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TÓPICOS EM ECONOMETRIA APLICADA

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por
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SUMÁRIO

Evaluating Different Approaches in Constructing Coincident and Leading Indices of Economic Activity for the Brazilian Economy

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Abstract

This paper has three original contributions. The first is the reconstruction of employment and income series to allow the creation of a new coincident index for the Brazilian economic activity. The second is the construction of a coincident index of the economic activity for Brazil, and from it, (re) establish a chronology of recessions in the recent past of the Brazilian economy. The coincident index follows the methodology proposed by The Conference Board and it covers the period 1980:1 to 2007:11. The third is the construction and evaluation of many leading indicators of economic activity for Brazil which fills an important gap in the Brazilian Business Cycles literature.

Keywords: Coincident and Leading Indicators, Business Cycles, Common Features, Latent Factor Analysis

J.E.L. Codes: C32, E32.

1 Introduction

An important concern of any modern society in what is the current "state" of economy and what should be the state of the economy in the near future. Entrepreneurs and individuals are interested in the question because their profits and welfare are, respectively, a function of it. Governments also have an interest in the subject for budgetary and welfare issues. Unfortunately, no one possesses a series that represents the "state of the economy" because it is a latent variable, i.e., it is non-observable.

Stock and Watson (1999) argue that, if we were to choose one variable to best represent the state of the economy, this variable would be the Gross Domestic Product (GDP). They claim that "[...] fluctuations in aggregate output are at the core of the business cycle so the cyclical component of real GDP is a useful proxy for the overall business cycle [...]". However, GDP is not readily available without measurement error, making it of little use for decision making in this context. The idea of bringing together information on GDP to construct coincident and leading indices for the U.S. is also present in Mariano and Murosawa (2003).

Including alternative information to estimate the state of the economy is also present in the recent effort of Issler and Vahid (2006). They argue that current U.S. research misses a vital piece of information on the state of the economy – the NBER dating committee decisions. They claim that, if "we are asked to construct an index of the health status of a patient, [and] we know that the best indicator of the health of the patient is the results of a blood test, [but] blood samples cannot be taken too frequently, and test results are only available with a lag, sometimes too long to be useful, [making our index] a function of variables such as blood pressure, pulse rate and body temperature that are readily available at regular frequencies. In order to estimate the best way to combine these variables into an index, would we (i) use the historical data on these variables only, or, (ii) use the historical blood test results as well? The answer is, obviously, the latter." Here, blood-test results play a similar role to the NBER dating committee decisions.

The lack of a direct measure of the state of the economy has led to the construction of proxies that can be used in real time. These are the so-called coincident indices of economic activity. From them we can also construct leading indices of observables that help predicting the current state of the economy – the so-called leading indices of economic activity.

With the exception to the work of Contador (1977) and Contador and Ferraz (1999), research on coincident and leading indices of economic activity in Brazil is fairly young and most of the literature dates from the 2000's. Chauvet (2001) and Picchetti and Toledo (2002) use common-factor models to generate a monthly coincident indicator of economic activity. Chauvet (2002) uses a two-state Markov Chain characterizing a recession or an expansion to propose a chronology for Brazilian business cycles. On a broader study, Duarte, Issler and Spacov (2004) evaluated three candidates for composite coincident indices: The Conference Board's (TCB's) index; Spacov's (2000) index, and Issler and Vahid's (2006) index. Using quadratic loss, the dating of these three indices was compared with that of a monthly proxy of Brazilian GDP, suggesting that the Brazilian coincident index should use the methodology put forth by TCB.

Unfortunately, part of this recent research effort in Brazil came to a halt because of the recent redesign of the official employment survey conducted by IBGE – Monthly Employment Survey (*Pesquisa Mensal do Emprego*) – which provides monthly Brazilian data on employment and labor income. Indeed, the change in the survey design in 2002 is so drastic that it eliminates long-span time-series on employment and income, which are crucial series for business-cycle research using TCB- and NBER-oriented methods.

The first goal of this paper is to resume business-cycle research in Brazil using these methods, which proved to be valuable after the empirical results in Duarte, Issler and Spacov. Indeed, one of the main challenges of Brazilian business-cycle research is to back-cast currently available income and employment series to be able to form a long enough coincident index with the usual series used in TCB's method – industrial production, sales, income and employment. Here, we devote a great deal of effort in reconstructing employment and income using a novel State-Space representation. It is based on the interpolation method proposed by Mönch and Uhlig (2005): a very flexible setup that allows the estimation of a wide range of models. As usual, estimation of the unobserved components in these models is performed employing the kalman filter.

Once we obtain a long enough span of the usual series used in TCB's method, we compute a new composite coincident index of Brazilian economic activity. Its dating of recessions is compared with those in Duarte, Issler and Spacov and with those implied by the monthly GDP estimate computed by Issler and Notini (2008).

Our last contribution is regarding the construction of leading indices of economic activity to track the composite coincident index proposed here. Although coincident indices have been relatively well studied in Brazil, leading indices have not. In constructing leading indices we take into account three interesting and novel features in Brazilian business-cycle research: (i) we consider using Granger (1969) causality tests, as well as novel alternative criteria in choosing candidate series to be included in leading composite indices; (ii) we investigate the ability of survey-based time series to lead our composite index; and, (iii) we compare the survey-based composite leading indices with standard leading indices.

Although comparisons are based on a variety of features of the dating properties of these different indices, our decision to validate the current composite index is mostly based on a variant of the QPS quadratic-loss statistic proposed by Diebold and Rudebusch (2001).

Empirical results obtained here are compared with the previous literature on Brazil. In evaluating different results and techniques used in constructing coincident and leading indices, we borrow from the almost century-long debate on this issue that

has been present in the U.S. economy, and a similar half-century or older debate in Europe.

This article is organized as follows. Section 2 contains a brief review of the international and the Brazilian literature. Section 3 presents the Kalman filter model. Section 4 presents the data and the main results. Section 5 concludes.

2 Literature Review

2.1 The International Experience

There has been a fair amount of research on cyclical indicators since the pioneering work of Arthur F. Burns and Wesley Mitchell, which lead to their classic book on business cycles – Burns and Mitchell (1946). Their work has led to the construction of composite indices of leading, coincident, and lagging indicators of economic activity. While their research on the subject was focused on the U.S. economy, it soon become apparent that these methods had the potential to be applied on what we now label a "global scale." Indeed, European research based on their methods gained momentum after WW-II, while the same happened in Latin America after inflation stabilized in the region by the second half of the 1990's.

The National Bureau of Economic Research (NBER) was founded in 1920 and started the work of dating the U.S. business cycles very early in the 20th Century. They are responsible for the development of methods detection the turning points in the level of an economic series (or in its logs) – classical business-cycle analysis – and for the detection of turning point on an isolated cyclical component (a detrended series) – growth-cycle analysis.

The NBER Business-Cycle Dating Committee is responsible for the U.S. business cycles dating since 1978. The most educated estimate of U.S. turning points is embodied in the binary variable announced by the NBER Business Cycle Dating Committee. The NBER Dating Committee summarizes its deliberations as:

> "The NBER does not define a recession in terms of two consecutive quarters of decline in real GNP. Rather, a recession is a recurring period of decline in total output, income, employment, and trade, usually lasting from six months to a year, and marked by widespread contractions in many sectors of the economy."
> (Quoted from http://www.nber.org/cycles.html)

The problem with the NBER committee deliberations is its lag – usually six months to one year after a turning point has occurred. This makes it of little practical use for instant or direct decision-making purposes. The final decision is a consensus between different visions of the experts present in the Dating Committee meeting (a total of 7 experts on business-cycle dating). These deliberations can be viewed as a result of a survey involving a group of very educated business-cycle researchers. It is exactly this character that makes it an interesting variable for the purposes of CIRET.

The first constructed coincident index of U.S. economic activity was implemented by the Census Bureau, a task that was later transferred to The Conference Board (TCB) – a non-profit private entity whose main purpose is to do research on this field. Since 1995, by order of the Department of Commerce of the U.S., TCB established a series of leading, coincident, and lagging indicators of economic activity. The coincident indicator is an average of the four coincident series – production, income, sales and employment. TCB uses a simple average of the standardized differenced (logged) series, which is a way of treating equally the fluctuations of all four series in computing the index. TCB approach is somewhat heuristic, since it requires no estimation of a formal econometric model. Despite that, it works surprisingly well in practice; see the comparison in Issler and Vahid (2006) using the TCB index and alternative econometric-based indices in trying to replicate the NBER dating decisions.

As an alternative to heuristic methods such as TCB's, several authors have proposed methods of building indices supported by sophisticated econometric and statistic techniques. Stock and Watson (1998a, 1998b, 1998c, 1989, 1993a) were the first to apply the tools of modern time-series econometrics to build an approach able to construct leading and coincident indices; to detect turning points of economic activity; and to predict the probability of a recession. Their models formalize the idea that the reference cycle is best measured by looking at co-movements across several aggregate time series, making their experimental index an estimate of the value of a single unobserved variable – "the state of the economy". The observable variables used in estimating the state of the economy are the usual coincident series: industrial production, income, sales and employment, which are forecast employing additional leading series.

An important empirical drawback in Stock and Watson's approach was its failure to detect the U.S. recession in 1990-1991. Many papers tried to improve on Stock and Watson's method, while keeping the formal building block of a structural econometric model. We review here just a few. Forni et al. (2000) proposed an alternative approach to Stock and Watson's which is very close to the latter in spirit. In its

more recent versions these authors build a dynamic common-factor model instead of a static one, i.e., based on current and lagged coincident series, not just current coincident series.

Chauvet (1998) improved on Stock and Watson's model with the inclusion of regime switching as proposed by Hamilton (1989). The idea is to capture asymmetries between expansions and contractions of the economic activity. It relies in the fact that contractions are more abrupt and shorter than expansions. Mariano and Murasawa (2003) extended Stock and Watson model in order to allow the use of mixed-frequency series, where GDP (quarterly measured) plays a central role. The coincident index is now the common factor of all four coincident series and also to interpolated monthly GDP, a sub-product of the analysis.

Finally, Issler and Vahid (2006) have a structural model for the NBER decisions, where the unobserved "state of the economy" is a function only of the cyclical behavior of the coincident series. They used canonical correlations analysis to filter out the noisy information contained in the usual four coincident series, building a composite coincident index that is matched to fit the information of the NBER decisions. Weights are estimated via an instrumental-variable Probit regression, which is then used to construct optimal coincident and leading indices (optimal 1-step ahead forecasts).

2.2 The Methodology of TCB

The ideas behind TCB's method are twofold: simplicity and robustness. Simplicity is used because they weight information in coincident and leading indices with equal weights, once one controls for the fact that different signals carry different information depending on their variance. One simple way to treat every series equally in this context is to standardize them, treating equally the standardized series. Robustness comes into play here, since standardizing is a way of robustly treating different realizations of the same random variable.

The coincident series is an equally-weighted linear combination of four coincident series (income (I_t), output (Y_t), employment (N_t), and sales (S_t)) once we control for the fact that the growth rate of these series have different variances. Hence, the coincident indicator uses weights constructed as:

$$\Delta \ln (CI_t) = \frac{1}{4}\left[\frac{\Delta \ln (I_t)}{\sigma_{\Delta \ln(I)}} + \frac{\Delta \ln (Y_t)}{\sigma_{\Delta \ln(Y)}} + \frac{\Delta \ln (N_t)}{\sigma_{\Delta \ln(N)}} + \frac{\Delta \ln (S_t)}{\sigma_{\Delta \ln(S)}}\right], \qquad (1)$$

where $\sigma_{\Delta \ln(I)}$, $\sigma_{\Delta \ln(Y)}$, $\sigma_{\Delta \ln(N)}$, and $\sigma_{\Delta \ln(S)}$ are respectively the standard deviations of income, output, employment, and sales growth. It is straightforward to construct the level series $\ln (CI_t)$ or CI_t once we posses $\Delta \ln (CI_t)$.

The leading series are usually chosen because they have *turning points* that happen *before* those of the level series $\ln(CI_t)$ or CI_t. To determine that, we first need a definition of "turning points" and of "before." In this literature, turning points are usually determined using an accepted algorithm for turning points or local minima and maxima of a time series – the Bry-Boschan algorithm, Bry and Boschan (1971). With turning points of the target variable and of the potential leading series in hand, all we have to determine is whether those of the potential leading series precede those of the target series, something a simple average of peaks and throughs precedence can determine. Leading series are those that downturn or upturn prior to the target series, on average. Once we determine the candidates of leading series, all we have to do is to combine them. Again, the TCB's methodology uses simplicity and robustness: all leading series are combined using a procedure similar to (1).

2.3 The Brazilian Experience

Contador (1977) was the first author to develop Brazilian coincident and leading indices of economic activity. He employs a myriad of methods, although has an intensive use of principal-component analysis. Alternatively, Spacov (2000) and Issler and Spacov (2000) use canonical correlation analysis to the same end, where the latter method solves the usual problem of "scale indeterminacy" found in principal-component analysis.

Chauvet (2001) uses principal-component analysis. To generate a monthly coincident indicator and an estimate of the probability of a recession in Brazil. Chauvet (2002) models the innovation in trend of Brazilian GDP as a two-state Markov Chain characterizing a recession or an expansion. In these two papers, she offers a chronology of Brazilian recessions. Picchetti and Toledo (2002) only take industrial production into account to propose a common-factor model for Brazilian (industrial) production. The unobserved component is estimated using the kalman filter, along the lines of Stock and Watson and Forni et al. (2000).

More recently, Duarte, Issler and Spacov (2004) evaluated three alternative coincident-index methods of economic activity for Brazil: TCB's index, whose instantaneous growth rate is a equally weighted combination of the standardized growth rate of the four coincident series (output, income, employment, and sales); Spacov's (2000) index, whose instantaneous growth rate is a weighted combination of the growth rate of the four coincident series, where canonical correlations are used to form weights; Issler and Vahid's (2006) index, whose instantaneous growth rate is a weighted combination of the growth rate of the four coincident series, where IV-Probit-regression coefficients are used to compute coincident-series weights. Using quadratic loss, the

dating of these three indices was compared with that of a monthly proxy of Brazilian GDP. The results suggest that the Brazilian coincident index should follow the methodology put forth by the TCB. Finally, based on this result, these authors propose a chronology of recessions for the Brazilian economy in the recent past.

A common problem in Brazilian statistical data is the constant revisions they are subjected to. In most instances, these revisions did not prevent the construction of a chained series. However, in 2002, the new redesign of *Pesquisa Mensal do Emprego (Monthly Employment Survey)* lead to a virtual discontinuity in the employment and income (labor income) series. Since these were two completely different survey designs, chaining the previous series with the new ones was not an option. This implied a halt in business-cycle research in 2002, unless we could back-cast the current series yielding a long enough time-series span for the study of business cycles. Indeed, this is exactly what we discuss next. In some sense, the current paper is an attempt to restart Brazilian business-cycle research post 2002, where newly reconstructed series are used to re-evaluate previous findings.

3 Back-Casting Using the Kalman Filter

In this section, we give a brief review of the Kalman filter model applied to back-cast two of our coincident series – employment and income. A detailed description of this technique can be found in Harvey (1989) or in Hamilton (1994).

Consider a vector of $n \times 1$ observables in period t – y_t, a $r \times 1$ vector of latent variables (non-observables) in period t – ξ_t, and a $k \times 1$ vector of predetermined variables in period t – x_t. A state-space representation is a way of summarizing the relationships between these 3 sets of variables, where the dynamic nature of the system is taken into account. In most applications, the state-space representation is linear, which leads naturally to the conditional log-likelihood of the system under Gaussian innovations and into a way of estimating the latent variables in the system. The latter is usually the ultimate goal of constructing such models.

The state-space representation considered here has a *state equation* and a *measurement equation*, respectively as follows:

$$\xi_{t+1} = \mathbf{F}\xi_t + v_{t+1} \tag{2}$$
$$y_t = \mathbf{A}'x_t + \mathbf{H}'\xi_t + w_t, \tag{3}$$

where $\mathbf{F}$, $\mathbf{A}'$, and $\mathbf{H}'$ are fixed coefficient matrices in this simplified setup, but could be time-varying in more elaborate applications. Indeed, we will make $\mathbf{H}'$ a time-varying matrix in back-casting employment and income for Brazil.

The state equation (2) describes the dynamics of the *state vector* (ξ_t) containing the latent variables we want to estimate. The observation equation (3) links the vector containing the observables y_t to the vector containing the pre-determined variables and the latent variables in the system.

The disturbances v_t and w_t are assumed to be orthogonal at all leads. Moreover, these error terms have a multivariate Normal distribution as follows:

$$\begin{pmatrix} v_t \\ w_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \begin{pmatrix} \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{pmatrix}\right), \tag{4}$$

which makes (2) and (3) to be a Gaussian conditional (linear) system in which estimation and forecasting can be based upon. The statement that x_t is predetermined (or "exogenous") means that x_t provides no information on v_{t+s} and w_{t+s}, $s \geq 0$, beyond that contained in $y_{t-1}, y_{t-2}, \cdots, y_1$. The coefficients matrices $\mathbf{F}$, $\mathbf{A}'$, and $\mathbf{H}'$, and the two variance-covariance matrices $\mathbf{Q}$ and $\mathbf{R}$ can be estimated by maximizing the conditional log-likelihood function of the system, given initial conditions on $\xi_{1|0}$ and on its variance-covariance matrix, labelled $\mathbf{P}_{1|0}$.

We are interested in the values of the unobserved state variable – ξ_t. We can forecast them based on the full set of data, which is called the smoothed estimate of ξ_t, or, we can forecast ξ_t using only data up to period $t-1$, which is called the filtered estimate. Both are presented, respectively, below:

$$\xi_{t|T} = \mathbb{E}\left(\xi_t \,|y_1, x_1, \cdots, y_T, x_T\right), \tag{5}$$

$$\xi_{t|t-1} = \mathbb{E}\left(\xi_t \,|y_1, x_1, \cdots, y_{t-1}, x_{t-1}\right). \tag{6}$$

Our starting point in using the kalman filter to back-cast the employment and income is the paper by Mönch and Uhlig (2005), where they used the filter to interpolate GDP from quarterly to monthly frequency. They assume that unobserved monthly GDP (labelled as y_t^+ here) follows an $AR(p)$ process explained by the exogenous regressors x_t and an $AR(1)$ error term:

$$\begin{aligned} \left(1 - \phi_1 L - \cdots - \phi_p L^p\right) y_t^+ &= x_t \beta + u_t \\ u_t &= \rho u_{t-1} + \varepsilon_t. \end{aligned}$$

They set the observed quarterly GDP (labelled as y_t here), simply as:

$$y_t = \sum_{i=0}^{2} y_{t-i}^+, \qquad t = 3, 6, 9, 12, \ldots \tag{7}$$

$$y_t = 0, \qquad \text{otherwise.} \tag{8}$$

Hence, quarterly GDP, which we can only observe on months $t = 3, 6, 9, 12$, is the sum of the corresponding monthly GDPs in that quarter. Otherwise, it is just zero. Notice that setting $y_t = 0$ for the months we do not observe GDP is a clever way of making quarterly GDP observable at the monthly frequency. The aggregation of monthly GDP can also be made averaging the y_t^+'s, i.e., as $y_t = \frac{1}{3}\sum_{i=0}^{2} y_{t-i}^+$.

If we assume that the polynomial $\left(1 - \phi_1 L - \cdots - \phi_p L^p\right)$ is of order one, i.e., $p = 1$, with coefficient ϕ, the state-space form of Mönch and Uhlig's problem is the following:

$$\xi_t = \begin{pmatrix} y_t^+ \\ y_{t-1}^+ \\ y_{t-2}^+ \\ u_t \end{pmatrix} = \begin{pmatrix} \phi & 0 & 0 & \rho \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \rho \end{pmatrix} \begin{pmatrix} y_{t-1}^+ \\ y_{t-2}^+ \\ y_{t-3}^+ \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} x_t\beta \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ 0 \\ 0 \\ \varepsilon_t \end{pmatrix} \quad (9)$$

$$y_t = \mathbf{H}_t'\xi_t, \quad (10)$$

where (9) and (10) are respectively the state and the observation equations and the matrix $\mathbf{H}_t'$ is time-varying, with the following format:

$$\mathbf{H}_t' = \begin{cases} \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}, & t = 3, 6, 9, 12, \ldots \\ \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, & \text{otherwise.} \end{cases} \quad (11)$$

One interesting feature of the approach in Mönch and Uhlig is that it encompasses several data interpolation models that are state-space based, summarized in Table 1 below:

Table 1 – Resulting Model as a Function of ϕ and ρ in (9)

Model	ϕ	ρ
Static model in levels with IID residuals	0	0
Static model in levels with AR(1) residuals (Chow and Lin, 1971)	0	free
Static model in 1st differences with IID residuals (Fernandez, 1971)	0	1
Dynamic model in levels with IID residuals (Mitchell et al., 2005)	free	0
Dynamic model in 1st differences with IID residuals	free	1
Dynamic model in levels with AR(1) residuals	free	free

To assess the quality of interpolation, Mönch and Uhlig follow Bernanke, Gertler, and Watson (1997) by using two R^2 measures of fit. Denoting by $\widehat{y_{t|T}^+}$ the smoothed

estimate of monthly GDP, and by $\widehat{u_{t|T}}$ the same estimate of the error term u_t, they consider:

$$
\begin{aligned}
R^2_{\text{level}} &= \frac{\text{VAR}\left(\widehat{y^+_{t|T}}\right)}{\text{VAR}\left(\widehat{y^+_{t|T}}\right) + \text{VAR}\left(\widehat{u_{t|T}}\right)}, \text{ and,} \\
R^2_{\text{diff}} &= \frac{\text{VAR}\left(\widehat{\Delta y^+_{t|T}}\right)}{\text{VAR}\left(\widehat{\Delta y^+_{t|T}}\right) + \text{VAR}\left(\widehat{\Delta u_{t|T}}\right)}.
\end{aligned}
$$

They claim it is more informative to report the R^2 in first differences since the same statistic in levels will always be close to unity.

We now adapt the state-space representation in (9) and (10) to the problem of back-casting a series which we observe part of its realizations but not all. In some sense, this is very close to the problem worked out in Mönch and Uhlig, since they only observe quarterly GDP for some but not all months of the year. Their solution was to set to zero the missing observations. This seems like a clever and natural solution. It shuts down the missing values of the observed quarterly series in monthly frequency that are used in forecasting the state variable. This same principle is applied here to construct back-cast estimates of employment and income for the Brazilian economy.

Suppose we posses a total of $t = 1, 2, \cdots, T^*, \cdots, T$, observations on x_t. However, for series y^+_t, we only posses data from $t = T^* + 1, \cdots, T$, with missing values from $t = 1, 2, \cdots, T^*$. This is exactly our setup for income and employment in this paper. If we set the order of the polynomial $\left(1 - \phi_1 L - \cdots - \phi_p L^p\right)$ to unity, i.e., $p = 1$, with coefficient ϕ, recalling that now we need not impose the time-aggregation restriction in (11), the state-space form of our problem collapses to the following:

$$
\begin{aligned}
\xi_t &= \begin{pmatrix} y^+_t \\ u_t \end{pmatrix} = \begin{pmatrix} \phi & \rho \\ 0 & \rho \end{pmatrix} \begin{pmatrix} y^+_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} x_t \beta \\ 0 \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t \end{pmatrix} \\
y_t &= \mathbf{H}'_t \xi_t, \qquad (12)
\end{aligned}
$$

where (**??**) and (12) are respectively the state and the observation equations and the matrix $\mathbf{H}'_t$ is time-varying, with the following format:

$$
\mathbf{H}'_t = \begin{cases} \begin{bmatrix} 1 & 0 \end{bmatrix}, \ t = T^* + 1, \cdots, T \\ \\ \begin{bmatrix} 0 & 0 \end{bmatrix}, \qquad \text{otherwise.} \end{cases} \qquad (13)
$$

The key to the problem lies in the choice for $\mathbf{H}_t'$ in (13). Here, we make the latent variable y_t^+ identical to y_t for the periods in which the latter is observed, with no error term. This has two consequences. First, the algorithm will forecast y_t^+ to be identical to y_t for $t = T^* + 1, \cdots, T$. Second, it will use the available data on employment (income) to estimate a model and will use this model to forecast the latent variable in the periods in which it is not observable, i.e., from $t = 1, 2, \cdots, T^*$. Under correct specification, this model can produce the optimal forecasts of the latent variable consistent with all available future information. That will be simply given by the smoothed forecast of y_t^+, i.e., by $\widehat{y_{t|T}^+}$.

4 Empirical Results

4.1 Data

An important part of this paper is the choice of the variables to be included in the coincident indicator. We follow the recent Brazilian experience: Duarte, Issler and Spacov (2004) and Spacov (2001). For output, labelled Y_t, we use industrial production, computed by IBGE, and available from 1980:1. There is not a long-span sales series in Brazil, we therefore follow Duarte, Issler and Spacov and use total Brazilian production of corrugated paper as a proxy for sales, labelled S_t, which is computed by ABPO. Employment, labelled N_t, is given by the total number of persons – 10 years old or older – that have a job. It is extracted from the *Monthly Employment Survey* computed by IBGE. Income is proxied by the labor income series, labelled I_t, extracted from this same *Survey*.

The last two series – employment and income – are only available from 2003 on, because of a drastic redesign of the *Monthly Employment Survey*. Here, we back-cast these series using a state-space representation estimated using the kalman filter.

4.2 The Coincident Series

As stressed above, one of the original contributions of this paper is to back-cast two of the coincident series for the Brazilian economy – income and employment. We used the techniques described in the previous section to back-cast them. In the current *Monthly Employment Survey*, income is available from 2002:2 on, while employment is available from 2002:3 on.

Back-casting was conducted in two steps. First we select the co-variate series, which could potentially explain the variations of income or employment. These co-variates are then used in the state-space regression, which is estimated using the

framework described above – based on the algorithm by Mönch and Uhlig (2005). Our setup allows for several different dynamic models to be estimated, all described in Table 1, depending on different values for the parameters ϕ and ρ.

We tested seven series as auxiliary regressors in the back-casting procedure, all available for the period 1980:1 to 2007:11. They are: industrial production, output in the process industry, corrugated paper production, car production, steel production, cement production, energy production, and the monthly real GDP series estimated by Issler and Notini (2008). The dependent variables and all co-variates entered in levels in the state space representation, which is estimated in all the six different versions described in Table 1. In addition to the co-variates listed above, our models also include eleven seasonal dummies. In Table 2, we present the R^2_{diff} measure of fit for each model described in Table1.

Table 2 – Employment and Income Resulting $R^2_{\text{diff}}$for each Model

Model	Employment	Income
Static model in levels with IID residuals	0.4979	0.1134
Static model in levels with $AR(1)$ residuals	0.4729	0.0425
Static model in 1st differences with IID residuals	0.0072	0.0000
Dynamic model in levels with IID residuals	0.0597	0.0827
Dynamic model in 1st differences with IID residuals	0.0000	0.0000
Dynamic model in levels with $AR(1)$ residuals	0.0000	0.0048

Our final choice of auxiliary variables and models was as follows. For employment (in logarithms) we choose only the monthly GDP series and energy production (in logarithms) as co-variates. For income (in logarithms), we selected only the paper production series and cement production (in logarithms) as auxiliary variables. In both cases, the model with the highest R^2_{level} and R^2_{diff} was the static model with i.i.d. errors, where set the parameters ϕ and ρ equal to zero.

All four coincident series used in this paper are plotted below, which includes the results of the back-casted series. All four series – Production (Y_t), Sales (S_t), Income (I_t) and Employment (N_t) – were seasonally adjusted using the X-12 procedure. For income and employment, the shaded areas in the graphs below depict the actual sample in which we observe them.

Figure 1: Industrial Production - In log and Seasonally Adjusted

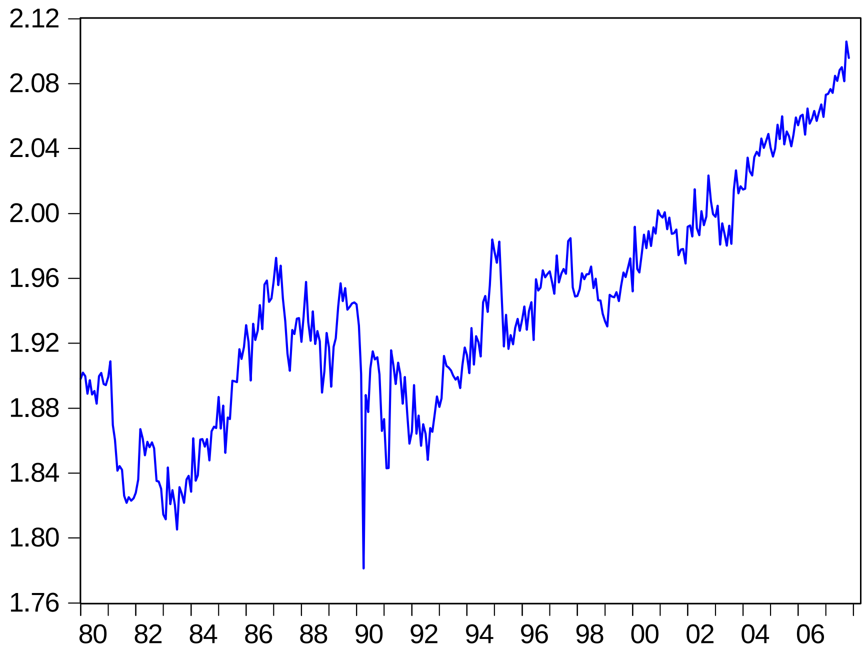

2.12
2.08
2.04
2.00
1.96
1.92
1.88
1.84
1.80
1.76
80
82
84
86
88
90
92
94
96
98
00
02
04
06

Figure 2: Sales - In log and Seasonally Adjusted

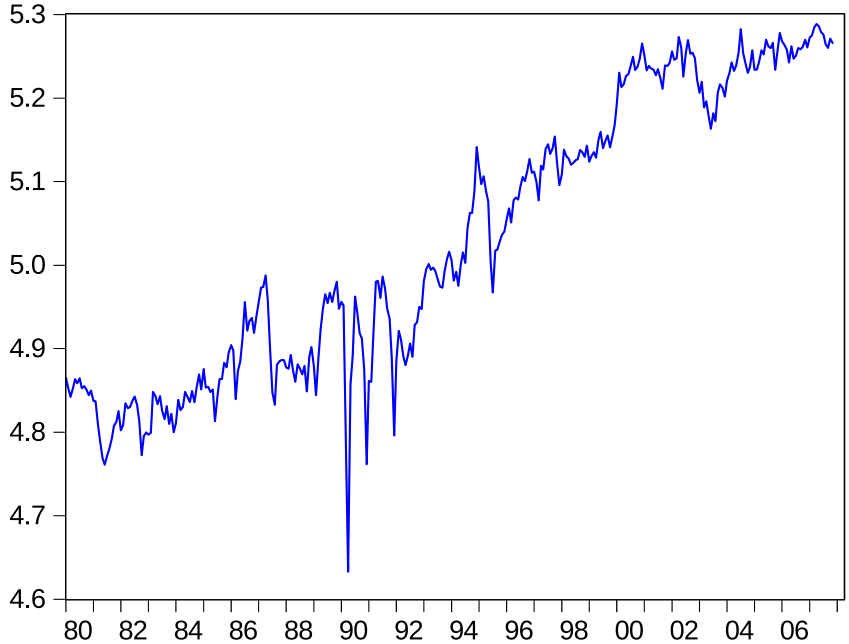
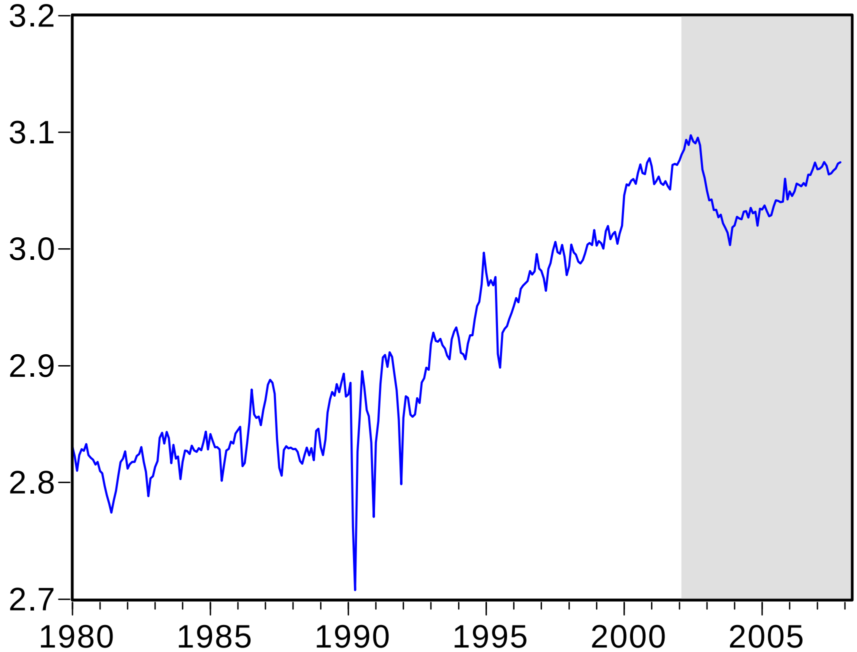
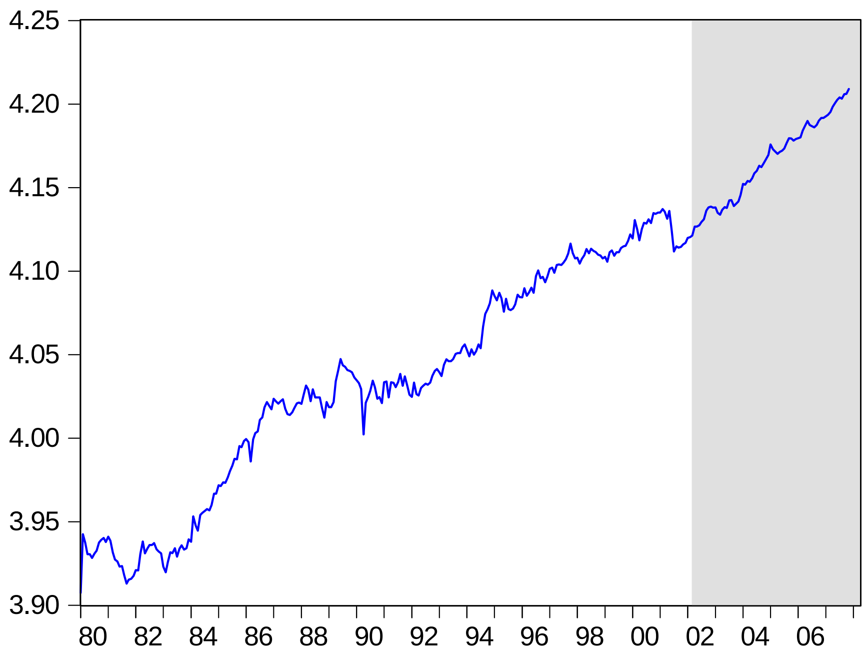


Figure 3: Income - In log and Seasonally Adjusted



Shaded areas depicts the actual sample

Figure 4: Employment - In log and Seasonally Adjusted



Shaded areas depicts the actual sample

All four coincident series were tested for unit roots. We used three different tests. On a preliminary basis, we used the Augmented Dickey-Fuller (ADF) test. Initial results were later examined in light of the results of the Phillips and Perron (1988) test and the stationarity test proposed by Kwiatkowski et al. (1992). All four coincident series showed signs of unit roots in testing and therefore were transformed into first differences (logs) prior to combination into a composite index.

Table 3: Coincident Series - Unit Root Tests

Variable	ADF		Kwiatkowski et. al	Phillips and Perron	
	t-statistic	p-value	LM-statistic	t-statistic	p-value
Employment	-0.62	0.86	2.13*	-0.55	0.88
Ind. Production	-0.49	0.89	1.78*	-0.87	0.80
Sales	-0.34	0.92	2.12*	-0.76	0.82
Income	-0.43	0.90	2.10*	-0.75	0.83

Notes:(i) ADF and Phillips and Perron H_0:series has a unit root; Kwiatkowski H_0:series is stationary.(ii)the asterisk (*) indicates that we reject the null hypothesis at 5%.

4.3 TCB's Coincident Index – $TCB - CI_t$

Using (1), we constructed a coincident index consistent with TCB's method, labelled $TCB - CI_t$, and plotted below. Next, we compare the turning-point dating of this index with that of two other indices: a monthly estimate of Brazilian GDP computed by Issler and Notini (2008) and the composite index previously proposed by Duarte, Issler and Spacov (2004), available until 2002:11. The latter also uses TCB's technique.

The turning points of these three composite indices were then compared using the Bry and Boschan (1971) and the Mönch and Uhlig (2005) dating algorithm, the latter being a slightly modified version of the former. Results in Table 4 show that the current dating using TCB's method yields results closer to the dating in Duarte, Issler and Spacov than to the dating of Brazilian monthly GDP. The most striking differences appear in the dating of the 1991 recession. The dating of Duarte, Issler and Spacov and of GDP encompass two recession episodes into one as compared to the dating of $TCB - CI_t$. It is also noteworthy that GDP misses the two last recessions as dated by $TCB - CI_t$ and by Duarte, Issler and Spacov's[1].

[1] This behavior – GDP missing the last two recessions – vanishes if one uses the modified Bry-Boschan dating method proposed in Mönch and Uhlig (2005) to date all three indices.

Figure 5: Coincident Index – Shaded Bry-Boschan Turning Points

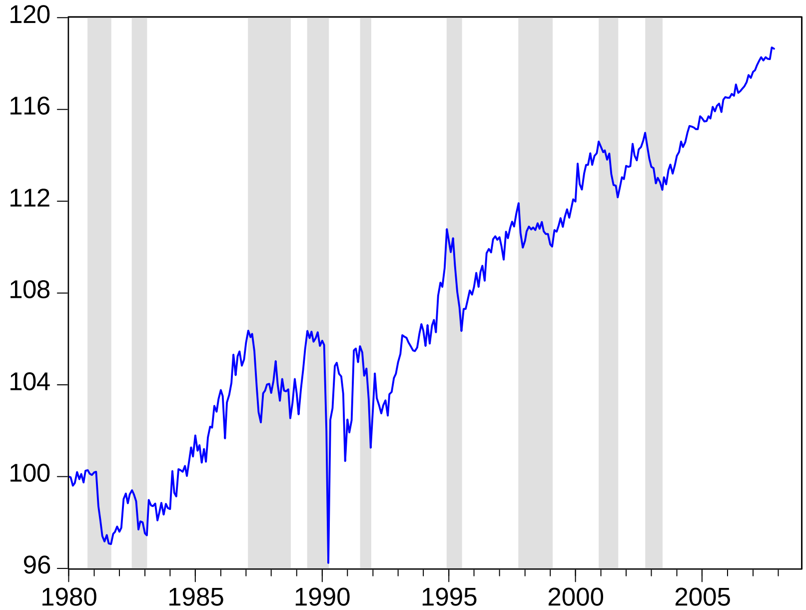


Table 4 – Turning-Point Comparisons Using Bry-Boschan Dating					
Peak Dates			Through Dates		
$TCB - CI_t$	Duarte et al.	Brazilian GDP	$TCB - CI_t$	Duarte et al.	Brazilian GDP
1980:10	NA		1981:09	NA	1981:11
1982:07		1982:6	1983:02	1983:10	1983:02
1987:02	1987:04	1988:3	1988:10	1989:02	1988:10
1989:06	1989:08	1989:6	1990:04		
1991:07			1991:12	1991:03	1991:12
1994:12	1995:03	1994:12	1995:07	1995:09	1995:07
1997:10	1997:10	1997:10	1999:02	1999:02	1999:01
2000:12			2001:09		
2002:10	2002:4		2003:06		

Notes: The analysis in Duarte et al. (2004) starts in 1982:05, therefore could not have dated the recession of 1980. Brazilian GDP dating uses the monthly series constructed by Issler and Notini (2008).

Given the results in Table 4, we can compute how frequent Brazilian recessions are. From 1980-2007:11 we have a total of 9 recessions. On average, we observe in this period one recession at approximately every 3 years and 3 months, which is substantially more frequent than the U.S. historical average of one recession about every 5 years. Recessions in Brazil also last longer than U.S. recessions: while ours last about 12 months, on average, U.S. recessions last typically from 6 months to one year, on average (in our sample period here – 1980:1 to 2007:11 – U.S. recessions lasted, on average, 9 months). Indeed, Duarte, Issler and Spacov make the point that this behavior may be due to hardships that the Brazilian economy has endured in the post-1980 era, where GDP growth declined form about 7% a year in real terms prior to 1980 to about 2.2% a year after 1980.

Table 5 below lists Brazilian recessions from 1980:1 to 2007:11 when the dating of turning points is made using the modified Bry and Boschan technique proposed by Mönch and Uhlig (2005). The latter takes into account asymmetry differences in peak and through dating, which may be at work to explain the difference in dating between the Bry and Boschan and the Mönch and Uhlig method. Here, the dating of peaks in $TCB - CI_t$ is identical to that in Brazilian GDP, whereas the dating of throughs is almost identical.

Table 5 – Turning-Point Comparisons Using Mönch and Uhlig Dating

Peak Dates			Through Dates		
$TCB - CI_t$	Duarte et al.	Brazilian GDP	$TCB - CI_t$	Duarte et al.	Brazilian GDP
1980:10	NA	1980:10	1981:09	NA	1981:11
1982:07		1982:07	1983:02		1983:02
1987:02	1987:04	1987:02	1988:10	1989:02	1988:10
1989:06	1989:08	1989:06	1990:04		1990:04
1991:07		1991:07	1991:12	1991:12	1991:12
1994:12	1994:12	1994:12	1995:07	1995:9	1995:07
1997:10	1997:10	1997:10	1999:02	1999:02	1999:01
2000:12	2000:12	2000:12	2001:09	2001:9	2001:09
2002:10		2002:10	2003:06		2003:03

Notes: The analysis in Duarte et al. (2004) starts in 1982:05, therefore could not have dated the recession of 1980. Brazilian GDP dating uses the monthly series constructed by Issler and Notini (2008).

Taking into account the overall results of the dating exercise shows that the back-casting of income and employment proposed in this paper has the following properties: (i) generates sensible results for those series in the back-cast period; (ii)

generates a sensible composite coincident index of economic activity. The latter is able to approximate reasonably well the turning points of monthly GDP and those of the TCB index using the retired income and employment series in Duarte, Issler and Spacov (2004). Of course, there are more similarities in turning-point dating when dating uses the technique proposed by Mönch and Uhlig.

We believe that the strategy we chose in this paper to construct a long span time-series for the Brazilian coincident indicator was the best possible. An alternative would be to chain the current employment and income series with their respective series retired by IBGE. Since the redesign of the *Monthly Employment Survey* was drastic, this procedure would chain completely different series. Another alternative would be to only use industrial production and sales to construct the composite index up to 2002:2, and then use the four usual series from 2002:3 onwards. This procedure would probably induce structural changes in mean and variance of the composite index after 2002:3.

4.4 The Composite Leading Indicator

Leading indicators are widely used in predicting turning points of business cycles in many countries. The selection of a leading indicator index involves three steps: (i) select an appropriate indicator as a measure of economic activity to be targeted, also called a reference series; (ii) select appropriate economic and financial indicators as predictors of the turning points of the reference series; (iii) combine the selected leading series in order to construct a composite leading index.

The first step was accomplished in the previous section, where we obtained a composite index of economic activity for the Brazilian economy after we back-cast the employment and income series. The next step is to select appropriate leading indicators as predictors of turning points. We search for series that satisfy the following conditions: (a) to be observable at a monthly frequency for the period 1980-2007:11; (b) timely data releases, and having small revisions regarding final data figures.

Recent research has shown that business-tendency survey data are particularly suitable for business cycle monitoring and forecasting. Business tendency surveys are conducted in all OECD member countries and have proved to be a cost-effective means of generating timely information on short-term economic fluctuations. In Brazil, the Brazilian Institute of Economics (IBRE) of Getulio Vargas Foundation (FGV) is a pioneering institution that computes surveys of economic activity. These include a survey of consumer expectations and another on business expectations on industrial production and related series: inflow of new orders, level of book orders, stocks of finished goods, etc.

From FGV's survey series and other Brazilian databases (IBGE, IPEADATA, and the Central Bank's), we selected 44 series that are candidates of being leading series of the coincident index. Our choice was guided by the international experience (Stock and Watson (1989, 1993)) and also by local experience (Duarte, Issler and Spacov (2004)).

A main issue regarding FGV's survey series is that they were computed on a quarterly frequency up to September 2005. From then on, surveys were then conducted on a monthly basis. Therefore, there is the need to interpolate the data on quarterly frequency to have an homogeneous series on a monthly basis. Our interpolation method was, again, Mönch and Uhlig's (2005).

All leading nominal series were deflated to reflect their purchasing power as of March, 2008. The deflator used was the Brazilian General Price Index "IGP-DI" – calculated by FGV. All series denominated in foreign currency were converted into Brazilian Reais at the prevailing exchange rate and subsequently deflated. All series were logged, unless logs could not be taken of the original series (potentially zero or negative figures). All series were also seasonally adjusted prior to the analysis using the X-12 procedure, whenever a seasonal pattern in them was detected.

With the exception of the survey-tendency series, all leading series were tested for unit roots. Survey series are bounded series, by construction. Therefore, they cannot posses a unit root, which leads to unbounded series in theory. To test for unit roots we used the Augmented Dickey-Fuller (ADF) test, the Phillips and Perron (1988) test, and the stationarity test proposed by Kwiatkowski et al. (1992). All series with a unit root were transformed into first differences (logs) prior to combination into a composite index[2].

In order to measure the quality with which a leading series correctly anticipates the "state of the economy" implied by the coincident series (recession or expansion), we use a criterion originally proposed by Diebold and Rudebusch (1999), and later employed by Zhang and Zhuang (2002) and Gallardo and Pedersen (1997). The Quadratic Probability Score, labelled as $QPS\left(h\right)$, is given by:

$$QPS(h) = \frac{\sum_{t=1}^{T} (P_t - R_t)^2}{T} \tag{14}$$

where P_t denotes the predicted state outcomes from a candidate leading indicator and R_t denotes the observed realizations of the reference series. Both are equal to one

[2] ADF unit-root test results are presented in Table A3 in the Appendix. Other test results are available upon request.

for a turning point and zero otherwise; T is the total number of sample observations, while h is the horizon in which the leading series potentially predicts the reference series. By construction, the value of $QPS(h)$ ranges between zero and one, with zero indicating a perfect fit for the the "state of the economy" of the reference series.

Next, we describe the basic criteria used to select the leading series that will compose our index. First, for each series, we calculate the optimum (minimum) $QPS(h)$ value, denoted by $QPS\,(h^{*})$, where h^{*} is the resulting optimum lag. To be a leading series candidate, the series must have $h^{*} > 0$ in $QPS\,(h^{*})$. This means that the series is leading, not lagging or coincident to reference series. Second, we apply Granger (1969) causality tests in order to examine whether the leading series precedes the reference series. We expect that a leading series Granger-causes the reference series but is not Granger-caused by it.

In the Appendix, Table A4 shows the $QPS\,(h^{*})$, h^{*}, and Granger-causality test results. The majority of the potential leading series do not Granger cause the coincident series. The exceptions are some FGV's survey series, in addition to SELIC – Central Bank's basic interest rate – and IBOVESPA – Brazilian Stock Market Index. From them, IBOVESPA shows promise, since its $QPS\,(h^{*}) = 24.5\%$, and $h^{*} = 5$. This means that, when we take the IBOVESPA index, with a lag of 5 months vis-à-vis period t, it correctly predicts 75.5% of the "state of the economy" as measured by the peak and though behavior of our our composite index. A slightly worse result in observed to the survey series on the production of real-estate inputs – $QPS\,(h^{*}) = 25.1\%$, and $h^{*} = 1$.

The $QPS\,(\cdot)$ statistic has only three series with values between 10 and 20% – intermediate-good production, consumer-good production, and inventories, and a few between 20 and 30%. The intersection of the two criteria above – "Granger causality" and "low $QPS\,(h^{*})$" – only has the IBOVESPA index and the production of real-estate inputs. Across all potential leading series the mean lag is 3, but the median and modal lag are 1. There are several interesting series which have $h^{*} > 1$ and a relatively low $QPS\,(h^{*})$: FGV's survey series on inventories ($QPS\,(h^{*}) = 17.6\%$), the IBOVESPA index ($QPS\,(h^{*}) = 24.5\%$), as well as a myriad of other FGV's survey series.

Looking at results in Table A4, there is no obvious way to select series to be in the composite index. We present next 10 *ad-hoc* criteria to select those series, the idea being that we want h^{*} to be high, $QPS\,(h^{*})$ to be low, and that a leading series Granger-causes the reference series but is not Granger-caused by it. We also investigate whether a series that has low h^{*}, with low $QPS\,(h^{*})$, would also have a relatively low $QPS\,(h)$ for higher values of h. The 10 criteria are listed below:

1. Select all series possessing $QPS(h)$ less than 0.4 and positive optimum lag;

2. Select all series possessing $QPS(h)$ less than 0.4;

3. Select all series that satisfied the Granger causality test criterion;

4. Select all series in the intersection between the first and third criterion;

5. Select all series in the intersection between the second and third criterion;

6. Select all Survey series that satisfied the Granger causality test criterion;

7. Select the five series in Table A4 that have the lowest $QPS(h)$ value;

8. Select the series for which h^* is between two and seven months and $QPS\,(h^*) < 0.3$;

9. Select the series for which h^* is between two and seven months;

10. Select survey series for which $QPS\,(h^*) < 0.3$.

Given these criteria, we computed 10 different composite leading indices of economic activity, labelled $LI_{i,t}$, $i = 1, 2, \cdots, 10$. We chose to combine leading series into the composite index using a counterpart of equation (1) – equal weights on standardized growth rates of the leading series[3].

Table 6, below, lists the values of QPS for each criterion listed above, computed for the optimum lag, i.e., $QPS\,(h^*)$.

Table 6 – Leading Indices: $QPS\,(h^*)$ computed using Mönch and Uhlig's Method			
Series	Description	$QPS\,(h^*)$	h^*
$LI_{1,t}$	$h^* > 0$ and $QPS < 0.4$	0.1910	1
$LI_{2,t}$	$QPS < 0.4$	0.1612	1
$LI_{3,t}$	Series that Granger cause the Coincident Index	0.2478	1
$LI_{4,t}$	Granger cause, $h^* > 0$ and $QPS < 0.4$	0.2358	3
$LI_{5,t}$	Granger cause and $QPS < 0.4$	0.2328	1
$LI_{6,t}$	Granger cause (FGV survey series only)	0.2657	1
$LI_{7,t}$	The five series which have the lowest QPS	0.1015	1
$LI_{8,t}$	h^* between 2 and 7 and $QPS \leq 0.3$	0.2507	4
$LI_{9,t}$	h^* between 2 and 7 (FGV survey series only)	0.2866	3
$LI_{10,t}$	$QPS \leq 0.3$ (FGV survey series only)	0.2507	3

[3]Tables A5 and A6 in Appendix compare the turning points data for each leading index and the turning points of the coincident index.

From the results in Table 6 $LI_{7,t}$ stands out as a candidate of composite index. The leading series in it have their $QPS\,(h^{*})$ between 11.04% and 23.28%. Despite that, the composite index has a $QPS\,(h^{*}) = 10.15\%$ – lower than the smallest $QPS\,(h^{*})$ of the series in it. The latter are all Industry Survey series: of the Consumer-Good Industry, Capital-Good, Real-Estate Input, Intermediary-Good and the Level of External Demand. The composite indices $LI_{2,t}$ and $LI_{1,t}$ also do well in terms of $QPS\,(h^{*})$ and can be considered as an alternative to $LI_{7,t}$.

Our next exercise is a dating exercise involving $TCB - CI_t$ and $LI_{i,t}$, $i = 1, 2, \cdots, 10$. We want to examine how well and how often these leading indices predict the turning points in $TCB - CI_t$. We are also interested in knowing whether they generate false predictions, i.e., predicting a non-existent peak or through in economic activity. We start with a 24-month window around period $\not t$, i.e., from $\not t - 12$ through $\not t + 12$, and consider turning points in $TCB - CI_t$ and in $LI_{i,t}$, $i = 1, 2, \cdots, 10$. From peak and through dates in $TCB - CI_t$ and $LI_{i,t}$, we are able to match peaks of $TCB - CI_t$ with peaks of $LI_{i,t}$, and throughs of $TCB - CI_t$ with throughs of $LI_{i,t}$. We can also compute the average lead in peak (or through) prediction for each episode, as well as to list false predictions of turning points.

Results of this exercise are presented in Tables 7 through 11 for $LI_{7,t}$, $LI_{2,t}$ and $LI_{1,t}$. The Appendix contains this exercise for the remaining leading indices.

Table 7 shows respectively the coincident index and $LI_{7,t}$ peaks and through dates. Peak prediction is much better done than through prediction: only one peak is lost and $LI_{7,t}$ anticipates the coincident-index peaks 2.5 months ahead, on average. For throughs, although none is lost, on three occasions through prediction of $LI_{7,t}$ occurs after the through itself, reflecting on an average lead of 0.33 months for through prediction.

Table 7 - Turning Points Comparisons					
Mönch and Uhlig (2005) Dates					
Peak Dates			Through Dates		
$TCB-CI_t$	$LI_{7,t}$	Lead	$TCB-CI_t$	$LI_{7,t}$	Lead
1980:10			1981:09	1981:09	0
1982:07	1982:03	4	1983:02	1983:06	-4
1987:02	1987:01	1	1988:10	1988:09	1
1989:06	1989:05	1	1990:04	1990:03	1
1991:07	1991:03	4	1991:12	1992:02	-2
1994:12	1994:11	1	1995:07	1995:06	1
1997:10	1997:03	7	1999:02	1999:01	1
2000:12	2000:11	1	2001:09	2001:09	0
2002:10	2002:09	1	2003:06	2003:07	-1

Table 8 performs the same analysis above for $LI_{1,t}$. The average lead for for peak prediction is again 2.5 months, while that for through prediction is 2.25 months. However, $LI_{1,t}$ predicts two extra peaks and three extra throughs than those observed on $TCB-CI_t$. This result is in contrast with that of $LI_{7,t}$, which predicted no extra peaks or throughs.

For $LI_{2,t}$ the results in Table 9 show an average lead for peak prediction of 1.88 months, with a lead of -0.75 months for through prediction, a very bad result for through prediction. Moreover, $LI_{2,t}$ predicts one extra peak and two extra throughs than those observed on $TCB-CI_t$. This result is in contrast with that of $LI_{7,t}$, which predicted no extra peaks or throughs.

Focusing on the overall results for turning-point prediction only, it is clear that $LI_{7,t}$ dominates either $LI_{1,t}$ or $LI_{2,t}$: all three missed one peak, but $LI_{7,t}$ predicted no extra peaks, while $LI_{2,t}$ predicted two extra peaks and $LI_{2,t}$ predicted one. Regarding throughs, all three composite indices did not miss any, while $LI_{7,t}$ predicted no extra throughs, which contrasts with the results for $LI_{1,t}$ and $LI_{2,t}$: three and two, respectively.

Table 8 - Turning-Point Comparisons					
Mönch and Uhlig Dates					
Peak Dates			Through Dates		
$TCB-CI_t$	$LI_{1,t}$	Lead	$TCB-CI_t$	$LI_{1,t}$	Lead
1980:10			1981:09	1981:04	5
1982:07	1982:02	5	1983:02	1982:09	5
	1984:07			1985:03	
1987:02	1986:09	5		1987:06	
1989:06	1989:05	1	1988:10		
1991:07	1991:06	1	1990:04	1990:03	1
1994:12	1994:11	1	1991:12	1991:11	1
1997:10	1997:09	1	1995:07	1995:06	1
2000:12	2000:07	5	1999:02	1998:09	5
2002:10	2002:09	1	2001:09	2001:09	0
	2004:06		2003:06	2003:06	0
				2005:1	

Table 9 - Turning-Point Comparisons					
Mönch and Uhlig Dates					
Peak Dates			Through Dates		
$TCB-CI_t$	$LI_{2,t}$	Lead	$TCB-CI_t$	$LI_{2,t}$	Lead
1980:10			1981:09	1981:08	1
1982:07	1982:02	5	1983:02	1983:06	-4
1987:02	1986:09	5		1987:06	
1989:06	1989:05	1	1988:10		
1991:07	1991:06	1	1990:04	1990:03	1
1994:12	1994:11	1	1991:12	1992:07	-7
1997:10	1997:09	1	1995:07	1995:06	1
2000:12	2000:12	0	1999:02	1998:12	2
2002:10	2002:09	1	2001:09	2001:09	0
	2004:08		2003:06	2003:06	0
				2005:01	

Tables 10 and 11 contain, respectively, peak and through dating statistics for all 10 composite leading indices. It becomes clear that the good $QPS(\cdot)$ statistic for $LI_{7,t}$ is a consequence of not predicting extra throughs and throughs and not missing

extra peaks and throughs vis-à-vis alternative indices.

All and all, considering the whole evidence in this section, we choose $LI_{7,t}$ to be our composite leading index of economic activity. Our choice is supported by a QPS value of 10.15%, meaning that this leading index provides wrong predictions of the state of the Brazilian economy only in 10.15% of the time.

There is a somewhat asymmetric behavior for $LI_{7,t}$ in terms of peak and through prediction: on average, while $LI_{7,t}$ predicts peaks with a two-and-a-half-month lead, it predicts throughs with a very small average lead of 0.37 months. Because of this behavior, we might want to consider either $LI_{1,t}$ as an alternative composite index for the purpose of through prediction only, since it leads $TCB-CI_t$ by 2.25 months.

Table 10 - Leading Indices: Peak Dating Comparisons

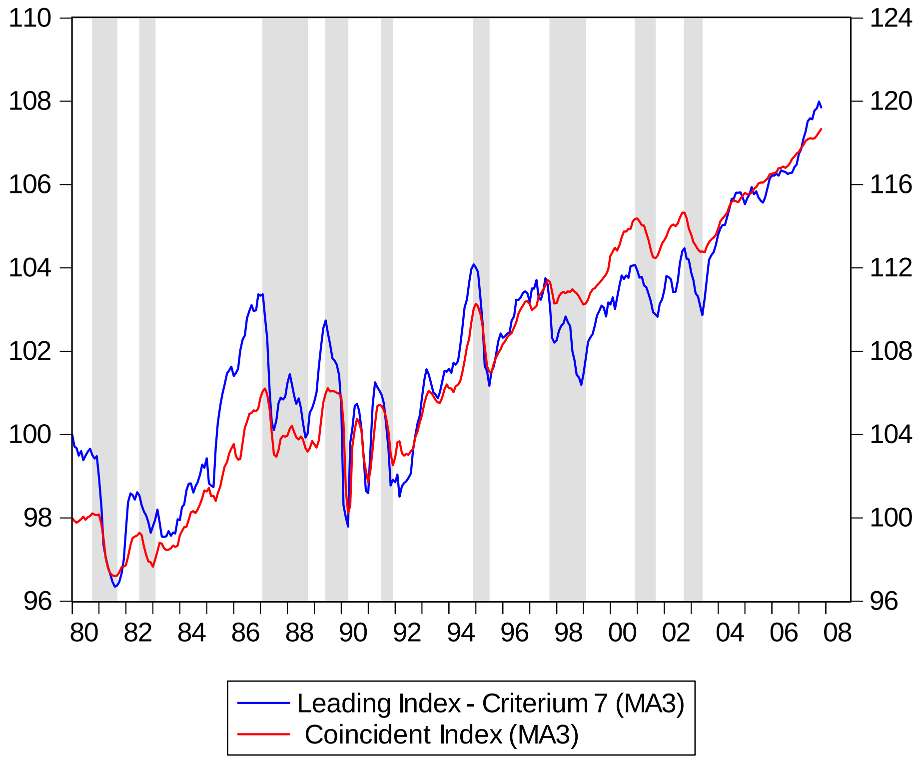
Mönch and Uhlig Dates

Index	# of Peaks	# Leading Peaks	# Missed Peaks	# Extra Peaks
$TCB-CI_t$	9	-	-	-
$LI_{1,t}$	10	8	1	2
$LI_{2,t}$	9	8	1	1
$LI_{3,t}$	7	6	3	1
$LI_{4,t}$	10	7	2	3
$LI_{5,t}$	8	6	3	2
$LI_{6,t}$	8	6	3	2
$LI_{7,t}$	8	8	1	0
$LI_{8,t}$	8	8	1	0
$LI_{9,t}$	10	7	2	3
$LI_{10,t}$	9	8	1	1

Table 11 - Leading Indices: Through Dating Comparisons				
Mönch and Uhlig Dates				
Index	# of Throughs	# Leading Throughs	# Missed Throughs	# Extra Throughs
$TCB - CI_t$	9	-	-	-
$LI_{1,t}$	11	8	1	3
$LI_{2,t}$	10	8	1	2
$LI_{3,t}$	8	7	2	1
$LI_{4,t}$	11	9	0	2
$LI_{5,t}$	9	7	2	2
$LI_{6,t}$	8	6	3	2
$LI_{7,t}$	9	9	0	0
$LI_{8,t}$	10	8	1	2
$LI_{9,t}$	11	8	1	3
$LI_{10,t}$	10	8	1	2

Finally, in the Figure below, we plot $TCB-CI_t$ and $LI_{7,t}$ smoothed by computing a three-month moving average. They have a straking similar behavior for the sample period covered in this paper. Of course, the original $LI_{7,t}$ leads the original $TCB-CI_t$ by 2.5 months for peaks and by 0.33 months for throughs.

Figure 6: Coincident and Leading Indexes



5 Conclusion

This paper has three original contributions. First, by back-casting the usual current employment and income series for Brazil, we allow business-cycle research in Brazil to resume using TCB and NBER oriented methods, which proved valuable after Duarte, Issler and Spacov (2004). Indeed, the main challenge of Brazilian business-cycle research was to be able to form a long enough coincident index with the usual series used in TCB's method – industrial production, sales, income and employment. Here, we devoted a great deal of effort in reconstructing employment and income using a novel flexible state-space representation based on the interpolation method of Mönch and Uhlig (2005).

Once we obtained a long enough span of the usual series used in TCB's method, we compute a new composite coincident index of Brazilian economic activity. Its dating of recessions is compared with those in Duarte, Issler and Spacov and with those implied by the monthly GDP estimate computed by Issler and Notini (2008).

Our last contribution is to propose a composite leading index of economic activity to track our composite coincident index. This is an important topic here, since Brazilian research had focused mainly on the construction of coincident indices. After a wide empirical search, we settled for a composite index that predicts correctly the "state of the economy" (expansion vs. recession), measured by our coincident index, almost 90% of the time. It misses one peak in economic activity and no through, while predicting no extra peaks or throughs. Moreover, on average, it leads the coincident index by 2.5 months for peaks and by 0.33 months for throughs. For anticipating throughs alone, an alternative composite leading index increases this lead to 2.25 months.

Finally, it is worth stressing that our choice of leading composite index – $LI_{7,t}$ – uses one series contained in the survey of industrial activity conducted by FGV: Level of Inventories. Since the criterion to choose the series in $LI_{7,t}$ was based solely on the five best values for $QPS(\cdot)$, it is interesting to find that one survey series made the top-five spots on that list.

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6 Apendix

6.1 The Bry and Boschan (1971) Algorithm

BRY BOSCHAN PROCEDURE FOR PROGRAMMED DETERMINATION OF TURNING POINTS

I. Determination of extremes and substitution of values

II Determination of cycles in 12-month moving average (extremes replaced)

A. Identification of points higher (or lower) than 5 months on either side

B. Enforcement of alternation of turns by selecting highest of multiple peaked (or lowest of multiple troughs).

III Determination of corresponding turns in Spencer curve (extremes replaced).

A. Identification of highest (or lowest) value within ± 5 months of selected turn in 12-month moving average.

B. Enforcement of minimum cycle duration of 15 months by eliminating lowerpeaks and higher troughs of shorter cycles

IV Determination of corresponding turns in short- term moving average of 3 to 6 months, depending on MCD (months of cyclical dominance).

A. Identification of highest (or lowest) value within ± 5 months of selected turn in Spencer curve.

V. Determination of turning points in unsmoothed series

A. Identification of highest (or lowest) value within ± 4 months, or MCD term, whichever is larger, of selected turn in short-term moving average.

B. Elimination of turns within 6 months of beginning and end of series.

C. Elimination of peaks (or troughs) at both ends of series which are lower (or higher) than values closer to end.

D. Elimination of cycles whose duration is less than 15 months.

E. Elimination of phases whose duration is less than 5 months.

VI. Statement of final turning points.

Source: Bry and Boschan (1971) page 21.

6.2 Additional Tables

Table A1: Leading Series

Series name	Description	Source
BASE_R	Monetary base	Bacen
SELIC_R	Selic interest rate	Bacen
M1_R	M1 money stock	Bacen
IBOV_R	Ibovespa index	Bovespa
EXP_PRECOS	Exports prices	Funcex
EXP_QUANTUM	Quantum of exports	Funcex
EXP_R	Exports (FOB)	Funcex
TTROCA	Terms of trade	Funcex
IMP_PRECOS	Imports prices	Funcex
IMP_QUANTUM	Quantum of imports	Funcex
IMP_R	Imports (FOB)	Funcex
CAMBIO_R	Exchange Rate	Bacen
NUCIFIESP	Manufacturing Industry	Fiesp
PROD_BC	Production - Consumer Goods	IBGE/PIM
PROD_BCD	Production - Consumer Durable	IBGE/PIM
PROD_BCND	Production - Consumption and Non Durable	IBGE/PIM
PROD_BI	Production - Intermediate Goods	IBGE/PIM
PROD_BK	Production - Capital Goods	IBGE/PIM
PRODINDT	Industrial production - processing industry	IBGE/PIM
PRODONI	Production - bus	IBGE/PIM
PRODVEI	Production - vehicles	Anfavea
PRODAUTO	Production - motors	Anfavea
PRODCAM	Production - trucks	Anfavea
SAL_R	Nominal Salary - industry	
PO	Staff employed - industry	Fiesp
HPP	Hours paid - industry	Fiesp
HTP	Hours worked in production - industry	Fiesp
ICMS_R		Confaz
INPC_R	National Consumer Price Index	
SPC		ACSP
IPA_R		FGV
FALENCIAS	Bankruptcy - Sao Paulo Capital	

Table A2: Survey Leading Series

Business Tendency Survey	Description	Source
NUCI_BR	Survey of Manufacturing Industry	FGV
NUCI_BC	Survey of Consumer Goods Industry	FGV
NUCI_BK	Survey of Capital Goods Industry	FGV
NUCI_MC	Survey of Construction Materials Industry	FGV
NUCI_BI	Survey of Intermediaries Goods Industry	FGV
DEMINT	Survey of Industry - Level of Internal Demand	FGV
DEMEX	Survey of Industry - Level of External Demand	FGV
DEMPREVINT	Survey of Industry - Internal Demand Forecast	FGV
DEMPREVEXT	Survey of Industry - External Demand Forecast	FGV
DEMGLOB	Survey of Industry - Level of Global Demand	FGV
DEMPREV	Survey of Industry - Global Demand Forecast	FGV
EMPPREV	Survey of Industry - Employment forecast	FGV
ESTOQUES	Survey of Industry - Level of Inventories	FGV
PRODPREV	Survey of Industry - Production Forecast	FGV

Table A3: Leading Series - ADF Unit Root Test

Series	t-statistic	p-value
BASE_R	-2.89	0.17
CAMBIO_R	-1.83	0.69
DEMGLOB	-2.92	0.16
DEMPREV	-4.42	0.00*
EMPPREV	-2.72	0.23
ESTOQUES	-5.06	0.00*
EXP_PRECOS	-0.16	0.99
EXP_QUANTUM	-2.71	0.23
EXP_R	-1.85	0.68
FALENCIAS	-2.38	0.39
HPP	-1.85	0.68
HTP	-1.99	0.61
IBOV_R	-3.45	0.04*
ICMS_R	-5.53	0.00*
IMP_PRECOS	-0.77	0.97
IMP_QUANTUM	-2.91	0.16
IMP_R	-2.61	0.28
INPC_R	-1.68	0.00*
IPA_R	-1.08	0.93
M1_R	-2.15	0.52
NUCI_BR	-3.46	0.04*
NUCIFIESP	-5.20	0.00*
PO	-1.11	0.92
PROD_BC	-2.96	0.14
PROD_BCD	-3.52	0.04*
PROD_BCND	-2.96	0.15
PROD_BI	-2.28	0.44
PROD_BK	-3.10	0.11
PRODAUTO	-6.14	0.00*

PRODCAM	-5.23	0.00*
PRODINDT	-2.54	0.31
PRODONI	-1.11	0.00*
PRODPREV	-3.29	0.07
PRODVEI	-7.12	0.00*
SAL_R	-3.44	0.04*
SELIC_R	-1.45	0.00*
SPC	-2.16	0.51
TTROCA	-4.28	0.00*
NUCI_BC	-2.40	0.14
NUCI_MC	-2.16	0.22
NUCI_BI	-2.88	0.04*
DeMINT	-3.04	0.03*
DEMEX	-5.24	0.00*
DEMPREVINT	-3.84	0.00*
DEMPREVEXT	-3.40	0.01*

Notes: (i) the specification of the test equation was chosen on the basis of the Schwartz Information Criterion; (ii) the asterisk (*) indicates that we reject the null hypothesis of a unit root at 5%.

Table A4: Leading Series - $QPS(h^*)$ and Granger Causality

Leading	**Optimum h^***	**Mín QPS-$QPS(h^*)$**	**Granger-Causes**
BASE_R	1	0.4567	B
DEMGLOB	3	0.2806	B
DEMPREV	4	0.2866	B
EXP_R	7	0.3642	N
EXP_QUANTUM	12	0.3104	N
HPP	1	0.4299	B
HTP	1	0.3940	N
IMP_R	1	0.3761	N
IPA_R	11	0.5164	N
M1_R	1	0.3373	C
NUCI_BC	1	0.3463	C
NUCI_BK	1	0.3134	C
NUCI_MC	1	0.2507	C
PO	1	0.4090	N
PRODAUTO	1	0.2149	B
PROD_BC	1	0.1254	B
PROD_BCND	1	0.2328	N
PROD_BI	1	0.1104	N
PROD_BK	1	0.3463	N
PRODINDT	1	0.3104	N
ESTOQUES	2	0.1761	B
IBOV_R	5	0.2448	C
ICMS_R	1	0.3134	B
INPC_R	5	0.4776	N
NUCI_BR	1	0.2746	B
NUCIFIESP	1	0.2478	B
PROD_BCD	1	0.2746	N
PROD_CAM	1	0.2627	N
PRODONI	1	0.4179	N

Notes: i) Statistics $QPS(h^*)$ and $h^*$are computed in accordance with the description of the equation (14) in the text. (ii) in the Granger causality test, the symbol C means that the leading series Granger-cause at least three out of four series that make up the coincident index with the reciprocal is not true. The symbol B means bi-directional causality in the Granger causality test. The symbol N indicates that the leading series not Granger cause the coincident series. The level of significance was set at 5% in these tests and the number

of lags tested was set at 3, 6, 12. To compute the results of the Granger test it was considered the existence of causality in at least one of these lags.

Table A4 (continuation)

Leading	**Optimum** h^*	**Mín QPS-**$QPS(h^*)$	**Granger-Causes**
PRODPREV	3	0.2507	N
PRODVEI	1	0.3224	N
SAL_R	1	0.3761	N
NUCI_BI	1	0.3224	B
CAMBIO_R	12	0.6000	B
EXP_PRECOS	3	0.3881	N
IMP_PRECOS	10	0.5045	N
SELIC_R	11	0.5821	C
TTROCA	2	0.3642	N
DEMEXT	6	0.3164	N
DEMINT	2	0.2746	B
DEMPREVEXT	4	0.3463	N
DEMPREVINT	4	0.2716	B
IMP_QUANTUM	1	0.3343	N
LN_SPC	1	0.2537	B

Notes: i) Statistics $QPS(h^*)$ and $h^*$are computed in accordance with the description of the equation (14) in the text. (ii) in the Granger causality test, the symbol C means that the leading series Granger-cause at least three out of four series that make up the coincident index with the reciprocal is not true. The symbol B means bi-directional causality in the Granger causality test. The symbol N indicates that the leading series not Granger cause the coincident series. The level of significance was set at 5% in these tests and the number of lags tested was set at 3, 6, 12. To compute the results of the Granger test it was considered the existence of causality in at least one of these lags.

Table A5: Leading Indices - Mönch e Uhlig Dates

Table A5 – Selected Leading Index					
Turning Points Comparatives– Mönch e Uhlig					
Peak dates			Through dates		
TCB - CI	LI3	Lag	TCB - CI	LI3	Lag
1980:10			1981:09		
1982:07			1983:02	1983:06	+4
1987:02	1986:09	-5			
1989:06	1989:03	-3	1988:10	1988:12	+2
1991:07	1991:04	-3	1990:04	1990:03	-1
1994:12	1994:11	-1	1991:12	1992:07	+7
1997:10	1997:08	-2	1995:07	1995:09	+2
2000:12	2001:03	+3	1999:02	1998:11	-3
2002:10			2001:09	2002:03	+6
	2004:09		2003:06		
				2005:12	

Table A5 – Selected Leading Index (continuation)					
Turning Points Comparatives– Mönch e Uhlig					
Peak dates			Through dates		
TCB - CI	LI4	Lag	TCB - CI	LI4	Lag
1980:10			1981:09	1981:03	-6
1982:07	1981:11	-8	1983:02	1982:10	-4
	1984:04			1985:03	
1987:02	1986:09	-5	1988:10	1988:07	-3
1989:06	1989 3	-3	1990:04	1990:02	-2
	1990:06				
1991:07			1991:12	1992:12	+12
1994:12	1994:09	-3	1995:07	1995:07	0
1997:10	1997:06	-4	1999:02	1998:11	-3
2000:12	2000:12	0	2001:09	2001:09	0
2002:10	2002:09	-1	2003:06	2003:01	-5
	2004:06			2005:03	

Table A5 – Selected Leading Index (continuation)					
Turning Points Comparatives– Mönch e Uhlig					
Peak dates			Through dates		
TCB - CI	LI6	Lag	TCB - CI	LI6	Lag
1980:10			1981:09		
1982:07	1982:04	-3	1983:02	1983:06	+4
				1987:06	
1987:02	1986:09	-5	1988:10		
	1988:03				
1989:06			1990:04	1990:03	-1
1991:07	1991:04	-3	1991:12	1992:06	+6
1994:12	1994:12	0	1995:07	1995:09	+2
1997:10	1997:06	-4	1999:02	1998:12	-2
2000:12	2000:12	0	2001:09		
2002:10			2003:06	2002:06	-12
	2004:09			2005:12	

Table A5 – Selected Leading Index (continuation)					
Turning Points Comparatives– Mönch e Uhlig					
Peak dates			Through dates		
TCB - CI	LI8	Lag	TCB - CI	LI8	Lag
1980:10			1981:09	1981:04	-5
1982:07	1982:04	-3	1983:02	1983:07	+5
				1987:7	
1987:02	1986:07	-7	1988:10		
1989:06	1989:04	-2	1990:04	1990:04	0
1991:07	1991:07	0	1991:12	1991:10	-2
1994:12	1994:10	-2	1995:07	1995:07	0
1997:10	1996:10	-12	1999:02	1998:10	-4
2000:12	2000:07	-5	2001:09	2001:07	-2
2002:10	2002:01	-9	2003:06	2003:07	+1
				2005:10	

Table A5 – Selected Leading Index (continuation)					
Turning Points Comparatives– Mönch e Uhlig					
Peak dates			Through dates		
TCB - CI	LI9	Lag	TCB - CI	LI9	Lag
1980:10			1981:09	1981:04	-5
1982:07	1982:01	-6	1983:02	1982:10	-4
				1985:03	
	1984:04			1987:07	
1987:02	1986:10	-4	1988:10		
1989:06	1989:04	-2	1990:04	1990:04	0
1991:07	1991:04	-3	1991:12	1991:10	-2
1994:12	1994:10	-2	1995:07	1995:07	0
1997:10	1996:10	-12	1999:02	1998:10	-4
	1999:10				
2000:12			2001:09	2001:10	+1
2002:10	2002:10	0	2003:06	2003:07	+1
	2004:07			2005:12	

Table A5 – Selected Leading Index (continuation)					
Turning Points Comparatives– Mönch e Uhlig					
Peak dates			Through dates		
TCB - CI	LI10	Lag	TCB - CI	LI10	Lag
1980:10			1981:09	1981:07	-2
1982:07	1982:04	-3	1983:02	1983:07	+5
				1987:07	
1987:02	1986:10	-4	1988:10		
1989:06	1989:04	-2	1990:04	1990:04	0
1991:07	1991:07	0	1991:12	1992:01	+1
1994:12	1995:01	+1	1995:07	1995:07	0
1997:10	1996:10	-12	1999:02	1998:10	-4
2000:12	2000:07	-5	2001:09	2001:10	+1
2002:10	2002:04	-6	2003:06	2003:07	+1
	2004:07			2005:10	

Table A6: Leading Indices - Bry-Boschan Dates

Table A6 – Selected Leading Index					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI1	Lag	TCB - CI	LI1	Lag
1980:10			1981:09	1981:04	-5
1982:07	1982:02	-5	1983:02	1982:09	-5
1987:02	1986:09	-5	1988:10	1987:06	-4
1989:06	1989:05	-1	1990:04	1990:03	-1
1991:07	1991:06	-1	1991:12	1991:11	-1
1994:12	1994:11	-1	1995:07	1995:06	-1
1997:10	1997:09	-1	1999:02	1998:09	-5
2000:12	2001:02	+2	2001:09	2001:09	0
2002:10	2002:09	-1	2003:06	2003:06	0
	2004:06			2005:10	

Table A6 – Selected Leading Index (Continuation)					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI2	Lag	TCB - CI	LI2	Lag
1980:10			1981:09	1981:08	-1
1982:07	1982:02	-5	1983:02	1983:06	+4
				1987:06	
1987:02	1986:09	-5	1988:10		
1989:06	1989:05	-1	1990:04	1990:03	-1
1991:07	1991:06	-1	1991:12	1992:07	+7
1994:12	1994:11	-1	1995:07	1995:06	-1
1997:10	1997:09	-1	1999:02	1998:12	-2
2000:12	2000:12	0	2001:09	2001:09	0
2002:10	2002:09	-1	2003:06	2003:06	0
	2004:08			2005:10	

Table A6 – Selected Leading Index (Continuation)					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI3	Lag	TCB - CI	LI3	Lag
1980:10			1981:09		
1982:07			1983:02	1983:06	+4
1987:02	1986:09	-5	1988:10		
1989:06			1990:04	1990:03	-1
1991:07	1991:04	-3	1991:12	1992:07	+7
1994:12	1994:11	-1	1995:07	1995:09	+2
1997:10	1997:08	-2	1999:02	1998:11	-3
2000:12	2001:03	-9	2001:09		
2002:10			2003:06	2003:06	0
	2004:09			2005:12	

Table A6 – Selected Leading Index (Continuation)					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI4	Lag	TCB - CI	LI4	Lag
1980:10			1981:09	1981:03	-6
1982:07	1981:11	-8	1983:02	1982:10	-4
	1984:04			1985:03	
1987:02	1986:09	-5	1988:10		
1989:06			1990:04		
1991:07			1991:12	1992:12	+12
1994:12	1994:09	-3	1995:07	1995:07	0
1997:10	1997:06	-4	1999:02	1998:11	-3
2000:12	2000:12	0	2001:09	2001:09	0
2002:10			2003:06		
	2004:06			2005:03	

Table A6 – Selected Leading Index (Continuation)					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI5	Lag	TCB - CI	LI5	Lag
1980:10			1981:09		
1982:07			1983:02	1983:06	+4
1987:02	1986:09	-5	1988:10		
1989:06			1990:04	1990:03	-1
1991:07	1991:06	-1	1991:12	1992:06	+6
1994:12	1994:11	-1	1995:07	1995:09	+2
1997:10	1997:06	-4	1999:02	1998:12	-2
2000:12	2000:12	0	2001:09		
2002:10			2003:06	2002:6	-12
	2004:09			2005:03	

Table A6 – Selected Leading Index (Continuation)					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI6	Lag	TCB - CI	LI6	Lag
1980:10			1981:09		
1982:07			1983:02	1983:06	+4
1987:02	1986:09	-5	1988:10	1987:06	-8
	1988:03				
1989:06			1990:04	1990:03	-1
1991:07	1991:04	-3	1991:12	1992:06	+6
1994:12	1994:12	0	1995:07	1995:09	+2
1997:10	1997:06	-4	1999:02	1998:12	-2
2000:12	2000:12	0	2001:09		
2002:10			2003:06	2002:6	-12
	2004:09			2005:12	

Table A6 – Selected Leading Index (Continuation)					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI7	Lag	TCB - CI	LI7	Lag
1980:10			1981:09	1981:09	0
1982:07	1982:03	-4	1983:02	1983:06	+4
1987:02	1987:01	-1	1988:10	1988:09	-1
1989:06	1989:05	-1	1990:04		
1991:07			1991:12	1992:02	+2
1994:12	1994:11	-1	1995:07	1995:06	-1
1997:10	1997:09	-1	1999:02	1999:01	-1
2000:12	2000:11	-1	2001:09	2001:09	0
2002:10	2002:09	-1	2003:06	2003:07	+1

Table A7 - Leading series that compound Leading Index number 1

DEMGLOB
DEMPREV
EXP_R
EXP_QUANTUM
IMP_R
M1_R
PRODAUTO
PROD_BC
PROD_BCND
PROD_BI
PROD_BK
PRODINDT
ESTOQUES
IBOV_R
ICMS_R
PROD_BCD
PRODPREV
EXP_PRECOS
TTROCA
DEMEX
DEMINT
DEMPREVEXT
DEMPREVINT
SPC

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. The leading series chosen are the ones which the QPS took less than 0.4 and the maximum lag was greater than zero.

Table A8 - Leading series that compound Leading Index number 2

DEMGLOB
DEMPREV
EXP_R
EXP_QUANTUM
HTP
IMP_R
M1_R
NUCI_BC
NUCI_BK
NUCI_MC
PRODAUTO
PROD_BC
PROD_BCND
PROD_BI
PROD_BK
PRODINDT
ESTOQUES
IBOV_R
ICMS_R
NUCI_BR
NUCIFIESP
PROD_BCD
PROD_CAM
PRODPREV
PRODVEI

Table A8: (continuation)

SAL_R
NUCI_BI
EXP_PRECOS
TTROCA
DEMEXT
DEMINT
DEMPREVEXT
DEMPREVINT
IMP_QUANTUM
SPC

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. The leading series chosen are the ones which the QPS took less than 0.4 and the maximum lag was greater than zero. The difference with the criterion 1 is choice of the optimum lag. In this case we exclude the zero lag from que optimum lag so the series show optimum lag above zero.

Table A9 - Leading series that compound Leading Index number 3

BASE_R
M1_R
NUCI_BC
NUCI_BK
NUCI_MC
IBOV_R
NUCI_BI
CAMBIO_R
SELIC_R
DEMEXT

Notes: (i) these series were selected by the Granger Causality Test criterion.(ii) we consider the causality test for the lag 3, 6 and 12. The series identified as causing the series of the index in any of these lags, was included in the composite leading index number 1.

Table A10 - Leading series that compound Leading Index number 4

M1_R
IBOV_R
DEMEXT

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. The select series were subject to the Granger causality test. (ii) The criterion is the intersection of the first and third criterion.

Table A11 - Leading series that compound Leading Index number 5

M1_R
NUCI_BC
NUCI_BK
NUCI_MC
IBOV_R
NUCI_BI
DEMEXT

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. The select series were subject to the Granger causality test. (ii) The criterion is the intersection of the first and third criterion.

Table A12 - Leading series that compound Leading Index number 6

DEMGLOB
DEMPREV
IBOV_R
PRODPREV
DEMPREVINT

Notes: (i) Survey series which granger causes the coincident index.

Table A13 - Leading series that compound Leading Index number 7

PROD_BI
PROD_BC
ESTOQUES
PRODAUTO
PROD_BCND

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. (ii) The index is formed by the five series which shown the lowest QPS.

Table A14 - Leading series that compound Leading Index number 8

DEMGLOB
DEMPREV
IBOV
PRODPREV
DEMPREVINT

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. (ii) Series which the QPS ≤ 0.3 and the optimum lag is in the open interval (2,7).

Table A15 - Leading series that compound Leading Index number 9

DEMGLOB
DEMPREV
PRODPREV
DEMEX
DEMPREVINT
DEMPREVEXT

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. (ii) Survey series which the optimum lag is in the open interval (2,7).

Table A16 - Leading series that compound Leading Index number 10

DEMGLOB
DEMPREV
ESTOQUES
NUCI_MC
PRODPREV
DEMINT
DEMPREVINT
NUCI_BR

(i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. (ii) Survey series which the QPS ≤ 0.3. (iii) with the exception of NUCI_MC these are the same series definied by criterion number 9.

Convergência em Clubes: uma Análise da Trajetória de Renda dos Estados Brasileiros

October 26, 2010

Resumo: Este artigo estuda o comportamento das trajetórias de rendas per capita para os estados brasileiros, no período de 1985 a 2006. O objetivo é identificar se existe uma tendência de convergência no longo prazo entre os estados, ou pelo menos dentre alguns deles. Utilizamos para isso o modelo e o teste econométrico proposto por Phillips e Sul (2007). Neste arcabouço as características idiossincráticas de cada elemento no painel são separadas de um componente comum, e a partir da variância destas características, ao longo do tempo, é que o teste econométrico rejeita ou não a hipótese de convergência. Este teste além de poder ser aplicado para a amostra inteira, pode ser aplicado em subamostras (formadas apenas por alguns estados) e nesse caso indica a convergência em clubes. Como resultado do teste concluímos que cinco grupos de convergência são formados no Brasil, além de duas unidades da federação divergirem das demais. Este resultado indica a existência de sete pontos de equilíbrio de longo prazo, reforçando uma ausência de convergência geral. Dentre todas as trajetórias de renda per capita, a do DF é a única que já estando acima das demais, continua com forte tendência de crescimento.

Abstract: This article intends to investigate convergence growth behavior among Brazilian states. For this we apply a new econometric method proposed by Phillips and Sul (2007). The method is composed by: a non-linear time-varying factor model, in which heterogenous and common behavior are easily represented, and an econometric convergence test, which investigates convergence. The approach proposed guarantees flexible behavior for the series allowing different transitions paths to ultimate converge. From their test also, one can determine overall convergent behavior, subgroup convergent behavior or divergent behavior, in which case no series seems to approach each other in the long run. Using per capita real GDP for 25 Brazilian states and the Federal District, we applied the test and conclude from the outcomes that five distinct convergence groups exist in our sample. Besides the groups two states have paths that completely diverge from the others. The result indicates seven distinct points of long run equilibrium, which emphasizes the idea of no convergence at all.

Palavras-chave: convergência de renda, modelo não-linear de fator comum variante no tempo.

Keywords: income convergence, non-linear dynamic common factor model.

J.E.L.: O15, O47, R10.

1 Introdução

[1]O conceito de convergência de riqueza entre países, estados, etc tem sido foco de atenção de estudos empíricos sobre crescimento econômico pelo menos nos últimos vinte anos. Crescimento econômico implica aumento de renda real per capita ao longo do tempo, o que pode indicar melhora na qualidade de vida dos indivíduos e por isso se dedica tanta atenção ao tema. Ainda relacionado a crescimento e convergência entre economias está a questão da conseqüente diminuição de desigualdade entre as regiões convergentes. Neste artigo investigamos a hipótese de convergência de renda real per capita entre os estados brasileiros analisando o período de 1985 a 2006, usando um modelo não-linear e variante no tempo proposto por Phillips e Sul (2007).

Apesar do tema convergência de renda já ter sido abordado em vários estudos anteriores, queremos verificar a contribuição deste método menos restritivo e mais parcimonioso para a análise. Além destas características o método de Phillips e Sul apresenta outras também desejáveis e igualmente importantes: baseia-se em informaçõs provenientes de painel, evita a falácia de Galton analisando convergência através da evolução da variância entre características idiossincráticas das séries, e descreve o equilíbrio no longo prazo de forma mais apropriada do que um modelo de cointegração.

O artigo de Phillips e Sul (2007) discute convergência em painel apresentando um modelo para descrever as trajetórias das séries e um teste econométrico para testar a hipótese de convergência entre elas. O modelo é não-linear e de fator comum variante no tempo, o qual seria adequado para modelar o comportamento heterogêneo de elementos distintos e ainda permitir uma evolução temporal deste comportamento. O teste econométrico é baseado numa regressão simples cujo objetivo é identificar convergência através de uma análise de variância entre as séries do painel. A hipótese nula supõe convergência na amostra. Se rejeitada, subamostras são escolhidas e testadas com a intenção de se verificar uma possível formação de convergência em clubes. Clubes delimitam subamostras convergentes, isto é encerram elementos cuja dispersão de seus componentes idiossincráticos diminui ao longo do tempo. Este teste não depende de características de estacionariedade das séries no painel, e essa é uma grande vantagem em relação a estudos anteriores.

Nosso resultado empírico mostra que cinco grupos de convergência estão sendo formados no Brasil, e além deles duas unidades da federação aparecem como divergentes. A existência de grupos, por definição, indica que a dispersão na distribuição de renda diminui entre alguns estados brasileiros (estados de um mesmo grupo), no entanto, persiste a nível nacional.

[1]Este artigo foi feito em co-autoria com o professor Luiz Renato Lima (EPGE-RJ)

Este artigo está organizado da seguinte forma: a seção 2 apresenta uma revisão de literatura sobre convergência de renda e cita estudos anteriores realizados para o Brasil. A seção 3 descreve um modelo de crescimento econômico. A seção 4 associa o modelo de fator comum dinâmico proposto por Phillips e Sul(2007) ao modelo de crescimento econômico e introduz o teste econométrico de convergência. A seção 5 analisa os resultados de convergência encontrados para os estados brasileiros. A seção 6 conclui nossa investigação resumindo os pontos relevantes da pesquisa. Dados e gráficos relacionados ao experimento são apresentados no apêndice.

2 Revisão de Literatura

Como um conceito econômico convergência significa regiões distintas que no longo prazo terão mesma taxa de crescimento. O primeiro modelo econômico a analisar convergência foi proposto por Solow em 1956, quando ele afirmou que a mesma taxa de crescimento econômico assintótica ocorre entre regiões com mesma tecnologia e preferências independentemente de suas condições iniciais (estoque de capital físico e humano). Baseado nessa abordagem neoclássica, Barro (1997) descreve convergência como consequência do retorno decrescente do capital. Regiões com menos estoque de capital relativo por trabalhador tendem a crescer mais rápido que outras onde o capital é abundante. Contrário as expectativas de Solow, Romer (1986) e Lucas (1988) propuseram um modelo de crescimento endógeno no qual retornos decrescentes do capital não são esperados e por isso convergência entre regiões não é observada no longo prazo.

Transformar esses conceitos econômicos em modelos econométricos para testar a hipótese de convergência tem levado a diferentes noções estatísticas de comportamento convergente: convergência absoluta, convergência condicional e convergência em clubes. Convergência absoluta ocorre quando as rendas per capita das economias se aproximam no longo prazo independente de suas condições iniciais; convergência condicional e em clube não assumem características estruturais similares mas dependem delas para alcançar convergência. Para a convergência em clube é obrigatório que as rendas per capita dos elementos do clube se aproximem no longo prazo.

A análise de convergência empírica começou com Barro (1991), Barro e Sala-i-Martin (1992). Estes autores propuseram uma regressão para análise de crescimento de países no cross-section, com a taxa de crescimento como variável dependente e renda inicial per capita como regressor principal (outros regressores foram incluídos na regressão para representar as condições iniciais, como: taxa de poupança, crescimento populacional, etc). De acordo com este modelo se o principal regressor tem um coeficiente negativo (β) países com menor renda inicial per capita crescerão mais rápido que os demais com renda inicial maior (tudo controlado pelas condições iniciais), o que sugere convergência entre os países no longo prazo. Este é o tipo de convergência conhecido por $\beta-$convergência.

As regressões usadas para o estudo de $\beta-$convergência foram aplicadas em muitas pesquisas brasileiras, das quais podemos mencionar: Azzoni (1999), Fer-

reira e Ellery Jr (1996), Schwartsman (1996) e Zini (1998). Estes estudos supõem que preferências e tecnologias são compartilhadas entre estados brasileiros e por isso não incluem variáveis para controlar por possíveis diferenças entre regiões. A possivel omissão de variáveis relevantes nestas regressões pode ter causado um viés na estimação do β. Arellano e Bond (1991) e Islam (1995) discutem este aspecto.

Em 2000, Menezes e Azzoni resolvem a possivel omissão de variáveis na regressão usando dados em painel com efeito fixo. Analisam convergência entre regiões metropolitanas brasileiras. Seus resultados mostram que a velocidade de convergência aumenta uma vez que características idiossincráticas são consideradas, provando que persistências nas diferenças institucionais e tecnológicas são responsáveis por divergências entre áreas metropolitanas. Também mostram que diferenças idiossincráticas não são significativas quando somente consideram áreas do norte e nordeste brasileiro na regressão; ou quando consideram áreas do sul e sudeste. Estes resultados sugerem que diferenças idiossincráticas são significativas apenas a nível nacional.

Uma crítica a técnica de $\beta-$convergência diz respeito às variáveis que devem ser incluídas na regressão (além da renda inicial). Esta é uma questão importante porque dependendo dos regressores incluídos o sinal de β (indicador de convergência) pode ser alterado.

Em 1993, Quah mostrou que a amplamente usada $\beta-$convergência sofre também da clássica falácia da teoria de regressão. Ele defende que um sinal negativo para β é perfeitamente consistente com ausência de convergência, no sentido que regiões podem não diminuir sua dispersão ao longo do tempo – a qual estaria relacionada ao conceito de $\sigma-$convergência. Como Quah (1993) apontou "não é apropriado desenhar implicações dinâmicas a partir de evidências cross-section"; esse fato é conhecido na literatura como falácia de Galton da regressão em direção a média. Portanto, Quah sugeriu que se analisasse diretamente as características das distribuições de renda no cross-section por vários anos e se computasse a função densidade a cada momento. Para analisar a probabilidade de transição (lei de movimento) de cada economia, Quah (1993b) usou a cadeia de Markov e técnicas de kernel estocástico (Quah, 1997) - que seria uma versão contínua da cadeia de Markov. Seus testes mostraram que economias ricas estão se tornando mais ricas enquanto economias pobres estão se tornando mais pobres, com as economias médias desaparecendo no longo prazo. Como conseqüência de seu trabalho a convergência total não é esperada no longo prazo, embora a convergência em grupos – sendo observados dois grupos – é plausível. Aplicando a metodologia de Quah (1997) para produto per capita brasileiro por município, Andrade e outros (2004) rejeitaram convergência total mas encontraram dois grupos – um que compreende municípios ricos e outro municípios pobres. Também concluíram que existe certa mobilidade apesar da persistência ser forte. Outros pesquisadores também aplicaram as idéias de Quah (1997) para investigar níveis de convergência no Brasil: Mossi e outros (2003), Magalhães e outros (2005) e Raul e Azzoni (2006). A maioria desses trabalhos mostram evidências da formação de dois clubes entre os estados brasileiros – um formado por estados do norte e nordeste e outro por estados

do sul e sudeste.

Concomitante as pesquisas de Quah, Carlino e Mills (1993) usaram séries de tempo para investigar convergência estocástica para regiões americanas. Eles encontraram fracas evidências de convergência, no entanto, quando consideraram existências de quebras nas tendências estocásticas a convergência não foi rejeitada. No Brasil, Barossi e Azzoni (2002) aplicaram a mesma abordagem de séries temporais, identificando pontos de quebra endogenamente, e analisaram convergência entre as regiões brasileiras. Concluíram que as regiões convergem, exceto a região norte; e numa análise individual concluíram que os estados do norte, centro-oeste e sudeste possuem rendas que convergem para o nível regional, enquanto que os estados do nordeste e sul não possuem tal comportamento.

Uma característica comum na análise de convergência de renda, observada empiricamente por diferentes métodos, é a inexistência de convergência total e a recorrente formação de clubes de convergência.

Métodos para analisar clubes foram propostos por Durlauf e Johnson (1995). Utilizando a técnica de árvore de regressão e um modelo que controla pela renda inicial per capita e taxa de alfabetização dos indivíduos. Nesta técnica a amostra é dividida em diferentes modelos lineares formando subamostras, e a convergência é analisada usando o conceito de $\beta-$convergência em cada uma das partes. A técnica de árvore de regressão foi mais tarde usada por Coelho e Figueiredo (2007) para verificar convergência entre municípios brasileiros e os resultados sugerem existência de oito clubes distintos.

Bernand e Durlauf (1996), Durlauf e Quah (1999) e Islam (2003) criticaram alguns pontos sobre os testes empíricos de convergência descritos até então. Primeiro, como testes diferentes checam diferentes aspectos de convergência, eles chegam a conclusões distintas sobre convergência total ou em clube. Segundo, como $\beta-$convergência é necessária mas não suficiente para convergência σ (Islam 2003), é possível que a dispersão entre as séries ditas $\beta-$convergentes não esteja diminuindo ao longo do tempo. Terceiro, abordagens que utilizam séries de tempo e outras baseadas em distribuição dependem de características muito específicas das séries subjacentes, como estacionariedade. Esta é a razão pela qual novos modelos continuam a ser desenvolvidos.

Em 2006, Ho propos um modelo para análise de comportamento dinâmico de séries de tempo; seu modelo é não-linear e consiste de uma regressão dinâmica em painel com efeito threshold. Neto e outros (2008) aplicaram este teste a estados brasileiros e identificaram a existência de dois clubes; um formado por norte e nordeste (incluindo Goias e excluindo Amazonas) e outro formado por estados do Sul e Sudeste, incluindo os estados de Mato Grosso, Mato Grosso do Sul, Amazonas e o Distrito Federal.

Phillips e Sul (2007) (de agora em diante, PS) propõem também um modelo não-linear e dinâmico, bem genérico, capaz de modelar uma grande diversidade de trajetórias temporais; e um teste econométrico para verificar a hipótese de convergência baseado na diminuição de dispersão entre as séries. Este teste além de testar convergência total, pode testar convergência em grupos. Seguiremos essa pesquisa para verificar o comportamento da renda per capita real dos

estados brasileiros.

3 Crescimento Econômico

Para analisar convergência econômica entre regiões distintas precisamos verificar o modelo de crescimento mais adequado para traçar as trajetórias observadas de renda das economias subjacentes. Segundo Parente e Prescott (1994), PS (2006) e assumindo progresso tecnológico em um modelo de crescimento neoclássico, o log da renda real per capita, $\log y_{it}$, pode ser escrito como:

$$\log y_{it} = \log y_i^* + [\log y_{i0} - \log y_i^*]e^{-\beta_{it}t} + \log A_{it} = a_{it} + \log A_{it} \qquad (1)$$

onde $\log y_i^*$ é o nível em estado estacionário do log da renda real per capita efetiva, $\log yi0$ é o log da renda real inicial efetiva, β_{it} é a velocidade da taxa de convergência variante no tempo, e $\log A_{it}$ é o log da acumulação tecnológica para economia i no instante t. Esta relação pode ser resumida por um componente em transição (a_{it}) e pelo *log* da tecnologia que inclui componentes permanentes. O $\log A_{it}$ ainda foi decomposto por PS (2006) considerando um elemento comum compartilhado entre as economias:

$$\log A_{it} = \log A_{i0} + \gamma_{it} \log A_t \qquad (2)$$

Dessa forma a tecnologia corrente de uma região depende de sua acumulação inicial e da tecnologia pública disponível. No entanto, a forma que cada economia absorve a tecnologia pública depende de características locais. Por exemplo, regiões mais desenvolvidas são normalmente as que lançam as inovações e portanto se beneficiam integralmente do processo; regiões em desenvolvimento normalmente precisam de um tempo para se preparar para o uso da tecnologia lançada (tempo de aprendizado, reestruturação) e por isso inicialmente tem menor benefício. Este mesmo mecanismo de absorção tecnológica pode ser aplicado dentro de um país, onde diferentes estados terão diferentes velocidades de aprendizado e adaptação a inovações, o que pode implicar grandes desigualdades entre eles.

Dessa forma o log do produto real pode ser descrito por:

$$\log y_{it} = a_{it} + \log A_{i0} + \gamma_{it} \log A_t \qquad (3)$$

onde um componente comum e outros idiossincráticos modelam a trajetória de crescimento.

O modelo econométrico que buscamos para analisar convergência de renda entre estados, utiliza-se da idéia que séries em painel podem ser decompostas em componentes idiossincráticos e comum, e é finalmente o comportamento do termo idiossincrático que determinará uma possível convergência no longo prazo. Na próxima seção descrevemos modelos econométricos de fator comum e mais especificamente o modelo que usaremos para análise de convergência.

4 Modelo Dinâmico de Fator Comum

A idéia do artigo de PS (2007) é a de modelar dados em painel de acordo com um modelo estendido de fator comum, no qual comportamentos heterogêneos transientes são possíveis mesmo quando a convergência é percebida no longo prazo. Como a teoria econômica, para se aproximar da realidade, espera que agentes (países, regiões) tenham comportamentos distintos, trabalhos empíricos, que procuram modelar indivíduos, devem apresentar mecanismos de representar heterogeneidade de agentes.

Um trabalho empírico popular que implementa comportamento heterogêneo envolve uma estrutura de fator comum e efeitos idiossincráticos. Um modelo simples é:

$$X_{it} = \delta_i \mu_t + \varepsilon_{it} \tag{4}$$

onde δit mede a parte sistemática idiossincrática de Xit em termos de um fator comum μ_t. Este fator comum pode representar o comportamento comum agregado de X_{it}, ou qualquer outra variável de influência comum no comportamento dos indivíduos. O modelo em (4) tenta capturar a evolução de X_{it} em relação a μ_t por meio de elementos idiossincráticos: δ_i o elemento sistemático (relacionado ao comportamento comum entre as séries), e ε_{it} o termo de erro.

O modelo de PS é uma extensão a este, propondo duas mudanças básicas para se analisar convergência; primeiro, permite-se que a parte idiossincrática relacionada ao componente comum evolua ao longo do tempo, tornando-se δ_{it} um coeficiente variante no tempo. Em seguida, modela-se δ_{it} como um componente aleatório que absorve o termo de erro da equação anterior. É este componente aleatório que permitirá um comportamento convergente em relação ao fator comum no longo prazo. O novo modelo pode ser resumido da seguinte forma:

$$X_{it} = \delta_{it} \mu_t \tag{5}$$

onde podemos notar que ambos os elementos são variantes no tempo. O componente de maior interesse para análise de convergência é o δ_{it} já que este componente é o que captura o comportamento idiossincrático das séries. Como δ_{it} não possui componente comum, ele pode ser representado por um modelo semi-paramétrico flexível, determinado por:

$$\delta_{it} = \delta_i + \sigma_i \varepsilon_{it} L(t)^{-1} t^{-\alpha} \tag{6}$$

onde δ_i é fixo, ε_{it} é iid(0,1) entre i e fracamente dependente ao longo de t, e $L(t)$ é uma função de crescimento lento, por exemplo $L(t) = \log t$, de forma que $L(t) \to \infty$ quando $t \to \infty$. Fica claro a partir da definição (6) que para todo $\alpha \geq 0$ os coeficientes δ_{it} convergem para δ_i; isto torna o valor de α um elemento de interesse para hipótese de convergência. Se a convergência não for rejeitada e $\delta_i = \delta_j$ para $i \neq j$ o modelo (6) permite períodos de transição onde $\delta_{it} \neq \delta_{jt}$ o que significa que divergências transitórias entre i's é possível, no entanto no longo prazo a convergência é esperada entre i e j.

Retomando nosso modelo de crescimento econômico, podemos reescrever o produto como em (3). Se ainda considerarmos que a tecnologia comum cresce a uma taxa constante a, nosso modelo de crescimento pode ser fatorado como:

$$\log y_{it} = (\frac{a_{it} + \log A_{i0} + \gamma_{it} \log A_t}{at})at = \delta_{it}\mu_t \tag{7}$$

onde δ_{it} pode ser modelado como em (6). Este termo (δ_{it}) representa a participação relativa do componente comum na economia i. Sua trajetória de transição indica como cada economia se aproxima ou se distancia da trajetória de crescimento comum determinada por μ_t. Durante a transição δ_{it} depende de condições iniciais, do estado estacionário da economia, e da velocidade de convergência para o estado estacionário (β_{it}). Como δ_{it} é o parâmetro que captura o comportamento idiossincrárico em relação a um fator comum, o foco da análise de convergência recai sobre ele diretamente.

O teste de convergência analisa exatamente a trajetória de δ_{it}. Séries cujos elementos idiossincráticos se aproximam com o tempo serão parte de um mesmo grupo. Note que com esse tipo de análise se identica clubes de convergência a partir do teste de hipótese, não sendo necessário incluir na regressão qualquer outra variável de controle.

4.1 Curva de Transição Relativa e Convergência

Para se estimar um modelo da forma (5) é necessário se impor alguma estrutura sobre δ_{it} e μ_t porque o número de observações no painel é normalmente inferior ao número de incógnitas no modelo. PS (2007) sugerem, com o propósito de simplificação, se usar uma versão relativa de δ_{it}, que chamaram coeficiente de transição relativo (h_{it}), e definiram como:

$$h_{it} = \frac{X_{it}}{\frac{1}{N}\sum_{i=1}^{N} X_{it}} = \frac{\delta_{it}\mu_t}{\frac{1}{N}\sum_{i=1}^{N} \delta_{it}\mu_t} = \frac{\delta_{it}}{\frac{1}{N}\sum_{i=1}^{N} \delta_{it}} \tag{8}$$

Por esta equação vemos que hit mede o coeficiente δit em relação a sua média no painel em t. Assim como δit, hit traça uma trajetória de transição para a economia i, mas faz isso em relação a média do painel. Pela definição de hit algumas propriedades seguem imediatamente: primeiro, a média no cross-section de hit é a unidade; segundo, se δit converge para δ, o coeficiente de transição relativa hit converge para a unidade. Neste caso, a variância de hit no cross-section converge para zero, isto é:

$$\sigma_t^2 = \frac{1}{N}\sum_{i=1}^{N}(h_{it} - 1)^2 \to 0 \text{ a medida que } t \to \infty \tag{9}$$

Este arcabouço permite que curvas de transição relativa sejam bem diferentes mesmo que finalmente convirjam. Note que a definição de equilíbrio relativo no longo prazo entre duas séries (X_{it}, X_{jt}) é dado por:

$$\lim_{k \to \infty} \frac{X_{it+k}}{X_{jt+k}} = 1, \text{ for all } i \text{ and } j \tag{10}$$

Esta definição garante convergência de variância em (5). De acordo com (5), (10) é equivalente a convergência do coeficiente do fator:

$$\lim_{k \to \infty} \delta_{it+k} = \delta \tag{11}$$

Assim, esperamos que a dispersão decresça no longo prazo para as curvas de transição relativas (h_{it}) convergentes. Ao analisarmos convergência em subamostras as mesmas considerações são válidas, o que significa que a variância em um grupo convergente é dada por:

$$\sigma^2_{tG} = \frac{1}{N_g} \sum_{i \epsilon N_g}^{N} (h^G_{it} - 1)^2 \to 0 \text{ a medida que } t \to \infty \tag{12}$$

onde Ng é o número de economias no grupo.

Essa diminuição da variância entre as trajetórias de transição relativa das economias é que é buscada pelo teste de convergência, tanto para não rejeitar a hipótese de convergência total quanto para não rejeitar convergência em subgrupos ou clubes.

Empiricamente, h_{it} pode ser calculada diretamente a partir dos dados na amostra. Para análise de longo prazo, como é o caso do estudo de convergência é conveniente se remover ciclos dos dados. PS (2007) sugerem o uso do filtro de Hodrick-Prescott. Este filtro pode ser aplicado em amostras pequenas de séries temporais e não assume nenhum comportamento especial para o termo μ_t, que pode ser um processo de tendência comum ou estocástica.

4.2 Teste de Convergência

O teste de convergência é baseado em uma regressão simples. A idéia é verificar se a variância das trajetórias de transição relativas ($\hat{h}_{it}$) convergem para zero à medida que $t \to \infty$. Esta idéia pode ser usada para o teste de convergência ou divergência total; neste último caso, a formação de clubes ainda deve ser considerada.

Pela definição de δ_{it} em (6) a hipótese nula para teste é dada por:

$$H_0 : \delta_i = \delta \text{ e } \alpha \geq 0 \tag{13}$$

enquanto a hipótese alternativa é:

$$H_A : \delta_i \neq \delta \text{ para todo } i \text{ ou } \alpha < 0 \tag{14}$$

Para a análise de convergência seguimos três passos. Primeiro construímos uma razão entre variâncias nos diferentes cross-sections (H_1/H_t), onde H_t representa a variância do coeficiente de transição relativo (h_{it}) em t:

$$H_t = \frac{1}{N}\sum_{i=1}^{N}(h_{it}-1)^2, h_{it} = \frac{Xit}{N^{-1}\sum_{i=1}^{N} X_{it}} \tag{15}$$

Se existe convergência, esperamos que a H_t diminua com o tempo, de forma que a razão entre as variâncias dos cross-section tende a infinito.

Em seguida, usamos o fato de que sob a hipótese de convergência ($h_{it} \to 1$ e $H_t \to 0$ a medida que $t \to \infty$ para um dado N) H_t tem forma logarítmica dada por:

$$\log H_t = -2\log L(t) - 2\alpha \log t + k + \varepsilon_t \tag{16}$$

Para termos razão de variâncias podemos reescrever esta fórmula subtraindo $\log H_1$ dos dois lados de (16), obtendo uma equação de regressão de onde estimaremos α. A equação seria:

$$\log(\frac{H_1}{H_t}) - 2\log L(t) = \hat{a} + \hat{b}\log t + \hat{u}_t \tag{17}$$

$$\text{para } t = [rT], [rT]+1, ...T \quad \text{com } r > 0$$

onde $\hat{b} = 2\hat{\alpha}$, e $\hat{\alpha}$ é uma estimativa de α em H_0; $L(t) = \log t$. Note que a estimação utiliza dados da amostra a partir de $[rT]$. Sugeri-se $r = 0.3$. Este valor foi obtido por uma simulação de Monte Carlo, na qual para valores de T moderados ($T \leq 50$), este valor de r ainda garante precisão de tamanho do teste para valores pequenos de α.

O último passo é verificar se convergência não é rejeitada dada a estatística-t de $\hat{b}$, estimada na regressão (17) e computada usando desvio-padrão de longo prazo. Se a estatística-t for menor que -1.65, rejeita-se a hipótese de convergência a 5%.

A rejeição de convergência no entanto não significa que não existe qualquer convergência no painel; a convergência em subgrupos deve ainda ser testada. Os passos anteriores servem para ambas as análises de convergência. No apêndice descrevemos um passo-a-passo do algoritmo de convergência de PS.

5 Resultados de Convergência para os Estados Brasileiros

Esta seção apresenta os resultados de convergência obtidos quando aplicamos o teste para os estados brasileiros. O painel que analisamos é composto pelo PIB real per capita de 1985 a 2006 de 25 estados brasileiros e do Distrito Federal. Os estados e algumas características das séries estão apresentadas no apêndice. Um resultado de convergência total indicaria que após ser extraído o componente de crescimento comum das séries, a variância dos componentes idiossincráticos decresce ao longo do tempo. Um resultado de convergência em clubes tem a mesma definição apenas se limita aos elementos do clube.

Começamos extraindo os ciclos de negócios das séries no painel aplicando o filtro de Hodrick-Prescott. O objetivo é eliminar componentes de curto prazo, já que a análise de convergência se refere ao longo prazo.

Em seguida construímos as trajetórias de transição relativa (h_{it}) segundo a equação (15). Algumas delas (restringimos as trajetórias no gráfico apenas para facilitar a visualização; não eliminamos trajetórias que influenciam a análise posterior) estão apresentadas no Gráfico 1.

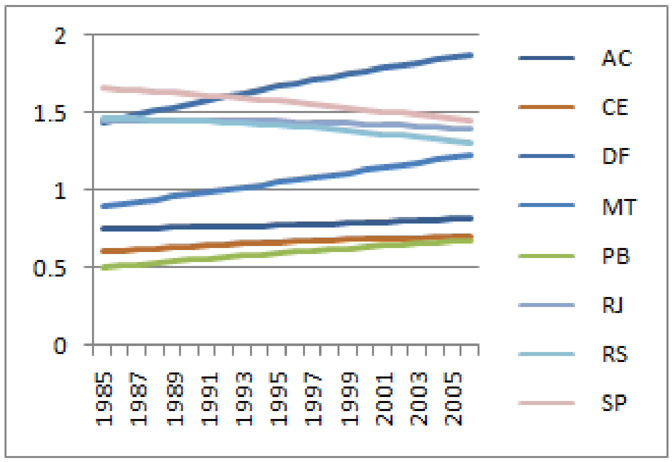

Gráfico 1: Trajetórias de transição de alguns estados

Nota-se que o DF (série mais acima) tem o maior PIB per capita relativo desde 1991, e parece que no longo prazo essa série diverge das demais; outras séries na parte superior (como SP, RJ) parecem ter sua participação na média diminuída, enquanto que o contrário ocorre com as séries na parte inferior, como AC e CE. Essa análise parcial sugere a existência de clubes de convergência.

Um ponto que pode ser notado quando todas as trajetórias de h_{it} são consideradas é que muitos estados brasileiros (11 no total) têm PIB per capita abaixo da média nacional desde 1985: MA, PA, PI, PB, AL, CE, RN, AC, PE, RR, BA; enquanto 9 estados e o Distrito Federal estão acima da média em todo o período: SP, RJ, ES, SC, RS, PR, MS, MG, AM; destes, apenas o DF apresenta tendência crescente. RO e SE estiveram pouco acima da média apenas nos 3 primeiros anos da amostra; a tendência para estes estados tem sido de convergência abaixo da média nacional.

Ao executarmos o teste econométrico de convergência para a amostra completa, obtivemos o seguinte resultado:

$$\log \frac{H_1}{H_t} - 2\log\log t = \underset{(-3.07)}{-0.19} - \underset{(-25.05)}{0.58}\log t \qquad (18)$$

O valor da estatística-t para o parâmetro α indica que convergência total é rejeitada mesmo a 1% para a amostra completa. Prosseguimos então com a

análise de convergência em subamostras. Os resultados das estatísticas passo-a-passo, bem como os gráficos de trajetórias de clubes, estão no apêndice. O resultado final pode ser resumido na Tabela I.

Tabela I: Resultados do teste de convergência

Clubes	Estados	b	$t-estat$
Clube 1	ES, GO, MT, RJ, SC, SP	0.51	6.20
Clube 2	PR, RS	1.08	7.04
Clube 3	AC, AM, MA, MG, MS, PB, RN, RO, RR	0.15	2.15
Clube 4	BA, CE, PE, PI, SE	0.45	6.52
Clube 5	AL, PA	0.03	0.37
Estados divergentes	AP, DF	-	-589.5

Em comparação com estudos anteriores, este teste se mostrou mais sensível a identificação de múltiplos equilíbrios de longo prazo. Não temos apenas dois clubes como na maioria dos estudos, mas cinco. Pela composição desses clubes percebe-se que um componente espacial influencia convergência. Note que o clube 2 é formado apenas por estados do sul enquanto o clube 4 apenas por estados do nordeste; já o clube 1 mistura estados ricos do sudeste (cuja participação na média nacional vem diminuindo), com estados em crescimento do centro-oeste e sul brasileiro. O clube 3 inclui estados cuja renda per capita é comparável a média nacional (MG, MS) e estados cuja participação na média vem aumentando; este clube, no entanto, se diferencia do anterior (clube 1) por apresentar convergência para um equilíbrio abaixo da média nacional (MA, RO, PB, AC, RN, RR). Logo a influência espacial não é a única a determinar convergência; outras características como instituições, nível de escolaridade, solos adequados a culturas lucrativas aproximam a renda per capita entre os estados.

Um ponto relevante ao observamos a formação dos clubes (e os gráficos no apêndice) é que em todos eles os estados que tinham maior participação na renda média nacional em 1985 continuam tendo a maior participação em 2006, no entanto, esta participação tem tendência decrescente. Isto indica que em nenhum dos clubes o equilíbrio de longo prazo se dá em um ponto acima da participação do estado mais rico na média em 1985.

6 Conclusão

Nosso objetivo foi o de estudar o comportamento de convergência da renda per capita estadual brasileira entre 1985 e 2006, por meio de uma abordagem mais genérica. Utilizamos o arcabouço de PS, que se baseia em: (a) dados provenientes de painel, (b) um modelo não-linear, variante no tempo, e (c) um teste econométrico capaz de rejeitar ou não a hipótese de convergência total, ou formação de clubes, baseado na evolução da variância. A vantagem desta abordagem em relação a anteriores é a de não ser necessário que se identifique variáveis que expliquem convergência na regressão de teste; e ainda, a de não assumir propriedades de estacionariedade ou não estacionariedade de componentes

subjacentes. A convergência segundo o teste de PS ocorre como consequência da diminuição da variância entre as trajetórias de h_{it} (no nosso caso, da participação da renda do estado na média nacional) na amostra.

Para que o arcabouço de PS possa ser aplicado é preciso que a variável de interesse possa ser decomposta em um componente comum e outro idiossincrático, o que mostramos ser possível para a renda per capita na seção 2.

Os resultados encontrados pelo teste econométrico apontam para a formação de múltiplos equilíbrios no longo prazo. Ao todo foram destacados 5 clubes de convergência além de dois elementos fora de quaisquer clubes. Verificando os estados que compõem cada clube, notamos que o componente espacial é importante mas não determinante para a formação dos clubes. Estados do norte e nordeste aparecem isolados nos clubes 4 e 5 convergindo para valores abaixo da média nacional; e misturados a MG e MS no clube 3, onde a renda per capita parece convergir no longo prazo para a média nacional. Estados do sul cuja renda per capita vêm diminuindo em relação a renda média nacional (PR, RS) compõem o clube 2. Já o clube 1 reúne os estados do sudeste (SP, RJ, ES), centro-oeste (GO, MT) e sul (SC) cuja renda per capita parece convergir para um valor acima da média nacional. O estado do AP foi discriminado como divergente por aumentar a variância das trajetórias de renda sempre que comparado a núcleos dos grupos. O mesmo acontece com o DF.

Pelos gráficos no apêndice notamos que uma tendência dos elementos em cada clube é a diminuição da participação dos estados mais ricos e o aumento da participação dos estados mais pobres na média da renda nacional. A única exceção é o Distrito Federal que diverge totalmente dos outros estados na amostra, se dirigindo para um nível de renda muito superior a média nacional.

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7 Apêndice

7.1 Apêndice A: Descrição dos Dados

Utilizamos as séries de renda real per capita dos estados brasileiros.

nome original da série: PIB Estadual per capita - R$ de 2000(mil) Deflator Implícito do PIB nacional.

fonte: IBGE (Instituto Brasileiro de Geografia e Estatística)

período: 1985 - 2006 (base anual)

Lista dos estados e sigla correspondente: Acre (AC), Alagoas (AL), Amazonas (AM), Amapá (AP), Bahia (BA), Ceará (CE), Distrito Federal (DF), Espírito Santos (ES), Goiás (GO), Maranhão (MA), Minas Gerais (MG), Mato Grosso do Sul (MS), Mato Grosso (MT), Pará (PA), Pernambuco (PE), Piauí (PI), Paraná (PA), Rio de Janeiro (RJ), Rio Grande do Norte (RN), Rondônia (RO), Roraima (RR), Rio Grande do Sul (RS), Santa Catarina (SC), Sergipe (SE), São Paulo (SP).

(*) O estado de Tocantins não foi incluído na pesquisa por ter sido criado apenas a partir de 1988, e só existirem dados para este a partir de 1989. Devemos considerar na análise que por esse mesmo motivo o estado de Goiás teve seu tamanho reduzido em 1988.

7.2 Apêndice B: Algoritmo de Convergência de Phillips e Sul

O algortimo de PS para testar convergência segue os passos abaixo:

passo 1: Ordenar dados. De acordo com a última observação no painel (período T) todas as séries são ordenadas de forma decrescente. Isso cria uma relação de vizinhança entre as séries. A idéia por trás da ordenação é a de aproximar séries que estão próximas em T porque se existe convergência esta é mais aparente dentre as últimas observações no painel.

passo 2: Determinar a série base. A série base é a primeira série cujo teste de convergência não rejeita sua convergência com a série subsequente. Se nenhuma série base for encontrada, o algoritmo indica divergência entre todas as trajetórias no painel e termina a execução.

passo 3: Determinar o núcleo. O teste econométrico (usando a série base e séries subsequentes uma a uma) é executado até que se rejeite convergência a 5% ($t_{\hat{b}} < -1.65$). O núcleo será formado pelas séries que estavam na regressão quando o maior $t_{\hat{b}}$ foi encontrado.

passo 4: Obter o complemento do núcleo. As séries que não estão no núcleo, formam o complemento do núcleo.

passo 5: Tentar estender o núcleo. O teste de regressão $\log t$ é executado, a cada momento utilizando como base de dados as séries do núcleo e apenas uma de seu complemento (o chamado núcleo aumentado). Os núcleos aumentados associados a testes com estatísticas $t_{\hat{b}}$ superior a um valor crítico (este valor deve ser maior que -1.65, normalmente se inicia com 0) formam o núcleo estendido.

Desta forma, o núcleo estendido é formado por todas as séries do núcleo original e as séries do complemento do núcleo para as quais a hipótese de convergência não foi rejeitada quando foram individualmente inseridas no núcleo aumentado. Por fim o teste $\log t$ é executado novamente usando o núcleo estendido; se $t_{\hat{b}} < -1.65$, a hipótese de convergência é rejeitada (porque mesmo que a convergência não tenha sido rejeitada individualmente esta não foi mantida com o núcleo estendido); neste caso é necessário reiniciar esse passo, aumentando o valor crítico (que originalmente era 0) em 0.5. No entanto, se $t_{\hat{b}} > -1.65$, a hipótese de convergência não foi rejeitada para o núcleo estendido, o que significa que este é um clube. Note que é possível que nenhum núcleo estendido exista; neste caso somente as séries do núcleo original formam um clube convergente. Depois de definido um clube, as séries que fazem parte deste devem ser retiradas da amostra, para que novos clubes sejam buscados.

passo 6: Buscar novos grupos de convergência. Se mais de uma série permanece na amostra, o teste de convergência deve ser aplicado usando todas elas. Se a convergência for rejeitada, o algoritmo deve ser reiniciado no passo 2. Se somente uma série permanece na amostra esta diverge dos demais elementos e se termina a execução.

Este algoritmo determina se existe convergência total ou em clubes e mesmo divergência entre as séries.

O algortimo de convergência determina um clube escolhendo séries cuja variância entre suas trajetórias de transição relativa e a trajetória de transição relativa do núcleo decresce com o tempo.

7.3 Apêndice C: Resultados do Teste PS

Aplicando a regressão $\log t$ para toda a amostra rejeitamos convergência total a 1%. Por isso, seguimos o algoritmo e continuamos a aplicar o teste de convergência em subamostras.

A Tabela C-I mostra os resultados de convergência para o grupo 1. Notamos que SP forma a base do grupo, e o núcleo além da base inclui RJ. No fim da execução do algoritmo, para a subamostra iniciada por SP, um grupo formado por : SP, RJ, SC, ES, MT, GO é determinado; sua estatística $t = 6.2$. As trajetórias fora deste grupo (complementares) quando utilizadas na regressão retornam estatística $t = -25.8$, i.e. rejeitam convergência. Neste caso subamostras dentro do grupo complementar devem ser investigadas para a busca de novos clubes.

Table C-I: Resultados parciais de convergência - grupo 1

Ordena		t valor			Teste
séries	UF	Passo 1	Passo 2	Grupos	$\log t$
1	DF		-51.4		
2	SP	Base	Núcleo	S_1	t_{S_1}
3	RJ	9.1	Núcleo	S_1	$= 6.2$
4	SC	4.9	4.9	S_1	
5	RS	-3.7	-12.5	S_1^c	
6	ES		5.5	S_1	
7	PR		-1.1	S_1^c	
8	MT		8.2	S_1	
9	MS		-12.1	S_1^c	
10	AM		-51.1	S_1^c	$t_{S_1^c}$
11	MG		-62.5	S_1^c	$= -25.8$
12	GO		2.8	S_1	
13	AP		-136.3	S_1^c	
14	RO		-14.2	S_1^c	
15	RR		-5.4	S_1^c	
16	SE		-29.8	S_1^c	
17	AC		-8.8	S_1^c	
18	BA		-32.8	S_1^c	
19	PE		-72.5	S_1^c	
20	RN		-11.8	S_1^c	
21	PA		-154.9	S_1^c	
22	CE		-18.2	S_1^c	
23	PB		-9.8	S_1^c	
24	AL		-31.1	S_1^c	
25	MA		-2.9	S_1^c	
26	PI		-6.5	S_1^c	

O gráfico 2 abaixo representa as trajetórias de transição relativas (h_{it}) de cada estado no gupo 1. Notamos que a convergência de renda se dá em um nível superior a renda média nacional.

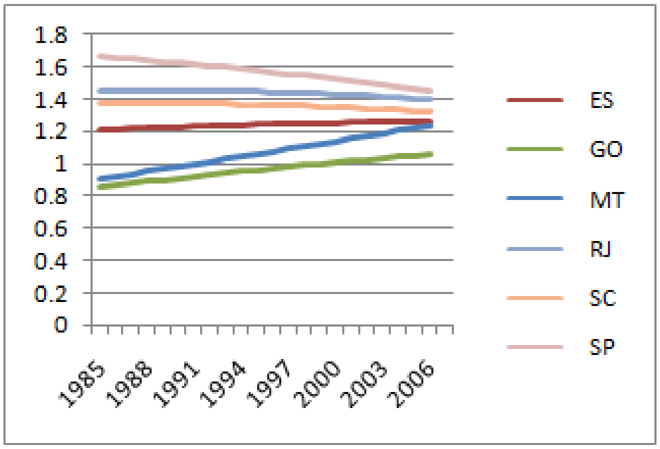

Gráfico 2: Trajetórias de transição relativas grupo 1

A Tabela C-II mostra os resultados de convergência para o grupo 2. Notamos que RS forma a base do grupo, e o núcleo além da base inclui PR. Nenhuma outra série pode ser inserida neste clube sem causar aumento de variância superior ao aceito pelo teste. Logo o grupo 2 é formado apenas por RS e PR, com estatística $t = 7.2$. As demais trajetórias fora deste grupo (complementares) quando utilizadas na regressão retornam estatística $t = -33.3$, i.e. rejeitam convergência. Neste caso subamostras dentro do grupo complementar continuam a ser investigadas.

Table C-II: Resultados parciais de convergência - grupo 2

Ordena		t valor			Teste
séries	UF	Passo 1	Passo 2	Grupos	$\log t$
1	DF		-112.1	S_2^c	
2	RS	Base	Núcleo	S_2	t_{S_2}
3	PR	7.1	Núcleo	S_2	$= 7.1$
4	MS	-0.1	-0.1	S_2^c	
5	AM	-14.3	-17.1	S_2^c	
6	MG		-11.4	S_2^c	
7	AP		-70.1	S_2^c	
8	RO		-9.2	S_2^c	
9	RR		-2.9	S_2^c	
10	SE		-22.4	S_2^c	$t_{S_2^c}$
11	AC		-4.7	S_2^c	$= -33.3$
12	BA		-24.1	S_2^c	
13	PE		-47.7	S_2^c	
14	RN		-7.3	S_2^c	
15	PA		-107.4	S_2^c	
16	CE		-12.6	S_2^c	
17	PB		-6.1	S_2^c	
18	AL		-25.7	S_2^c	
19	MA		-0.9	S_2^c	
20	PI		-3.9	S_2^c	

O gráfico 3 representa as trajetórias de transição (h_{it}) de cada estado do gupo 2. Notamos que estes estados do sul, têm sua participação na renda nacional diminuindo ao londo do período; mesmo assim ainda convergem para um nível de renda acima da média nacional.

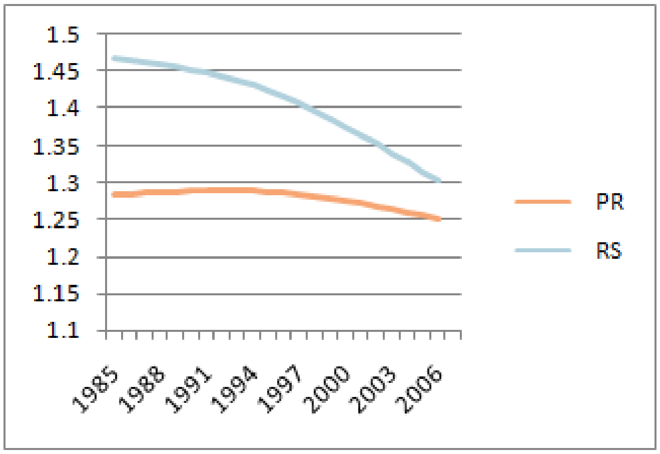

Gráfico 3: Trajetórias de transição relativas grupo 2

A Tabela C-III mostra os resultados do teste de convergência para uma amostra que exclui os estados já no grupo 1 e 2. Um terceiro grupo é identificado formado por: AC, AM, MA, MG, MS (base), PB, RN, RO, RR. A estatística $t = 2.1$ indicando convergência, mas a estatística do grupo complementar ainda indica divergência ($t = -902.3$). Novos grupos devem ser buscados dentro da subamostra resultante.

Table C-III: Resultados parciais de convergência - grupo 3

Ordena séries	UF	t valor Passo 1	Passo 2	Grupos	Teste $\log t$
1	DF		-72.6	S_3^c	
2	MS	Base	Núcleo	S_3	t_{S_3}
3	AM	6.8	Núcleo	S_3	$= 2.1$
4	MG	6.9	Núcleo	S_3	
5	AP	-19.5	-19.5	S_3^c	
6	RO		1.3	S_3	
7	RR		2.5	S_3	
8	SE		-13.3	S_3^c	
9	AC		2.8	S_3	
10	BA		-11.3	S_3^c	$t_{S_3^c}$
11	PE		-34.5	S_3^c	$= -902.3$
12	RN		0.6	S_3	
13	PA		-421.1	S_3^c	
14	CE		-3.1	S_3^c	
15	PB		0.2	S_3	
16	AL		-19.7	S_3^c	
17	MA		1.3	S_3	
18	PI		-0.5	S_3^c	

O gráfico 4 representa as trajetórias de transição (h_{it}) de cada estado do grupo 3. Estados nesse grupo parecem convergir para a renda média nacional.

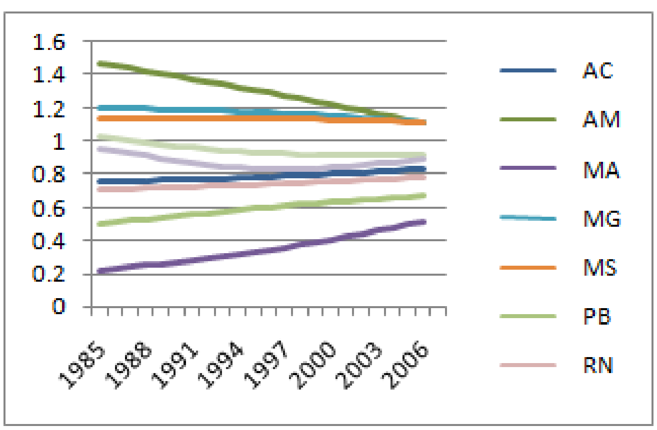

Gráfico 4: Trajetórias de transição relativas grupo 3

A Tabela C-IV mostra os resultados de convergência para o grupo 4. Notamos que SE forma a base do grupo, e o núcleo além da base inclui BA. A estatística $t = 6.5$ indica convergência para o grupo formado por: SE, BA, PE, CE, PI. As demais trajetórias fora deste grupo (complementares) quando testadas quanto convergência retornam estatística $t = -604.5$, i.e. rejeitam convergência.

Table C-IV: Resultados parciais de convergência - grupo 4

Ordena séries	UF	t valor Passo 1	Passo 2	Grupos	Teste $\log t$
1	DF		-108.1	S_4^c	
2	AP		-4.3	S_4^c	t_{S_4}
3	SE	Base	Núcleo	S_4	$= 6.5$
4	BA	39.9	Núcleo	S_4	
5	PE	5.6	5.6	S_4	
6	PA	-7.7	-9.9	S_4^c	
7	CE		12.6	S_4	
8	AL		-26.4	S_4^c	$t_{S_4^c}$
9	PI		6.4	S_4	$= -604.5$

O gráfico 5 representa as trajetórias de transição (h_{it}) de cada estado do gupo 4. Estados nesse grupo convergem para uma renda inferior a média nacional.

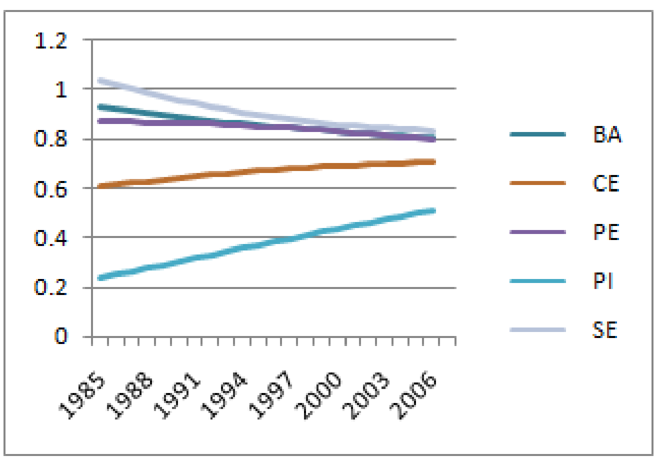

Gráfico 5: Trajetórias de transição relativas grupo 4

A Tabela C-V mostra os resultados do teste de convergência para os estados que ainda não pertencem a nenhum grupo. Um quinto grupo é identificado formado por: PA, AL; com estatística $t = 0.37$. Estados fora deste grupo apresentam estatística t que rejeita convergência ($t = -589.5$).

Table C-V: Resultados parciais de convergência - grupo 5

Ordena séries	UF	t valor Passo 1	t valor Passo 2	Grupos	Teste $\log t$
1	DF		-200.1	S_5^c	$t_{S_5^c}$
2	AP		-15.5	S_5^c	$= -589.5$
3	PA	Base	Núcleo	S_5	t_{S_5}
4	AL	0.3	Núcleo	S_5	$= 0.37$

O gráfico 6 apresenta as trajetórias de transição (h_{it}) de cada estado do gupo 5. Estados nesse grupo convergem para uma renda inferior a média nacional. Mas não podem ser incluídos no grupo 4 pois aumentam a variância para um nível intolerável em relação ao núcleo de 4.

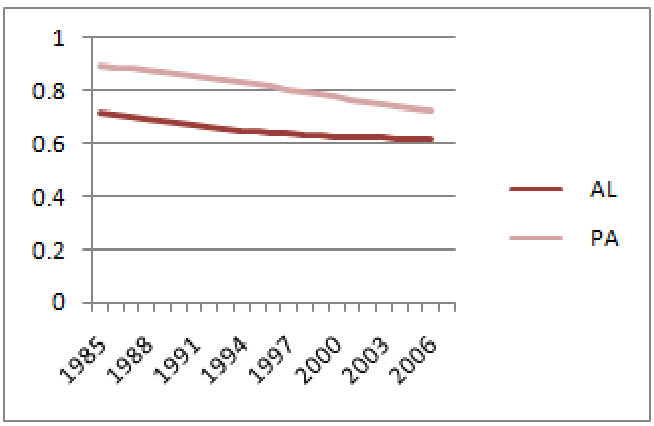

Gráfico 6: Trajetórias de transição relativas grupo 5

As trajetórias ainda não pertencentes a nenhum grupo são AP e DF. Quando testadas conjuntamente divergem ($t = -589.5$). O gráfico 7 apresenta as trajetórias de transição (h_{it}) de cada uma. O DF apresenta renda per capita quase duas vezes superior a média nacional e com tendência de crescimento, enquanto o AP permanece com renda próxima a média nacional, tendo decrescido sua participação a partir de 2000.

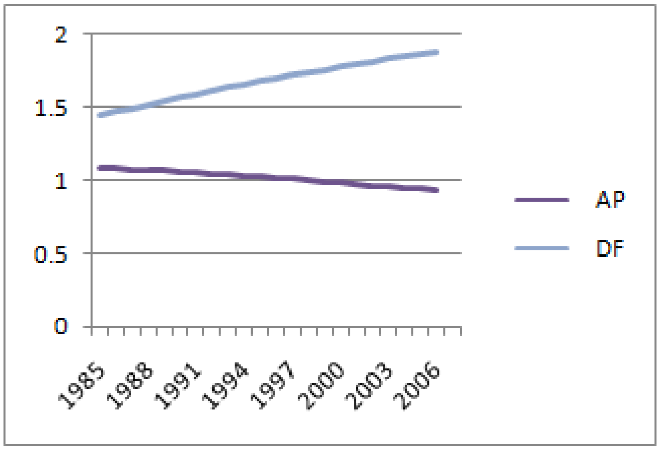

Gráfico 7: Trajetórias de transição relativas divergentes

Testing for Clubbing Behavior among Professional Forecasters

October 26, 2010

Abstract

This article studies the behavior of professional forecasters examining their predictions over time with the intention to investigate whether consensus or the mean forecast value is a significant summary of each panel. We start assuming that each individual's optimal forecast, given some weak conditions, can be disaggregated into common and idiosyncratic terms. Consequently we can apply a very general formulation proposed by Phillips and Sul's (2007) to analyze convergence. Their econometric model and regression-based test are appropriate to study a panel of heterogenous individual behavior and convergence or clusters' formations over time. We applied Phillips and Sul's (2007) model to the Survey of Professional Forecasters dataset (provided by the Federal Reserve Bank of Philadelphia) to study forecasters' behavior when their predictions of 4 American macroeconomic indicators (which are among the most related to the economic activities) are followed: inflation rate (measured by the consumer price index), industrial production, real gross domestic product and unemployment rate. From the econometric test's results we have some evidence of clustering formations among forecasters for all the variables and although no overall convergence is detected, divergence is rare among individuals; hence consensus prevails.

Keywords: Behavior of professional forecasters, clustering formation, non-linear time-varying factor model.

J.E.L. Codes: C23, D71, D81

1 Introduction

[1]Macroeconomic variables forecast receives a great deal of attention from both public and private sectors. Decision makers in both areas typically guide themselves using the information provided by panels of forecasters. Inflation, unemployment, industrial activity rate and real gross domestic product are among the economic indicators followed closely by agents who take decisions based on the economic outlook. Usually, in a panel of predictions, the mean is used as a summary of the forecasters' belief, as if this average was an appropriate aggregator of the whole information set. In this paper we aim to investigate anonymous forecasters behavior, specifically, if their predictions over time will lead to convergence or clusters formation among them. Furthermore we want to check if a special kind of cluster, one that converges around the mean forecast, is observed in the panels and holds the majority of individuals' predictions in which case the mean forecast represents a good measure of consensus.

Studies related to consensus behavior sometimes define consensus according to different properties. In 1985, Zarnowitz studied forecasts in the ASA-NBER panel, by comparing the performance of the mean forecast value with that of individual forecasters when confronted with the actual value observed afterwards. In his study he defined consensus as the mean forecast observed in the panel. Lahiri and Teigland (1987) and Schnader and Stekler (1991) associated consensus with some distribution properties of the forecast series, which should be unimodal, symmetric, and at least as peaked as the normal density for consensus to exist. They tested their concept using ASA-NBER survey panel and found a lack of consensus for many macroeconomic forecasts.

Using the mean value definition, Gregory, Smith and Yetman (2001) developed a framework to formally test the appropriatness of the mean as an indicator of consensus. According to their model the consensus hypothesis is not rejected if individuals' forecasts give the same weight to some common component, and differ from the sample average at any point in time only by an additive, mean zero, idiosyncratic term. In their study, consensus was described as convergence to average forecast.

The way we define consensus in this paper also refers to the capacity of the average to summarize the content of a sample. As Gregory et al (2001)

[1]This article was jointly made with Luiz Renato Lima and Murillo Campello.

we also focus on convergence to the average forecast over time, which means we also modeled forecasters' behavior in accordance to some econometric equation. Nevertheless we propose a distinct approach based on a much more general framework developed by Phillips and Sul (2007). Our contribution in this article is to adapt forecasters' predictions to their model, and then, after running their econometric convergence test, assess the use of the mean sample value as an adequate consensus measure.

Phillips and Sul's (PS, hereafter) framework is composed by a non-linear time-varying factor model and a regression-based test. The former is used to describe a wide range of possible time paths which can be decomposed into a common and an idiosyncratic component. The latter is used to appraise the possible existence of convergence clusters in a sample; the test ultimately assesses the behavior of the idiosyncratic components in each time path. These two parts of their framework are very suitable to our analysis. First, individual's behavior need some flexibility to be modeled. Second, as individual's behavior may be decomposed into common and idiosyncratic terms (which will be seen later), PS's econometric convergence test can be appropriately used to check convergence and divergence in the panel. What is more, their test has the advantage of doing without any stationarity properties of the underlying series. Hence, as PS's framework lends itself perfectly to modeling forecasters' behavior, we chose to follow them.

In order to consider whether the mean sample value is an appropriate measure of consensus, we just need to verify the econometric test's result. If the test cannot reject that the majority of paths in the sample converges towards the average sample value in the long run, we assume consensus cannot be rejected; otherwise (if many convergence clusters are identified; or many individuals diverge; or clusters diverge from the sample average) we assume consensus is rejected in the panel, and the sample mean should not be considered as an appropriate summary of forecasters' predictions.

The data we use to analyze macroeconomic forecasters' behaviors are provided by the Survey of Professional Forecasters. This survey is held since 1990 by the Federal Reserve Bank of Philadelphia, and was formerly (since 1968:Q4) conducted by the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER). It is a quarterly survey, and contains predictions for 27 economic variables; some of them were recently included. We chose the following variables to investigate forecasters' predictions: consumer price index (CPI), industrial production, real gross domestic product (RGDP) and unemployment rate. These were chosen basi-

cally because they closely reflect economic activity, and as such are frequently checked by economic agents.

The convergence test empirically showed that overall convergence behavior was not observed for any of the variables under analysis. Yet, clustering formation could not be rejected and divergent elements accounted for less than 20% of the panel's members. Furthermore, there is one club in each panel that encompassed the majority of forecasters and converged around the mean forecast, ensuring that the average is an adequate indicator of consensus among individuals' predictions.

This article is organized as follows. Section 2 outlines a behavior theory developed to provide an economic rationale to explaining forecasters' attitudes. Section 3 describes the theoretical model that depicts forecasts' time paths and a way to disaggregate them into common and idiosyncratic components. Section 4 considers PS's econometric model and why we can apply it to study forecasters' predictions' behavior. Section 5 addresses the properties of a panel data which accomodate convergence theory and that is used by PS's (2007) econometric test. Section 6 explains PS's econometric convergence test and an algorithm to apply this test repeatedly in order to investigate cluster convergence behavior in a panel. Section 7 provides the results obtained when the convergence algorithm is applied to the Survey of Professional Forecasters. Section 8 presents some concluding remarks. Data and some graphs related to the experiment are reported in appendix C.

2 Related Behavior Theory

In order to explain forecasters' behavior some theories were developed in earlier studies. In 2004, Otaviani and Sϕrensen formulated two theories to interpret the strategic behavior of professional forecasters. In their first theory, in which forecasters competed in a *forecasting contest* with pre-determined rules, they concluded that in equilibrium forecasters differentiated their predictions from competitors by heavily considering their private information than they would in an honest report of their expectations. According to the *reputational cheap talk* theory, in which forecasters wish to promote themselves as being well-informed, market will evaluate them in view of their prediction and the realized state, there is an incentive to herd, which is self-defeating. The incentive to herd comes from the fact that more talented forecasters are on average closer to the realized state. In both theories (the

contest and the reputational) there is an incentive to deviate from honesty, although in the former agents differentiate from each other, while in the latter they herd. These strategic behavior theories consider that individuals are identified when responding a survey. As argued by Croushore (1997), "some [survey] participants might shade their forecasts more toward the consensus (to avoid unfavorable publicity when wrong), while others might make unusually bold forecasts, hoping to stand out from the crowd".

In 2007, Batchelor also studied forecasters' behavior and prediction bias focusing on macroeconomic variables' forecasts. Bias was defined as the final outcome minus the consensus forecast, i.e. bias should be insignificant whenever herd toward the realized value exists. He used data of real GDP and inflation forecasts for all the G7 economies. In several countries he found evidence of bias in the consensus forecast mainly for real GDP growth. However, he stated that the existence of bias does not imply irrational behavior of forecasters. Earlier models were proposed to explain rational bias in the finance and economics literature. Batchelor and Dua (1990b) suggested an explanation which is consistent with rational behavior in a market where forecasters compete to sell their services (as in the *forecasting contest theory*). The biases in favor of optimism and pessimism were attempts by forecasters to differentiate their products, and the failure to converge towards consensus reflected this product differentiation. In 1996, Laster et al developed a model in which the wages firms pay their forecasters are a function of their accuracy as well as the publicity they generate for their employers by being correct. Heuristically, if all forecasters have the same information and prefer accuracy to publicity (as in the *reputational cheap talk* theory) their forecasts will cluster tightly around a consensus. In contrast, when publicity is the aim (differentiation is crucial) forecasters will not want to be in the cluster because their forecasts would attract little or no attention.

Since then, many articles have recognized that forecasters operate in markets, and are subject to incentives of various kinds that might bias their predictions even if information is processed rationally. Although biases were observed in these studies, all of them examined panels in which individuals or firms were identified by the surveys.

In this article we examine a survey which does not disclose forecasters' identities, called anonymous surveys (we can only follow forecasters' predictions because each individual is associated with a number). We hypothesize that when this is the case agents' predictions should be highly correlated across forecasters. Even if consensus is not achieved by an overall convergent

behavior, the majority of agents may compose a cluster. The difference when analyzing an anonymous panel is that as individuals are not identified they have no incentive to "brand" (or differentiate) their product, as publicity is impossible; and at the same time, they have no incentive to convey any private information in their shared forecasts. These two reasons point in the same direction and tend to inhibit outlying predictions, hence reinforcing convergence toward consensus.

The next section presents how a forecasting panel should be modeled in order to achieve the decomposition degree appropriate to apply PS's (2007) framework.

3 The Theoretical Model

In order to study forecasters' behavior, we need a model that represents their predictions over time. The following notation is used in this paper to simplify all the succeeding equations: N individuals observe each variable, which is indexed by i , during T time periods, indexed by t. These indexes come as subscripts of the series under study.

Suppose an agent wants to predict some stationary univariate time series $\{y_t\}_{t=1}^{\infty}$ based on an information set available at time $t-h$, which we will call $\mathcal{F}_{t-h}$. Let $F_{t,t-h}$ denote the cumulative distribution of y_t based on $\mathcal{F}_{t-h}$ and $\hat{y}_{t,t-h}$ the point forecast of y_t conditional on the same information set ($\mathcal{F}_{t-h}$). Each individual makes his optimal forecast of $y_{t,t-h}$, namely $\hat{y}^i_{t,t-h}$, minimizing some loss function L^i. Suppose also that his loss function depends only on the forecast error $e^i_{t,t-h} \equiv y_t - \hat{y}^i_{t,t-h}$. [2]

Definition 1 (Quantile) $\tau_i \equiv \Pr[y_t \leq \hat{y}^i_{t,t-h}|\mathcal{F}_{t-h}] = F_{t,t-h}(\hat{y}^i_{t,t-h})$, in which τ_i corresponds to the conditional probability that y_t is less than the optimal forecast made by the ith individual at time t having information set $\mathcal{F}_{t-h}$.

One can rewrite this definition using the inverse of the cumulative distribution function, and obtain: $\hat{y}^i_{t,t-h} = F^{-1}_{t,t-h}(\tau_i)$. Based on this formula the optimal forecast of an individual conditional on the information set available at time $t-h$ corresponds to the conditional quantile function of y_t at τ_i:

$$\hat{y}^i_{t,t-h} = Q_{y_t}(\tau_i|\mathcal{F}_{t-h}), \text{ for some } \tau_i \in (0,1) \tag{1}$$

[2] According to Patton and Timmermann (2007) this is not a very restrictive assumption as many common loss functions take this form.

This result was first presented by Weiss (1996) and recently by Patton and Timmermann (2007), who showed that as the loss function depends only on the forecast error, the optimal forecast is any measure that summarizes the conditional distribution of the variable under study. Having a continuum of agents, all quantiles of y_t will be described by each individual's choice of optimum value.

Assumption 1: Any quantile of the conditional distribution predicted by the ith individual can be written as the product of another statistic of the conditional distribution (without loss of generality we will use $E(.|.)$) which does not depend on the individual, and an idiosyncratic component that does; namely,

$$f_{it} = \hat{y}^i_{t,t-h} = Q_{y_t}(\tau_i|\mathcal{F}_{t-h}) = \delta_{it} E(y_t|\mathcal{F}_{t-h}) \tag{2}$$

where δ_{it} adjusts the common expected value of y_t to the quantile predicted by the ith individual. We will use f_{it} to represent the forecast of individual i at time t to simplify notation.

From this theorectical model one can decompose individual's forecasts into two distinct parts: one that is completely left for the individual to define (δ_{it}) and the other that is a common factor among them. The common factor is the conditional expectation of the predicted variable given the information available at the moment the prediction is made.

In the next section we describe the econometric model (PS) to investigate convergence behavior among series in a panel where their paths can be modelled in accordante to equation (2), i.e. whenever the panel's series can be decomposed into idiosyncratic and common components.

4 The Econometric Model and Assumptions

An econometric model developed by PS (2007) studies convergence behavior among the series in a panel, whenever its content can be decomposed into an idiosyncratic and a common component. As we described in the previous section, one can model forecasts to follow this approach.

According to PS's (2007) transition modelling technique both common and individual specific components' paths are disaggregated and studied separately in a non-linear time varying factor model. Their approach offers flexibility in idiosyncratic behavior over time while retaining some commonality across the panel by means of an unknown common component.

The model is suitable for both heterogeneous transient behavior and convergence behavior in the long run. One important characteristic that empirical panel models must incorporate to better fit reality is the ability to treat heterogeneous behavior, as the economic theory expects agents to have different conducts. Using the notation we mentioned in the previous section, we present some convenient models.

One popular empirical panel model that implements heterogeneous behavior involves a common-factor structure and idiosyncratic effects. Many research papers in this field are striving to match the econometric methods to theory and to the needs of empirical studies. One simple example of a single-factor model would be:

$$X_{it} = \delta_i \mu_t + \varepsilon_{it} \tag{3}$$

where δ_i measures the systematic idiosyncratic part of X_{it} in terms of the common factor μ_t. The common factor (μ_t) may represent the aggregate common behavior of X_{it} , or any common variable of influence on individual's behavior, such as interest rate, exchange rate or any representative agent's conduct. The model in (3) seeks to capture the evolution of X_{it} in relation to μ_t by means of its idiosyncratic elements: δ_i the systematic element (that weights the common factor for each individual), and ε_{it} the error term.

PS (2007), using panel models, propose an extension to this factor model in two different ways. First, they allow the idiosyncratic part to evolve over time, and as such they accomodate the heterogeneous agent behavior and evolution in this behavior, adding a time component to the latter's purely idiosyncratic element. They called δ_{it} the time-varying factor loading coefficient. They further allow δ_{it} to absorb a random element, the error term in (3); this random component permits a convergence behavior in δ_{it} over time in relation to the common factor μ_t. Their new model can be summarized by the following equation:

$$X_{it} = \delta_{it} \mu_t \tag{4}$$

where we can note that both elements (δ_{it}, μ_t) are time varying, but only the first has idiosyncratic nature. This model can be applied directly to our forecast panel, as forecasts can be decomposed into an idiosyncratic dynamic term and a common component term:

$$f_{it} = \delta_{it} E(y_t | \mathcal{F}_{t-h}) = \delta_{it} \mu_t \tag{5}$$

The component of main interest to convergence studies is δ_{it} because this is the element that captures idiosyncratic behavior. One can interpret δ_{it} as the relative share of the common component (μ_t) that affects individual i at time t. PS (2007) define δ_{it} by a semiparametric form implying non-stationary transitional behavior:

$$\delta_{it} = \delta_i + \sigma_i \varepsilon_{it} L(t)^{-1} t^{-\alpha} \tag{6}$$

where δ_i is fixed, ε_{it} is iid(0,1) accross i and weakly dependent over t, and $L(t)$ is a slowly increasing varying function, for example $L(t) = \log t$, so that $L(t) \to \infty$ as $t \to \infty$. It is clear from the definition (6) that for all $\alpha \geqslant 0$ the loading coefficients δ_{it} converge to δ_i, which turns the value of α a null hypothesis of interest. If this hypothesis holds and $\delta_i = \delta_j$ for $i \neq j$ the model allows transional periods where $\delta_{it} \neq \delta_{jt}$ which means that transitional divergence across i is possible, although in the long run convergence is expected between i and j.

The convergence test for the panel is the second contribution of their research. PS (2007) develop an econometric regression-based test of convergence for the element δ_{it} where the null hypothesis is $H_0 : \delta_{it} \to \delta$ for some δ as $t \to \infty$. The main features of this approach can be listed as: first, the test does not make any assumptions about trend stationarity or stochastic nonstationary in X_{it} or μ_t. Second, the model they propose (4) has a very general form which permits a variety of different paths for δ_{it}. By describing the paths of δ_{it} this model allows a complete analysis of the idiosyncratic behavior of each forecaster.

Consequently, as one can write forecasts using the product of an idiosyncratic element and a common component, PS's non-linear time-varying model and its associated regression-based test of convergence is an appropriate framework to investigate forecasters' behavior.

In the next section we present (a) how to model the components of a series in order to estimate the idiosyncratic part and (b) the characteristics associated to the panel's series that allows checking convergent behavior and group formation among them.

4.1 Relative Transition Curve and Convergence

To estimate the model proposed in equation (4) we need to impose some structure on δ_{it} and $E(y_t|\mathcal{F}_{t-h})$ (corresponds to μ_t) because usually there are

fewer observations in the panel than the number of unknowns in the model. For simplification purposes, PS suggest the use of a relative version of δ_{it}, which they call relative loading or relative transition coefficient (h_{it}), and define as:

$$h_{it} = \frac{f_{it}}{\frac{1}{N}\sum_{i=1}^{N} f_{it}} = \frac{\delta_{it}E(y_t|\mathcal{F}_{t-h})}{\frac{1}{N}\sum_{i=1}^{N} \delta_{it}E(y_t|\mathcal{F}_{t-h})} = \frac{\delta_{it}}{\frac{1}{N}\sum_{i=1}^{N} \delta_{it}} \tag{7}$$

From this formula one can say that h_{it} measures the loading coefficient δ_{it} in relation to its panel average at time t (or, the idiosyncratic behavior at time t in relation to the average idiosyncratic behavior in t). Like δ_{it}, h_{it} traces out a transition path of the ith forecaster, but does so in relation to the panel average. Note that if h_{it} converges to 1 for some individual (or a group of individuals) in the panel, his forecast converges to the average forecast in the sample, which implies consensus behavior by the definition of consensus we follow. We will check this result after appllying PS's convergence econometric test.

In order to study the econometric test one needs to observe some properties of h_{it} that follow from its definition in (7): first, the cross sectional mean of h_{it} is unity; second, if δ_{it} converges to δ for all individuals (the null hypothesis), the relative transition coefficient h_{it} converges to unity, in which case we have overall consensus in the panel. In this case also the cross sectional variance of h_{it} converges to zero, that means:

$$\sigma_t^2 = \frac{1}{N}\sum_{i=1}^{N}(h_{it} - 1)^2 \to 0 \text{ as } t \to \infty \tag{8}$$

This convergence framework allows different relative transition curves during the process of convergence, until it ultimately converges (this is overall convergence).

Note that the definition of relative long-run equilibrium between two series (f_{it}, f_{jt}) :

$$\lim_{k\to\infty} \frac{f_{it+k}}{f_{jt+k}} = 1, \text{ for all } i \text{ and } j \tag{9}$$

guarantees variance convergence as stated in (8). According to (4), (9) is equivalent to a convergence of the factor loading coefficients:

$$\lim_{k\to\infty} \delta_{it+k} = \delta \tag{10}$$

As stated, we would expect dispersion decreases for the relative transition curves along time if convergence is the final result.

Using the same equations presented in this section we could observe convergence behavior among some elements of the sample, forming a group, in which case, we would state that:

$$\sigma_{tG}^2 = \frac{1}{N_g} \sum_{i\epsilon N_g}^{N} (h_{it}^G - 1)^2 \to 0 \text{ as } t \to \infty \tag{11}$$

where N_g is the number of individuals in the group.

This variance decrease (associated to the relative transition path) is the property the authors use to test the null hypothesis of convergence in a regression based test; it is also the hypothesis used to group agents into convergence clusters. Hence, as long as the test identifies some individuals with decreasing variance related to their relative idiosyncratic behavior, it points out the individuals that attribute similar weights to the common component and therefore forms a group.

Empirically, h_{it} can be calculated directly from a sample, but for a long run analysis, as is the case of convergence studies, it will be convenient to remove business cycle components from the data, to keep (4) appropriate. PS (2007) suggest the use of the Hodrick-Prescott filter which will give an estimate of the product $\hat{\theta}_{it} = \widehat{\delta_{it}\mu_t}$. The Hodrick-Prescott filter is suitable for short time series data, and it does not assume any special form of μ_t, which can be a stochastic or deterministic trend process. From the estimate of the product $(\widehat{\delta_{it}\mu_t})$ by the filter, we build a transition coefficient $(\hat{h}_{it})$ which is exactly the term we want to model the behavior. Under the assumption that estimation errors of the estimated common component $(\hat{\theta}_{it})$ are asymptotically dominated by μ_t the consistency of $\hat{h}_{it}$ is easily shown.

5 The Convergence Test

The statistical test of convergence is based on a simple regression equation. The idea behind this equation is to verify whether the variance of the idiosyncratic component $(\hat{h}_{it})$ converges to zero as $t \to \infty$. A noteworthy point

to bear in mind is that this idea is suitable to test both overall convergence and overall divergence. In this last case some convergence clusters may be identified, which indicates variance decreases within groups, however variance never reaches zero when the overall sample is considered.

From the definition of δ_{it} (6) and its convergence aspects the test proposed has power against divergence when $\delta_i \neq \delta$ as well as when $\alpha < 0$. The null hypothesis of interest is given by:

$$H_0 : \delta_i = \delta \text{ and } \alpha \geq 0 \tag{12}$$

while the alternative is: $H_A : \delta_i \neq \delta$ for all i or $\alpha < 0$.

They follow three steps to analyze convergence in the panel. First, they construct the cross sectional variance ratio H_1/H_t, where:

$$H_t = \frac{1}{N}\sum_{i=1}^{N}(h_{it}-1)^2, h_{it} = \frac{f_{it}}{N^{-1}\sum_{i=1}^{N} f_{it}} \tag{13}$$

If convergence exists we expect that H_t decreases over time, hence the cross sectional variance ratio goes to infinity.

Second, they run the following regression and compute a t-statistic for the coefficient $\hat{b}$ using the long run variance of the regression residuals: [3]

$$\log(\frac{H_1}{H_t}) - 2\log L(t) = \hat{a} + \hat{b}\log t + \hat{u}, \tag{14}$$

$$\text{for } t = [rT], [rT]+1, ...T \quad \text{with } r > 0 \tag{15}$$

where we can set $L(t) = \log t$; and the coefficient $\hat{b} = 2\hat{\alpha}$, where $\hat{\alpha}$ is an estimate of α in H_0. Note that this regression uses data from a fraction ($[rT]$) of the sample until its end. The authors suggest $r = 0.3$. This value was proposed after a Monte Carlo simulation, in which for small or moderate T (that is, $T \leq 50$),.this value for r still secures size accuracy in the test for small α values.

The third and last step is to verify, based on the t-statistic of $\hat{b}$ (computed using a HAC standard error), if convergence is not rejected, that is, if $\alpha \geq 0$. The null hypothesis of convergence is rejected if $t_{\hat{b}} < -1.65$, at a 5% level.

[3] For a complete explanation about how to achieve the following regression equation refer to Phillips and Sul (2007) mathematical appendix section.

The rejection of the null hypothesis does not mean there is no convergence; there may be so in subgroups of the panel. To find out if this is the case, an algorithm which determines groups of convergence behavior based on the sample data and the same regression procedure is described in the appendix.

In the next section we present the dataset we used to investigate convergence behavior among forecasters.

6 Empirical Analysis

We use a panel of forecasters' predictions provided by the Survey of Professional Forecasters. This is the oldest quarterly survey of macroeconomic forecasts in the United States. It was first implemented in 1968 by the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER) until 1990 when The Federal Reserve Bank of Philadelphia took over the report.[4] Every three months, the professional forecasters make predictions about 27 economic variables including output, inflation, unemployment, and industrial production. For each variable, the panelists forecast its value for different time horizons: current quarter (defined as the quarter in which the survey is conducted) and for the four quarters after the current one.

In this paper, we focus on forecasts made for the current quarter. It is important to mention that for the surveys conducted after the 1990:Q2, the Federal Reserve Bank of Philadelphia has set the deadlines for responses to be submitted no later than the third week of the second month of each quarter. The panelists' main characteristic is that they are economists of the private sector. As the title of the survey suggests, they are professional forecasters, they produce economic variables forecasts regularly as part of their jobs. The Federal Reserve Bank of Philadelphia keeps the number of panelists in the 30s.

One important characteristic of this survey is the anonymity it provides to its respondents. Each forecaster's identification is kept confidential although an identification number connects forecasts made by the same individual across years. The anonymity is designed to encourage people to provide their best forecasts to their personal capacities, with the possibility that their

[4] For more information about the survey refer to http://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/

prediction may not necessarily coincide with their firm's. The negative side of anonimity is that no one has an incentive to convey any private information into the forecasts as their privilege of having them will not be recognized.

Although the survey began in 1968, we found some obstacles to building balanced panels of data that conforms to our objective, which is: track a great number of individuals (N) and the same individuals for the maximum time period (T). The first obstacle is that for the years the survey was conducted by NBER-ASA the same identification number could represent different forecasters (this is documented in the survey's manual), only after 1990, when the survey was taken over by the Philadelphia Federal Reserve Bank, there was guarantee that the same identification number always represents the same individual, and is never reused. Hence, our panel is restricted to the last 19 years of reports. The second obstacle is that even restricting the sample to the last 19 years respondents vary over the life of a survey: some interrupt their participation and others do not participate every time the survey is held. As we want to extract a common component from all the forecasts in the sample, we need to form a balanced panel with the largest possible period and individuals been tracked. Because of these two obstacles our panels do not include all quarters from 1990 to 2009, and they may be different among distinct macroeconomic variables (different in either number of individuals forecasters (N) and time span (T)).

From the dataset provided by the Federal Reserve Bank and after building balanced panels we choose the following variables to investigate forecaster's behavior: real gross domestic product (RGDP), consumer price index (CPI), unemployment (UNEMP), and industrial production (INDPROD). These were chosen because they are usually refered to as closely related to economic activity. Observing the deadline rule fixed by the Federal Reserve Bank, forecasts made for the current quarter are informed at least one month and a half (nearly 45 days) before the end of the quarter.

Before applying the convergence algorithm, we removed any business cycle component from all of the macroeconomic series under ananlysis. To this end we used the Hodrick-Prescott filter. The filter multiplier was set to 1600, the value recommended for quarterly data.

After these steps we obtained balanced panels with no business cycle; we are then apt to investigate convergence behavior using PS's econometric convergence test. The panels' main characteristics and the results of the convergence algorithm are reported in the next section. For a detailed description of the original dataset see appendix A.

6.1 Results

In this section we present the whole procedure implemented to study convergence behavior among individuals in the Survey of Professional Forecasters (SPF). We start with a description of how the four forecasters' panels (one for each economic variable: CPI, industrial production, RGDP and.unemployment) were extracted from the SPF and prepared to be used as input to the convergence algorithm; then we report and analyze the results of this algorithm when applied to each panel. To shed light on how these outcomes were finally achieved we also report the intermediate algorithm's results and some graphs for all the panels in appendix C.

In order to apply the the convergence algorithm we first need to prepare the dataset provided by the SPF. As discussed in the previous section, preparing the dataset means first identifying balanced panels for each variable while keeping the dimensions of the panel (N and T) above some threshold value. This threshold condition is used to avoid panels with few individuals (N) or few time periods (T) . We choose 10 as a lower bound value for both N and T. Among the balanced panels selected, we considered only those with the highest number of individuals for each variable.

Table I reports the results of this procedure. As we can see from column 2, the largest balanced panels we could extract from the SPF have a varying number of forecasters (N) for each variable: for example, the RGDP has the maximum number of individuals in its panel which is 17. Although the panels differ in the number of individuals, all of them have the same time dimension (column 3) and were obtained within the same time span (including all the quarters in this period except 2007:Q3), but this happened accidentally. From column 6 we note also that all the series could be transformed into its log form except CPI, which contains some negative prediction values.

Table I: Properties of the largest balanced panels

Series	N	T	Time span*	Observations	Function applied
CPI	16	11	2005:Q2 to 2008:Q1	176	none
INDPROD	15	11	2005:Q2 to 2008:Q1	165	log
RGDP	17	11	2005:Q2 to 2008:Q1	187	log
UNEMP	16	11	2005:Q2 to 2008:Q1	176	log

Notes:
N denotes the maximum number of individuals in the panel;
T denotes the maximum number of periods observed;
*Time span does not include 2007:Q3;
Observations are equal to N * T.

As a second preparatory step, before running the algorithm, any temporary component should be removed from the panels' series as convergence analysis is related to long run behavior. This was accomplished by the use of the Hodrick-Prescott filter.

After these two steps have been taken (we have balanced panels with smoothed series) we can apply the convergence test for each of the four forecasters' panels. The first algorithm's step tests the overall convergence; if this hypothesis cannot be rejected, then a sole convergent group (one formed by all forecasters in the panel) is assumed to exist, and this implies consensus all the more evidently. Although, if the hypothesis is rejected, the following algorithm's steps will test convergence in subsamples of the panel, in order to investigate if convergence cannot be rejected in clusters. In the end, the algorithm reports overall convergence or convergence groups with possibly divergent elements outside them, or overall divergence among forecasters.

Table II summarizes the results obtained by the convergence test. The first column presents the series' names, and the second major column presents statistics related to the convergence test when all forecasters are included in the regression analysis (ie. overall convergence test's outcome). These statistics are: the number of individuals in the panel (N) and the t value of the term in the regression equation associated with convergence behavior. Remember that a t value lower than -1.65 indicates rejection of the null hypothesis (convergence hypothesis) at 5% significant level. The next two major columns also present the same statistics when the regression is executed within a convergent and divergent (non-convergent) subpanel respectively. For the convergent panels (column 3) two additional informations

are presented: the number of convergent subpanels identified in the whole sample and the approximate value of the relative transition parameter (h_{it}) in each subpanel. This last value is the one that might show some evidence of a consensus subgroup in the panel, which is a group whose forecasts are becoming close to the average forecast of the whole sample; by definition of h_{it} this means that its value is becoming close enough to 1.

Note that although the number of forecasters in a subpanel may vary, the number of periods (T) remains the same - which means subpanels only restricts the N dimension, named N_g when reporting elements in convergent groups and N_d when refering to divergent elements.

Following the analysis of the table content we reject the null hypothesis of overall convergence among forecasters for all of the macroeconomic series, as the t values reported for all of them (column 3) are less than the critical value (-1.65). However, from the second major column (named Convergent Groups), we notice evidence of clustering behavior among individuals as in each subpanel the hypothesis of convergence could not be rejected (t value$_g$ above the critical value). Also, we can see that one of the convergent groups for each variable has a h_{it} close to 1, i.e. its forecasts are becoming close to the average forecast in the long run. From this perspective we can accept the mean forecast as a representative measure of consensus.

We can also note from this table that consensus groups have the majority number of individuals in their respective panel: for the CPI this number reaches 62%, for the other variables this amount increases to more than 80%. Both real GDP and industrial production have only one convergent group with, respectively, 82% and 86% of the forecasters; few individuals have diverging predictions (17% and 13%). Unemployment also has one convergent group with 81% of the forecasters, although this is not the only convergent group (there is another smaller one) less than 7% of the forecasters diverge. For the CPI predictions, as we mentioned before, 62% of the agents form a convergent group wich comes close to the average forecast; although this is the major group, more than 18% of the forecasters diverge. Importantly, from this picture we draw the conclusion that, for all the variables, consensus (or the mean forecast value) represents a significant statistic. Although we can note that CPI forecasters are more uncertain about the short run than forecasters of the other three variables. This may just reflect the fact that CPI is a more volatile series than others.

Table II: Outcome of PS's convergence test

	Overall Sample		Convergent Groups				Divergent Members	
Series	N	t value	No. of groups	N_g	t value$_g$	h_{it}	N_d	t value$_d$
CPI	16	-113.80	2	3	9.74	> 1	3	-21.39
				10	5.56	≈ 1		
INDPROD	15	-5.57	1	13	-0.64	≈ 1	2	-12.43
RGPD	17	-4.91	1	14	5.90	≈ 1	3	-15.81
UNEMP	16	-7.43	2	13	-0.10	≈ 1	1	NA
				2	3.45	< 1		

This table contains the results of the convergence algorithm when applied to some specific groups of individuals: (1) overall sample, (2) convergent groups, (3) divergent individuals. Overall Sample columns' present the total number of individuals in the sample (N) and t value of the term in the regression that is associated to convergent behavior. Convergent Groups columns' present the number of convergent groups identified by the algorithm, the number of individuals in each convergent group (N_g) and the t-statistic of the term in the regression that is associated to the convergent behavior of the group (t value$_g$). Divergent Individuals columns' present the number of individuals that do not belong to any group (N_d) and the t-value of the term in the regression that is associated to convergent behavior considering a sample with only the divergent individuals. A t-value $<$ -1.65 rejects the convergence hypothesis.

Given the econometric test's outcome our hypothesis that consensus prevails in an anonymous panel could not be rejected. From all the groups identified, we note that clustering behavior characterizes the conduct of the majority of agents in the panels investigated. Therefore, clubbing formation (with or without consensus) qualifies macroeconomic forecasters' behavior in anonymous panels. As consensus groups prevail, the hypothesis that individuals herd exist in anonymous panels cannot be rejected.

To make clear how the algorithm achieved the outcomes in Table II, we present a detailed description of each algorithm's step and its partial outcomes in appendix section C.

7 Conclusion

In this paper, we investigated the behavior of some macroeconomic variables' forecasters, with the aim of examining if their predictions tend to have similar convergence characteristics in the long run and may lead to a consensus when agents are not identified in the surveys. As forecasts can be disaggregated into some common and idiosyncratic components - generalizing the idea that each individual's optimal prediction represents a conditional quantile of the common variable under analysis - we could use a very suitable econometric model proposed by PS (2007), in which a series is decomposed in exactly those parts. Along with the model, these authors also proposed a regression based-test capable of rejecting the null hypothesis of convergence for the series' time paths. Their formulation is particularly useful to measuring individual's transitional behavior over time relative to some common trend component; further their econometric convergence test provides the basis for a clustering algorithm, which can determine club formation and/or divergence behavior among the panel's series. As a byproduct of the regression-based test, a relative factor-loading coefficient is yielded for each convergence club. From this value we are able to determine if consensus is a significant measure to summarize the panels' forecasts.

Our dataset comes from the Survey of Professional Forecasters, which has been provided by the Federal Reserve Bank of Philadelphia since the 1990's. We analyzed the panel of forecasters who predicted CPI, RGDP, industrial production and unemployment. These series were chosen because they are closely related to economic activity.

The results we obtained from the convergence test indicated that although overall convergence could not be observed for any of the series under analysis, clustering formation could not be rejected and consensus clusters were observed in all of them. From the graphs (in appendix C) and numbers in Table II we observe that the consensus groups contain the majority of individuals in the sample, while divergent elements account for less than 20% of the panel's members. Unemployment's mean forecast achieved the highest consensus. RGDP, industrial production and CPI also have consensual predictions, although CPI presents a relatively lower convergence in relation to others.

This clustering behavior and consensus achievement by the majority of forecasters in each panel were our expected result. Since our dataset relies on anonymous predictions, individuals are not encouraged to show a dis-

tinguished knowledge as this will not be recognized. Moreover they have little incentive to convey their private information as this may benefit less informed competitors. Both of these reasons lead forecasters towards consensus behavior.

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8 Appendix

8.1 Appendix A: Data Description

The series of the Survey of Professional Forecasters we present the results in the paper are listed in the following table:

Table A-I: Economic variables under study

Variable	Description
CPI	Forecasts for the CPI inflation rate.
	Seasonally adjusted, annual rate, percentage points.
	Quarterly forecasts are annualized quarter-over-quarter percent changes.
INDPROD	Forecasts for the quarterly average level of the index of industrial production.
	Seasonally adjusted, index, base year varies
RGDP	Forecasts for the quarterly level of real GDP.
	Seasonally adjusted, annual rate, base year varies.
UNEMP	Forecasts for the quarterly average unemployment rate.
	Seasonally adjusted, percentage points.

8.2 Appendix B: Phillips and Sul's algorithm

Phillips and Sul's algorithm to test convergent behavior in a panel runs as follows:

step 1: Ordering the data. According to the last forecast of each individual in the panel (forecasts made at T) all the series should be ordered in a decreasing fashion; this builds some neighboring relation among those series. The idea behind this ordering is that when there is convergence as $T \to \infty$, it is usually more apparent at the last observation.

step 2: Determining a base series. The base series is the first forecast series in the panel such that the test cannot reject the convergence hypothesis (i.e., $t_{\hat{b}} > -1.65$) if the series is included in the log t regression with its subsequent neighbor . If no base series is found, the algorithm indicates divergence behavior among individual's predictions and finishes execution.

step 3: Determining a core group. Run the $\log t$ regression using the base series and its subsequent neighbor. Keep running the regression each time by adding the next neighboring series until the $t_{\hat{b}} < -1.65$. The core is determined from the series that were in the regression with the greatest $t_{\hat{b}}$ value.

step 4: Obtaining a complementary core group. The series that were not in the core group, should be added in a complementary core.

step 5: Trying to extend the core. Run the $\log t$ regression test each time using as the dataset the core and a different series in the complementary core - datasets formed in this way are called augmented cores. Take the datasets

associated to each test that yields a $t_{\hat{b}}$ value above some critical value (here this value should be greater than -1.65, usually it starts from 0) and form an extended core. Accordingly, the extended core contains all the series originally in the core and the series in the complementary core for which the convergence hypothesis was not rejected when they were individually added to the core. Again, run the $\log t$ regression test, using the extended core as the dataset; if the $t_{\hat{b}} < -1.65$, the null of convergence is rejected (because although each augmented core did not reject convergence, the extended version did). We should then restart this step increasing the critival value (that was originally 0) in increments of 0.5. Although, if the $t_{\hat{b}} > -1.65$, the null of convergence will not be rejected for the extended core, which means the extended core is a convergent club. Note that, it may be possible that no extended core exists, if this is the case only the series originally in the core group forms a convergent club. After the club is identified, drop those series from the core and complementary core, if any.

step 6: Searching for new groups of convergence. If more than one series remains, apply the same regression based test to all of them. If convergence is rejected, restart the algorithm in step 2. If only one series remains stop the convergence test. This is a divergent series.

This algorithm can determine if overall convergence, subgroup convergence, or divergence among the forecasters exists in a sample. It is a very convenient method as it uses the same regression and test procedure to group series in a panel. According to the algorithm, a group is composed by elements whose relative transition curves' variance decreases over time. Although the variance is at its minimum value when the panel contains only the core group, some series from the complementary core can still extend the initial core without causing the regression test to reject the convergence hypothesis.

8.3 Appendix C: Phillips and Sul's Algorithm's Results

This appendix reports the step-by-step results of the Phillips and Sul's algorithm when applied to the balanced panels of CPI, industrial production, RGDP and unemployment.

8.3.1 Consumer Price Index Results

Considering the balanced panel with forecasts for the CPI as the dataset, the algorithm checks overall convergence and if this is rejected it searches for club formation. The algorithm runs as follow:

First, the estimated equation to test overall convergence with $r = 1/3$ is:

$$\log \frac{H_1}{H_t} - 2 \log \log t = \underset{(42.0)}{0.74} - \underset{(-113.8)}{0.98 \log t}$$

From the $t_{\hat{b}}$ statistics obtained for the $\log t$ term the null hypothesis of overall convergence in the panel is rejected at the 5% level. Thereby, the next algorithm's steps examines convergence in subsamples of CPI's forecasters panel.

Table C-I reveals the intermediate outcomes of each algorithm's step.

The first result is shown in column 1 and it comes from the ordering of all the forecasters' predictions according to its last time series observation. Based on this ordering, the second step determines a base series - the one for which the $\log t$ test could not reject the convergence hypothesis with its following neighbor. For the CPI's forecasters' panel this happens for ID 483. The next step (step 3) indicates the core, which is a group of consecutive series, including the base series, with maximum $t_{\hat{b}}$ that was found before the regression test rejects the null hypothesis. From column 3 we can see that the t statistics related to this step are $t_{\hat{b}(ID483,ID535)} = 4.3$, $t_{\hat{b}(ID483,ID535,ID512)} = 9.7$, $t_{\hat{b}(ID483,ID535,ID512,ID543)} = -3.0$; hence the core group is taken to be ID 483, ID 535 and ID 512. As a subproduct of this step, we have a complementary core formed by the series that are not in the core (step 4). The next step then is to augment the core each time adding to it a series that is in the complementary core and recalculating the $t_{\hat{b}}$ statistics. These are presented in column 4. Remember that in this step the critical value should be higher than -1.65 to prevent rejection of H_0 when the core is finally extended. After the $t_{\hat{b}}$ is calculated for each augmented core, we extend the core including all the series that obtained a $t_{\hat{b}}$ above the critical value when individually added to the core.

We can see that for the CPI forecasters' panel none of the series in the complementary core could extend the core in the beginning. Hence the first core group is also the first convergence club and its t statistic is shown in column 5 ($t_{S_1} = 9.7$). For the remaining series (in the complementary club) the t statistics clearly rejects overall convergence ($t_{S_1^c} = -37.4$) although club

convergence may exist. To check this possibility, the algorithm restarts. The second core comprises the first five remaining forecasters (ID 543, ID 463, ID 484, ID 521, ID 472) as the maximum $t_{\hat{b}}$ statistic is obtained when the fifth series is added to the regression ($t_{\hat{b}(ID543,ID463,ID484,ID521,ID472)} = 8.7$). This same core is extended to include the next five series which results in a positive $t_{\hat{b}}$ statistics when used in regressions with the augmented core. To properly consider the extended core a convergence club, we check its overall $t_{\hat{b}}$ statistic, as it does not reject the null of convergence (it is $t_{S_2} = 5.56$) this extended core is the second convergence club. Again, the series outside the two clubs form a group that rejects the overall convergence hypothesis ($t_{S_2^c} = -21.4$) and when the algorithn restarts it could not find any base series among them, as such, we say the remaining series (ID 528, ID 516, ID 526) diverge. And the algorithm stops.

Table C-I: Convergence algorithm's intermediate results for CPI

Last T		t value			$\log t$		t value			$\log t$
Order	ID	Step 1	Step 2	Club	Test	ID	Step 1	Step 2	Club	Test
1	id483	Base	Core	S_1						
2	id535	4.3	Core	S_1	t_{S_1}					
3	id512	9.7	Core	S_1	$= 9.7$					
4	id543	-3.0	-3.0	S_1^c		id543	Base	Core	S_2	
5	id463		-4.1	S_1^c		id463	4.3	Core	S_2	
6	id484		-8.8	S_1^c		id484	2.7	Core	S_2	
7	id521		-5.4	S_1^c		id521	0.4	Core	S_2	
8	id472		-8.5	S_1^c		id472	8.7	Core	S_2	
9	id424		-8.0	S_1^c		id424	2.5	2.5	S_2	t_{S_2}
10	id527		-15.0	S_1^c	$t_{S_1^c}$	id527	1.2	1.8	S_2	$= 5.56$
11	id446		-29.1	S_1^c	$= -37.4$	id446	1.2	3.8	S_2	
12	id510		-34.4	S_1^c		id510	4.7	9.4	S_2	
13	id539		-79.5	S_1^c		id539	5.6	3.6	S_2	
14	id528		-42.3	S_1^c		id528	2.3	-1.3	S_2^c	
15	id516		-102.4	S_1^c		id516	-18.0	-48.7	S_2^c	$t_{S_2^c}$
16	id526		-36.5	S_1^c		id526		-35.2	S_2^c	$= -21.4$

Figures 1 and 2 show the relative transition paths of forecasters within each club. From Figure 1 we can note that individuals in the first club present forecasts that are above the average sample value, as their h_{it} paths converge to some value greater than 1. One characteristic that is intrinsic to a club is

that the variance among the individuals' predictions within a club decreases in the long run, this comes as a result of the regression test.

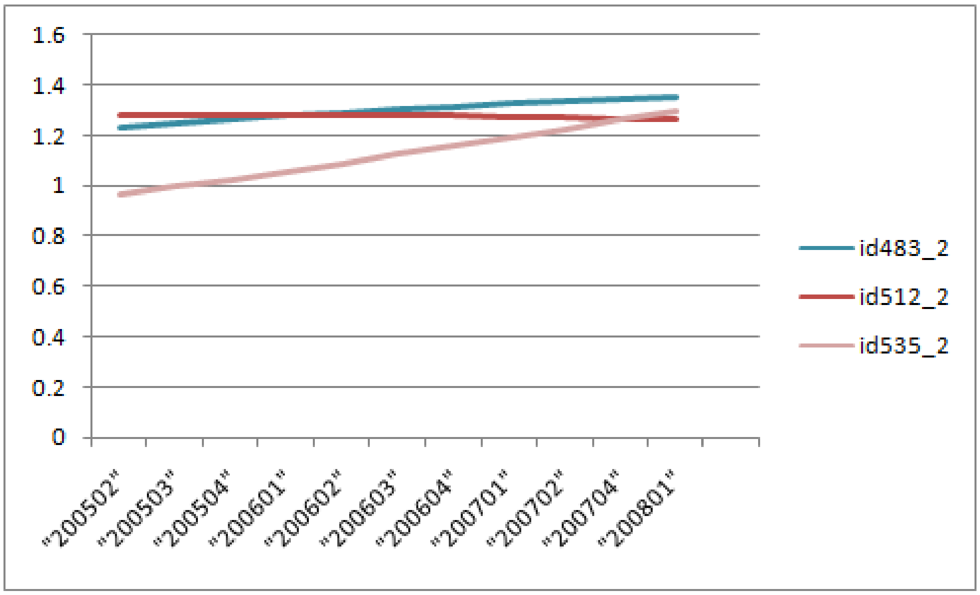

Figure 1: CPI's relative transition paths within club 1.

From Figure 2 we note that the majority of the CPI's forecasters are within the second club, and these forecasters made predictions close to the average sample value as the h_{it} paths approaches one. This is the so-called consensus group.

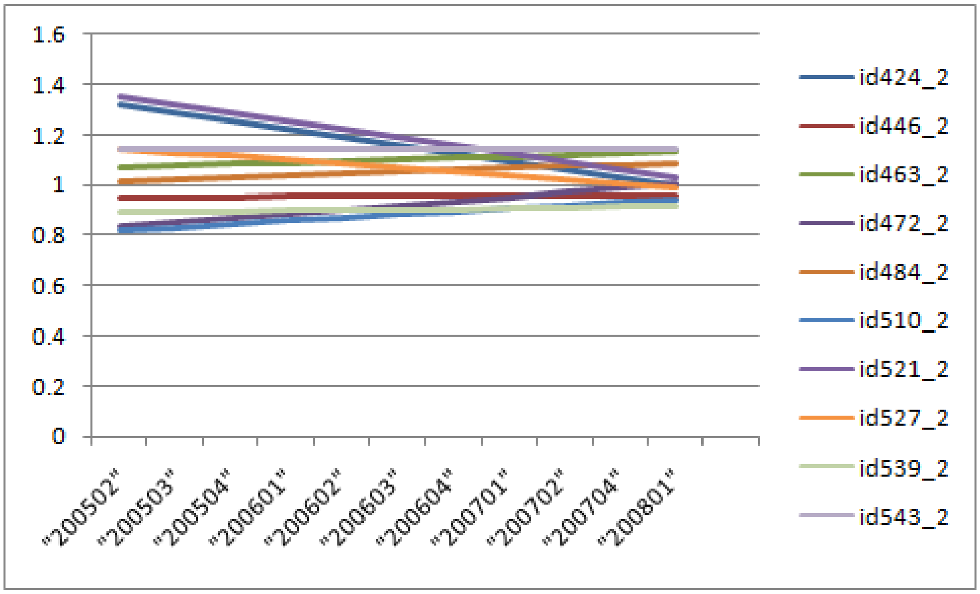

Figure 2: CPI's relative transition paths within club 2.

Figure 3 shows the relative transition paths of 3 divergent forecasters. All of them seem to make predictions below the average sample value, although none of them seem to converge in the long run.

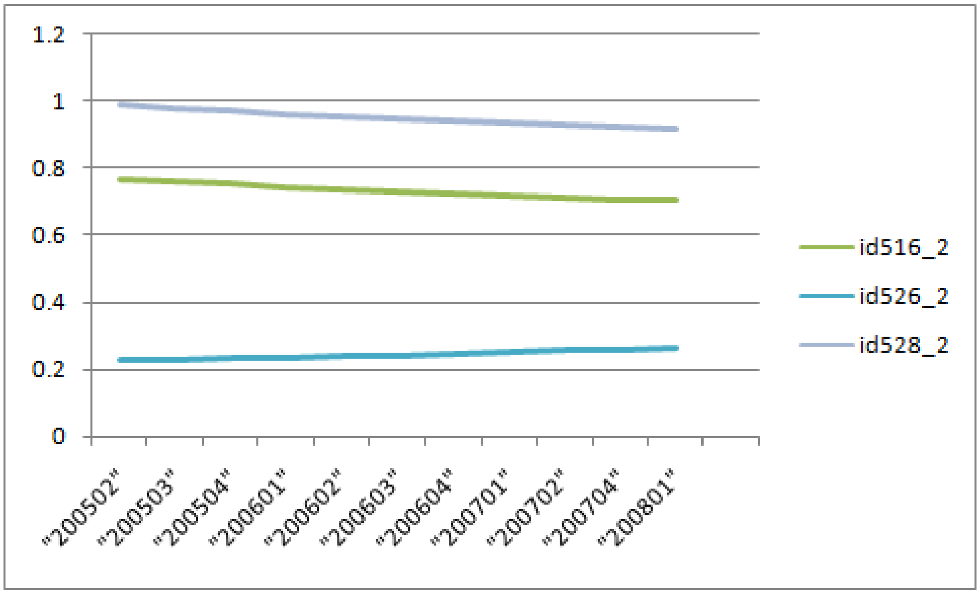

Figure 3: CPI's relative transition paths of divergent forecasters.

The same layout used to present CPI results is used to present industrial production, unemployment rate and real gross domestic product partial outcomes.

8.3.2 Industrial Production Results

Applying Phillips and Sul's (2007) convergence test for industrial production's forecastings, which encompasses 15 individuals in the panel, we obtained the results presented in Table C-II. Column 1 shows the first algorithm step which order all of the series according to their last observation in time. Column 3 identifies the base series and the t statistics of an increasing core group. Column 4 reports the core series and the t statistics of an augmented core, which is formed by the core series and each time a different series of the complementary core (series outside the core). From the t statistics of the augmented core, series for which the null of convergence was not rejected are included in the core, leading to the extended core. Column 6 shows the t statistics of this extended core which is $t_{S_1} = -0.6$. As this t statistic is

above the critical value (-1.65) the extended core is considered the first convergence club. This first club contains 13 individuals whose forecasts seems to converge in the long run. Two individuals rest outside this first club; when the convergence test is applied on them the null hypothesis of convergence is rejected ($t_{S_1^c} = -12.4$); these forecasters are therefore considered divergent elements of the panel.

Table C-II: Convergence algorithm's intermediate results for industrial production

Last T Order	ID	t value Step 1	t value Step 2	Club	$\log t$ Test
1	id446			S_1	
2	id521	Base	Core	S_1	$t_{S_1} = -0.6$
3	id535	3.7	Core	S_1	
4	id472	2.9	Core	S_1	
5	id527	3.3	Core	S_1	
6	id543	4.2	Core	S_1	
7	id424	7.3	Core	S_1	
8	id510	7.2	Core	S_1	
9	id528	8.9	Core	S_1	
10	id463	5.3	5.3	S_1	
11	id539	2.1	6.5	S_1	
12	id483	3.9	7.6	S_1	
13	id484	1.2	4.5	S_1	
14	id516	-2.1	-1.1	S_1^c	$t_{S_1^c} = -12.4$
15	id526		-4.2	S_1^c	

Figure 4 shows the relative transition paths of the forecasters in the first club. Note that the spread among the elements in a club decreases in the long run. This is an expected result for the groups identified by the $\log t$ regression test.

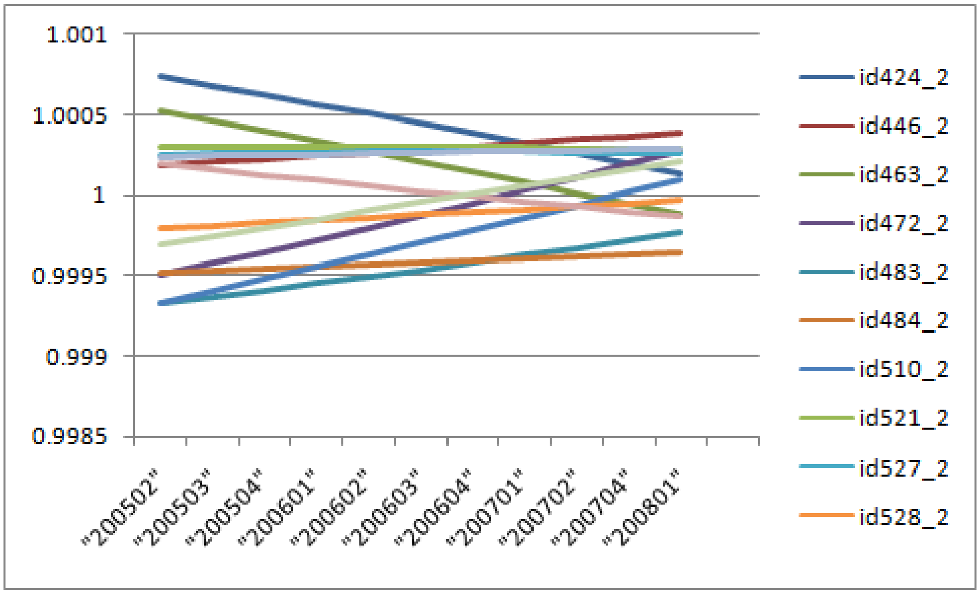

Figure 4: Industrial production's relative transition paths within club 1.

Figure 5 shows the relative transition paths of the 2 divergent forecasters. Note that their predictions are distancing from the average sample and also

from each other.

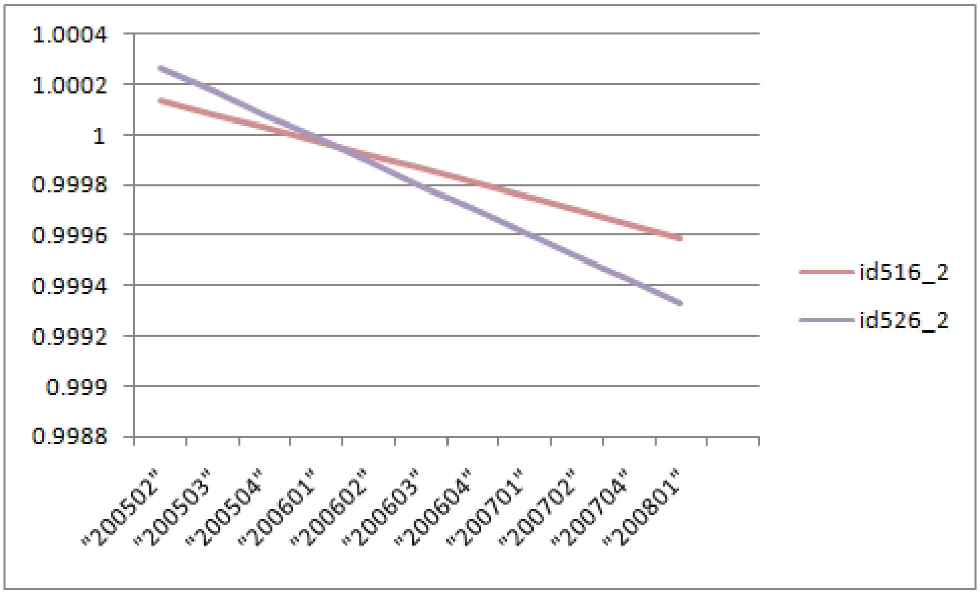

Figure 5: Industrial production's relative transition paths of divergent forecasters.

8.3.3 Real Gross Domestic Product Results

The RGDP panel has 17 forecasters. For this panel one convergence club was identified with 14 individuals. The remaining 3 (id512, id543 and id526) diverged; the algorithm could not find a base series among them. The intermediate results of the convergence algorithm are presented in Table C-III. This table has the same layout used to present partial outcomes in the preceding subsections.

Table C-III: Convergence algorithm's intermediate results for RGDP

Last T		t value			$\log t$
Order	ID	Step 1	Step 2	Club	Test
1	id512		-3.4	S_1^c	
2	id539	Base	Core	S_1	$t_{S_1} = 5.9$
3	id463	-1.2	Core	S_1	
4	id527	4.1	Core	S_1	
5	id528	6.1	Core	S_1	
6	id472	6.9	Core	S_1	
7	id521	8.9	Core	S_1	
8	id446	9.4	Core	S_1	
9	id424	4.2	4.2	S_1	
10	id535	2.7	3.9	S_1	
11	id484	3.8	9.5	S_1	
12	id483	2.5	2.8	S_1	
13	id532	1.6	2.3	S_1	
14	id516	2.2	8.1	S_1	
15	id543	0.3	-0.3	S_1^c	$t_{S_1^c} = -15.8$
16	id526	-0.8	-0.7	S_1^c	
17	id510	0.1	4.9	S_1^c	

Figure 6 depicts the relative transition paths for the first convergence

group.

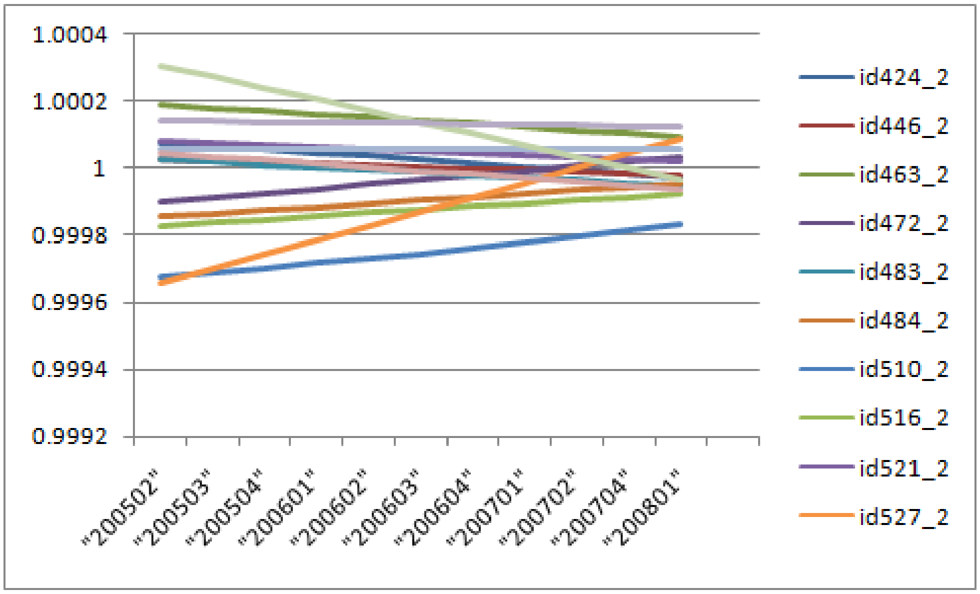

Figure 6: RGDP's relative transition paths within club 1.

Figure 7 shows the relative transition paths for the divergent series. We can clearly note that divergence occurs if we consider this whole group, although it seems that 2 of the individuals in this group converged (id526, id543). To shed more light on this issue, we took a careful look at their series (the numbers), which indicated that the variance was increasing over time: it was initially in the order of 10e-13 and subsequently changed to 10e-10 until it finished the period at 10e-09. Hence, the econometric convergence test is very sensible to variance surges and this is the reason why convergence was rejected even for these 2 individuals' time paths.

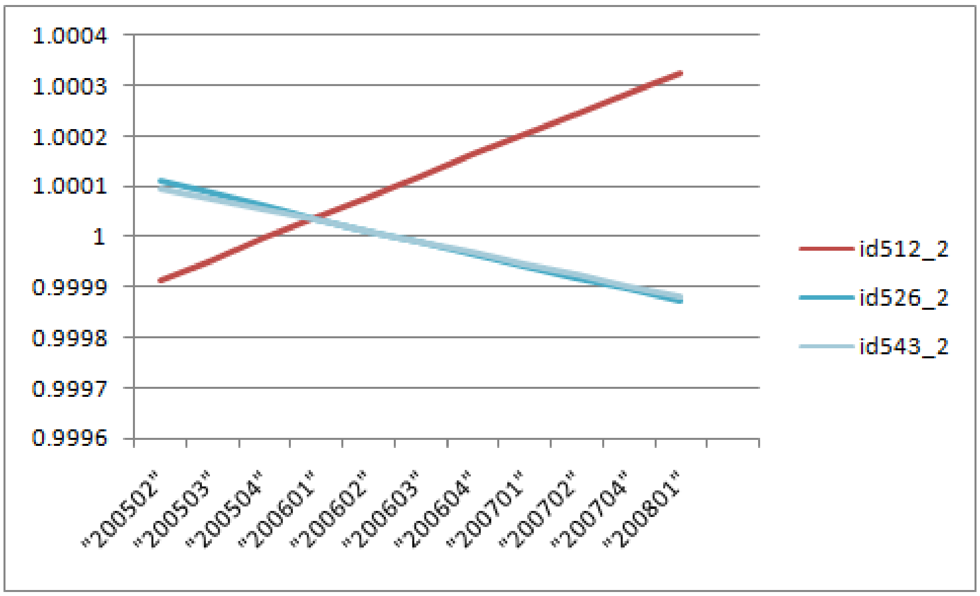

Figure 7: RGDP's relative transition paths of divergent forecasters.

8.3.4 Unemployment Results

The unemployment panel has 16 forecasters. For this panel two convergence clubs were identified, and one individual diverges. The critical value used in the second step of the first convergence analysis was 2.5. This was because for critical values lower than this (starting with 0) the extended core rejected the null hypothesis of convergence. Table C-IV presents the results of each algorithm's main step; the table uses the same layout used previously in this section. From the table content we observe that 2 convergent groups were not rejected in the sample. The first one was composed by 13 elements. Three individuals remained outside the first group and the convergence hypothesis was rejected when all of them were considered in the subsample. Although when they were checked in pairs, the hypothesis of convergence could not be rejected for 2 of them; hence, only 1 element diverged.

Table C-IV: Convergence algorithm's intermediate results for Unemployment

Last T Order	ID	t value Step 1	t value Step 2	Club	$\log t$ Test	ID	t value Step 1	t value Step 2	Club	$\log t$ Test
1	id446	Base	Core	S_1						
2	id528	3.5	Core	S_1	t_{S_1}					
3	id526	4.3	Core	S_1	$= -0.1$					
4	id543	5.7	Core	S_1						
5	id510	4.9	Core	S_1						
6	id527	5.6	Core	S_1						
7	id521	6.2	Core	S_1						
8	id484	8.5	Core	S_1						
9	id535	7.7	Core	S_1						
10	id424	9.6	Core	S_1						
11	id532	9.4	9.4	S_1						
12	id512	1.8	2.5	S_1						
13	id483	-0.5	2.0	S_1^c		id483		-10.2	S_2^c	
14	id539	-2.2	5.5	S_1						
15	id472		-2.2	S_1^c	$t_{S_1^c}$	id472	Base	Core	S_2	t_{S_2}
16	id463		-4.4	S_1^c	$= -10.2$	id463	3.4	Core	S_2	$= 3.4$

Figure 8 and Figure 9 present the relative transition paths of the first and the second convergence clubs respectively.

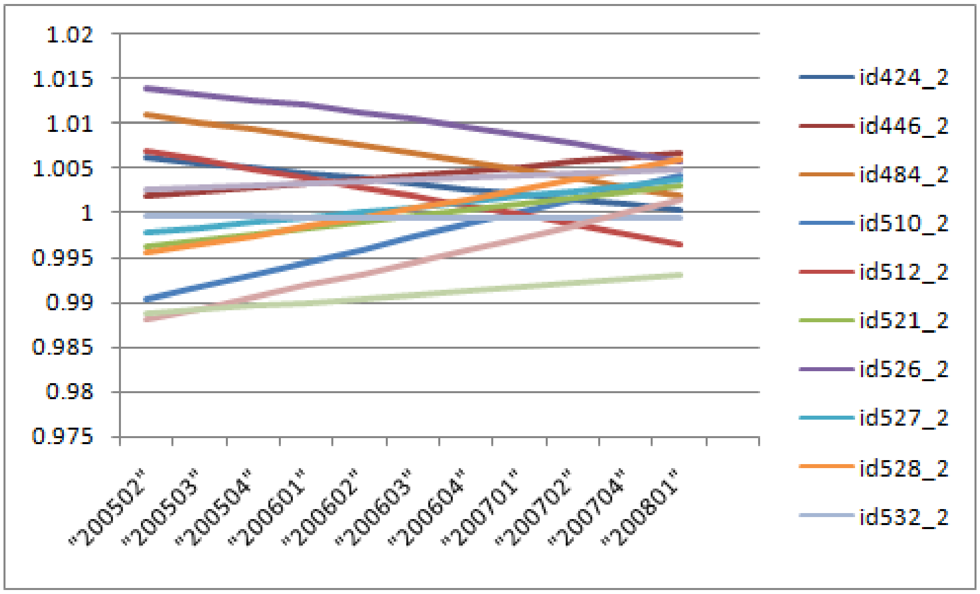

Figure 8: Unemployment's relative transition paths within club 1.

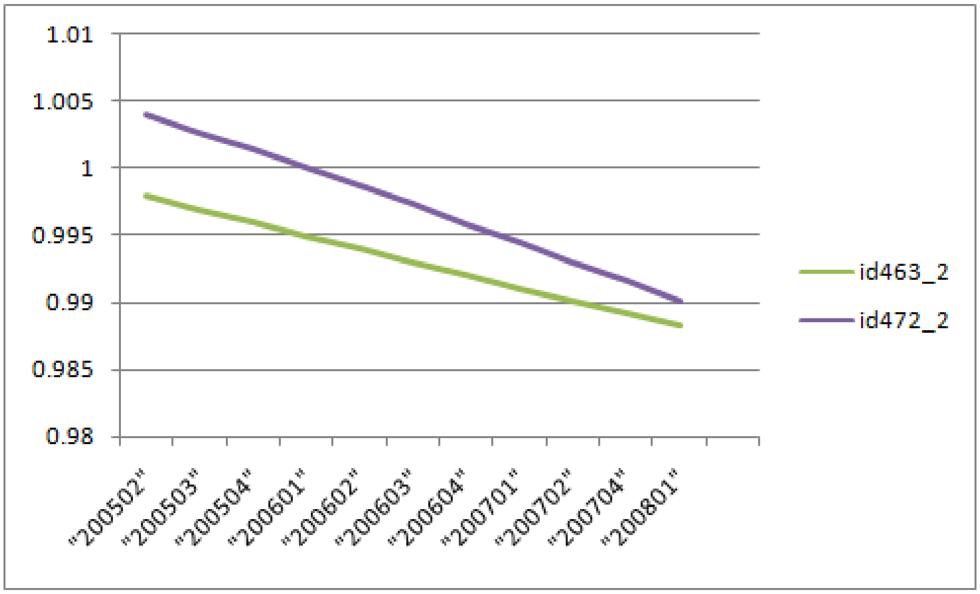

Figure 9: Unemployment's relative transition paths within club 2.

Figure 10 shows the sole individual to diverge in the panel.

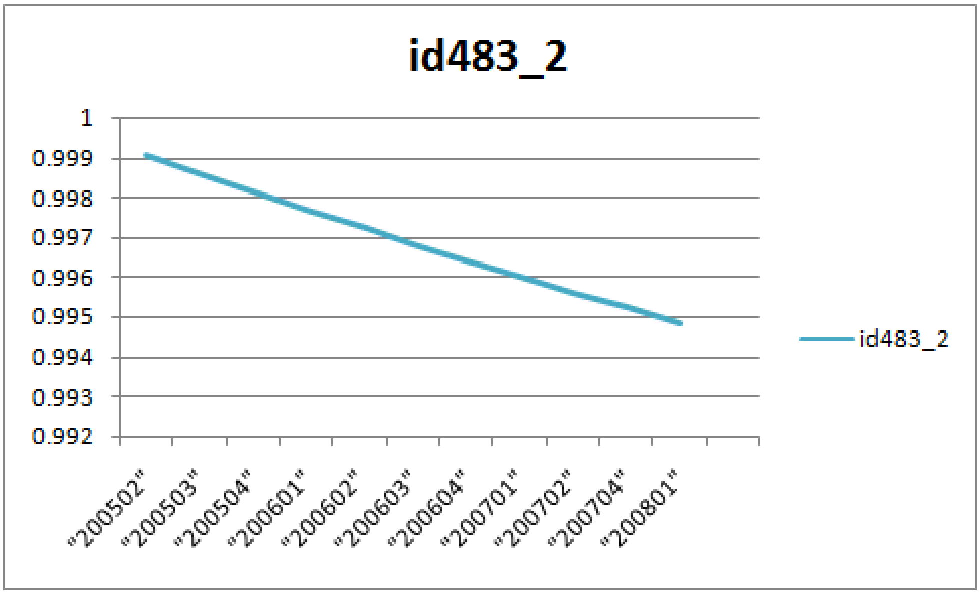

Figure 10: Unemployment's relative transition paths of divergent forecaster.