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Mauro Cazzaniga

Trade credit in a developing country: the role of large suppliers in the production network

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Dissertação apresentada à Escola de Economia de São Paulo como requisito à obtenção de título de mestre em Economia.

Orientador: Prof. Dr. Pierluca Pannella

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Resumo

Em países em desenvolvimento, empresas pequenas podem ter custos proibitivos para empréstimos bancários. Neste ambiente, o crédito comercial (vendas a prazo entre empresas) em uma rede de produção de firmas pode mitigar sua necessidade de liquidez, mas também pode ser uma fonte adicional para a propagação de choques. Nós utilizamos um modelo de rede de produção com firmas grandes e pequenas em cada setor em que no equilíbrio, em linha com a evidência empírica, empresas maiores são provedoras líquidas de crédito comercial para os clientes menores. Firmas grandes competem à la Cournot: elas escolhem a produção e a fração de crédito ótimas dadas as quantidades escolhidas por outras empresas, e incorrem em um custo de monitoramento para recuperar o crédito. As vendas a prazo podem ter um efeito benéfico na economia uma vez que reduzem a necessidade das empresas por capital de giro. Nós calibramos o modelo com dados da economia brasileira e encontramos que crédito comercial endógeno amplifica choques de produtividade. As relações geradas pelo crédito comercial amplifica choques financeiros para firmas grandes em setores upstream, mas os mitiga se as firmas estão em setores downstream.

Palavras-chave: crédito comercial, redes de produção, competição imperfeita.

Abstract

In developing countries, small companies can face a prohibitive cost of bank credit. In this environment, trade credit in the input-output network of firms can mitigate their liquidity needs but may also be an additional source of shock propagation. We build a production network model with large and small companies in multiple sectors where, in equilibrium, and in line with empirical evidence, large suppliers are net providers of trade credit to small clients. Large companies compete a la Cournot: they choose their optimal production and amount of trade credit given the quantities chosen by other companies and incur monitoring costs to recover credit. Trade credit can have a beneficial effect on the economy since it reduces firms' need for working capital. We calibrate the model to Brazilian data and find that endogenous trade credit amplifies productivity shocks. Trade credit linkages amplify financial shocks to large firms in upstream sectors, while they are mitigated if the firms are in downstream sectors.

Keywords: Trade Credit, Input-Output Network, Imperfect Competition.

JEL Classification: E23, E44, L13, L14, O41

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1 Introduction

It is widely documented how credit markets can charge different interest rates for similar loan transactions (Banerjee (2001), Banerjee and Duflo (2005), Gilchrist, Sim and Zakrajšek (2013)). This dispersion is particularly extreme in developing countries where smaller firms usually have very limited access to finance.[1] For these firms, trade credit represents a way to substitute for the lack of bank credit (Petersen and Rajan (1997)). This paper presents a model of a sectoral input-output network with an endogenous supply of trade credit between small and large firms facing different bank interest rates. Our goal is to analyze the role of large firms as credit providers and measure the loss from interest rate dispersion for the case of Brazil.

Our theory is motivated by two main facts regarding Brazil. The first one is the striking dispersion of credit costs between small and large firms. The average annual interest rate faced by a small Brazilian firm[2] was around 45% over the last 5 years, well above the central bank interest rate which oscillated between 2% and 13%.[3] Cavalcanti et al. (2021) show that the interest rate of a firm with 300 employees is 3 percentage points lower than the one of a firm with 30 employees and 5.5 percentage points lower than the one of a firm with 3 employees.

The second fact is that this dispersion in financing costs is reflected in the structure of the trade credit network as large firms are net providers for smaller firms.[4] This polarization in the trade credit market is substantially wider if compared to advanced economies. Figure 1 shows that the median net trade credit (Accounts Receivable minus Accounts Payable over the total current assets) of large companies[5] as a share of total current assets is higher in Brazil if compared to Europe and the US in most sectors. A similar relation is observed when we consider the aggregate net trade credit position by sector (that is, the sum of accounts receivable minus accounts payable over the sum of the total current assets of large firms in the data), which is shown in Figure 2.

Based on this evidence, we develop a theory of trade credit in production networks in which large firms supply trade credit enjoying some market power. The framework builds on existing static models of trade credit in economies with sectoral input-output

1 See Cavalcanti et al. (2021) for the case of Brazil.
2 Annual revenues below $920.000 US dollars.
3 The average interest rate for small enterprises is calculated by the Serviço Brasileiro de Apoio às Micro e Pequenas Empresas. See https://datasebrae.com.br/paineltaxasdejuros.
4 In Table 4 in the Appendix, we show that there exists a significant and sizeable firm-level negative correlation between net trade credit positions and interest rates for a sample of Brazilian firms.
5 Large firms are defined with the Brazilian criterion of yearly revenue over BRL 300 million. Considering the average exchange rate of 3.94 BRL/USD in 2019, this amounts to a threshold for large firms of $76 million.

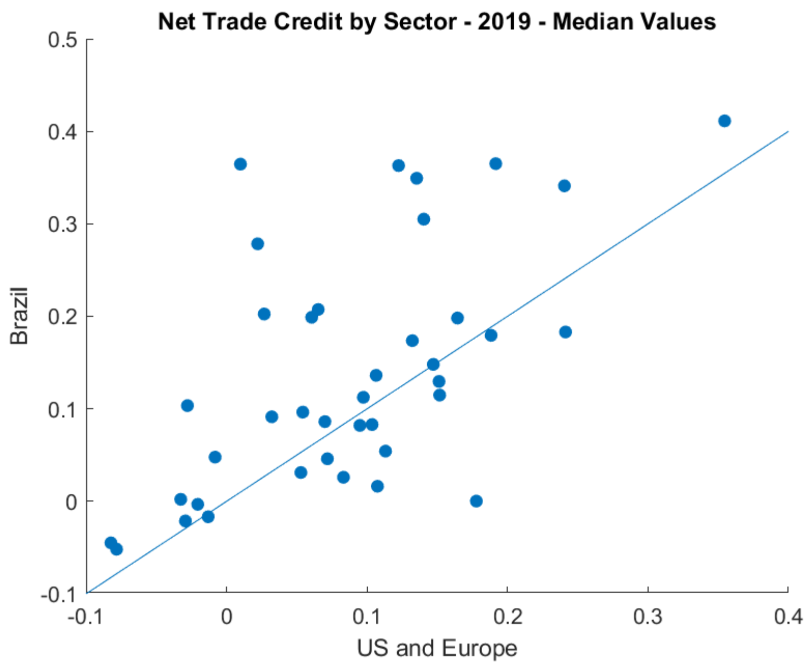

Figure 1 – Median share of net trade credit of large firms by sector. Data are from SP Global.

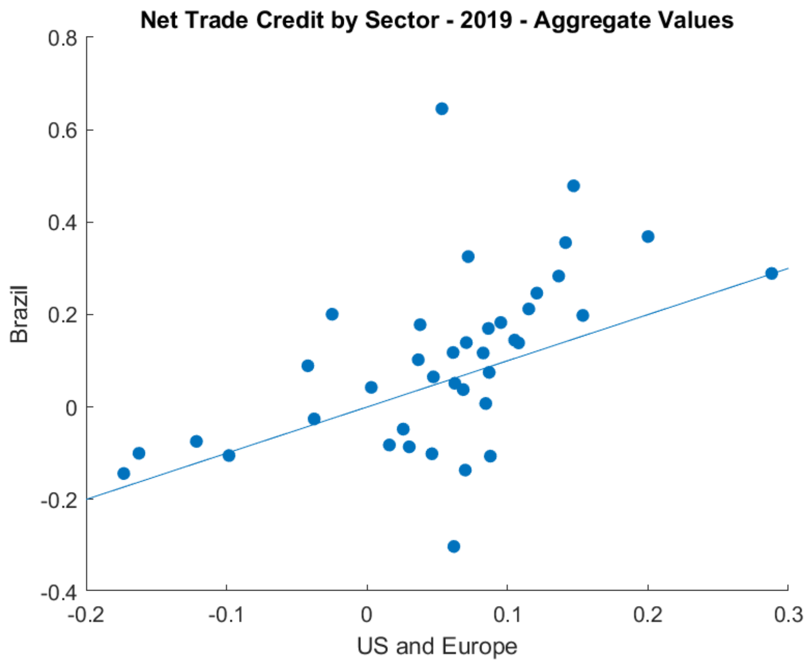

Figure 2 – Aggregate share of net trade credit of large firms by sector. Data are from SP Global.

linkages, such as Luo (2020) and Altinoglu (2021). Firms in each sector purchase from and provide inputs to other sectors but they all face a working capital constraint limiting their operations. In order to overcome this constraint, they can obtain a loan from the financial sector or delay a fraction of payments to their suppliers. An essential modification with respect to the existing literature is that we add firms of different sizes in each sector. Large firms can obtain credit at a lower interest rate. In addition, while small firms are atomistic and perfectly competitive, large firms compete a la Cournot when choosing their supply of goods to each sector and the share of sales on which they allow for delayed payment. Therefore, the amount of trade credit is an endogenous object.

In order to recover a delayed payment, a firm must incur a monitoring cost. This cost increases in the size of sales and the share of trade credit offered to the buyers. The model is quite tractable: we can derive a system of two equations solving for the share of trade credit and the optimal sales of goods between each sectoral linkage. Given the solutions for sales and trade credit, the rest of the model can be expressed as a function of sector-to-sector specific distortions.

In equilibrium, large firms provide net credit to small firms and they can indeed mitigate their financial constraints. When the interest rate for large firms increases, the shock is amplificated as it propagates downstream as their supply of trade credit is reduced. However, it is also mitigated when it travels upstream, as the supplier firms increase their trade credit supply to the affected sector. Since trade credit also depends on monitoring that has a decreasing marginal cost on sales, the decrease in production resulting from a negative shock also counterbalances the increase in trade credit supply from the higher rates. All these effects interact with the traditional propagation through the input-output linkages.

The presence of endogenous trade credit and its interaction with sales also means that productivity shocks have effects on the trade credit shares in the economy. Since the marginal monitoring cost decreases with sales, a positive productivity shock leads to an increase in trade credit supply, amplifying its effect.

In order to evaluate how beneficial is the provision of trade credit from large firms, we calibrate our model to Brazilian data and compare the welfare cost of interest rate shocks to small and large firms in our economy with endogenous trade credit and a modified economy in which trade credit is not allowed. We find that the presence of trade credit always reduces the loss from higher interest rates for small firms for the parameters considered. In particular, the effect is larger when the shock hits upstream sectors. Trade credit amplifies the effect of a rise in the interest rate of large firms if the shock hits sectors that have larger net credit positions.

Related Literature. The paper belongs to the literature on production networks started with Jr and Plosser (1983) and recently developed by Acemoglu et al. (2012), Baqaee and Farhi (2018), Baqaee and Farhi (2019), and Bigio and La'o (2020).[6]

More broadly, it relates to the literature studying the role of financial frictions on misallocation (Hsieh and Klenow (2009), Restuccia and Rogerson (2008), Jones (2011)).

[6] Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) provide an extensive review on this literature.

Baqaee and Farhi (2020) build a general theoretical framework for analyzing the effects of distortions on output and TFP in network economies, even when considering nonlinear (that is, non-Cobb-Douglas) production networks. Bigio and La'o (2020) map distortions to financial frictions and conduct a quantitative exercise using data from the US economy to measure how frictions are able to amplify shocks in a network economy. Liu (2019) presents the concept of distortion centrality, which relates to how distortions accumulate via the input-output linkages of the economy and serves as a summary statistic for the effects of shocks or subsidies to frictions. We model distortions as being the result of financial frictions along with markups due to imperfect competition between large firms.

More specifically, in the model we add trade credit between suppliers and buyers of inputs. Altinoglu (2021) models firms with a cash-in-advance constraint, fixed trade credit shares, and a collateral borrowing constraint. Financial shocks via a tightening of the borrowing constraint propagate by tightening the constraints of upstream firms.

We use a similar framework as Luo (2020), which models an economy with working capital constraints where trade credit shares are fixed and costs are transmitted to clients via price discrimination. The paper also considers the impact of interest rate shocks by adding an adjustment cost of trade credit that allows it to vary from its initial point, as well as the possibility of firms postponing credit repayments.

Bocola and Bornstein (2023) build a dynamic model where oligopolistic suppliers sell intermediary goods to perfectly competitive final goods producers. There is a lag between production and payments, which creates the need for credit contracts. Here, trade credit depends on the reputation of the final goods producer, since the firm can default on payments, and the authors study the impacts of changes in credit intermediation costs and borrowing constraints (although they do not model interest rates explicitly). Hardy and Simonovska (2023) consider a stylized model with two firms, a large supplier which can borrow in foreign currency while the small firm can only borrow at higher rates, studying how trade credit can also serve as a risk-sharing mechanism for exchange rates.

We distinguish between perfectly competitive small firms and oligopolistic large firms, allowing us to take into account the interest rate dispersion between firm sizes as well as sectors. In our contribution, the working capital financing need arises from a timing mismatch between input delivery and payment: companies must pay inputs in advance, but they do not have access to all of their clients' payments in advance. Large firms choose their production and share of trade credit given the quantities of other firms, facing a monitoring cost for giving credit. This allows us to derive endogenously the level of trade credit as a function of the interest rate spread applied to large versus small firms, in comparison with other models where trade credit shares are taken as fixed.

Finally, the paper is also related to the empirical literature on trade credit. Petersen and Rajan (1997) and Demirguc-Kunt and Maksimovic (2001) show how firms can act as credit providers when financial constraints limit the provision of bank credit. Particularly, Petersen and Rajan use data from US firms to show that trade credit use varies according to firm size, with large firms being relevant suppliers of trade credit. In the short run, trade credit can also mitigate the costs of financial crises as shown by Garcia-Appendini and Montoriol-Garriga (2013). Jacobson and Schedvin (2015) show how corporate failures propagate upstream through trade credit linkages. Garcia-Marin, Justel and Schmidt-Eisenlohr (2019) proposes a model of trade credit in which firms learn about their trading partners and rationalize the relationship between trade credit and markups in the case of Chile.

The paper is organized as follows. Section 2 gives an overview of the model and derives the main conditions describing the effect of the interest rate dispersion on trade credit and final output. Section 3 describes the calibration of the model and the evaluation of welfare losses. Section 4 concludes.

2 Model

The economy is made of M sectors producing intermediate goods indexed by i, with $i \in I = \{1, ..., M\}$. Intermediate goods are used as inputs for the production of other intermediate goods and for the production of a final consumption good. There exist two types of intermediate goods in each sector: those produced by “small"firms and those produced by “large"firms. A representative firm aggregates the sectoral goods into the final one according to:

$$Q = \prod_{i \in I} \left[(c_{i,L})^{\nu_i} (c_{i,S})^{1-\nu_i} \right]^{\psi_i}, \quad \text{with} \quad \psi_i \geq 0 \quad \text{and} \quad \sum_{i \in I} \psi_i = 1,$$

where the subscripts $\{S, L\}$ denote the goods purchased from small and large firms.

Small firms behave as perfectly competitive while large firms compete a la Cournot. There is a number N_i of large firms in a sector i. We describe below the problem of an intermediate firm.

2.1 The problem of an intermediate firm

A large firm n in sector i produces using labor and other intermediate goods j as inputs according to the following production function:

$$y_{i,L,n} = A_{i,L} (h^{i,L,n})^{\alpha_i} \prod_j \left[(x_{j,L}^{i,L,n})^{\nu_{i,L}} (x_{j,S}^{i,L,n})^{1-\nu_{i,L}} \right]^{\sigma_j^i},$$

where $h^{i,L,n}$ is the labor used by the firm n and $x_{j,L}^{i,L,n}$ is the amount of intermediate goods used by the firm and produced by large firms in sector j. The parameter σ_j^i is the share of intermediate inputs from sector j used by sector i.[1]

The model is static. However, in order to introduce a role for credit, we assume that there exists a timing friction between the payment of the inputs and production. Specifically, firms face a working capital constraint:

$$(1 - \theta_{j,L}^{i,L}) p_{j,L}^{i,L} x_{j,L}^{i,L} + (1 - \theta_{j,S}^{i,L}) p_{j,S}^{i,L} x_{j,S}^{i,L} + w_i h^{i,L,n} \leq$$

$$(\kappa_{i,L} - \theta_{i,L,n}^{j,L}) p_{i,L,n}^{j,L} x_{i,L,n}^{j} L + (\kappa_{i,L} - \theta_{i,L,n}^{j,S}) p_{i,L,n}^{j,S} x_{i,L,n}^{j,S} + D_{i,L,n}.$$

On the left-hand side, we have the total advanced payment of inputs. $\theta_{j,L}^{i,L}$ is the share of payment for goods produced by large firms in sector j on which the firm obtains a delayed payment. $(\kappa_{i,L} - \theta_{i,L,n}^{j,L})$ represents the share of sales for which the firm can obtain working capital from selling to large firms in sector j. $\kappa_{i,L}$ represents the fact that the firm

[1] For each variable, subscripts refer to the supplier while superscripts refer to the buyer.

does not receive all of its sales payments in advance; this ensures that the constraint is binding with debt $D_{i,L,n} > 0$. While $\kappa_{i,L}$ is an exogenous parameter, $\theta^{j,L}_{i,L,n}$ is chosen by the firm. $D_{i,L,n}$ is the amount of short-term credit obtained by the financial sector. We do not model the lending choice of banks, but we assume that the firm obtains credit at a rate $r_{i,L}$. Notice that the prices of the output, $p^{j,L}_{i,L,n}$ and $p^{j,S}_{i,L,n}$, can be firm-specific since they change with the supply of trade credit. However, we will focus on symmetric equilibria in which all firms offer the same combination of prices and trade credit.

When offering trade credit, a firm must pay a monitoring cost. This cost increases in the total amount of trade credit given, $\theta^{j,L}_{i,L,n}p^{j,L}_{i,L,n}x^{j,L}_{i,L,n}$, but with a decreasing marginal effect. The cost also depends on the share of trade credit $\theta^{j,L}_{i,L,n}$. Specifically, the total cost of offering a delayed payment to large firms in sector j is:

$$c(\theta^{j,L}_{i,L,n})^{\eta}\left(\theta^{j,L}_{i,L,n}p^{j,L}_{i,L,n}x^{j,L}_{i,L,n}\right)^{\gamma}.$$

with $\gamma < 1$ such that the marginal cost is decreasing. The monitoring cost ensures the possibility of an interior solution for the trade credit shares.

For example, for two sales where the trade credit has the same value but represents a different share of the total amount, the firm would have different monitoring costs. We can think that monitoring trade credit that represents a higher fraction of a sale requires more effort as the share of inputs used in the client's activities that had their payment delayed will also be higher.

Large firms compete a la Cournot: they choose their optimal production and supply of trade credit, given the choice of the other competitors and internalizing the effect on the price of their output. The demand function faced by a large firm in sector i selling to small firms in sector j is:

$$p^{j,S}_{i,L} = \frac{\Omega^{j,S}_{i,L}}{1+(1-\theta^{j,S}_{i,L,n})r_j}\frac{1}{\left(\sum_k^{N_i} x^{j,S}_{i,L,k}\right)^{1-\nu_{j,S}\sigma^j_i}}.$$

This function can be obtained from the optimal choice of inputs. Below we derive the solution for $\Omega^{j,S}_{i,L}$. In addition, each intermediate firm also chooses the optimal quantity to sell to the final producer given its demand function:

$$p_{i,L} = \frac{\Psi_{i,L}}{\left(\sum_k^{N_i-n} z_{i,k} + z_{i,n}\right)^{1-\nu_i\psi_i}}.$$

We will focus on equilibria in which all working capital constraints are binding. Therefore, a large firm that maximizes its final profits has the following objective function:

$$\max \sum_j^M \left[1+(\kappa_{i,L}-\theta^{j,L}_{i,L,n})r_{i,L}\right]\frac{\Omega^{j,L}_{i,L}}{1+(1-\theta^{j,L}_{i,L,n})r_{j,L}}\frac{x^{j,L}_{i,L,n}}{\left(\sum_k^{N_i-n} x^{j,L}_{i,L,k} + x^{j,L}_{i,L,n}\right)^{1-\nu_{j,L}\sigma^j_i}}$$

$$+\sum_{j}^{M}\left[1+(\kappa_{i,L}-\theta_{i,L,n}^{j,S})r_{i,L}\right]\frac{\Omega_{i,L}^{j,S}}{1+(1-\theta_{i,L,n}^{j,S})r_{j,S}}\frac{x_{i,L,n}^{j,S}}{\left(\sum_{k}^{N_i-n}x_{i,L,k}^{j,S}+x_{i,L,n}^{j,S}\right)^{1-\nu_{j,S}\sigma_i^j}}$$

$$-\sum_{j}^{M}c(\theta_{i,L,n}^{j,L})^{\eta}\left(\frac{\theta_{i,L,n}^{j,L}\Omega_{i,L}^{j,L}}{1+(1-\theta_{i,L,n}^{j,L})r_{j,L}}\frac{x_{i,L,n}^{j,L}}{\left(\sum_{k}^{N_i-n}x_{i,L,k}^{j,L}+x_{i,L,n}^{j}\right)^{1-\nu_{j,L}\sigma_i^j}}\right)^{\gamma}$$

$$-\sum_{j}^{M}c(\theta_{i,L,n}^{j,S})^{\eta}\left(\frac{\theta_{i,L,n}^{j,S}\Omega_{i,L}^{j,S}}{1+(1-\theta_{i,L,n}^{j,S})r_{j,S}}\frac{x_{i,L,n}^{j,S}}{\left(\sum_{k}^{N_i-n}x_{i,L,k}^{j,S}+x_{i,L,n}^{j}\right)^{1-\nu_{j,S}\sigma_i^j}}\right)^{\gamma}$$

$$+(1+\kappa_i r_i)\frac{\Psi_i z_{i,L,n}}{\left(\sum_{k}^{N_i-n}z_{i,k}+z_{i,n}\right)^{1-\nu_i\psi_i}}$$

$$-(1+r_{i,L})w_i h^{i,L,n}-\sum_{j}^{M}[1+(1-\theta_{j,L}^{i,L})r_{i,L}]p_{j,L}^{i,n}x_{j,L}^{i,L,n}-\sum_{j}^{M}[1+(1-\theta_{j,S}^{i,L})r_{i,L}]p_{j,S}^{i,n}x_{j,S}^{i,L,n}$$

$$+\mu_{i,L}[A_{i,L}(h^{i,L,n})^{\alpha_i}\prod_{j}\left[(x_{j,L}^{i,L,n})^{\nu_{i,L}}(x_{j,S}^{i,L,n})^{1-\nu_{i,L}}\right]^{\sigma_j^i}-\sum_{j}x_{i,L,n}^{j,L}-\sum_{j}x_{i,L,n}^{jS}-z_{i,L,n}].$$

The first two lines are the total sales to large and small firms. The third and fourth lines are the monitoring costs incurred from supplying trade credit to large and small firms. The fifth line is the sales to the final producer, while the sixth line is the total costs of inputs.

From the first order condition with respect to $\theta_{i,L,n}^{j,S}$ and $x_{i,L,n}^{j,S}$ we obtain the following system of two equations:

$$c\,(\eta+\gamma)\,(1+r_{j,S})\left(\theta_{i,L}^{j,S}\right)^{\eta+\gamma-1}-c\eta r_{j,S}\left(\theta_{i,L}^{j,S}\right)^{\eta+\gamma}=$$

$$[(1+\kappa_{i,L}r_{i,L})r_{j,S}-(1+r_{j,S})r_{i,L}]\left(p_{i,L}^{j,S}x_{i,L}^{j,S}\right)^{1-\gamma} \tag{2.1}$$

and

$$\frac{\mu_{i,L}}{p_{i,L}^{j,S}}=\left[1-\frac{1}{N_i}(1-\nu_{j,S}\sigma_i^j)\right]\left(1+(\kappa_{i,L}-\theta_{i,L}^{j,S})r_{i,L}-\gamma c\left(\theta_{i,L}^{j,S}\right)^{\eta+\gamma}\left(p_{i,L}^{j,S}x_{i,L}^{j,S}\right)^{\gamma-1}\right) \tag{2.2}$$

From these two equations, we can solve for the optimal share of trade credit and sales as a function of parameters and prices $p_{i,L}^{j,S}$.

The first equation describes the optimal choice of the share of trade credit from a large firm in sector i to a small firm in sector j. Notice that it always increases in $r_{j,S}$, while it decreases in $r_{i,L}$. In particular, a necessary condition to supply trade credit is that $r_{i,L}<\frac{r_{j,S}}{1+(1-\kappa_{i,L})r_{j,S}}$. That is, when the firm's client is in a situation of increased financial distress, the supplier increases trade credit since it internalizes that the demand will have a higher response. In contrast, when the supplier is more financially constrained, the cost of postponing payments increases, leading to a decrease in credit supply.

The share $\theta_{i,L}^{j,S}$ also increases in the size of sales. This occurs because an increase in sales reduces the marginal cost of raising the share of trade credit. Therefore, while an increase in the interest rate of a buyer induces a higher supply of trade credit, a reduction in sales counteracts this effect. We can think of this as an increasing returns-to-scale component of trade credit.

The second equation describes how the supplier discriminates prices by establishing the relation between the marginal cost, $\mu_{i,L}$, and the price charged to clients. The first term in multiplication refers to the markup generated from the Cournot competition between large firms. The second term illustrates that both the opportunity cost and the monitoring cost are passed on to clients via price discrimination, which is in line with the literature that trade credit has an implicit cost to buyers that is bundled in prices, such as in Petersen and Rajan (1997) and Luo (2020).

From the first order condition with respect to intermediate inputs and sales to the final producer, we find that

$$\Omega^{i,L}_{j,L} = \left[1 - \frac{1}{N_i}(1 - \nu_i \psi_i)\right](1 + \kappa_i r_i)\nu_{i,L}\sigma^i_j p_{i,L} A_i (h^{i,L})^{\alpha_i} \prod_j \left[(x^{i,L,n}_{j,L})^{\nu_{i,L}}(x^{i,L,n}_{j,S})^{1-\nu_{i,L}}\right]^{\sigma^i_j}.$$

The problem of a small firm is similar to the one of a large firm, but considering the limit of $N_i \to \infty$. However, in our quantitative exercise, the cost of credit for these firms is always higher or equal to the one of their buyers. Therefore, in equilibrium, they always choose not to offer any trade credit.

2.2 Market clearing and equilibrium

All intermediate good markets clear so that the total production equals the demand from all firms and sectors plus the demand from the final producers:

$$Y_{i,L} = c_{i,L} + \sum_j^M X^{j,L}_{i,L} + \sum_j^M X^{j,S}_{i,L}.$$

The total number of workers is fixed for each sector. However, a worker can choose to work in a small or a large firm:

$$H_i = H_{i,L} + H_{i,S}.$$

In equilibrium, small firms maximize their profits given prices, while large firms internalize the demand function of their buyers. Wages inside each sector must be equalized.

2.3 Closing the Model

Once we solve for the optimal $\theta^{j,S}_{i,L}$ and $\left(p^{j,S}_{i,L} x^{j,S}_{i,L}\right)^{1-\gamma}$, the demand of intermediate inputs from each class of firms in one sector to another class of firms in other sectors can be expressed as a linear function of distortions and total sales as in most traditional models of production networks. Specifically, the demand from large firms in sector i of goods produced by large firms in sector j is:

$$p_{j,L}x^{i,L}_{j,L} = \phi^{i,L}_{j,L}\nu_{i,L}\sigma^i_j p_{i,L}y_{i,L}$$

Note that ϕ, the distortion, can be interpreted as the inverse of the markup charged in this specific sector-size link transaction. When $\phi = 1$, we are at the efficient economy; the lower the value of ϕ, the further we move from the efficient equilibrium.

We can model our economy as a standard network economy with $2M$ sectors, where each sector-size pair is considered its own sector. This allows us to obtain a $2M \times 2M$ matrix of distortions for each of the links in the economy.

From the supplier's first order conditions, we can obtain a relation between the marginal cost and the price charged to the final consumers, which then allows us to express $p^{i,L}_{j,L}$ as a markup over the final consumer price $p_{j,L}$. The expression for the distortion for a large firm in sector i buying from a large firm in sector j is given by:

$$\phi^{i,L}_{j,L} = \left[1 - \frac{1}{N_j}(1 - \nu_{i,L}\sigma^i_j)\right]$$

$$\frac{1 - \frac{1}{N_i}(1 - \nu_i\psi_i)}{1 - \frac{1}{N_j}(1 - \nu_j\psi_j)} \frac{1 + \kappa_i r_{i,L}}{1 + \kappa_j r_{j,L}} \frac{1 + (\kappa_j - \theta^{i,L}_{j,L})r_{j,L} - c\gamma\left(\theta^{i,L}_{j,L}\right)^{\eta+\gamma}\left(p^{i,L}_{j,L}x^{i,L}_{j,L}\right)^{\gamma-1}}{1 + (1 - \theta^{i,L}_{j,L})r_{i,L}} \tag{2.3}$$

The first terms in the multiplication reflect markups charged due to the market structure and the working capital constraint. $\left[1 - \frac{1}{N_j}(1 - \nu_{i,L}\sigma^i_j)\right]$ is the markup generated from the Cournot competition for sales from j to i, while $1 - \frac{1}{N_i}(1 - \nu_i\psi_i)$ distorts the revenue of the client due to the markup charged from the final consumer by large firm i (and analogously for the term $1 - \frac{1}{N_j}(1 - \nu_j\psi_j)$ which distorts $p_{j,L}$ and represents the markup charged by j from the final consumer). Both $1 + \kappa_i r_{i,L}$ and $1 + \kappa_i r_{j,L}$ result from the working capital financing constraint.

Note that the last term, however, expresses how trade credit affects distortions in the economy. The denominator, $1 + (1 - \theta^{i,L}_{j,L})r_{i,L}$, reflects how a higher share of trade credit improves the client's working capital constraint. The numerator $1 + (\kappa_j - \theta^{i,L}_{j,L})r_{j,L} - c\gamma\left(\theta^{i,L}_{j,L}\right)^{\eta+\gamma}\left(p^{i,L}_{j,L}x^{i,L}_{j,L}\right)^{\gamma-1}$ represents the marginal trade credit costs that are passed on to the client via price discrimination.

Similarly, we can express the other distortions considering all combinations of small and large firms. The expression for a large firm i buying from a small firm j consists of

$$\phi^{i,L}_{j,S} = \left[1 - \frac{1}{N_i}(1 - \nu_i\psi_i)\right]\frac{1 + \kappa_i r_{i,L}}{1 + r_{i,L}} \tag{2.4}$$

Where we have the markups from the Cournot competition in i and from the working capital constraint for i.

For a small firm i buying from a large firm j,

$$\phi_{j,L}^{i,S} = \frac{1 - \frac{1}{N_j}(1 - \nu_{i,S}\sigma_j^i)}{1 - \frac{1}{N_j}(1 - \nu_j\psi_j)} \frac{1 + \kappa_i r_{i,S}}{1 + \kappa_j r_{j,L}} \frac{1 + (\kappa_j - \theta_{j,L}^{i,S})r_{j,L} - c\gamma\left(\theta_{j,L}^{i,S}\right)^{\eta+\gamma}\left(p_{j,L}^{i,S}x_{j,L}^{i,S}\right)^{\gamma-1}}{1 + (1 - \theta_{j,L}^{i,S})r_{i,S}} \tag{2.5}$$

This is almost the same as Equation 2.3, except that since small firms are not oligopolistic, the expression lacks the markup $1 - \frac{1}{N_i}(1 - \nu_i\psi_i)$ charged from the final consumer.

Finally, for a small firm i buying from another small firm j, we have only the markup from the working capital constraint.

$$\phi_{j,S}^{i,S} = \frac{1 + \kappa_i r_{i,S}}{1 + r_{i,S}} \tag{2.6}$$

Given the equilibrium distortions ϕ_i^j derived here, the Domar weights (industry sales as a share of total GDP) can be computed as a function of distortions, intermediate share parameters (σ_i^j and ν_i) and final share parameters (ψ_i) as in a standard model of production networks.

Let λ denote the $2M$ size vector of Domar weights, where λ_i is the Domar weight of the large firm in sector i and λ_{i+M} of the small firm in sector i. If we construct the vector of "extended"final demand shares,

$$\Psi = \begin{bmatrix} \nu\psi \\ (1-\nu)\psi \end{bmatrix}$$

And the "extended"input-output matrix, which will be size $2M \times 2M$,

$$W = \begin{bmatrix} \phi_L^L \nu_L \sigma & \phi_S^L (1 - \nu_L)\sigma \\ \phi_L^S \nu_S \sigma & \phi_S^S (1 - \nu_S)\sigma \end{bmatrix}$$

We can obtain the following relation from the market clearing condition for intermediary goods, which resembles an inverse Leontief matrix:

$$\lambda' = \Psi'(I - W)^{-1}$$

In the next section, we calibrate the model using data for Brazil and evaluate the role of trade credit in mitigating the cost of interest rate dispersion between large and small firms.

3 Numerical Exercise

We consider an economy with 40 sectors. The division of sectors and the corresponding Brazilian industry codes (CNAE codes) can be found in Appendix B. We excluded banking and finance, real estate activities, and public services. The matrix of technical coefficients (including labor shares) is then set to the Brazilian input-output matrix, as well as the final demand shares. The input-output matrix is published by the Brazilian Institute of Geography and Statistics (IBGE). Table 1 shows a summary of values and sources used to calibrate parameters.

Parameter	Value
σ_i^j	From the IO Table
ψ_i	From the IO Table
$r_{i,L}$	Inferred from interest expenses
$r_{i,S}$	Inferred from firm size distribution
$\kappa_{i,L}, \kappa_{i,S}$	Mean cash to current assets ratio
ν_i	0.75
$\nu_{i,L}$	0.7
$\nu_{i,S}$	0.4
γ	0.5
η	1.5

Table 1 – Parameter calibration values.

We obtain the technical coefficient σ_i^j by dividing the sales from j to i by the total expenses for sector i, such that the production technology has constant returns to scale.

Interest rates for large firms are calculated using income statement and balance sheet data. We use data from Factset for Brazilian firms and calculate the interest rate paid by a firm by dividing interest expenses by total debt. Then, the rates are averaged by sector.

For small firms, we do not have firm-level balance sheet data available such that we can infer the different interest rates by sector. Instead, we use a measure based on the average interest rates for working capital loans with maturity of up to one year, which are calculated by SEBRAE[1] based on loan data from the Central Bank. For each sector, we use administrative data from RAIS (Annual Relation of Social Information) and classify firms as large if they have over 250 employees. We split the other firms into microenterprises (up to 9 employees), small firms (up to 49 employees), and medium firms (up to 249 employees) and obtain the proportion of each firm size by sector. The interest rate is then calculated

[1] https://datasebrae.com.br/paineltaxasdejuros

using a weighted average of the rates for medium firms (18%), small firms (33%), and microenterprises (35%).

κ is calibrated to match the average cash to current assets ratio for each sector, as it represents how much the firm can finance expenses without recurring to working capital. We also assume that $\kappa_{i,S} = \kappa_{i,L}$ for each sector i.

We assume that the final consumer spends 75% of output in goods from large firms, which is around the share that they represent in the GDP.[2] Large firms buy 70% from other large firms and small firms buy 40% from large firms.

We set $\gamma = 0.5$ and $\eta = 1.5$ for all sectors. Since they sum to 2, this turns Equation 2.1 into a quadratic equation in θ, which means we can find an analytical solution, which is presented below. This greatly speeds up the computation time.

$$\theta_{i,L}^{j,S} = \frac{1}{2-\gamma}\left[\frac{1+r_{j,S}}{r_{j,S}} - \sqrt{\left(\frac{1+r_{j,S}}{r_{j,S}}\right)^2 - \frac{2\left(p_{i,L}^{j,S}x_{i,L}^{j,S}\right)^{1-\gamma}}{c_i}\left[1 - r_{i,L}\left(\frac{1+r_{j,S}}{r_{j,S}} - \kappa_{i,L}\right)\right]}\right]$$

The use of $\gamma = 0.5$ allows us to have greater sensitivity of trade credit to sales.

The number of large firms N_i is set such that the model markup over final consumer prices

$$\left[1 - \frac{1}{N_i}(1 - \nu_i\psi_i)\right](1 + \kappa_{i,L}r_{i,L})$$

matches the average markup (calculated as sales over total costs) for each sector in the data.

For each sector, we define the net trade credit as the average value of Accounts Receivable minus Accounts Payable, over the total Current Assets. We set the cost parameters c_i in order to match the net trade credit in the model to the data. The value in the model is defined as follows:

$$NetTC_i = \frac{\sum_j \sum_s (\theta_{i,L}^{j,s} p_{i,L}^{j,s} x_{i,L}^{j,S}) - \sum_j (\theta_{j,L}^{i,L} p_{j,L}^{i,L} x_{j,L}^{i,L})}{p_{i,L}x_{i,L}}$$

Figure 3 shows the match between the net trade credit positions in the data and in the model.

[2] https://www.sebrae.com.br/sites/PortalSebrae/ufs/mt/noticias/micro-e-pequenas-empresas-geram-27-do-pib-do-brasil,ad0fc70646467410VgnVCM2000003c74010aRCRD

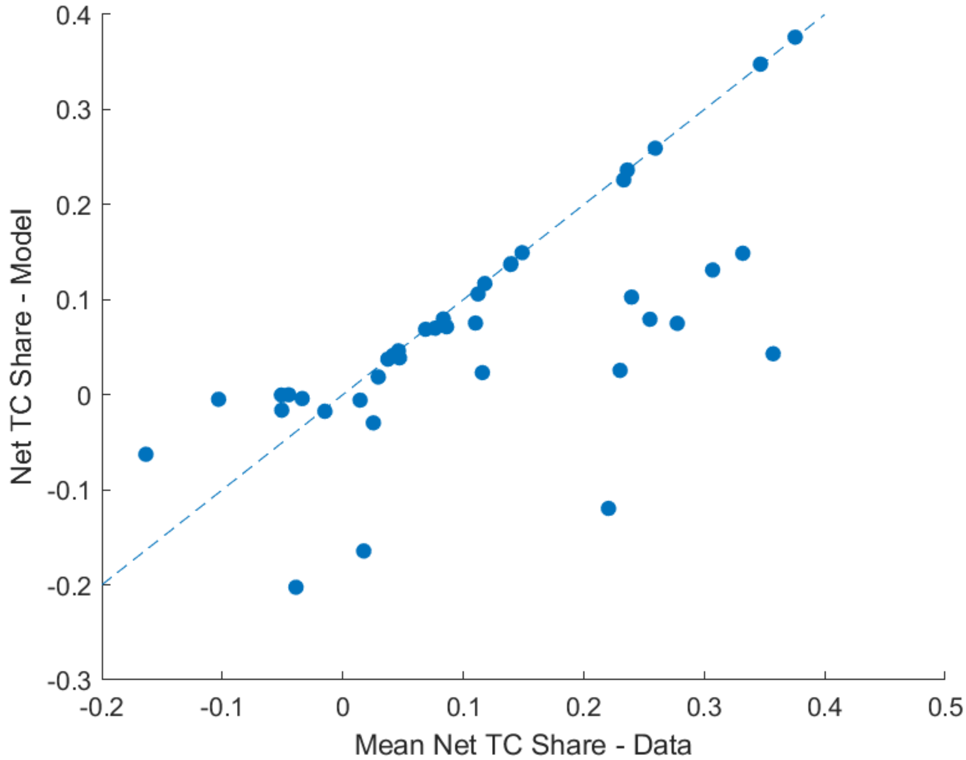

Figure 3 – Match between TC positions in model and data.

3.1 Trade Credit and Misallocation

Baqaee and Farhi (2020) and Bigio and La'o (2020) establish a theoretical framework for network economies where misallocation can be represented by sectoral distortions - in our model, represented by ϕ, described in Equations 2.3 through 2.6. These distortions can also be understood as the inverse of markups charged in prices.

In this section, we investigate the relationship between trade credit, financial shocks, and distortions and attempt to illustrate how it can reduce misallocation. Figure 4 plots Equation 2.5 for a specific sectoral link, namely the sectoral distortion of a small firm in sector 15's (Metal Industry) purchases from a large firm in sector 14 (Metallurgy), $\phi_{14,L}^{15,S}$ as a function of the trade credit share $\theta_{14,L}^{15,S}$. We obtain the value of the distortion by computing the general equilibrium keeping the values of $\theta_{14,L}^{15,S}$ fixed.

Note that the curve is concave. The initial portion is increasing since trade credit is beneficial to the client as it helps offset the working capital constraint even though they pay higher prices. However, as θ increases, the marginal costs passed to the client via price discrimination increase more than the gains from working capital, which means the curve reaches a decreasing portion.

Note that the interest rate shock to the client changes the curve's position. It tightens the client's working capital constraint due to worse working capital loan conditions,

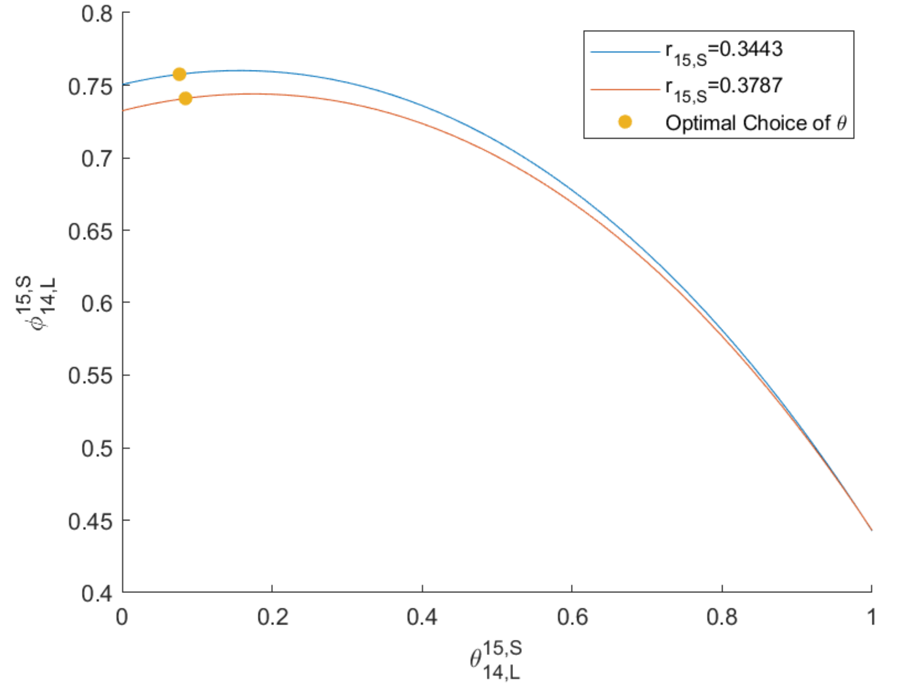

Figure 4 – Sectoral distortion from sector 14 to 15 as a function of trade credit share.

resulting in a lower value for the distortion (that is, a higher markup).

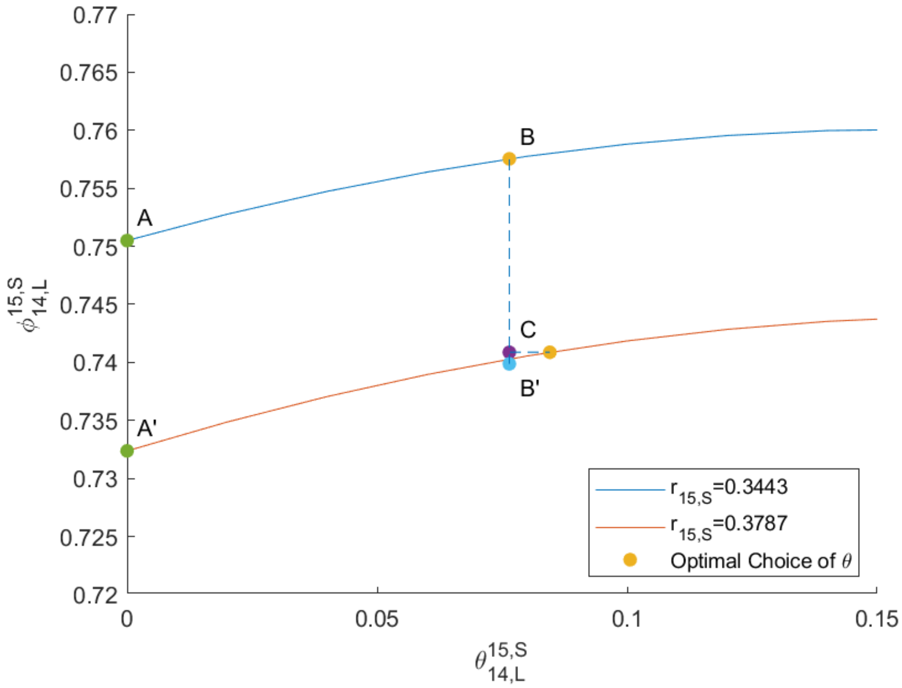

Figure 5 – Sectoral distortion from sector 14 to 15 as a function of trade credit share (close-up).

Figure 5 illustrates the role of trade credit in misallocation. Without trade credit, that is, when $\theta = 0$, the value of the distortion goes from point A to A'.

If we hold trade credit shares fixed from before the shock, we go from point B to B'. Note, however, from Figure 4 that the distance between the curves decreases as θ increases. We have then that $B - B' < A - A'$; which means that the presence of trade credit, even when it does not readjust endogenously, reduces the impact of the shock.

The more interesting result is that given that trade credit shares are endogenous, an increase in the client's interest rate leads to an increase in trade credit supply as the selling firm internalizes the opportunity to increase demand. This means that the impact is further mitigated as $B - C < B - B'$.

The shape of the curve for distortion is affected by the parameters η, γ, and c - the lower the marginal cost of trade credit, the larger the increasing part of the curve.

For sectors with a low enough cost parameter c, we find that the curve has no decreasing part for the $[0, 1]$ range for trade credit shares. Figure 6 illustrates the distortion in the reverse direction, that is, a small firm in sector 14 buying from a large firm in sector 15. As $c_{15} < c_{14}$, the marginal cost is lower and so the client is less burdened by higher prices.

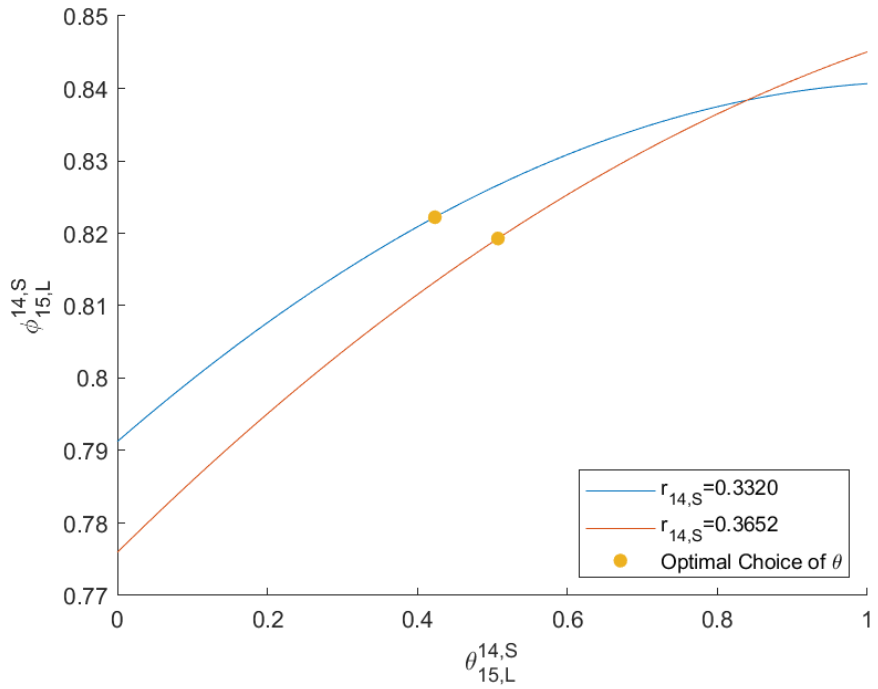

Figure 6 – Sectoral distortion from sector 15 to 14 as a function of trade credit share.

The analysis in the case of an interest rate shock to suppliers is similar. Figure 7 plots the distortion for a small firm in sector 15 buying from a large firm in sector 14 (the same as in 4), but shows the effects of a shock to the supplier's (sector 14) interest rate. In this case, note that the distance between the curves increases as θ increases; so the higher the share of trade credit, the more a shock to the supplier affects the client.

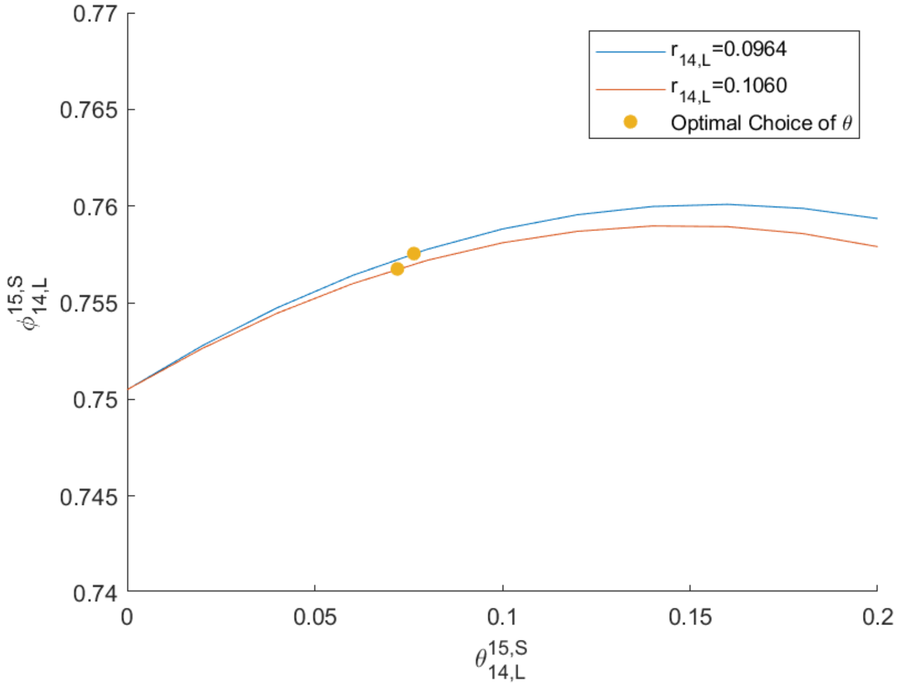

Figure 7 – Sectoral distortion from sector 14 to 15 as a function of trade credit share (shock to the supplier).

3.2 Output Loss from Interest Rate Spread

Using the model calibrated to Brazilian data, we first conduct a numerical exercise where we calculate the output loss from the spread in interest rates between large and small firms. We start with an economy where interest rates can differ between sectors, but small firms face the same rates as large firms, and then compare it to the economy with interest rates calibrated as described previously. The objective of this exercise is to quantify the misallocation generated by interest rate spreads, and how this interacts with trade credit.

We consider three different scenarios: the first is the economy described in the previous section, where firms can endogenously choose their supply and demand of trade credit. In the second scenario, the shares of trade credit are fixed to the levels when there is no spread. In the third scenario, we assume there is no trade credit allowed. The results are shown in Table 2.

Scenario	Output Loss
No TC	-2.93%
Fixed TC	-2.91%
Endogenous TC	-2.63%

Table 2 – Output loss generated by interest rate spreads.

The output loss from the interest rate spread between firm sizes in the case where there is no trade credit is approximately -2.93% and decreases only to -2.91% when we hold trade credit shares fixed from the scenario with no spread. The fact that there is almost no gain from trade credit comes from the fact that, in our counterfactual where rates are the same for both firm sizes, firms choose to give almost no credit to their clients.

In contrast, when there is an endogenous choice, the output loss is reduced to -2.63%, as firms increase their trade credit supply when their clients are in greater financial distress. This illustrates that the impact of high interest rates or high spreads, such as in developing countries, for example, is partly mitigated by larger firms acting as financial intermediaries and credit suppliers.

3.3 Effects of Financial Shocks

In the next exercise, we compute the output loss generated by shocks to sectoral interest rates by comparing the three different scenarios previously presented (economy without trade credit, holding credit fixed, and considering the endogenous choice).

We find that adverse financial shocks to small firms are always mitigated by the endogenous choice of trade credit for the parameters considered: large firms increase their supply of credit in response to the shock, providing more liquidity to their affected clients. This also means that the effect is larger when trade credit is endogenous in comparison to when it is fixed. Figure 8 compares the percentage output loss in the three scenarios considered for a 1% increase in interest rates for each sector. The x-axis represents the output loss in the scenario for the economy without trade credit.

Recall equation 2.3, which specifies the expression for demand distortions between sectors. The increase in the interest rate increases the distortion. However, when we have trade credit, we can see that the value of ϕ increases in relation to when θ is set to zero: clients' demand is higher than the scenario without credit, which explains the lower output loss. This effect is even greater when trade credit is endogenous since suppliers choose a new θ such that it decreases the distortion after the interest rate increase.

The mitigation effect is greater if the small firms have higher accounts payable: sectors that receive less trade credit before the shock will receive less liquidity. For small firms, this is related to the sector's size and its position: sectors with a higher Domar weight (the sales to GDP ratio), which are typically more upstream, tend to receive more credit. This is illustrated in Figure 9, which relates the magnitude of the output loss in the scenario with endogenous trade credit over the one without trade credit to the accounts payable of small firms.

In comparison, when small firms are hit with a positive interest shock (which we

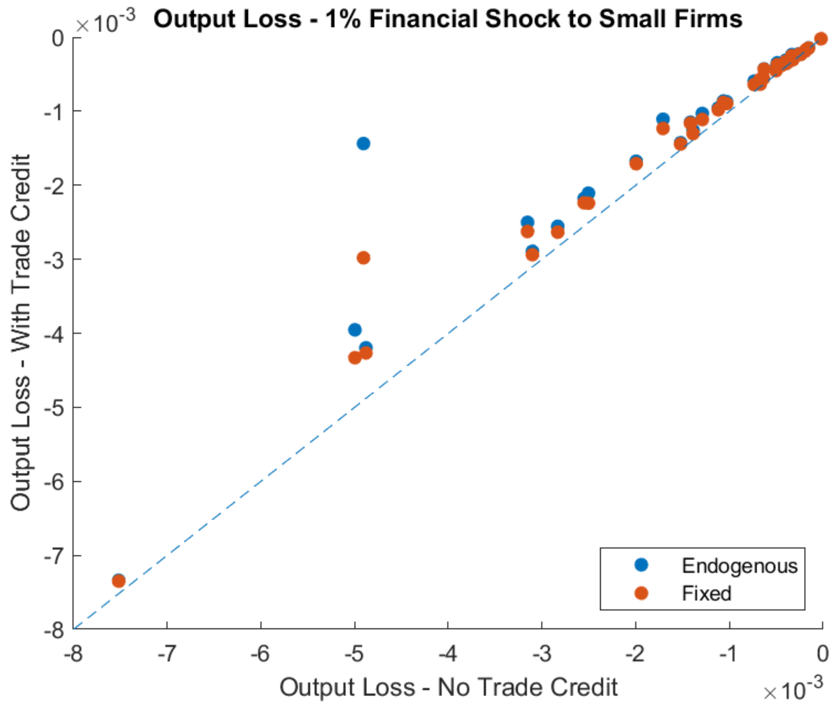

Figure 8 – Output loss from shock to small firms in specific sectors.

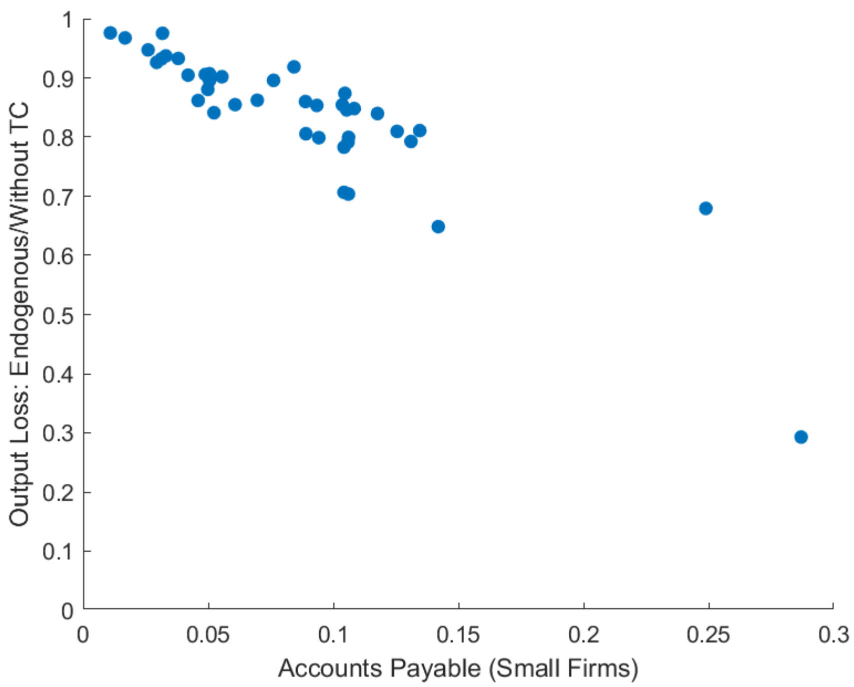

Figure 9 – Relation between shock mitigation and small firms' accounts payable.

could think of as a subsidy, for example), we also have that the magnitude of the output gain is smaller when we have trade credit. Analogously to the case of a negative shock, this can be explained both due to the reduction of credit supply and the fact there is less room to reduce misallocation.

When we consider adverse financial shocks for large firms, however, we find that the presence of trade credit can either amplify or mitigate the negative effects on output. Figure 10 illustrates the effects of a 1% increase on large firms' interest rates for each sector, considering the three scenarios presented.

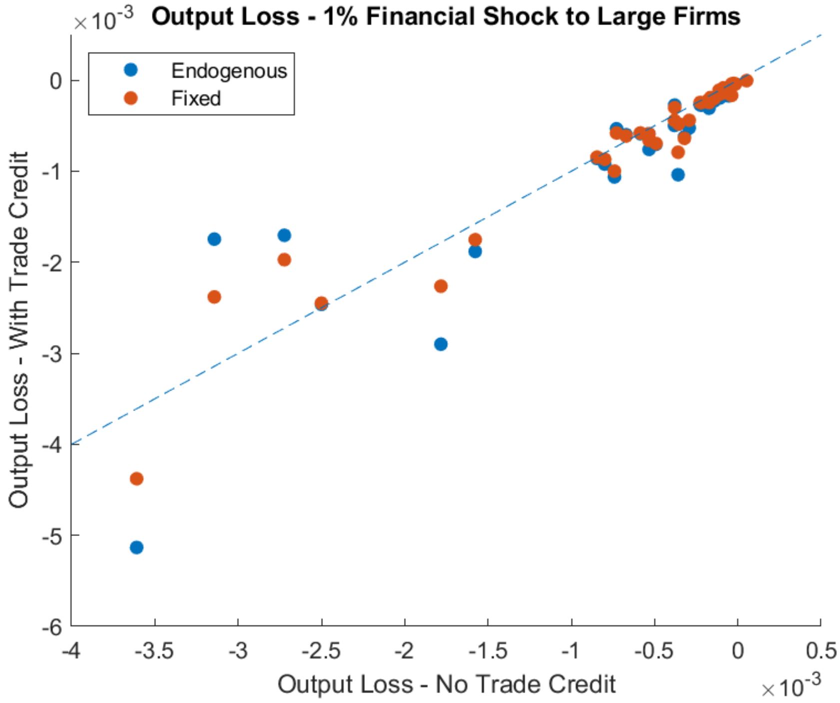

Figure 10 – Output loss from shock to large firms in specific sectors.

We can have both effects due to large firms being both suppliers and receivers of trade credit. On the one hand, since they also receive credit, we have the same mitigating effect as the shock to small firms: large companies in other sectors will increase their supply, which increases demand compared to the scenario when $\theta = 0$. So, this propagates mostly in the upstream direction of the affected sector, since its suppliers mitigate the adverse effect on demand.

On the other, we have an amplifying effect that propagates in the downstream direction, as now we have a reduction both in trade credit and sales that affect the clients of the affected sector negatively. Thus, it shows the response to a financial shock to large firms can vary depending on their position along the supply chain: we expect that for downstream sectors, which have a lower net trade credit position, the upstream mitigation effect would dominate the shock; and for upstream sectors, with a higher net trade credit position, the downstream amplification propagation would dominate.

In fact, this is what we see in Figure 11, where sectors that are net debtors (negative net position) tend to show that the shock is mitigated in the scenario with endogenous credit, in comparison to shocks that are amplified when considering that the affected sector is a net creditor (positive net position).

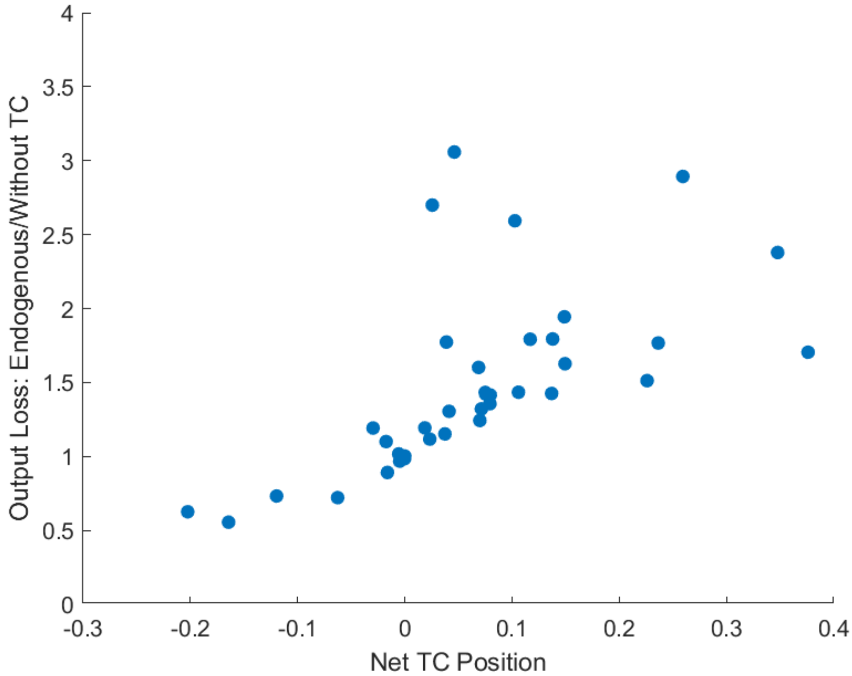

Figure 11 – Relation between shock amplification and net trade credit.

The model has some implications regarding financial shocks. First, it implies that both sectoral and size distributions of firms are relevant for the propagation of financial shocks, given that interest rates interact with trade credit. Second, trade credit is a relevant metric to evaluate the mitigation or amplification of shocks. We also find that this is related to a sector's position in the supply chain: downstream sectors such as services and retail are net debtors and so financial shocks to them are mitigated.

Note that this is particularly relevant if we consider policies that offer subsidized interest rates for large firms, for example. It implies that positive shocks to upstream firms are amplified via the trade credit channel, while the opposite happens considering downstream firms.

3.4 Effects of Productivity Shocks

We also consider how productivity shocks can interact with trade credit. Given that we consider endogenous trade credit shares that also depend on sales, a novel feature of our model is that trade credit supply may also change in response to this type of shock.

Given that the relationship between shares and sales is positive, we would expect that trade credit amplifies productivity shocks, given that the rise in sales would also lead to an increase in trade credit shares. Although we do find this effect, for the parameters used in the calibration, however, it is relatively small.

Scenario	Shock Amplification
Total Effect (Endogenous TC over no TC)	0.19%
Sales Effect (Fixed TC over no TC)	0.06%
Trade Credit Effect (Endogenous TC over Fixed)	0.14%

Table 3 – Amplification of productivity shocks due to trade credit.

Table 3 shows that shocks are amplified by 0.19% in magnitude when trade credit is endogenous in relation to the scenario without trade credit. Given that we are in a Cobb-Douglas economy, productivity shocks propagate uniformly across sectors, as shown by Baqaee and Farhi (2019), and thus this factor is the same independent of the size or sector of the affected firms.

We can decompose this amplification into two separate effects. Consider a positive productivity shock to a sector. This increases the sector's production and also lowers its prices, leading to higher sales throughout all sectors of the economy. Recalling Equations 2.3 and 2.5, this reduces the marginal monitoring cost which is passed on to firms via price discrimination. This is the first effect, which we can calculate by measuring the amplification when shares are kept fixed.

This, however, explains only one third of the overall effect (0.06% of the 0.19%). If we then account for the variation of trade credit shares due to the lower monitoring cost, we see that it is responsible for the majority of the amplification. We considered how higher sales reduced the marginal monitoring cost; this, in turn, will also increase the trade credit supply. Higher trade credit shares will help with firms' constraints and generate more positive impacts, which explains how this mechanism can amplify shocks.

Even if the final effect is small, this seems to suggest that taking endogenous trade credit into account is relevant for the propagation of productivity shocks. A possible extension of this model could consider non-linear generalized production networks, such as incorporating CES production functions, which could lead to amplification results that vary depending on firm sector and size.

4 Conclusion

This paper proposes a model of endogenous trade credit in a multisector environment with interest rate dispersion and input-output linkages. In each sector, there are small and large firms. The first are price takers, while the second choose production and the share of trade credit internalizing the demand function of all their clients. Since large firms can obtain bank credit at a lower price, in equilibrium they provide trade credit to small firms. This is in line with empirical evidence showing that in Brazil large firms are net providers of trade credit. The model includes both upstream and downstream propagation effects through the trade credit linkages.

We calibrate the model to Brazilian data and show that the presence of trade credit always mitigates the aggregate effect of an increase in interest rates for small firms in specific sectors. However, a change in the interest rate spread can either amplify or mitigate the effect depending on the sector. The framework can be used to run additional quantitative exercises related to productivity and financial shocks. In particular, we can quantify the distributional effect of these shocks across different sectors and firm sizes.

Bibliography

Acemoglu, D.; Carvalho, V. M.; Ozdaglar, A.; Tahbaz-Salehi, A. The network origins of aggregate fluctuations. *Econometrica*, Wiley Online Library, v. 80, n. 5, p. 1977–2016, 2012.

Altinoglu, L. The origins of aggregate fluctuations in a credit network economy. *Journal of Monetary Economics*, Elsevier, v. 117, p. 316–334, 2021.

Banerjee, A. V. Contracting constraints, credit markets and economic development. *Credit Markets and Economic Development (September 2001)*, 2001.

Banerjee, A. V.; Duflo, E. Growth theory through the lens of development economics. *Handbook of economic growth*, Elsevier, v. 1, p. 473–552, 2005.

Baqaee, D.; Farhi, E. Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics*, v. 135, n. 1, p. 105–163, 2020.

Baqaee, D. R.; Farhi, E. *Macroeconomics with heterogeneous agents and input-output networks*. 2018.

Baqaee, D. R.; Farhi, E. The macroeconomic impact of microeconomic shocks: beyond hulten's theorem. *Econometrica*, Wiley Online Library, v. 87, n. 4, p. 1155–1203, 2019.

Bigio, S.; La'o, J. Distortions in production networks. *The Quarterly Journal of Economics*, Oxford University Press, v. 135, n. 4, p. 2187–2253, 2020.

Bocola, L.; Bornstein, G. The macroeconomics of trade credit. *NBER Working Paper 31026*, 2023.

Carvalho, V. M. From micro to macro via production networks. *Journal of Economic Perspectives*, v. 28, n. 4, p. 23–48, 2014.

Carvalho, V. M.; Tahbaz-Salehi, A. Production networks: A primer. *Annual Review of Economics*, Annual Reviews, v. 11, 2019.

Cavalcanti, T. V.; Kaboski, J. P.; Martins, B. S.; Santos, C. *Dispersion in financing costs and development*. 2021.

Demirguc-Kunt, A.; Maksimovic, V. Firms as financial intermediaries: Evidence from trade credit data. World Bank, Washington, DC, 2001.

Garcia-Appendini, E.; Montoriol-Garriga, J. Firms as liquidity providers: Evidence from the 2007–2008 financial crisis. *Journal of financial economics*, Elsevier, v. 109, n. 1, p. 272–291, 2013.

Garcia-Marin, A.; Justel, S.; Schmidt-Eisenlohr, T. Trade credit, markups, and relationships. CESifo Working Paper, 2019.

Gilchrist, S.; Sim, J. W.; Zakrajšek, E. Misallocation and financial market frictions: Some direct evidence from the dispersion in borrowing costs. *Review of Economic Dynamics*, Elsevier, v. 16, n. 1, p. 159–176, 2013.

Hardy, F. E. S. B.; Simonovska, I. Trade credit and exchange rate risk pass through. *NBER Working Paper 31026*, 2023.

Hsieh, C.-T.; Klenow, P. J. Misallocation and manufacturing tfp in china and india. *The Quarterly journal of economics*, MIT Press, v. 124, n. 4, p. 1403–1448, 2009.

Jacobson, T.; Schedvin, E. V. Trade credit and the propagation of corporate failure: An empirical analysis. *Econometrica*, Wiley Online Library, v. 83, n. 4, p. 1315–1371, 2015.

Jones, C. I. Intermediate goods and weak links in the theory of economic development. *American Economic Journal: Macroeconomics*, v. 3, n. 2, p. 1–28, 2011.

Jr, J. B. L.; Plosser, C. I. Real business cycles. *Journal of political Economy*, The University of Chicago Press, v. 91, n. 1, p. 39–69, 1983.

Liu, E. Industrial policies in production networks. *The Quarterly Journal of Economics*, Oxford University Press, v. 134, n. 4, p. 1883–1948, 2019.

Luo, S. Propagation of financial shocks in an input-output economy with trade and financial linkages of firms. *Review of Economic Dynamics*, Elsevier, v. 36, p. 246–269, 2020.

Petersen, M. A.; Rajan, R. G. Trade credit: theories and evidence. *The review of financial studies*, Oxford University Press, v. 10, n. 3, p. 661–691, 1997.

Restuccia, D.; Rogerson, R. Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic dynamics*, Elsevier, v. 11, n. 4, p. 707–720, 2008.

APPENDIX A – Relation Between Trade Credit and Interest Rates

The relation between higher interest rates and lower trade credit supply has been documented in the literature. We find evidence of this relation using firm-level quarterly data for Brazilian firms from Factset in the 2017 to 2019 period.

Although we do not have information on interest rates contracted by firms, we construct a proxy that is also used to calibrate the model using interest expenses over total debt. We construct the net trade credit position measure as accounts receivable minus accounts payable over the total current assets. We add sector and period fixed effects.

	Net TC Position
Quarterly Interest Rate	-0.50052^{***}
	(0.174475)
Sector FE	Yes
Quarter FE	Yes
R^2	0.31
Observations	4670

Table 4 – Relation between net trade credit position and interest rates. Standard errors are robust. *** $p < 0.01$

APPENDIX B – List of Sectors and CNAE Codes

The 40-sector division of the economy is done according to the following correspondence between sectors and Brazilian 2-digit industry codes (CNAE). We also present the calibration for some of the model's variables.

Sector	CNAE Codes	κ_i	$r_{i,S}$	$r_{i,L}$
Agriculture	01-03	0.29	0.34	0.09
Oil and Gas	05-06	0.23	0.34	0.24
Mining	07-09	0.35	0.34	0.14
Food Industry	10-12	0.25	0.34	0.15
Textile and Clothing Industry	13-15	0.11	0.34	0.17
Wood Industry	16	0.17	0.34	0.21
Paper Industry	17	0.28	0.33	0.08
Printing Industry	18	0.33	0.35	0.11
Fuel Industry	19	0.21	0.32	0.08
Chemical Industry	20	0.15	0.33	0.19
Pharmaceutical Industry	21	0.24	0.32	0.11
Plastics and Rubber Industry	22	0.02	0.33	0.44
Non-Metallic Minerals Industry	23	0.18	0.34	0.12
Metallurgy	24	0.16	0.33	0.1
Metal Industry	25	0.02	0.34	0.13
Electrical Machines and Equipment	26-27	0.16	0.35	0.15
Mechanical Machines and Equipment	28	0.1	0.34	0.11
Vehicles and Parts	29-30	0.28	0.34	0.09
Furniture and Other Industries	31-32	0.12	0.34	0.49
Machine Repair and Maintenance	33	0.21	0.35	0.04

Table 5 – Sector and CNAE code correspondence.

Sector	CNAE Codes	κ_i	$r_{i,S}$	$r_{i,L}$
Energy	35	0.27	0.35	0.1
Water and Waste Management	36-39	0.24	0.34	0.1
Construction	41-43	0.11	0.34	0.12
Vehicles and Parts Retail	45	0.13	0.33	0.23
Retail and Wholesale	46-47	0.2	0.34	0.14
Land Transportation	49	0.3	0.34	0.09
Water Transportation	50	0.2	0.35	0.03
Air Transportation	51	0.31	0.34	0.13
Storage and Post	52-53	0.49	0.35	0.17
Food Service and Hotels	55-56	0.29	0.35	0.22
Radio, TV and Editing	58-60	0.46	0.35	0.12
Telecommunications	61	0.29	0.34	0.12
Information Technology	62-63	0.36	0.35	0.16
Professional Services	69-75	0.42	0.35	0.12
Non-Real Estate Rentals	77	0.33	0.35	0.1
Administrative Services	78-82	0.22	0.34	0.21
Private Education	85	0.29	0.34	0.15
Private Health	86-87	0.17	0.35	0.1
Arts, Culture and Sports	90-93	0.54	0.35	0.22
Other Services	94-96	0.22	0.35	0.14

Table 6 – Sector and CNAE code correspondence, continued.

APPENDIX C – Detailed Derivation

In this section, we give a more detailed view on how to and solve the model.

First, we consider the small firm's problem:

$$\begin{aligned} \max \quad & p_{i,S} A_{i,S} (h^{i,S})^{\alpha_i} \prod_j \left[(x^{i,S}_{j,L})^{\nu_{i,S}} (x^{i,S}_{j,S})^{1-\nu_{i,S}} \right]^{\sigma^i_j} - w_i h^{i,S} - \sum_{s\in\{S,L\}} \sum_j p^{i,S}_{j,s} x^{i,S}_{j,s} - r_{i,S} D_{i,S} \\ \text{s.t.} \quad & \sum_{s\in\{S,L\}} \sum_j (1-\theta^{i,S}_{j,s}) p^{i,S}_{j,s} x^{i,S}_{j,s} + w_i h^{i,S} \leq \kappa_{i,S} p_{i,S} y_{i,S} + D_{i,S} \end{aligned}$$

The working capital restrictions always binds (with $D_{i,S} > 0$ given the inclusion of κ). This means we can rewrite the problem substituting for $D_{i,S}$ as:

$$\max \quad \overbrace{[1+\kappa_{i,S} r_{i,S}] p_{i,S} \underbrace{A_{i,S} (h^{i,S})^{\alpha_i} \prod_j \left[(x^{i,S}_{j,L})^{\nu_{i,S}} (x^{i,S}_{j,S})^{1-\nu_{i,S}} \right]^{\sigma^i_j}}_{y_{i,S}}}^{\text{Small firms have no market power}}$$

$$-(1+r_{i,S}) w_i h^{i,S} - \sum_{s\in\{S,L\}} \sum_j^M [1 + (1-\theta^{i,S}_{j,s}) r_{i,S}] p^{i,S}_{j,s} x^{i,S}_{j,s}$$

From the first-order conditions in relation to inputs, we can obtain the demand function:

$$p^{j,S}_{i,L} = \frac{p_{j,S} A_{j,S} (h^{j,S})^{\alpha_j} \prod_k \left[(x^{j,S}_{k,L})^{\nu_{j,S}} (x^{j,S}_{k,S})^{1-\nu_{j,S}} \right]^{\sigma^j_k}}{1 + (1-\theta^{j,S}_{i,L,n}) r_j} \frac{\nu_{j,S} \sigma^j_i}{\left(\sum_k^{N_i} x^{j,S}_{i,L,k} \right)^{1-\nu_{j,S}\sigma^j_i}}.$$

Final consumers solve the following problem:

$$\max \prod_{i\in I} \left[(c_{i,L})^{\nu_i} (c_{i,S})^{1-\nu_i} \right]^{\psi_i} - \sum_{s\in\{S,L\}} \sum_i p_{i,s} c_{i,s}$$

Since large firms also have market power over the final consumer, this yields the following demand function:

$$p_{i,L} = \frac{\nu_i \psi_i Q}{\left(\sum_k^{N_i - n} z_{i,k} + z_{i,n} \right)^{1-\nu_i\psi_i}}.$$

Here, $c_{i,s}$ refers to the final consumer's demand for good i, s while $z_{i,s}$ refers to the supply. Q is final output.

We can then set up the large firm's problem as stated in section 2. The next step is solving for the general equilibrium using the expressions for the distortions/inverse markups. From the large firm's first-order condition in relation to $z_{i,L}$:

$$\mu_{i,L} = (1 + \kappa_{i,L} r_{i,L}) \left[1 - \frac{1}{N_i} (1 - \nu_i \psi_i) \right] p_{i,L}$$

Substituting for the marginal cost in equation 2.2, we obtain a relation between the discriminated price $p_{i,L}^{j,S}$ and the final consumer price $p_{i,L}$. The final step is substituting $p_{i,L}^{j,S}$ in the demand function. We then obtain the following simplified demand function using final consumer prices as reference:

$$x_{i,L}^{j,S} = \phi_{i,L}^{j,S} \nu_{j,S} \sigma_i^j \frac{p_{j,S} y_{j,S}}{p_{i,L}}$$

With $\phi_{i,L}^{j,S}$ given as in equation 2.5.

We have the following market clearing condition for intermediary goods:

$$y_{i,L} = c_{i,L} + \sum_{s \in \{S,L\}} \sum_j x_{i,L}^{j,s}$$

We can substitute the expressions for intermediate input demands:

$$y_{i,L} = \frac{\nu_i \psi_i Q}{p_{i,L}} + \sum_{s \in \{S,L\}} \sum_j \phi_{i,L}^{j,s} \nu_{j,s} \sigma_i^j \frac{p_{j,s} y_{j,s}}{p_{i,L}}$$

$$p_{i,L} y_{i,L} = \nu_i \psi_i Q + \sum_{s \in \{S,L\}} \sum_j \phi_{i,L}^{j,s} \nu_{j,s} \sigma_i^j p_{j,s} y_{j,s}$$

Divide by Q to obtain

$$\lambda_{i,L} = \nu_i \psi_i + \sum_{s \in \{S,L\}} \sum_j \phi_{i,L}^{j,s} \nu_{j,s} \lambda_{j,s}$$

This sets up a linear system which we can solve for the Domar weights λ.

Next, we detail the computational procedure used to solve the model. We use an iterative procedure such that the values of the trade credit shares converge.

We begin by calculating the initial value for the model where $\gamma = 1$; note from equations 2.1 and 2.3 that the values of theta and distortions depend only on parameters in this case. This allows us to obtain the Domar weights using the general equilibrium relations.

Using the Domar weights, we can calculate labor demands by equating wages across firm sizes. We make a first guess that the total output is one. Substituting the demands in the production functions, we obtain a log-linear system that can be solved for the values of each sector's production. From the Domar weights, prices and final demands are calculated, which are then used to calculate the total output. This is iterated until the output converges.

Next, the demand functions can be used to calculate intermediate input sales $p_{j,s}^{i,s} x_{j,s}^{i,s}$. We use these values to compute trade credit shares when $\gamma < 1$. We then run the model once more to obtain the intermediate input sales and repeat the process until trade credit shares converge.