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A monetary mechanism for sharing capital: Diamond and Dybvig meet Kiyotaki and Wright*

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Abstract

A model is presented in which banks accept deposits of fiat money and intermediate capital. Although theories about the coexistence of money and credit are inherently difficult, the model offers a simple explanation for the dual role of financial institutions: Banks are well monitored, and can credibly allow fiat-money withdraws to whom needs it, thus qualifying to become safe brokers of idle capital. The model shares some features with those of Diamond and Dybvig (1983) and Kiyotaki and Wright (1989).

1 Introduction

In this paper, we show in a simple model that the provision of inside money should be coordinated with the intermediation of capital, in contrast to well known proposals for regulating the financial system as a separation of money and credit.[1] The model builds on the sharing of storable goods, emphasized by Diamond and Dybvig (1983), and the creation of inside money that appears in recent extensions of the model of Kiyotaki and Wright (1989).[2]

Our model does not allow for general money holdings or for interest payments on deposits. Despite this lack of realism, we believe its main finding is robust to more general specifications simply because it is quite intuitive and easy to state. Banks are well monitored, and can credibly promise future access to fiat-money withdraws to clients who turn out to need it. The ability to pay interests would reinforce the fact that banks can become both conservative issuers of inside money and trustworthy receivers of idle capital. Therefore, the dual role of issuing money and intermediating capital is well suited to banks.

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[1] Commercial banks have historically played active roles in both payment systems and capital intermediation. Not surprisingly, episodes of bank failure have led Friedman (1959) and others to express concerns that movements in the demand for money would interact with credit activities in undesirable ways, generating financial fragility.

[2] See Cavalcanti, Erosa, and Temzelides (1999).

The paper is organized as follows. The environment with monitored banks and a non-bank public is described in section 2. In section 3, we present a benchmark version of the model where the non-bank public uses outside money and there is no intermediation. This set of allocations is helpful for showing that in the limit, when there is no trade risk, inside money is unnecessary. In section 4 we weaken the perfect anonymity of the non-bank public, by introducing the concept of credit lines restricted to binary credit records. We derive a necessary and sufficient condition for credit to be implementable. We then find sufficient conditions under which inside-money allocations have the banking sector also intermediating capital. Section 5 concludes. All the proofs appear in the appendix.

2 The environment

Time is discrete and the horizon is infinite. There is one type of divisible and perishable consumption good per date, but people rank goods in odd dates differently from goods in even dates. There is also a limited supply of perfectly durable, indivisible and productive assets, called capital. These assets are non reproducible. There is a $[0, 2]$ continuum of each of 2 types of people. Each type is specialized in consumption and production: a type e person consumes even-date goods and produces odd-date goods, and a type d person consumes odd-date goods and produces even-date goods. Each type maximizes expected discounted utility, with discount factor $\beta \in (0, 1)$. We find it useful to have a notation for the two-period discount factor, $\delta \equiv \beta^2$. We also find it convenient to refer to a type e individual in an even (odd) date, or a type d individual in an odd (even) date, as a *consumer* (*producer*).

People meet randomly in pairs and face idiosyncratic preference shocks. The utility from consumption is $\varepsilon u(y)$, where ε is the *iid* shock with support in $\{0, 1\}$, u is the utility function, and y is the amount consumed. The probability of $\varepsilon = 1$ is $\pi \in (0, 1]$. Individuals without capital cannot produce. Those holding one unit of capital can produce any choice of $y \in \mathbb{R}_+$ units of the corresponding date good, at a utility cost normalized to be y itself. Utility in a period is thus $\varepsilon u(y)$ when consuming, and $-y$ when producing. The function u is defined on $[0, \infty)$, is increasing and twice differentiable, and satisfies $u(0) = 0$, $u'' < 0$, $u'(0) = \infty$ and $u'(\infty) < 1$.

The $[0, 2]$ continuum of each type is further divided into two groups of equal measure, defined by the amount of information publicly available about their histories. The society is able to keep a public record of the assets, actions and shock realizations of the first group, called bankers. Regarding the other group, the non-bank public, or non bankers, for short, the society can keep a record of their announcements to bankers. More precisely, at date 0, each non banker $i \in [0, 1]$ is assigned a unique password number $s = f(i)$ according to a function f known to bankers. Although each identity i is private information, bankers can record messages from non bankers declaring a pair (i, s) in a given date. We shall see that bankers can be regulated so as to keep f private to the banking

sector.

In each period, people are twice randomly matched in pairs. First each non banker is matched with a banker of the same type. In these meetings there is no scope for consumption and production, although announcements can be recorded, and capital and monetary assets, to be defined below, can change hands. Then, in a second meeting, each type e banker is matched with a type d banker, and each type e non banker is matched with a type d non banker. In the second meeting, the realization of preference shocks occur and production takes place.

We assume that people cannot precommit to future actions, so that those who produce or give up assets have to get a future reward for doing so. As in Cavalcanti and Wallace (1999), bankers can be induced to produce and transfer assets without receiving something tangible in exchange, because they can be rewarded and punished in the future for actions they take currently. Unlike their environment however, non bankers here can in principle transfer assets to bankers without receiving something tangible, although that cannot happen in meetings between two non bankers, where they must receive something tangible in order to produce.

We assume that bankers have a technology that permits them to create indivisible, perfectly durable and uniform objects called notes at any time. We also assume that in a meeting with production, capital units can be transferred only after production takes place, so that capital cannot perform as a medium of exchange. To keep the model simple, we assume that each person can carry from one meeting to the next a pair of capital and money in $\{0,1\}^2$, that is, at most one unit of capital and at most one unit of money. We let $k \in [0,4]$ be the total measure of the existing supply of capital. When $k=4$, each person starts at date 0 with one unit of capital and the model more closely resembles typical random-matching specifications. The measure of capital allocated to bankers is k_b, and that allocated to non bankers is k_n, with $k = k_b + k_n$.

We only consider steady states with people of types e and d treated symmetrically within the same sector. We can anticipate that the non-bank sector features a measure of potential producers, p, those with capital and without money, a measure of potential consumers, q, and a level of production y_n. The sector's welfare, U_n, is defined as the sum of non-bank expected utility

$$U_n = \frac{1}{1-\beta}\pi pq[u(y_n) - y_n], \tag{1}$$

that is, the present discounted value of the product of two terms: a measure of trade frequency, πpq, and a measure of social gains per meeting, $u(y_n) - y_n$. It can be shown that this value, divided by two, correspond to the expected discounted utility, faced by each non banker at the very first date of the economy. This interpretation of U_n requires that types, and stationary asset and credit holdings, are allocated randomly to non bankers at the first date, according to the stationary distributions chosen as steady states.[3]

[3] This is equivalent to assuming the the first date of the model is even with probability one

Finally, when an allocation of scarce resources across sectors is to be discussed, the overall objective function of the planner problem is assumed to be $\min\{U_b, U_n\}$, where U_b is a measure of welfare for the bank sector, that also receives the same expected utility interpretation. The expression for U_b can be shown to equal the right-hand side of (1), with y_n replaced by bank's production, p replaced by the measure of capital allocated to bank producers, and for q equal to one (money holdings is not a constraint for bankers).

3 Benchmark allocations

In our benchmark allocations, non bankers are never handed out passwords. Our goal is to specify first the set of symmetric and stationary allocations satisfying sequential, individual rationality constraints. We call them participation constraints, as in the mechanism design approach of Cavalcanti and Wallace (1999).

By assumption, there is no production in the first round of meetings, when bankers and non bankers meet, because the individuals in these meetings are interested in consuming goods of the same type, that they cannot produce. Without passwords, non bankers are completely anonymous, and could only transfer capital in exchange for money. However, money holders are not interested in buying capital from bankers because they already have the money to buy goods, and will thus prefer to wait for the second round of meetings with other non bankers. It follows that the absence of passwords shuts down capital trade between bankers and non bankers.

Without capital transfers and production across the bank and non-bank sectors, note issue by bankers to non bankers is not consistent with a steady state. Indeed, bankers have nothing to offer to non bankers when it comes the time to retire or destroy such notes. We use that fact and also ignore note issue in this section, so that the bank sector is in isolation, given an arbitrary endowment of bank capital, k_b.

With the deterministic pattern of consumption and production dates, there is room for reallocating capital from producers to consumers after production takes place. The planner thus recommends that every bank producer transfers his or her capital holdings to the consumer at the end of the meeting, if the consumer does not have a unit of capital already. We can assume without loss of generality that $k_b \leq 1$, since capital is not scarce if $k_b \geq 1$.

A banker that defects from an allocation can be punished with autarky because other bankers can be instructed to not produce for him. Incentive constraints require expected utilities to be above that of autarky, which is zero. Let v_b denote the expected discounted utility of a banker consumer, and w_b denote that of a banker producer with capital. The stationary values v_b and w_b

half, and odd with probability one half. The type notations, d and e, are thus not necessary in the description of allocations. It is also not important to associate π to preference risk or impatience, as in Diamond and Dybvig (1983). The same formulation would go through if we have assumed productivity risk.

satisfy, for a given level of bank production y_b,

$$v_b = k_b[\pi u(y) + \beta w_b] + (1 - k_b)\delta v_b \tag{2}$$

and

$$w_b = \pi[-y_b + \beta v_b] + (1 - \pi)\beta v_b. \tag{3}$$

The participation constraints are

$$u(y_b) + \beta w_b \geq 0 \tag{4}$$

and

$$-y_b + \beta v_b \geq 0, \tag{5}$$

since the payoff from defection is zero.

Since $\delta = \beta^2$, equation (2) indicates that with probability $1-k_b$ the consumer waits for two periods for a chance to consume, and cannot produce next period because he or she does not receive capital from a producer currently. There is a measure of $1 - k_b$ producers without capital, so that the welfare sum is

$$U_b = v_b + k_b w_b + (1 - k_b)\beta v_b, \tag{6}$$

and U_b equals the right-hand side of (1) for $q = 1$ and $p = k_b$.

Definition 1 *An allocation y_b is implementable with capital $k_b \leq 1$ available to bank producers if there exists (v_b, w_b) such that (2-5) hold.*

Having allocated the maximum amount of capital to producers, the optimum y_b is defined in what follows.

Bank problem *Maximize U_b by choice of an implementable allocation y_b with capital k_b.*

The problem of maximizing U_b is thus equivalent to maximizing $u(y_b) - y_b$ subject to the participation constraints (4-5). Since (5) implies $w_b \geq 0$ and thus (4), only ((5) needs to be considered . After solving the Bellman equations for v_b and w_b, the producer's participation constraint is easily found to be equivalent to

$$u(y_b) \geq \frac{y_b}{\beta}[(1 - \delta)\frac{1}{\pi k_b} + \delta]. \tag{7}$$

The optimum production level corresponds to the minimum between the y_b that satisfies this constraint with equality, and the first-best level of production, the y^* such that $u'(y^*) = 1$. The following lemma states that the problem of allocating capital in the bank sector imposes the same restrictions as an increase in preference risk in the problem without capital scarcity.

Lemma 1 *The benchmark optimum for banks only depends on π and k_b by the way of the product πk_b.*

We now turn to study non bankers, also in isolation from bankers. We shall see that non bankers need to use money, and that capital scarcity affects the way money is distributed. We assume a symmetric distribution of outside money which is not affected by bank behavior.

We can anticipate that non bankers without money are given priority for receiving capital at date 0, and that the initial measure of non bankers without capital receives a unit of money. In order to put capital to its best use, the planner recommends that one unit of money be exchanged for a level of output y_n, when $\varepsilon = 1$, together with a unit of capital. If some individuals must hold money and capital, a possibility discussed in the next section, the planner suggests that money buy just goods if the consumer has capital already.

Given the above considerations, when describing desirable allocations, it suffices then to distinguish four values for non bankers: v_n is the value of a consumer with money (with or without capital); $\bar{v}_n$ is that of a consumer without money and with capital; w_n is that of a producer without money and with capital; and $\bar{w}_n$ is that of a producer with money (with or without capital). If p is the measure of producers with capital and without money, and q is the measure of consumers with money (with or without capital), then, for a given level of non-bank production,

$$v_n = \pi p[u(y_n) + \beta w_n] + (1 - \pi p)\beta \bar{w}_n \tag{8}$$

and

$$w_n = \pi q[-y_n + \beta v_n] + (1 - \pi q)\beta \bar{v}_n, \tag{9}$$

hold. Current consumers without money have to wait for the next period, when they become producers, to engage in trade, so that $\bar{v}_n = \beta w_n$. As a result of the unit upper bound on money holdings, the same applies to current-period producers with money, so that $\bar{w}_n = \beta v_n$. Hence, one can summarize the non-bank Bellman equations in matrix notation as

$$M \begin{bmatrix} v_n \\ w_n \end{bmatrix} = \begin{bmatrix} \pi p u(y_n) \\ -\pi q y_n \end{bmatrix}, \tag{10}$$

where

$$M = \begin{bmatrix} 1 - \delta + \delta \pi p & -\beta \pi p \\ -\beta \pi q & 1 - \delta + \delta \pi q \end{bmatrix}. \tag{11}$$

The participation constraints for non bankers assume that defection on the part of non bankers goes unpunished because such defection does not become part of a public record. Thus, the participation constraints are simply that trade is weakly preferred to leaving the meeting with what was brought into the meeting. There are two such constraints, one for the consumer and one for the producer:

$$u(y_n) + \beta w_n \geq \beta \bar{w}_n \tag{12}$$

and

$$-y_n + \beta v_n \geq \beta \bar{v}_n. \tag{13}$$

It follows from (8-9) that the non-bank participation constraints are equivalent to the requirement that $v_n \geq 0$ and $w_n \geq 0$.

The measures p and q have to be consistent with stationarity. If there are $1-p$ potential producers with money in the current period, while πpq producers (without money) engage in trade in the current period, then next's period measure of consumers with money is given by $1-p+\pi pq$. The stationarity requirement for q is thus

$$q = 1 - p + \pi pq, \text{ with } p,\ q \in [0,1], \tag{14}$$

which also implies that for p, namely $p = 1 - q + \pi pq$. There is also a capital constraint: the measure of producers without money and with capital, plus the measure of consumers without money and with capital, cannot exceed k_n. Since these measures correspond, respectively, to p and $1-q$, the capital constraint is

$$p + 1 - q \leq k_n. \tag{15}$$

We have thus chosen to examine, in the absence of credit, the following class of allocations.

Definition 2 *An allocation (y_n, p, q) is implementable with capital k_n if (14-15) holds and there exists a nonnegative solution (v_n, w_n) to (10).*

The sum of expected utilities for non bankers, U_n, is given by

$$U_n = qv_n + (1-q)\bar{v}_n + pw_n + (1-p)\bar{w}_n. \tag{16}$$

Substituting the expressions for v_n, w_n, $\bar{v}_n$ and $\bar{w}_n$ in (16) yields equation (1). The non-bank production problem is the following.

Outside-money problem *Maximize U_n by choice of an implementable allocation (y_n, p, q) with capital k_n.*

The participation constraint for the producer, $w_n \geq 0$, is equivalent to the inequality $\beta v_n \geq y_n$. Since $y_n \geq 0$, then $w_n \geq 0$ implies $v_n \geq 0$. Solving now for w_n and v_n in (10), for a given y_n, yields, after some simple algebra, the condition that $w_n \geq 0$ if and only if

$$u(y_n) \geq \frac{y_n}{\beta}[(1-\delta)\frac{1}{\pi p} + \delta], \tag{17}$$

when p is positive.

Therefore, the non-bank optimality problem is that of maximizing $pq[u(y_n) - y_n]$ subject to the producer's participation constraint, (17), the stationarity requirement that $q = 1 - p + \pi pq$, and the capital constraint $p + 1 - q \leq k_n$. For

β sufficiently high and $k_n \geq 1$, the participation constraint does not bind and the solution is given by $u'(y_n) = 1$ and $p = q$ satisfying $\pi p^2 - 2p + 1 = 0$. This choice of (p, q) corresponds to the distribution of outside money that maximizes the flow of trade πpq. When (17) is violated for such a p and the first-best level of output, satisfying $u'(y_n) = 1$, then the social planner has to trade-off a reduction in the social surplus, $u(y_n) - y_n$, and in the trade volume πpq, for an increase in p which weakens the participation constraint. The capital constraint however determines a maximum feasible $p + 1 - q$ as the intersection of a straight line with the graph of $q = 1 - p + \pi pq$ in the (p, q) plane.

The fact that non bankers need to use money also implies that they cannot share capital as efficiently as the bank sector.

Lemma 2 *If the distribution of capital is constrained by* $k_n \leq k_b < 1$, *and* $\pi \in (0, 1]$, *then* $U_n < U_b$ *holds in the constrained optimum.*

The case in which capital is not scarce is also instructive. It highlights the role of shocks regarding the difference between U_b and U_n, because $U_b = U_n$ would tell us that outside money is working perfectly well.

Proposition 1 *Assume that* k_b, $k_n \geq 1$. *If* $\pi = 1$, *then optimization of benchmark allocations yields* $U_n = U_b$. *Hence outside money is essential (and inside money is not) for these parameters. However, when* $\pi < 1$, *that optimization yields* $U_n < U_b$, *and* U_n *is increasing in* π.

In face of lemma 2, maximization of the economy-wide welfare, $\min\{U_b, U_n\}$, for $\pi < 1$, would require a greater allocation of capital to the non-bank sector, namely, $k_n > k_b$ such that $U_b = U_n$.

When $\pi < 1$, the use of outside money in the non-bank sector is such that a measure of capital remains idle in the hands of consumers, who were not able to sell capital in the previous period. There is also in every period a measure of producers with money and without capital. We shall show that inside money can reduce this problem when we allow non bankers to build a credit record with the bank sector. Even when capital is not scarce, but $\pi < 1$, inside money in connection with personalized credit can insure non bankers against the risk of unsuccessful trade attempts.[4]

4 Credit allocations

Optimum allocations correspond to the best possible use of recorded histories. For the purposes of this paper, and to keep the dimensionality of the problem tractable, we restrict attention to a simple scheme of binary credit records. The planner assigns to each non banker a *balance* $z \in \{0, 1\}$ and instructs bankers to issue inside money, upon request, to non bankers with $z = 1$. When money

[4]If non bankers could store an unlimited number of notes, the bank sector would have to offer interest rates on depositors in order to induce the participation of non bankers.

is issued, the record is updated to $z = 0$. When a non banker with $z = 0$ makes a deposit, the money deposited is destroyed and his record is updated to $z = 1$.

As in the previous section, we choose a notation ignoring capital holdings, and show later that incorporating capital scarcity in the analysis can still be accomplished with the simple notation. We distinguish the following non-bank values after meetings with bankers take place, but before they are matched with other non bankers: v_{nz} is the value of a consumer with money and credit record z, and w_{nz} is that of a producer without money and credit record z. It is also useful to think of z as money *deposited* in the bank sector. We could also assign a value to consumers without money, say $\bar{v}_{nz}$. It just happens, however, that if $z = 0$ then the consumer cannot buy goods in the current period, or make a deposit into his or her account in the next period, and thus $\bar{v}_{n0} = \beta w_{n0}$. If $z = 1$, the consumer has no incentives to make a withdraw from a banker in the next period, when he or she becomes a producer. Hence $\bar{v}_{n1} = \beta w_{n1}$. Likewise, we could also have assigned a value to producers with money, $\bar{w}_{nz}$. If $z = 0$, the producer will need the money anyway, so he or she makes no deposit next period and $\bar{w}_{n0} = \beta v_{n0}$. If $z = 1$, the producer cannot improve his or her record further, and essentially for the same reason, $\bar{w}_{n1} = \beta v_{n1}$.

The values v_{nz} and w_{nz} should hence satisfy the following system of equations for a given y_n:

$$M \begin{bmatrix} v_{n0} \\ w_{n0} \end{bmatrix} = \begin{bmatrix} \pi p u(y_n) \\ -\pi q y_n \end{bmatrix} + x \begin{bmatrix} (1-\pi p)\beta(w_{n1} - \beta v_{n0}) \\ 0 \end{bmatrix} \tag{18}$$

and

$$M \begin{bmatrix} v_{n1} \\ w_{n1} \end{bmatrix} = \begin{bmatrix} \pi p u(y_n) \\ -\pi q y_n \end{bmatrix} + \begin{bmatrix} 0 \\ (1-\pi q)\beta(v_{n0} - \beta w_{n1}) \end{bmatrix}, \tag{19}$$

where M is as defined in the benchmark case, p and q are the respective measures of producers without money and of consumers with money, integrated over the distribution of credit records, and x is a variable associated to the availability of capital and should momentarily be considered identical to one. The case of capital scarcity, which requires $x < 1$, is discussed later. The terms multiplying $1-q\pi$ and $1-p\pi$ in the first and fourth equations are written assuming that consumers with good credit, who need money, agree to withdraw from the banker and have the record updated to $z = 0$; and that producers with bad credit, who were not able to spend money in the previous period, agree to deposit with the banker and have their record updated to $z = 1$. The participation constraints regarding these transactions are, respectively,

$$v_{n0} \geq \beta w_{n1} \tag{20}$$

and

$$w_{n1} \geq \beta v_{n0}. \tag{21}$$

The first inequality assures that a non banker is willing to borrow when there is an opportunity, a constraint that is easily satisfied by a stationary allocation with discounting. We call the second inequality the *deposit* constraint. It assures

that a non banker is willing to deposit money with a banker, and to become a producer currently without money, just for the sake of improving his or her credit record. It can be easily verified that this constraint is equivalent to the requirement that a producer with good credit is willing to produce in exchange for money, namely,

$$y_n \leq \beta(v_{n1} - v_{n0}).$$

The participation constraint for producers with bad credit is

$$w_{n0} \geq 0, \tag{22}$$

which is the same as the inequality $\beta v_{n0} \geq y_n$. Finally, there are participation constraints for consumers, requiring that v_{n1} and v_{n0} be nonnegative, and which are again implied by the producer's constraints.

Next, we discuss feasible measures of producers without money and consumers with money, p and q. It is intuitive that a credit allocation in this framework can increase *both* measures. In order to keep the set of feasible measures tractable, we require that the distribution of deposits be constant and the same for both consumers and producers, so that $\frac{p_1}{p} = \frac{q_1}{q}$. With this additional requirement, we have the following lemma.

Lemma 3 *The set of stationary measures* (p, q, x)*, associated to the system (18-19), is fully described by the equality*

$$q = 1 - p + \pi pq + A_x(p, q), \text{ with } p, q \in [0, 1], \tag{23}$$

where $A_x(p, q) = xq(1-\pi p)p(1-\pi q)/[xq(1-\pi p)+p(1-\pi q)]$ *defines a concave function in the* (p, q) *plane.*

The lemma shows that credit allows for an increase, when compared to equation (14), in the set of feasible measures of potential producers and consumers. In the proof, it is used the fact that a measure of producers, in proportion to $p(1 - \pi q)$, fail to acquire money, but is able to make withdraws at the next date because they have good records. Likewise, a measure of consumers, in proportion to $q(1-p\pi)$, fail to spend their money holdings, and is able to make deposits at the next date in order to leave the bad-record state. When $x = 1$ and $p = q$, half of the non-bank public holds a bad record, and the other half holds a good record.

The following lemma follows from the fact that the only potentially binding constraint for non banks is the deposit constraint (20).

Lemma 4 *The non-bank production* y_n *satisfies the participation constraints (20-22) and the system (18-19) if and only if*

$$u(y_n) \geq \frac{y_n}{\beta}[(1-\delta)\frac{1}{\pi p} + \delta + \frac{\det(M)}{(1-\pi q)\delta(\pi p)^2}]. \tag{24}$$

It is shown in the proof that the determinant of the matrix M, $\det(M)$, is positive and converges to zero as β approaches one. Notice also that x does not appear directly in the inequality (24). In fact, the deposit constraint is not affected by x because x shows up in equation (18) only multiplying $w_{n1} - \beta v_{n0}$, the term to be solved for when studying the deposit constraint. It follows that for β high enough, the deposit constraint does not bind for $y_n = y^*$ such that $u'(y^*) = 1$, the first-best level of production.

We now turn to the intermediation of capital. Inside-money is destroyed in a credit allocation when a non-banker producer with $z = 0$ makes a deposit. The only reason for this non banker to actually make a deposit, instead of holding on to money and waiting to become a consumer next period, is the potential gain of producing again currently to acquire more money. Without capital, the non banker will choose not to deposit. A necessary condition for a credit allocation to be implementable, therefore, is that depositors have access to capital.

We should now let p denote the measure of producers with capital and without money, integrated over states z. Regarding the set of consumers without money, it turns out that they will be all in state $z = 0$ in the steady state, since the ones in state $z = 1$ are able to withdraw from the bank and have money for the meetings with other non bankers. As a result, the measure of consumers without money, $1 - q$, is also understood to have capital. It is thus necessary to allocate capital at least to a measure of p producers and $1 - q$ consumers. The capital constraint now reads

$$p + 1 - q + \lambda(1 - p + q) \leq k_n, \tag{25}$$

where λ is the measure of non bankers with money and capital.

We shall discuss only two possible cases, $\lambda = 1$ and $\lambda = 0$. If $\lambda = 1$, so that $x = 1$ and the bank sector is not reallocating capital, then the capital constraint (25) requires $k_n = 2$, that is that all non bankers hold capital. As capital becomes scarce and below some critical point, the reduction in the capital allocated to bankers makes credit suboptimal. Hence, as k is reduced continuously to the point where $\lambda = 0$ becomes optimal, a point where $k_n > 1$ and $p = q$ is still feasible, then the extra capital that becomes available as λ shifts from 1 to 0 can be allocated to the bank sector. If β is sufficiently high, so that the participation constraints do not bind with $p = q$, then bank intermediation with some $x > 0$ makes credit attain a higher welfare due to the increases in p and q allowed by having $A_x(p, q) > 0$ in equation (23).

We have thus already set the main elements for showing that intermediation of capital can be desirable. Further characterization of the optimal x would depend on how much intermediation imposes a cost to bankers since, as assumed in section 2, bankers meeting with depositors are themselves producers. Intermediation takes capital away from bank producers and tend to reduce bank welfare. That discussion would depend too much on details of the model, and go beyond the scope of this paper. Also, the advantage of restricting attention to $\lambda \in \{0, 1\}$ is that either all non bankers with money hold capital, or none of them do. As a result, we do not need an extra notation for telling apart the

consumers with money and capital from the consumers with money only. We present in a lemma below, for completeness, the full description of the allocation of capital in the bank sector when intermediation takes place.

We let the fraction of bank producer holding capital at the beginning of a period, before transfers to non bankers take place, be denoted $\tilde{k}_b$. If a request of capital from a depositor is agreed with probability $\theta \in [0, 1]$, then

$$x = \lambda + (1 - \lambda)\tilde{k}_b\theta. \tag{26}$$

The values of $\tilde{k}_b$ and θ consistent with stationarity are as follows.

Lemma 5 *Capital intermediation with probability θ is feasible if, for p_b, ε and τ in $[0, 1]$,*

$$k_b = p_b + \varepsilon, \tag{27}$$

$$\tilde{k}_b = \frac{p_b}{1 - \tau}, \tag{28}$$

$$\tau = \frac{\varepsilon(1 - p_b)}{p_b + \varepsilon(1 - p_b)} \tag{29}$$

and

$$\tau = \theta(1 - \lambda)\frac{pq(1 - \pi q)(1 - \pi p)}{p(1 - \pi q) + xq(1 - \pi p)}. \tag{30}$$

In the proof of the lemma, it is used the fact that, with intermediation, bank capital needs to be split between a fraction of producers, p_b, and a fraction of consumers, ε, because there is a constant inflow of capital into the bank sector that cannot be transferred to producers in the same period. Moreover, in order that these fractions are kept stationary, a bank producer must transfer his capital with probability τ given by equation (29). Equation (30) is the requirement that τ coincides with the probability that depositors request capital, $\theta(1 - \lambda)$, times the measure of depositors in a given period, which is given by the fraction in the right-hand side of (30).

Definition 3 *A credit allocation $(y_b, y_n, p, q, x, \lambda, \theta, \tilde{k}_b)$ is implementable with $k = k_b + k_n$ and $\lambda \in \{0, 1\}$ if (23-30) hold and y_b is implementable for bankers when the capital allocated to bank producers is $\tilde{k}_b$.*

It is clear that the expressions for U_b and U_n remain unchanged. Hence the optimum problem is stated as follows.

Welfare problem *Maximize* $\min\{U_b, U_n\}$ *by choice of a credit allocation $(y_b, y_n, p, q, x, \lambda, \theta, \tilde{k}_b)$ with capital $k = k_b + k_n$.*

Proposition 2 *There exists an open interval $K \subset (0, 3)$ of capital levels such that, if $k \in K$ and β is sufficiently high, then capital intermediation is essential, in the sense that bankers trade capital with non bankers with positive probability in an optimum.*

Although the proof of proposition 2 is restricted to the case $\lambda \in \{0, 1\}$, we believe that the argument holds more generally. Because $A_x(p, q) < q$, allocating an unit of capital to consumers with money creates less deposits than allocating that unit to the bank sector. The difficulty with this more general discussion is that intermediation causes a cost to banks. That cost can be made arbitrarily small by choosing x close to zero, so that the extra bank capital generates an welfare improvement when the alternative is $\lambda = 0$. If the alternative has $\lambda \in (0, 1)$, further restrictions on the other parameters of the model may prove necessary.

5 Concluding remarks

We have shown that inside-money allocations may include capital intermediation if capital is sufficiently scarce. Is is instructive to consider how allocations would look like if the agents intermediating capital could not issue money. The non-bank public would demand some compensation in order to give capital away. If capital is scarce, the bank sector cannot promise returning capital in the near future for sure. The ability to issue money thus facilitates receiving capital from the public.

We have restricted allocations to a simple scheme in which the non-bank public is monitored with two levels of credit ratings. A more sophisticated arrangement would let the public make more deposits than what we have allowed. However, bankers would continue to distribute capital more efficiently than non bankers, weakening capital constraints, as a result of information asymmetries. It is thus reasonable to conjecture that intermediation would survive to more sophisticated monitoring arrangements, as long as non bankers remain partially anonymous, a necessary condition for fiat money to be essential in trade.

Our results require a notion of capital scarcity. But caution should be used with a literal interpretation. If capital were divisible, and its marginal product always positive, then scarcity would always exist in a sense, because some people would be more inclined to use capital than others. We have not pursued such an avenue for obvious reasons of tractability.

As a by-product, our model confirms the essentiality of uncertain time profiles of consumption, like in Diamond and Dybvig (1983), for a role of inside money. As shown, when there is no trade risk, outside money performs well in our model. There are thus important implications from removing consumption randomness in models of money.

When capital intermediation takes place in an optimum, it does so by refluxing capital from consumers to producers, and in this sense, the banking system can be considered illiquid. We have only discussed equilibria where intermediation is never disrupted. If consumers stop transferring capital to bankers at some date, the bank system would have difficulties in honoring intermediation in the future. Discussing banking crisis is arguably problematic with mechanism design, and lies beyond the scope of this paper. The model can provide,

however, for new insights in comparison to Diamond and Dybvig (1983), since our bankers are never illiquid in the sense of fiat assets.

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Appendix

Proof of lemma 1
The result follows directly from the inequality (7), and from the fact that the frequency of bank trade is $\pi \min\{k_b, 1\}$. □

Proof of lemma 2
A straightforward comparison of inequalities (7) and (17), when the constraints (14) and (15) are taken into account, reveals that the bank constraint set is strictly larger than that of non bankers for $k_n \leq k_b < 1$. In addition, the capital constraint for non bankers imply $pq < k_b$ for $k_n < k_b$. As a result,

$$(1-\beta)U_n = \pi pq[u(y_n) - y_n] < \pi k_b[u(y_b) - y_b] = (1-\beta)U_b$$

follows for the optimum choices of y_n and y_b, given k_n and k_b, which proves the result. □

Proof of proposition 1
The non-bank constraint set is increasing in p and coincides with that of bankers if and only if $p = 1$. It follows from the stationarity restrictions on p that, if $\pi < 1$, then $p = 1$ only if $q = 0$. Moreover, when participation constraints do not bind, although production levels are the same in both sectors, the frequency of trade is higher in the banking sector, since the stationary values of p, such that $p = q$, are less than 1 by a difference that is decreasing in π. □

Proof of lemma 3

If p_1 denotes the current measure of producers without money and with deposits in the bank, then $p_1(1-\pi q)$ is the fraction of those making withdraws next period, so that $q = 1 - p + \pi pq + p_1(1-\pi q)$ in the steady state. Similarly, if q_0 is the measure of consumers with money and $z = 0$, then $p = 1 - q + \pi pq + xq_0(1-\pi p)$, so that $p_1(1-\pi q) = xq_0(1-\pi p)$ must hold. Using the latter expression, together with $m \equiv \frac{p_1}{p} = \frac{q_1}{q} = 1 - \frac{q_0}{q}$, to solve for m, yields equation (23). Equation (23) itself can be written in two different ways:

$$(x + 1 - x)q(1-\pi p) = 1 - p + A_x(p,q) \text{ and } p(1-\pi q) = 1 - q + A_x(p,q),$$

so that multiplying both sides of both equations by the denominator, call it D, of $A_x(p,q)$, and rearranging terms, yields

$$[xq(1-\pi p)]^2 = D[1 - p - q + \pi pq + xq(1-\pi p)] \text{ and } [p(1-\pi q)]^2 = D(1-q).$$

Since $[DA_x(p,q)]^2 = [xq(1-\pi p)]^2[p(1-\pi q)]^2$ then

$$A_x(p,q) = (1-q)^{\frac{1}{2}}[1 - p - q + \pi pq + xq(1-\pi p)]^{\frac{1}{2}}$$

indicates that $A_x(p,q)$ equals the composition of two strictly concave functions. Therefore, (23) defines a concave function in the (p,q) plane. $\square$

Proof of lemma 4

The first part of the proof is to show that satisfaction of the deposit constraint implies $v_{n0} \geq \beta w_{n1}$. Solving for v_{n0} and w_{n1} in (18-19), under the assumption that $w_{n1} - \beta v_{n0}$ is a nonnegative constant, implies after some simple algebra, that $v_{n0} - \beta w_{n1}$ is positive. Hence, $w_{n1} - \beta v_{n0} \geq 0$ implies that the unique values solving (18-19) are all nonnegative, and thus the other participation constraints are satisfied.

For showing that the deposit constraint is equivalent to (24), we proceed as follows. To save on notation below, we write $u = u(y_n)$, $\rho = \pi p$, $\xi = \pi q$, $P = 1 - \delta + \delta\rho$, $Q = 1 - \delta + \delta\xi$ and $\mu = \det(M) = PQ - \delta\rho\xi$. Now (18-19) defines a system for v_{n0} and w_{n1} in two equations that can be written as $\det(C)[v_{n0}\ w_{n1}]^T = CS$, where $S = M[\rho u\ ,\ -\xi y_n]^T$,

$$C = \begin{bmatrix} \mu + (1-\xi)\delta P & (1-\rho)\beta Q \\ (1-\xi)\beta P & \mu + (1-\rho)\delta Q \end{bmatrix},$$

and $\det(C) > 0$. As a result,

$$\begin{aligned} \det(C)(w_{n1} - \beta v_{n0}) &= \begin{bmatrix} (1-\delta)\beta[-\xi P - \delta\xi(1-\rho)] & \mu \end{bmatrix} S \\ &= \begin{bmatrix} -(1-\delta)\beta\xi & \mu \end{bmatrix} \begin{bmatrix} Q\rho u - \beta\rho\xi y_n \\ \beta\rho\xi u - P\xi y_n \end{bmatrix} \end{aligned}$$

so that $w_{n1} \geq \beta v_{n0}$ if and only if $\beta\rho u\delta\rho(1-\xi)(1-\delta) \geq y_n[(1-\delta)\mu + \delta\rho\mu - (1-\delta)\delta\rho\xi]$. Using now $\mu = PQ - \delta\rho\xi = (1-\delta)[1 - \delta + \delta\rho + \delta\xi(1-\rho)]$ completes the proof. $\square$

Proof of lemma 5

Let p_b denote the fraction of bank producers with capital, and q_b denote the fraction of bank consumers without capital, both measured at the second round of meetings, when banks produce and consume. Then $k_b = p_b + 1 - q_b$, and for $q_b = 1 - \varepsilon$ equation (27) holds. Moreover, if τ is the probability that a bank producer with capital transfers capital to a non banker, then $p_b = (1 - \tau)\tilde{k}_b$, so that (28) holds. Also, if α is the probability that a bank consumer without capital receives capital from a non banker, then stationarity requires $q_b = (1 - \alpha)(1 - p_b + p_b q_b)$ and $p_b = (1 - \tau)(1 - q_b + p_b q_b)$. These expressions can be rewritten as $\alpha q_b = (1 - \alpha)(1 - p_b - q_b + p_b q_b)$ and $\tau p_b = (1 - \tau)(1 - p_b - q_b + p_b q_b)$, so that $\alpha q_b(1 - \tau) = \tau p_b(1 - \alpha)$. This condition, together with $q_b = (1 - \alpha)(1 - p_b + p_b q_b)$, for $q_b = 1 - \varepsilon$, implies (29). Finally, according to lemma 3, the probability that a banker producer meets with a depositor is given by the third term in the right-hand side of (30). $\square$

Proof of proposition 2
When participation constraints allow $y_n = y^*$ and $\lambda = 1$, the welfare problem maximizes pq subject to (23) and (25), for $k_n = 2$. According to lemma 2, as k is sufficiently reduced, maximizing $\min\{U_b, U_n\}$ implies $k_b < 1$. Now, the level curves of pq in the (p, q) plane are differentiable and strictly convex, while (23) defines a strictly concave constraint on the same plane. Since preferences are also continuous and differentiable, welfare varies continuously with k while $\lambda = 1$ remains optimal. Hence, there is $\bar{k} < 3$, such that for $k = \bar{k}$, there are two allocations, one with $\lambda = 1$, and another with $\lambda = 0$, that attain the same optimum welfare. If $x > 0$ in the latter allocation, there is nothing else to prove. If, by the contrary, $x = 0$ in that allocation, which, by continuity, features $k_n > 1$, then it also has $p = q$ because that maximizes pq and it is feasible with $k_n > 1$. Thus some non-bank capital remains idle with consumers with money, and can be transferred to the bank sector. Since, again by continuity, $k_b < 1$, then this idle capital can be transferred to the bank sector, with a part used to support intermediation with a small $x > 0$, through a small τ and small ε, without reducing the measure of producers with capital, that is, such that p_b remains the same. That reallocation of capital, by making x positive, increases both p and q. As β has been chosen sufficiently high so that participation constraints do not bind, then welfare increases, contradicting that $x = 0$ is optimal. $\square$