 5.** *Assume that the central bank has strong capital concerns ($\phi > \bar{\phi}$). If the central bank's holdings of long term bonds in the beginning of period $t$ is high enough ($b > \bar{b}$), then minimize the loss function* (7) *when ($\phi > \bar{\phi}$) subject to* (3)*,* (4)*,* (15) *and* (6) *is equivalent to minimize* (7) *when ($\phi = 0$) subject to* (3)*,* (4)*, and* (17)*. Where,*

$$0 \leq i_t \leq i_{t|t-1} \tag{17}$$

Let the central bank set target $b^{tg} = \bar{b}$, in the beginning of period $t$ all information that matters to the private sector determine $i_{t+1|t}$ is whether the central bank will accomplish or not the target in the end of the period. If $b \geq b^t$, then, in period $t+1$, the central bank sets the policy rate by minimizing (7) subject to (3), (4) and (17) as if ($\phi = 0$). If $b < b^t$, it is subject to (3), (4) and (6). Hence, we can assume that the private sector expects that the central bank meets the target with fixed probability $q$ and denies on it with probability $1-q$ without much loss of generality. In this case we can extract the endogenous variable, $b$, from the space state of the economy.

**Proposition 6.** *Assume that the central bank has strong balance sheet concerns ($\phi > \bar{\phi}$) and fix the target policy $b^{tg} = \bar{b}$. Then, if policy is conducted under discretion, for all $i_{t|t-1} \geq 0$ and all $b > 0$ we have $i_{t+1|t}\left(i_{t|t-1}, b, s_t^h\right) = \rho$.*

We only sketch the proof here [9]. Assume that the central bank sets the policy rate for period $t+1$ by minimizing (7) subject to (3), (4), (6) and (15). If $s^h_{t+1}$ and $b' \leq \bar{b}$ we can disregard restrictions (3), (6) and the the parameter $\phi$ in (7). In this case, Proposition (1) assures $\bar{i}_{t+1} = \rho$. Moreover, if $s^h_{t+1}$ and $b' > \bar{b}$, we can solve the easier problem proposed by lemma (5), which solution is given by $\bar{i}_{t+1} = i_{t+1|t}$. Therefore, equation (16) imply that $i_{t+1|t} = qi_{t+1|t} + (1-q)\rho$, which is uniquely solved for $i_{t+1|t} = \rho$, as we wanted to show.

**Proposition 7.** *Assume that the central bank has strong balance sheet concerns ($\phi > \bar{\phi}$) and fix the target policy $b^{tg} = \bar{b}$. Then, if policy is conducted under discretion, for all $i_{t|t-1} \geq 0$ and all $b > 0$ we have $i_{t+1|t}\left(i_{t|t-1}, b, s^l_t\right) = \frac{(1-p)(1-q)}{1-(1-p)q}\rho$.*

We only sketch the proof here [10]. Assume again that the central bank sets the policy rate for period $t+1$ by minimizing (7) subject to (3), (4), (6) and (15). If $s^l_{t+1}$, then, for all $b' > 0$, we have $\bar{i}_{t+1}\left(i_{t+1|t}, b', s^l_{t+1}\right) = 0$. To see this, note that this problem is the same solved in Proposition (3) further restricted by $\phi > 0$ and (15). Then, because the value of the loss functions in these two problems equals when $\bar{i}_{t+1} = 0$, it must be the optimal choice. Moreover, Proposition (6) assures $\bar{i}_{t+1}\left(i_{t+1|t}, b', s^h_{t+1}\right) = i_{t+1|t}$ for all for all $b' \geq \bar{b}$ and $\bar{i}_{t+1}\left(i_{t+1|t}, b', s^h_{t+1}\right) = \rho$ for all $b' < \bar{b}$. Then, equation (16) imply that,

$$\begin{aligned} i_{t+1|t}\left(i_{t|t-1}, b, s^l_t\right) &= (1-p)\left[q\bar{i}_{t+1}\left(i_{t+1|t}, b', s^h_{t+1} | b' \geq \bar{b}\right) + (1-q)\bar{i}_{t+1}\left(i_{t+1|t}, b', s^h_{t+1} | b' < \bar{b}\right)\right] + \\ &+ p\bar{i}_{t+1}\left(i_{t+1|t}, b', s^l_{t+1}\right) \\ &= (1-p)\left[qi_{t+1|t} + (1-q)\rho\right] \end{aligned}$$

That can be uniquely solved by $i_{t+1|t}\left(i_{t|t-1}, b, s^l_t\right) = \frac{(1-p)(1-q)}{1-(1-p)q}\rho$, as we wanted to show.

To understand the mechanism through which the TAPP may affects market expectations, suppose that in the beginning of period 1 the private sector forms period 2 interest rate expectation assessing what the central bank optimally does in the case that the economy recovers or not. If the economy does not recover the central bank certainly keep low interest rates. If the economy does recover the central bank might raise it. Thus, off equilibrium, asset markets price long term bonds according to an "average" expectation about next period's interest rate. But what happens if the central bank engages in a large scale purchase of this asset in the end of the period? Unlike the classic problem with no capital concerns, if the economy recovers in period

[9] See details in the appendix
[10] See details in the appendix

2, the central bank is unwilling to set the high rate that the public expected in the beginning of the period 1 because it entails an expressive capital loss. Under the rational expectations hypothesis, the private sector anticipates it and revises its expectations downward so as to match the optimal rate in each state of nature.

We can now assess how the TAPP affects equilibrium prices and allocations in period 1 when the economy falls in a liquidity trap. We solve the model for $\pi_1$, $\tilde{y}_1$, $\pi_{2|1}$, and $\tilde{y}_{2|1}$ as function of the exogenous realization of the natural interest rate, $r_1^n < 0$, and the endogenous expectations, $i_{2|1}$, assuming that the central bank has strong capital concerns ($\phi > \bar{\phi}$), that it conducts monetary policy setting the policy rate under discretion and fixing a target $b^{tg} = \bar{b}$,

$$\begin{bmatrix} \pi_{2|1} \\ \tilde{y}_{2|1} \\ \pi_1 \\ \tilde{y}_1 \end{bmatrix} = \psi \begin{bmatrix} p\kappa \\ p(1-\beta p) \\ \kappa \\ (1-\beta p) \end{bmatrix} r_1^n + q(1-p)\kappa\psi \begin{bmatrix} 1 \\ p + \kappa^{-1}(1-\beta p - \kappa p \sigma^{-1}) \\ 1 + (1-\beta)p + \kappa\sigma^{-1} \\ 1 + (1-\beta p)\kappa^{-1} \end{bmatrix} \left(\rho - i_{2|1}(s_1^l)\right) \tag{18}$$

Note that the second term of the RHS of (18) vanishes away as $q \to 0$ and the equilibrium converges to the discretion equilibrium as in Proposition 3. If, however, we consider $q > 0$ and plug $i_{2|1}$ suggested by Proposition (7) in (18), for any $p > 0$, period 1 inflation and output gap grow together with $q$.

To illustrate our findings we calibrate and solve the model. We follow Eggertsson and Woodford (2003) to interpret periods as quarters, and assume coefficients values of $\sigma = 0.5$ and $\beta = 0.99$. The assumed value of the discount factor implies a long term real rate of interest of equal to four percent per annum so we map $\rho = 1$. Also, we set the relative weight that the central bank assigns to output gap, $\lambda$, to one and the probability of recovery, $(1-p)$, to 0.25 so the expected crisis extension is about 4 quarters. To push the economy to the ZLB we set $r_t^n(s^l) = -1$. The assumed value of $\kappa = 0.02$ is consistent with empirical estimate of Rotemberg and Woodford (1997). The assumed value of $\sigma = 0.5$ represents a relatively low degree of interest-sensitivity of aggregate expenditure. We prefer to bias our assumptions in the direction of only a modest effect of interest rates on the timing of expenditure, so as not to exaggerate the size of the output contraction that is predicted to result from an inability to lower interest rates when the zero bound binds.

Figure (1) plots $i_{2|1}(s_1^l)$ against $q$. When no targeted asset purchases are in play, $q = 0$, the interest rate expectation is just the same the original model, ($\phi = 0$), under discretion, where $i_{2|1} = (1-p)\rho$; as it rises, however, this expectation falls continually down to $i_{2|1} = 0$, when

$q = 1$, which is the level of interest rate that the central bank would like to commit if it had the chance. Accordingly, period 2 inflation and output expectations also rise driving ex ante period 1 real interest rate down. The fall in the real interest rate stimulates spending supporting a higher level of period 1 output and price level.

Figures (2) and (3) plot the equilibrium actual and expected inflation and output gap on the QE probability respectively. As Bernanke et al. (2004) and Krishnamurthy and Vissing-Jorgensen (2011) empirical research suggest, the figures show that this variables all move together, so the QE can mitigate the deflationary pressures that the economy suffers as it falls in to the liquidity trap. This effect, nevertheless, seems to be moderate as suggested by the recent empirical literature [11].

The central bank purchase program lowers the interest rate expectation because it is a mechanism that allows the central bank to credibly commit to low interest rates contingent to states of nature in which the natural rate turned back positive. In order words, the program induces the private sector to believe that the central bank will keep low interest rates after the economy recovery. Thus, the higher the likelihood of recovery, the more important is the QE to fight deflation and output loss. To illustrate it, we repeat the exercise when the economy gets out the trap with higher probability, $(1 - p) = 0.4$, and reproduce the equilibriums outcome in figures (4) and (5). Clearly, the QE effects on the model's outcome in greater under this situation.

## 7 Conclusion

In this work we derived outcomes and welfare properties for a standard New-Keynesian model when the Zero Lower Bound is a relevant restriction to monetary policy under two equilibrium assumptions: discretion and commitment. We found that the model's welfare outcome is greater under the commitment than the discretion equilibrium and the former is not time consistent. Then we introduced a financially inndependent and capital concerned central bank to the model and show how it can use a Targeted Asset Purchase Program as a mechanism to commit itself to future actions. Under this set up, the central bank is able to implement an incentive compatible equilibrium that is more desired than the discretion outcome with no role for unconventional policy. As a matter of policy, we argue that our findings rationalize recent unconventional measures taken by the central bank as a useful additional dimension of monetary policy if it is used to shape policy expectations.

[11] See Vayanos and Vila (2009), Chen et al. (2012) and Krishnamurthy, A. and Vissing-Jorgensen, A. (2011).

## 8 Figures

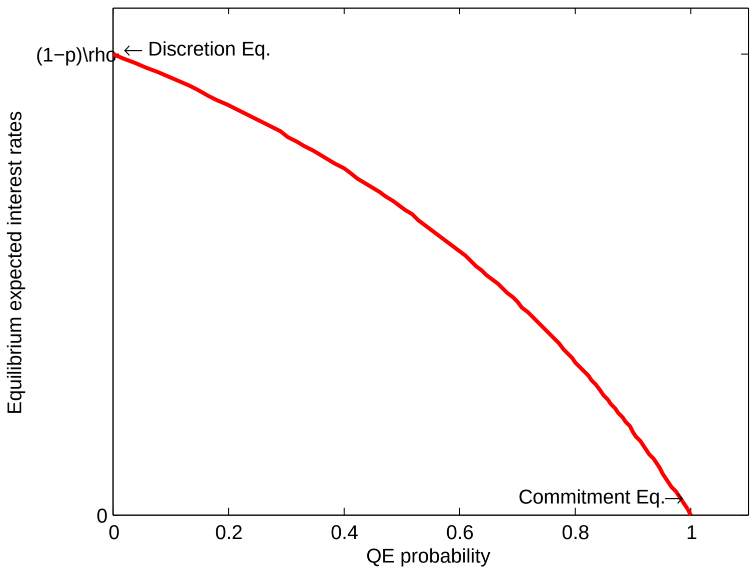


Figure 1: Equilibrium interest rate expectation on QE probability

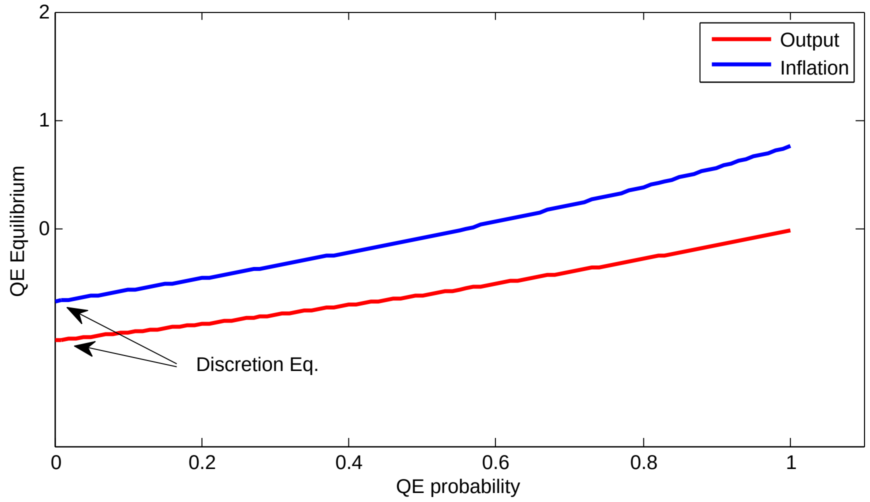


Figure 2: Equilibrium inflation and Output gap on QE probability ($p = 0.75$).

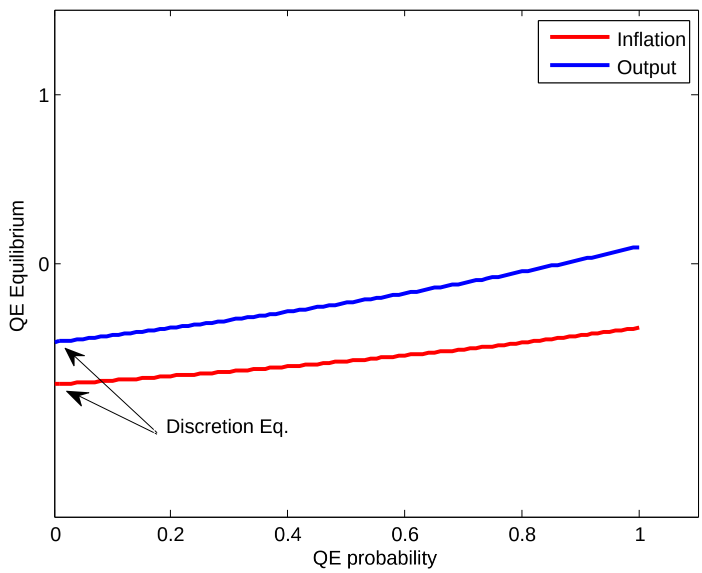


Figure 3: Equilibrium Expected Inflation and Output gap on QE probability ($p = 0.75$).

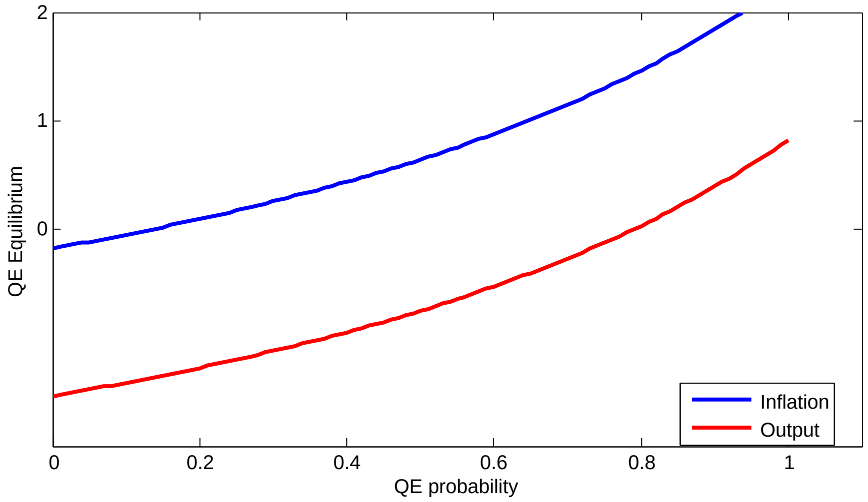


Figure 4: Equilibrium inflation and Output gap on QE probability ($p = 0.5$).

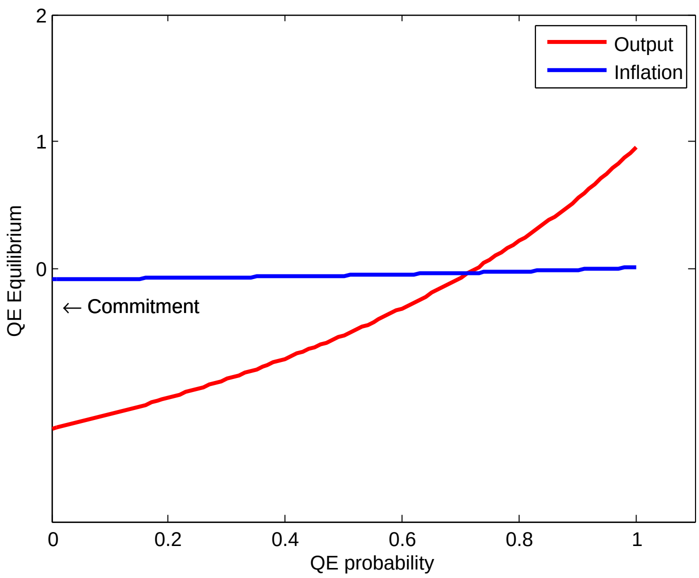


Figure 5: Equilibrium inflation and Output gap on QE probability ($p = 0.5$).

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