

Nº 214

ON THE DIFFERENTIABILITY OF THE
CONSUMER DEMAND FUNCTION

Prof: Paulo Klinger Monteiro
Mário Rui Páscoa
Sergio Ribeiro da Costa Werlang

Maio de 1993

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Paulo Klinger Monteiro
IMPA, Rio de Janeiro

Mário Rui Páscoa
Faculdade de Economia, Universidade Nova de Lisboa

Sergio Ribeiro da Costa Werlang
Fundação Getúlio Vargas, Rio de Janeiro

ABSTRACT

For strictly quasi concave differentiable utility functions, demand is shown to be differentiable almost everywhere if marginal utilities are pointwise Lipschitzian. For concave utility functions, demand is differentiable almost everywhere in the case of differentiable additively separable utility or in the case of quasi-linear utility.

ADDRESS FOR CORRESPONDENCE:

Mário R. Páscoa
Faculdade de Economia
Universidade Nova de Lisboa
Travessa Estevão Pinto (Campolide)
1000 Lisboa
Portugal

I. INTRODUCTION

Katzner (1968) showed that demand functions derived from quite nice utility functions, even of class C^2 , need not be differentiable everywhere. For strictly quasi-concave C^2 utility functions, such that the marginal utilities are always strictly positive and the indifference hypersurfaces never intersect the boundary of the positive orthant, the demand function is of class C^1 at a bundle x if and only if the bordered Hessian determinant does not vanish at x . This condition holds on an open dense set of bundles and a corresponding open dense set of full measure of prices and income. Differentiability of demand almost everywhere is therefore the most that can in general be expected, even for C^2 utility functions.

A more general result by Debreu (1972) established that for C^1 preferences, demand is smooth at the inverse image of a bundle x if and only if the indifference hypersurface has nonzero Gaussian curvature at x . Moreover, Debreu (1976) showed that continuous differentiability of demand does not require the existence of a C^2 utility function without critical point; the necessary condition is that utility is of class C^1 with nonvanishing derivative.

Rader (1973, 1979) established sufficient conditions on utility functions for demand to be differentiable on almost every price vector and income. Using Sard's (1958) lemma on critical values of differentiable mappings, not necessarily of class C^1 , Rader (1979) was able to replace the C^2 condition on utility by the assumption of twice - differentiability.

Our first result in this paper goes a step further by assuming that utility functions are differentiable, with strictly positive pointwise Lipschitz marginal utilities. Our other results address the case of concave utility functions, showing that the desired almost everywhere differentiability of demand holds for quasi-linear concave functions and for additively separable strictly concave differentiable functions. Rader (1973) had addressed also the concave case and showed that here it sufficed to guarantee that demand is locally Lipschitz in income.

II. DEFINITIONS AND AUXILIARY RESULTS

The consumer utility function u has \mathbb{R}_+^n as its domain and income is normalized to be one. The demand correspondence is given by $p \rightarrow d(p) = \{x \geq 0: x \text{ maximizes } u(y) \text{ subject to } p \cdot y = 1\}$. In this section we enunciate some basic results to be used later on.

We start with a pointwise version of Rader's (1973) theorem 1:

Lemma 1: Suppose $u: \mathbb{R}_+^n \rightarrow \mathbb{R}$ is strictly quasi concave and continuously differentiable. Let $f: \mathbb{R}_+^n \rightarrow \mathbb{R}_+^n$ be given by $f(x) = \nabla u(x) / (x \cdot \nabla u(x))$. If $p \in \mathbb{R}_{++}^n$ is such that u is twice differentiable at $d(p) \gg 0$ and $f'(d(p))$ is nonsingular, then demand is differentiable at p .

Proof: This lemma is a direct consequence of the facts that $d = f^{-1}$, f is an homeomorphism and $f'(d(p))$ is nonsingular. In fact, an homeomorphism with nonsingular derivative at a point is such that the inverse is differentiable at the inverse image of this point (see Dieudonné (1960), for example).

Now we recall the strongest result by Sard on the measure of the set of critical points of a function.

Lemma 2: (Sard (1958)): Suppose $\Omega \subseteq \mathbb{R}^n$ is open and $f: \Omega \rightarrow \mathbb{R}^n$. Let $B = \{x \in \Omega: f'(x) \text{ exists and is singular}\}$. Then $f(B)$ is a null set.

(This is a corollary to theorem 1 in page 254 of Sard (1958)); notice that f does not even need to be continuous outside of B).

Next, we enunciate a recent result that will be used in our discussion of Rader's (1973) condition that demand maps null sets into null sets.

Lemma 3: (Villani (1984)): Let $U \subseteq \mathbb{R}^n$ be open and $\varphi: U \rightarrow \mathbb{R}^n$ be continuous, one-to-one and differentiable almost everywhere. Then φ^{-1} takes null sets into null sets if and only if φ' is nonsingular almost everywhere.

We will appeal in our proofs to the following consequence of Fubini theorem:

Lemma 4: Consider the measure spaces $(X, \mathcal{A}, \lambda)$ and (Y, \mathcal{B}, μ) such that X and Y are Polish spaces and μ is a nonatomic measure. If $f: X \rightarrow Y$ is measurable, then $\text{graph}(f)$ has null $\lambda \times \mu$ measure.

Proof: Let $G: X \rightarrow X \times Y$, given by $x \rightarrow (x, f(x))$. Now G is one to one and by Kuratovsky theorem $\text{Im}(G) = \text{graph}(f)$ is a measurable set. Now $(\lambda \times \mu)(\text{graph}(f)) = \int \mu(\{y: (x, y) \in \text{graph}(f)\}) d\lambda(x) = \int \mu(\{f(x)\}) d\lambda(x) = 0$.

Finally, let us recall that a vector-valued function f defined on an open set of \mathbb{R}^n is said to be pointwise Lipschitzian at x_0 if there exists a constant K and a neighborhood N of x such that $\|f(x) - f(x_0)\| \leq K \|x - x_0\|$, for every $x \in N$.

III. RESULTS

Our first result extends Rader's (1973) theorem 1, by replacing the assumption of twice - differentiable of utility with the weaker condition of pointwise Lipschitzian marginal utilities.

Theorem 1: If the utility function is differentiable on \mathbb{R}_{++}^n with pointwise Lipschitzian derivative and such that $\nabla u(x) \gg 0$ for every $x \in \mathbb{R}_{++}^n$, then the demand function is differentiable in almost every price p in $\{p \gg 0: d(p) \gg 0\}$.

Proof: Demand is given by a function whose inverse is the mapping $f: x \rightarrow \nabla u(x)/(x \cdot \nabla u(x))$, which is well defined since the denominator is never

zero. Our first claim is that f is pointwise Lipschitzian. In fact, $\|f(x) - f(x_0)\| = (\nabla u(x) - \nabla u(x_0)) / (x_0 \cdot \nabla u(x_0)) + \nabla u(x) [(x \cdot \nabla u(x))^{-1} - (x_0 \cdot \nabla u(x_0))^{-1}] \leq K_{x_0}^{(1)} \|x - x_0\| + \nabla u(x) [(x \cdot \nabla u(x))(x_0 \cdot \nabla u(x_0))]^{-1} [x \cdot \nabla u(x_0) - x_0 \cdot \nabla u(x)] \leq K_{x_0}^{(1)} \|x - x_0\| + K_{x_0}^{(2)} [\|x - x_0\| \nabla u(x) + x_0 (\nabla u(x) - \nabla u(x_0))] \leq (K_{x_0}^{(1)} + K_{x_0}^{(3)}) \|x - x_0\|.$

Now, a consequence of the claim is that f is differentiable almost everywhere and that the set of nondifferentiability of f is mapped into a null set N . By lemma 2, f has a null set C of critical values and therefore the set of regular values of f is $\mathbb{R}_{++}^n \setminus (N \cup C)$, a set of full measure. Then, by lemma 1, $d = f^{-1}$ differentiable for every $p \in \mathbb{R}_{++}^n \setminus (N \cup C)$ such that $d(p) \gg 0$. \square

As a corollary we obtain Rader's (1973) theorem 1: when the utility function twice differentiable, demand is differentiable in almost every price in $\{p \gg 0: d(p) \gg 0\}$. In fact, a differentiable function is pointwise Lipschitzian.

If we assume u differentiable in \mathbb{R}_+^n then a stronger result is true (which extends Rader's (1979) theorem 2 and its first corollary on D^2 utility to the more general case of pointwise Lipschitzian marginal utilities).

Theorem 1': Add to the assumptions of theorem, the hypothesis that u is continuously differentiable on \mathbb{R}_+^n . Then the demand function is differentiable for almost every price.

Proof: from theorem 1 it suffices to prove that demand is a.e. differentiable in $\{p \gg 0: d(p) \gg 0\}$. Using Kuhn-Tucker's theorem we have that $x = d(p)$ if and only if $p \cdot x = 1$ and $f(x) \leq p$. Hence if $f_K(x) < p_K$ then $x_K =$

0. Therefore, for any set of indices I , the set $A_I = \{p \gg 0: f_i(d(p)) < p_i \text{ for } i \in I \text{ and } d_l(p) > 0 \text{ for } l \notin I\}$ is open. In this set, $d(p)$ is the solution of $\max \{u(x): x_i = 0 \text{ for } i \in I, \sum_{l \notin I} p_l x_l = 1\}$. Hence theorem 1 applied to $u|_{\mathbb{R}_+^I}$ ensures that demand is differentiable almost everywhere in A_I .

It remains to consider the set $E = \bigcup_{i=1}^n \{p \gg 0: f_i(d(p)) = p_i \text{ and } d_l(p) = 0\}$. However, if $d_l(p) = 0$ it is true that d is a function d^i of p_{-i} alone, where $p_{-i} = (p_1, \dots, p_{1-i}, p_{i+1}, \dots, p_n)$. But in this case d is mapping a null set because each set $\{p \gg 0: p_i = f_i(d^i(p_{-i}))\}$ is the graph of a measurable function and is therefore a null set, by lemma 4. \square

In the proof of theorem 1 it was fundamental that $f = d^{-1}$ takes null sets into null sets (this is usually called Lusin's condition). The following example shows that in general f may not have this property.

Example: Take a function $g: [0, +\infty] \rightarrow [0, +\infty]$ continuous, strictly decreasing such that $g' = 0$ a.e. (for example, let g be the inverse of the increasing function H in Sacks(1937), p. 101, given by $H(x) = \sum_{n=1}^{\infty} F(nx)/2^n$, where F is Cantor's function on $(0, 1)$).

Now, let G be a primitive of g and define the utility function by $u(x, y) = G(x) + G(y)$. Clearly, u is concave and continuously differentiable. The function f is given by $f(x, y) = (g(x), g(y)) / (xg(x) + yg(y))$. It is easy to check that f' is singular a.e.. Since f is an homeomorphism, f is open and therefore its range has positive measure. But lemma 2 implies that the image of the set of critical points of f is a null set. This proves that f takes a null set (i.e., the complement of the critical points of f) onto a non-null set.

As discussed in Rader (1973) it is desirable that demand takes null sets into null sets. The above example gives us also a demand function that does not satisfy Lusin's property and is an alternative to Rader's (1973) example (pp. 924-925). In fact, the above example fails the following necessary and sufficient condition for Lusin's property on demand.

Theorem 2: Suppose f given by $f(x) = \nabla u(x)/(x \cdot \nabla u(x))$ is differentiable almost everywhere. The demand function d takes null sets into null sets if and only if f' is non singular almost everywhere.

(Proof: this is an immediate consequence of lemma 3)

Our next results deal with the case of concave utility functions. We will show that in this context it is possible to drop, in some special cases, (i) the assumption made on marginal utilities or even (ii) the assumption that utility itself is always differentiable. In case (i) we will look at additively separable utility and in case (ii) we will look at quasi-linear utility.

Theorem 3: Suppose $u: \mathbb{R}_+^2 \rightarrow \mathbb{R}$, given by $u(x, y) = G(x) + H(y)$, is strictly concave, differentiable and that $g \equiv G' > 0$, $h \equiv H' > 0$. Then the demand function is differentiable almost everywhere.

Before proving this theorem we have to introduce some notation. Let $g: [0, \infty) \rightarrow \mathbb{R}$ and $x \geq 0$, we define $\bar{D}g(x) = \limsup_{x' \rightarrow x} (g(x') - g(x))/(x' - x)$ and $\underline{D}g(x) = \liminf_{x' \rightarrow x} (g(x') - g(x))/(x' - x)$. We have that g is differentiable at x if and only if $\underline{D}g(x) = \bar{D}g(x)$.

Proof of theorem 3: (i) Let $(x, y) = d(p, q)$ and $(x', y') = d(p', q')$. We will start by establishing the following identity:

$$\begin{aligned} & \left(\frac{g(x') - g(x)}{x' - x} + \frac{g^2(x)}{h^2(y)} \cdot \frac{h(y') - h(y)}{y - y'} \right) (x' - x) = \\ & - \frac{h(y') - h(y)}{y - y'} \cdot \frac{g(x)}{h(y)} \cdot [x'(p' - p) + y'(q' - q)] + \frac{h(y')}{q'} (p' - p) \cdot \frac{g(x)}{h(y)} (q' - q) \quad (\text{A}) \end{aligned}$$

In fact, $g(x)/h(y) = p/q$, $g(x')/h(y') = p'/q'$ and let $dg \equiv g(x') - g(x)$, $dh = h(y') - h(y)$; then $dg = (p'/q') h(y') - (p/q) h(y) = (p'/q' - p/q) h(y') + (p/q)dh$, that is, $(x' - x)dg/(x' - x) = (p'q - pq') h(y')/q'q + (p/q) (y' - y) dh/(y' - y)$. Now $1 = p'x' + q'y' = (p' - p)x' + (q' - q)y' + p(x' - x) + q(y' - y) + 1$ and substituting above we get: $(x' - x)[(g(x') - g(x))/(x' - x) + (p/q)^2 (h(y') - h(y))/(y' - y)] = (p'q - pq') h(y')/q'q - (p/q^2) (dh/(y' - y)) [(p' - p)x' + (q' - q)y']$.

(ii) Now, to prove the differentiability of demand it suffices to prove the differentiability of the mapping $(p, q) \rightarrow x(p, q) \equiv d_1(p, q)$. By Stepanov's theorem (see Saks (1937), IX, p. 310), it is enough to show, for almost every (p, q) , that $\limsup_{(p', q') \rightarrow (p, q)} (x' - x)/(|p' - p| + |q' - q|) < \infty$. From the above identity and the fact that g and h are decreasing we have the following implication: $\limsup_{(p', q') \rightarrow (p, q)} |x' - x|/(|p' - p| + |q' - q|) = \infty \Rightarrow \bar{D}g(x) = 0 = \bar{D}h(y)$. Therefore it suffices to prove that the set $P = \{(p, q): \bar{D}g(x) = 0 = \bar{D}h(y), \text{ where } (x, y) = d(p, q)\}$ is null.

If $g'(x)$ and $h'(y)$ exist and $d^{-1}(x, y) \in P$, then $g'(x) = 0 = h'(y)$. This implies that the Jacobian of f at (x, y) is zero (recall the definition of f , lemma 1) and, Sard's (1958) theorem, the set $\{(p, q): (x, y) = d(p, q), g'(x) = 0 = h'(y)\}$ is null. Therefore, it suffices to prove that $P_g = \{(p, q): (x, y) = d(p, q), \bar{D}g(x) > \underline{D}g(x)\}$ and $P_h = \{(p, q): (x, y) = d(p, q), \bar{D}h(y) > \underline{D}h(y)\}$ are null.

(iii) We will prove that P_g is a null set. The proof for P_h is similar. Notice that $P_g = \{(\frac{g(x)}{h(y)}, q, q): q(xg(x) + yh(y)) = h(y), \bar{D}g(x) > \underline{D}g(x)\}$. Define $\psi(y) = h(y) (1 - qy)$. The function ψ is strictly decreasing and continuous; call $\theta = \psi^{-1}$. Then $P_g = \{(\frac{1 - qy}{x}, q): \theta(qx g(x)) = y, \bar{D}g(x) > \underline{D}g(x)\}$. By Fubini theorem it suffices to prove for almost every $q > 0$ that $\hat{P}_g = \{(\frac{1 - qy}{x}, \theta(qx g(x)) = y, \bar{D}g(x) > \underline{D}g(x)\}$ is null.

Since g is differentiable almost everywhere, the set $E = \{x: \bar{D}g(x) > \underline{D}g(x)\}$ is null. Now, by item 4.6, p. 271 of Saks (1937), if a function has at each point of a set a finite Dini derivative, then the function satisfies Lusin's property on the set. So, if the mapping $x \rightarrow R(x) = [1 - q\theta(qx \wedge g(x))]/x$ has for each point of E a finite Dini derivative, then P_g is null.

Let us calculate one such derivative. Call $z = xg(x)$, and $z' = x'g(x')$. Then it is true that

$$\frac{R(x') - R(x)}{x' - x} = -R(x') - q^2 \frac{\theta(qz') - \theta(qz)}{q(z' - z)} \cdot \frac{z' - z}{x' - x} \quad (B)$$

In fact, $x'R(x') - xR(x) = -q\theta(qz') + q\theta(qz)$, then $(x' - x)R(x') + x(R(x') - R(x)) = q(\theta(qz) - \theta(qz'))$ and the above identity follows.

Notice that $z' - z = (x' - x)g(x') + x(g(x') - g(x))$, so $(z' - z)/(x' - x) = g(x') + x(g(x') - g(x))/(x' - x)$. Then,

$$\frac{R(x') - R(x)}{x' - x} = -R(x') - q^2 \frac{\theta(qz') - \theta(qz)}{q(z' - z)} \cdot [g(x') + x \frac{g(x') - g(x)}{x' - x}] \quad (C)$$

Now, let $u = \theta(qz)$ and $u' = \theta(qz')$, then $(u' - u)/q(z' - z) = ((\varphi(u') - \varphi(u))/(u' - u))^{-1} = [((h(u') - h(u))/(u' - u)) (1 - qu') - qh(u)]^{-1} < 0$. So $|(u' - u)/q(z' - z)| \leq (qh(u))^{-1}$. Therefore for each $x \in E$, identity (C) implies that $x\bar{D}R(x) \leq -R(x) + qh(u)^{-1} (g(x) + x\bar{D}g(x)) < \infty$ \square

Our last result addresses the case of a quasi-linear utility function and does not require differentiability of this function on its entire domain:

Theorem 4: If the utility function $u: \mathbb{R}_+^n \rightarrow \mathbb{R}$ is given by $u(x) = x_1 + v(x_2, \dots, x_n)$, where v is a strictly concave function, then, for almost every price in $\{p \gg 0: d(p) \gg 0\}$, the demand function is differentiable.

Proof: This result can easily be established appealing to results on concave conjugacy and to Alexandroff's (1939) theorem. Let v_* be the concave conjugate function of v , that is, $v_*(z) = \sup_{x_1} \{v(x_1) - \langle z, x_1 \rangle\}$; the function v_* is concave. Let us denote by $Gg(x)$ the set of supergradients of a concave function g at x , that is $Gg(x) = \{z \in \mathbb{R}^n: g(y) \leq g(x) + \langle z, y - x \rangle, \forall y\}$. Now, $x_1 \in Gv_*(z)$ if and only if $z \in Gv(x_1)$, (as can be easily seen using known results as convex conjugacy, see Rockafellar (1970) 23.5.1).

If at prices p , the optimal solution x is interior, then $\hat{p} \equiv p_1/p_1 \in Gv(x_1)$ and, therefore, $x_1 \in Gv_*(\hat{p})$. Now, v is strictly concave on every convex subset of $\{x_1: Gv(x_1) \neq \emptyset\} \subseteq \text{dom } v$ and therefore v_* is differentiable throughout $\text{int}(\text{dom } v_*)$ (see Rockafellar (1970) 26.3). Then, $x_1 = \nabla v_*(\hat{p})$.

Now, by Alexandroff's (1939) theorem, the concave function v_* is twice differentiable almost everywhere on $\text{int}(\text{dom } v_*)$ and therefore, the demand function will be differentiable in almost every price in $\{p \gg 0: d(p) \gg 0\}$. In fact, outside of a null set, $\nabla x_1 = \nabla^2 v_*(\hat{p})$ and $x_1 = 1 - \sum_{j \neq 1} x_j p_j / p_1$. \square

References

- Alexandroff, A. D. (1939): "Existence of the Second Differential of a Convex Function Almost Everywhere in its Domain and Some Related Properties of Convex Surfaces" (Russian), Leningrad State University, Annals 37, Mathematical Series, 6, pp. 3-35.
- Debreu, G. (1970): "Economies with a Finitive Set of Equilibria", *Econometrica* 38.
- Debreu, G. (1972): "Smooth Preferences", *Econometrica* 40.
- Debreu, G. (1976): "smooth Preferences: A Corrigendum", *Econometrica* 44.
- Dieudonné (1960): "Foundations of Modern Analysis", Prentice - Hall.
- Katzner, D. (1968): "A Note on the Differentiability of Consumer Demand Functions", *Econometrica*, vol. 36, No. 2.
- Rockafellar, T. (1970): "Convex Analysis", Princeton University Press.
- Sard, A. (1958): "Images of Critical sets", *Annals of Mathematics*, vol.68, No. 2.
- Sard, A. (1942): "The Measure of the Critical Points of Differentiable Maps", *Bulletin of the American mathematical Society*, 48.
- Saks, S. (1937), *Theory of the integral*, Hopner Publishing Company, 2^o ed.
- Villani, A. (1984), On Lusin's condition for the inverse function, *Rendiconti del Circolo Matematico di Palermo*, serie II, tomo XXXIII, pp 331-335.

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