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Fernando Siqueira dos Santos

Three Essays on Macroeconomics

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Tese apresentada à Escola de Economia de São Paulo para obtenção do título de doutor em economia

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Prof. Marcio Holland de Brito

Prof. Vladimir Kuhl Teles

Prof. Paulo Piccheti

Prof. Joaquim Pinto de Andrade

Prof. Ricardo Dias de Oliveira Britto

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Resumo

Esta tese é composta por 3 estudos empíricos sobre macroeconomia. O primeiro ensaio discute a persistência da inflação no Brasil. O segundo estudo analisa o produto potencial e, principalmente, a questão taxa neutra de juros no Brasil, tema fundamental para a condução da política monetária. O último trabalho discute a questão da paridade entre os juros no Brasil e no exterior.

O primeiro ensaio desta tese estima a taxa real de juros de equilíbrio no Brasil durante o período 1997-2012 usando diversas metodologias. Os resultados mostram alguma diferença nas estimativas da taxa de juros de equilíbrio dependendo da especificação utilizada, principalmente na modelagem da Curva IS. A mensuração do hiato do produto não é o principal responsável pelos resultados encontrados para a taxa de juros de equilíbrio. A estimação conjunta do PIB potencial e taxa neutra de juros não leva a resultados muito diferentes dos obtidos estimando a taxa neutra isoladamente. Independente do modelo utilizado, os resultados indicam redução na taxa de equilíbrio no Brasil nos últimos anos.

O segundo ensaio estima a persistência da inflação no Brasil tanto em termos agregados quanto desagregados. O trabalho ainda compara a persistência da inflação no Brasil com a persistência em outros países emergentes. Os resultados indicam que a persistência da inflação no Brasil é maior do que em outros países, mas este resultado não é obtido para todos os métodos de estimação utilizados. A persistência no núcleo da inflação é maior do que na "inflação cheia". Apesar da persistência elevada, nossos resultados indicam que a expectativa de inflação é uma variável mais importante na determinação da inflação corrente do que a inflação passada.

O terceiro ensaio analisa a diferença entre as taxas de juros no Brasil e no exterior, particularmente nos EUA, e evidências de fluxos de investimentos locais ou estrangeiros para explorar o diferencial de juros. Os resultados indicam que os fluxos de investimento estrangeiro tiveram pouco impacto nas taxas de juros no Brasil. Medidas de risco-país e risco cambial são importantes para explicar o diferencial de juros sendo que as medidas de risco-país parecem ter sido mais importante no início de nossa amostra enquanto as medidas de risco cambial foram mais importantes nos últimos anos. Medidas de risco cambial, particularmente a volatilidade do câmbio ajudam a explicar a falta de convergência dos juros no Brasil com os juros praticados no exterior. Apesar da elevada volatilidade da taxa de cambio, uma simples estratégia de comprar Real (BRL) e investidor no mercado local de juros (estratégia similar a aplicar no contrato futuro de Real) teria gerado um índice de Sharpe tão elevado ou maior do que o observado em estratégias mais sofisticadas envolvendo diversas moedas.

Abstract

This thesis is composed of 3 empirical studies on macroeconomics. The first essay studies the persistence of inflation in Brazil. The second essay studies the concepts and measures of potential output and neutral interest rate, two fundamental pillars in the conduct of monetary policy. The third essay studies the parity of local and foreign interest rates.

The first essay measures the neutral real interest rate for Brazil during 1997-2012 using different methodologies. The results show some difference in the estimates of the natural interest rate in Brazil depending on the specification of the IS curve and its explanatory variables. Measurement of the output gap is not a source of divergence among our estimation of natural rate as different methodologies yields similar values for the output gap. Joint estimation of the inflation and output cycles leads only to small difference in the output gap estimates and hence on natural interest rate. Finally, our results indicate that the impact of monetary policy on output gap increased during the last years.

The second essay analyzes inflation persistence in Brazil. Both aggregate and disaggregated inflation persistence are computed. We also compare inflation persistence in Brazil with estimates for other emerging countries with a long history of high inflation. The results indicate that inflation persistence in Brazil is higher than in other emerging markets. Core inflation presents more inflation persistence than headline inflation, particularly due to the exclusion of the low persistence food items. Despite the large persistence in Brazilian inflation, disaggregated data are more sensible to expected inflation than lagged inflation and thus indicates a major role for forward looking behavior.

The third essay studies the difference between interest rates in Brazil and other countries, particularly US, and evidence of arbitrage investments aiming at exploring this difference. Our results indicate that there is important evidence of foreign investment inflows to Brazil but the impact of these flows are not sufficient to reduce local interest rates substantially. Both country and currency risk are important determinants of the interest rate difference between Brazil and other countries but exchange rate risk, particularly exchange rate volatility, plays a major role in avoiding full interest rate convergence. Despite the large BRL volatility, a simple strategy of going long BRL + local rate (similar to buy BRL forward contracts) would have generated large Sharpe ratios, closer or higher than Sharpe ratios generated from more complex strategies involving long position on high yield currencies and short position on low yield currencies.

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1 Potential Output and Neutral Interest Rate in Brazil

1.1 Introduction

In the last ten years the conduct of monetary policy in Brazil has been designed mainly by adjusting interest rates to inflation and the output gap. This movement reflects the emergence of Taylor Rules as one of the best way to conduct or at least describe the conduction of monetary policy. The adoption of inflation target regimes also favored the use of monetary policy rules in the way described by Taylor (1993). A key aspect of Taylor rules is to adjust interest rates to inflation and output gap. The reasoning behind this postulate is that real interest rates must be increased or decreased to make inflation converge to the target. Even when there is no explicit target, the monetary authority can use this framework. Indeed, Taylor (1993) shows that this kind of simple rule fits the conduct of monetary policy in US, a country known for its option to not adopt the inflation target framework.

A key variable in this framework is the so called neutral or equilibrium real interest rate (REIR). Theoretically, the neutral rate of interest is the rate at which real output equals potential output or, in other words, the output gap is zero. A zeroed output gap is consistent with no inflation pressures according to the Phillips Curve theory. Over the last years, some monetary policy rules evolved in a way to bypass the need to know the level of REIR. One of this monetary policy rule is to increase real interest rate when one believes aggregate demand is greater than aggregate supply. This is known as first difference rules, as it doesn't use the level of interest rate as the policy instrument, but its first difference. Even this being a way of conducting monetary policy, it is of course a second best choice. Knowing the level of REIR is the best way of conducting monetary policy.

These two concepts, neutral interest rate and output gap, plays a major role in the conduct of monetary policy not only in Brazil but in several other countries. These are part of what Blinder (1998) calls fundamental concepts in modern macroeconomics and have been widely studied and discussed in the last years. In this paper we focus on neutral interest rate determination. More specifically, we apply statistical methods to extract the neutral rate of interest in Brazil based on some measures of inflation and output gap.

In this paper we treat output gap as given in several estimations. In other words, we do not always try to joint determinate output gap (or potential output) and neutral interest rate. We are aware of a possible failure of our natural interest rate measure as a good proxy but we judge the benefit of joint determination can be small in the case of Brazil. Due to the high variability of inflation over the last years and also due to the low explanatory power of the Phillips Curve, the joint estimation of a "Phillips Curve consistent" output gap and the neutral interest rate from a IS curve can lead to a process of error accumulation and barely reliable estimates. Considering these difficulties, made worse in Brazil due to a small sample of data, we propose some models to extract the neutral interest rate from the data.

Our results indicate that the neutral interest rate decreased in Brazil over the last years. However, there is a large degree of uncertainty about the level of the neutral rate. Different specifications for the IS curve lead to different estimation results for the neutral interest rate. Different methodologies used for extracting the output gap lead to very similar results and we see this as an indication that future extensions of this paper would envisage more structural determination of the IS curve rather than proposing new methods to extract the output gap. Our results also indicate that the impact of monetary

policy on output gap increased over the last years. The coefficients obtained from the IS Curve and Phillips Curve in this paper are close to the ones observed in other countries and also close to the ones estimated by other authors for Brazil.

The reduction in neutral interest rate in Brazil is an expected result. As has been extensively mentioned by the Brazilian Central Bank, the overnight interest rate (Selic) was reduced from more than 20% in the late 90's to less than 10% without a huge acceleration in inflation (or GDP growth). This result is consistent with a reduction in the neutral interest rate over the last years. The contribution of this papers lies mainly on proposing a method for quantifying this reduction.

1.2 Brief Literature Review

The literature on real interest rates is vast and we will discuss just a small part of it in this section. We will discuss three different kind of literature on neutral real interest rates: i) theoretical, ii) empirical and iii) empirical with Brazilian data.

The most recent studies on real equilibrium interest rate rely on the work of Woodford (2003). Woodford (2003) uses a "wicksellian" approach to define the real neutral interest rate. Wicksell described the neutral rate of interest in three ways: a) the rate of interest that equates savings with investment, b) the marginal product of capital and c) the rate that is consistent with price stability. Muinhos and Nakane (2006) present empirical evidence focused on Brazilian data on these three definitions of natural interest rate.

The concept is an important part of the modern macroeconomic theory represented by the Dynamic Stochastic General Equilibrium Models (DSGE). Books like Woodford (2003) and Gali (2008) presents some examples of these models. In these models, the neutral rate has an exact expression and will depend on some details (or hypothesis) in the model. In the most conventional models, the natural rate will depend on the rate of time preference of households and their willingness to substitute consumption across time. This conventional expression appears not only in books like Gali (2008) and Woodford (2003) but also in several empirical papers like Laubach and Williams (2003).

Our empirical exercise will be based on this concept of neutral rate, the rate that "closes" the output gap. This is also close to the definition of neutral interest rate used by Blinder (1998), where the neutral interest rate comes from the IS curve. The distinction comes from the fact that we impose as little as possible model structure in our estimations (only a measure of the output gap and a measure of interest rate in most of the specifications). Also, in our estimations the neutral rate is assumed to vary slowly over time. In the DSGE models, the variables that determine the natural rate are clearly specified and the value is consistent with equilibrium output every period (or period by period). In modeling the "neutral" interest rate using statistical tools we are more closely obtaining a rate to which the economy is trending over time (Amato, 2005).

Many empirical works on real interest rates had been published in last years. Some of them offers some theoretical background, like Laubach and Williams (2003), Neiss and Nelson (2003) and Blanchard and Summers (1984), while others focuses on the statistical procedures aiming at extract the neutral real interest rate from the data. In most cases, the estimation of the output gap and the neutral interest rate are made jointly by estimating a small scale macroeconomic model. In most cases, the macroeconomic models include both the potential output and the neutral interest rate as latent variables and are estimated using standard Kalman filter techniques. This is the approach followed by

Laubach and Williams (2003). We will not name other empirical works here. In most cases, the Laubach and Williams (2003) procedures are simply replicated or slightly modified.

In Brazil, there are only a few papers on the subject. This small number of empirical work doesn't mean the subject is not important. References to the equilibrium rate of interest are common in the minutes of Brazilian Central Bank meetings. It is of common knowledge that the neutral real interest rate in Brazil hover around 8%. This number is the mode of several different model specifications in Muinhos and Nakane (2006). Barcelos Neto and Portugal (2009) also provides estimates for the equilibrium interest rate in Brazil using a methodology that is similar to the one presented in this paper. The natural rate estimated by Barcelos Neto and Portugal (2009) is close to 7% for the period jan-00 to dec-05 (see section 5 and 6 for our own results) and the close to 9.5% using a structural macroeconomic model. The results from Barcelos Neto and Portugal (2009) do not present any evidence of a trend (downward or upward) in the natural interest rate.

1.3 Potential Output Measures

As one can observe from the previous discussion in the introduction of this paper, the concept of potential output is deeply connected with the concept of neutral real interest rate. Indeed, several authors use a positive theory to define the equilibrium rate of interest: the REIR is the rate of interest that equalizes output and potential output. In this section we briefly discuss the evolution of Brazilian gross domestic product (GDP) and also factors affecting potential GDP growth. The idea behind this session is to give more facts related to the Brazilian economy to improve the discussion of the results regarding our estimates of potential output and REIR in the next sections. We divide the discussion into two themes: structural measures of potential output and statistical measures.

1.3.1 Structural Measures of Potential Output

In the literature of growth accounting, GDP growth is usually decomposed into the contributions of labor, capital and productivity. In this sub-section we briefly discuss labor and capital stock data for Brazil.

Labor force growth has been diminishing in Brazil in the last years. This is a result of lower population growth over the last 15 to 20 years: according to IBGE (Brazilian Institute for Geography and Statistics), population growth was above 2% in the early 80's and diminished to close to 1% in the last years. Working age population growth (population between 15 and 64 years) diminished from close to 2,5% in the early 80's to less than 1% in the last years.

As a result of lower population growth and higher GDP growth, the unemployment rate in Brazil has fallen during the last ten years (this is clearer after 2002). Using data from PNAD, the unemployment rate in the late 90's was above 12% and diminished in the following years to reach values as low as 6%. This pattern is also clear using other source of data. Using the data from Dieese/Seade Employment Survey (covering only São Paulo metropolitan area) the unemployment rate fell from close to 14% in the late 80's to 8% in 2010. Using the Dieese/Seade unemployment rate for metropolitan areas[1], our preferred measure, the unemployment rate fell from close to 12% in the late 90's to close to 7% in 2010. Using data from the IBGE Employment and Unemployment Survey, the unemployment rate fell from close to 14% in 2003 to 6% in 2010.

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[1] The metropolitan areas are: São Paulo, Belo Horizonte, Porto Alegre, Distrito Federal, Recife and Salvador.

The other variable usually included in the growth accounting literature is capital stock. The construction of capital stock data is more complicated than estimate population or working age population in a country. It is difficult to track investments and mainly determine the depreciation of capital. Also it is difficult to estimate the "initial" capital stock, the stock of capital at some point in the past from where we use a law of motion based on investments and depreciation to update the values of capital stock.

Besides that it is difficult to compare capital stock among countries. Different authors can use different methods to track the stock of capital in the economy and this can lead to different results. The stock of physical capital in Brazil has been estimated by Morandi and Reis (2004) and also Gomes et all (2005). Morandi and Reis (2004) show that the Brazilian capital stock is not very different from other countries, particularly more developed countries like USA, Japan or the European countries. On the other side, Gomes et all (2005) include in their calculation a "wasting" factor. The implication of this factor is to diminish the capital stock, i.e., the capital stock is lower than you could infer using the investment data along with a depreciation factor. According to these authors, this adjustment factor is particularly important in period of great public investment share in total investments. In the Morandi and Reis (2004) study, the capital-output ratio is close to 3x in the recent period, while in Gomes et all (2005) it is only close to 2x. In advanced economies, this relation hovers between 3x to 4x according to Morandi and Reis (2004). It means that the Brazilian capital stock lies in the lower bound of more developed countries at best when adjusted for this "waste factor".

We bypass the problem of "measuring capital stock right" using data on the rate of utilization of capital. This variable is considered a good proxy for inflation pressures as the work of Gordon (1998) indicates. This variable has also been used in Brazil and is always mentioned in inflation reports or minutes of the Brazilian Central Bank meetings.

The output gap derived from the production function approach can be described as follows. Let Y_t be the output any some time interval *t, K* is the capital stock and *L* is the labor force. The output is produced using a production function like:

$$Y_t = A_t F(K, L) \tag{1.1}$$

Where *A* is the productivity factor and *F* can be expressed as the traditional Cobb-Douglas function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1.2}$$

Now, assume there is a *non-accelerating inflation rate of capital utilization,* $\overline{N}_t$, and also a *non-accelerating rate of inflation rate unemployment* (NAIRU), $\overline{U}_t$. The potential output in this case is given by:

$$Y_t = A[K_t \overline{N}_t]^{\alpha}[L_t(1-\overline{U}_t)]^{1-\alpha} \tag{1.3}$$

and finally assume that productivity is constant during our sample period. Using this formulation and these assumptions, the output gap y_t can be written as:

$$y_t = \alpha(N_t - \overline{N}_t) + (1-\alpha)(\overline{U}_t - U_t) \tag{1.4}$$

The values of y_t is computed calibrating the value of α from data on national accounts and value of potential utilization rate of capital and NAIRU are estimated using statistical techniques like HP or BP filter or even simpler approaches like time trends.

The figures bellow present the evolution of the main variables in this production function approach to determine the output gap. We used the Hodrick-Prescott filter to detrend the series. We used a three months average to construct quarterly data for unemployment rate and capital utilization. The data on labor share is available only on an annual basis. To construct a quarterly series we first created a quarterly series assuming the labor's share was constant over the quarters. Then we used a centered 8 quarters moving average (three leads and three lags).

Fig. 1.1a: Capital Utilization Rate (N_t)

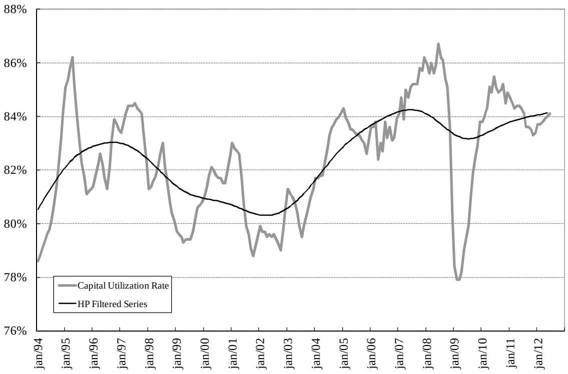

Fig. 1.1b: Unemployment Rate (U_t)

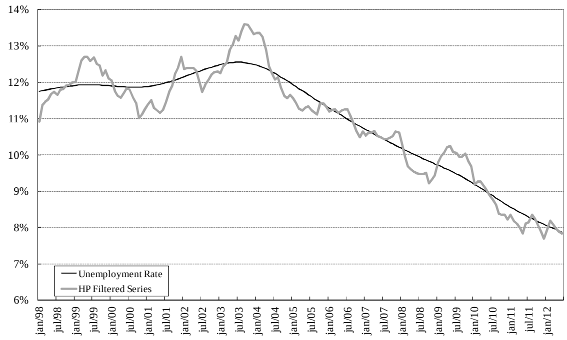

Fig. 1.1c: Labor Share ($1\text{-}\alpha_t$)

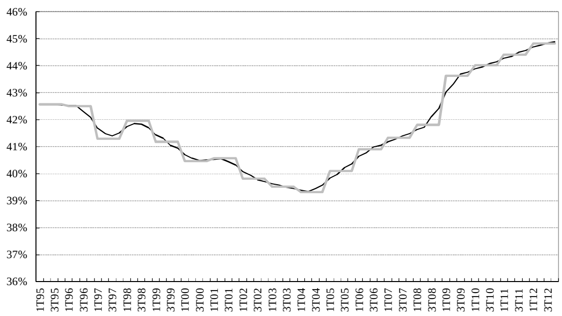

Fig. 1.1d: Output Gap (y_t)

1.3.2 Statistical Potential Output Measures: HP Filter and Unobserved Components Decompositions

1.3.2.1 Hodrik-Prescott Filter

There are several different statistical methods to measure the potential output of an economy. The most common way is to use the Hodrick-Prescott filter. The basic procedure of the HP filter consists of decomposing any time series x_t into a growth component g_t and a cyclical component c_t:

$$x_t = g_t + c_t\,, \quad t = 1,...,T \tag{1.5}$$

The cyclical component has average zero over long periods. The growth component is referred as the trend of the variable and the cyclical component is the temporary deviation from this trend. The growth component is relatively "smooth" and the equation for decomposing x_t is given by:

$$\underset{\{g_t\}}{Min}\left\{\sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T}\left[\left(g_t - g_{t-1}\right) - \left(g_{t-1} - g_{t-2}\right)\right]^2\right\} \tag{1.6}$$

Hodrick and Prescott (1997) propose $\lambda = 1600$ for quarterly series. To set this value, they assume as a reference a 5% value for the cyclical component and a 0,125% value for the variation of the growth rate ($\Delta^2 g_t$). Also, they take into account the fact that if both c_t and $\Delta^2 g_t$ had normal distribution with standard deviations of σ_1 and σ_2,respectively, the solution to the problem would be $\sqrt{\lambda} = \sigma_1 / \sigma_2$. So, using the reference values just mentioned they set $\sqrt{\lambda} = 5/0{,}125$, or, $\sqrt{\lambda} = 40$.

1.3.2.2 Structural Time Series Model

The Hodrick-Precott procedure is somewhat "deterministic" in the sense that some parameters are not estimated but rather imposed into the model. Harvey and Jaeger (1993) propose a different way to decompose x_t into trend and growth components using an unobserved components model (UCM). The model proposed by Harvey and Jaeger is a simplified version of the Structural Time Series Model (STM)[2] which can be expressed as:

$$x_t = \mu_t + \gamma_t + \psi_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2), \quad t = 1, \ldots, T \tag{1.7}$$

$$\mu_t = \mu_{t-1} + \beta_t + \eta_t, \quad \eta_t \sim NID(0, \sigma_\eta^2) \tag{1.8}$$

$$\beta_t = \beta_{t-1} + \zeta_t, \quad \zeta_t \sim NID(0, \sigma_\zeta^2) \tag{1.9}$$

$$\gamma_{1,t+1} = -\gamma_{1,t} - \gamma_{2,t} - \gamma_{3,t} + \omega_t, \quad \omega_t \sim NID(0, \sigma_\omega) \tag{1.10}$$
$$\gamma_{2,t+1} = \gamma_{1,t} \text{ and } \gamma_{3,t+1} = \gamma_{2,t}$$

$$\psi_t = \rho \cos \lambda_c \psi_{t-1} + \rho \sin \lambda_c \psi_{t-1}^* + \vartheta_t \tag{1.11}$$
$$\psi_t^* = -\rho \sin \lambda_c \psi_{t-1} + \rho \cos \lambda_c \psi_{t-1}^* + \vartheta_t^*$$

Equations (1.8) and (1.9) describes the stochastic process for the *level* or *trend* (μ_t) and *slope* (β_t) respectively. Equation (1.10) describes the modeling of *seasonality* (γ_t). The stochastic *cycle,* ψ_t, is described by *sine* and *cosine* functions, the frequency of the cycles, λ_c, and a damping factor, ρ.

The HP filter is a particular case of this more general formulation when $\sigma_\eta^2 = 0$ and $\sigma_\varepsilon / \sigma_\zeta = 1600$ in the model without the seasonal component. Harvey and Jaeger (1993) argue that this is a good

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[2] *Harvey and Jaeger (1993) do not include the seasonal factor since they use seasonally adjusted series. Hodrick and Prescott (1997) also used seasonally adjusted data. We will also use seasonally adjusted data in most estimation, particularly the ones involving GDP.*

approximation for the US, particularly in the period studied by Hodrick and Prescott, but are not good for some other countries. The estimated coefficients using Brazilian GDP data is presented in table 1.1. The plot of the estimated cycle is provided in the end of this section together with other measures of the cycle component of GDP. The results indicate one of the restrictions imposed by the HP filter ($\sigma_\eta^2 = 0$) but not the other ($\sigma_\varepsilon / \sigma_\zeta = 1600$). Despite the difference being large, the estimated cycle component is similar using either STM or HP filter as we shows in the end of this section. In the Harvey and Jaeger (1993) paper, they estimated $\sigma_\varepsilon / \sigma_\zeta$ 78 for US GDP and the cycle component is also similar to the cycle estimated by the HP filter[3].

Table 1.1 – Brazilian GDP: Estimates of Structural Time Series (Unobserved Components) Model Paramaters (1Q96 to 3Q12)*

	σ_ε^2	σ_η^2	σ_ζ^2	σ_ω^2	$\sigma_\varepsilon / \sigma_\zeta$	σ_ϑ^2	ρ	$2\pi / \lambda_c$
Brazilian GDP	1,496	0,000	0,066	n.a.	22,699	6,186	0,845	11,3
Brazilian GDP (not seas. adj.)	0,954	0,000	0,058	0,383	16,362	7,410	0,839	11,1

**Variances multiplied by 10^5. $2\pi/\lambda_c$ is the frequency of the cycle in quarters.*

***Seasonal adjustment as part of the STM.*

1.3.2.3 Autoregressive Model

Finally, there is another way to calculate output gaps we consider in this paper. The model is an unobserved components specification with permanent and transitory components. The permanent component is associated with GDP level (or "growth" in the Hodrick-Prescott language) while transitory component is the cycle component. Despite being very simple, this specification is among the most tractable one that allow imposing consistency between output gap measures and inflation dynamics[4]. The model is described as:

$$
\begin{aligned}
x_t &= \mu_t + z_t + \varepsilon_t \\
\mu_t &= \mu_{t-1} + \beta_t + \eta_t \\
\beta_t &= \beta_{t-1} + \zeta_t \\
z_t &= a_1 z_{t-1} + a_2 z_{t-2} + \omega_t
\end{aligned}
\qquad (1.12)
$$

The consistency between output gap and inflation can be imposed by estimating this model jointly with the Phillips curve. This model is similar to the Structural Time Series model (STM) presented earlier in subsection 1.2.2. The output gap (or cyclical component, z_t) in this model follows an AR(2)

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[3] *The estimated parameters appear in the table 1 of Harvey and Jaeguer (1993).*

[4] *This model is attributable to Watson (1986) and Kuttner (1994). The link between inflation and this measure of output gap will be presented in the next sections.*

process. The AR(2) structure is arbitrary. The common explanation for choosing this structure is that it allows more persistence in the cyclical component than an AR(1) specification. Also, since this is a cyclical or transitory component, the random walk structure is not applicable. We tested both an AR(1) and an AR(2) specification with quarterly GDP data and the results were similar. The AR(2) specification is the conventional formulation used in empirical research following Kuttner (1994) and Watson (1986).

We also estimated this model with a deterministic slope. The slope variable usually has a very low variance, particularly in short samples like ours. The deterministic slope model is also used by several authors as discussed in Orphanides and van Norden (2002) and Stock and Watson (1988). The deterministic slope model has less parameters to be estimated and can be useful in modeling Brazilian GDP. Using the total sample for Brazilian GDP as available from IBGE since 1Q96, the stochastic slope formulation fit the data better than the deterministic slope formulation. Using data starting in 1999, the two models lead to very similar results as the estimated variance of the slope component is close to zero. Note that the slope component is an approximation for the underlying growth of the economy. Using large samples, it is possible to identify periods of lower or higher growth. But in short samples this task is more difficult. In the case of Brazil, using data from 1996 the model estimate an increasing slope component: average GDP growth was low in the late 90's and increased after 2003 up to 2008.

Multivariate versions of the HP filter have also been proposed in the literature as a way to improve the HP filter as a measure of output gap consistent with inflation dynamics. The model described above however is simpler and easier to estimate and have been more widely used than the multivariate version of the Hodrick-Prescott filter.

In table 1.2 we present the summary statistics for the AR(1) and AR(2) model specification using quarterly GDP. The data used were seasonally adjusted by IBGE and we don't need to include the seasonal component as part of the model.

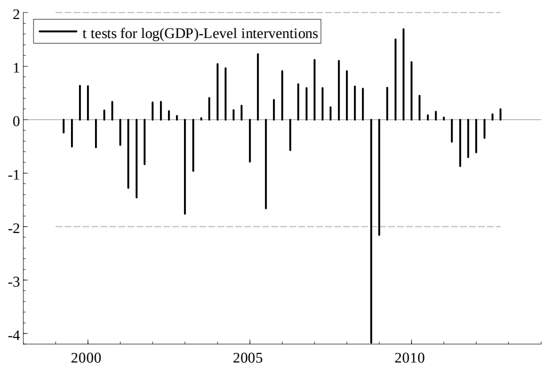
Table 1.2 – Brazilian GDP: Estimates of Structural Time Series (Unobserved Components) Model Paramaters (1Q96 to 3Q12)*

	σ_{ε}^{2}	σ_{η}^{2}	σ_{ζ}^{2}	σ_{ω}^{2}	AR1	AR2	AIC	SBC
Brazilian GDP	0,010	0,237	0,000	52,637	1,209	-0,365	-9,105	-8,994
Brazilian GDP	0,004	7,142	0,000	19,290	0,850		-8,884	-8,774

**Variances multiplied by* 10^5.

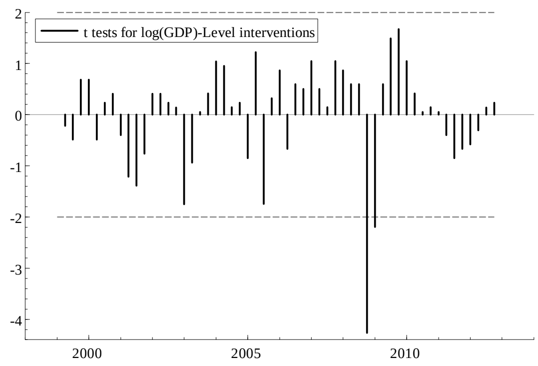
A possible problem with the structural time series (STM) approach is the existence of structural breaks in the series. In Fig. 1.2 we present a structural break test based on the auxiliary residuals. This is the standard test for structural break for models in State Space form. The auxiliary residuals are standardized residuals from the smoothed state. In our case, the auxiliary residuals are $\overline{\eta}_t / \sqrt{\operatorname{var}\left(\overline{\eta}_t\right)}$ where $\overline{\eta}_t$ are the residuals of the level state equation, $\overline{\mu}_t$.

Fig. 1.2a: Structural Break in Level



Stochastic slope, AR(1) formulation

Fig. 1.2b: Structural Break in Level



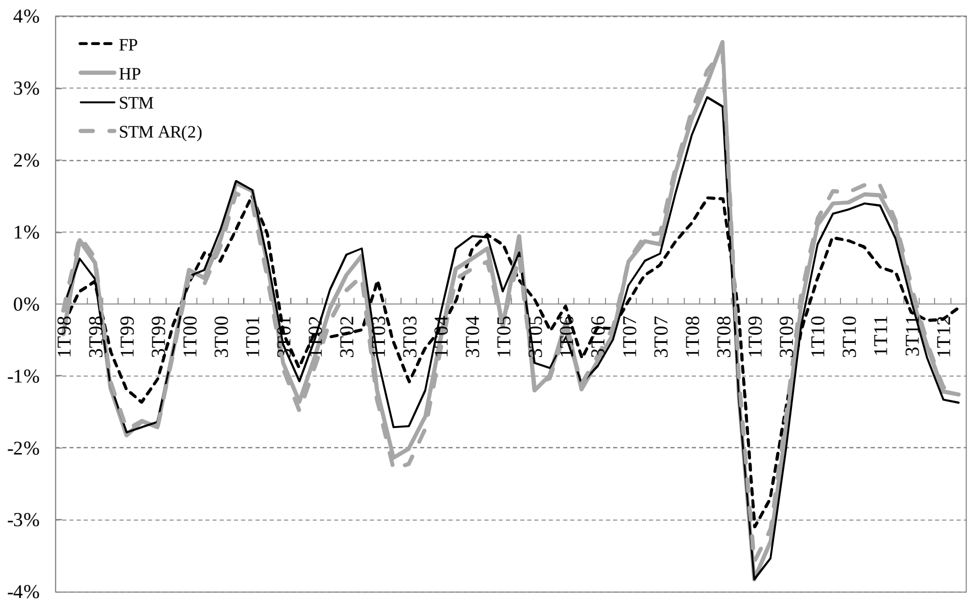
Deterministic slope, AR(1) formulation

1.3.2.4 Statistical Potential Output Measures: Concluding Remarks

In this section we presented several ways to extract "trend" and "cyclical" components from time series, particularly GDP. In the previous pages, we opt to use the conventional or original notation in every case. Note however that the components are similar in every case. As already stressed by Harvey and Jaeger (1993), the HP filter is a special case of the STM model[5]. In each of these models, the dependent variable is being decomposed into "trend" and "cyclical" components. The HP filter name the trend as "growth" component following the tradition of Real Business Cycle Theory (RBC) to focus on long term growth. The STM model and the AR(2) formulation uses a random walk to model the trend.

Despite these differences in statistical formulation and notation, these methods yield similar results. The figure bellow presents the cyclical component of each of these three methods.

Figure 1.3: GDP cycles – HP and UCM (STM) decomposition



[5] *More precisely, the HP filter can be written as a STM model with some restrictions on the parameters.*

1.4 Data Description

We construct our measure of output gap, *y*, using both the production function approach and statistical methods presented earlier. Capacity utilization is provided by FGV and unemployment rate by SEADE. The main advantage of using the Dieese/Seade unemployment rate is it longer data sample. The latest IBGE Employment and Unemployment survey starts in 2001 while the Dieese/Seade sample starts in 1998[6]. We also used the Brazilian GDP "quantity index" for the statistical measures of potential GDP.

For the interest rate series we use the 6 month swap rate as a measure of nominal interest rate. The reason for using the 6 months swap instead of the 12 months swap is the higher liquidity of this rate at the first half of our sample period. This rate was also used by the Brazilian Central Bank in their simulations in the first part of our sample. More recently BCB started to use the 12 months swap rate due to the good liquidity of this instrument. The difference between 6 months and 12 months swap rate was low during most part of our sample, including the second half (post 2005). The reason is that both rates include a term premium (or market premium) and also the forecast ability of market participants is low. The 12 months swap rate is more volatile in the first half of our sample but the level of both series are similar.

We use the 12 months ahead inflation expectation as a proxy for inflation expectations to form our measure of real interest rate. This is the simplest measure of inflation expectation and the one commonly used in empirical works in Brazil. It is also commonly used by the Brazilian Central Bank along with the current and next year inflation expectation.

The other variables used are the CRB commodity price index measured in R$. This is a measure of external price inflation and will be used in the specification of the Phillips curve. We also used a measure of external GDP growth (or external output gap) provided by OECD. The reason behind using this variable is to control the IS specification for other determinants of GDP growth (besides local interest rate) like higher foreign demand and also higher liquidity in global capital markets. An alternative way to consider these forces into our model would be to include a measure of interest rate in USD available for Brazilian companies or a more detailed specification of demand, including a specific equation for exports. The problem with these alternatives is the small sample size: the more variables we include into the model, the lower the power of the statistical tests and the lower the confidence on the estimated coefficients. The inclusion of external GDP growth was also included in several other empirical works both for Brazil or other countries. Finally, the importance of external GDP growth precedes the 2008 financial crisis as the work of Holland and Santos (2008) shows.

At this point it is important to mention what variables we do not include. The main variables used in empirical work not incorporated in our models are the real exchange rate misalignment and also a measure of fiscal imbalance (or fiscal instance). Determining exchange rate misalignment is a difficult task and there are few works on the subject in Brazil. Despite this, this variable could have played an important role in determining the output gap particularly in the period of fixed exchange rate regime. We consider the inclusion of exchange rate misalignment as a future refinement of this work. Studies on cyclically adjusted fiscal imbalances are even scarcer in Brazil.

[6] *We use the Dieese/Seade survey for several metropolitan areas, similar to the survey performed by IBGE.*

1.5 A Simple Model for Neutral Interest Rate Determination

1.5.1 Introduction

In this section we sketch a baseline new Keynesian macroeconomic model. Our intent is just to explain the theory behind the econometric exercise we will present in the next pages. We will not get into the details of the model and particularly on its premises and assumptions. The discussion is based in Gali and Gertler (2007) but there are several good references for the new keynesian model including graduate textbooks like Romer (2011), Gali (2008) and Woodford (2003). The traditional model, in its reduced form representation, is composed by tree equations: an aggregate demand equation (IS curve), an aggregate supply equation (Phillips curve) and a monetary policy reaction function (Taylor rule).

In a very simple form, the model can be written as[7]:

$$y_t = a_1 E_t y_{t+1} + a_2 (r_t - \bar{r}_t) + u_t \qquad \text{(IS equation)}$$
$$\pi_t = b_1 \pi_{t-1} + (1 - b_1) E_t \pi_{t+1} + b_2 y_{t-1} + e_t \qquad \text{(Phillips curve)}$$
$$i_t = c_1 i_{t-1} + (1 - c_1)(\bar{r}_t + \bar{\pi}_t + c_1 y_{t-1} + c_2 (E_t \pi_{t+1} - \bar{\pi}_t)) + v_t \qquad \text{(Taylor rule)}$$

Despite this being a simple model, there are several complication we will treat in our empirical tests. First, there is lags between changes in the real interest rate gap, $r_t - \bar{r}_t$, and its impact on the output gap. Second, the model as expressed above considerer that only current interest rate gap is important for the determination of the output gap but a more general formulation includes all expected future interest rate gaps as being the determinant of current output. We will use long term rates as a proxy for expected future rates.

In the Phillips curve equation we are only including the output gap and lagged inflation and expected inflation as explanatory variables for current inflation rate. In open economies, the external inflation is also an explanatory variable. And, as we mentioned in the IS equation case, there may be lags at which the output gaps affects inflation, so lagged output gap could also be an explanatory variable.

In our estimations, we will not always rely on joint estimation of this model. Our main objective is to make inference about the level of $\bar{r}_t$, the neutral real interest rate. This variable appears in the Taylor rule equation and also in the IS equation. Note, however, that the Central Bank do not know this level but also need to make some inference about it. It can be the case that the Central Bank do not follow a Taylor rule just like the one specified above or that the level of the neutral interest rate inferred by the Central Bank is different from the one we can infer using our specification for the IS curve. Taking this into consideration, we will not include the Taylor rule in most of our models. We estimate a neutral interest rate implicit in Selic target rate in the final part of this section. This is the only part of this paper where we estimate a Taylor rule and we estimate this equation on a standalone basis, i.e., without including of any other equation in the model.

Also, the joint estimation of the model can sometimes be misleading, particularly in cases where the Phillips curve cannot be easily estimated or in cases where the Phillips curve does not apply at all.

---

[7] *In this section and in the following sections, y_t will denote the output gap. In some of the previous examples, this variable represented the output. As we already mentioned, we are following the conventional notation in this paper. In some examples presented in section 2, we used the same notation used in the papers we were discussing.*

This seems to be the case of Brazil at least in the first half of our sample. Besides being in a process of disinflation that the output gap alone could not be able to explain, there were several episodes of exchange rate devaluation with sensible effects on inflation and also supply shocks like the energy shortage of 2001. Note that these are some identifiable challenges to estimate a Phillips curve in Brazil. Several others, less easily identifiable can be also present in the data.

We incorporate the external output gap, y_t^*, into the model in some specifications to capture the effects of a growing world economy into the Brazilian economy. Holland and Santos (2008) shows that this variable is statistically significant as a determinant of Brazilian output gap between 1996 and 2007. In the last years, several studies analyzed the synchronization of business cycles around the world like Kose, Prasad and Torrones (2003). We will not review this literature. It suffices to say that business cycles synchronization have been high for a long time and seems to have increased in the last decade. Among the possible explanations for this phenomena are the increasing commercial and financial integration. The debt and inflation crisis in Brazil in the 80's and 90's may have caused a reduction in the correlation between Brazilian GDP growth and GDP growth in other countries. During the late 90's and 00's this correlation may have increased again. Another explanation for the global business cycle synchronization is the occurrence of a common, global shock. The oil shocks of the 70's is one example of such a shock. More recently, the subprime crisis can be considered another example of a global shock. Independent of the causes, the synchronization of the Brazilian and world GDP growth seems to be an important source of information in the evolution of output gap in Brazil. Taking this into account, we include a measure of world output gap in our model as explanatory variable in some of our estimations.

1.5.2 IS Curve and Phillips Curve: Single Equation Estimation

In order to estimate a State Space model for the neutral interest rate it is important to define variance ratios between the equations's errors and also have some initial guess for the values of the parameters, including equation coeficients. Considering this, we present the estimation results for single equation estimation of the IS Curve and Phillips Curve using the same data we will use for the estimation of the State Space model. We used a HP filtered series in the real interest rate ($\overset{=}{r}_t$) as a proxy for the neutral interest rate in this first step[8].

Our estimated IS Curve is summarized by:

$$y_t = \underset{(0.128)}{0.934}\, y_{t-1} - \underset{(0.125)}{0.409}\, y_{t-2} - \underset{(0.065)}{0.152} \sum_{j=1}^{2} \frac{(r_{t-j} - \overset{=}{r}_{t-j})}{2} + u_t \qquad (1.13)$$

$$DW = 1.96, \quad SER = 0.0087$$

[8] *We denote $\overset{=}{r}_t$ the neutral rate derived from statistical methods like HP filter and $\bar{r}_t$ the neutral rate derived using an economic model.*

And our estimated Phillips Curve is summarized by:

$$\pi_t = \underset{(0.0032)}{0.0055} + \underset{(0.110)}{0.131}\pi_{t-1} + \underset{(0.272)}{0.519}\pi^E_{t-1} + \underset{(0.012)}{0.037}\pi^*_{t-1} + \underset{(0.063)}{0.068}\, y_{t-2} + e_t \qquad (1.14)$$

$$\overline{R}^2 = 0.669, \quad DW = 1.74, \quad SER = 0.0063$$

In our preferred specification, we do not include the measure of external output gap nor the dummy variable for the 4Q08, representing the 2008/09 crisis. In some specifications, we add these variables and the results are presented in the next sections. The reported coefficients for the IS Curve is based on the output gap measured using the HP filter. The results are similar when we use the production function approach but the standard error of the equation is lower in this case due to the lower volatility of this measure of output gap. We used the mean of two lags of the interest rate gap in our specification. We do this to reduce the number of parameters being estimated. Laubach and Williams (2003) among others use this specification. Some other authors like Garnier and Wilhelmsen (2005) use several lags of the interest rate gap. This specification is not appropriate to our small sample.

1.5.3 IS Curve and the Neutral Rate of Interest: the Simplest Model

The simplest model we want to estimate is given by just an IS equation and a random walk specification for the neutral interest rate. For illustration purpose, let's ignore for a while the fact that we use the average of the last two lags of the interest rate gap. This "simplest" model can be written as:

$$\begin{aligned} y_t &= \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y^*_t + \alpha_4 (r_{t-1} - \overline{r}_{t-1}) + u_t \\ \overline{r}_t &= \overline{r}_{t-1} + v_t \end{aligned} \qquad (1.15)$$

In the Kalman Filter specification, the first equation in (1.15) is the "signal" or "observation" equation and the second equation is the "state" or "transition" equation. In order to estimate this model using the Kalman Filter, some adjustments can be useful. First we drop out the constant term α_0 and rewrite the signal equation as:

$$y_t = -\alpha_4 \overline{r}_{t-1} + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y^*_t + \alpha_4 r_{t-1} + u_t \qquad (1.16)$$

Dropping out the constant term do not seem to be a problem since the variable has a zero mean in almost all specification for the output gap. Indeed, in our model the only variable with mean different from zero is the interest rate.

Define $\alpha_4 \overline{r}_{t-1} = \overline{r}^*_{t-1}$ to obtain:

$$\begin{aligned} y_t &= \overline{r}^*_{t-1} + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y^*_t + \alpha_4 r_{t-1} + u_t \\ \overline{r}^*_t &= \overline{r}^*_{t-1} + v^*_t \end{aligned} \qquad (1.17)$$

Written this way, the model can be estimated using the Kalman Filter procedure. It is important to note that the variance of the error term in the neutral interest rate equation has changed. Since $\alpha_4 \overline{r}_{t-1} = \overline{r}^*_{t-1}$, $\sigma^2_{v*} = \alpha_4^2 \sigma^2_v$. It means that the variance of the error term is lower in this state variable. As stressed by several authors, the maximum likelihood estimator (MLE) is biased when the one

equation variance is much larger than the variance of the other equations. This is called “pile-up effect”. To bypass this problem, some author recommends alternative estimators or the imposition of variance ratios. We will use variance ratios based on estimates of the equation on a standalone basis and filtering tools. This strategy is discussed by Laubach and Williams (2003) and used more explicitly by Mesonier and Renne (2007). This simple model with an exogenous output gap is used by Cour-Thimann *et al.* (2006) and used by the Brazilian Central Bank (BCB)[9]. We will use two lags of the interest rate like we did in (1.13) to estimate the model (1.17).

To estimate the model by the Kalman Filter, we used as a reference value for α_4 and σ_u^2 the coefficient presented in (6.1). For σ_v^2 used as a reference the variance of the “trend” component of a HP filtered sequence of the real interest rate. With these values, we impose a ratio between the variances in the two equations (σ_{v*}^2 and σ_u^2). This restriction is important only when the variance in one equation is much larger than the variance of the other equations. This was the case in our estimation. Without this restriction the optimization procedure did not reach convergence in our estimation. This is not a particular problem of our sample. Laubach and Williams (2003) discuss this problem in their estimation and also gives more information about the causes of this lack of convergence. We also tried ratios twice as large as the reference ratio proposed the single equation estimation and the results were similar. Different ratios lead to more or less volatile neutral interest rate series but similar coefficients. The downward trend in the estimated neutral rate is present with lower and higher variance ratios.

The figure below presents the results for the estimation of the neutral real interest rate in Brazil. The results differs depending on the specification of the IS curve and shows a larger divergence in the end of the sample, particularly just after the 2008/09 crisis. The inclusion of the foreign output gap makes the estimated natural interest rate more volatile. The inclusion of a dummy variable for the fourth quarter of 2008, the most acute moment of the sub-prime crisis, makes the natural interest rate less volatile. The international output gap, y*, is significant and the estimated coefficient varies from 0,3 in the model without the dummy variable to 0,2 in the model with the dummy. This impact is close to the one estimated by Holland and Santos (2008).

Table 1.3: Estimation Results of System (1.17) – 1Q99 to 3Q12

	Coefficient	Std. Error	z-Statistic	Prob.
α_1	0.933424	0.125566	7.433753	0.0000
α_2	-0.427405	0.111971	-3.817105	0.0001
α_4	-0.105809	0.076531	-1.382559	0.1668
$log(\sigma^2_u)$	-9.510592	0.180236	-52.76748	0.0000
	Final State	Root MSE	z-Statistic	Prob.
$-\alpha_4 r_T*$	0.005950	0.003159	1.883558	0.0596

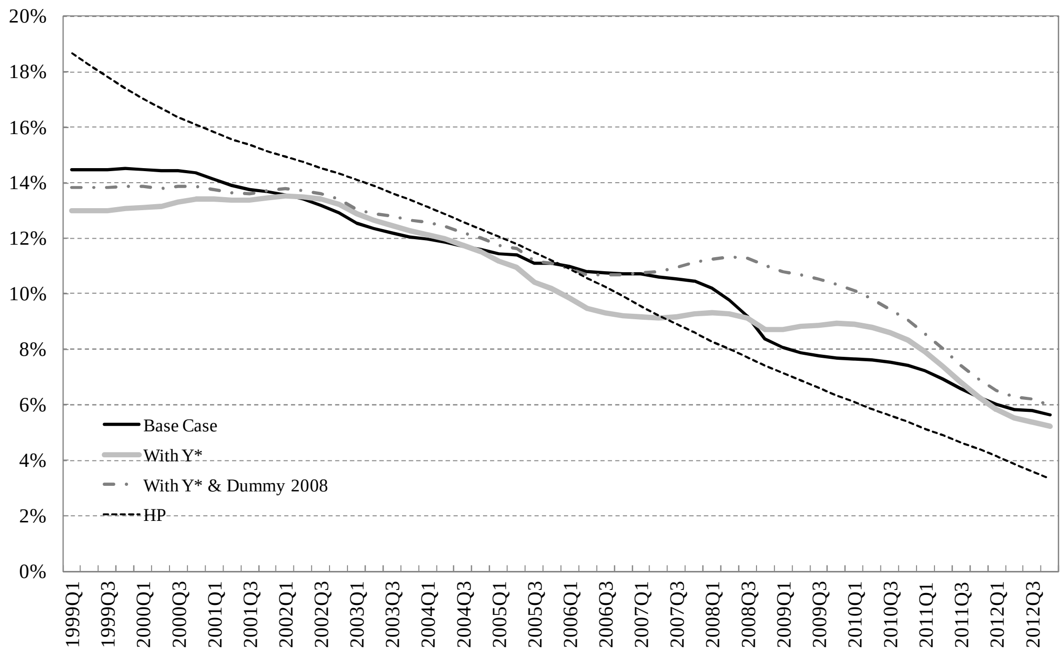
The estimated persistence of the output gap is close to 0,5 ($\alpha_1 + \alpha_2 = 0,506$). This is lower than the

[9] *In the 2Q12 and 2Q10 Inflation Report, the BCB presents a small macroeconomic model with an exogenous output gap. Like in our model, the neutral rate is determined using the constant and some other parameters in the IS equation.*

estimated persistence reported by other authors using data from US or other countries. For instance, Rudebuch and Svensson (1999) estimated a similar IS curve for US and the estimated persistence of the output gap is close to 0,9. The estimated persistence provided by Laubach and Williams (2003) is also close to 0,9. Garnier and Whilhelmsen (2005) estimated a similar model using data from US, Euro Area and Germany and the estimated persistence of the output gap is close to 0,9 using US data, close to 0,8 using data from Germany and Euro Area.

The estimated impact of the interest rate gap on the output gap is close to -0,1. This impact is close to the one estimated by Laubach and Williams (2003) and Rudebusch and Svensson (1999) using US data. We note however that in our case the estimated impact is not statistically significant at a 10% level. This result does not change materially if we include the dummy variable or the global output gap: in these cases, the estimated impact is significant at a 15% level but still not significant at a 10% level. The results changes when we consider a shorter sample by excluding the first years of our estimation period (for instance, 1999). We discuss this further in the next subsection. The small impact of the interest rate gap on the output gap is not a Brazilian phenomenon: Goodhart and Hoffman (2005) discuss the small impact of interest rate on the IS equation in several countries.

Fig. 1.4: Neutral Real Interest Rate Estimates for Brazil ($\bar{r}_t$ ***)***



Our results favor the hypothesis of a reduction in the equilibrium real interest rate in Brazil. The estimated equilibrium real interest rate (fig. 1.4) is as large as 14% in the beginning of the sample and diminishes to values close to 6% at the end of the sample. It is also worth mentioning the large dispersion of the estimation results at the beginning and at the end of the sample. This result seems to arise from the large variability of all variables at these points. Another important factor to consider is the exchange rate misalignment and also the fiscal instance at these moments. We did not include any of these variables in our estimation but we recognize that both variables are also important determinants of GDP growth and output gap. Both the exchange rate and the fiscal policy changed significantly during the quarters covered in our sample, particularly in the beginning and in the end (during the crisis period). We consider a more structural model for the IS curve as a possible future refinement of this paper.

At this point it is important to mention that the above estimates of the neutral rate include a term premium or a risk premium besides the expected future interest rates determined by the monetary authority. We didn't modeled the term premium in our estimation since we are dealing with neutral "effective" real interest rates instead of focusing on a real interest rate associated solely with the expected short term rates.

1.5.4 The Impact of Monetary Policy on Output Gap

There has been a lot of discussion during the last quarters on the impact of monetary policy on economic activity and particularly on output gap. The Brazilian Central Bank argued several times that the impact of monetary policy on output gap increased during the last years due to more credibility, increase in the credit/GDP ratio, among other factors[10]. In this section we perform the same "random coefficient approach" to estimate the impact of monetary policy on output gap.

To perform this calculation, we cannot use the estimated equilibrium interest rate estimated in the previous section since it was already extracted from a random coefficient estimation. Also, joint estimation of two random coefficients could not be performed since it would be difficult to reach convergence or the estimated standard deviations would be too large. To bypass these difficulties, we used the HP filter to construct the natural rate, denoted by $\overset{=}{r}_{t-1}$. We see the joint estimation of the neutral rate and the impact of interest rate on the output gap as a possible refinement of this paper. The availability of more data in the next year may facilitate this refinement.

To measure the impact of monetary policy on the output gap we estimated a time-varying coefficient model for the output gap as follow:

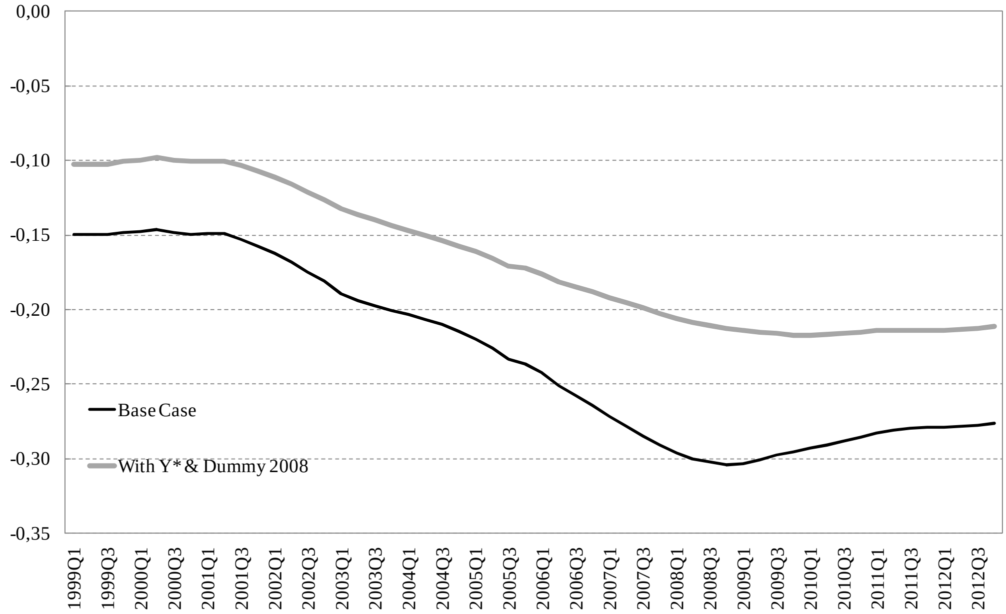
$$
\begin{aligned}
y_t &= \alpha_1 y_{t-1} + \alpha_1 y_{t-1} + \alpha_3 y_t^* + \alpha_{4,t} \sum_{j=1}^{2} \frac{(r_{t-j} - \overset{=}{r}_{t-j})}{2} + u_t \\
\alpha_{4,t} &= \alpha_{4,t-1} + v_t
\end{aligned}
\qquad (1.18)
$$

Table 1.4: Estimation results of system (1.18) – 1Q99 to 3Q12

	Coefficient	Std. Error	z-Statistic	Prob.
α_1	0.855497	0.152278	5.617991	0.0000
α_2	-0.316208	0.159131	-1.987102	0.0469
σ_u	-9.565882	0.265439	-36.03796	0.0000
$log(\sigma^2_v)$	-7.098783	4.147100	-1.711746	0.0869
	Final State	**Root MSE**	**z-Statistic**	**Prob.**
$\alpha_{4,T}$	-0.275956	0.140428	-1.965105	0.0494

[10] *This discussion was present in most of the meeting minutes of 2010 and 2011 and also present in several Inflation Reports in 2010 and 2011.*

Fig. 1.5: Impact of monetary policy on output gap ($\alpha_{4,t}$)



Our results indicate that the impact of monetary policy on output may have indeed increased in the last years. The estimated impact of monetary policy, measured as the "real interest rate gap", increased from close to -0,1 in the beginning of our sample to close to 0.25 at the end of sample. The estimation results are consistent with the evidence provided by other authors. Holland and Santos (2008) provided estimates for this coefficient close to 0.07 but using a sample that ended in 2008. If this coefficient increased in the last years as our results indicates, the 0.11 presented in the estimation of the system (1.18) is consistent with previous estimation. For United States, Rudebuch and Svensson (1999) also report a coefficient close to 0.10.

Most empirical work on the equilibrium interest rate are concerned with the estimation of the equilibrium rate and much less concerned with the estimated impact of "real interest rate gap", $R_{t-1} - \overline{R}_{t-1}$, on the output gap. This makes it difficult to compare our result with time-varying coefficient with other evidence. Boivin and Giannoni (2006) estimates a more structural model for United States and finds values for the impact of the real interest rate gap between 0.49 and 0.66, which is larger than to the values estimated here, even at the final part of our sample. Also, in this estimation with US data, the impact of monetary policy is larger using the real interest rate gap than the impact estimated using variation on the real interest rate.

We finish this sub-section discussing again the objective of estimating the impact of the interest rate gap on the output gap. We already discussed several challenges to estimating the neutral rate in Brazil like the small sample problem and now the time-varying parameter problem. The small sample creates several well-known problems in econometrics. In our case, we stress the fact that using a small sample make it difficult or even prohibit the inclusion of explanatory variables. As we discussed earlier, we are including the minimum number of lags of explanatory variables and avoiding including additional variables like the fiscal instance or a measure of exchange rate misalignment. The time-varying parameter problem will not be addressed directly in this paper since it requires more parameters to be

estimated which is troublesome particularly in the next section when we estimate a joint econometric model including the Phillips curve, the IS curve and the structural model of GDP.

1.5.5 Central Bank Reaction Function Approach

Another approach we can use for estimating the neutral rate of interest is to use the Central Bank reaction function (or Taylor rule). The reaction function can be written as:

$$i_t = \alpha i_{t-1} + (1-\alpha)(i_t^* + \beta_\pi(\pi_t^e - \overline{\pi}_t) + \beta_y y_t) + \varepsilon_t \qquad (1.19)$$
$$i_t^* = i_{t-1}^* + u_t$$

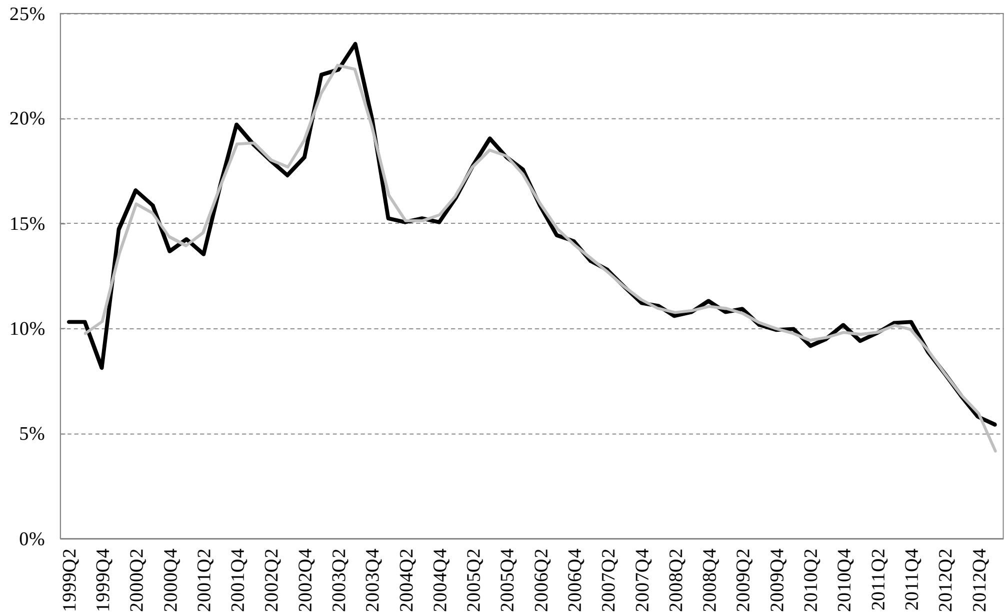
In this case, we are modeling a nominal (i_t^*) instead of real natural interest rate. We will assume the neutral real interest rate is given by $\overline{r}_t = i_{t-1}^* - \overline{\pi}_t$. This approach to the Central Bank target to real interest rate is similar to the presented in the original paper of Taylor (1993). The estimation a nominal "neutral" interest rate is more problematic when inflation target is not constant or long run inflation expectation is volatile. This seems to be the case in Brazil in the first half of our sample. Considering this, we will focus our discussion on the results for the last part of our sample.

The results of our estimation indicates that the "target" rate implicit in the estimated Taylor rule diminished in the last years from a maximum of more than 20% to close to 10.5%. Note that the inflation target in Brazil in the last years had been fixed at 4.5%. This indicates that the target "real" interest rate was close to 6% in the last years. This result is line with the indications presented in the Copom minutes: the minutes did not mention a specific value, but the minutes indicated sometimes when real rates were close to 6% that the real rate was close to the equilibrium levels. More important, the minutes indicated several times that the equilibrium real interest rate in Brazil was trending lower which we judge is consistent with the Central Bank targeting a lower real interest rate.

Table 1.5: Estimation results of system (1.19) – 1Q99 to 3Q12

	Coefficient	Std. Error	z-Statistic	Prob.
α	0.554908	0.042547	13.04231	0.0000
β_π	1.325374	0.389416	3.403492	0.0007
β_y	0.727678	0.656254	1.108836	0.2675
$log(\sigma^2_\varepsilon)$	-10.48780	0.629929	-16.64918	0.0000
$log(\sigma^2_u)$	-7.741062	0.381549	-20.28852	0.0000
	Final State	Root MSE	z-Statistic	Prob.
i_T^*	0.054418	0.023377	2.327778	0.0199

Fig. 1.6: Natural Interest Rate - Central Bank Reaction Function Approach



1.6 Joint Estimation of Inflation, REIR gap and Output Cycles

This is the main part of this paper. In this section we jointly estimate the output gap, represented by the cyclical component of GDP and also the neutral rate of interest. We begin by estimating the output gap jointly with the Phillips curve and later we include the neutral interest rate equation into the model. We already mentioned that the joint estimation is difficult because of several factors like possible changes in coefficients values over the sample period, large residuals in some equations and also small sample. The Inflation Target regime was adopted in 1999 and before that the reference interest rate was not the monetary policy instrument. So, before 1999 it is difficult to rely on the reference rate to infer the neutral interest rate. The Real Plan was implemented in 1994 and inflation fell abruptly right after that. But the inflation rate in the first years of the Real Plan was still high and inflation presents a clear downward trend up to 1996 or 1997. Finally, the GDP series have been revised over the last years and the new methodology provides quarterly GDP data only from 1996. These limitations will have an important role in our empirical exercise presented in this section.

1.6.1 Joint Estimation of Output Gap and Phillips Curve: The AR(2) Model

Before we proced to the estimation of the complete model, we estimate the structural time series model (STM) for GDP jointly with the Phillips curve. The objective of this subsection is to introduce the main difficulties of estimation the STM using the State Space formulation and also discuss the merits of including the Phillips curve in the estimation of the output gap.

The simple model we will estimate can be written as:

$$\pi_t = a_\pi + b_\pi \pi_{t-1} + b_{\pi*}\pi^*_{t-1} + b_y z_{t-2} + \varepsilon^\pi_t \quad (1.20a)$$

$$y_t = \mu^*_t + z_t + \varepsilon^y_t \quad (1.20b)$$

$$\mu^*_t = \mu^*_{t-1} + \beta_{t-1} + \varepsilon^\mu_t \quad (1.20c)$$

$$\beta_t = \beta_{t-1} + \varepsilon^\beta_t \quad (1.20d)$$

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \varepsilon^z_t \quad (1.20e)$$

This model was firstly estimated using data starting in 1996 and we do not include the inflation expectation variable since this data started to be collected by the Central Bank in 1999. We used the lagged output gap with two lags as suggested using the single equation estimation. In this model, several parameters are constrained to one and are not estimated. But even with this simplification, several parameters need to be estimated: 5 variances and 5 coefficients. This is a large number of parameters considering that this "long" sample has only 66 observations.

We also estimated this model with an exogenous real equilibrium interest rate (REIR) gap. In this case, we rewrite equation (1.20e) as:

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \phi_3 \sum_{j=1}^{2} (r_{t-j} - \overline{\overline{r}}_{t-j})/2 + \varepsilon^z_t \quad (1.20e')$$

Our proxy for the real equilibrium interest rate in this case is the HP filtered real interest rate. The estimation results of these models are presented in table 1.6.

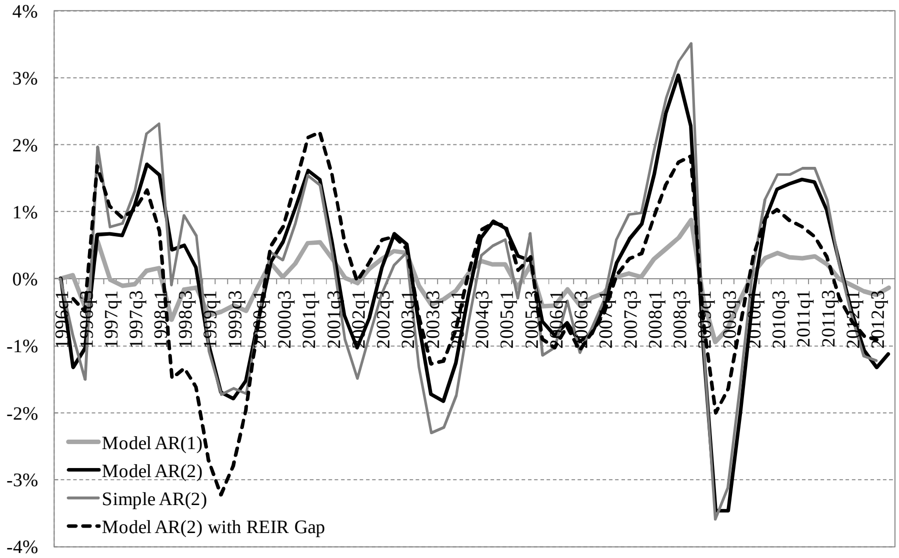
Table 1.6: Estimation Results of System (1.20) – 1Q96 to 3Q12

	AR(1) Model		AR (2) model		AR (2) model + REIR Gap	
	Coef.	Std. error	Coef.	Std. error	Coef.	Std. error
a	0,0083	0,0016 ***	0,0081	0,0015 ***	0,0083	0,0015 ***
b_π	0,3776	0,0924 ***	0,3880	0,0846 ***	0,3718	0,0863 ***
$b_{\pi*}$	0,0195	0,0074 ***	0,0191	0,0074 ***	0,0210	0,0075 ***
b_y	0,1291	0,4717	0,0112	0,0617	0,0806	0,0799
$\phi 1$	0,6739	0,5104	1,2530	n.a.	0,8669	0,4276 ***
$\phi 2$			-0,5904	n.a.	-0,1470	0,4016
$\phi 3$					-0,1424	0,0697 **
Variances ($x10^5$)						
μ^*	11,8100	12,2300	0,0000	0,7480	7,0000	9,2100
β	0,0079	0,0614	0,0690	0,0561 *	0,0319	0,0759
z	3,2700	10,4600	5,3800	1,8200 ***	4,9300	6,3500
π	3,8200	0,7470 ***	3,9200	0,6770 ***	3,7900	0,6760 ***
y	0,0000	n.a.	2,9000	1,1200 ***	0,0001	3,1600

The AR(1) model lead to poor results. Some parameters of the model could not be estimated and the cyclical component of output is less persistent than in other specifications of the model. The AR(2) specification of the cyclical component of output with the Phillips curve lead to a similar result to the model without inflation (see tables 1.2 and 1.6). This can be seen from fig. 1.7: there is a large correlation between the estimated cyclical component of output from these two models. Adding the REIR gap to the model leads to some small differences in the coefficients of the model and also on the

dynamics of the output gap: the estimated output gap in this case is more volatile in the first part of the sample and less volatile in the final years. However, the impact of REIR gap on the cyclical component of output is closer to the single equation estimation of the IS Curve.

Figure 1.7: Estimation Results of System (1.20) – output gap



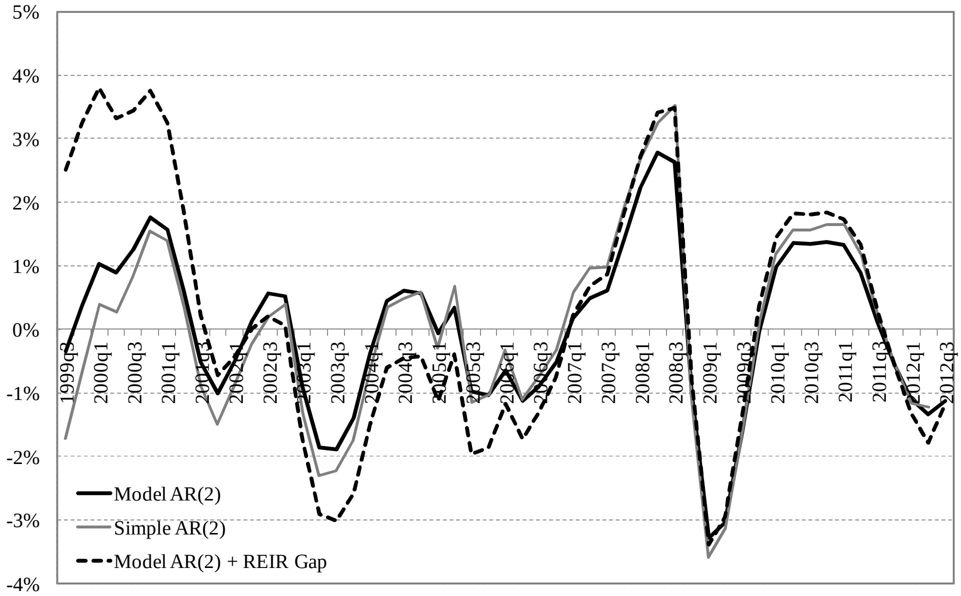
We also estimated a simpler version of the model using data starting in 1999. In this case, we made more simplification into the model. The main simplification is to use a deterministic slope formulation. In this case, $\beta_t=\beta=cte$. Doing this, we eliminate one parameter in the estimation process, $var(\beta_t)$. This simplification is also important to facilitate the convergence of the model parameters. We include the inflation expectation variable in the Phillips Curve equation. The estimation results are presented in table 1.7 and figure 1.8.

Table 1.7: Estimation Results of System (1.20) – short sample (1Q99 to 3Q12)

Deterministic trend, AR(2) model			Deterministic trend, AR(2) model + REIR Gap		
	Coef.	Std. error		Coef.	Std. error
a	0,0035	0,0030	*a*	0,0034	0,0029
$b\ \pi$	0,1390	0,1096	$b\ \pi$	0,1470	0,1076
$b\ \pi e$	0,6564	0,2908 **	$b\ \pi e$	0,6497	0,2893 **
$b\ \pi *$	0,0351	0,0083 ***	$b\ \pi *$	0,0344	0,0084 ***
$b\ y$	0,0543	0,0601	$b\ y$	0,0303	0,0354
c (level)	0,0087	0,0006 ***	*c (level)*	0,0093	0,0003 ***
$\phi 1$	1,2455	0,1873 ***	$\phi 1$	1,1125	0,1818 ***
$\phi 2$	-0,5888	0,1973 ***	$\phi 2$	-0,3773	0,1698 **
			$\phi 3$	-0,2490	0,0783 ***
Variances ($x10^5$)			*Variances* ($x10^5$)		
$\mu *$	3,1700	3,2200	$\mu *$	0,0000	n.a.
z	5,6000	3,3500 **	z	6,2800	2,6400 ***
π	2,5900	0,5080 ***	π	2,6200	0,5060 ***
y	0,0156	1,0500	y	0,6000	1,0600

The estimated impact of the output gap on inflation is lower in this model than in the single equation estimation presented in section 1.5.2 despite the estimated coefficients of the IS Curve being similar to those obtained from the single equation estimation. The impact of REIR gap is higher in this short sample estimation: this result was expected due to the increase in the impact of monetary policy on output gap discussed earlier. The cyclical component of output is more volatile in the first part of this short sample. This can be related to the high volatile inflation data in this first part of the sample.

Figure 1.8: Estimation Results of System (1.20) – output gap (short sample)



1.6.2 The complete model

We will use the AR(2) model to joint estimate the output gap and equilibrium interest rate. The complete model can be written as:

$$\pi_t = a_\pi + b_\pi \pi_{t-1} + b_{\pi*}\pi^*_{t-1} + b_y z_{t-1} + \varepsilon^\pi_t \qquad (1.21a)$$

$$y_t = \mu^*_t + z_t + \varepsilon^y_t \qquad (1.21b)$$

$$\mu^*_t = \mu^*_{t-1} + \beta_t + \varepsilon^\mu_t \qquad (1.21c)$$

$$\beta_t = \beta_{t-1} + \varepsilon^\beta_t \qquad (1.21d)$$

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \phi_3 \sum_{j=1}^{2}(r_{t-j} - \overline{r}_{t-j})/2 + \varepsilon^z_t \qquad (1.21e)$$

$$\overline{r}_t = \overline{r}_{t-1} + \varepsilon^r_t \qquad (1.21f)$$

This model is similar to the one presented in section 1.3. The main differences are: i) we included a Phillips curve in the model, ii) the real interest rate gap enters into the cyclical component of GDP as a explanatory variable and iii) the equilibrium interest rate is modeled as random walk in the model instead of using some predetermined value. Equations (1.21a), (1.21b) and (1.21e) are observation equations and equations (1.21c), (1.21d) and (1.21f) are state equations. Equations (1.21a) to (1.21e)

form the standard structural time series model for GDP with the only difference being the regression effect given by $\varphi_3(r_{t-1} - \overline{r}_{t-1})$.

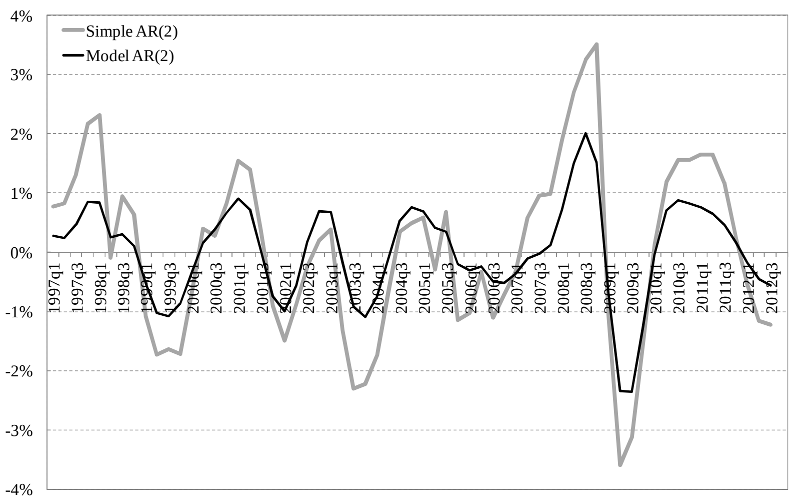
As we already mentioned, we estimate some of these variables previously to create initial values for some parameters of the model. Equations (1.21a) and (1.21d) were estimated using simple OLS regressions. The structural time series model was estimated using the Kalman filter. The model is complex and several different parameters need to be estimated. This problem is discussed in more detail by Orphanides and von Norden (2002). Due to the large number of parameters being estimated, we estimated the model using the long sample (1996 to 2012). The main advantage of using the long sample is to better modeling GDP components, including the output gap. The main disadvantage of using this sample is the difficulty in estimating the Phillips curve. Inflation was very volatile in the first half of our sample and there is not inflation expectation data in this part of the sample.

In table 1.8 we present the results from the estimation of the complete model. The results are similar to the ones presented earlier. The estimated coefficient of output gap on inflation is close to the estimated in other specifications of the model. This coefficient is not statistically significant at conventional levels. This result is similar to the ones presented earlier both with output gap measures generated in the model and output gap measures generated outside the model. The similarity in these results may arise as a consequence of the estimation process: in all cases, the initial values of parameters where chosen using the single equation results presented earlier. Besides that, the AR(2) specification for the cyclical component of output was also used in most of estimations performed in this paper.

Table 1.8 – Estimation results

	AR(2) Model		
	Coef.	Std. error	
$b\,\pi$	0,6404	0,0403	***
$b\,\pi*$	0,0426	0,0125	***
$b\,y$	0,1417	0,1436	
$\phi\,1$	0,7009	0,5325	
$\phi\,2$	-0,2066	0,3830	
$\phi\,3$	-0,2098	0,1897	
Variances (x10^5)			
$\mu*$	0,0590	13,3600	
β	0,2860	0,3400	
z	5,4700	6,4700	
π	8,8500	1,5700	***
y	1,7300	4,1500	
r	5,4000	n.a.	

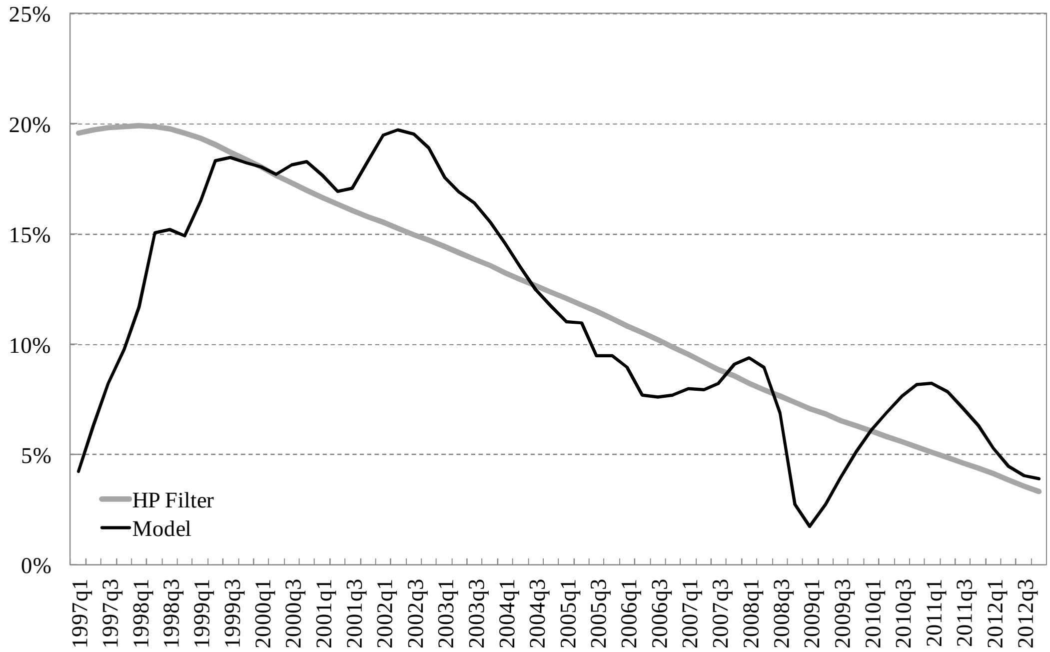
Figure 1.9 – Estimation results of system (1.21) – output gap



We present the estimated neutral real interest rate in figure 1.10 together with the HP filtered real interest rate data. The estimated neutral interest rate is more volatile than the HP filtered data. The estimated REIR is low at the beginning of the sample and increases in the early 90's. This result seems to be a consequence of the estimation processes that ties the REIR to the observed real interest rate with the ϕ_3 coefficient in (1.21d). The real interest rate at the beginning of our sample is low due to

large inflation rather than low nominal interest rate as in the final part of the sample. According to our estimation, the REIR decreased in Brazil over the last years, particularly after 2002 (this occurs in almost all estimations). There is a large decrease in the estimated REIR during the financial crisis of 2008/9. We attribute this result to the sharp fall in output in this period. We tried to estimate the model using a dummy variable in this period and also a measure of foreign output gap as in section 1.5. However, in this case, the estimation procedure did not converge to any interpretable result and the results of this estimation were not reported.

Figure 1.10 – Estimation results of system (3.21) – neutral interest rate



1.7 Real Interest Rates: International Comparison

1.7.1 Overview

In this section we discuss the evolution of real interest rates in the world over the last 20 to 30 years. The discussion is brief and serves just as an introduction to the theme. The objective of this section is just to introduce possible explanations for the reduction in the neutral rate of interest in Brazil over the last years. We do not, however, present empirical evidence for any of these potential explanations for the reduction in interest rates in the last years. Segura-Ubiergo (2012) presents an empirical investigation on the determinants of real interest rates with data covering the period from 1980 to 2009 for a large group of countries with focus in Brazil. In section 1.7.3 we discuss the evolution of the marginal productivity of capital, a variable that is often used as a proxy for real equilibrium interest rate.

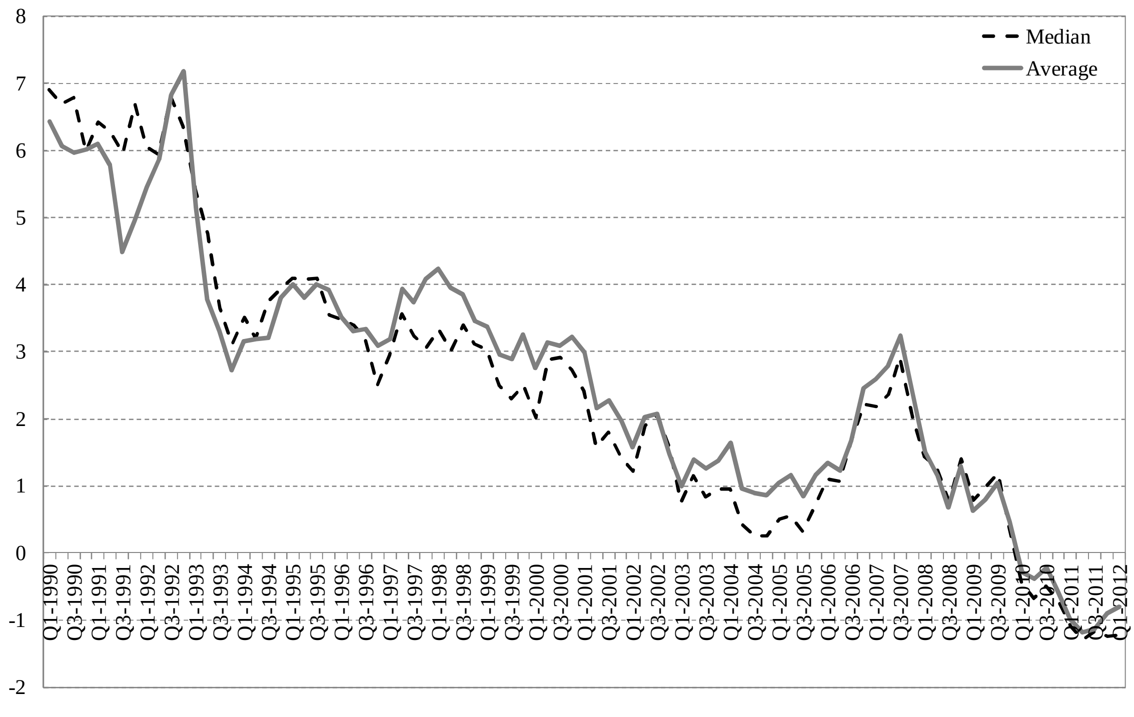
Interest rates were high during the 80’s and also 90’s. Several authors offer an overview of interest rates across countries like Blanchard and Summers (1984), Barro and Sal-i-Martin (1990) and Chadha

and Dimsdale (1999). Among the common explanation for high interest rates in the 80's and in the 90's are the large budget deficits in several countries, tight monetary policy, high inflation and also inflation and overall economic uncertainty. Other common explanation is the opportunity of high returns on investments.

Over the last ten years, interest rates diminished in several countries, particularly after the global crisis of 2008/09. In this period, low interest rate is mainly a result of loose monetary policy which is a consequence of low GDP growth worldwide. In some countries, yields on bonds have raised after the crisis as a consequence of large deficits and debts as percentage of GDP increasing risk premiums. However, this did not happen in the largest countries like US and Germany. Also, emerging markets like the so called BRICs still have bellow average deficits and debts. The rise in bond yields has been concentrated in some European countries.

Among the common explanation for this reduction in real interest are the reduction on government debt and deficits over the last years, particularly in the 90's, the reduction in macroeconomic volatility and particularly the reduction in inflation and inflation volatility or inflation risk. This reduction in overall macroeconomic volatility is called "great moderation" by several authors. The absence of global shocks like the oil shocks of the 70's and also the debt crisis of the 80's is a potential explanation for this lower volatility.

Fig. 1.11: World (OECD) Real Interest Rate (in %)



***Countries**: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Iceland, Ireland, Israel, Italy, Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, United Kingdom and United States. Real rate calculated using last twelve months inflation and quarterly short term interest rates. All data from OECD Stats (stats.oecd.org).*

Some authors like Goodfriend (2007) attribute the lower volatility to better macroeconomic policies. According to him, some influential theories include the Rational Expectations Theory and the

abandonment of the idea of a permanent trade-off between inflation and unemployment, leading to less active monetary policy in the late 80's. The introduction of Inflation Targeting regime in several countries and the compromise with low and steady inflation also helped in this process of reducing macroeconomic volatility. Another important factor related to all of these others was the improvement in communication and increased credibility of police makers. Using the "financial" view of market interest rates as risk-free rates plus market premiums for risk factors, the reduction in inflation risk could have caused a reduction in inflation risk premium and also the overall reduction in macroeconomic and benchmark interest rate volatility could have caused a reduction in overall risk premium. The reduction in government debt and deficits could have lead to a reduction in default premiums.

Another explanation for lower rates is the reduction in the productivity of capital. We discuss this point in more detail in the next subsection along with data on worldwide productivity of capital.

1.7.2 The Country Risk Premium Approach

A common approach to infer about the neutral rate in a small open economy is to decompose the local rate into the international risk-free rate plus a country risk premium. This is the standard approach in international economy textbooks. Note that this is not one of the determinants of the neutral interest rate according the tradition Wicksell view. This view is based on some aspect on free capital mobility among countries and also on the notion of financial arbitrage among market participants. This view is usually expressed in the Copom minutes in Brazil and we will discuss it briefly here. In Santos (2012) this view is discussed in more detail. This "financial" view of interest rate can be expressed as:

$$i_t = i_t^* + \Delta e_t^E + \theta_t \qquad (1.22)$$

Where i_t is the local interest rate, i_t^* is the international benchmark, Δe_t^E is the expected exchange rate depreciation and θ_t is the country risk premium. If we assume that exchange rates are well represented by a random walk, $\Delta e_t^E = 0$ and we can focus on the other two components of the relation: the international benchmark and the country risk premium.

Usually, the international benchmark is the FED Funds Rate or the US Treasury rates from long term bonds. Whatever this rate is, it probably decreased over the last years as we discussed in the previous sub-section. For instance, the long-term treasury rates in US hovered around 5% to 6% before the 2008/09 crisis and lately have been close to 2%.

The country risk premium also lowered in the last years. As we will present in more detail in Essay 3, the Brazilian country risk measured by the EMBI+ decreased from more than 5% in 2004 to 2% over the last two to three years. The 5 year *Credit Default Swap (CDS) Rate*, another measure of country risk, decreased from more than 5% in 2004 to close to 1% in the last two years. Note that the reduction in i_t^* and θ_t are close to 4% each one. Using (7.1) and these measures of international interest rate and country risk, the reduction in local rate should be close to 8%. This is approximately what happened in this period: the Selic rate was close to 15% in 2004 and decreased to close to 8% in 2012.

1.7.3 The Marginal Product of Capital

As we have illustrated in the last pages, several explanations have been proposed for the reduction of interest rates over the last years, including lower inflation risk, lower exchange rate risk and lower

debt and deficits in several countries since the 90's. Another possible explanation is the reduction in the productivity of capital. The productivity of capital plays a central role in the determination of the equilibrium interest rate. However, this variable is not readily available as data on inflation or debt or deficits. Kfoury and Nakane (2006) offer an explanation of equilibrium interest rates based on the productivity of capital.

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha} \tag{1.23}$$

This is the same expression as (3.2) with the difference that now we include the subscript *i* to denote the country. With this production function, the marginal product of capital is given by:

$$\frac{\partial Y_{i,t}}{\partial K_{i,t}} = \frac{\alpha A_{i,t} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}}{K_{i,t}} = \alpha \frac{Y_{i,t}}{K_{i,t}} = \frac{\alpha}{K_{i,t} / Y_{i,t}} \tag{1.24}$$

Another common form to express relations (1.23) and (1.24) is to use capital ($k_{i,t} = K_{i,t} / L_{i,t}$) and output per worker ($y_{i,t} = Y_{i,t} / L_{i,t}$). The reason behind this formulation is that output per worker is directly associated with capital per worker. Also, under certain circumstances or assumptions, the marginal product of capital should be higher in countries where the stock of capital is lower. This proposition follows two assumptions: the production function has decreasing returns to scale ($\partial^2 y / \partial k^2 < 0$) and there is some form of restriction to capital flows that avoid the equalization of capital's return among countries. The figures bellow present the output per worker and capital per worker relation using data from the Penn World Table version 7.1 and also the marginal product of capital. The capital stock was calculated using the perpetual inventory method in the same way as in Caselli and Feyrer (2007).

Some complications arise in the computation of (1.24). First, the capital share, α, may change over time and its calculation may differ among countries. Caselli and Feyrer (2007) use the same source for all α_i but do not discuss the possibility of this variable changing over time. Another complication is how to account for different price levels of output and investment goods. When the price level of output, P_y, is growing faster than the price level of capital, P_k, the return on capital investment is larger than specified in (1.24). Difference in the relative price of capital among countries makes it harder to compare the marginal product of capital among countries. We will discuss the evolution of the relative price of capital in Brazil and also the evolution of capital's share in our empirical exercise. But this analysis is just a complement to the empirical analysis of the neutral rate in Brazil from the previous section. A more detailed investigation of the marginal product of capital in Brazil would need to be done separately as new paper.

We presented the evolution of the labor share, $1-\alpha$, in section 1.3. As one can see, the labor share increased over the last years which implies the capital share decreased. Over the last years, particularly after 2004, there were a sensible acceleration in the growth rates of workers and also earnings per workers. This rise occurred after a period of high unemployment and very low earnings per worker in the late 90's and early 2000's. Another possible explanation for the increase in labor share is the "formalization" of workers if the labor share reported in National Accounts do not account for part of the self-employed workers. This increase in labor share in Brazil is a local phenomenon: on average, the labor share decreased in the world as pointed by several authors like Rodriguez and Jayadev (2010).

Fig. 3.12a: Capital and output per worker

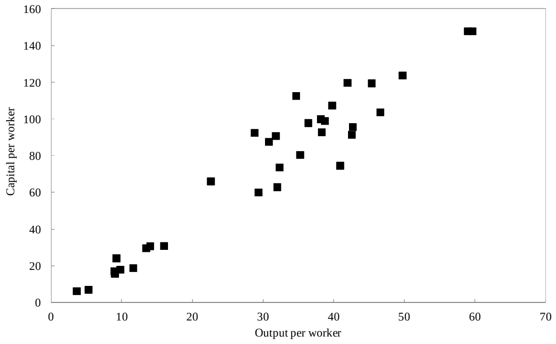
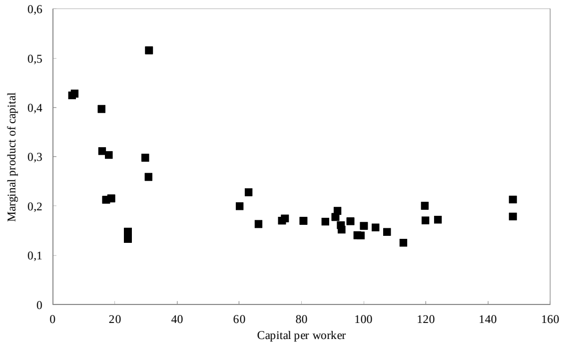


Fig. 3.12b: Marginal product of capital



Source:Penn World Table 7.1

Using this naïve decomposition with data from PWT 7.1 it is difficult to have some indication of why interest rate is so high in Brazil. In most cases, the marginal product of capital lies between 10% and 20%[11]. This statement is more appropriate to countries with higher capital per worker but several countries with low capital per worker also have lower interest rate than Brazil.

1.8 Final Remarks from Essay 1

In this paper we used different methodologies to estimate the output gap and the neutral interest rate. Our results indicate little difference between the output gap estimates obtained from different methods. In particular, we call attention to the small difference from estimates obtained from purely statistical methods and more structural methods.

Our results indicate that the neutral real interest rate in Brazil decreased in the last years. However, the size of this reduction differs depending on the specification of the IS curve. The results also points to a high variability of the equilibrium real rates, particularly in the final part of our sample. This result can be attributed to the high volatility of output in Brazil and also in other countries following the financial crisis of 2008. The inclusion of the foreign output gap and also a dummy variable for the 2008/09 crisis lead to a sensible impact on the estimates of the neutral interest rate and also on the coefficients of the IS curve. In our estimations of the IS curve, we did not include measures of the fiscal instance or real exchange rate misalignment. We judge this could be a direction for further research.

The estimation of a structural time series model for GDP jointly with the Phillips curve and the IS curve lead to results that are similar to the obtained from the equations being estimated independently. In particular, the output gap estimated using the structural time series model alone or jointly with the IS curve are very similar. As a consequence, the estimation of the output gap jointly with the Phillips curve does not meaningfully change the coefficients in the Phllips curve.

Despite these uninspiring results we judge the availability of more information in the next years would improve the estimation results since several simplifying assumptions were made in order to reduce the loss of degrees of freedom in the estimation. The joint estimation can be performed only after 1999 when the inflation targeting regime was implemented and the interest rate started to be used as the monetary policy instrument. This restriction creates a small sample of no more than 56 observations in

[11] *Our estimates are higher than those presented by Caselli and Feyrer (2007) using other version of PWT.*

2012 to be used to estimate more than 10 parameters. Besides that, there is some evidence of time-varying parameters, particularly in the first years of our sample.

The causes of this reduction in the natural interest rate are not clear. Interest rates is going lower in the world since early 90's and real interest rates became negative in several countries after the 2008/09 financial crisis. Besides that, several measures of Brazilian country risk like the EMBI+ or the *Credit Default Swap* (CDS) rate declined over the last years. This is consistent with the view of interest rate convergence. The reduction in the marginal product of capital is another explanation for the reduction in neutral interest rate. However, our descriptive analysis of the marginal product of capital in several countries and also in Brazil over the last years does not indicate that the marginal product of capital decreased.

2 Why is Brazilian Inflation so High? Inflation Persistence in Brazil and other Emerging Markets

2.1 Introduction

After reaching a hyperinflation environment for some years, Brazilian inflation decreased to more normal levels after the introduction of the Real Plan in 1994. Despite the sharp fall in Brazilian inflation and the convergence to the level observed in the emerging market economies, it is still higher than those observed in developed countries.

The aim of this paper is to compare inflation persistence in Brazil with other emerging markets, particularly those with a long history of high inflation. We consider this a first step to better understand what lies beneath the higher inflation in Brazil. Other important differences may be due to larger marginal costs (positive output gaps), larger pass-through of commodity price shocks, larger inflation expectations, among others. All these aspects may be further investigated in future works related to Brazilian inflation. Larger inflation persistence could help explain why Brazilian inflation is higher than in other countries: As the country evolves from a high inflation to a low inflation environment, large persistence rates could make this process slower in some countries.

Table 2.1: Inflation in Selected Emerging Markets (1980-2010)

	1980 to 1994	1995 to 2010	1995 to 1999	2000 to 2004	2005 to 2010
Brazil	**782,1**	**10,7**	**19,4**	**8,7**	**5,0**
Chile	20,1	4,2	6,0	2,8	3,8
Colombia	24,4	9,7	17,9	7,3	4,7
Czech Republic	n.a.	4,0	7,5	2,6	2,7
Hungary	14,5	10,0	18,9	7,1	5,1
Israel	81,9	4,0	8,2	1,6	2,4
Mexico	51,5	11,2	24,5	6,0	4,4
Peru	856,6	4,4	8,4	2,4	2,6
Poland	91,0	7,5	16,4	4,4	2,6
South Africa	13,9	6,5	7,3	5,5	6,6
Turkey	58,6	40,3	81,0	37,7	8,7
Mean	**199,5**	**10,2**	**19,6**	**7,8**	**4,4**
Median	**55,0**	**7,5**	**16,4**	**5,5**	**4,4**

Source: IMF

In this paper we focus on emerging markets inflation, particularly on countries with a history of high inflation in recent years. The question we address is why Brazilian inflation is higher than inflation rates in other emerging markets. We do not exactly aim to answer why inflation is higher in emerging markets. We think this could be an important step towards understanding why some countries reach lower and stable inflation than others.

We do not deal with some aspects of inflation that could explain large inflation in some countries such as the behavior of inflation expectations and the effect of indexation. Indexation mechanisms are related to inflation persistence but they are not the same. The main reason not to deal with these questions is the lack of comparable variables in each country. Therefore, a few countries have data on

inflation expectations and usually the methodology varies from country to country; that makes comparisons very difficulty. The characterization of inflation expectations data in each country, its evolution and impacts on current inflation is undoubtedly a subject that should be further investigated. In this paper we use inflation expectation data only for the estimation of the Phillips curve in Brazil. And, as we already mentioned, a deeper analysis of inflation expectations in Brazil and other countries is left for future research.

The countries we include in our sample of emerging markets with a history of high inflation in recent years are: Brazil, Colombia, Peru, Chile and México in Latin America; South Africa in Africa; Czech Republic, Poland and Hungary in Europe and Israel in Asia. All these countries have a history of high or hyperinflation early in the 90's and they have all pursued low inflation through an inflation target regime during the last ten years or more. We do include neither developed countries nor emerging countries in Asia in our sample due to their long history of low inflation.

Our results indicate that inflation persistence in Brazil between 1995 and 2011 is larger than estimated for other emerging markets in the same period. This result is valid both for headline inflation and core inflation measures and is based in simple autoregressive models. Using the Hansen (1999) method of *median unbiased estimation* the coefficient is close to our sample mean of the emerging markets inflation persistence. The inflation persistence in Brazil is also close to persistence estimated for developed countries by other authors.

Our result indicates that core inflation is more persistent than headline inflation. This result arises as a consequence of the exclusion of food products from core inflation. Regarding the determinants of inflation persistence, our results are inconclusive as the persistence of most variables used in our Phillips Curves estimation is very close to each other.

Despite the relatively large inflation persistence in Brazil, our proxy for expected inflation has a larger coefficient than lagged inflation in our estimated Phillips Curve. This result is more pronounced when we use disaggregated than aggregated data. Our results indicate that disaggregated inflation persistence is much lower than aggregated inflation persistence. This result is also reported by other author's estimation of disaggregated inflation persistence for other countries (mainly developed countries). This can be a result of common factors present at aggregated data but not identifiable in disaggregated data as indicated by Granger (1987). In such cases, inflation expectations may capture only this common factor which could explain our results. We let this question as an issue for future research.

2.2 Phases in Brazilian Inflation after the Real Plan

In this section we use an unobserved components model of inflation to explain the evolution of Brazilian inflation since the Real Plan by using monthly data since 1995. Many authors have already described the evolution of Brazilian inflation using small sample periods such as Minela *et al* (2008). In general, authors recognize at least two different periods in Brazilian inflation after the Real Plan: a) the first years after the introduction of the Plan when inflation was still high but tending downwards (in the 10-year period before the Real Plan inflation was higher than 100% per year on average) and b) the following years, when the Real Plan was already consolidated and inflation remained low for some time. There are also some interesting episodes during these periods, e.g. the introduction of the Inflation Targeting Regime in 1999 and the confidence crisis of 2002.

Figure 1 shows the evolution of the underlying level of Brazilian inflation estimated by using an unobserved components model. It is clear that inflation diminished in the years that followed the introduction of the Real Plan in 1994 and increased after the exchange rate devaluation in the beginning of 1999. Another increase in inflation occurred in the confidence crisis of 2002. After 2003, inflation had remained between 1% and 2% per quarter.

The figures indicates that Brazilian inflation present a clear seasonal pattern: inflation is higher at year end and also at the beginning of each year. The seasonal pattern is more pronounced in the second half of our sample.

We did not test for structural breaks in Brazilian inflation data. We will include a time trend for the first half of our sample (up to 4Q98) in our estimations of the Phillips curve whenever this is possible. We will also include seasonal dummy variables or AR(4) models as our preferred specification due to the seasonality of inflation in Brazil. As a robustness test, we also use the AIC as a criterion for choosing the appropriate lag length in our Phillips curve estimation.

2.3 Inflation Persistence in Brazil: Aggregated Data

2.3.1 Introduction

Inflation persistence has been widely studied in developed countries but not so much in developing countries. Levin and Piger (2004) and O'Reilly and Whelan (2005) are good references for the study of inflation persistence in advanced economies. Gerlach and Tillmann (2011) studies inflation persistence in Asian countries. In Brazil, there are some studies of inflation persistence by using long memory (ARFIMA) models like Figueiredo and Marques (2009) and Rebelo *et al* (2009) and also by using autoregressive models without long memory and structural models (Phillips curve) like Oliveira and Petrassi (2010).

A useful way to study inflation persistence is by estimating a simple autoregressive model and computing the sum of coefficients on lagged inflation:

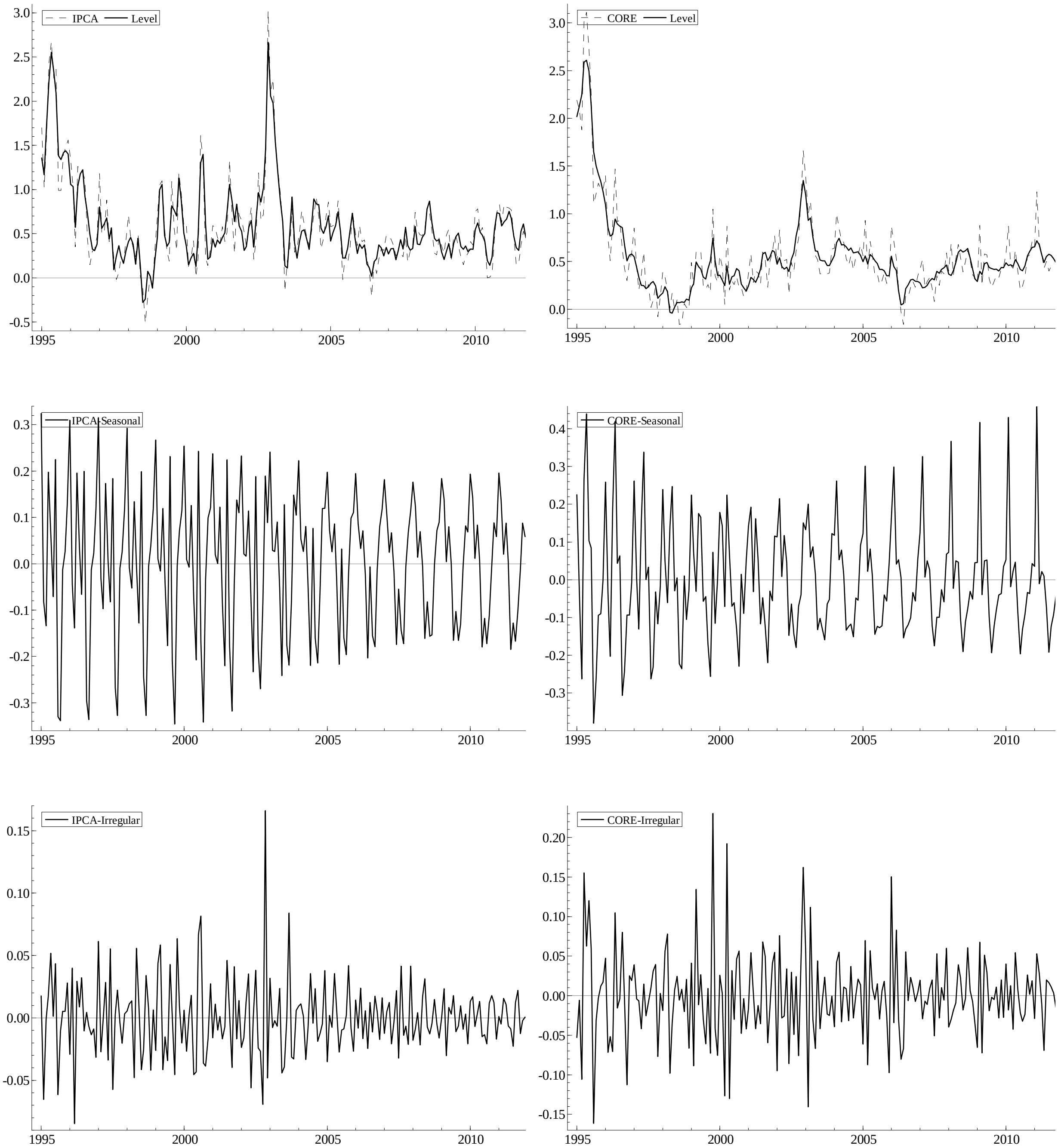
$$\pi_t = a + \sum_{i=1}^{p} \lambda_i \pi_{t-i} + e_t \qquad (2.1)$$

Where π_t is the quarterly inflation rate. Another specification that is useful to analyze inflation persistence is rewriting the inflation process as:

$$\pi_t = a + \rho \pi_{t-1} + \sum_{i=1}^{p} c_i \Delta \pi_{t-i} + e_t \qquad (2.2)$$

By this equation, we can focus on the parameter ρ. The main advantage of this specification is that it is easier to study the evolution of inflation persistence as it is measured by just one coefficient instead of "*p*" parameters as in the *AR(p)* model.

Figure 2.1: Inflation Decomposition - Structural Time Series Model (1995-2011)



Note: Structural Time Series (STM) model for Brazilian inflation. The left column presents the estimated components of headline inflation while the right column presents the core inflation. Inflation components are Level (first row), Seasonal component (second row) and Irregular component (third row).

We also estimated an *AR(1)* model with seasonal dummies. The advantage of this model is that we preserve the sample size in the estimation. With the use of an *AR(p)*, the first *"p"* observations is lost in the estimation process as *"p"* lagged values for the exogenous variable are necessary to initiate the estimation. This will be important for the estimation of disaggregated data in the next section. We estimated this model also with aggregated data for the sake of comparability in the later section of this study. The model with seasonal dummy variables can be written as:

$$\pi_t = a + \lambda \pi_{t-1} + \sum_{q=2}^{4} d_q T_q + e_t \qquad (2.3)$$

Where *q* refers to the quarters and *T* is the dummy variable: *q=2* indicates the second quarter of the year and so on up to the fourth quarter (*q=4*). Our measure of persistence in this case is the coefficient "*b*".

An issue usually addressed by other authors is the possibility of structural breaks in the series or model coefficients. Since we are using a small sample, the chances that a structural break occurred at some point of the sample is small. Even so, we estimate some of the structural parameters as random walks (time varying coefficient) to increase the confidence in the results.

We also include a dummy variable at the beginning of the sample in some estimations due to the initial effects of the stabilization program in 1994. The Real Plan was introduced in the beginning of 3Q94 and inflation was reduced from more than 10% per month to less than 2% in the following months (this can be clearly seen in figure 1). Our dummy variable goes up to the end of 1998. It is not clear however if the low inflation in 1997 and mainly in 1998 was due to continued effect of the stabilization plan or a consequence of low GDP growth in Brazil and also in the world as a result of several financial crises that occurred in 1997 and 1998. Global GDP growth was below the average in 1998 and commodity prices fell that year, contributing to the low inflation in Brazil (exchange rate was fixed at that moment).

Finally, we estimated this simple model including a measure of output gap and also a measure of "external" inflation and a measure of inflation expectation. Historically, many empirical works in Brazil used the exchange rate depreciation as one of the inflation determinants. Despite this being a good proxy for the 1999 and 2002 episodes of exchange rate depreciation, this is a poor proxy in other periods as this measure of foreign inflation does not capture the "global inflation" factor. We experiment with two different measures of global inflation: The CRB commodity price index and the average CPI inflation in the OECD countries. Ciccarelli and Mojon (2010) show that the "global inflation" is an "attractor" of local inflation for several countries. Ciccarelli and Mojon (2010) use a factor model to derive their global inflation factor but call attention to the fact that the average inflation is a good approximation to the global factor[12]. The results for the estimations using the "global inflation" variable are not reported: When this measure of global inflation is used, it leads to a highly significant coefficient and also makes almost all other coefficients in the model not significant at usual levels. We interpret this as one more indication that inflation is a "global" phenomenon in some aspects. But this also indicates that this measure of global inflation is not exogenous or an independent variable. Both the Brazilian inflation and the global inflation (OECD median inflation, as we defined earlier) are determined by the same factors, which characterize an endogenous regressor.

[12] *This seems clear in figure 1 in Ciccarelli and Mojon (2010).*

The use of commodity price index is a better proxy for the effects of changes in “external inflation” during the 1995-2011 period as Brazil had a fixed exchange rate regime in the first part of the sample and changes in foreign prices in domestic currency where dominated by changes in foreign prices in the external currency. In 1999 and also 2002 due to the large depreciation of the Real, tradable prices (and also regulated prices) presented high rates of inflation. In other periods, like in 2004-2005 and later in 2007-2008, the exchange rate appreciation was not sufficient to avoid an escalation of tradable prices as commodity prices surged around the world. Some theoretical papers also refer to the import price as a measure of external inflation. We consider the commodity price index as a wider definition of external inflation and preferred in our estimations. In our estimations we use the average of commodity prices and exchange rate in the last month of the quarter: $P_t^{\$*} = E_t P_t^*$ and $\pi_t^* = \ln(E_t P_t^*) - \ln(E_{t-1} P_{t-1}^*)$ where $P_t^{\$*}$ is the commodity price index in dollar and E_t is the exchange rate in R$/US$.

Our measure of output gap is derived using the deviation of GDP from a trend (HP filter). Several authors have used the variation in labor share as a measure of marginal cost. Data on labor costs in Brazil are available only after 2001 due to a change in the employment and unemployment survey. Allowing for the output gap as an addition explanatory variable is important due to the fact that some models attribute the persistence of inflation to the persistence of the driving variable, namely the output gap (or marginal cost). This is called “inherited” persistence. Some authors such as Fuhrer (2006) discuss this hypothesis.

Finally, we included the “twelve months ahead” inflation expectations (π^E) provided by the Brazilian Central Bank - BCB as a measure of inflation expectation. The inflation expectation is the main determinant of inflation in most economic models since the appearance of the “rational expectations” models in the 60’s. This variable is provided by the BCB on a daily basis and we used the average of the last month of the quarter as our measure of inflation expectation.

We also experimented using this variable with two lags instead of just one as is the common practice. Indeed, we consider this our preferred specification. The reason is clear: The output gap used in the Phillips curve equation is y_{t-1}, which is known by market participants at time *t-1*. Therefore, exchange rate and commodity prices movements are known at time *t-1* (we use π_{t-1} and π_{t-1}^* in the Phillips curve estimation). In this case, the result of using π_{t-1}^E instead of π_{t-2}^E is clearly to overestimate the impact of π^E in the determination of π and underestimate the impact of other variables. Indeed, in this case, the impact of other variables is just the impact not pondered into π_{t-1}^E. This could include, for instance, some kind of market participants’ uncertainty about the true Phillips curve or sluggish adjustment of market participants forecast projections.

There is another obvious explanation for not using π_{t-1}^E: It may also capture influences already known to affect inflation in period t. For instance, some already announced price adjustment (this is particularly true for monitored prices) and also any other kind of price shocks known at time t-1. Therefore, if market participants know inflation is persistent in a way consistent with an autoregressive model like the ones we are considering in this paper, the inclusion of π_{t-1}^E in the estimation together with π_{t-1} will also tend to underestimate the impact of π_{t-1} in the Phillips curve equation.

Considering this complete specification for the Phillips curve, the equation we estimate can be written as:

$$\pi_t = a + \rho\pi_{t-1} + \phi\pi^E_{t-1,t+4} + \beta\pi^*_{t-1} + \gamma y_{t-1} + e_t \qquad (2.4)$$

Dummy variables for each quarter are also incorporated into this equation to control for seasonal effects.

2.3.2 *Inflation persistence using aggregated data*

In table 2.2 we provided a summary of the main statistics of dependent and independent variables used in our estimations. The main highlight of this table is the high autocorrelation of all inflation data[13]. The autocorrelation of external price index is relatively small. This is an expected result as the variable is composed of exchange rate and commodity prices and both variables can be considered a financial asset and so their movements are close to random walks.

The estimation results for inflation persistence using aggregated data are presented in table 2. Overall, the inflation persistence estimates are quite similar if we use the *AR(1)* or the *AR(4)* model. The inclusion of the dummy variable leads to an increase in the inflation persistence estimate. This is not different from results reported by other authors using different techniques. Many authors report results indicating larger inflation persistence when the sample period is shorter: In most of these studies, the shorter period accounts for possible breaks in the inflation process. In this paper, we used a dummy variable instead to control for possible breaks or changes in the inflation process.

Table 2.2 – Summary Statistics of Variables: 1Q1995 to 4Q2011

	IPCA	Core	Monitored Prices	Free Prices	Tradables	Non Tradables	Output gap	Expected Inflation*	External Inflation
	π	π	π	π	π	π	y	π^E	π^*
Mean	1,8%	1,7%	2,6%	1,6%	1,4%	1,9%	0,1%	1,3%	3,0%
Median	1,4%	1,4%	1,7%	1,3%	1,1%	1,2%	0,2%	1,2%	3,5%
Std Dev	1,4%	1,4%	2,5%	1,4%	1,6%	2,0%	1,4%	0,4%	11,8%
Corr(t,t-1)	0,57	0,75	0,50	0,53	0,20	0,70	0,62	0,67	0,07

**From 1Q00 to 1Q11; inflation expectation was divided by 4 to convert into quarterly data*

Inflation persistence is larger for core inflation than headline inflation. Core inflation excludes food prices and also monitored prices, two groups with low inflation persistence. The inflation persistence of food prices will be discussed in the next section. The low persistence of monitored prices seems to arise from the fact that this group of prices are indexed to two different price indexes, the consumer price index (IPCA) and also the "general price index" (IGP), which is a mixture of producer price index, consumer price index and also construction cost index. The indexation mechanism also changed

[13] These inflation groups are not completely different from one another. For instance, core inflation excludes monitored prices and part of the food product in the CPI (IPCA) basket. So, core inflation includes most of the free prices or non-tradable prices. Non-tradable prices are also free prices and so on. Despite this overlap, the segmentation is important since these groups are usually referred to in macroeconomic analysis both by the Central Bank and by market participants.

during the last years due to the inclusion of productivity gains clauses aimed at reducing the indexation and due to the change in the composition of the baskets of price index used by regulators for determining the monitored prices adjustments. In other words, the low persistence of monitored prices inflation does not mean that this group of price does not have strong mechanisms to propagated inflation shocks.

Table 2.3 – Inflation persistence estimates

	IPCA	**Core**	**Monitored Prices**	**Free Prices**	**Tradables**	**Non Tradables**
a) Autogressive Model (eq. 1 and eq. 2)						
AR (4)	0,430	0,502	0,529	0,367	0,309	0,312
ρ	0,408	0,538	0,492	0,361	0,317	0,507
b) Autoregressive model plus dummy variable for 1996:1 to 1998:4						
AR (4)	0,514	0,692	0,418	0,477	0,290	0,692
ρ	0,488	0,681	0,352	0,499	0,277	0,695
c) Autoregressive model plus dummy variable for 1996:1 to 1998:4 and output gap						
ρ	0,513	0,699	0,332	0,546	0,303	0,705
γ	0,039	0,026	-0,076	0,085	0,063	0,131
d) Complete Model (eq. 4)						
ρ	0,474	0,704	0,301	0,520	0,285	0,714
γ	0,008	0,015	-0,135	0,058	0,047	0,124
$\gamma/(1-\rho)$	0,015	0,051	-0,193	0,121	0,066	0,434
β	0,028	0,024	0,040	0,030	0,066	0,012
$\beta/(1-\rho)$	0,053	0,081	0,057	0,063	0,092	0,041

Notes: inflation persistence estimates using models (1), (2) and (4) using quarterly inflation data from 1Q95 to 4Q11.

The inclusion of the output gap does not lead to a sensible change in the persistence estimates. The coefficient of output gap has an “unexpected” sign in the monitored prices equation. This result is not completely unexpected since monitored prices inflation depends largely on past inflation: If output gap leads to an overall inflation rise in year one and then due to a more rigid monetary policy the output gap turns negative in next year, the monitored prices inflation will rise when the output gap had already fallen. Despite this “unexpected” sign, the coefficient is not significant. The impact of output gap is larger on free prices and particularly in non-tradable prices as expected. The estimated impact on core prices is also small. This result seems at odds with the large impact of output gaps on free prices, which comprehends the majority of the core inflation components.

In the last group of coefficients estimates in table 2.3 we present the result from the estimation of a Phillips curve with external price inflation. Again, the difference in the persistence coefficient from previous estimations is minimal. The largest difference between persistence estimates arise when we allow for a time trend in the first part of our sample. This result is in line with several other papers on the subject that consider breaks in the inflation process. In our model, the time trend for the post-stabilization period is used to represent this break.

In this last group of coefficient estimates we include the estimated long-run impact ($\gamma/(1-\rho)$) and also $\beta/(1-\rho)$). The output gap coefficient, γ, is smaller when we include the external inflation variable but in some cases this difference is very small (we did not make statistical tests for this difference). The impact of output gap is larger for non-tradable inflation (services represent a good portion of this group) and negative for monitored prices as was the case in the previous set of coefficients estimates. The impact of the external price inflation is larger on tradable prices and lower for non-tradable prices, two results that are in line with conventional theory.

In the estimation results provided in table 3, the lag structure was chosen to 4 (*AR(4) model*) due to the possible presence of seasonality in the dependent variable. A possible refinement of this estimation would be to select the best lag structure according to some information criteria, like the AIC and the SBC. In the following table we report the inflation persistence of models (b) and (d) using the AIC to chose the most appropriate model. We tested lag structures from 4 to 1 in all cases. We report both the chosen lag structure and the respective inflation persistence measure. Another refinement would be to adjust the data for seasonal factors before estimating the model. In next table report the results of these two refinements. We used the Arima X-12 procedure to created seasonally adjusted inflation data.

The results are similar in qualitative terms: the groups with the lowest inflation persistence are the same independent of the model being used. Tradable goods present the lowest persistence while non-tradable the largest. The inflation persistence of monitored prices is not particularly high. A possible explanation is the non-linear indexation mechanism in many items: some prices are not adjusted every year and in many cases the indexation may be related to the General Price Index (IGP) which includes producer, consumer and construction prices. In other cases, the revision of prices includes productivity targets that must be met by companies and affects the size of overall price adjustments.

The seasonal adjustment does not change the size of inflation persistence in a meaningful way with the exemption of tradable and monitored prices.

Table 2.4 – Inflation persistence estimates

	IPCA		Core		Monitored Prices		Free Prices		Tradables		Non-Tradables	
AR (4)		0,519		0,683		0,435		0,475		0,290		0,698
AIC	AR(3)	0,456	AR(4)	0,683	AR(1)	0,303	AR(3)	0,433	AR(1)	0,175	AR(4)	0,699
AIC (S.A. data)	AR(1)	0,480	AR(1)	0,639	AR(1)	0,395	AR(1)	0,421	AR(1)	0,269	AR(3)	0,736
Complete Model												
ρ	AR(3)	0,420	AR(4)	0,665	AR(1)	0,202	AR(3)	0,411	AR(1)	0,056	AR(4)	0,704
γ		0,046		0,041		-0,115		0,084		0,036		0,139
$\gamma/(1-\rho)$		0,079		0,123		-0,144		0,143		0,038		0,469
β		0,030		0,021		0,051		0,029		0,053		0,013
$\beta/(1-\rho)$		0,052		0,062		0,064		0,049		0,056		0,045
Complete Model (seasonally adjusted inflation)												
ρ	AR(1)	0,366	AR(4)	0,694	AR(1)	0,293	AR(3)	0,462	AR(4)	0,218	AR(3)	0,737
γ		0,009		0,026		-0,090		0,076		0,082		0,036
$\gamma/(1-\rho)$		0,015		0,086		-0,128		0,140		0,105		0,137
β		0,029		0,022		0,050		0,033		0,048		0,019
$\beta/(1-\rho)$		0,046		0,070		0,070		0,061		0,062		0,072

Notes: inflation persistence estimates using models (1), (2) and (4) with quarterly inflation rates from 1Q95to 4Q11. The number of lags in each model was chosen using the AIC criteria except for the "naïve" AR(4) described in the first line of the table.

2.3.3 The impact of expected inflation

The Brazilian Central Bank has been collecting inflation expectations from market participants since the end of 1999. We will use the "twelve months ahead" inflation expectation as our measure of inflation expectation in this sub-section. This measure is the most observed and also the one widely used in empirical exercises. Since this data is only available from 2000, we restrict our sample in this case to 1Q00 to 2Q12[14] and use our seasonally adjusted inflation data. Due to the lower sequence of data in this case, we estimate only the more parsimonious AR(1) specification.

We restrict our estimation to the *headline* IPCA and core inflation. We do this particularly because our inflation expectation measure refers to headline inflation and this measure of inflation expectation may not be good for groups representing a small portion of the index.

The results indicate that inflation expectation is significant when we use π^E_{t-1} in our estimation. Our measure of inflation expectation is based on the average of the last month of the quarter it can capture price adjustments already known to affect inflation in the next quarter. Also, since some variables affect inflation with a lag, the inflation expectation collected at the end of the quarter may incorporate several shocks or variables that will affect inflation in the next quarter. The standard Phillips curve estimation with Brazilian data lead to the following result when we use core inflation as our measure of inflation:

$$\pi_t = \underset{(0,002)}{0,005} + \underset{(0,169)}{0,255}\pi_{t-1} + \underset{(0,281)}{0,444}\pi^E_{t-1} + \underset{(0,004)}{0,003}y_{t-1} + \underset{(0,006)}{0,018}\pi^*_{t-1}, \quad R^2 = 0,64, \quad SE = 0,0037$$

The results are very different when we use π^E_{t-2} or π^E_{t-3} as our measures of inflation expectations. In these cases, the coefficient on expected inflation is non-significant and have a negative sign.

2.3.4 Time-varying inflation persistence

In this subsection we assess the evolution of inflation persistence during the last years. We do so by using a random coefficient approach for the inflation persistence coefficient. The random coefficient is estimated by using the Kalman filter. In order to reduce the number of parameters to be estimated, we adjusted the inflation series to eliminate the seasonal factors. The seasonal adjustment was made by using the X-12 procedure but the results are pretty much the same if we had used the unobserved components models previously discussed. Using the seasonally adjusted series, the model we estimate is:

$$\hat{\pi}_t = \lambda_t \hat{\pi}_{t-1} + \gamma y_{t-1} + e_t \qquad (2.5)$$
$$\lambda_t = \lambda_{t-1} + \varepsilon_t$$

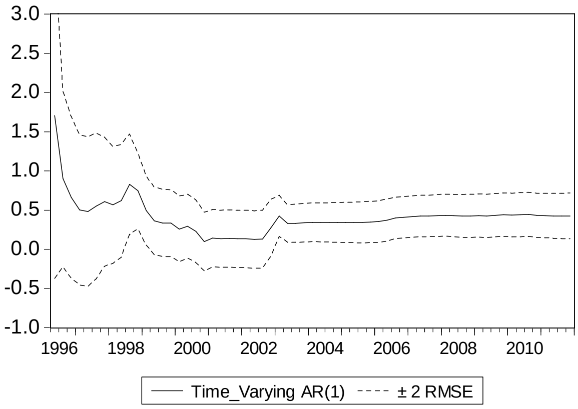
[14] *In most of the empirical exercises, our sample starts in 1Q95 with estimation starting in 1Q96 due to the use of 4 lags in some models. In this section, we adjust the inflation rate for seasonal factors using the X-12 procedure. The "twelve months ahead" inflation expectation is available at the BC starting in 2001. We used the "current year" and "next year" inflation expectation to form the "twelve months ahead" expection for the first twelve months of our sample. In this case only, we extended the sample to the last data available, 2Q12.*

Where $\hat{\pi}_t = \pi_t - \overline{\pi}$ is the deviation of current inflation from the average inflation. We included the dummy variable in all estimations. The reason to use deviation from mean is to eliminate the constant coefficient in the equation to be estimated. The constant coefficient in the above equation usually leads to more volatile time-varying coefficients and can also lead to non-convergence in some specifications.

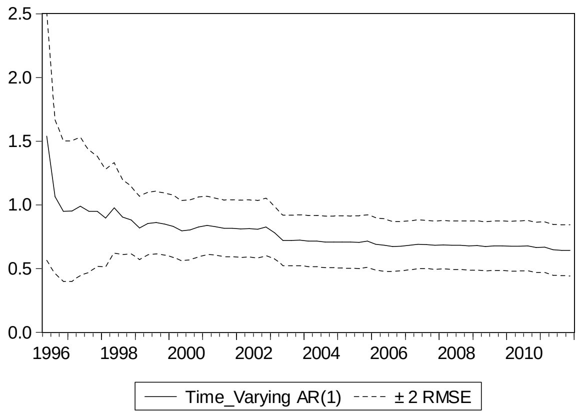
The results from the time-varying coefficient display no clear trend in our sample (figures 2a). For headline inflation, the coefficient is indeed larger in the second half of our sample. The results do not change significantly if we include more variables in the Phillips curve equation. The results for core inflation show less variability and also no clear trend in the upward or downward in our sample period.

Figure 2.2a: time-varying AR(1) model (random walk coef.)

Headline inflation (filtered state)



Core inflation (filtered state)

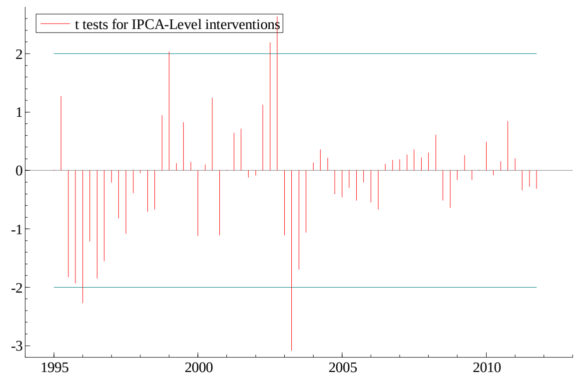


We also estimated the structural time series model using quarterly data (the results in figure 1 used monthly data). The structural time series model decompose the inflation rate into Level, Seasonal and Irregular components. The structural break test in this case is based on a t-Test on the residuals from the Level equation. The results differ a little if we use core or headline inflation. In both cases there is some evidence of structural break at the beginning of the sample and also between late 2002 and early 2003, the period marked by a large increase in inflation, inflation expectations and also large exchange rate depreciation. In this period there were some changes in the indexation mechanism for some monitored prices.

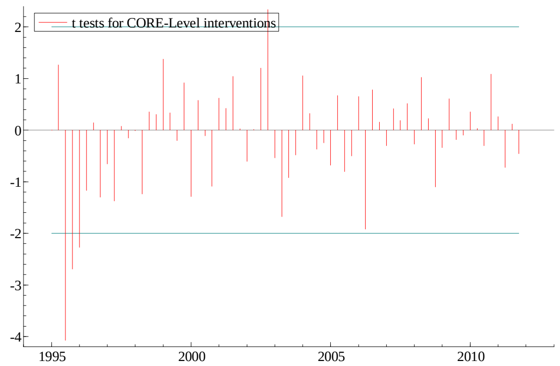
We didn't treat possible breaks in the inflation process in the estimation results presented in tables 2.3 and 2.4. We will discuss further a possible break in the inflation process using a dummy variable in the next section when we deal with disaggregated data.

Figure 2.2b: Auxiliary residuals from the local level model

Headline inflation



Core inflation



2.3.5 Sources of inflation persistence in aggregated data

The previous section presented evidence that inflation persistence in Brazil differs if we include or do not include explanatory variables in the model. This result can be explained by using the "inherited" versus "intrinsic" hypothesis for inflation persistence.

Let us consider a simple rational expectations model for inflation (usually called New Keynesian Phillips Curve, NKPC) in which the output gap follows an AR(1) process[15]:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma y_t + e_t \qquad (2.6)$$
$$y_t = \rho y_{t-1} + u_t$$

In this case, it is possible to show that:

$$\pi_t = \frac{\gamma\rho}{1-\rho\beta} y_{t-1} + \kappa_1 e_t + \kappa_2 u_t \qquad (2.7)$$

Where parameters κ_1 and κ_2 are functions of the other parameters in the first equation. It is clear from the equation above that the "persistence" of the inflation variable depends on the degree of persistence of the output gap. This is called the "inherited" inflation persistence. If we assume $e_t \equiv 0$, it is also possible to express the inflation equation as:

$$\pi_t = \gamma \pi_{t-1} + \kappa u_t \qquad (2.8)$$

Again, the inflation persistence will depend on the persistence of the driving forces of the inflation process. If we use more complex specification like equation (2.4), it gets very difficult to get an analytical solution for the inflation equation. Despite that, it is possible to get some indication about

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[15] *This example is based on Fuhrer (2006).*

the forces behind the inflation persistence by looking at the impact of each variable that entered into the Phillips curve equation, its coefficient and is autocorrelation.

With the use of the results from table 2.3 and summary statistics of the variables in table 2.2, it is difficult to infer the causes of inflation persistence in Brazil. Except for external price inflation, the explanatory variables are all highly persistent and have almost the same standard deviation. The expected inflation variable is the most persistent variable but this is clearly a result of using expected inflation for the next twelve months on a quarterly basis. The results indicate that more information is still needed for us to have a better understanding of the high inflation persistence in Brazil.

2.4 Inflation Persistence in Brazil: Disaggregated Data

2.4.1 Introduction

Aggregate inflation dynamics depends on the dynamics of disaggregated inflation dynamics. But this natural result alone does not help in our task of understanding the forces behind inflation dynamics and particularly inflation persistence. Another result of the aggregation process is that the persistence of aggregated data will have a "tilt" towards the most persistent individual series. Cecchetti and Debelle (2006) offer a good explanation for this result and also a graphical illustration of it. Granger (1980) and Zaffaroni (2004) give a more theoretical explanation about the consequences of aggregation of individual series. In these papers, the importance of the distribution of individual persistence parameters is commonly expressed as the individual series being stationary or not having a mass in distribution at the unit root point.

2.4.2 Inflation Persistence in Disaggregated Data without Common Factors

In this sub-section we estimate inflation persistence for disaggregated data. Several papers have demonstrated that aggregate inflation persistence is larger than disaggregated inflation persistence. We calculate disaggregated inflation persistence for 342 products using IPCA data during 3Q1999 to 1Q2010. Since 1995 there has been three changes in IPCA components: In 1996, in 1999 and 2006. The items in the 1999 and 2006 poll are pretty much similar but the items in the 1996 poll are very different and were not included in this analysis. The 1999 poll had 511 items and the 2006 poll had 384 items. There are 341 common items among these two samples. Among these 341 items, 9 were excluded due to lack of dynamics and unreliable results from the autoregressive models used in estimation. Most of these items are monitored prices where price changes occur seldom and several entries are zeroed. Due to this, the *AR(1)* or *AR(4)* models used were not reliable. In order to check for common items we used the items coding provided by IBGE. The final sample consists of 332 inflation series from 3Q1999 to 1Q2011.

The table bellow presents the descriptive statistics of the disaggregated data. As one can see in such table, the median inflation is close to the aggregate inflation (both weighted, "original data" or unweighted) and is much more volatile and presents less autocorrelation than the aggregate inflation. Klenow and Kristosov (2008) present similar results for US inflation data: In their sample, the persistence of inflation is much larger in the aggregate than in the disaggregated data. The same results were found by Clark (2006) also using data for US inflation.

In this section we estimate several simple models of inflation dynamics (or Phillips curves) by using disaggregated data. The objective of these estimations is to understand the differences in inflation persistence in different inflation groups. This is important due to the impact that highly persistent

series can have on the aggregated data. Granger (1980), Zaffaroni (2004) and Pesaran (2003) have shown that under certain assumptions, the dynamics of the aggregated data can be analyzed from the disaggregated data. In particular, by considering a simple *AR(1)* model for inflation dynamics in disaggregated data, Granger (1980) shows that the aggregated data can present long memory. This result was later refined by Zaffaroni (2004) and Pesaran (2003). We will use some results presented in Pesaran (2003) to discuss our disaggregated results and its implication for aggregated inflation persistence. Debelle and Ceccheti (2008) discuss in a much less rigorous way the impact of aggregation of inflation series with different persistence on aggregated inflation dynamics. The general results of these papers can be summarized as follows: a) assuming independence of all disaggregated inflation series, the aggregated inflation persistence will be determined by the distribution of the inflation persistence of the individual series and b) also depending on the distribution of the individual series, the aggregated series can display long memory. The result expressed in (a) is very intuitive while (b) is not. In both cases, a beta distribution is used to characterize the distribution of the *AR(1)* parameters for individual series and it is possible to show that the presence or not of long memory can be inferred from specific parameters of the beta distribution.

Table 2.5: Summary Statistics of Disaggregated Inflation Data

	Mean	Std Dev	Corr(t,t-1)
Agreggate	1,65%	1,12%	0,447
Unweighted Aggregate	1,97%	1,50%	0,252
Minimum	-1,92%	0,67%	-0,545
Median	1,77%	2,80%	0,072
Maximum	12,06%	50,33%	0,843

The table bellow presents the results for several specification of the Phillips. In all cases, except for model 5, we present the coefficients for the aggregated specification and also the mean and median coefficients of the disaggregated specification. We will focus our discussion on models 1, 2 and 5. Each model presents estimations results for individual inflation items specifications as follows: Let $\pi_{i,t}$ be the inflation rate of product *i* at the quarter *t* (one of the 332 products we are using in this section as we defined in the beginning of this sub-section). We estimate product specific Phillips curves similar to equations (1) to (4) for each product *i*. This can be written as:

$$\pi_{i,t} = \lambda_i \pi_{i,t-1} + \phi \pi^E_{t-1,t+4} + \beta \pi^*_{t-1} + \gamma y_{t-1} + u_{i,t}, \quad \text{i=1,...,332} \tag{2.9}$$

For each specification, we present the mean and the median of the *AR(1)* coefficient and also the mean and the median of the individual coefficients of the other explanatory variables. The "aggregated" row represents the coefficient estimated by using the aggregated inflation data, both weighted and unweighted. The results are not exactly the same as in table 2 for headline inflation due to the different sample periods: In table 2.2, we estimated the models using the whole sample (1Q95 to 1Q11) and for disaggregated data we are using just data from 3Q99 to 1Q11 (this is the data available for disaggregated data). We estimated the equations one at once by using a simple routine written to estimate each of the 332 equations and then store the coefficients in a specific matrix.

The estimation results in model 1 shows that inflation persistence in disaggregated data is much smaller than in aggregated data. This result is similar to the one reported by Clark (2006) and also Klenow and Krystosov (2005) for US data.

The inclusion of the output gap in the inflation equation lowers the persistence of inflation both in aggregated data equation and in disaggregated data. In opposition to what happens to inflation persistence, the impact of the output gap is larger in disaggregated data than aggregate data. This is an indication of omitted variable bias in the other equations.

In model 4 we also add inflation expectation and commodity prices in the inflation equation. The results indicate a larger role for expected inflation than lagged inflation. The impact of output gap is lower than in model 2 and the impact of commodity prices is small. This low impact of commodity prices can be considered a puzzle since both exchange rate movements and also commodity prices inflation are considered relevant variables in inflation dynamics almost everywhere.

Table 2.6 – Estimation results: disaggregated inflation persistence

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8*
AR(1)								
Aggregate	0,476	0,469	0,346	0,090	0,225			
Unweighted Agg.	0,355	0,334	0,197	0,020	0,097			
Disaggr. Mean	0,131	0,115	0,059	0,071	0,046			
Disaggr. Median	0,146	0,116	0,049	0,036	0,030			
Output gap								
Aggregate		0,069	0,065	0,017	0,031	0,002	0,038	0,020
Unweighted Aggregate		0,116	0,129	0,064	0,057	0,028	0,079	0,058
Disaggregated Mean		0,160	0,155	0,075	0,083	0,028	0,079	0,058
Disaggregated Median		0,057	0,047	0,011	0,060	0,030	0,063	0,051
Commodity Prices								
Aggregate				0,060	0,061	0,063	0,065	0,039
Unweighted Aggregate				0,073	0,078	0,072	0,076	0,045
Disaggregated Mean				0,035	0,077	0,072	0,076	0,045
Disaggregated Median				0,016	0,047	0,037	0,044	0,016
Expected inflation (t-1) & (t-2)			**t-1**	**t-1**	**t-2**	**t-1**	**t-2**	**t-2**
Aggregate			1,103	0,972	0,125	1,134	-0,840	0,218
Unweighted Aggregate			1,373	1,013	0,054	1,542	-1,079	0,147
Disaggregated Mean			1,294	0,902	0,182	1,542	-1,079	0,148
Disaggregated Median			1,177	0,995	0,433	1,172	-0,639	0,270
Lagged Aggreg. Inflation								
Aggregate						-0,148	0,646	0,009
Unweighted Aggregate						-0,366	0,674	-0,066
Disaggregated Mean						-0,366	0,674	-0,065
Disaggregated Median						-0,059	0,766	0,113

**Identical to model 7 excepts for the Inclusion of a dummy variable for 4Q02 and 1Q03*

In model 5 we included the lagged aggregated inflation as an explanatory variable. The reason behind this is quite simple: Indexation mechanisms are almost always related to lagged "aggregated" inflation and it is considered a "conventional wisdom" that several indexation mechanisms are still in practice in Brazil not only among "monitored" or "regulated" prices but also among "free" or "non-monitored"

prices. The results of model 5 contradict this conventional wisdom: Expected inflation is more important than lagged inflation as an explanatory variable for disaggregated inflation dynamics.

The estimated impact of expected inflation is lower when we consider $\pi^E_{t-2,t+4}$ instead of $\pi^E_{t-1,t+4}$. Our interpretation to this results is that expected inflation incorporates several price adjustments announced in advance and also some "macro trends" or "common factors" in the sense of Granger (1987) already known to affect inflation in the next quarter. In other words, our measure of expected inflation[16] may be at the same time a driving force of inflation and a variable that is being impacted by current inflation. If this is true, this could be an important cause of inflation persistence in Brazil. Nevertheless we recognize that this explanation is just "tentative" and we consider this hypothesis as a direction for further research on inflation persistence in Brazil. If we run a regression of the variation in inflation expectation ($\pi^E_{t,t+4} - \pi^E_{t-1,t+4}$) on the variation of the underlying level of inflation ($\pi^L_t - \pi^L_t$), we find regression coefficients as large as 0.5 by using core inflation. When using headline inflation, the impact is lower - close to 0.15.

$$\pi^E_{t,t+4} - \pi^E_{t-1,t+4} = \underset{(0,072)}{0,504}(\pi^L_t - \pi^L_{t-1}) + e_t \tag{2.10}$$

The impact of different weights is attributed to different items in the aggregation process as fig. 2.3 shows. There is no correlation between the weight of each item and its estimated persistence. The distribution of inflation persistence in model 1 is depicted in fig. 4. The distribution is skewed to the left with mean of 0.131 and median of 0.157. These values are much lower than the *AR(1)* coefficient from the aggregated equation (close to 0.5). The persistence of unweighted aggregated inflation is lower than the persistence of weighted inflation (the weighted inflation is actual data informed by IBGE). In the next section we compare this distribution with the Beta distribution and the values upon which a long memory in aggregated series could arise.

Figure 2.3: AR(1) Persistence (model 1) x Weight

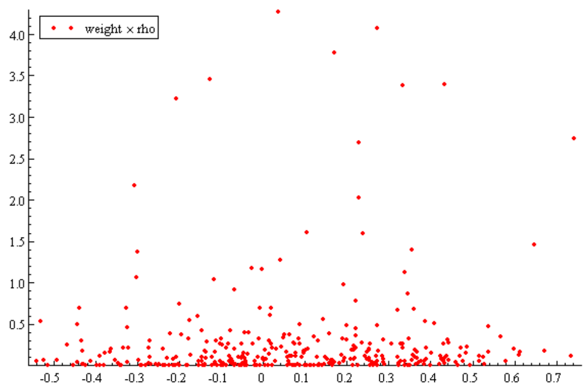
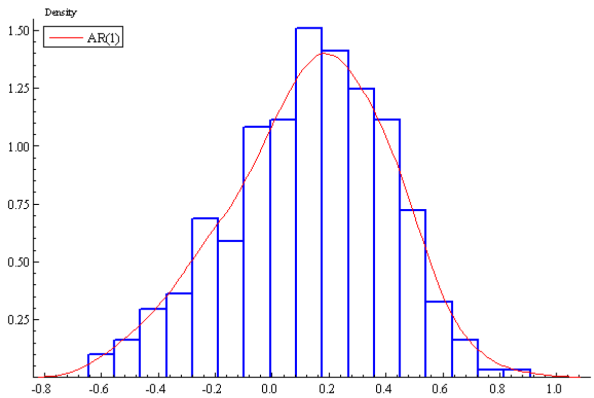


Figure 2.4: AR(1) Persistence (Model 1)



[16] *This is also the measure commonly used by the Central Bank.*

The figures (2.5), (2.6), (2.7) and (2.8) present the estimated coefficients for expected inflation, lagged inflation, output gap and the R^2 of each equation in model 5. The figures just make it easier to see the impact of each variable in disagreed inflation dynamics as summarized in table 2.6.

For each graph we present the estimated coefficient in the *y axis* followed by the group from which the specific product belongs to. We did not write the exact name of the product because it would be almost impossible for anyone to read it. Therefore, the several bars labeled "food" are indeed the several products belonging to the food group in the same order as presented by IBGE in its inflation (IPCA) releases. For example, the very first bar represents the coefficients for "rice" (FOOD group) and the last bar represents "telecommunication handset" (COMMUNICATION group). Since the communication group has only 5 products, in most graphs the last label that appears is EDUCATION.

Figure 2.5: Expected inflation coef. (model 6)

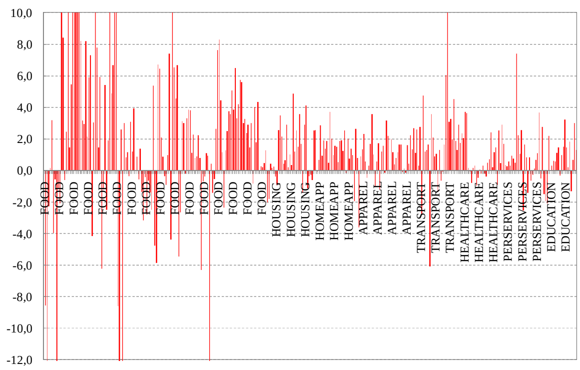


Figure 2.6: Lagged inflation coef. (model 6)

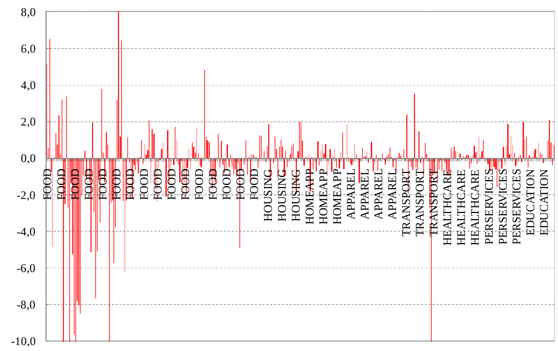


Figure 2.7: Output gap coef. (Model 6)

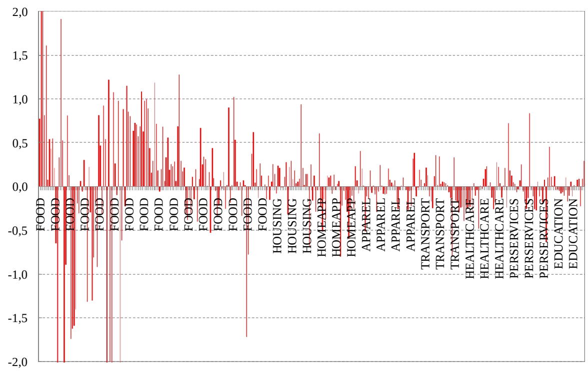
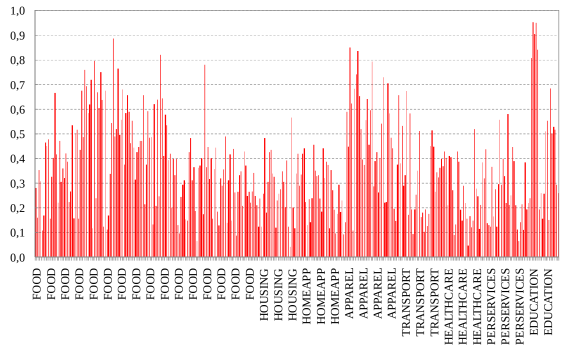


Figure 2.8: R^2 (Model 6)



In figure (2.8) we present the R^2 for model 6. The explanatory power is lower for disaggregated data than for aggregated inflation data. By using model 6, the mean R^2 is just 0.41. The result is very different from the ones observed by other authors using a similar dataset for other countries. For instance, the mean R^2 in Gianoni *et al* (2010) is just 0.15 when using a factor model[17] for disaggregated data. The R^2 rises close to 0.70 when using aggregated data. In Chudik and Pesaran (2011) the estimated R^2 varies from 0.36 to 0.39 by using an autoregressive model for disaggregated data and the estimated R^2 rises to values between 0.48 and 0.56 when the author includes "common factors" in the estimated model. Gianonni *et al* (2010) use US data and Chudik and Pesaran (2011) use data from Germany, France and Italy.

[17] *The factors are extracted from a large dataset of macroeconomic variables. Most of the variables are economic activity indicators. The nominal interest rate is also used as a factor.*

Figure (2.9) shows the dispersion of the estimated coefficient of lagged inflation and expected inflation on disaggregated inflation dynamics. There is a clear negative relation between the coefficients: Items with strong response to lagged inflation present a low response to expected inflation. This result was somehow expected: When inflation is stable, the sum of coefficients of lagged inflation and expected inflation must not exceed one in modulus[18]. The estimated impact of inflation expectation is usually larger than the impact of lagged inflation (both last 12 months cumulated inflation and the AR(1) coefficient). Among the several different estimated equations, the lagged inflation coefficient is larger than the estimated coefficient on expected inflation in almost 80 equations out of 332. The products that show greater impact of lagged inflation are evenly distributed across the several inflation groups.

Figure 2.9: Dispersion of expected inf. coefficient (X-axis) and lagged inf. coefficient (Y-axis)

Figure 2.9a: Model 6

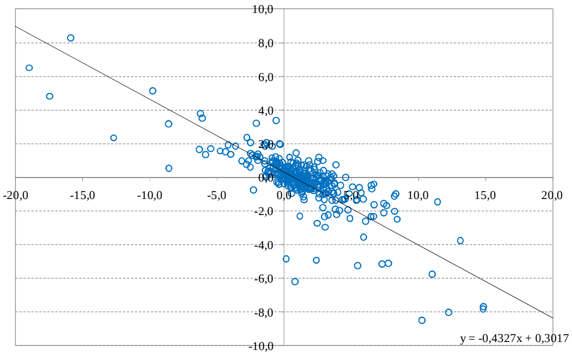
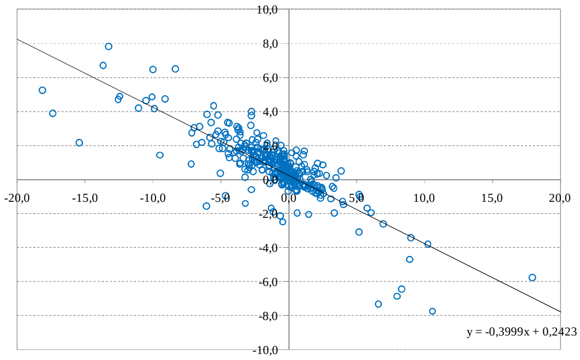


Figure 2.9b: Model 7



2.4.3 Inflation Persistence in Disaggregated Data: The Relevance of Common Factors

A possible explanation for the low R^2 of the estimated equation in the previous sub-section is the presence of common factors at the cross-section of the estimated model. This problem was first studied by Granger (1987). The presence of common factors at the cross section can explain the existence of a good fit of statistical models at the aggregate level but almost no fit at the disaggregated level.

Let us consider a simple dynamic linear model for disaggregated data:

$$\pi_{i,t} = \lambda_i \pi_{i,t-1} + \alpha_i \eta_t + u_{i,t} \qquad (2.11)$$

Where $\pi_{i,t}$ is the inflation rate in the item *i* at the time interval *t* and η_t is the common factor for all items at t. This model is similar to the ones presented previously except for the inclusion of a common factor η_t at every cross-section. There is a large literature on estimation and inference in such kind of specification and we will not review the different approaches and results from this literature[19].

---

[18] *In almost 40 equations out of 332 the sum of coefficients on lagged inflation and expected inflation exceeds one in modulus. We did not perform any test to check if the sums are statistically different from one.*

[19] *Coakley et al (2002) and Chudik and Pesaran (2011) are the most similar research we found. Pesaran (2006) presents a more theoretical discussion of the subject.*

Coakley *et al* (2002) propose using principal components techniques to construct instruments for η_t. Pesaran (2006) proposes the average of the disaggregate variable, $\bar{\pi}_t = N^{-1}\sum_i \pi_{i,t}$, as an instrument for η_t. This approach is similar to model 6, model 7 and model 8 in our previous estimations. The difference is that we used the lagged (and aggregated) dependent variable instead of contemporaneous aggregated variable. We will use the principal components approach to derive the common factor at the cross-section of our data. In order to do so, we will used model 1 and model 5 residuals to estimate the principal components. In model 1 we incorporated no "macro" structure to each individual inflation series whereas in model 5 we incorporated several variables that could be a proxy for the common factor. This could indicate higher and lower bonds common factors. However, the two factors are pretty much similar to a correlation close to 0.9. Due to this result, we will only present results with the use of the common factor extracted from residuals of model 5. After extracting the common factor, we estimate the model again by using the augmented equation:

$$\pi_{i,t} = \lambda_i \pi_{i,t-1} + \alpha_i f_t + u_{i,t} \qquad (2.11')$$

Where f_t is the common factor extracted from the residuals of model 5.

Overall, the inclusion of the common factor does not cause great changes in the estimated coefficients of the model. When we use the common factor into model 6, the common factor is almost always non-significant and the model results do not change much: The estimated coefficients on lagged and expected inflation are very similar and the R^2 is also similar. When we include the common factor into model 1, the results change but not much. There is a sensible increase in the R^2 but this can be explained by the simple structure of the model and its lack of macroeconomic variables as explanatory variables. The t-statistic is larger than two for several products indicating this common factor is significant in most regressions.

2.4.4 Relation to long memory models

In the last years several papers estimated long memory models for aggregated inflation data on the grounds of the results presented by Granger (1980)[20]: Assuming a simple autoregressive model for disaggregated data, the aggregated data may present long memory depending on the distribution of the autoregressive coefficients of disaggregated data. This results is not only applicable to inflation but for any aggregated data. See Pesaran (2003) and Zaffaroni (2004) for more examples.

Let y_{it} represent the value of the y variable observed at time t for i-th individual[21]. Thus let us consider that the dynamics of this variable follows an AR(1) model for all individuals:

$$y_{it} = \lambda_i y_{i,t-1} + u_{it}, \ \text{i=1,2, ..., N}, \quad \text{t=1,2, ..., T} \qquad (2.12)$$

Therefore, let us assume λ_i follows a Beta distribution of the second type on the range (0,1):

[20] *Pesaran (2003) and Zaffaroni (2004) present some refinements of the Granger (1980) results.*

[21] *This explanation for the long memory result arising from aggregation is based in Pesaran (2003).*

$$f(\lambda)=\frac{2}{B(p,q)}\lambda^{2p-1}(1-\lambda^2)^{q-1},\quad 0\le\lambda\le 1 \tag{2.13}$$

In this case:

$$\lambda^j=\frac{B(p+j/2,q)}{B(p,q)} \tag{2.14}$$

And for a large *j*:

$$\lambda^j=(p+j/2)^{-q} \tag{2.15}$$

In this case, the aggregated data, $Y_t=\sum_{i=1}^{N} y_{it}/N$ will present long memory when $0<q<1$.

By using the results from the previous section for disaggregated data, we could approximate a Beta distribution for the AR(1) coefficients as a form to estimate the parameters *p* and *q* of the Beta distribution and check if it is consistent with long memory models for aggregated inflation data in Brazil. Usually, the long memory hypothesis is tested directly from the aggregated data. In some circumstances, for instance in our small sample of quarterly inflation data, this approach can be considered an alternative to the more traditional tests of long memory in time series data.

It is clear from figure (2.4) that the case proposed initially by Granger (1980) does not apply to our estimation results. The distribution of the $\lambda's$ are not restricted to the interval [0,1] and the Beta distribution cannot be applied. It does not mean that aggregated inflation does not have long memory. The generalization of long memory proposition is not an easy task and we do not intend to do it in this paper.

2.5 Inflation Persistence in Emerging Markets

In this section we study inflation persistence in other developing countries. The sample of countries we use is based on two aspects related to the history of these countries. Firstly, we include in our sample countries with a history of high inflation in the recent past, particularly in the 90's. Secondly, we include in our sample countries that introduced a clear strategy to pursue monetary stability, in particular by using the inflation target regime. These countries resemble Brazil and are more prone to show the same problems in dealing with inflation cycles and also inflation expectations. Therefore, high inflation can lead to several mechanisms of indexation similar to the ones observed in Brazil and thus lead to high inflation persistence.

Table 2.6 presents the results for inflation persistence estimates for our sample of developing countries. The estimated inflation persistence is larger in Brazil than for most of other developing countries considered in our sample. Only Poland shows inflation persistence larger than the one estimated for Brazil. It is important to notice the low inflation persistence for most of the countries. This result seems to indicate that inflation persistence is similar for most of the countries. The difference in inflation dynamics among emerging countries and developed countries may lie on other aspects of the inflation dynamics and not on persistence.

Table 2.5: Summary Statistics of Emerging Markets Inflation

	Headline Inflation			Core Inflation		
	Mean	Std Dev	Corr(t,t-1)	Mean	Std Dev	Corr(t,t-1)
Chile	3,8%	1,8%	0,443	2,3%	1,2%	0,337
Czech Republic	4,0%	2,4%	0,174	3,9%	2,6%	-0,019
Hungary	8,9%	3,9%	0,643	8,5%	4,0%	0,734
Israel	3,6%	2,5%	0,362	3,3%	2,6%	0,379
Mexico	10,1%	5,5%	0,760	9,6%	5,3%	0,775
Poland	6,4%	3,6%	0,650	6,0%	3,3%	0,869
Turkey	29,3%	12,2%	0,849	29,1%	12,1%	0,913
South Africa	5,6%	2,1%	0,592	n.a.	n.a.	n.a.
Colombia	8,7%	4,0%	0,411	5,2%	1,9%	0,045
Peru	4,0%	2,0%	0,565	n.a.	n.a.	n.a.
Brazil	7,3%	2,7%	0,714	7,3%	2,7%	0,714
Mean	8,3%	3,9%	0,560	8,4%	4,0%	0,527
Median	6,4%	2,7%	0,592	6,0%	2,7%	0,714

Source: OECD and authors' calculation. All data based on annualized quarterly inflation rates.

Several authors have used Hansen (1999) bootstrap method to estimate the mean of the autoregressive coefficients in dynamic models like the ones we have estimated so far. The first reason to use Hansen (1999) is the bias in OLS estimation. The second reason is to construct confidence intervals to the estimated coefficients. Considering these reasons we have also used Hansen (1999) method for emerging markets inflation (using this method also make it easier to compare our results with those reported by other authors). All models were estimated including both a constant and a time trend. The time trend is not a dummy variable as in the previous estimation (results from table 6). This makes the results not directly comparable. A possible future refinement of this estimation would be to reduce the estimation interval to exclude the stabilization period in most countries (usually from 1995 to 2000). The constant term and trend coefficients were not reported.

In table 2.7 we report the results of emerging markets estimates. In figure 2.10 we show both coefficients estimates for both emerging and developed countries. Using this alternative estimation procedure, inflation persistence is lower than average in Brazil and not higher as in other cases. This can be a result of the estimated model, which did not allow for structural break or a dummy trend variable. In many emerging markets, the estimated 90% confidence interval for ρ is very large. Again, this can be a result of the post stabilization period, which could have made the coefficients unstable in the first part of our sample.

Table 2.6: Inflation persistence estimates for emerging markets

	Headline Inflation		Core Inflation	
	ρ	AR(4)	ρ	AR(4)
Chile	0,151	0,375	0,223	0,295
Czech Republic	-0,076	0,174	0,328	0,505
Hungary	0,482	0,508	-0,041	0,236
Israel	-0,226	0,022	0,226	0,404
Mexico	0,438	0,423	0,488	0,487
Poland	**0,543**	**0,670**	**0,708**	**0,650**
Turkey	0,393	0,411	0,209	0,198
South Africa	0,368	0,448	n.a.	n.a.
Colombia	0,310	0,412	0,461	0,682
Peru	-0,052	0,033	n.a.	n.a.
Brazil	**0,594**	**0,598**	**0,681**	**0,692**
Mean	0,266	0,370	0,365	0,461
Median	0,368	0,412	0,328	0,487

Notes: inflation persistence coefficient based on models (1) and (2) using data from selected emerging markets from 1Q00 to 4Q11.

The estimated confidence interval for emerging markets is usually larger to the estimated for developed countries by Levin and Piger (2004): in our 11 countries sample, the average interval is 0.51while in the Levin and Piger (2004) sample of 12 countries, the average interval is 0.38.

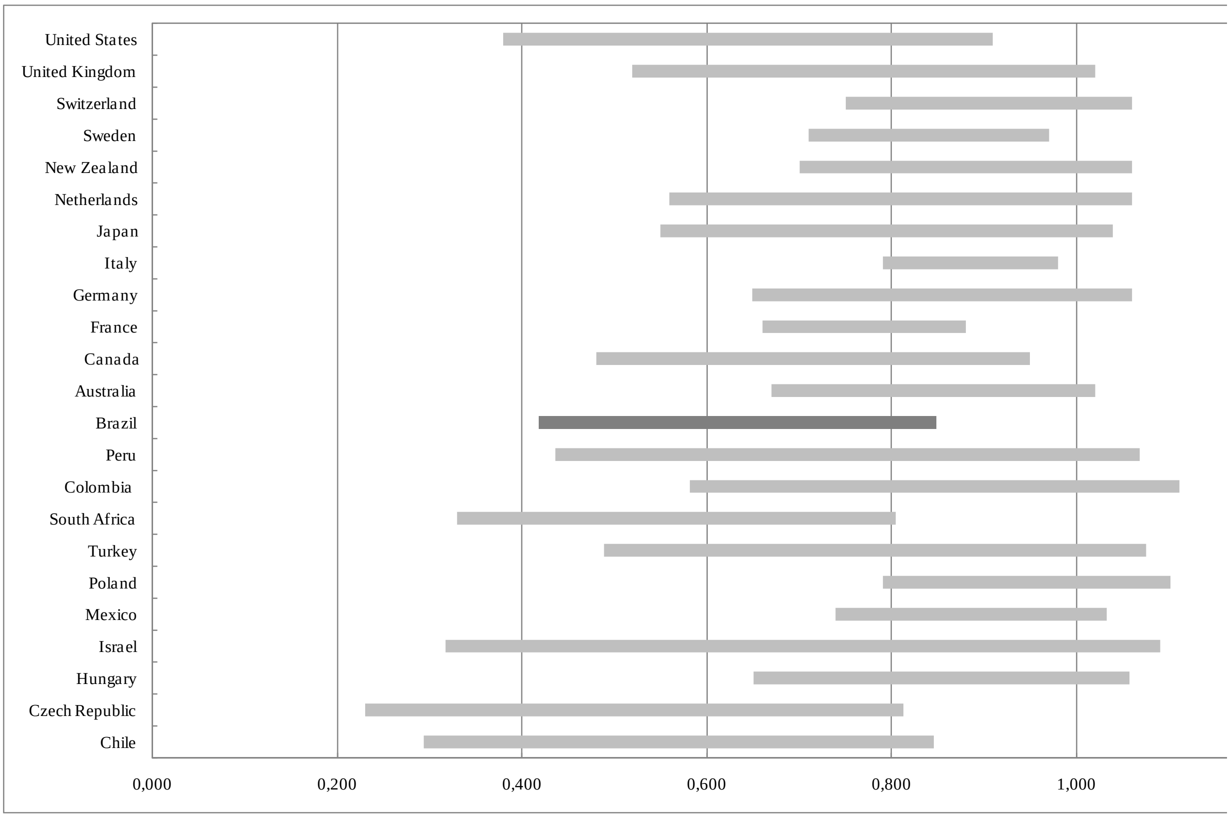
Table 2.7: Inflation persistence in emerging markets – Hansen (1999) median unbiased approach

	Coef. Estimate		Grid Confidence Interval (90%)	
	ρ	S.E.	lower	higher
Chile	0,453	0,141	0,294	0,846
Czech Republic	0,408	0,147	0,230	0,812
Hungary	0,732	0,086	0,651	1,058
Israel	0,487	0,172	0,317	1,091
Mexico	0,797	0,061	0,740	1,032
Poland	0,837	0,084	0,791	1,102
Turkey	0,608	0,135	0,489	1,076
South Africa	0,480	0,124	0,330	0,804
Colombia	0,686	0,132	0,581	1,112
Peru	0,574	0,136	0,436	1,069
Brazil	0,547	0,110	0,418	0,848
Mean	0,601	0,121	0,480	0,986
Median	0,574	0,132	0,436	1,058

Notes: inflation persistence coefficient based on model (1) using Hansem (1999) medium unbiased estimation and bootstrap confidence interval.

Using Hansen (1999) method, the inflation persistence is higher for all countries but Brazil. This result was expected since OLS estimation of autoregressive models are biased downward as expressed by Hansen (1999) among others.

Figure 2.10: Inflation persistence in emerging markets and developed countries*



**The results for emerging markets are those from table 7. The results from developed countries were extracted from Levin and Piger (2004), figure 2.*

2.6 Final Remarks from Essay 2

This paper estimates inflation persistence for Brazil using both aggregated and disaggregated data and also estimates inflation persistence for other emerging countries. The results can be summarized as follows: inflation persistence in Brazil is larger than in other emerging markets when traditional OLS estimation is used. Using the median unbiased estimation, inflation persistence in Brazil is not different from other countries.

Disaggregated inflation persistence is much lower than aggregated inflation persistence. Such results hold for both the mean and median of disaggregated inflation persistence. Despite the large role for lagged inflation in aggregated inflation dynamics, our estimates indicate that expected inflation plays a major role in disaggregated inflation dynamics in Brazil.

Our results did not identify the sources of inflation persistence. One of main difficulties in identifying the sources of inflation persistence is the similar levels of autocorrelation in the variables we used as inflation determinants. The most persistent variable in our study is inflation expectation but this

variable is very persistent by construction (we used inflation expectation for the next twelve months on at quarterly frequency). In this paper we did not analyze carefully the existence of common factor among disaggregated inflation data. One possible explanation for the large persistence in aggregated inflation in Brazil is the presence a common persistent factor in disaggregated inflation. We consider this possibility as a refinement of this paper.

In sum, our results points out that inflation persistence in Brazil is larger than in other countries but this result is not common to all methods we used. There are still several questions that should be addressed before calling inflation persistence a critical problem in Brazil.

3 Interest Rate Convergence in Brazil: Country Risk Premium, Currency Premium and Limits to Arbitrage

3.1 Introduction

According to most macroeconomic textbooks, if the local interest rate is greater than the international interest rate, capital would flow to the higher interest rate country until the difference goes to zero. In practice this hardly happens. Despite some evidence of interest rate convergence (or transmission) among countries as pointed out in Frenkel, Schmukler and Serven (2004), interest rate differences among countries remains in place for several years. Some possible explanations for this empirical fact include greater probability of default among higher interest rate countries, expected currency depreciation, market segmentation, and limits to arbitrage, among other explanations.

The concept of "limits to arbitrage" and also "risk arbitrage" will be very important throughout the paper. The plain vanilla example usually given in introductory economics and finance textbooks of arbitrage opportunities only exists in rare circumstances. For instance, the covered arbitrage operation usually yields no gain as the forward exchange rates are priced according to the interest rate differential[22]. Schleifer and Vishny (1997) called "risk arbitrage" the operations that are not exactly risk-free. This includes for instance, investing in high yield currencies like the Real. The textbooks arbitrage is flawed in another aspect: funds available for speculators are limited. Usually, fund managers use other people's money to engage in arbitrage operations. Several papers already demonstrated that funding constraint is usually biding. This funding constraint explains why some prices deviated from fundamentals in the crisis of 2008 and also in other crises. An example of this is Griffoli and Ranaldo (2011). Adrian and Shin (2010) also presents evidence of liquidity and funding constraints on financial intermediaries positioning.

The aim of the present paper is to study the reasons behind the non-convergence of interest rate rates in Brazil to levels observed in developed countries and even developing countries (for instance Mexico, Chile, Colombia and Peru have had interest rates lower than 10% and sometimes lower than 5% for several years). The results are hardly conclusive considering the ample scope of the paper. However, we present some evidence of exchange rate and to a lesser extent country risk as factors behind the larger interest rate in Brazil. Another contribution of this paper is to present a clear presentation of the main investment opportunities regarding the Brazilian fixed income market and the existing risky arbitrage opportunities.

Another problem with this concept of "interest rate convergence" is that in fact, there is not one "interest rate" but several different rates in each country. In this paper we will study short term rates most of the time but we will also study the dynamics of long term rates. In practice, short term rates are less risky (considering volatility as a measure of risk) and is more used in arbitrage operations. However, it is a conventional wisdom in Brazil that foreign investors represent a large fraction of trades in long term bonds and also long term interest rate derivatives. Taking this into account, we

[22] *Some authors present evidence that covered interest arbitrage was possible (forward price deviated from the interest rate differential) during the crisis in 2008. However, moments like the final months of 2008 are an exception. It is interesting to note that prices exceptionally deviating from fundamentals by a large amount are one of the results of the model in Schleifer and Vishny (1997).*

study the impact of foreigners' position in interest rate futures on the shape of the yield curve and also two-year rates (long term for Brazilian standards).

Over the last few years, the Sharpe ratio of carry trades involving both long position in the BRLxUSD (or simply BRL) exchange rate and a long (receiver) position in a local fixed income instrument (or fixed income derivative like interest rate futures[23]) have been high. As the local interest rate has been higher than interest rates of other countries, the main risk factor over the last years was exchange rate volatility. Our results indicate that even adjusting for the high BRL volatility, carry trades involving the BRL yielded a positive risk adjusted gain. We will examine FX volatility in the last section of this paper. Contrary to common statements made by the local press, the BRL is not the most volatile exchange rate among the most traded currencies. High yield currencies like ZAR, PLN and CZK presented higher volatility over the last eight years. Also contrary to local "conventional wisdom", the BRL did not present the largest "beta" among the most negotiated currencies. Despite being among the most volatile and highest beta currency, several other currencies are more volatile and present a larger beta. However, the BRL presented one off the largest appreciation over the last years, particularly in the first part of our sample (2004 to 2007). In synthesis, the BRL presented a low beta and a large alpha in the first part of our sample and a low alpha and large beta in the second part of our sample (2008 to 2011). In any case, the Sharpe ratio of simple strategies involving the BRL was large over the entire sample period. The explanatory power of this simple APT model for the BRL is lower than average, particularly in the first part of our sample and the explanatory power of this model is highly dependent on the currency in the denominator of the FX pair.

We summarize our results as follows: a) there are several indications of arbitrage operations involving the BRL and local rates both by foreign and local investors; b) the returns of investing in the BRL + local rate was positive on average and the Sharpe ratio was also large; c) the BRL volatility was the most important risk factor over the last years but several forms of interventions or restriction on trading activity or investment positions also presented a risk; d) the volumes traded or invested by foreign investors in one of the liquidity interest rate markets was not sufficient to cause discernible increases or decreases in local interest rates. This paper is organized as follow: in section 3.2 we present a brief discussion of the literature related to this paper. In section 3.3 we discuss a simple analysis of interest rate differences among countries and FX futures pricing and the evolution of spot and forward BRL rates and presented some evidence of positive carry trade returns for foreign investors or local investors with long positions in the BRL (short USD). In section 3.4 we study a foreign investor's position on local interest rate instruments and its impacts on local rates. In section 3.5 we study country and currency risks faced by foreign investors investing in Brazil. Section 3.6 presents our conclusions for this chapter.

3.2 Brief Literature Review

In this section we briefly discuss some relevant studies on covered and uncovered interest rate parity, capital flows and the more recent theme of "carry trade". Most of the older research focused on interest rate parity and capital flows as these two subjects are connected as mentioned in the introduction: differences in interest rates leads to a capital flow from the low interest rate country to the high interest rate country. Lately, the literature has focused on so called "carry trade" activity. This

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[23] *When using interest rate futures, the equivalent position to a long position in a fixed income security is a "receiver" position. We may use this term in the following pages instead of "long" interest rate futures. Note that these terms represent the same position being the "receiver" the usual term among market participants.*

recent literature has some differences with the 80's and 90's literature as it clearly recognizes that there are limits to arbitrage and, more important, the arbitrage opportunities in investing in high yield countries are not "risk-free" as usually expressed in textbooks. The risks arise not only from possible defaults (default risk or country risk are usually recognized in theory) but also from interest rate and exchange rate volatility, decreasing liquidity, market to market adjustments on positions and possible redemptions of funds from clients, among others.

3.2.1 Covered and uncovered interest parity and currency risk premium

Frankel (1995) proposes several measures of financial integration, including covered and uncovered interest parity. The author's results indicate that real interest differences among countries can be large and persistent. Regarding the covered interest differential, the author's results indicate that there are differences too but he attributes it to a "country premium".

Frankel et al. (2004) study the correlation/co-movement between short term interest rates in several countries and conclude that changes in interest rates in industrial countries are transmitted to developing countries. The impact is large even for countries with floating exchange rate regimes. The authors argue that the result is an indication of integration in financial markets, high correlation in business cycles among different countries is an indication that rates in developing countries are dependent on interest rates in developed or industrial countries. The authors do not study the size of the difference in local and international interest rates.

3.2.2 Capital Flows in the 90's

Large capital inflows to Latin America is not new phenomenon. Several papers describe earlier episodes of capital inflows to emerging markets, including Latin America countries in the '80s, early '90s, and also late '90s. In many cases, the inflows are associated with both better economic prospects in emerging markets, including structural reforms, improvements in the Balance of Payments, etc. Besides that, the situation in global financial markets and particularly in developed markets like US played an important role in explaining the inflows of capital to emerging markets.

Diaz-Alejandro (1984) describes the external crisis in the early '80s. According to Diaz-Alejandro, the "*breakdown of international financial markets and an abrupt change in conditions and rules for international lending*" explains the severity of the crisis in Latin America in the early '80s. In this case, the major form of capital inflows was external loans particularly borrowed by local banks instead of the portfolio inflows predominant in later episodes of capital flows. As the author argues, the tightening affected all countries in a similar manner despite the efforts made in terms of structural reforms or external positions. The importance of external condition was large also as the source of the inflows: the end of financial repression in the US in the late '70s explains why several countries in Latin America (Latam), particularly Chile and Argentina according to Diaz-Alejandro, were able to receive massive lending from international investors.

Calvo, Leiderman and Reinhart (1993) analyses the large capital inflows into Latin America countries in the early '90s. Several characteristics of this episode resemble the more recent episodes including: lower yields on external debt (lower risk premiums), exchange rate appreciation, and also exchange rate intervention by major central banks in the region, stock market appreciation, etc. Also in this case, the differences in macroeconomic performance were not a driver of these flows. As Calvo et al (1994) describe it, "*capital is returning to Latin American countries despite wide differences in macroeconomic policies and economic performance across the region*". Calvo et al (1994) construct principal components from two sets of data from ten Latin American countries: real exchange rate and

foreign reserves. The first and second components explain a large portion of the variance of these two variables in these ten countries. The author attributes this result to the fact that “external factors” were the main source of capital inflows. The real exchange appreciation is a consequence of these flows and the accumulation of international reserves is a consequence of the policy response in these countries[24].

3.2.3 “Carry-trade” and capital flows in the last decade

The recent literature has focused on the theme of carry-trades: inventing in high yield (high interest rate) currencies with money borrowed from low yield currencies. Note that this strategy assumes from the beginning that it is possible to earn positive returns investing in high interest rate countries with funds obtained from countries with low interest rates. Frankel (2007) notes that capital flows in the ‘90s were also associated with the name “carry trade” but, at the time the nomenclature was more common among market participants. In the last five years several studies were released with a better explanation of the operation, including its risks. Also, the very low interest rate in US in the last years (summing to the already low interest rate in Japan and Switzerland) appear to have contributed to the increase in carry-trade operations and consequently to the increase in empirical research on this subject.

Clarida, Davis, and Pedersen (2009) show that carry trades returns are negatively related to exchange rate volatility. In particular, the authors show that for a large number of currencies, high interest rate is associated with appreciating currencies. While in moments of high exchange rate volatility the expected result based on uncovered interest rate parity arises: high interest rate currencies depreciate.

Brunnermeyer, Nagel, and Pedersen (2009) show that the negative return in high volatility regimes in carry trades is associated with funding liquidity and crash risk in these investments. This result is in line with the findings of Ranaldo and Soderlind (2010): some currencies, like the Japanese Yen and Swiss Franc tend to appreciate in moments of high exchange rate volatility and also moments of low liquidity in financial markets.

Both Brunnermeyer et al (2009) and Clarida et al (2009) call attention to the asymmetric distribution of exchange rate variation in many countries. This characteristic of FX returns is related to episodes of carry-trade “unwind” according to these authors. This point is also made by Frankel (2007). The common explanation to this fact is the leverage effect, similar to the already well-documented asymmetric distribution of equity return: due to leveraged positions, investors may be forced to “stop” long positions in assets when the value of this asset falls. When the asset price rises, investors with long position are not forced to “stop the gain”.

Burnside, Eichenbaum, Kleshchelsky, and Rebelo (2011) show that carry trade strategies yield a positive return on average. This return is uncorrelated with common risk factors observed in other financial instruments like equity.

[24] Brazil is part of these 10 countries together with Argentina, Bolivia, Chile, Colombia, Ecuador, Mexico, Peru, Uruguay and Venezuela. The accumulation of international reserves in the early 90’s was important in Brazil as it helped to introduce the fixed exchange rate in the first years of the Real Plan.

3.3 Spot and forward exchange rates and interest rate parity relations

3.3.1 Foreign and local interest rates: the covered and uncovered interest parity

Uncovered interest parity states that local interest rates, i, must be equal to international interest rates, i^*, plus expected exchange rate variation, $E_t \Delta s_{t+1}$. This statement is based on simple arbitrage operations: let S_t be exchange rate measured in REAL/USD[25] and i_t the local interest rate (Certificado de Depósito Interbancário – CDI; local interbank rate) and i_t^* the US interest rate (T-Bills for instance). Then, a US investor would borrow $1 and buy S_t Reais and earn $S_t(1+i_t)$ after one period. The amount is than converted in USD to get: $S_t(1+i_t)/S_{t+1}$. The amount should be equal to the cost incurred by the American investor, $1+i_t^*$. This is usually stated as a simple equation:

$$S_t(1+i_t)/S_{t+1} = 1+i_t^* \quad (3.1)$$

This is the ex-post result of the arbitrage operation. This is not a risk-free operation: at the initiation of the operation the investor knows the interest rate in both local and foreign markets (i_t and i_t^*) and also knows the spot exchange rate, S_t. But he cannot be sure about the exchange rate at maturity (or "exit" exchange rate, S_{t+1}). Several authors like Clarida et al (2009) and Brunnermeyer and Pedersen (2009) have argued that carry trades increase in moments of low volatility in exchange rates. This result is a consequence of the risks associated in investing in high yield countries like Brazil. A possible way to diminish the risks is to make "covered arbitrage" using exchange rate forward contracts. As we discuss the latter, this is not such a clear source of arbitrage opportunity as many would argue: the exchange rate forwards are usually priced using the interest rate differential, eliminating any gain in the operation.

The uncovered interest parity is usually stated by means of expectations:

$$S_t(1+i_t)/E_t S_{t+1} = 1+i_t^* \quad (3.2)$$

Where $E_t S_{t+1}$ is the expectation at time t for the exchange rate at time t+1.

The covered interest parity differs from the uncovered parity only as the investor does not assume the risk of variations in the exchange rate, instead he buys the forward exchange rate

$$S_t(1+i_t)/F_{t,t+1} = 1+i_t^* \quad (3.3)$$

As we already know, the local interest rate was higher than the foreign interest rate in the last years, $1+i > 1+i^*$. So, for this relation to hold, we need a) $S_t / E_t S_{t+1} < 1$, or b) $S_t / F_{t,t+1} < 1$. These two inequalities do not represent the same relation. In (a) it is implicit that the local currency (the Real in our example) is expected to depreciate. In (b) it is implicit that the Real trades at a forward discount to the USD (or the USD trades at a forward premium in relation to the Real).

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[25] *Since our interest in on the evolution of Brazilian interest rates and exchanges rates, we will also use Brazilian convention in our illustrations.*

We show in the next sections that a) investors did not expect a depreciation of the Brazilian Real during all the period analyzed and b) the Real trades at a forward discount relative to the US dollar.

The forward discount arises as a non-arbitrage condition used to price exchange rate forward contracts:

$$F_{t,t+1} = S_t(1+i_t)/(1+i_t^*) \qquad (3.4)$$

$F_{t,t+1}$, S_t are expressed in local currency per unit of the foreign currency (LC/FC)

This is the standard formula for pricing FX futures or forwards and used in practice by market participants.

In Brazil, the foreign interest rate used in practice to price exchange rate future contracts is the local interest rate denominated in USD, the so called "cupom cambial". The name arises from the fact that foreign currency denominated local bonds or contracts are settled in Reais. Therefore, this kind of bond more precisely pays the exchange rate variation plus an interest rate or yield. The "cupom cambial" is this extra payment embedded in dollar denominated local bonds or contracts. The mechanics of this derivative is similar to a floating rate FX swap (local currency floating rate and foreign currency fixed rate, the swap rate). We explain this further in the next sub-section.

3.3.2 Foreign and local interest rates: the foreign currency denominated local bonds

Several emerging countries issue bonds denominated in foreign currency, mainly in USD. This phenomenon is usually described as "original sin": many emerging markets are not able to issue debt in their own currency. The existence of foreign denominated debt traded in the local market is also useful to study interest convergence.

In Brazil, bonds denominated in USD were issued for several years by the Brazilian government (including the Brazilian Central Bank and the Brazilian Treasury). The amount of this kind was large in the period of fixed exchange rate regime as investors were concerned about possible currency devaluation. The amount was also high in the first years of the floating rate regime.

More recently the Brazilian government has issued more inflation-linked bonds and also zero coupon bonds. Despite this change in the composition of Brazilian government debt, the local bourse, BMF&Bovespa, developed interest rate derivatives with the same characteristics of foreign currency denominated local bonds. The contract, called DDI, pays an interest rate (called *cupom cambial*) plus the variation in the BRL/USD exchange rate. In mathematical terms, the total return of this contract can be written as:

Total return of dollar denominated debt instruments: $\hat{i}_t + \Delta S_{t+1}$ (3.5)

The interest rate $\hat{i}_t$ is the *cupom cambial* or the local interest rate in USD. The conventional expression for the forward rate using local variables is then:

$$F_{t,t+1} = S_t(1+i_t)/(1+\hat{i}_t) \qquad (3.6)$$

Note that in this case the arbitrage is less risky compared to investing in the local denominated debt. As already mentioned works like Clarida et al. (2009) and Brunnermeyer et al (2009) show that FX

volatility is one of the most important determinant of carry trade results. By eliminating the FX risk, the dollar denominated local bonds offer a less risky arbitrage operation than the local rate debt instrument. At this point, it is also important to mention that all the operation mentioned above are "risky" arbitrage operations. The concept of risk-free arbitrage operations or risk-free arbitrage opportunities is almost inexistent in practice[26].

3.3.3 The forward premium puzzle in Brazil: early evidence of positive carry trade returns

The forward premium in different countries had been widely studied in the last years. In Brazil, some studies on this subject are Chrity, Garcia and Medeiros (2006) and Garcia and Lowenkron (2006). In both cases, the authors show that forward FX prices are a bad predictor of future values of the spot rate.

In practice, spot and forward exchange rates move in tandem, especially short term forward rates (in Brazil, forward rates expire monthly making the shortest term for forward rate always less than one month). However, the difference between spot and forward rates usually do not provide good information about the expected path for spot exchange rates. This empirical evidence dates back to Fama (1984). Note that the forward premium puzzle implies positive carry trade returns: if the high yield currency does not depreciate as predicted by the uncovered interest rate parity, there is a positive carry gain if the investor borrows in the low yield currency and invest the proceeds in the high yield currency. This implication had not been stressed by early studies on the forward premium but has been pointed out in recent studies on carry trades.

As one can see in fig.1, the spot and forward rates of BRL/USD moved very much close each other over the last years. A simple regression of the form:

$$s_{t+1} = \alpha + \beta f_{t,t+1} + e_{t+1} \qquad (3.7)$$

Would lead to values of α close to zero and β close to unity (we use lower case letters to denote the natural logarithm of variables). However, this is not the best way to infer the accuracy of forward rates as a predictor of future spot rates. The usual equation uses the variation in spot rates and the forward premium or discount:

$$s_{t+1} - s_t = \Delta s_{t+1} = \alpha + \beta(f_{t,t+1} - s_t) + e_{t+1} \qquad (3.8)$$

In this case the results are very poor and sometimes the estimated beta coefficients are even wrongly signed. This result is often called "forward premium puzzle". The variable $(f_{t,t+1} - s_t)$ is the forward premium for the currency in the numerator (forward discount for the currency in the denominator, i.e., the BRL usually trade at a discount to the USD as usually $f_{t,t+1} > s_t$).

Estimating equation (3.8) using monthly data from jan-04 to dec-11 leads to the following result:

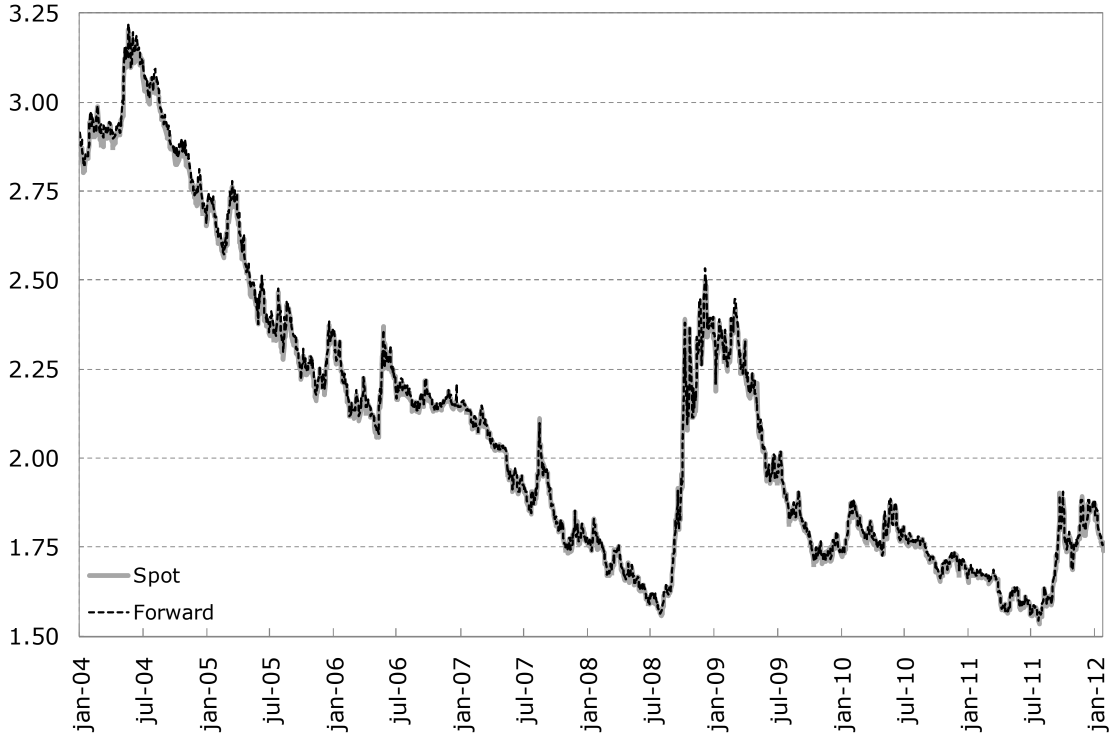
[26] *For further discussion of this point see Schleifer and Vishny (1997). Considering that most arbitrage opportunities are "risky investments", the Sharpe ratio has been used to evaluate the attractiveness of these opportunities.*

$$\Delta s_{t+1} = \underset{(0,0123)}{-0,0022} \underset{(1,4420)}{-0,2927}(f_{t,t+1} - s_t) + e_{t+1} \qquad (3.8')$$

$$\sigma_e = 0,0452 \qquad R^2 = 0,0004$$

This result is in line with the literature on the forward premium as the β coefficient is *wrong signed* considering the expectation hypothesis for forward rates. Note however that the coefficients are not significant. This result is different from those reported by Chrity et al (2006) who finds the positive and significant value for beta [β=0,33(0,14)] for the period dec-95 to dec-98.

Fig. 3.1: Spot and forward exchange rate



Notes: the data were provided by Bloomberg. The forward rate is the generic first contract.

The weak relation between the forward discount on the BRL/USD exchange rate and spot exchange rate variation can be seen using a graphical depiction of these two variables as in the figure 3.2b. The plot of exchange rate variation with expected exchange rate variation does not present a clear pattern between these two variables as well (fig. 3.2a). Also, no clear pattern emerges when we use the forward premium and future spot rate variation. As we show in the next pages, the forward rate is closely related to the interest rate differential as explained in section 3.3. The BRL traded at a forward discount to the USD in every month in our sample as can be seen in fig. 3.2a and 3.2b[27].

Expression (1.6) is based on the notion that forward rates are based on expected future spot rates. On the other hand, we showed in the third section of this paper that the non-arbitrage condition for forward rates implies that the forward premium is related to the interest rate differential as:

[27] *Chrity et al (2006) present evidence that this pattern was also present in previous years.*

$$f_{t,t+1} = s_t + i_t - i_t^* + e_t \tag{3.9}$$

$$f_{t,t+1} - s_t = i_t - i_t^* + e_t \tag{3.9'}$$

The relation is not so widely studied as an equation (3.7) or (3.8) but is widely used as a reliability check in empirical works using forward rates. Burnside et al (2011) for instance rank interest rate differentials by the forward premium in their empirical investigation of carry trade returns. We estimate this equation using both the libor rate as the reference USD rate as in equation (3.4) and with the *cupom cambial* as the reference rate in USD (eq. 3.6). Note that we are using log variables in (3.9).

Fig. 3.2a: Forward discount & expec. FX variation

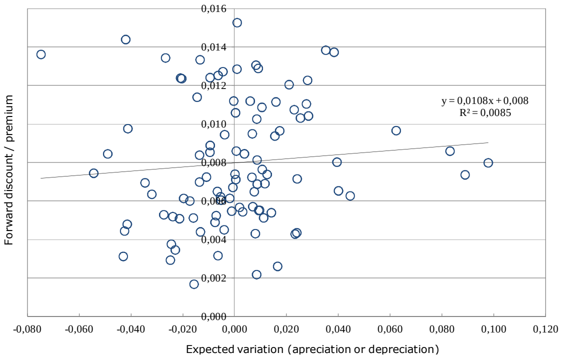
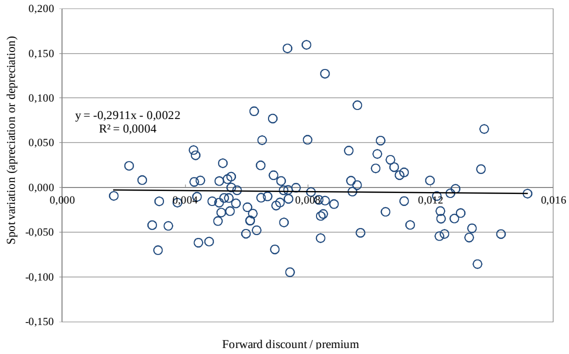


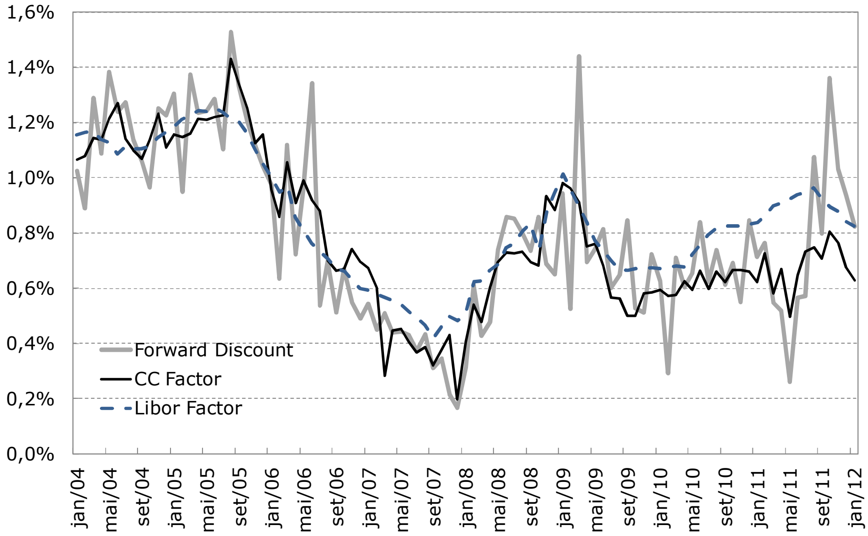
Fig. 3.2b: Forward discount & spot FX variation



Notes: the data were provided by Bloomberg. Expected exchange rate variation (appreciation or depreciation) was provided by the Brazilian Central Bank through Focus Survey.

The unit root tests for the forward premium and interest rate differences are presented in the Appendix. There is indication that all variables have a unit root. For the forward premium the presence of a unit root is less clear. The ADF test rejects the unit root hypothesis at the 5% level the KPSS test reject the hypothesis of stationarity at the 5% level. We will treat all variables are non-stationary and estimate (3.9) using both OLS and error-correction models.

Fig. 3.3: Forward discount & Interest rate differential



Notes: "CC Factor" is the interest rate differential using the cupom cambial as the USD rate as in equation 3.6. The "Libor Factor" is the interest rate differential using the libor rate as the USD rate as in equation 3.4.

The results of our estimation for equation (3.9) are presented in the table 1a. The results indicate that both Libor and *cupom cambial* are good explanatory variables for the forward premium, as expected. In both cases, the coefficient is very close to one. The R^2 is larger in the model with the *cupom cambial* as an explanatory variable and the coefficient is closer to one is this case, indicating a better fit of this variable. This is in line with market participants' indications.

We also estimated the model using an error-correction formulation. The unit root test indicated the presence of unit root in the interest rate differentials and was inconclusive in the forward premium variable and we will treat all variables as I(1). The cointegration test indicates the presence of one cointegration relation between the forward premium and the interest rate differences (*CC Factor and Libor Factor*) as expected.

Table 3.1a: OLS specification for the forward discount ($f_{t,t+1} - s_t = \alpha + \phi(i_t - i_t^*) + e_t$)

	Model 1	Model 2
Constant	0,00013	-0,0018
	(0,00051)	(0,00076)
CC Factor	0,9959 ***	
	(0,0619)	
Libor Factor		1,1589 ***
		(0,0874)
σ_ε	0,00168	0,00192
R^2 *adjusted*	0,7310	0,6478

Notes: "CC Factor" is the interest rate differential using the cupom cambial as the USD rate as in equation 3.6. The "Libor Factor" is the interest rate differential using the libor rate as the USD rate as in equation 3.4. In both estimations, the dependent variable is the forward discount, f_t-s_t, as in 3.9'. All variables are expressed in percent per month. Standard deviation inside the parenthesis.

Table 3.1b: Error-correction model for the forward discount ($f_{t,t+1} - s_t = \alpha + \phi(i_t - i_t^*) + e_t$)

Model 1			**Model 2**		
Error-correction relation			Error-correction relation		
fp	*ccf*	*c*	*fp*	*liborf*	*c*
1	-0,9692	-0,000035	1	-1,1053	0,00147
	(0,073)			(0,0987)	
Lagged error-correction deviation			Lagged error-correction deviation		
fp	*ccf*		*fp*	*liborf*	
-0,8502	-0,0573		-0,8032	-0,0381	
(0,1599)	(0,0820)		(0,1331)	(0,0270)	

Notes: "CC Factor" is the interest rate differential using the cupom cambial as the USD rate as in equation 3.6. The "Libor Factor" is the interest rate differential using the libor rate as the USD rate as in equation 3.4. In both estimations, the dependent variable is the forward discount, f_t-s_t, as in 3.9'. All variables are expressed in percent per month. Standard deviation inside the parenthesis.

The summary of error correction formulation is presented in table 3.1b. The results are not materially different from the OLS estimation. It is important to mention that in both estimations the deviation from the cointegration relation is significant for the dynamics of the forward premium but not for the interest rate difference. This is an indication that the forward premium or exchange rate adjusts to the interest rate movements but not the opposite. We treat this result with caution since this could arise as a result of measuring errors in variables, particularly in the forward premium.

A possible drawback of this estimation is that the forward premium is not exogenous to the *cupom cambial* rate. These two variables can be both endogenous as the market for *cupom cambial* rate adjusts to movements in the forward premium and the forward premium could also adjust to changes in the *cupom cambial*. Indeed, as the market for forward USDBRL exchange rate is larger than the market for *cupom cambial,* it is probably more accurate to say that the forward premium is exogenous to the *cupom cambial* but not the opposite.

3.3.4 Final remarks

This section presented empirical evidence on some very elementary results of interest rate parity tests and forward exchange rate determination. Our results indicate that forward exchange rates are not good predictors of future spot rates and that forward rates are priced according to the interest rate differential (covered interest parity). This is an import result for our main question (why interest rates in Brazil do not converge to international levels) as it indicates that there are not risk-free investments opportunities involving Brazilian assets like deviations from the covered interest parity. Investors aiming at benefiting from the high interest rate environment in Brazil must bear some risk, mainly from exchange rate variation.

3.4 Risky arbitrage: carry-trades and factor models of exchange rates

The most common form of risky arbitrage with interest rates in different countries is so called “carry trade”. In this operation, investors go “long” in high-yield currencies and “short” low-yield currencies. Contrary to the covered interest parity, in this kind of operation the investors do not hedge the exposure by selling exchange rate forwards. As we mentioned, since forward contracts are priced taking into consideration the interest rate difference, selling forward contracts would eliminate the gain related to the interest rate differential.

Several authors show that “carry trade” strategies generated positive returns in the last years. Among others, studies on carry–trade include Burnside et al (2011). But this result is implicit to the “forward exchange bias” documented as earlier as in the ‘70s. The forward bias or forward puzzle is originated from the hypothesis that forward exchange rates should indicate expected future spot exchange rates: if a forward exchange rate is being traded at a discount, it is expected that the spot rate will depreciate in the future. In practice, what happens is that the exchange rate trading at a discount usually appreciates. As we mentioned earlier, currencies that trade at a discount are those where local interest rates are higher.

The well-documented “forward puzzle” indicates that high-yield currencies do not depreciate on average. If the spot rate does not depreciate and the local interest rate is higher, there is a positive carry return from investments in the so-called high yield currencies.

So, if there is a positive return, why does the interest rate not fall to a point where the positive excess return is canceled? There are several possible explanations for this. The most common relates the risk-

return relationship in this kind of operation. Some authors, like Lyons (2001), argue that the Sharpe ratio on carry trades may be low, diminishing the appetite for this operation. Another explanation is that there are several kinds of investors in the financial market with different demand motives related to the same asset. There are also different strategies related to the same asset. For instance, several asset managers have offered both "yield" investments like the ones related to carry trades and "momentum" stile investments based on short-term past returns of a given currency.

Historically, interest rates presented low volatility, especially short-term rates. On the other hand, exchange may exhibit large swings. Indeed, exchange rate volatility has been widely studied in the last years. As a consequence, the main risk related to carry trades relies on exchange rate volatility. This result is well documented in Clarida et all (2009). In this section we study the characteristics of both risk and return from carry trades involving the BRL from the point of view of foreign investors. In section 5 we study arbitrage operations from the point of view of local investors.

3.4.1 The Carry-Trade and the Sharpe ratio

The following equations showed in this section related spot and forward exchange rates and also interest rate differences. These are mathematical formulas used to price forward rates that will be used in the next sections. Another important concept that will be used in the next sections is the carry-trade and another important concept is the Sharpe ratio. The Sharpe ratio is widely used to analyze risky investments carry trades.

The basic idea of carry-trades is to buy forward exchange rates in countries with high interest rates and fund the investment with short positions in currencies with low interest rates. The carry-trade payoff can be expressed as:

$$y_t\left[(1+i_t^*)\frac{S_{t+1}}{S_t}-(1-i_t)\right] \tag{3.10}$$

Where:

$$y_t=\begin{cases}+1 & \text{if} \quad i_t<i_t^* \\ -1 & \text{if} \quad i_t>i_t^*\end{cases} \tag{3.11}$$

This strategy can also be constructed using the forward premium as a reference instead of looking at interest rate differences spot exchange rates. In this case, the strategy consists of selling currencies that are at a forward premium: $F_t>S_t$. This is a simpler way to look at currencies that offer a larger interest rate assuming that the covered interest parity holds (equation (3.9)). Just to make the strategy clear for now, let us look at the Brazilian Real (BRL) example. The interest rate in Brazil is among the highest in the world and the Real trades at a discount to the US Dollar most of the time. In this case, $F_t>S_t$. A simple carry-trade strategy in this case would be to buy sell forward contracts on the USD/BRL exchange rate contract. The investor will sell this contract by $\$F_t$ and expiration its value will be $\$S_T$. In simple terms, the payoff is positive when you buy a currency that is trading at a discount and this currency do not depreciate up to the maturity of the contract. In the case of the Real, the forward currency traded at a discount over the last few years (the USD traded a premium) but the Real indeed appreciated in this same time period. So, a simple strategy of selling forward USD/BRL contracts would produce positive payoffs.

Several authors like Burnside et all (2011), Lustig et all (2011) and Verdelhan (2012) form portfolios of currencies based on their forward premium from the highest to the lowest. A typical strategy is to form 6 portfolios and form long position in the portfolio with the largest discount and short the portfolio with the largest premium. We will use the data provided by Verdelhan (2012) since it is the most recent study. But the payoffs provided by these authors are very similar.

The Sharpe ratio is then used to address the attractiveness of carry trade investments. The variable has been measured and used both by academics like Burnside et all (2011) and also by market participants as expressed by Lyons (2001). The Sharpe ratio is measured as the excess return from an investment divided by the standard deviation of returns. The standard deviation is a usual measure of risk for any investment following the CAPM.

Sharpe Ratio = $(R_t - i_t^{RF}) / \sigma_t$ (3.12)

Where R_t is the total return from a risky investment, i_t^{RF} is the risk free rate and σ_t is the standard deviation of returns.

3.4.2 Risk and return in carry trades with Brazilian assets

The main difficulty in calculating "theoretical" Sharpe ratios refers to the interest rate instrument being used. We will assume an investor borrows and invests in one month Libor rate and one month CDI rate respectively. Some authors use 3 month rates but this makes little difference since most of the variation in returns comes from variations in exchange rates. At this point it is important to clarify the sources of risk and return: in a typical carry trade, investors expect to earn the interest rate differential (main source of gain) but the risk of this operation lies mainly in the exchange variation. Consider for example the result from a carry trade with the characteristics given above: if the one month rate is "contracted" at both the long and short side of the operation there is no interest rate risk when the operation is carried up to maturity. Also, as the money is borrowed from short term rates, the variation in these rates during the "life" of the contract tend to be small (variations in these rates is also a source of risk if the investor expects to close the operation before the maturity of these interest rate contracts, but the risk is lower than the risk associated with exchange rate variations).

On the other hand, the exchange rate variation tends to fluctuate during the life of the operation. This is the main risk for the carry trade. Several authors show that carry trades results are positive in moments of low volatility in exchange rates. Taking the perspective of US investor investing in Brazil it is easy to understand the sources of risk and return in a carry trade operation. Let S be the exchange rate between US dollars and Brazilian Real and let i be the interest rate in Brazil (high yield currency). The gross return at time T on the long position of the carry trade will be $S_0(1+i_0)^T / S_T$. The gross return on the short position will be $(1+i_0^*)^T$. Where S_0, i_0, i_0^* denotes variables at the initiation of the operation.

Since we know Brazil is the "high yield" currency, we have $i_0 > i_0^*$. The return to the foreigner investor will be exactly this interest rate differential if the currency does not appreciate or depreciate between t and T *(i.e.,* $S_0=S_T$*)*. If the Real appreciates, the gain for the foreigner investor will be even greater than the interest rate differential as $S_0 / S_T > 1$. On the other hand, the return will be lower when the Real depreciates. This simple example shows that even investors aiming at only benefiting

from higher interest rate in Brazil would care about the movements in exchange rate. Market participants sometimes call these strategies as “income strategies” since the main expected source of return is the interest rate differential. As a result of this, inflows to high yield currencies tend to be higher in moments of low exchange rate volatility.

An important characteristic of currencies subject to carry trades as the “yield currency” is its variation asymmetry: these currencies may suffer from large reversals of carry trade operations or “unwind” of carry trades, resulting in relatively fast exchange rate depreciations as investors sell the yield currency and buy back the “funding currency”. The asymmetry in exchange rate variation is usually explained as a result of levered investors and risk management. When the position is yielding a gain, there is “no need to hurry” but when the position starts, it yields a loss of a certain amount, a stop-loss order may be given by the investor. If the order is large enough to depreciate the assent even more, this may trigger stop-loss orders from other investors.

In the figure bellow we present rolling 21 days returns on the BRL as a yield currency and the USD as a funding currency (we will refer to this single currency carry trade strategy as *BRL Strategy*). The returns are positive on average (+1,4), but highly volatile and subject to large negative returns eventually. As already described, most of the volatility in returns arise as a result of exchange rate variation. We construct this example using spot exchange rate without adjusting for bid and ask spread and one month rate as a measure of interest rate. Interest rates are originally presented as annual rates and they were converted into daily rates using a 252 day basis.

In order to make our example more comparable with other studies with construct monthly returns using one month rates (CDI in Brazil and Libor for US) together with one month spot exchange rate variation and also present results from two currency funds: Deutsche Harvest G10 Index from Deutsche Bank and UBS V24 Carry Index from UBS. Using these funds as benchmark, the return and Sharpe Ratio on the BRL Strategy is much greater: the annualized return from 2004 to 2011 is close to 21% on the BRL and 7% and 2% on Deutsche and UBS respectively. The Sharpe Ratio on the BRL is 1,37 and 0,99 and 0,21 on the Deutsche and UBS respectively. The returns and Sharpe Ratio we calculated for these two funds match the numbers presented in marketing materials provided by these banks and discard miscalculation as a possible explanation for the large difference in returns and Sharpe Ratios.

Fig. 3.4a: 1m rolling “BRL” carry trade return*

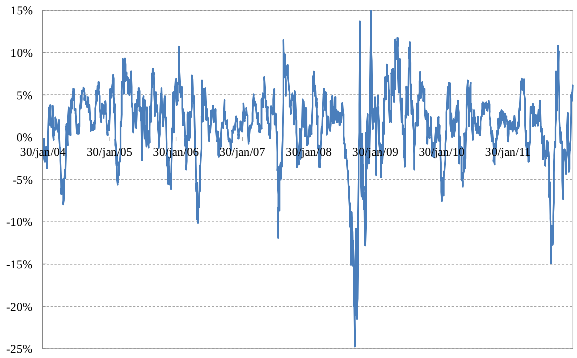
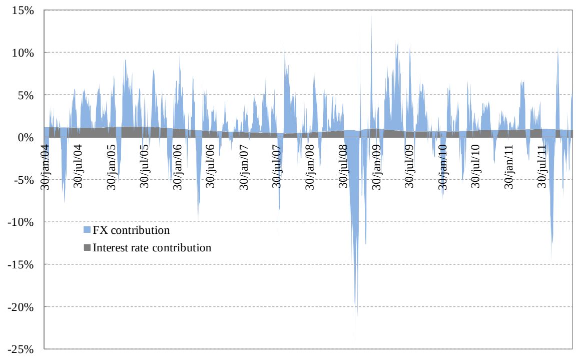


Fig.3.4b: “BRL” carry trade return decomposition



*Notes: *funding currency: USD; investing currency: BRL. The graphs are based on simulated results. The “BRL” index was simulated using the BRL/USD exchange rate and the local (CDI) and foreign (Libor) interest rate.*

A possible explanation for these differences would lie on bid and ask spread and trading costs that were not included in our BRL Strategy index. Papers like Burnside et al (2011), Menkhoff et al (2011), Lyons (2001) among others present results for carry trade strategies both with and without bid and ask spreads. In their results, the reduction in returns attributable to bid and ask spreads is usually smaller than 1% and the increase in volatility is close to 1% both at an annual basis. Using these values as a reference, the Sharpe Ratio on the BRL Strategy would be close to 1,2, still larger than the ones observed in the funds we are using as benchmark.

In a 2007 study on carry trades[28], Goldman Sachs presented results based on simulated carry trade strategies similar to the ones we are using and similar to the strategies from the funds we are taking as benchmark. In their study, allowing for emerging market currencies in the portfolio increases both the return and Sharpe Ratio: the average annual return is 4,8% from 1987 to 2006 for the G10 portfolios and 16,7% for the Global portfolio (includes emerging markets). The Sharpe Ratio is close to 2,0 in the Global portfolio and 1,6 in the G10 portfolio. The results are in line with the conventional wisdom about large returns in emerging markets.

If we break our sample in two parts, from 2004 to 2007 and from 2008 to 2011 the results are a little bit different. Both the returns on BRL Strategy and the funds are larger in the first half of the sample: 28%, 15% and 6% for BRL, Deutsche and UBS respectively. The Sharpe Ratio is also larger: 2,2 for the BRL and 2,3 and 0,8 for Deutsche and UBS respectively. These values are also close to the ones presented by Goldman Sachs for the period 2004 to 2006. The values presented by Goldman Sachs are also close to the ones reported by Burnside et al (2010).

Fig. 3.5a: Carry-trade index

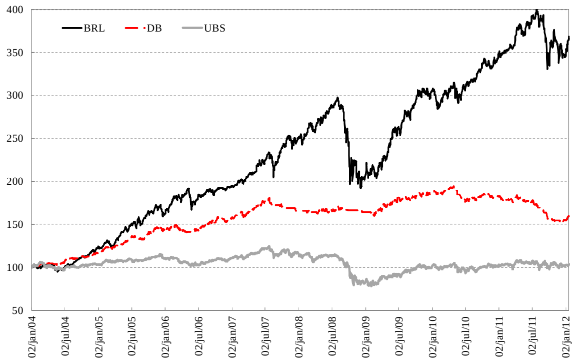
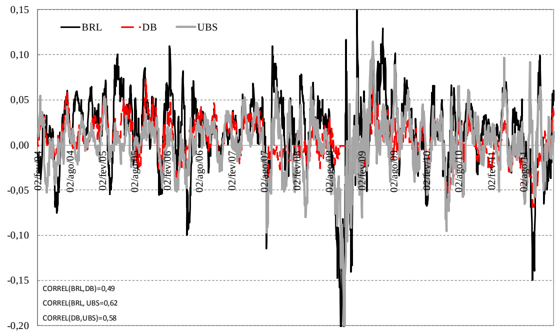


Fig. 3.5b: Carry-trade 1 month rolling return



Notes: the data for DB and UBS carry trade index are provided by Bloomberg. The "BRL" index was simulated using the BRL/USD exchange rate and the local (CDI) and foreign (Libor) interest rate.

These results indicate that the BRL or carry trades involving the BRL offered investors a positive return and in many cases greater than those observed in strategies involving currencies in developed countries. The Sharpe Ratio is also greater in the BRL Strategy. The result is also an indication of the attractiveness of the BRL as an investment: it is a standard result in finance that diversification yields to better risk-return trade-offs. The Sharpe Ratio on the simple BRL Strategy described above was close to or higher than the ratios in diversified portfolios.

[28] *Can the Carry Trade Carry On? Global Economics Paper n. 156. The sample period is 1987 to 2006.*

3.4.3 Exchange rate risk premium and the "Dollar factor"

According to the conventional theory, exchange rates are expected to depreciate in countries with higher interest rates. This means that the interest rate differential is a compensation for expected exchange rate depreciation and on average there would be no gain in investing in high yield countries (this is the notion of uncovered interest parity). We already presented evidence that this is not corroborated by the data. On average, exchange rates appreciated in countries with higher interest rates: this is another way to express the forward exchange premium anomaly discussed in section 3.3 and the negative β usually estimated in empirical works.

The fact that investing in high interest rate countries yields a gain on average does not mean this is a risk free operation. Exchange rates in high yield currencies tend to be more volatile and also present positive skewness (Brunermeyer et all, 2009), meaning that a large depreciation is more probable than a large appreciation. These facts together can be simply stated as: investing in high interest rate countries leads to a gain on average at the cost of being exposed to large losses eventually. In the picture bellow we plot together volatility and skewness of monthly exchange rate variation from jan-04 to dec-11. The picture illustrates the positive correlation between volatility and skewness in exchange rate data.

Past studies of the exchange rate risk premium in Brazil as Garcia and Olivares (2001) relied on statistical approaches to extract expected depreciation from financial instruments in a time of fixed exchange rate regime. Since Brazil had a floating exchange rate in our sample, this approach is not possible here. We will rely on some measures of expected depreciation but we attributed most of the risk in exchange rate to its volatility and skewness, two measures that are completely unimportant in the period of fixed exchange rate.

The Sharpe ratio described in section 3.4.1 is a way to put together these two concepts: average gain and average risk. The Sharpe ratio however does not take into account the skewness or downside risk of the investment. As one can see in fig. 3.6a, the skewness of the BRL is among the largest in our sample currencies. Brunnermeyer et all (2009) present indications that large positive skewness is an indication of net long position from speculators: as investors unwind the carry trade, the currency being disinvested by these speculators suffers losses (depreciation) that can trigger "stop loss" orders from other investors leading to further depreciation. The spiral effect does not happen when the position is yielding gain since there is no imposition of "stop gain" in most institutions. The stop loss order can be a consequence and exigency of the risk management setting of the institution.

Fig. 3.6a: FX volatility and skewness

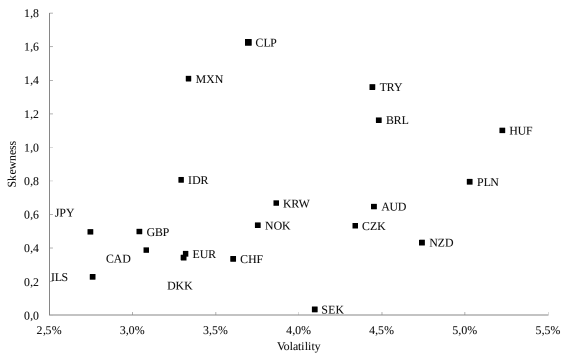
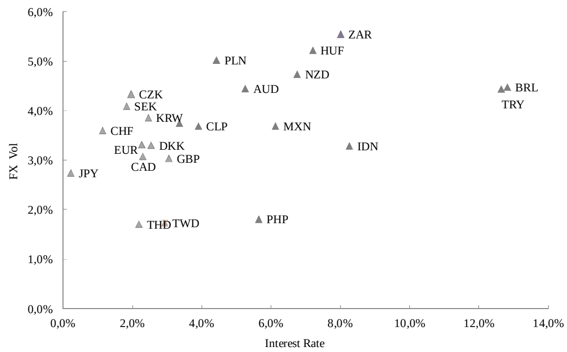


Fig. 3.6b: FX volatility and short term rates



Another form to express exchange rate volatility is systematic and unsystematic volatility using a market model. The concept of "beta" on exchange rates has been widely used by market participants

despite no clear explanation about what is the explanatory variable. Some candidates are measures of market risk appetite on risky assets like equities or bonds, the dollar index or other measures extracted from exchange rates. We estimated the model with two distinct measures of "market factor": the median from our sample currencies and the principal component extracted from our sample currencies. Let x_t be the market factor and $y_{i,t}$ the return (variation of spot exchange rate) of exchange rate *i*. Using these variables we can run a simple market model for each exchange rate:

$$y_{i,t} = \alpha + \beta x_t + u_{i,t} \tag{3.13}$$

At this point one would wonder why this could be useful. A well know result form market models like (3.13) is that the variance of returns, $y_{i,t}$, can be decomposed into market factors and in this case "currency specific" factors[29]:

$$\sigma_{y_i}^2 = \beta^2 \sigma_x^2 + \sigma_{u_i}^2 \tag{3.14}$$

This decomposition can be used to indicate if the large volatility of the BRL is due to country specific factors (σ_u) or "leverage" (β) on the market factor.

In the CAPM model, the x_t is a proxy for the market portfolio. In the FX model, there are some proxies for x_t as the DXY index (also known as US dollar index) and the TWI (trade weighted US dollar index). The disadvantage of using these indexes is that they are based on a small group of currencies and give a large weight to an even smaller group of currencies. The DXY is a weighted average of the EUR (58%), GBP (12%), JPY (14%), CHF (4%), CAD (9%) and SEK (9%). The TWI includes the AUD (4%) and the weights are better distributed but this index is heavily weighted on just two currencies, EUR (37%) and CAD (30%) due to large commerce with US.

We use a large set of currencies to extract x_t using both principal components methods and sample median. Our sample currencies are listed below. The sample is based on countries with a floating exchange rate regime and also currencies that play an important role in international FX market. Among the most traded currencies, we do not include the CNY since China uses a fixed or strictly managed exchange rate regime. The median and principal component proxy calculated from our sample of currencies yielded very similar results: the two sequences have a correlation above 0,98. The result is very similar if we normalize the exchange rate variable or not. The main difference between the two variables (median and principal component) is the large volatility of the principal component sequence. This is a result of the larger weight attributed to more volatile sequences in the principal component method vis-à-vis the simple median. The results presented bellow are based on the median FX variation as a proxy for x_t ($x_t = median(y_{i,t})$). If we use the principal component extracted x_t, the result is qualitatively the same (currencies with the highest/lowest betas and highest/lowest R^2 are in the same position using any measure of x_t). The β values are different: since the principal component method yields a more volatile sequence for x_t, the b using this variable is lower for any y_t.

The figure and table bellow summarize the results. The beta for the BRL is above one and above average as expected but the value is not in the top quartile. Other currencies like the Hungarian Florint

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[29] *This is result arise assuming* $\text{cov}(x,u) = 0$.

(HUF) and the Polish Zloty (PLN) present a large hoarding into the common factor than the BRL. Also, the Australian and New Zealand dollar (NZD) present a larger beta than the BRL. The results also present evidence of the influence of fundamentals and risk factors on betas: commodity currencies like the BRL, AUD, NZD and even NOK present larger betas than average. The betas are also larger at low rated currencies like HUF, PLN, CZK and TRY. These countries also have larger interest rates.

The residuals of these models are not homocedastic: the adherence of the data to the model is better during the second part of the sample. The results do not change significantly if we use the sterling pound as a base currency: since the US dollar depreciated against most currencies during the last years, some critics would argue that our results could indicate only sensitivity to US dollar depreciation instead of a market model. A possible extension of our estimation would be to include other variables like commodity index and measures of market risk premium on fixed income securities as explanatory variables.

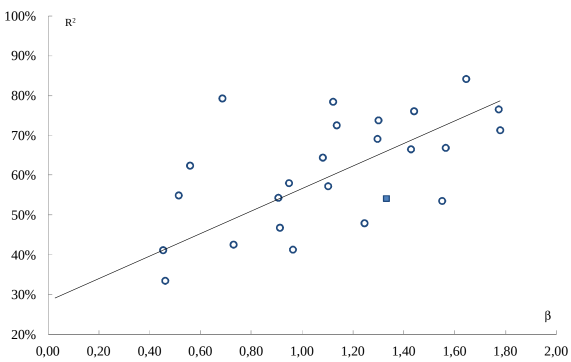
The result for the BRL also indicates a low explanatory power of this model considering the high value of the beta coefficient. The R^2 for the BRL is among the lowest in the sample. The result may arise as a consequence of FX interventions during the most part of our sample and also as a result of the transformations the country suffered in the last years, particularly in the first part of our sample. The BRL suffered a large depreciation in 2002 and the reversal of this depreciation took long to occur, as investors were cautious about the new government during the first years of its mandate (2003-2006).

A possible refinement of our results would be to divide the sample into two or more parts and investigate the stability of the beta coefficients. The table with estimated coefficients is presented in the appendix. The results indicate that large beta currencies in the first part of the sample are also large beta currencies in the second part of the sample: the group of commodity currencies like ZAR, AUD and NZD and high risk currencies like PLN, CZK, HUF and TRY are among the largest beta in both parts of the sample. The explanatory power of this simple model is larger in the in the second half of the sample. This result could arise due to more widespread use of this idea among market participants after some years of "learning" or due to the larger volatility in the second part of the sample and particularly global shocks that could have affected most currencies in a similar way. For instance, the second part of the sample includes both the subprime crisis and the Euro Zone crisis.

Table 3.2: FX beta model (USD)

Currency	Beta	R^2	Currency	Beta	R^2
HUF	1.778	71.4%	CHF	1.101	57.3%
PLN	1.772	76.6%	MXN	1.08	64.5%
AUD	1.644	84.2%	CLP	0.9625	41.4%
NZD	1.5635	66.9%	CAD	0.947	58.1%
ZAR	1.5495	53.6%	IDR	0.9112	46.9%
SEK	1.439	76.1%	GBP	0.905	54.4%
CZK	1.427	66.6%	ILS	0.7289	42.6%
BRL	1.331	54.1%	SGD	0.685	79.3%
NOK	1.299	73.8%	MYR	0.5577	62.5%
KRW	1.295	69.2%	TWD	0.513	55.0%
TRY	1.244	48.0%	PHP	0.46	33.5%
EUR	1.121	78.5%	THB	0.4518	41.2%
DKK	1.1347	72.6%	JPY	0.028	-1.0%

Fig. 3.7: FX beta model (USD)



Notes: data provided by Bloomberg. The results are based on OLS estimation of equation (3.13) using monthly data using the median variation of the USD against these 26 currencies as the "x" variable.

The result for the BRL is different: the first part of the sample is marked by a large alpha and a low beta. Most of the estimated alphas are not significant in both part of the sample. The estimated alpha for the BRL is significant at a 10% significance level in the first part of the sample and the value is a huge 0,76% per month. In the second part of the sample, the alpha is much smaller and the beta is larger. Part of the explanation for the large alpha in the first part of the sample can be attributed to the large depreciation of the BRL in late 2002 and early 2003. This could lead to a depreciated currency in real terms that lead to a subsequent appreciation. The causes for the depreciation in 2002/03 was strictly local, this result do not appear in other currencies.

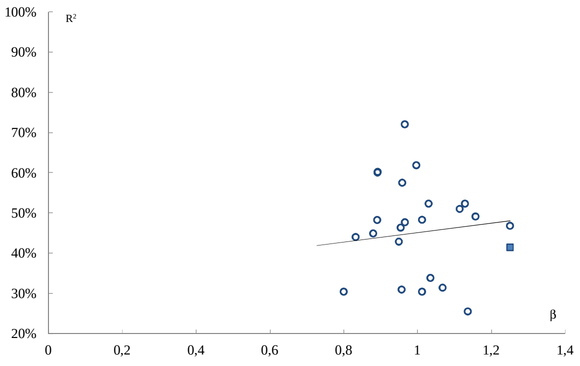
We also include other explanatory variables in this FX Beta model: the VIX volatility index and also the CRB commodity index. Both variables are non-significant in most of the estimations. This result indicated that the beta alone explain most of the variation in the currencies in our sample. This does not mean that variations in volatility or commodity prices are not important for currency pricing. Instead, indicates that these effects are well captured by the beta.

Another refinement would be to replicate the same exercise using a different currency as the basis for our estimations. We chose to use the GBP as the base currency instead of the USD due to its high liquidity and the results are replicated in the table below. The results are not as good as the USD results. The betas are closer to one and the explanatory power of the model is overall between 30% and 60% versus an explanatory power between 40% and 80% when we use the USD as the denominator currency. The betas are lower for other European currencies like HUF and CZK and PLN. Large beta, non-European currencies keep their large beta status but the estimated beta are lower than the ones estimated with the USD as the basis currency. The second replication of the beta model shows that a good part of the good explanatory power of the model lies in the appreciation/depreciation of the USD.

Table 3.3: FX beta model (GBP)

Currency	Beta	R^2	Currency	Beta	R^2
HUF	1.0111	30.5%	CHF	0.9484	43.0%
PLN	0.956	31.0%	MXN	1.029	52.4%
AUD	1.1132	51.1%	CLP	1.2495	46.9%
ZAR	1.1351	25.6%	CAD	0.8789	45.0%
NZD	1.034	33.9%	IDR	1.1562	46.9%
SEK	0.9648	47.8%	GBP	-	-
CZK	0.799	30.5%	ILS	1.0112	48.4%
BRL	1.2507	41.5%	SGD	0.9644	72.1%
NOK	0.8897	48.3%	MYR	0.9957	62.0%
KRW	1.1276	52.4%	TWD	0.9575	57.6%
TRY	1.067	31.5%	PHP	0.9534	46.4%
EUR	0.8911	60.3%	THB	0.8314	44.1%
DKK	0.8905	60.1%	JPY	0.7276	14.2%

Fig. 3.8: FX beta model (GBP)



Notes: data provided by Bloomberg. The results are based on OLS estimation of equation (3.13) using monthly data using the median variation of the british pound against these 25 currencies as the "x" variable.

In synthesis, what can we infer from the estimations in this section? The main message is that the BRL is in fact a "high volume" currency and also present high skewness, making investments in BRL denominated instruments risky. However, the BRL is not amongst highest beta currency. An

explanation for this fact is the relatively high specific risk (σ_u in our model). We acknowledge that over the last years the estimated β increased and idiosyncratic risk diminished, indicating the movements in BRL replicated more closely the median market movement (amplified by a large β). The result for the second part of our sample seems to corroborate the concerns usually presented by representatives of the Brazilian government stating that the BRL is subject to speculative capital and large swings derived from international market factors. This result seems to be a consequence of high interest rates in Brazil and elevated participation of commodities among Brazilian exports: we did not test this hypothesis but our results indicates that high yield (low credit rating) countries and commodity exports are among the highest beta countries/currencies in our sample. This result is in line with widespread market commentaries and technical (non-academic) research.

3.4.4 A factor model for the Brazilian Real

A novel literature surged in the last few years for the determination of exchange rates using factors like a dollar factor (similar to the one presented in the previous subsection) and also a carry trade factor. For a recent exploration of these factors see Vederlhan (2012). We estimated the following factor model for the Brazilian Real:

$$\Delta s_{t+1} = \alpha + \beta(i_t - i_t^*) + \gamma(i_t - i_t^*)Carry_{t+1} + \delta Carry_{t+1} + \varphi Dollar_{t+1} + \varepsilon_t \qquad (3.15)$$

Where $Carry_t$ and $Dollar_t$ refer to the carry trade factor and the dollar factor respectively. The carry trade factor is an index formed using portfolios (baskets) currencies with the largest interest rate and currencies with the lowest interest rates. We will use data from Verdelhan (2012) but similar data is constructed by Burnside et all (2011) and Lustig et all (2011). The dollar factor is the average change of the dollar against other currencies. Also in this case we use the data provided by Verdelhan (2012) but similar data was presented in the last subsection.

The results for the estimation of (3.15) are provided bellow. The coefficient on the carry factor is positive and statistically significant. The coefficient on the dollar factor is also significant. The coefficient on the interest rate differential is still not significant in this estimation.

We start including only the carry factor. This variable is statistically significant and has the expected sign. The coefficient on the carry factor is high and close to the highest estimated by Verdelhan (2012) for other countries. The estimated coefficient on the dollar factor is lower than the one we estimated in the previous section.

$$\Delta s_{t+1} = \underset{(0,003)}{-0,002} + \underset{(0,136)}{0,939}\, Dollar_{t+1} + \underset{(0,099)}{0,387}\, Carry_{t+1} + \varepsilon_t \qquad (3.16a)$$

$$\sigma_e = 0,0293 \quad \mathrm{R}^2 = 0,585$$

The inclusion of the interest rate differential does not lead to any improvement in the estimation results and this variable is not statistically significant.

$$\Delta s_{t+1} = \underset{(0,008)}{0,006} - \underset{(0,894)}{0,861}(i_t - i_t^*) + \underset{(0,111)}{0,388}\, Carry_{t+1} + \underset{(0,156)}{0,944}\, Dollar_{t+1} + \varepsilon_t \qquad (3.16b)$$

$$\sigma_e = 0,0295 \quad \mathrm{R}^2 = 0,588$$

The inclusion of an interaction term involving the interest rate differential and the carry factor is also not statistically significant.

$$\Delta s_{t+1} = \underset{(0,008)}{0,005} - \underset{(0,908)}{0,791}(i_t - i_t^*) + \underset{(0,375)}{0,291} Carry_{t+1} + \underset{(0,156)}{0,944}\, Dollar_{t+1} + \tag{3.16c}$$

$$+\underset{(5,124)}{1,164}(i_t - i_t^*) Carry_{t+1} + \varepsilon_t, \qquad \sigma_e = 0,0297 \qquad \mathrm{R}^2 = 0,588$$

This is another indication of speculative positioning on the Real that sums to the evidence provided in the previous sections. Compared to the results of Verdelhan (2012) our results share several similarities like the large "beta" on the dollar factor for Eastern Europe currencies like the Polish Zloty and the Hungarian Forint. Additionally, our results indicate a relatively low loading factor of the Real on the dollar factor and a relatively large loading on the Carry factor. The high load on the Carry factor was expected due to the high interest rate in Brazil. The low load factor on the Dollar factor was not expected.

3.4.5 Summing up

This section provided some evidence of a positive return of speculative investments involving the Brazilian Real and the large interest rate in Brazil. Also, there is strong evidence that the Brazilian Real is positively correlated with measures of carry-trade returns. In the next sections we look for some evidence of impact of foreign flows on the local interest rate. Returning to our original question (the reasons behind the lack of convergence of the local interest rate to the international interest rate), the results of this section are still far from conclusive. We interpret our results as an indication of that foreign investors (as well local investors) try to make profits using simple strategies involving long positions in high yield Brazilian financial instruments. In the next section we try to look for evidence that these flows may have some impact on local interest rates.

The discussion in this section is not exactly a theory of interest parity determination. It is more related to theories of exchange rate than interest rate parity. However, any theory interest rate parity will in include exchange rate dynamics.

3.5 Interest rate differential: is it country or currency risk?

In the previous sections we showed that Brazilian interest rates were higher than interest rates in other countries, particularly in US. This difference is larger for interest rates in Reais than interest rates in USD. In this last section we discuss the concepts of country and currency risks and the evolution of these risks in Brazil in the last years.

3.5.1 Country risk premium

In this section we assume rates in US play the role of risk-free rates. In this case, a simple decomposition of interest rates in Brazil would include the risk-free rate (US benchmark rate) plus a risk premium as follows:

$$i_t = i_t^* + \theta_t \tag{3.17}$$

Where θ_t is the risk premium and i_t and i_t^* are the local (BR) and foreign (US) rates.

Note that this is a rough approximation considering the evidence presented in previous sections where different sources of risk where presented. Now considerer that Brazil also issues debt instruments denominated in US dollars both in Brazil and in foreign markets. The USD rate of Brazilian debt can be shown as follows:

$$\hat{i}_t = i_t^* + \hat{\theta}_t \tag{3.18}$$

In this case, the risk premium $\hat{\theta}_t$ is closely related to the country risk premium as $\hat{i}_t$ (*cupom cambial*) and i_t^* (local US rate) is denominated in the same currency (US dollar). These two variables usually denominate onshore and offshore dollar rates. This risk premium was low at beginning of our sample (between late 2003 and 2004) and increased in the last part of our sample. In true, this risk premium took negative values at the very beginning of our sample. A possible explanation for this evolution is the more active approach from BCB at the exchange rate market, including buying and selling US dollars in the spot and future (swap) market, and the restrictions imposed on foreign investors investing in Brazil fixed income market and restrictions on local banks position in foreign exchange contracts, particularly short position on USD.

Another breakdown used to infer the size of the country risk premium is to use the difference between yields on Brazilian debt denominated in US dollars negotiated in US (offshore) and US yields (T-bills or Treasuries). This decomposition can be written as:

$$\hat{i}_t^* = i_t^* + \hat{\theta}_t \tag{3.19}$$

As we did earlier, we use a "hat" to indicate dollar denominated yields on Brazilian securities. $\hat{i}_t^*$ is best represented by C-Bonds or Global Bonds. These bonds are denominated in US dollar and negotiated in the external market and issued according the foreign rules. The country risk premium $\hat{\theta}_t$ is a rough approximation of country risk measures like the EMBI (Emerging Market Bond Index) spread calculated by JP Morgan. The EMBI risk measure is based on differences between emerging market bond yields and Treasuries or T-Bills yields on the same maturities and also adjusted for different durations of the these bonds associated to the different pattern of coupon payments.

In the last years, the market for Credit Default Swap (CDS) became more liquid and the swap rate in these derivatives is also used as a reference for default risk. The increased importance of CDS market became evident in the recent European crisis as the most cited measure of country risk was the CDS rates in countries like Greece, Portugal and others.

Using both EMBI spreads or CDS rates leads to same qualitative result in terms of evolution the Brazilian country risk: there was a sharp decrease during the last years. The country risk premium measured using offshore dollar yields was large in the early 2000's, particularly during the financial crisis in 2002. The fall in the country premium is consistent with the upgrades Brazil received from several rating agencies in the last years.

Several authors already noted that part of variation in country risk measures is due to better risk appetite from investors and does not represent better fundamentals. Taking this into consideration we present the difference between Brazil and Global Embi spread over the last 7 years in our table of country risk measures. Even in this case, there was a large fall in Brazilian country risk.

There is no exact way to decompose the interest rate difference between Brazil and USA or any other country. However, it is possible to approximate the difference using local and foreign interest rates and other variables. The approximation was done by Garcia and Olivares (2001) for instance. We now propose an approximation using six months and one one year rates. The problem with longer rates is

that it will not be available in Brazil in the early part of our sample. If we choose even shorter rates, some variables we use to decompose the differential would make no sense. For instance, there is no measure of default risk for one month or one day period. Even for a period of six months there will not be a default measure. On the other hand, some measures of exchange rate risk for periods longer than one year are not available.

Table 3.4: Summary statistics of country risk measures

	2004	2005	2006	2007	2008	2009	2010	2011
CDS 5Y	5,43%	3,28%	1,41%	0,89%	1,87%	2,11%	1,21%	1,33%
Embi Br	5,41%	3,99%	2,36%	1,81%	3,02%	3,07%	2,04%	1,94%
Embi Global	4,36%	3,16%	1,98%	1,88%	3,93%	4,81%	2,87%	3,29%
Embi Br - Global	1,05%	0,83%	0,38%	-0,07%	-0,91%	-1,74%	-0,83%	-1,36%
CC 1M: spread over Libor	-0,28%	-0,06%	-0,45%	1,21%	0,92%	1,20%	1,71%	2,89%
CC 3M: spread over Libor	-0,37%	-0,09%	-0,26%	0,92%	1,09%	0,95%	1,34%	2,80%
CC 6M: spread over Libor	-0,10%	-0,13%	-0,32%	0,88%	1,66%	1,15%	1,33%	2,72%

Notes: data provided by Bloomberg. The data are year averages of the daily data.

Taking all these difficulties into account, we propose a breakdown based on some known variables and also into "other" factors that may prevent full converge. These other factors may account for transaction costs, taxes, low liquidity in some financial instruments used in the decomposition, risks associated with government intervention and also errors in variables.

$$local\ rate\ =\ foreign\ rate\ +\ default\ risk\ +\ FX\ \ risk\ +\ other\ risk\ factors \tag{3.20}$$

We use as a reference foreign interest rate the Libor rate. Our preferred measure of default risk is the CDS rate as it is a direct measure of default risk perceived by investors and our measure of exchange rate risk will be the *CALL* and *PUT* option prices on the BRL/USD rate. The measure is better than expected depreciation since there could be differences between expected exchange depreciation and appreciation among those providing information to the Brazilian Central Bank (usually some economists working at banks, brokerage houses or hedge funds) and investors that in fact can make investments locally or in foreign countries. Also, the option price is the hedge price of exchange rate variation and can be seen as a price to avoid exchange rate risk. We do this decomposition using six month and one year rates. We do not use longer-term rates due to the low liquidity of longer rates in Brazil especially in the beginning of our sample and low liquidity of options for longer maturities. The FX risk is the net cost of buying a CALL and selling a PUT[30]. On the other hand, the measures of default risk are not appropriate for short term.

The results in fig. 3.9a and 3.9b indicate that a large part of the interest rate differential can be attributable to the exchange rate risk. The FX risk is based in the CALL and PUT options as we already explained. The CALL and PUT options are derived from the standard Black-Scholes formula and in this case the interest rate differential plays a key role in this price together with the exchange rate volatility. The average FX risk in our sample is 4,5%. The lowest value was observed in 2007 when the interest rate difference and FX volatility was the lowest in the sample.

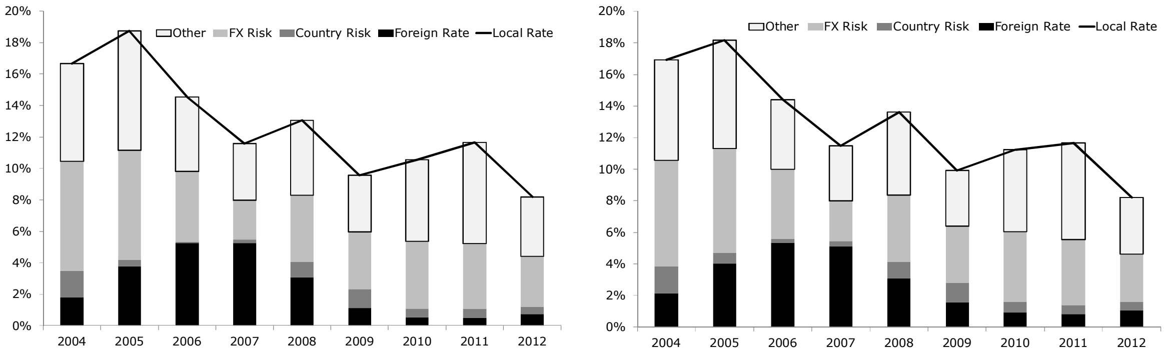
The variation in country risk explains just a small part of the interest rate difference in our sample period. This variable was much more important in the years of 2001 to 2003 when the country risk

[30] *Burnside et all (2010) also studies hedged portfolios like this.*

reached more than 20%. We used the CDS rate as a measure of country risk. The six month country risk measure was created using a linear interpolation of one, two, three, four and five years rates.

Fig. 3.9: Interest Rate Decomposition (eq. 3.20)

Fig. 3.9a: 6 months rate ***Fig. 3.9b: 1 year rate***



Notes: data provided by Bloomberg. The Local Rate is taken from the CDI curve and the foreign rate is the USD Libor rate. The Country Risk is the CDS rate. We used a linear interpolation of the 1,2, 3, 4 and 5 year rates to get a 6 months CDS rate. The FX risk is the price of an "at the money" CALL and PUT options (expressed in percentage of the spot exchange rate). All data are year average of monthly data.

According to this breakdown, a large portion of the interest rate difference is due to the "other factors". The portion of the interest rate difference was relatively low in 2006 and 2007 but increased in the last years.

3.6 Short-term foreign capital flows and interest rate

The concept of arbitrage embodies flow of funds to some investment in order to eliminate this arbitrage opportunity. Many authors that arbitrage opportunity may sometimes exist for a small period of time and small trades can eliminate the arbitrage gain. On the other hand, the difference in interest rate between Brazilian assets and interest rates in other assets has been large for long period.

In this section we look for evidence of the investors funds flowing into Brazil to exploit the gains of the high interest rates. We use mainly two different sources of data: the Brazilian Central Bank data, including the Balance of Payments accounts and local banks net position in USD assets, and data obtained from BMF&Bovespa data on foreign exchange and interest rate derivatives.

We also address the impact of foreign flows on interest rates in Brazil. The impact of foreign investors on local rates is not a new event and has been studied in other countries like in the US between 2004 and 2006. In the US, yields on long term Treasuries fell even after a series of increases in short term (fed funds) rates starting in the end of 2004. The lack of response of long-term rates to the rise in fed funds was called a "conundrum" at the time and several studies offered some explanations for this event. Most of these explanations relied on large flows of foreign investors buying Treasuries securities. "Reserve rich" emerging countries like China and oil exporters were among the possible large buyers of treasuries according to most of these studies.[31]

[31] *Warnock and Warnock (2009) studied the impact of foreign investors on interest rates in US and is the paper most close to ours. Backus and Wright (2007) also discuss the impact of foreign flows into US interest rates.*

3.6.1 *Balance of Payments Data*

The figure bellow presents the evolution of foreign investors' flows to equity and fixed income securities in Brazil. The data is taken from the Brazilian Balance of Payments. We use quarterly data to make it easier to see important trends and episodes in the last fifteen years. In the first five years, portfolio invesment flows into fixed income securities were larger due to the very high interest rate in Brazil (above 40% in some moments) and also due to the low liquidity of the Brazilian equity market. From 2007 to 2011 portfolio flows into equity investments is larger than flows into fixed income securities. This reflects the large amount of IPOs in this period and the increased liquidity of Brazilian stock market. The rise in commodity prices may have played an important role in these flows as commodity producers are the largest companies in Brazil: Vale (mining) and Petrobras (oil) shares represents close to 30% of Ibovespa index and also MSCI index. Other commodity producers, like steel and pulp producers, represent another good portion of these indexes.

3.6.2 *Futures and Derivatives Bourse (BMF&Bovespa) Data*

A possible drawback of this data is that it do not include positions into fixed income or exchange rate derivatives. Exchange rate futures in Brazil are several times more liquid than spot exchange rate and interest rate futures are the most traded derivative at BMFBovespa (even more than FX futures). Considering this, we also look at foreign investor positions in fixed income and exchange rate derivatives.

There are significant differences between data on foreign exchange and interest rate futures. Usually, most of the open contracts on exchange rate refer to the so-called "active" contract or the closest to maturity contract. On the other hand, for interest rate futures the most liquid contract is just a small part of the total contracts and the available data from BMFBovespa refers to total contracts without any distinction between long or short maturities, duration, DV1 or any other better measure of foreign investors position. Anecdotic evidence is that foreign investors buy more long-term contracts than short-term contracts. However, there is no empirical evidence of this due to lack of data.

At this point, some could argue that foreign investors would be better off investing in short term rates to exploit the interest rate differential without bearing the risk associated with the larger duration or term premium implicit in long rates. The main explanation for this not happening is the implicit yield on forward currency contracts as described at the beginning of this work. The forward price on exchange rate is derived from the interest rate differential. So, a simpler strategy to receive the interest rate differential is to buy forward exchange rate contracts (buy Reais or sell US dollars in the futures market) and roll the contract at expiration.

Fig. 3.10a: Portfolio flows: fixed income securities

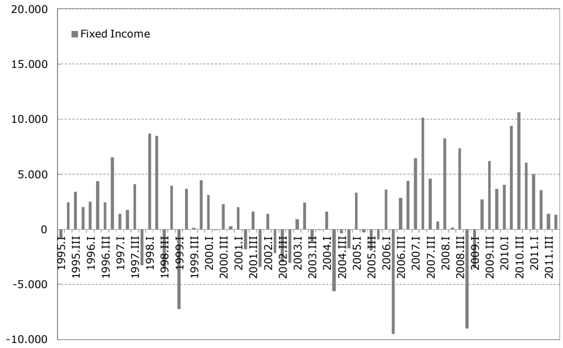
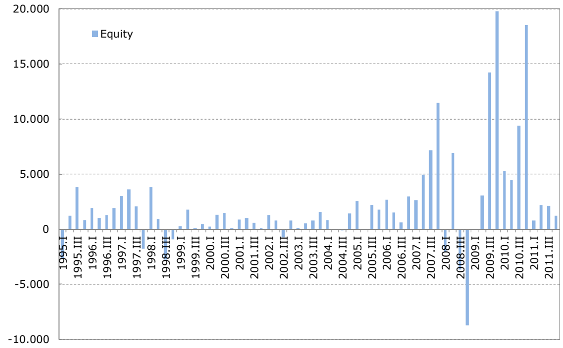


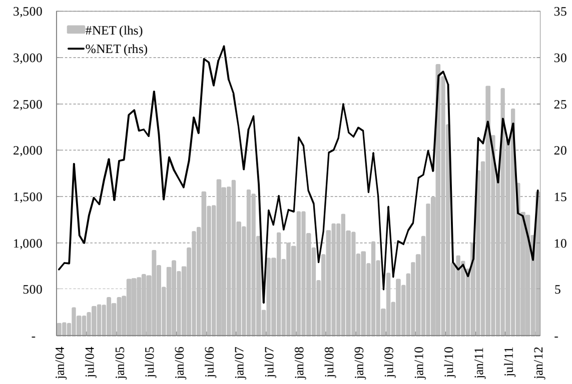
Fig. 3.10b: Portfolio flows: equity investments



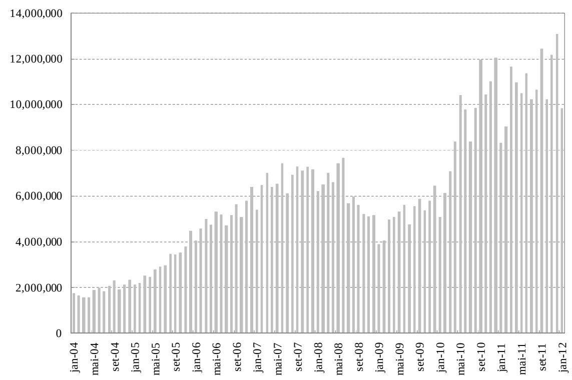
Notes: data in US$ millions. The data were provided by Brazilian Central Bank.

Fig. 3.11: Foreign investors position

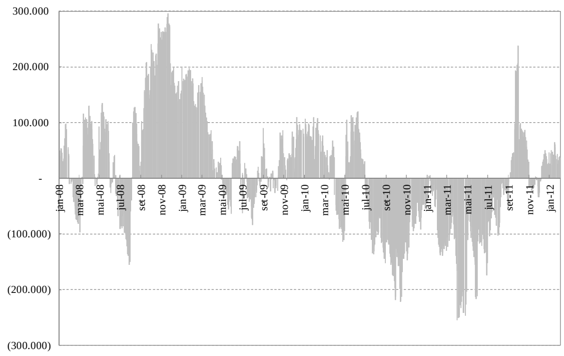
a) Interest Rate Futures (Foreigners)



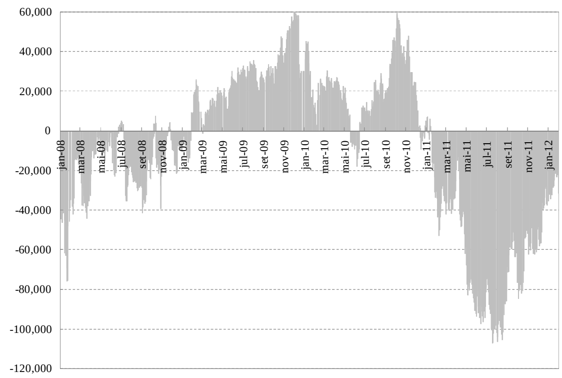
b) Interest Rate Futures (#Total)



a) FX Futures (#contracts)



a) Ibovespa Futures (#contracts)



Notes: data provided by BM&FBovespa.

3.6.3 Short Term Flows and Interest Rates

In this subsection we estimate the impact of foreign investors flows on interest rate. In particular, we use data from foreign investors' positions on interest rate futures. The interest rate futures negotiated

at BMF are some of the most liquid instruments in Brazilian fixed income market and the data available is one of the most accurate regarding the type of investor.

To estimate the impact of foreign flows on local rates we use data from the Brazilian term structure of interest rates together with data on inflation and GDP growth and also foreign flow information. This is a tentative approach to incorporate foreign flows into traditional macroeconomic modeling of the yield curve. We start the study by breaking down the yield curve into level (L), slope (S) and curvature (C) factors using splines:

$$i_{m,t} = L_t + S_t m + C_t m^2 + e_{m,t} \tag{3.21}$$

The variables L, S, C are the coefficients of the regression of interest rates at every point in time, t, on a constant and the variables "maturity", *m*, and squared maturity, m^2. Some authors use a simpler approach and define the slope as the difference between the longest and shortest rates divided by the longest maturity in the yield curve and the curvature as the difference between the average of the longest and shortest maturity and some intermediary rate. This method is called "empirical" method by some authors. The results are similar as pointed out in Santos (2006). However, we prefer to use the spline method since the relation between the factors in this method is not deterministic as in the "empirical" method.

Table 3.5 presents a statistical summary of the variables and the evolution of the term structure factors. The level factor is close to the Selic rate. This result is in line with several different researches available using Brazilian data or data observed in other countries. The slope factor is always positive. This can be interpreted as an evidence of greater term premium in higher rates. The slope is lower in periods when the Selic rate is falling reflecting market anticipation of future monetary policy actions. The curvature factor is very small but negative during the full sample.

We estimated separated equations for each factor in the yield curve and use as explanatory variables the difference between twelve months ahead inflation expectation and the targeted inflation ($\hat{\pi}_t = \pi^e_{t,t+12} - \pi^{BC}_{t,t+12}$, sometimes called expected inflation gap), a measure of output gap formed using the monthly economic activity indicator constructed by the Brazilian Central Bank (known as *IBC-Br*). This indicator is the best proxy available for monthly GDP and we formed a measure of monthly output gap using the HP filter.

We also included a measure of global inflation formed using the three months cumulated inflation from the CRB commodity index measured in Brazilian Real. This variable captures both the pressures arising as a consequence of exchange rate variation (depreciation or appreciation) and the impact of commodity price shocks. This is a better proxy for inflation pressures on tradable prices than the variation of exchange rate alone especially for the period post 2003. For each of the factors, $F = \{L, S, C\}$, we estimate the following single equation:

$$F_t = \alpha_0 + \alpha_1 y_t + \alpha_2 \hat{\pi}_t + \alpha_3 \pi^* + \alpha_4 f_t + \varepsilon_t \tag{3.22}$$

We also included a measure of interest rate surprise from the Copom decisions. This variable is the difference between market expectation of the next Copom decision and the actual decision: let i_t be target Selic rate set by Copom at date *t* and effective from *t+1* up to the next Copom meeting and

i_t^e the expected Selic rate. Then $i_t - i_t^e$ is a measure of monetary policy surprise, which could have a significant impact on market rates at that month[32].

Table 3.5: Statistical summary of interest rate data

	1M	2M	3M	6M	9M	12M	24M	L	S	C
Mean	13,23%	13,21%	13,24%	13,34%	13,45%	13,53%	13,79%	12,01%	0,61%	-0,05%
Median	12,29%	12,19%	12,08%	12,09%	12,36%	12,42%	12,79%	11,26%	0,60%	-0,04%
St. Dev.	3,21%	3,19%	3,16%	3,04%	2,92%	2,80%	2,48%	3,12%	0,22%	0,01%
Autocorr.	0,993	0,994	0,993	0,989	0,984	0,978	0,957	0,991	0,798	0,804
Asym.	0,539	0,566	0,566	0,545	0,525	0,529	0,606	0,522	0,177	0,260
Kurt.	-0,732	-0,709	-0,688	-0,727	-0,789	-0,854	-0,804	-0,640	0,918	1,971

Fig. 3.12a: Level (L) factor

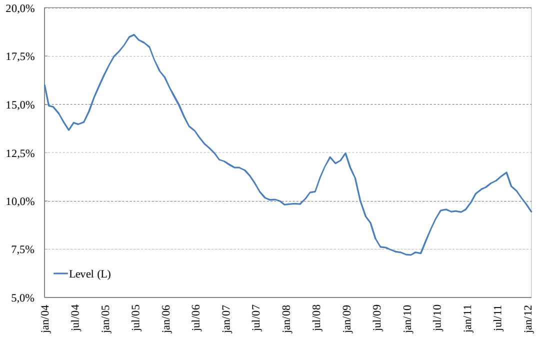
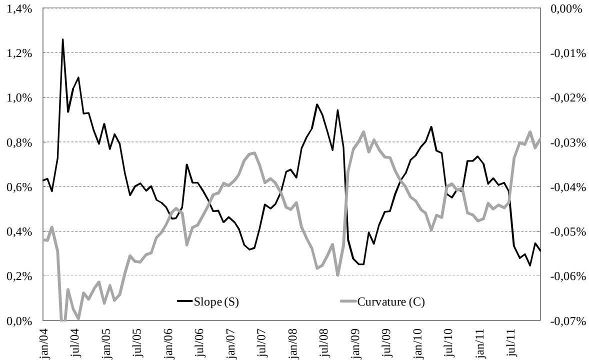


Fig. 3.12b: Slope and curvature factor



Notes: data from Bloomberg. The factors were calculated by the author following equation 3.21.

The results are presented in the table 3.6. The estimated coefficients on the macroeconomics variables are almost all in line with theory: positive output and expected inflation gaps are associated with larger Level and Slope factor. However, the estimated coefficient of inflation gap on the Slope factor was not significant at reasonable levels. The estimated impact of commodity price inflation was negative in the level factor. This is at odds with the conventional theory. A possible explanation for this result is that market inflation expectations already capture the effect of commodity price shocks. Another explanation is that the variable is not well constructed. The impact of interest rate surprises is significant at the 1% level in all factors and is close to one in the Level factor (the estimated coefficient was 0,75 with a standard deviation of 0,25 meaning one cannot reject the hypothesis of the coefficient being equal to one at reasonable confidence levels).

The estimated impact of foreign position on interest rate futures is not significant in most regressions and less so in the Slope and Curvature factors. The estimated impact of foreign flows is small but significant in the Level factor. The estimated coefficient is not significant at the Slope and Curvature equations and we will restrict our comments to the Level equation.

[32] Up to 2005 there were monthly Copom meetings. Nowadays, the meetings occur at intervals of circa 45 days. The best measure of market expectation of Copom decision is the implicit rate from interest rate futures contract. This measure is much more complicated to construct. We proceed assuming market rates are well approximated by economists expected rates available at the BCB surveys.

Table 3.6: Foreign flows impact on local term structure factors ($F_t = \alpha_0 + \alpha_1 y_t + \alpha_2 \hat{\pi}_t + \alpha_3 \pi^* + \alpha_4 f_t + \varepsilon_t$)

	L	L	L	S	S	C	C
F_{-1}	0,973 ***	0,962 ***	0,933 ***	0,626 ***	0,619 ***	0,6180 ***	0,5994 ***
	(0,011)	(0,011)	(0,019)	(0,074)	(0,071)	(0,068)	(0,0582)
cte	0,000	0,001	0,006	0,002 ***	0,002 ***	-0,00015 ***	-0,00016 ***
	(0,001)	(0,001)	(0,003)	(0,000)	(0,000)	(0,000)	(0,0000)
y	0,113 ***	0,100 ***	0,092 ***	0,020 ***	0,018 **	-0,0007 *	-0,00047
	(0,019)	(0,019)	(0,019)	(0,008)	(0,007)	(0,000)	(0,0003)
π	0,284 ***	0,293 ***	0,345 ***	0,020	0,018	-0,0022 **	-0,00217 **
	(0,051)	(0,048)	(0,055)	(0,021)	(0,020)	(0,001)	(0,0010)
π^*	-0,010 **	-0,0119 **	-0,0127 **	0,006 ***	0,005 ***	-0,00027 **	-0,0002 **
	(0,004)	(0,005)	(0,005)	(0,002)	(0,002)	(0,000)	(0,0000)
f	0,009 *	0,011 **	0,011 **	0,001	0,001	-0,00011	-0,00014 *
	(0,005)	(0,005)	(0,005)	(0,002)	(0,002)	(0,000)	(0,0000)
$i-i^*$		0,751 ***	0,648 ***		0,272 ***		-0,0242 ***
		(0,253)	(0,255)		(0,088)		(0,0000)
trend			-0,000035 *				
			(0,00002)				
R^2 adj	0,991	0,992	0,992	0,708	0,733	0,704	0,780

Notes: F={L, S, C}. See full description in the main text. This result is based on the flow% measure of foreign investors in local interest rate instruments. The results using flow# are similar and will not be reported.

A possible explanation for the positive coefficient of foreign flows on the Level factor is that these flows may be endogenous to the interest rate in Brazil, being greater in moments of higher rates or increasing rates. The first objective of foreign investors is to earn the interest rate difference between low rates in their own country (example United States o Europe) and high interest rates in Brazil. In these cases, their position would be higher in moments when the difference between Brazilian rates and rates in other countries are greater. Also, increasing rates can have a positive effect in foreign flows if investors expect that high and increasing rates have a positive impact on exchange rates. As we already demonstrated in previous sections, the main risk to investors looking to earn the positive interest rate difference between low and high yield countries is the exchange rate variation. So if one believes higher interest rates will have a positive affect on exchange rate, moments of increasing rates can be seen a less risky environment for investing in high yield countries.

We addressed these questions first estimating a simultaneous equation model allowing the foreign flows to be endogenous variable depending on the interest rate difference and also on a measure of risk aversion denoted by *RL*.

$$i_t = \alpha_0 + \alpha_1 y_t + \alpha_2 \hat{\pi}_t + \alpha_3 \pi^* + \alpha_4 f_t + \varepsilon_t \qquad (3.23)$$
$$f_t = \phi_0 + \phi_1 (i_t - i_t^*) + \phi_3 RL_t + e_t$$

Using this approach the results were not satisfactory particularly due to the difficulty in reaching significant coefficients on the flow equation. Overall, the results indicated that neither the local or external interest rate was significant. Also, the risk aversion variables were not significant in some

variations of (3.23) allowing for other variables or lagged dependent variables on the right hand side of (3.23). The results of these regressions are not presented in this paper.

As an alternative to the simultaneous equation model, we used a VAR model with the flow equation and the local rate as dependent variables and several other variables as exogenous variables. The exogenous variables include both variables theoretically important for the local interest rate like the inflation and output gap, interest rate surprises, commodity price inflation among others and also include variables that should be important for the flow equation like the risk aversion index and foreign interest rate (note that for foreign investors, what we refer to as foreign interest rate can be the local interest rate or the benchmark rate - Libor). The VAR model can be represented as:

$$Y_t = A + BY_{t-1} + CX_{t-1} + \zeta_t \tag{3.24}$$

Where $Y=[i,f]$ and X includes the variables listed in (3.23). The estimated coefficients of this VAR model are presented in the appendix.

Using the VAR approach, the results for the investor flow variable are more interesting. As it happened in the simultaneous equation model, the flow variable is more difficult to model than the interest rate variable. The R^2 is smaller and several variables are not significant in the flow equation. The coefficients are presented in the Appendix and we will focus on some coefficients and impulse response in this section. We estimated the VAR using both 2 year rates as the local interest rate and also using the slope defined earlier and also using a simple measure of slope: 2Y rates less 1M rates. The *flow* variable was represented by both the number of contracts (*flow#)* and the percent of open contracts on interest rate futures (*flow%)* but always one at once in the system. We summarize the results in the next lines.

Our results indicate that the impact of interest rate shocks on foreign investors' flows into CDI futures contracts is small and not significant. A possible explanation is the misspecification of the model. In the model we included variables that proxy for liquidity conditions in the international market, risk premiums, among others. However, the variables are all approximations for the theoretical variables, specially the variables that proxy for liquidity conditions, risk appetite and returns in alternative assets. A possible refinement of this model is to include a better measure of yields in alternative investments (here we use only medium term, 2Y, rates in US).

We also estimated the model without including our measure of Copom surprises (difference between expected and actual Copom decision regarding the Selic rate) but, the results did not change significantly: despite this variable being one of the most significant in the model, the explanatory power of the model is barely affected when we exclude this variable making the residuals of the equations with and without it very similar to each other.

An interesting question we should answer at this point is how much money is needed to change local interest rates by some specific amount, for example 1 basis point? Considering the results from fig. 10d, the estimated amount is indeed very large. The standard deviation in the *flow#* equation represents 342 thousand contracts (close to 20% of the average position). The estimated impact of 1σ of *flow#* on 2Y interest rate is close to only 0,25% (or 25 bp). Despite being statistically significant, the estimated impact of foreign flows on local interest rates is minor. It is also important to mention

that the notional value of this 1σ is very large: 342 thousand contracts represent more than R$ 30 billion[33].

Fig. 3.13a: Response of FLOW% to BR2Y

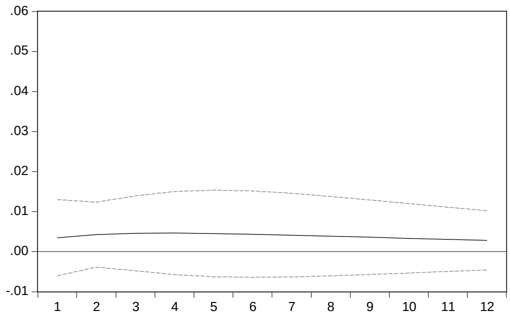


Fig. 3.13b: Response of BR2Y to Flow%

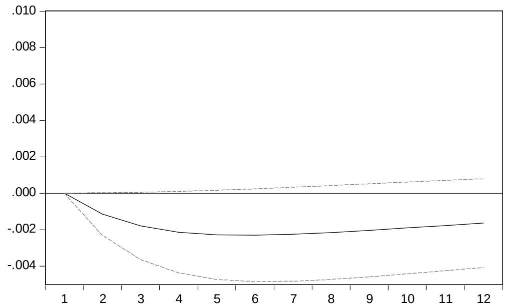


Fig. 3.13c: Response of FLOW# to BR2Y

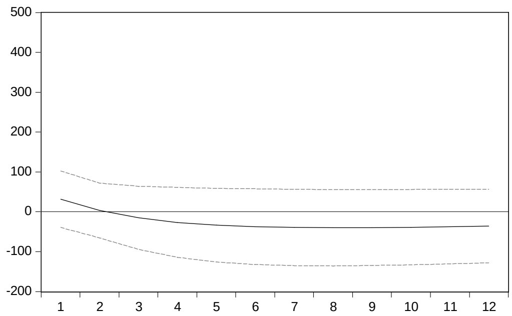
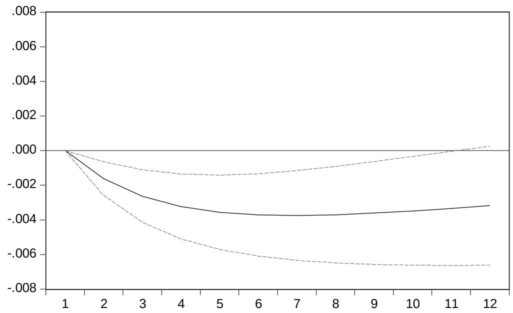


Fig. 3.13d: Response of BR2Y to Flow#



3.7 Final Remarks from Essay 3

Our results indicate that BRL forwards are priced according to the interest rate differential, as predicted by standard financial models (non-arbitrage conditions). Expected future exchange rates play no role in determining the FX forward price. In this financial model, the local rate in US dollar, the so-called *cupom cambial*, is more important than the Libor rate.

Foreign flows to fixed income securities had a small impact on interest rates in Brazil. Data available for foreign position into interest rate futures indicated that this position was "net long" since 2004. However, increases or decreases in this position have had only a limited impact on the yield curve or long rates (2 years). We interpret this result as an indication of limited funds available to extinguish the arbitrage opportunity. As we have stressed before, this is not a risk free arbitrage opportunity. Considering this, even individual investors would have to limit the size of their position.

Our results also indicate that the Sharpe ratio for investments on the BRL is greater or at last equal to Sharpe ratios for carry trade funds and also for portfolios formed based on the interest rate differential calculated in papers like Burnside et all (2011). The volatility of the BRL is larger than average and represent a risk for foreign investors investing in Brazil. However, over the last seven years, the risk-return trade-off on the BRL was attractive relative to similar strategies based on carry trades.

[33] Since we do not have the open position by tenure, we cannot infer the duration of this position. The notional value is not the value of the flows to the country since investors need only some part of this value to be put on guarantee (margin account) to trade interest rate futures.

A large part of BRL movements can be explained by above average sensitivity to global FX movements measured by a common component extracted from a large sample of currencies. This large hoarding on the global FX factor may be a result of high interest rate in Brazil and net commodity exports. The high liquidity of BRL, particularly on the futures market may have a positive impact on these results. These characteristics of the BRL are more pronounced in the second part of our sample.

Now returning to the title of the present paper, what seems to be the best explanation for the lack of convergence of local rates to international (say USD) rates: country risk, currency risk or limits to arbitrage? There is not a clear answer even considering the extensive research on the subject presented in this paper. But, after acknowledging the limitations of our study and the complexity of the subject, we are prone to argue in favor of currency risk and also limits to arbitrage. There are no risk-free investment opportunities for foreign investors to earn the high interest rate differential between BRL interest rates and interest rates in other countries and the main risk for foreign investors over the last years was the high BRL volatility (currency risk). Also, a large amount of money is necessary to change local interest rates in the futures market. Finally, even local institutions have limitations on foreign investments (institutional setting imposed by BCB or other local government institution) and earn the higher USD rate available in the local market (*cupom cambial*). Institutional limits for foreign investors are also set by the local government concerning foreign investors' flows to Brazil. The country risk diminished by several times in the last years and is not relevant for this discussion.

3.8 Appendix from Essay 3: Unit root tests and other estimation results

3.8.1 Unit Root Tests

We present here the unit root tests for several variables used in this paper. The order follows the order in with each variable appears in the text. We performed Augmented Dickey-Fuller (ADF) test in all cases and the KPSS test in cases where the ADF test seems inconclusive.

Table A.1 presents the unit root test for variables used in section 1.3. There is evidence of unit root in both interest rates differences (CC Factor and Libor Factor). In the case of the forward premium, we can reject the hypotheses of unit root at the 5% level. Using the KPSS test on the FP variable, we can reject the hypothesis of stationarity at the 5% level. The results are inconclusive for this variable.

Table A.1: Unit Root Test

	FP	CC Factor	Libor Factor
Lag	1	0	2
Constant, Trend	C	C	C
ADF Statistic	-2,903	-2,186	-1,637
p-value	0,0486	0,2128	0,4599

Table A.2 presents the unit root tests for variables used in section 1.6. The results indicate that the estimated level (L) of the yield curve and the inflation expectation variable are not stationary. In both cases, the KPSS would indicate the opposite. The level variable is expected to show a significant persistence due to “interest rate smoothing” by the Central Bank. The inflation expectation variable is the “next twelve months” inflation expectation calculated on a monthly basis. This seems to explain why the variable is highly persistent. We will consider both variables stationary.

Table A.2: Unit Root Test

	L	S	C	Flow#	Flow%	y	0^e
Lag	2	0	0	0	0	0	1
Constant, Trend	C, T	C	C	C, T	C	C	C
ADF Statistic	-3,070	-4,049	-4,747	-3,795	-4,301	-2,906	-1,858
p-value	0,1195	0,0018	0,0002	0,021	0,0008	0,0484	0,3506

3.8.2 VEC coefficients (from section 1.6)

Vector Error Correction Estimates
Sample (adjusted): 2003M11 2012M01
Included observations: 99 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
FP(-1)	1.000.000	
CCF(-1)	-0.969221	
	(0.07309)	
	[-13.2613]	
C	-0.000355	
Error Correction:	D(FP)	D(CCF)
CointEq1	-0.850205	-0.057353
	(0.15996)	(0.08204)
	[-5.31496]	[-0.69910]
D(FP(-1))	-0.209040	0.014895
	(0.11157)	(0.05722)
	[-1.87360]	[0.26031]
D(CCF(-1))	0.031751	-0.224355
	(0.23008)	(0.11800)
	[0.13800]	[-1.90134]
C	-7.76E-05	-7.99E-05
	(0.00019)	(9.5E-05)
	[-0.41722]	[-0.83827]
R-squared	0.495702	0.047928
Adj. R-squared	0.479777	0.017862
Sum sq. resids	0.000323	8.50E-05
S.E. equation	0.001845	0.000946
F-statistic	3.112.690	1.594.112
Log likelihood	4.848.079	5.509.162
Akaike AIC	-9.713.292	-1.104.881
Schwarz SC	-9.608.439	-1.094.396
Mean dependent	-6.43E-05	-6.58E-05
S.D. dependent	0.002558	0.000955

Vector Error Correction Estimates
Sample (adjusted): 2003M11 2012M01
Included observations: 99 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
FP(-1)	1.000.000	
LIBORF(-1)	-1.105.299	
	(0.09875)	
	[-11.1924]	
C	0.001417	
Error Correction:	D(FP)	D(LIBORF)
CointEq1	-0.803278	-0.038112
	(0.13312)	(0.02703)
	[-6.03416]	[-1.41019]
D(FP(-1))	-0.171879	0.006203
	(0.09958)	(0.02022)
	[-1.72606]	[0.30682]
D(LIBORF(-1))	-0.659612	0.338617
	(0.50710)	(0.10295)
	[-1.30076]	[3.28911]
C	-0.000114	-3.14E-05
	(0.00019)	(3.8E-05)
	[-0.61497]	[-0.83512]
R-squared	0.507630	0.169505
Adj. R-squared	0.492081	0.143279
Sum sq. resids	0.000316	1.30E-05
S.E. equation	0.001823	0.000370
F-statistic	3.264.805	6.463.204
Log likelihood	4.859.928	6.438.434
Akaike AIC	-9.737.228	-1.292.613
Schwarz SC	-9.632.374	-1.282.128
Mean dependent	-6.43E-05	-5.07E-05
S.D. dependent	0.002558	0.000400

3.8.3 VAR coefficients (from section 1.6)

Vector Autoregression Estimates
Sample (adjusted): 2004M02 2012M01
Included observations: 96 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	i	*f (%)*
i (-1)	0,939058	0,296219
	-0,05731	-0,3718
	[16.3848]	[0.79672]
f (-1)	-0,021566	0,61125
	-0,01238	-0,08034
	[-1.74142]	[7.60841]
C	0,009551	0,034946
	-0,00727	-0,04715
	[1.31409]	[0.74117]
y	0,114737	-0,066373
	-0,06476	-0,42014
	[1.77159]	[-0.15798]
$\P^E$	0,01688	0,585511
	-0,24538	-1,5918
	[0.06879]	[0.36783]
$i - i^E$	0,591798	-6,280164
	-0,66358	-4,30472
	[0.89183]	[-1.45890]
VIX	0,000259	-0,001146
	-0,0002	-0,0013
	[1.29147]	[-0.88054]
VIX(-1)	-0,000164	-0,000457
	-0,00015	-0,001
	[-1.06214]	[-0.45515]
EMBI	0,276906	1,575952
	-0,28052	-1,81976
	[0.98713]	[0.86602]
EMBI(-1)	-0,23549	-2,214878
	-0,29192	-1,89376
	[-0.80668]	[-1.16957]
*i**	0,823316	-5,803027
	-0,42183	-2,73649
	[1.95176]	[-2.12061]
i (-1)*	-0,788539	6,174456
	-0,41319	-2,68045
	[-1.90840]	[2.30351]
R-squared	0,935613	0,557494
Adj. R-squared	0,927181	0,499547
Sum sq. resids	0,004138	0,174133
S.E. equation	0,007019	0,04553
F-statistic	110,9639	9,620733
Log likelihood	346,2747	166,7717
Akaike AIC	-6,964056	-3,22441
Schwarz SC	-6,643513	-2,903866
Mean dependent	0,136395	0,175834
S.D. dependent	0,026009	0,06436
Determinant resid covariance (dof adj		1,00E-07
Determinant resid covariance		7,68E-08
Log likelihood		513,8787
Akaike information criterion		-10,20581
Schwarz criterion		-9,564719

Vector Autoregression Estimates
Sample (adjusted): 2004M02 2012M01
Included observations: 96 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	i	*f (#)*
i (-1)	0,869823	-3188,245
	-0,05827	-2967,07
	[14.9270]	[-1.07454]
f (-1)	-4,40E-06	0,746605
	-1,30E-06	-0,06806
	[-3.29114]	[10.9702]
C	0,022041	969,8213
	-0,00821	-417,865
	[2.68577]	[2.32090]
y	0,116017	1,624413
	-0,06106	-3109,25
	[1.89992]	[0.00052]
$\P^E$	0,134593	7190,02
	-0,23885	-12161,7
	[0.56350]	[0.59120]
$i - i^E$	0,488038	-206,9911
	-0,63545	-32355,4
	[0.76802]	[-0.00640]
VIX	0,000287	-7,665398
	-0,00019	-9,75504
	[1.49606]	[-0.78579]
VIX(-1)	-8,27E-05	-4,788122
	-0,00015	-7,69766
	[-0.54688]	[-0.62202]
EMBI	0,226586	11346,68
	-0,26925	-13709,4
	[0.84156]	[0.82766]
EMBI(-1)	-0,246842	-19392,39
	-0,27926	-14219,1
	[-0.88393]	[-1.36383]
*i**	0,752927	-42567,19
	-0,4048	-20611,6
	[1.85999]	[-2.06520]
i (-1)*	-0,749159	40537,25
	-0,39478	-20101,4
	[-1.89765]	[2.01664]
R-squared	0,940908	0,725472
Adj. R-squared	0,93317	0,689522
Sum sq. resids	0,003798	9845514
S.E. equation	0,006724	342,3573
F-statistic	121,5917	20,17998
Log likelihood	350,3941	-690,0507
Akaike AIC	-7,049877	14,62606
Schwarz SC	-6,729333	14,9466
Mean dependent	0,136395	1050,052
S.D. dependent	0,026009	614,4183
Determinant resid covariance (dof adj.		5,194032
Determinant resid covariance		3,976681
Log likelihood		-338,6977
Akaike information criterion		7,556202
Schwarz criterion		8,197289

3.8.4 *FX Beta model: additional results*

Full Sample Estimation Results

	α	s.e	t-stat.	β	s.e	t-stat.	σ	R^2
AUD	-0.0009	0.0018	-0.49	1.558	0.068	22.79	1.76%	84.4%
BRL	-0.0026	0.0032	-0.82	1.236	0.121	10.25	3.11%	52.0%
CAD	-0.0011	0.0021	-0.53	0.889	0.078	11.33	2.02%	57.0%
CHF	-0.0013	0.0023	-0.55	1.066	0.089	12.02	2.28%	59.9%
CLP	0.0001	0.0029	0.05	0.896	0.111	8.06	2.86%	40.0%
CZK	-0.0007	0.0023	-0.29	1.403	0.089	15.71	2.30%	71.9%
DKK	0.0013	0.0015	0.84	1.117	0.059	18.89	1.52%	78.8%
EUR	0.0013	0.0016	0.86	1.122	0.059	18.88	1.53%	78.7%
GBP	0.0028	0.0020	1.39	0.877	0.077	11.36	1.99%	57.2%
HUF	0.0041	0.0027	1.52	1.716	0.103	16.73	2.64%	74.4%
IDR	0.0020	0.0025	0.82	0.843	0.095	8.87	2.45%	44.8%
ILS	-0.0004	0.0021	-0.18	0.709	0.079	8.94	2.04%	45.1%
JPY	-0.0031	0.0028	-1.09	0.056	0.107	0.52	2.76%	-0.8%
KRW	0.0014	0.0022	0.64	1.209	0.086	14.13	2.20%	67.4%
MXN	0.0038	0.0021	1.81	0.994	0.081	12.32	2.08%	61.1%
MYR	-0.0011	0.0011	-0.98	0.518	0.043	12.06	1.11%	60.1%
NOK	0.0007	0.0018	0.38	1.253	0.070	17.84	1.81%	76.8%
NZD	0.0005	0.0027	0.17	1.495	0.104	14.43	2.67%	68.3%
PHP	-0.0018	0.0015	-1.18	0.394	0.058	6.79	1.50%	31.9%
PLN	0.0018	0.0024	0.76	1.694	0.091	18.58	1.50%	31.9%
SEK	0.0016	0.0018	0.86	1.400	0.070	19.98	1.80%	80.6%
SGD	-0.0019	0.0009	-2.04	0.643	0.035	18.40	0.90%	77.9%
THB	-0.0017	0.0013	-1.24	0.432	0.052	8.38	1.33%	41.9%
TRY	0.0049	0.0033	1.48	1.146	0.127	9.00	3.28%	45.5%
TWD	-0.0004	0.0012	-0.39	0.486	0.045	10.91	1.15%	55.2%
ZAR	0.0044	0.0039	1.13	1.550	0.148	10.47	3.79%	53.4%
Mean	0.0005	0.0022	0.1134	1.027	0.082	12.987	2.09%	57.51%

Full Sample Estimation Results with Additional Explanatory Variable

	α	s.e	t-stat.	β	s.e	t-stat.	CRB	s.e	t-stat.	VIX	s.e	t-stat.	σ	R^2
AUD	-0.0011	0.0018	-0.63	1.451	0.107	13.51	0.011	0.044	0.25	0.0008	0.0004	1.85	1.75%	84.5%
BRL	-0.0031	0.0031	-0.99	0.944	0.188	5.01	-0.047	0.076	-0.61	0.0016	0.0007	2.23	3.07%	53.0%
CAD	-0.0012	0.0021	-0.56	0.904	0.126	7.18	0.005	0.051	0.10	-0.0001	0.0005	-0.18	2.05%	55.7%
CHF	-0.0012	0.0023	-0.55	1.378	0.135	10.21	0.153	0.055	2.79	-0.0012	0.0005	-2.25	2.20%	62.7%
CLP	-0.0002	0.0029	-0.08	0.724	0.176	4.11	-0.017	0.071	-0.24	0.0010	0.0007	1.47	2.87%	39.6%
CZK	-0.0004	0.0023	-0.19	1.670	0.138	12.12	0.058	0.056	1.03	-0.0015	0.0005	-2.72	2.25%	73.2%
DKK	0.0013	0.0015	0.85	1.271	0.092	13.83	0.088	0.037	2.36	-0.0005	0.0004	-1.38	1.50%	79.4%
EUR	0.0013	0.0015	0.86	1.270	0.093	13.72	0.085	0.038	2.26	-0.0005	0.0004	-1.34	1.51%	79.3%
GBP	0.0033	0.0019	1.76	1.014	0.111	9.13	-0.095	0.045	-2.10	-0.0015	0.0004	-3.52	1.81%	64.5%
HUF	0.0042	0.0027	1.53	1.874	0.163	11.48	0.059	0.066	0.89	-0.0007	0.0006	-1.11	2.66%	74.0%
IDR	0.0014	0.0024	0.61	0.510	0.142	3.58	-0.033	0.058	-0.57	0.0020	0.0006	3.62	2.32%	50.3%
ILS	-0.0005	0.0021	-0.26	0.689	0.127	5.42	0.013	0.052	0.26	0.0002	0.0005	0.39	2.07%	43.6%
JPY	-0.0029	0.0026	-1.10	0.508	0.158	3.21	0.125	0.064	1.95	-0.0023	0.0006	-3.80	2.58%	11.6%
KRW	0.0010	0.0022	0.45	1.059	0.132	8.03	0.040	0.053	0.74	0.0012	0.0005	2.42	2.15%	69.1%
MXN	0.0035	0.0020	1.72	0.672	0.122	5.50	-0.104	0.049	-2.10	0.0015	0.0005	3.19	1.99%	64.4%
MYR	-0.0013	0.0011	-1.15	0.498	0.067	7.40	0.035	0.027	1.28	0.0003	0.0003	1.30	1.10%	60.7%
NOK	0.0008	0.0019	0.43	1.286	0.112	11.53	-0.032	0.045	-0.71	-0.0004	0.0004	-0.99	1.82%	76.5%
NZD	0.0002	0.0027	0.07	1.504	0.164	9.17	0.080	0.067	1.21	0.0004	0.0006	0.64	2.67%	68.2%
PHP	-0.0019	0.0015	-1.25	0.443	0.092	4.80	0.046	0.037	1.22	-0.0001	0.0004	-0.20	1.50%	31.0%
PLN	0.0020	0.0024	0.81	1.807	0.145	12.47	0.006	0.059	0.10	-0.0007	0.0006	-1.30	2.36%	77.9%
SEK	0.0015	0.0019	0.79	1.450	0.111	13.03	0.062	0.045	1.36	0.0000	0.0004	0.09	1.81%	80.4%
SGD	-0.0020	0.0009	-2.17	0.710	0.054	13.15	0.058	0.022	2.63	-0.0001	0.0002	-0.51	0.88%	78.7%
THB	-0.0016	0.0013	-1.18	0.592	0.079	7.50	0.034	0.032	1.06	-0.0009	0.0003	-2.89	1.29%	45.1%
TRY	0.0045	0.0034	1.34	1.007	0.201	5.01	0.039	0.082	0.47	0.0011	0.0008	1.47	3.28%	45.6%
TWD	-0.0005	0.0012	-0.40	0.506	0.071	7.08	0.003	0.029	0.10	-0.0001	0.0003	-0.45	1.16%	53.8%
ZAR	0.0040	0.0039	1.04	1.362	0.232	5.88	0.016	0.094	0.17	0.0014	0.0009	1.50	3.78%	53.7%
Mean	0.0004	0.0021	0.0669	1.042	0.128	8.580	0.026	0.052	0.612	0.0000	0.0005	-0.0953	2.09%	60.6%

2004 to 2007 Estimation Results

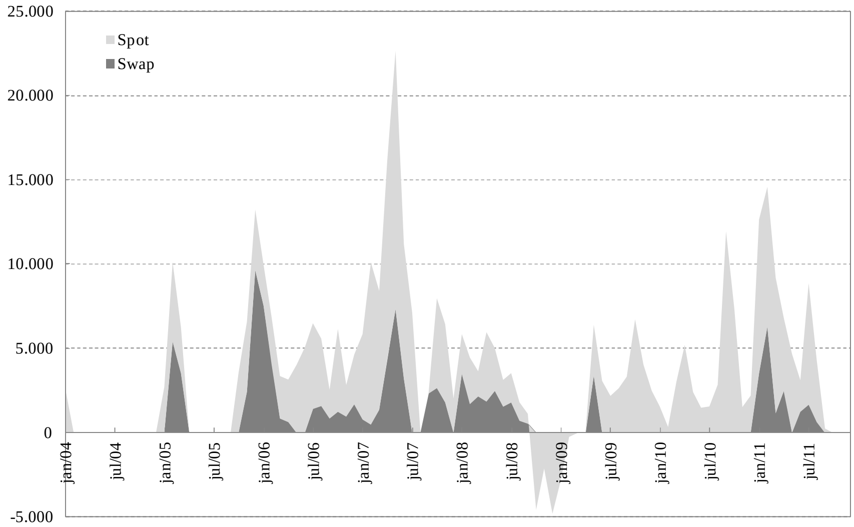
	α	s.e	t-stat.	β	s.e	t-stat.	σ	R^2
AUD	0.0025	0.0026	0.98	1.465	0.170	8.62	1.71%	61.4%
BRL	-0.0076	0.0048	-1.59	0.642	0.314	2.05	3.15%	6.5%
CAD	-0.0020	0.0033	-0.60	0.871	0.220	3.95	2.22%	24.1%
CHF	0.0034	0.0021	1.63	1.382	0.137	10.12	1.37%	68.8%
CLP	-0.0008	0.0032	-0.26	0.573	0.211	2.72	2.12%	12.2%
CZK	-0.0018	0.0027	-0.66	1.509	0.180	8.37	1.81%	60.0%
DKK	0.0017	0.0016	1.07	1.269	0.107	11.85	1.08%	75.2%
EUR	0.0017	0.0016	1.02	1.262	0.108	11.71	1.08%	74.7%
GBP	0.0017	0.0022	0.79	1.205	0.142	8.49	1.43%	60.7%
HUF	0.0027	0.0032	0.84	1.699	0.211	8.06	2.12%	58.2%
IDR	0.0041	0.0034	1.21	0.510	0.223	2.29	2.24%	8.5%
ILS	-0.0006	0.0023	-0.24	0.584	0.155	3.77	1.56%	22.3%
JPY	0.0042	0.0031	1.35	0.908	0.206	4.41	2.07%	28.6%
KRW	-0.0021	0.0019	-1.11	0.860	0.124	6.94	1.25%	50.7%
MXN	0.0006	0.0024	0.25	0.243	0.157	1.54	1.58%	2.9%
MYR	-0.0017	0.0012	-1.44	0.252	0.077	3.28	0.77%	17.5%
NOK	0.0015	0.0031	0.47	1.391	0.203	6.86	2.04%	50.0%
NZD	0.0031	0.0040	0.78	1.592	0.261	6.09	2.63%	44.0%
PHP	-0.0051	0.0020	-2.50	0.180	0.135	1.34	1.35%	1.7%
PLN	-0.0016	0.0030	-0.53	1.770	0.197	8.97	1.35%	63.3%
SEK	0.0039	0.0024	1.61	1.625	0.160	10.18	1.98%	69.1%
SGD	-0.0009	0.0011	-0.78	0.649	0.072	9.06	0.72%	63.8%
THB	-0.0003	0.0016	-0.18	0.776	0.106	7.35	1.06%	53.5%
TRY	0.0008	0.0052	0.15	1.086	0.343	3.17	3.45%	16.4%
TWD	0.0011	0.0016	0.67	0.556	0.108	5.13	1.09%	35.5%
ZAR	0.0090	0.0055	1.65	2.092	0.361	5.80	3.63%	41.5%
Mean	0.0007	0.0027	0.1759	1.037	0.180	6.236	1.80%	41.2%

2008 to 2011 Estimation Results

	α	s.e	t-stat.	β	s.e	t-stat.	σ	R^2
AUD	-0.0046	0.0025	-1.81	1.588	0.076	20.98	1.77%	90.1%
BRL	-0.0004	0.0043	-0.10	1.328	0.128	10.40	2.99%	69.1%
CAD	-0.0005	0.0027	-0.17	0.885	0.080	11.04	1.88%	71.6%
CHF	-0.0047	0.0041	-1.15	1.017	0.122	8.31	2.87%	58.6%
CLP	-0.0005	0.0049	-0.09	0.950	0.148	6.43	3.46%	45.7%
CZK	0.0007	0.0039	0.17	1.374	0.117	11.79	2.73%	74.2%
DKK	0.0014	0.0027	0.54	1.088	0.080	13.59	1.88%	79.3%
EUR	0.0015	0.0027	0.56	1.094	0.080	13.59	1.89%	79.3%
GBP	0.0051	0.0034	1.49	0.805	0.101	7.94	2.37%	56.4%
HUF	0.0053	0.0045	1.19	1.712	0.133	12.83	3.13%	77.3%
IDR	-0.0014	0.0037	-0.39	0.914	0.111	8.24	2.60%	58.2%
ILS	-0.0009	0.0035	-0.24	0.729	0.105	6.94	2.46%	49.5%
JPY	-0.0073	0.0043	-1.71	-0.098	0.127	-0.77	2.98%	-0.9%
KRW	0.0033	0.0040	0.83	1.262	0.120	10.49	2.82%	69.4%
MXN	0.0039	0.0032	1.23	1.130	0.095	11.95	2.22%	74.7%
MYR	-0.0017	0.0019	-0.91	0.567	0.056	10.15	1.31%	68.0%
NOK	0.0004	0.0023	0.19	1.227	0.069	17.90	1.61%	86.9%
NZD	-0.0018	0.0040	-0.46	1.483	0.118	12.56	2.77%	76.6%
PHP	0.0004	0.0023	0.16	0.419	0.068	6.16	1.59%	43.5%
PLN	0.0053	0.0038	1.39	1.662	0.114	14.63	1.59%	43.5%
SEK	0.0002	0.0028	0.07	1.363	0.085	16.07	1.99%	84.3%
SGD	-0.0029	0.0015	-1.92	0.644	0.045	14.29	1.06%	80.9%
THB	-0.0018	0.0021	-0.85	0.365	0.063	5.82	1.47%	40.7%
TRY	0.0085	0.0045	1.88	1.136	0.135	8.42	3.16%	59.3%
TWD	-0.0018	0.0017	-1.01	0.476	0.052	9.20	1.21%	63.5%
ZAR	0.0021	0.0056	0.37	1.460	0.167	8.75	3.91%	61.2%
Mean	0.0003	0.0033	-0.0288	1.022	0.100	10.681	2.30%	63.9%

3.8.5 Intervention on the BRL

BRL Intervention: US$ millions



Administrative measures

- External borrowing by local companies: tax increased from 5% to 5,38% in dec-07 if debt maturity is lower than 3 months. Increased to 6% in 2011 and maturity was also increased several times until reach up to 5 years in early 2012.

- Fixed income (for foreign investors): 1,5% tax in mar-08. This tax was zeroed in oct-10 and then charged again in sep-09. The tax rate was increased in 2010 first to 4% and then to 6%.

-Equity investments (foreign investors): 2% tax charged starting in sep-09. This tax was zeroed in dec-11.

-Bank's net position on USD: 1% tax charged starting in jul-11.

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