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Redistribution with Labor Market Frictions*

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Abstract

How should search frictions in the labor market affect distributive policies? Can we assess current real-world policies? After building a framework for answering these questions we show that any constrained efficient allocation must satisfy the following set of testable restrictions: i) earnings and employment probability must be co-monotone, ii) wedges on taxable income and employment probability must have the same sign; and; iii) wedges at the bottom of the distribution of income should be positive. Labor income tax schedules and unemployment benefits are shown not to suffice for implementing constrained efficient allocations. Firms can nonetheless be provided incentives to generate the efficient supply of vacancies using informationally feasible tax instruments. We devise a method for the quantitative assessment of inefficiency, calibrate our model to the U.S. economy, and find that it is possible to increase government revenues by 3.48% while preserving everyone's utility. ***Keywords:*** *Mirrlees' problem; Directed Search.* ***JEL Classification:*** *D82, H21.*

THANKS in great part to the use of tax perturbation methods, a burgeoning body of recent work – Lehmann et al. (2011); Geromichalos (2015); Kroft et al. (2020); Hummel (2019) – has uncovered important effects of taxes on employment with subtle consequences for distributive policy. Yet, however relevant for assessing the optimal design of existing policies, the tax perturbation approach is ill-equipped to address constrained efficiency. Since this approach takes the set of tax instruments as given, it cannot answer the question of what is feasible if more instruments are given to the planner. For this, a framework is needed

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which combines the informational structure that precludes first-best redistribution and the labor market frictions that lead to unemployment. Offering such a framework is the purpose of this paper.

We assume that the labor market is decentralized and frictional; as in Golosov et al. (2013), it is characterized by competitive search (Shimer (1996); Moen (1997); Mortensen and Wright (2002)). Firms post vacancies which are contract offers comprising an output to be produced by a worker and the earnings to which the worker is entitled if the output is produced. Each contract thus defines a sub-market. Posting a vacancy/contract is costly for firms. Once posted, the probability that a vacancy is filled depends on the sub-market tightness. Workers in turn apply to vacancies that maximize their expected utility. A matching function summarizes the process that brings together firms and workers interested in the same labor contracts. Not all workers willing to accept a posted contract find a job, and not all vacancies with interested candidates are filled.

In contrast with Golosov et al. (2013), we assume that workers are heterogeneous, whereas firms are homogeneous. This allows us to consider distributive policies aiming not at the residual income emphasized by Golosov et al. (2013) but at income inequality related to differences in workers' productivities to be addressed.[1]

Most current policies create distortions in the vacancy creation margin. However, simply noting that such a margin is distorted says nothing about inefficiency. While wedges naturally arise in an economy in which distortionary taxes are imposed, a basic tenet of second-best analysis – e.g., Lipsey and Lancaster (1956) – is that counting distortions is not the proper way of doing welfare analysis. Because we fully specify the informational structure that underlies policy choices we are able to characterize constrained efficient allocations and assess optimal wedges.

Two wedges distinguish constrained efficient and the first-best allocations. First, is the effort wedge, which is the difference between the marginal rate of substitution between effort and consumption and the marginal rate of transformation of labor and output. Second, is the vacancy wedge, the difference between the marginal social cost and benefit of increasing the probability of employment through the creation of new vacancies. We show that, if an allocation is constrained Pareto efficient, then: (i) the probability of finding a job and the output produced per worker in each sub-market are co-monotone; (ii) effort and vacancy wedges share a common sign; and (iii) wedges are positive at the bottom of the income distribution even when the underlying (Paretian) objective entails finite inequality aversion.[2] Workers who find it easier to produce, also display a stronger preference for being employed. This leads to complementarity in manipulating the two margins: effort, and

[1]This brings our work closer to that of Mirrlees (1971); Diamond (1998); Saez (2001).

[2]Seade (1977) proves that, as long as the planner's objective assigns non-negative but finite weight on all agents in a Mirrlees' economy unless there is bunching, marginal tax rates are zero at the bottom of the distribution.

employment. Whether too much or too little effort and employment must be addressed depends on the specific underlying objective, of which we are agnostic. Independently of the objective, however, the existence of unemployed workers means that even for the least productive (employed) workers, it is possible to find a positive mass of agents for which the marginal value of consumption is no smaller than theirs. This is what makes it worthwhile to pay the welfare cost of distorting low productivity workers' allocation.

An important aspect of restrictions (i) to (iii) is their strong empirical content. In particular, (i) can be directly verified with the appropriate data, while (ii) can be checked by a suitable calibration of the underlying model. Testing efficiency is, therefore, not only feasible but also relatively straightforward. Beyond testing, we devise a simple approach for quantifying the inefficiencies of any observed allocation. In our baseline calibration for the U.S. economy, we find that the use of optimal policies leads to a 3.48% increase in net revenue for the Government while preserving the expected utility of all workers.

Optimal policies induce higher employment for low productivity agents and lower for high productivity agents. It is important also to emphasize that our framework allows us to handle the endogeneity of wages which, as in Acemoglu and Shimer (1999), arise due to general equilibrium effects.[3] Pre-tax *wage* inequality decreases under the optimal policy.

While our main approach takes whatever choices society has made and simply asks whether welfare improvements are possible, we also derive the optimal policy and devise a numeric method for the case in which we are given the planner's objective. The formula for optimal effort wedges are very similar to optimal labor wedges of economies without labor market frictions; Pareto weights must, however, be adjusted to account for the fact that not all agents are employed. We express optimal tax formulae using elasticities of taxable income as in Saez (2001) and use them to adapt the Mankiw et al.'s (2009) numeric procedure to compute the Utilitarian optimum.

With regards to implementation, we show that not all incentive feasible allocations can be implemented by policies that rely on an unemployment benefit and a non-linear tax schedule. Adding a tax/subsidy on vacancy creation to the set of policy instruments does permit optimum allocations to be implemented. While there are many different ways an allocation can be implemented, it is possible to rely on a tax system in which labor income taxes only depend on each worker's earnings and effort wedges *are* marginal income tax rates.

The rest of the paper is organized as follows. Following a brief literature review, we describe the model economy in Section 1. Section2 contains the main results of the paper. In 2.1 the program used to characterize efficient allocations is presented, and its solution derived. In 3 a non-linear income tax schedule, combined with informationally feasible unemployment benefits, is shown not to suffice for implementing the constrained efficient allocation. Examples of feasible instruments capable of doing it are presented. Section 4

[3]General equilibrium effects like the ones considered here are very hard to handle with a perturbation approach, as Sachs et al.'s (2020) *tour de force* reminds us.

assesses the costs for the U.S. government of not using such instruments. This work is mostly concerned with efficiency regardless of a society's fairness concerns. Yet, for completeness, in Section 2.2 we discuss the consequence of labor market frictions for the commonly studied Utilitarian and Rawlsian metrics. Section 6 concludes. The appendix contains all proofs.

Literature Review

The interaction between taxes and labor market frictions in equilibrium unemployment models has been the object of academic interest since Pissarides (1985). Golosov et al. (2013); Schaal and Taschereau-Dumuouchel (2014); Geromichalos (2015); Boadway and Cuff (2016); Lehmann et al. (2016); Kroft et al. (2020); Hummel (2019) are recent representative works on the topic.

Lehmann et al.'s (2006) pioneering work applies a mechanism design approach to an optimal taxation problem in a labor market characterized by search frictions. The assumption of ex-post wage bargain leads to surpluses for both the firm and the worker.[4] The mechanism allows for the extraction of information from the pair, which, however cooperative, still have conflicting interests. The authors consider extensive margin responses only, whereas we take the intensive margin adjustments into account. We assess how labor *market frictions alone* affect optimal tax prescriptions and create extensive margin distortions. Moreover, in our model firms commit to contract. This eliminates all surpluses beyond the informational rents that accrue to workers.

Boadway and Cuff (2016) combine insurance and redistribution concerns in policy design by assuming a heterogeneous mix of workers. They do not consider intensive margin adjustments but allow for non-participation by assuming that agents are heterogeneous in their participation costs. As already mentioned, in the spirit of the majority of the directed search literature, we assume that firms can commit to posted wages, whereas wages are determined through bargain in Boadway and Cuff (2016).

Closest to our work are Kroft et al. (2020) and Hummel (2019). Kroft et al. (2020) is a prime example of a burgeoning literature aiming at providing general principles and at offering direct policy recommendations which rely on estimable sufficient statistics. Their work relies on a perturbation/sufficient statistic approach to decompose the effect of tax policies on micro (i.e., individual) responses and macro (i.e., equilibrium) responses in a very general model. They consider only extensive margin responses, whereas intensive margin responses are essential for our framework.

In concurrent work, Hummel (2019) also follows a perturbation/sufficient statistic approach to study optimal taxation in an economy with a directed search. In a setting in which a heterogeneous mix of workers choose labor supply in the extensive and intensive mar-

[4]Following up on their very original contribution, Lehmann et al. (2011) extends the model to allow for endogenous participation.

gins, Hummel (2019) uses policy elasticities to derive testable necessary conditions for tax systems to maximize a utilitarian criterion. We, in contrast, characterize the whole set of constrained efficient allocations using a mechanism design approach along the lines of Werning (2007). We show that efficiency imposes restrictions on observable allocations akin to those derived by Werning (2007). Beyond these, we obtain a novel set of testable restrictions which arise due to labor market frictions—see findings (i)–(iii) in the introduction.

Our work also clarifies the set of policy instruments necessary to implement second-best allocations. As we show, the combination of labor income taxes and unemployment benefits is not sufficient. In this sense, our numeric findings represent gains above the ones that can be found in the important complementary works mentioned above.

Geromichalos (2015) studies how taxes used to finance an unemployment insurance program affect efficiency. As in our case, a directed search model is used. Contrary to what we do here, Geromichalos (2015) assumes that workers are identical and focuses on the externalities created by the unemployment insurance funding itself. We provide a complete characterization of these externalities and compute them at the optimum in our numeric exercises. Indeed, when firms must bear the full costs of posting vacancies the implicit vacancy wedge is non-zero; private and social costs do not coincide.

Another important reference is Guerrieri et al. (2010), which studies economies with adverse selection and frictions due to competitive search.[5] Like in our case, firms post contracts to which workers must self-select. They prove equilibrium existence and uniqueness in a setting with common values. They also show that equilibria will not be efficient in general. In our work, we consider private values that would lead to the first-best were it not for the labor market friction and society's distributive goals. We focus on optimal policy, in particular, characterizing constrained efficient allocations and evaluating the costs of not using all the publicly available information. Finally, to focus on the consequences of labor market frictions for distributive policies, we refrain from addressing the type of ex-post wage dispersion and residual inequality that motivates Golosov et al. (2013). We do so not because we think this aspect is unimportant but because it already has been competently addressed in Golosov et al. (2013) and, most importantly, because by combining these two dimensions of policy in a single framework, we lose tractability.

[5] Guerrieri (2008) also considers directed search with private information, but this pertains to the quality of the match, information that is known only after the match takes place.

1 Environment

The economy is inhabited by a continuum of agents with preferences defined over consumption, c, and effort, n, represented by

$$\mathcal{U}(c,n,\theta) := \varphi(c) - \theta\eta(n),$$

where $\varphi : \mathbb{R}_+ \mapsto \mathbb{R}$ is a smooth, increasing, and strictly concave function, and $\eta : \mathbb{R}_+ \mapsto \mathbb{R}$ is a smooth, increasing, and strictly convex function, with $\varphi', -\varphi'', \eta', \eta'' > 0$.

Heterogeneity across agents is captured by the disutility of effort parameter θ, which follows a distribution $F(\cdot)$ which has a support in the $[\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} > 0$ – a compact interval. The associated density, $f(\cdot)$, $f(\theta) > 0 \; \forall \; \theta$, is common knowledge. Each person's θ is her private information.

The production side of the economy is as follows. One unit of effort, n, produces one unit of output, z, measured in units of consumption. Market frictions are captured by the assumption that opening a vacancy is costly. We follow most of the directed-search literature and assume that a firm can create (at most) one job opportunity by paying a fixed cost $\kappa > 0$.

A job opportunity is a contract specifying an output, z, to be produced and a payment y to which the worker is entitled if z is produced. We endow each firm with the ability to commit to contract offers.[6] Job opportunities, (z, y), are publicly observable. Each pair (z, y) defines ***a sub-market***. A worker who applies to a given job has a probability p of finding a job, where p is a function of λ, the workers-to-vacancies ratio for that sub-market.

Similarly, when deciding which sub-market to enter and whether to enter at all, a firm takes as given the ratio λ of workers applying for the same job and vacancies at each sub-market. Intuitively, one can imagine that it is not too difficult for a worker to observe a contract offer. But, without a centralized mechanism coordinating search efforts, nothing prevents multiple workers from applying for the same offer. This leads some workers to remain unemployed and some vacancies not to be filled simultaneously.

Although we could focus on the map from λ to p, it will be more convenient to use the map $\lambda : [0, 1) \to \mathbb{R}_{++}$, where $\lambda(p)$ represents the workers-to-vacancies ratio in a sub-market in which workers find employment with probability p, or, more often, with $\vartheta(p) := \lambda\left(p\right)^{-1}$, the function defining the vacancies-to-workers ratio, in a sub-market in which workers find employment with probability p. We assume that ϑ is a strictly convex, strictly increasing, and twice differentiable function satisfying $\vartheta(0) = 0, \lim_{p\uparrow 1} \vartheta(p) = \infty$.

Policy The economy is also inhabited by a government that designs distributive and insurance policies to maximize some welfarist objective.

[6]Renegotiation does not occur after a match is realized, and, thus, the wage compression effect highlighted in Hungerbühler et al. (2006) and Lehmann et al. (2016) does not play a role here.

Standard Policy We initially consider a policy comprised of a non-linear income tax schedule, $T(\cdot)$, and unemployment benefits, $c^u(\theta)$. This will allow us to define the potential misalignment between private and social objectives in the form of effort and employment probability wedges in an often analyzed setup.

We shall refer to this policy as ***a standard policy***.

Using the following transformations in variables: $u(\theta) := \varphi(c(\theta))$, $h(\theta) := \eta(z(\theta))$, $\underline{u}(\theta) = \varphi(c^u(\theta))$. We define an allocation as a mapping $\Theta \mapsto [0,1] \times \mathbb{R}^3$ from types to tuples $(p, u, h, \underline{u})$. Associated with an allocation, $\big(p(\theta), u(\theta), h(\theta), \underline{u}(\theta)\big)_\theta$, is expected utility profile $(w(\theta))_\theta$, where

$$w(\theta) = p(\theta)\left[u(\theta) - \theta h(\theta)\right] + (1 - p(\theta))\,\underline{u}(\theta) \tag{1.1}$$

is the expect utility of a type θ worker.

If we implicitly define the functions, C and N through $C(\varphi(c)) := c$ for all c and $N(\eta(z)) := z$, for all z, then it is possible to define a ***feasible allocation*** as an allocation which satisfies the resource constraint,

$$\int \left\{ p(\theta)\left[N\left(h(\theta)\right) - C(u(\theta))\right] - (1 - p(\theta))C(\underline{u}(\theta)) - \frac{\kappa}{\lambda\left(p(\theta)\right)} \right\} f(\theta)d\theta \geq G, \tag{1.2}$$

for some exogenous G.

Equilibrium under a Standard Policy For a given standard policy one may establish through the labor income tax schedule, $T(\cdot)$, the cost $\chi : \mathbb{U} \mapsto R$ of delivering a flow utility of consumption u; $\varphi\left(\chi(u) - T\big(\chi(u)\big)\right) = u$ for all $u \in \mathbb{U}$. In what follows, we use the pair $(\chi(\cdot), \underline{u}(\cdot))$ to represent our standard policy.

A firm can enter (create a vacancy in) any sub-market θ provided it pays the fixed cost κ. If a standard policy is in place, a firm that decides to serve type θ workers maximize expected profits, $\lambda(p)p\left[N(h) - \chi(u)\right]$, subject to delivering expected utility $w(\theta)$ to these agents.

Note that firms take the equilibrium expected utility profile $(w(\theta))_\theta$ as given. Also, no incentive constraints are imposed on the firm. In fact, this is a private values environment in a competitive setting: *conditional on the labor contract*, $\big(\chi(u), N(h)\big)$, the firm's profit is independent of the type of workers who accept it. A firm will only create a vacancy $\big(\chi(u), N(h)\big)$ in sub-market θ if $\lambda(p(\theta))p(\theta)\left[\chi(u) - N(h)\right] + \kappa \leq 0$. Free entry guarantees that, for all θ, this will hold as an equality.

A standard policy affects the firm's optimization problem through two different channels. First, $\chi(\cdot)$ substitutes for $C(\cdot)$ as the cost function associated with the utility from consumption u. Second, $\underline{u}(\theta)$ affects the expected utility that the firm must deliver to a type θ worker. $\chi(\cdot)$ differs from $C(\cdot)$ because it takes into account the labor income tax schedule set in place by the government.

Using $\left(\chi(u(\theta)), N(h(\theta)\right)_\theta$ to define the set of equilibrium labor contract offers and $\left(p(\theta)\right)_\theta$ the probabilities associated with each of these sub-markets, then a type θ worker's decision is simply

$$\theta \in \underset{s\in\Theta}{\operatorname{argmax}}\, p(s)[u(s) - \theta h(s)] + [1 - p(s)]\underline{u}(s) \quad \forall\theta.$$

If a policy $(\chi(\cdot), \underline{u}(\cdot))$ induces an allocation satisfying (1.2) we call it a ***feasible policy***.

Given a feasible standard policy, $(\chi(\cdot), \underline{u}(\cdot))$, we rely on the fact that, for every θ, an equilibrium allocation solves program $\mathcal{P}^{EQ}$,

$$\max_{p,h,u} p\,[u - \theta h - \underline{u}(\theta),]$$

subject to

$$N(h) - \chi(u) \geq \frac{\kappa}{\lambda(p)p},$$

to characterize these allocations.

The first order conditions for program $\mathcal{P}^{EQ}$ yield

$$\theta\chi'(u(\theta)) = N'(h(\theta)),$$

and

$$\chi'(u(\theta))\,[u(\theta) - \theta h(\theta) - \underline{u}(\theta)] + N(h(\theta)) - \chi(u(\theta)) = \kappa\vartheta'(p(\theta)).$$

The first equation simply shows that the firm does not distort effort beyond the distortions imposed by the policy. As for the second equation, on the right-hand side, we have the marginal cost of increasing the probability of employment through vacancy creation. On the left-hand side, the private marginal benefit. The direct benefit is given by $N(h(\theta)) - \chi(u(\theta))$, the net revenue per vacancy. Whenever $u(\theta) - \theta h(\theta) > \underline{u}(\theta)$, there is also an indirect benefit. A small increase in p increases w, allowing the firm to save resources spent on each employed worker.

Wedges To assess potential inefficiencies raised by a standard policy we define the relevant ***wedges*** in our model economy. These wedges are differences between private and society's costs and benefits of adjusting the different margins. To derive the wedges below, we fix the unemployment insurance $\underline{u}(\theta)$ of type θ and require that this allocation maximizes the government budget constraint (1.2) subject to delivering s utility (1.1) to type θ. The marginal cost of effort must then equal its marginal benefit and the marginal resource brought by an increase in the queue length for type θ must equal its marginal cost,

$$p(\theta)\,[N\,(h(\theta)) - C(u(\theta))] - (1 - p(\theta))C(\underline{u}(\theta)) - \frac{\kappa}{\lambda\,(p(\theta))}.$$

In line with the definition above, for any allocation, we define, at each productivity level, θ, two wedges, the ***effort wedge***,

$$\tau_n(\theta) := 1 - \frac{\theta C'(u(\theta))}{N'(h(\theta))}, \tag{1.3}$$

and, the ***vacancy wedge***,

$$\tau_p(\theta) := N(h(\theta)) - [C(u(\theta)) - C(\underline{u}(\theta))] + \\ C'(u(\theta))[u(\theta) - \theta h(\theta) - \underline{u}(\theta)] - \kappa\vartheta'(p(\theta)). \tag{1.4}$$

The effort wedge is standard; it is what one calls labor wedge in models in models with frictionless labor markets. It measures the resource impact of a small, utility-preserving, increase in $h(\theta)$. To hold expected utility constant we adjust $u(\theta)$ and keep $p(\theta)$ and $\underline{u}(\theta)$ fixed.

Similarly, to define the vacancy wedge at an allocation, $(u(\theta), h(\theta), p(\theta), \underline{u}(\theta))_\theta$, we consider the resource impact of an expected utility preserving marginal increase in p at the type θ sub-market. Again, $u(\theta)$ is adjusted to hold $w(\theta)$ constant, while $h(\theta)$ and $\underline{u}(\theta)$ remain fixed.

To increase $p(\theta)$, a direct cost of vacancy creation, $\kappa\vartheta'(p(\theta))$, must be incurred — the rightmost term in the right hand side of (1.4). Such increase in $p(\theta)$ leads to a resource gain $N(h(\theta)) - C(u(\theta)) + C(\underline{u}(\theta))$ given by, on the one hand, the difference between what type θ produces conditional on finding a job, $N(h(\theta))$, and what he or she consumes, $C(u(\theta))$, and, on the other, by the resources that are saved by lower unemployment benefit payments, $C(\underline{u}(\theta))$. Beyond this, whenever $u(\theta) - \theta h(\theta) \neq \underline{u}(\theta)$, hence type θ is not fully insured, there is an expected utility change which must be compensated with a change in $u(\theta)$. The variation in resources used by this change in $u(\theta)$ accounts for the second to last term in the right hand side of (1.4).

Standard Policy's Wedges We can use the solution to $\mathcal{P}^{EQ}$ to show that, in the equilibrium associated with a standard tax system, the effort wedge is given by

$$\tau_n^{EQ}(\theta) = \frac{\chi'(u(\theta)) - C'(u(\theta))}{\chi'(u(\theta))} = T'(\chi(u(\theta))), \tag{1.5}$$

using the definition, $\chi(u) - T(\chi(u)) = C(u), \forall u$.

As for the vacancy wedge, it is of the form

$$\tau_p^{EQ}(\theta) = \chi(u(\theta)) - [C(u(\theta)) - C(\underline{u}(\theta))] - \\ [\chi'(u(\theta)) - C'(u(\theta))][u(\theta) - \theta h(\theta) - \underline{u}(\theta)], \tag{1.6}$$

which can also be written as

$$\tau_p^{EQ}(\theta) = T(\chi(u(\theta))) + C(\underline{u}(\theta)) - \frac{T'(\chi(u(\theta)))}{1 - T'(\chi(u(\theta)))} C'(u(\theta)) \left[u(\theta) - \theta h(\theta) - \underline{u}(\theta) \right]. \quad (1.7)$$

It is apparent from (1.7) that the vacancy wedge is typically different from zero. There are two fiscal externalities. The first, captured by the first two terms in (1.7) is positive; a higher $p(\theta)$ increases tax revenues and reduces unemployment benefits. Such social benefits are not taken into account by firms in their decision to post vacancies. The second, captured by the last term in (1.7), is negative if marginal tax rates are positive. From the firm's perspective, an increase in p reduces the flow utility of employment, $u(\theta) - \theta h(\theta)$, that the firm must deliver to the worker to guarantee that he or she attains the expected utility $w(\theta)$. However, a fraction of the resource saved is not a social cost, for it returns to society in the form of higher taxes.

The vacancy wedge therefore *increases with total taxes and decreases with marginal tax rates*. At the margin, higher total taxes lead firms to generate fewer vacancies and higher marginal tax rates lead firms to generate more vacancies than what would be desirable in the first best. Of course, a reference to the unconstrained optimum is not the best guide to policy-making if such an optimum is not attainable in the first place.

2 Constrained Efficient Allocations

In a fully efficient allocation, all wedges are zero. Away from first best, basic second-best ideas – Lipsey and Lancaster (1956) – suggest that distortions should be spread in all margins to minimize overall welfare losses. Therefore, to assess whether the distortions found in any given tax system are at the optimal level, we need to characterize the set of constrained efficient allocations by being explicit about the informational frictions that preclude one from reaching the first best.

Our approach for evaluating tax policies has two parts. First, we derive general characterization results for efficient allocations. These results provide restrictions on the observable features of allocations and/or policies that are necessary for constrained efficiency. Second, in Section 4 we derive a practical procedure for quantifying the potential gains from welfare-improving reforms.

Before all this can be accomplished we must specify the informational structure of the problem. We have already argued that the planner can observe whether a worker is employed or unemployed, and, if employed the worker's earnings. Moreover, we assume that every job offer is publicly observed, which allows the planner to recover the equilibrium value of p for each sub-market.

Planner's Problem We consider a mechanism under which each agent announces a type, θ, and is assigned: i) a specific market to which to apply; ii) how much to produce, $N(h(\theta))$, in case a job offer is received; iii) a flow utility of consumption $u(\theta)$ to which one is entitled conditional on working (and producing $N(h(\theta))$), and; iv) an unemployment utility $\underline{u}(\theta)$ if the application is made but an offer is not received. Hence, associated with each firm and/or market is a probability of receiving a job offer, $p(\theta)$, and an expected utility, $w(\theta)$, at a truth-telling equilibrium for the direct mechanism.

An allocation is ***incentive-feasible*** if it satisfies the incentive compatibility constraint,

$$\theta \in \operatorname*{argmax}_{s\in\Theta} p(s)[u(s) - \theta h(s)] + [1 - p(s)]\underline{u}(s) \quad \forall\theta,$$

and the resource constraint (1.2).

An incentive feasible allocation is ***efficient*** if there is no other incentive-feasible allocation that delivers the same expected utility $w(\theta)$ for all θ using fewer resources.[7]

Noting that $w(\theta) := p(\theta)\left(u(\theta) - \theta h(\theta)\right) + (1 - p(\theta))\underline{u}(\theta)$, incentive compatibility is equivalent to the envelope condition, $\dot{w}(\theta) = -p(\theta)h(\theta)$ for all θ, and the monotonicity condition, $p(\theta)h(\theta)$ decreasing.

The monotonicity constraint guarantees that the local second-order necessary condition is also satisfied at any allocation satisfying the first-order condition of the worker's optimization problem. It is also possible to show that, as in Mirrlees (1971), the first- and second-order conditions guarantee not only a local but also a global maximum. A novel feature of the problem studied here is that the second-order condition entails monotonicity on $p(\theta)h(\theta)$ not on $p(\theta)$ and $h(\theta)$ separately.

Another constraint on informationally feasible allocations arises because the planner does not observe whether an agent has applied to a job, or if she applied, to which job she did apply. In practice, this means that an agent may claim to have applied to a job in any sub-market but not to have received an offer. This creates a moral hazard problem that restricts the type of unemployment benefits that can be used.

Moral Hazard Without any moral hazard considerations, the cheapest way to deliver a certain amount of expected utility $w(\theta)$ is by offering full insurance: $u(\theta) = \underline{u}(\theta)$ for all θ. To implement this full insurance (for each type) allocation, the planner must not only be able to observe whether an agent actually applied to a specific type of vacancy but also be assured that no job offers are rejected. We assume that this is *not* the case.

If a worker can claim to have applied to a vacancy $(N(h(\theta)), \chi(u(\theta)))$ – or actually apply to such a vacancy and reject all received job offers — and by doing so, guarantee herself a

[7]Lemma 3 shows that surpluses can be returned to agents to increase everyone's utility in an incentive-compatible way.

utility $\underline{u}(\theta)$ without working, the constraint

$$w(\theta) \geq \sup_{\tilde{\theta}} \underline{u}(\tilde{\theta}), \quad \forall \theta, \tag{2.1}$$

must be added to the planner's program.

Because $w(\theta)$ is decreasing in θ, imposing $w(\bar{\theta}) \geq \sup_{\tilde{\theta}} \underline{u}(\tilde{\theta})$ is both necessary and sufficient to take the constraint on unemployment insurance into account.

2.1 Optimality Conditions

We now derive a set of efficiency conditions for the planner's program. Our focus is on providing objective criteria for using observable statistics to assess whether an allocation is rationalizable as the choices of a planner endowed with a Paretian objective.

We start with Lemma 1, which shows that while (2.1) does not rule out unemployment benefits that vary with θ, it renders any such policy inefficient.

Lemma 1. *In any solution to the planner's problem, $\underline{u}(\theta) = \underline{u}$ for almost every θ.*

The reasoning behind this lemma is straightforward. If we can find a positive measure of workers such that $\underline{u}(\theta) < \sup_{\tilde{\theta}} \underline{u}(\tilde{\theta})$, then those workers must be enjoying a utility from consumption when employed, $u(\theta)$, strictly greater than $\underline{u}(\theta)$. It is then possible to save resources by providing additional insurance to each type in this set without changing the agent's expected utility via an increase in unemployment consumption and a decrease in employment consumption. Due to additive separability between labor and consumption, such change has the same impact on agents independent of their types; it does not affect incentives.

Lemma 1 implies that if an allocation is constrained efficient, unemployment benefit must be independent of θ. For any allocation, $(p(\theta), u(\theta), h(\theta))_\theta$, the expect utility of a type θ worker is, therefore, $w(\theta) = p(\theta)\left[u(\theta) - \theta h(\theta)\right] + (1 - p(\theta))\, \underline{u}$.

The next proposition shows that an efficient allocation must be such that the least productive (highest θ) agent for whom $h > 0$, must be indifferent between working and remaining unemployed.

Proposition 1. *If $\hat{\theta}$ is the largest type such that $h(\theta) > 0$, then any efficient allocation must be such that $\underline{u} = u(\hat{\theta}) - \hat{\theta} h(\hat{\theta})$.*

If $\underline{u} < u(\hat{\theta}) - \hat{\theta} h(\hat{\theta})$, then $\underline{u} < u(\hat{\theta})$, in which case, the government can save resources by increasing the unemployment utility $\underline{u}$ by some small ε while decreasing the utility from consumption while employed of each type θ by $\varepsilon\left(1 - p(\theta)\right)/p(\theta)$.

To further characterize constrained efficiency, we note that an allocation that delivers an expected utility profile $(\varpi(\theta))_\theta$ is constrained efficient if and only if it solves a dual program

— denoted $\mathcal{P}^{EF}$ — of the form

$$\max \int \left\{ p(\theta) \left[N\left(h(\theta)\right) - C\left(\frac{w(\theta) - \underline{u}}{p(\theta)} + \theta h(\theta) + \underline{u} \right) \right] \right.$$
$$\left. - (1 - p(\theta))C(\underline{u}) - \frac{\kappa}{\lambda\left(p(\theta)\right)} \right\} f(\theta) d\theta, \quad (2.2)$$

subject to, for all θ,

$$\dot{w}(\theta) = -p(\theta)h(\theta), \quad (2.3)$$

$$p(\theta)h(\theta) \text{ decreasing}, \quad (2.4)$$

and,

$$w(\theta) \geq \varpi(\theta). \quad (2.5)$$

$\mathcal{P}^{EF}$ is an optimal control program with controls $h(\theta)$ and $p(\theta)$, state $w(\theta)$, and inequality constraints on $w(\theta)$. For tractability, we ignore the possibility of bunching and restrict our attention to $\mathcal{C}^2$ solutions, which satisfy the monotonicity condition and display strictly positive levels of consumption and effort for every type.[8] By solving for the Lagrange multiplier associated with the incentive constraint of Program $\mathcal{P}^{EF}$ we derive the effort wedge at the constrained efficient allocation.

Proposition 2. *The effort wedge at the constrained efficient allocation is*

$$\frac{\tau_n^\star(\theta)}{1 - \tau_n^\star(\theta)} = \frac{F(\theta)}{f(\theta)\theta} \frac{1}{C'(u(\theta))} \mathbb{E}\left[C'(u(\tilde{\theta})) - \bar{\alpha}(\tilde{\theta}) \middle| \tilde{\theta} \leq \theta \right], \quad (2.6)$$

where $\mathbb{E}$ is the expectation operator and $\bar{\alpha}(\theta)$ is the normalized (by $f(\theta)$) Lagrange multiplier associated with (2.5).

While the effort wedge formula (2.6) looks exactly as the one that would arise from a traditional Mirrlees' problem in which the labor market frictions are absent, marginal tax rates cannot be determined as if there were no frictions. Note, in particular, that $\alpha(\theta)$, which encodes the shadow cost of delivering expected utility for a type θ agent must incorporate the fact that utility is delivered not only when agents work but also when they cannot find a job. By changing the focus from efficiency to optimality under pre-defined social objective, in Section 2.2, we derive an explicit expression for optimal taxes for the case of a Utilitarian planner – equation (2.8). It expresses optimal tax formula as the sum of a traditional Mirrlees term and an adjustment for labor market frictions. For now, just note that the sign of $\tau_n^\star(\theta)$ depends on the average difference between the marginal cost of delivering the flow utility

[8]Since we restrict our attention to $\mathcal{C}^2$ allocations, inequality constraints on state variables add no complexity to our approach (see Seiestad and Sydsæter (1987), chap. 5).

$u(\theta)$ to agent θ when he or she works and $\alpha(\theta)$, the Lagrange multiplier on his/her promised *expected utility*, $\omega(\theta)$. An important use of resources, the payment of unemployment benefits, is not counted when we integrate over $C'(u(\theta))f(\theta)$. For now, we explore formula (2.6) to derive testable optimality condition.

Corollary 1. *It a tax system implements an efficient allocation, then the effort wedge $\tau_n^\star(\cdot)$, if differentiable in θ, is such that,*

$$\dot{\tau}_n^\star(\theta)\theta + \tau_n^\star(\theta)\frac{N''(h(\theta))h(\theta)}{N'(h(\theta))}\frac{\dot{h}(\theta)\theta}{h(\theta)} + \tau_n^\star(\theta)\frac{\dot{f}(\theta)\theta}{f(\theta)} \leq 1 - \tau_n^\star(\theta), \quad \forall\theta. \tag{2.7}$$

Equation (2.7) provides a restriction on the set of incentive tax schedules; no Paretian objective can rationalize a tax schedule that does not satisfy (2.7). The expression is identical to condition ($\bigstar$) in (Werning, 2007, p. 5). Any tax system that fails Werning's (2007) efficiency test also fails ours.[9]

Werning's is a concave program, which implies that the condition is both necessary and sufficient; any allocation satisfying (2.7) can be rationalized by a Paretian objective. Our program, in contrast, need not be concave. Despite this, we are able to show that, for any given $\underline{u}$, an allocation which satisfies the necessary conditions for the Hamiltonian associated program $\mathcal{P}^{EF}$ also satisfies the conditions for Arrow's sufficiency theorem – see Appendix G. Equation (2.7) follows from these conditions.

Naturally, without any information about vacancy creation, we cannot be assured that an allocation satisfying (2.7) is efficient. We shall provide restrictions on $p(\theta)$ that an efficient allocation must display. For now, it is worth noting that a weaker notion of efficiency can be guaranteed if one only is given information that pertains to (2.7). Say that an allocation is ***p*-efficient** if it is not possible to increase the welfare of a positive measure of agents without reducing the welfare of another positive measure set of agents when we optimize with respect to $(u(\theta), h(\theta))_\theta$ holding the mapping $p : \Theta \mapsto [0, 1]$ fixed.

Corollary 2. *An incentive-feasible allocation is p-efficient if and only if the associated effort wedge satisfies (2.7).*

It is important to bear in mind is that in the definition of p-efficiency the entire mapping, $(p(\theta))_\theta$, is held fixed.[10] Corollary 2 states that *if $(p(\theta))_\theta$ is not altered by changes in the labor income tax schedule*, then any allocation satisfying (2.7), and only those satisfying (2.7), are

[9]To assess (2.7), one uses information on output which in Werning (2007) is the same as earnings. With labor market frictions this is no longer the case: earnings and output will differ, in general. The alternative efficiency test, $\Lambda(\theta)$ non-decreasing in θ, for $\Lambda(\theta)$ defined in (4.2), uses earnings, instead.

[10]Needless to say, inequality (2.7) does *not* define an efficiency condition for a standard tax system. In the absence of additional instruments to 'control' $p(\cdot)$ the labor income tax must play the additional role of targeting $p(\cdot)$. In Section 5 we show how this extra role that taxes must play affect optimal tax design under an optimal standard *progressive* system.

efficient. If the tax system satisfies (2.7) and $\underline{u}$ satisfies Proposition 1, then there is a Paretian objective *and a function* $(\bar{p}(\theta))_\theta$ for which $\left((\tau_n^\star(\theta))_\theta, \underline{u}\right)$ is optimal. Of course, an optimal allocation would also have an optimally designed $(p(\theta))_\theta$.

Towards full (constrained) efficiency, we note that $(\bar{p}(\theta))_\theta$ can, in practice, be recovered from the data thus generating another set of restrictions that an allocation must satisfy for it to be rationalized as the solution to a $\mathcal{P}^{EF}$ program. Showing what these other restrictions are is our task in what follows.

First, recall that in Mirrlees (1971), incentive compatibility requires monotonicity in $h(\theta)$. Hence, in $u(\theta)$. Here, in contrast, an implementable allocation must be monotonic in $p(\theta)h(\theta)$, not necessarily in each one separately.[11] The next proposition shows that *efficiency* implies monotonicity in both $p(\theta)$ and $h(\theta)$.

Proposition 3. *If an allocation $(p(\theta), u(\theta), h(\theta))_\theta$ is efficient, then both $h(\theta)$ and $p(\theta)$ are decreasing in θ.*

Labor is less costly for lower types. Hence, conditional on being employed, one would expect optimal allocation to include higher labor supply by those types. Moreover, because matches with such types are more productive, efficiency considerations require that they are formed with higher probability. The logic is simple, for any given bundle, the marginal increase in expected utility from an increase in p is higher for a low θ type.

Most characterization results in the literature regard the sign of labor wedges. As we have seen, in our model, two important margins should be considered: the effort margin and the firm entry margin. The next proposition shows that the effort and the vacancy wedges have the same sign *at all productivity levels* and *for any Pareto-efficient allocation*.

Proposition 4. *At a constrained efficient allocation*

$$\tau_p^\star(\theta) = \tau_n^\star(\theta) N'(h(\theta))h(\theta), \quad \forall\theta.$$

Proposition 4 offers a tight connection between the two relevant wedges. And while our focus on efficiency precludes us from pinning down a specific sign for the wedges, a strong empirical content arises from the fact that marginal tax rates are easily observed.[12] The government distorts effort downward only when leaving smaller information rents to more productive types is desirable. The same rationale implies that employment probabilities should also be distorted downward: a more productive worker who deviates and chooses the

[11] Moreover, incentive compatibility does not imply co-monotonicity of u and h, since a third variable, p, also plays a role in defining w.

[12] The well-known findings regarding the sign of effort wedges concern a specific point in the Pareto frontier. In Mirrlees' (1971) original paper, a proof for positive wedges under a Utilitarian objective is provided for the case of separable preferences. Seade (1982); Ebert (1992); Werning (2000) provide proofs for general preferences. The Rawlsian case also proved positive wedges to be optimal (see, e.g., Phelps (1973); Boadway and Jacquet (2008)).

sub-market of a less productive worker will be hurt by both too little effort and too frequent unemployment since the utility difference she obtains for the two states is larger than the one experienced by the less productive worker.

2.2 Frictions and Optimal Tax Formulae

We have thus far focused on offering testable restrictions that *any* efficient allocation must possess. This agnostic position regarding the Planner's objective has the important advantage over optimal taxation exercises of exposing problems that any Planner should be willing to correct. The drawback is that it produces fewer prescriptions regarding what optimal policies ought to be. Also important from our perspective is that we have not explored the question of how the presence of labor market frictions changes policy prescriptions, an issue to which we now turn.

In Appendix A we show that the formula for the optimal labor income tax for a planner with a Weighted Utilitarian objective is

$$\frac{\tau_n^\star(\theta)}{1-\tau_n^\star(\theta)} = \frac{F(\theta)}{f(\theta)\theta}\frac{1}{C'(u(\theta))}\Bigg\{\mathbb{E}\left[C'(u(\tilde\theta)) - a(\tilde\theta)\mathbb{E}\left[C'(u(\theta))\right]\middle|\tilde\theta \le \theta\right] + A(\theta)\mathbb{E}\left[\left[C'(u(\theta)) - C'(\underline{u})\right]\left[1-p(\theta)\right]\right]\Bigg\}, \quad (2.8)$$

where $A(\theta) = \int_{\underline{\theta}}^{\theta} a(\tilde\theta)f(\tilde\theta)d\tilde\theta$, $a(\theta) \ge 0$, $A(\bar\theta) = 1$, is the exogenous weight attached to agent θ.

The first line in (2.8) is exactly the one we find in Mirrlees (1971). The second contains the new terms introduced by the presence of labor market frictions. It leads to higher marginal tax rates. As one increases the tax on agent θ the benefits are measured not only by the redistribution across working agents but also towards agents that were not able to find a job.

An immediate consequence of this expression is that a well-established finding from optimal tax theory is overturned. Indeed, when inequality aversion is positive and finite, the non-negative marginal tax rate is zero at the bottom unless there is bunching in a pure Mirrlees setting – Seade (1977); Ebert (1992). With labor market frictions the marginal tax rate faced by the least productive agent, $\bar\theta$, is strictly positive.

Proposition 5. *If an allocation maximizes a weighted utilitarian objective,*

$$\int a(\theta)w(\theta)dF(\theta),$$

$a(\theta) \ge 0$, $\int a(\theta)dF(\theta) = 1$, *then*

i) $\tau^n(\bar{\theta}) > 0$, $\tau^p(\bar{\theta}) > 0$, *and*

ii) $\tau^n(\underline{\theta}) = \tau^p(\underline{\theta}) = 0$.

Proposition 5 does not restrict the set of weights, except for non-negativity which is required for Pareto efficiency. This social objective covers a large subset of all constrained efficient allocations. Indeed, whenever the set of attainable utilities is convex, then all constrained efficient allocations are the solution of a weighted utilitarian program.

The typical argument for zero taxes at the bottom is as follows. Consider the impact of slightly increasing marginal tax rates at an income level z'. This introduces distortion and, hence, welfare loss in the bundle of agents at this income level. On the other hand, the extra revenue raised from agents earning more than z' allows one to spare agents earning z' or less from paying more taxes. Hence, if society weights the difference between total welfare gains for those below z' and losses for those above z' less than the welfare cost imposed on agents exactly at z', this tax increase is worth making. At the bottom, the mass of agents at or below z' is, by definition, zero, which, because of finite inequality aversion, means zero social weight. Hence, there is no gain to offset the welfare loss. Here, revenues raised from an increase in the marginal tax rate mean that we need not reduce the unemployment benefit. The unemployment benefit has a welfare value that is always positive because it applies to the whole distribution of agents. Hence, positive marginal tax rates remain optimal even at the bottom.

For a Rawlsian metric, the optimum in a Mirrlees' setting also displays positive taxes at the bottom; the logic above does not apply. The reason being that any sacrifice in utility is justified if it increases the utility of the least favored agent. Proposition 6 shows that another property of optimal taxes remains true in our setting: taxes are non-negative everywhere.

Proposition 6. *An allocation that solves a Rawlsian objective exhibits positive effort and vacancy wedges at all productivity levels, except the very top.*

Note that an allocation solves the Rawlsian program if it solves the cost minimization program, $\mathcal{P}^{EF}$, when constraint (2.5) is replaced by

$$w(\bar{\theta}) \geq \varpi(\bar{\theta}). \tag{2.9}$$

It corresponds to a revenue maximization objective (Piketty (1997)), which is associated with the maximum size of the government.[13] Proposition 6 extends Mirrlees' (1971) findings to our setting.

[13] In a Mirrlees' setting, very detailed characterizations are available for the Rawlsian objective. In particular, Kanbur and Tuomala (1994); Boadway and Jacquet (2008); Hellwig (2010) document the relationship between the underlying distribution of productivities and the properties of optimal tax schedules.

3 Implementation

Having characterized constrained efficient allocations we now ask what policy instruments are needed to decentralize them. As always, there are many different combinations of instruments that may be used, so a great deal of our analysis will focus on assessing whether often examined real world instruments suffice.

We start with standard systems, which are the most commonly assessed in policy evaluation.[14]

Under any given standard system, $\big(\chi(\cdot), \underline{u}(\cdot)\big)$ vacancy wedges are of the form (1.7). In Proposition 7, below, we show such a system cannot implement efficient allocations.

Proposition 7. *The allocation $(p^\star(\theta), h^\star(\theta), u^\star(\theta))_\theta$, which solves the planner's program, $\mathcal{P}^{EF}$, cannot be implemented using only a non-linear labor income tax schedule.*

Proposition 1 has established that $u(\bar{\theta}) - \bar{\theta}h(\bar{\theta}) = \underline{u}$ at the optimum. The least skilled worker is indifferent between finding a job or not doing so. Hence, if p cannot be controlled by the planner, firms have an incentive to offer jobs for low-skilled workers that require less effort but are associated with longer queues and a higher probability of unemployment.[15] This suffices to preclude efficiency.

Proposition 7 and all the discussion that followed convey the fact that if we rely on the taxation of labor earnings and incentive feasible unemployment benefits it will not be possible to attain constrained efficiency. The natural following step is to devise policies that directly target the vacancy wedge.

Taxing Firms The tax schedules to which Proposition 7 refer are based on labor earnings only. In a directed search environment, however, all offered contracts (z, y) are publicly observed. Because the information is public, the planner can identify the sub-market in which a firm participates and tax firms based on this information. The question we ask now is whether augmenting the set of policy instruments by taxing firms allows incentive feasible allocations to be decentralized.

In Proposition 8 we first offer a positive answer to this question. A tax that depends on both y and z can decentralize any implementable allocation. We also show that if we restrict these taxes to take the special form of a tax on realized gross profits, $z - y$, then not all incentive-feasible allocations can be implemented.

Proposition 8. *i) Assume that the government can tax a firm conditional on its offer, (z, y). Then every incentive-compatible allocation can be implemented. ii) Assume that the government can only use a tax on the worker's income and a tax, T_π, on the firm's gross profits,*

[14] See, for example, Geromichalos (2015); Hummel (2019); Lehmann et al. (2006, 2011, 2016).

[15] Recall that we are restricting attention to economies in which employed workers produce a strictly positive amount of output.

$\pi = N(h) - \chi(u)$. *Then, there are incentive-compatible allocations that cannot be implemented.*

What we learn from Proposition 8 is that *any* incentive-feasible allocation can be implemented by a tax that uses information on both y and z in arbitrarily interdependent ways.[16] However, if we restrict the way we combine the information produced by these variables, as when we use labor income and gross profit taxes then not all incentive-feasible allocations can be decentralized. To understand this latter result, note that asking whether an incentive-compatible allocation can be implemented boils down to asking whether one can generate a tax system on labor income and a firms' realized gross profits for which firms will not want to deviate to a different allocation. Of course, there are many allocations that can be implemented, e.g., all the equilibrium allocations under a standard system. What we are able to show, only working with local conditions is the existence of incentive-compatible allocations for which the use of these instruments would imply that the firms' problem is not locally concave.

As usual, there are many different ways of decentralizing an incentive feasible allocation. Proposition 12 in the online appendix presents a simple tax system according to which income taxes are paid by the firm.[17] Hence, firm's taxes are given by a function $T^f(u,h)$. Such taxes on firms may be chosen in such a way that effort wedges are marginal income tax rates, and allowing for a more detailed characterization of an optimal system. Interestingly, by imposing $\theta\chi'(u(\theta)) = N'(h(\theta))$ for all θ, we are able to partially characterize the taxes on firms, $T^f(u,h)$. In particular, for all $\theta \in \left(\underline{\theta}, \overline{\theta}\right)$, it must be the case that $T^f_u(u(\theta),h(\theta)) = -\theta T^f_h(u(\theta),h(\theta))$.

For the rest of this section we focus on such a tax system and in particular on $\tau^\star_p(\theta) - \tau^{EQ}_p(\theta)$: the deviation from optimal marginal value from a vacancy that should be induced via taxes to implement an optimal tax system. Focusing on this term allows us to assess whether vacancy creation must be incentivized — $\tau^\star_p(\theta) - \tau^{EQ}_p(\theta) > 0$ — or deincentivized — $\tau^\star_p(\theta) - \tau^{EQ}_p(\theta) < 0$ — by this tax on firms.

Recall that $\tau^\star_p(\theta)$ is the efficient vacancy wedge, i.e., the vacancy wedge at the solution to the planner's program $\mathcal{P}^{EF}$. We have shown it to be: $\tau^\star_p(\theta) = h(\theta)\left[N'(h(\theta)) - \theta C'(u(\theta))\right]$. Using $N'(h(\theta)) = \theta\chi'(u(\theta))$, the firms' first order condition, and $\tau^{EQ}_p(\theta)$, the equilibrium wedge (1.6), we get

$$\tau^{EQ}_p(\theta) - \tau^\star_p(\theta) = \chi(u(\theta)) - [C(u(\theta)) - C(\underline{u})] - \frac{\tau^\star_n(\theta)}{1-\tau^\star_n(\theta)} C'(u(\theta))\,[u(\theta) - \underline{u}]\,. \quad (3.1)$$

[16] As usual, there are many different implementations for any incentive-feasible allocation. Also important, a posted job offer, (z, y), allows the planner to recover the equilibrium p for the associated sub-market. Similarly, knowledge of p and y allows z to be recovered which permits implementation. Knowledge of any two of the three variables (p, z, y) allows for implementation. This is the content of Proposition 11, in the online appendix.

[17] It is rather simple to construct an analogous system in which taxes are remitted by the workers.

The difference $\tau_p^{EQ}(\theta) - \tau_p^\star(\theta)$ is zero if and only if the right hand side of (3.1) is zero. The first three terms on the right-hand side of (3.1) are easy to grasp. Firms take the private cost $\chi(u)$ into account not $C(u) - C(\underline{u})$; the vacancy creation condition internalizes neither the fact that taxes return to society nor fact that a worker who does not find a job obtains unemployment benefits which are costly to society. This leads to under-provision of jobs that must be corrected by subsidies to vacancy creation. The second effect is that whenever the utility of workers exceed those of unemployed a higher probability of lending a job creates some space for saving resources on the amount a worker must be paid to attain the same expected utility, an effect that was already explained in the context of Section 1.

There is, however, a difference. To grasp its logic, recall the definition of a vacancy wedge from (1.4) as the resource impact of a welfare-reserving marginal increase in $p(\theta)$, and allow all other variables to optimally adjust. The cost of a welfare-preserving perturbation, (dp, dh, du), for a θ type is

$$\big[C(u) - N(h) - C(\underline{u}) + \vartheta'(p) - C'(u)\left[u - \theta h - \underline{u}\right]\big]dp + p\big[\theta C'(u) - N'(h)\big]dh.$$

In a first best world, the last term vanishes due to the efficient choice of effort. In contrast, when incentive constrains are taken into account we choose $pdh = -hdp$ to preclude the reform from attracting different types, i.e., to guarantee that the expected utility at the new bundle is kept constant for all $\hat{\theta}$ and not only θ. Imposing $pdh = -hdp$ in the equation above leads to $\big\{C(u) - N(h) - C(\underline{u}) + \vartheta'(p) - C'(u)\left[u - \underline{u}\right] + N'(h)h\big\}dp$ that, at an optimum, must be zero for all dp.[18] A complete characterization of $\tau_p^{EQ}(\theta) - \tau_p^\star(\theta)$ requires knowing (or solving for) the constrained efficient allocation.

We conclude this section by showing that even partial knowledge of policy allows us to say more about $\tau_p^{EQ}(\theta) - \tau_p^\star(\theta)$. First, we relate progressivity with changes in $\tau_p^{EQ}(\theta) - \tau_p^\star(\theta)$, then we show how even for the very canonical case of linear taxes its sign cannot be determined without further knowledge of the environment.[19]

Remark 1. *If we differentiate $\tau_p^{EQ}(\theta) - \tau_p^\star(\theta)$ with respect to θ, terms cancel and we obtain*

$$\dot{\tau}_p^{EQ}(\theta) - \dot{\tau}_p(\theta) = -\left[\chi''(u(\theta)) - C''(u(\theta))\right]\left[u(\theta) - \underline{u}\right]\dot{u}(\theta).$$

Since

$$\chi''(u) - C''(u) = \frac{C'(u)}{1 - T'(\chi(u))}\left\{\frac{T''(\chi(u))}{1 - T'(\chi(u))}\frac{C'(u)}{1 - T'(\chi(u))} + T'(\chi(u))\frac{C''(u)}{C'(u)}\right\},$$

[18] This provides the heuristic for equation (4.3), formally proved in the appendix.

[19] We claim not that the primitives of most real-world economies are such that one can find Pareto weights that rationalize progressivity. Our discussion here is only illustrative of how the design of these two instruments are intertwined. In fact, if the support of f is compact, the system cannot be progressive in all its range if marginal tax rates are ever positive. The empirically relevant question is whether for an arbitrarily small value of $\underline{\theta}$ the schedule is progressive in the range of observed earnings.

then, provided that $\dot{u}(\theta) < 0$, a sufficient condition for $\tau_p^{EQ}(\theta) - \tau_p^{\star}(\theta)$ to be increasing in θ is that $T''(\chi(u)) \geq 0$, i.e., that the system be progressive.

How about levels? Noting that the taxes paid by a type θ worker are given by $\chi(u(\theta)) - C(u(\theta))$, the first three terms in the right hand side of (3.1) push towards incentivizing vacancy creation. However, the last term precludes us from signing the whole expression whenever $\tau_n^{\star} > 0$. The logic is easier to grasp in the context of linear taxes where the right hand side of (3.1) becomes

$$\frac{1}{1-\tau}\Big\{C(\underline{u}) - \tau\big[C(\underline{u}) - C(u(\theta)) + C'(u(\theta))\left[\underline{u} - u(\theta)\right]\big]\Big\}.$$

while the first term, is always positive, the term inside brackets is negative due to the convexity of $C(\cdot)$.

4 Quantitative Assessments

The purpose of this section is twofold. First, we devise a procedure for evaluating the inefficiency of any observed allocation and apply it to the U.S. economy. Second, we derive the optimal tax system under the assumption that we know the planner's objective.

The main advantage of the first approach is that it does not require making any assumption about the government's objective but that it is Paretian. The main drawback is that the reform we use to assess inefficiency finds the least costly allocation that delivers the same utility that agents attain at the baseline. It cannot rule out the existence of an allocation that uses even fewer resources and increases the utility of a positive measure of agents. Fortunately, if the latter is true the post-reform allocation will fail the efficiency tests previously derived, and we will know that we will have only found a lower bound for the potential gains. As for the second, the main advantage is that we are guaranteed to arrive at an efficient allocation. The main drawback is that we must be given the planner's objective.[20]

For all that follows we take for granted that we know the primitives, $(\varphi(\cdot), \eta(\cdot), \lambda(\cdot), f(\cdot), \kappa)$, of the economy we study. In the actual implementation we calibrate the parameters of the economy.

4.1 Calibration

If we were given the equilibrium utility profile, $(\varpi(\theta))_\theta$, then by solving the program $\mathcal{P}^{EF}$ we would find the least amount of resources that are required to deliver such profile. Comparing it with the current resource use the amount of inefficiency of any current policy can be assessed.

[20] It should also be pointed out that the first approach is substantially less computationally intensive.

The practical implementation of this program however requires us to deal with a couple of issues. First, is the fact that in practice we may not know all the primitives of the economy. Second is the question of how to recover the equilibrium utility profile $(\varpi(\theta))_\theta$.[21]

We recover all the information used for the three assessments above in the context of the U.S. economy. We assume that we know $\varphi(\cdot)$, $\eta(\cdot)$ and $\lambda(\cdot)$, and assume that the U.S. uses a standard policy where labor income taxes can be well approximated by the Musgrave (1959); Feldstein (1969); Bénabou (2000); Heathcote et al. (2017) functional form — henceforth, "HSV" approximation. We recover the other primitives and the equilibrium utility profile, $(\varpi(\theta))_\theta$, from the data. We find that it passes the co-monotonicity test (Proposition 3) but not the common sign restriction on wedges (Proposition 4). Once a flag is raised indicating that allocations are not efficient the next question is to assess how large the losses are.

We calibrate the benchmark economy and recover the utility profile, $(\varpi(\theta))_\theta$. Knowledge of $f(\cdot)$, $\lambda(\cdot)$, and κ, therefore, allows us to conduct any of the assessments previously described. More specifically, we assume that we know all the parameters of preferences and the matching function. We then assume that the government relies on a standard policy with HSV income tax schedule, solve for the equilibrium and use the data to recover the parameters for the tax function, the distribution of skills, κ, $\big(p(\theta), h(\theta), u(\theta), \underline{u}(\theta)\big)_\theta$.

By restricting the instruments we are, according to Proposition 7, ruling out the efficiency of observed allocations by fiat. That is, the procedure we use for retrieving the utility profile from the data relies on assumptions that are not compatible with efficiency. Not surprisingly the allocation will *not* pass the efficiency tests. It is the quantitative importance not the existence of inefficiency that is of concern in the assessment we make here. Of course, our method could be used if an arbitrary profile, $(\varpi(\theta))_\theta$, were given and the question of efficiency would be meaningful.

Characterization is accomplished by solving the program $\mathcal{P}^{EQ}$.[22] The solution to this program will generate the utility profile $(\varpi(\theta))_\theta$ to be used as explained before. In what follows, we specify the functional forms we use in the quantitative analysis and explain how we calibrate the parameters such that the model economy matches key statistics of the U.S. economy.

We assume $\varphi(c) = c^{1-\sigma}/(1-\sigma)$, $\eta(n) = n^{1+\gamma}/(1+\gamma)$ and $p = (1+\lambda)^{-1}$, which implies $\lambda(p) = 1/p - 1$. The coefficient of relative risk aversion, $\sigma = 2$, is consistent with Conesa et al. (2009) see Table 1. Disutility of effort exhibits a constant intensive margin Frisch elasticity. We choose $\gamma = 2$ such that the Frisch elasticity is consistent with the majority of the related literature and the estimates in Kaplan (2012).

We assume that the distribution of earnings is lognormal. We normalize its mean to

[21] Note that if one is only interested in knowing whether an allocation is efficient or not, the informational requirements for the tests developed in Section 2 can be shown not to be too demanding.

[22] That is, for every θ, an equilibrium allocation solves the dual program, $\max_{(p,y,z)} p\left[\varphi(y - T(y)) - \theta\eta(z) - \underline{u}(\theta)\right]$, subject to the non-negative profit constraint, $\lambda(p)p\big[z - y\big] \geq \kappa$.

1 and calibrate its standard deviation to match the ratio of household income of the 80th percentile to that of the 20th percentile. The data on income inequality was taken from Guner et al. (2016). Parker (1999) shows that the lognormal distribution does not adequately fit the tails of the income distribution. To circumvent this issue, we truncate 2.0% of the extreme observations from each tail. Figure 2 shows the calibrated and the actual income distribution.

Table 1: **Parameter calibration**

Parameter	Value	Source or target
σ	2.0	Conesa et al. (2009)
γ	2.0	Kaplan (2012)
Income distribution, y	–	Guner et al. (2016)
τ	0.064	Share of taxes paid
ζ	0.89	Share of taxes paid
κ	0.007	Employment rate of 0.93
ϕ	0.30	avg. replacement rate

Moreover, to parameterize the U.S. tax system, we rely on the HSV functional form $T(y) = y - \zeta y^{1-\tau}$. ζ, which captures the need for revenue, defines the level of the average tax rate, whereas τ controls the curvature of the tax function. If $\tau = 0$, then the tax scheme is flat. A higher τ implies more progressivity. We calibrate these parameters to match the share of taxes paid by the income quintile. The data were taken from Guner et al. (2016). Figure 1 shows that the model replicates the actual average tax schedule well.

We allow $\underline{u}$ to depend on θ in the benchmark to capture the fact that unemployment insurance benefits are increasing with income. In particular, we specify that $\underline{u}(\theta) = \hat{\underline{u}} + \phi u(\theta)$ and then choose the parameters $\hat{\underline{u}}$ and ϕ to match a 100% replacement rate for the lowest type and an average replacement rate of 45%.

The parameter κ is calibrated as follows. Solving program $\mathcal{P}^{EQ}$ for the functional forms we are using here, one finds

$$\frac{\bar{u}}{A(y)} - \frac{1-(1-\tau)(1-\sigma)}{(1-\tau)(1-\sigma)}y + \frac{y^2}{\kappa} = \left[\frac{\gamma}{1+\gamma}A(y)^{\frac{1}{\gamma}} + \frac{2A(y)^{\frac{1}{\gamma}}y}{\kappa}\right]\theta^{-\frac{1}{\gamma}} - \frac{A(y)^{\frac{2}{\gamma}}}{\kappa}\theta^{-\frac{2}{\gamma}}, \tag{4.1}$$

where $A(y) := (1-\tau)\zeta^{1-\sigma}y^{(1-\tau)(1-\sigma)-1}$.

Equation (4.1) provides an analytic expression for θ as a function of $\hat{\underline{u}}$, ϕ, κ, and y. Given the distribution of earnings, if we knew κ, $\hat{\underline{u}}$ and ϕ, we can recover, for each y, the preference parameter θ compatible with such choice. We observe the employment rate, $p(\theta)$, for the average, not for each θ. Thus, we calibrate κ so that the average $p(\theta)$ in the model matches the aggregate employment rate in the data.

Figure 2 decomposes the expected utility of all different types between utility conditional

on being employed and utility if unemployed. For very high and very low productivity agents, the difference is not too large. For the former, this is due to a very low probability of unemployment, whereas for the latter it is because employment utility is not too different from unemployment utility.

For the benchmark allocation, both effort and the probability of employment are monotonic in productivity, which means that the allocation satisfies the necessary condition for efficiency in Proposition 3. However, as one can see in Figure 3, for very high productivity agents, the effort wedge is positive, whereas the vacancy wedge is negative, which violates one of the efficiency conditions in Proposition 4.

4.2 Efficiency Tests

The first efficiency test consists of simply asking whether the probability of employment and output are increasing in w (decreasing in θ). Figure 1 shows that this is the case for the benchmark economy.

To apply the efficiency test (2.7) from Corollary 1 we note that if $\alpha(\theta)$ is the Lagrange multiplier associated with constraint θ, then $\alpha(\theta) \geq 0$ and $\alpha(\theta)[w(\theta) - \varpi_{\mathcal{T}}(\theta)] = 0$ are, respectively, the multiplier non-negativity and the complementary slackness constraints of the original program. For all θ let $\Lambda(\theta) = \int_{\underline{\theta}}^{\theta} \alpha(s)ds$. Non-negativity of α is, of course, equivalent to monotonicity of $\Lambda(\theta)$, defined as

$$\Lambda(\theta) := \int_{\underline{\theta}}^{\theta} C'(u(s))f(s)ds - \frac{\tau_n(\theta)}{1-\tau_n(\theta)} C'(u(\theta))f(\theta)\theta, \quad \forall\theta, \tag{4.2}$$

as we show in the appendix. Inefficiency is apparent in Figure 7 as one sees $\Lambda(\theta)$ declining for low wages.

Because the allocation violates p-efficiency, then it cannot be efficient, and we need not check the efficiency condition from Proposition 4.

It is important to emphasize that to obtain the taxes, distribution, and output used in this section we have calibrated the U.S. economy under the assumption that the government uses a standard policy. In Proposition 7 we have shown that these policies are not to be capable of implementing the optimum. Hence, the tests were bound to produce rejections, and at this point should be taken only for its illustrative purpose. Of course, the tests are possible for any observed $\big(h(\theta), f(\theta), \tau(\theta)\big)_\theta$, possibly leading to other outcomes.

4.3 A very simple procedure for quantifying inefficiencies.

A very low-cost procedure can be used to calculate lower bound for efficiency gains. We compare the revenues raised by the current system to those raised under an alternative system that delivers *the same utility profile* while satisfying the first-order conditions for the

planner's program, $\mathcal{P}^{EF}$. This is where the simplicity and the caveat both come from. By delivering the same utility profile we are able to derive an allocation satisfying the first-order conditions at essentially no computational cost.

Program $\mathcal{P}^{EF}$ is analogous to the program solved by Werning (2007) to assess the constrained efficiency of real-world tax schedules. Section 2 uses the first-order conditions for $\mathcal{P}^{EF}$ to check whether the observed schedules are rationalized by an optimization program leading to Pareto-efficient allocations, like in Werning (2007). Although the program is analogous, our procedure is of a different nature. We use the 'observed' utility profile $\varpi_{\mathcal{T}}(\theta)$ in the first-order conditions associated with $\mathcal{P}^{EF}$ to derive a new allocation and ask whether it raises more revenue than the current schedule. Doing so allows us to assign numbers to the observed inefficiency. [23]

Combining (A.7) and (A.8), in the appendix, we obtain, after some substitutions,

$$N\left(h(\theta)\right) - C\left(u(\theta)\right) + C(\underline{u}) + C'\left(u(\theta)\right)\left[u(\theta) - \underline{u}\right] - \kappa\vartheta'(p(\theta)) = N'(h(\theta))h(\theta), \quad (4.3)$$

whose heuristic we offer on page 20. If the promise keeping constraint, $w(\theta) \geq [\varpi_{\mathcal{T}}(\theta)$ binds for all θ we can use $u(\theta) = \theta h(\theta) - \underline{u} + [\varpi_{\mathcal{T}}(\theta) - \underline{u}]/p(\theta)$, and $\dot{\varpi}_{\mathcal{T}}(\theta) = -p(\theta)h(\theta)$ to obtain $h(\theta)$ as a function of $\varpi_{\mathcal{T}}(\theta)$, $\dot{\varpi}_{\mathcal{T}}(\theta)$ and $\underline{u}$. That is, for any $\underline{u}$ and a given path for $\varpi_{\mathcal{T}}(\theta)$, equation (4.3) is a function of $h(\theta)$ only: for a given $\underline{u}$, we use (4.3) to solve for $h(\theta)$ as a function of $\varpi_{\mathcal{T}}(\theta)$ (and $\dot{\varpi}_{\mathcal{T}}(\theta)$), which we recover from the data. Finally, note that $\underline{u}$ is itself a policy choice that must satisfy $\underline{u} = u(\bar{\theta}) - \hat{\theta}h(\bar{\theta})$ if at the optimum all types work with positive probability.

Of course, we could have used the same equations to solve for $p(\theta)$ and $u(\theta)$ instead. Hence, if we observe $\varpi_{\mathcal{T}}(\theta)$, we can recover the allocation satisfying the program's first-order condition as a function of $\underline{u}$. Follows from this that all we need to implement the assessment above is to have a procedure to extract, $\forall\, \theta$, the equilibrium utility, $\varpi_{\mathcal{T}}(\theta)$, from the data – we use such a procedure in Section 4.1. The downside of this approach is that we cannot rule out the possibility of finding an allocation that raises more revenue *and* delivers strictly more utility: the allocation we find generates exactly $(\varpi(\theta))_\theta$ which means that we are not ruling out the possibility of there being another allocation with $w(\theta) > \varpi(\theta)$ in a set of positive measure associated with even more tax revenues. The efficiency tests from Section 4.2 can be used to assess whether this is the case.

[23]Because we do not impose monotonicity when we derive the allocation satisfying the first-order conditions for $\mathcal{P}^{EF}$, we must check ex-post whether it satisfies (2.4). This issue does not arise if our only aim is to test the efficiency of current allocations: chosen contracts are, of course, incentive compatible.

4.4 Numeric Results

Table 2 and Figures 3 and 4 display our main findings. The first two columns of the table refer to the calibrated economy, whereas the last four columns consider two counterfactual worlds for which the baseline policy is characterized by either less (columns 3 and 4) or more (columns 5 and 6) progressive schedules.

Table 2: **Welfare Gains and Progressivity** The first two columns display the relevant statistics for the tax system calibrated to match the data for the U.S. economy ($\tau = 0.064$) and the optimum. The next four columns display the analogous statistics for two alternative specifications for the tax system, with the parameters re-calibrated to match the data for the U.S. economy: first for a linear tax schedule ($\tau = 0.00$) and then for a more progressive system ($\tau = 0.12$).

	$\tau = 0.064$		$\tau = 0.00$		$\tau = 0.12$	
Variable	Benchmark	Optimal	Benchmark	Optimal	Benchmark	Optimal
Average income	1.00	1.020	1.000	1.012	1.000	1.029
Income Gini	0.538	0.525	0.538	0.528	0.538	0.522
Average consumption	0.862	0.878	0.862	0.874	0.862	0.879
Consumption Gini	0.508	0.516	0.538	0.542	0.482	0.492
Average effort	1.275	1.276	1.278	1.278	1.278	1.279
Average prob of employment	0.935	0.933	0.935	0.936	0.935	0.930
Average gov. revenue	0.138	0.142	0.138	0.139	0.138	0.149
$\%$ change	-	3.48%	-	0.71%	-	8.41%

Let us for now focus on columns 1 and 2 ($\tau = 0.064$), our main findings. Compared to the benchmark, an efficient policy is characterized by higher, and slightly more unevenly distributed income. Average consumption is also higher but less unevenly distributed at the optimum than at the benchmark. This is accompanied by a very small *increase* in unemployment and (essentially) stability in effort conditional on employment. Most gains, therefore, arise from a redistribution of effort across agents. By moving from the current to the optimal policy, it is possible to increase government revenues by 3.48%.

The top left panel of Figure 3 displays the effort and vacancy wedges at the benchmark and at the optimum. Both at the benchmark and at the optimum, effort wedges are positive for all levels of income. They are increasing in income—decreasing in θ—at the benchmark since taxes are progressive. In contrast, they display a more complex pattern at the optimum, decreasing for low levels of income and increasing for intermediate to higher levels of income.

The top right panel displays the equilibrium wedge and a counterfactual optimal vacancy wedge, $\tau_n^{EQ}(\theta)N'(h^{EQ}(\theta))h^{EQ}(\theta)$. This is the wedge that should have been created by a policy that produces effort wedges $\tau_n^{EQ}(\theta)$ and induces the effort profile, $(h^{EQ}(\theta))_\theta$. The counterfactual wedges are not too different, as a percentage of income, from those at the optimal allocation. In contrast, actual wedges have a different sign and are substantially larger in absolute value. Since effort wedges are always positive, the benchmark policy fails the

efficiency test of proposition 4. It is important to bear in mind that negative vacancy wedges are *not* subsidies to participation. Such distinction, which should be clear from our characterization of equilibrium wedges – equation (1.7), is apparent in the benchmark: although vacancy creation is neither directly incentivized nor disincentivized, vacancy wedges are almost never zero. Figure 6, instead, compares firms' gross profits, $z - y$, and the overhead labor costs, $\kappa/(1-p)$. This difference, which is always 0 zero at the benchmark, is negative for low-paying jobs and positive for high-paying jobs. To provide a reference of how policy should be used to correct for the externalities at the extensive margin.

Policy changes induce gross wages to vary. These general equilibrium effects are displayed in the bottom-right panel of Figure 3 where the ratio between y/e, actual wages, and w an agent's productivity, but labeled wages in all figures, is displayed. Two things are worth noting. The first is that wages are higher (lower) for low (high) productivity agents at the optimum. Second, the ratio is declining in productivity except for the very low productivity agents. The presence of general equilibrium effects poses important challenges to the use of perturbation methods, which our approach easily handles for the type of question we ask here.[24]

Figure 4 displays the percentage variation between the baseline and the optimal output (top left) consumption (top right) and taxes paid (bottom left). Also, the bottom-right panel indicates the utility conditional on working at the benchmark and at the optimum. The optimal policy induces a small decline in the utility conditional on working for all but the least productive agents, which is compensated by an increase in employment, to keep expected utility at the benchmark level. The apparent contradiction with taxes on vacancy creation – Figure 6 – is solved once we recall that unemployment insurance is lower for high productivity agents at the optimum than at the benchmark.

We have shown the maximum revenues attained by any allocation such that $w(\theta) = \varpi_{\mathcal{T}}(\theta)$ for all θ. It may be the case that an alternative allocation raises even more revenues by *increasing* the utility of a positive measure of agents. To assess whether this is the case we take the following approach. Figure 7 displays the behavior of $\Lambda(\theta)$ at the benchmark and at the optimum. Non-monotonicity is apparent at the optimum too, thus showing that even better allocations are possible.

Progressive Income Taxes Following Lockwood and Manning's (1993) original insights, the role of labor income tax progressivity on unemployment has been the focus of a growing research agenda – e.g. Lehmann et al. (2016); Hummel (2019).[25] Our framework allows us to assess the impact of progressivity not only on unemployment but also on efficiency,

[24]Recent work by Sachs et al. (2020) is an important methodological breakthrough to handle these challenges.

[25]In our model search is competitive, hence the effects of progressivity on bargaining emphasized by Lockwood and Manning (1993); Lehmann et al. (2016); Hummel (2019) do not arise here.

the main focus of our work. We consider two counterfactual exercises in which we keep the calibration for the fundamentals used in our main exercise, but change the tax schedule progressivity, as captured by τ. First, we assume $\tau = 0$, that is, linear taxes, and then a more progressive system, $\tau = 0.12$. As one can see from (3.1) average taxes discourage the creation of vacancies. Interestingly, however, higher marginal tax rates lead firms to oversupply vacancies, which is in line with the findings in the literature.

As for efficiency, the bottom line in Table 2 shows that gains are substantially larger—more than twice as large—than what we have calculated for the U.S. economy when the baseline schedule is more progressive. In contrast, if the baseline schedule is linear, revenue gains from moving to an optimal are about one-third of what we have measured for the U.S. economy. The impact on employment, effort, and consumption is qualitatively similar across the different specifications for the tax schedule (Figure 8).

Robustness To assess the sensitivity of our findings we have repeated our exercises using two different choices for the preference parameters. We first held σ fixed at its baseline value, 2, and increased γ to 3. We have then recalibrated the remaining parameters the same way we did in our baseline exercise. The third and fourth columns of Table 3 display our findings. As one can see, the welfare gains are not very sensitive to changes in γ.

Table 3: **Robustness** The first two columns reproduce the findings for our baseline calibration. The next two columns report the findings for an economy in which disutility of effort is changed by increasing γ, while holding $\sigma = 2$. The last two columns report our findings for an economy with $\sigma = 1.5$ and $\gamma = 2$.

	Baseline calibration		$\gamma = 3.0$		$\sigma = 1.5$	
Variable	Benchmark	Optimal	Benchmark	Optimal	Benchmark	Optimal
Average income	1.00	1.020	1.000	1.017	1.000	1.008
Income Gini	0.538	0.525	0.538	0.527	0.538	0.535
Average consumption	0.862	0.878	0.862	0.875	0.862	0.869
Consumption Gini	0.508	0.516	0.509	0.516	0.508	0.512
Average effort	1.275	1.276	1.248	1.249	1.107	1.107
Average prob of employment	0.935	0.933	0.935	0.936	0.935	0.943
Average gov. revenue	0.138	0.142	0.138	0.142	0.138	0.139
% change	-	3.48%	-	2.84%	-	1.75%

Next, we reduce σ from 2 to 1.5, while holding γ fixed and proceed as we have done when γ was changed. The last two columns of Table 3 display our findings. In contrast with γ, inefficiency is significantly lower in the benchmark if $\sigma = 1.5$. A larger value for γ reduces the compensated elasticity of taxable income – see definition in Appendix D – but leaves the value of insurance, for a fixed allocation, equal. A reduction in σ in contrast, while increasing the same elasticity, greatly reduces the importance of insurance. The latter aspect appears to be more important to explain inefficiency.

4.5 Optimal Tax Systems

To derive the optimal schedule, we must assume that we know the planner's objective. For concreteness, we assume that the planner is Utilitarian. Next, we adapt Mankiw et al.'s (2009) algorithm to our setting. Beyond the fact that this is by now a well-known algorithm, thus simplifying communication, the fact that the algorithm uses a tax perturbation approach allows for a direct connection to the applied literature pioneered by Piketty (1997); Saez (2001).

To connect our model with Mankiw et al. (2009) we consider iso-elastic preferences and use the distribution of skills, w, where $w = \theta^{-1-\gamma}$. Next, because $y \neq z$, all relevant elasticities depend on both, which potentially increases the algorithm's complexity. As it turns, this is not the case. The formula used in the algorithm for the calculation of optimal marginal income tax in the algorithm is

$$\frac{T'(y(w_i))}{1-T'(y(w_i))} = \frac{\zeta(y(w_i))}{\varepsilon^c(y(w_i))}\left[\frac{u_c(c(w_i))}{w_i\pi(w_i)}\right]\left[\sum_{w_i}^{w_N}\frac{\pi(w_i)}{u_c(c(w_i))} - (1-\Pi(w_i))\sum_{w_1}^{w_N}\frac{\pi(w_i)}{u_c(c(w_i))}\right.$$
$$\left. +(1-\Pi(w_i))\sum_{w_1}^{w_N}\left(\frac{\pi(w_i)}{u_c(c(w_i))} - \frac{\pi(w_1)}{u_c(c(w_1))}\right)(1-p(w_i))\right], \quad (4.4)$$

where $\pi(w_i)$ is the discretized density at productivity level w_i, $\Pi(w_i)$ the associated cumulative distribution and the elasticities in the formula above are where[26]

$$\varepsilon^c(y) = \left.\frac{d\ln z}{d\ln(1-\tau)}\right|_{v=\bar{v}}, \quad \zeta(y) = \frac{d\ln z}{d\ln w}.$$

The term in the second line of (4.4) is added to Mankiw et al.'s optimal tax formulae to account for labor market frictions. Much of the added complexity from labor market frictions is avoided as follows. In Mankiw et al.'s algorithm, $1+\varepsilon(y(w))$, where $\varepsilon(y(w))$ is the uncompensated elasticity of taxable income, substitutes for the cross-sectional elasticity, $\zeta(y(w))$, in the numerator in the first term in the right hand side of (4.4). Without labor market frictions, $\zeta(y(w)) = 1+\varepsilon(y(w))$, which explains its use since Saez's pioneering work.[27]

While this equivalence ceases to be valid with labor market frictions, what is of the essence, as Jacquet et al. (2013) has pointed out, is the ratio $\zeta(y(w))/\varepsilon^c(y(w))$, which arises when one replaces the endogenous distribution of earnings by the exogenous distribution of

[26] Note also that we define elasticities as a function of earnings, instead of output. In reality, they are a function of both — see Online Appendix D.

[27] To be precise, this equivalence requires one using Saez' (2001) definition of elasticities. The ratio $\zeta(y(w))/\varepsilon^c(y(w))$ is, in contrast, the same, independently of whether we use Saez' elasticities or the elasticities defined in Jacquet et al. (2013); Scheuer and Werning (2017).

types,
$$\frac{1}{\varepsilon^c(y)}\frac{1}{\varphi(y)y} = \frac{\zeta(y(w))}{\varepsilon^c(y(w))}\frac{1}{f(w)w}.$$

More importantly, we prove that, even with labor market frictions, $\zeta(y(w))/\varepsilon^c(y(w)) = 1+\gamma$, where γ^{-1} is the Frisch elasticity of labor supply. This allows us to run the algorithm using information on $y(w)$ and $p(w)$ only, leaving $z(w)$ in the background.

We follow the calibration presented in Mankiw et al. (2009).[28]. In Figure 9, we display the optimal tax schedule for different employment rates (left figure). For the sake of comparison, we also show the tax schedule for the model without frictions, where we use Mankiw et al.' formulae. It can be seen that labor market frictions increase the marginal tax rates. In Figure 9, we also show the probability of employment and the distribution of wages (right figure). Note that the density is concentrated at the wages up to 200, which implies that the marginal tax rates at the upper end of the distribution are not affected even for high levels of frictions.

5 Optimal Standard Policy

A standard policy is not capable of implementing all incentive-feasible allocations — Proposition 7. Indeed, by denying the planner the use of the additional policy instruments described in Proposition 8 one forces labor income taxes to play the additional role of trying to induce the socially optimal amount of vacancy creation. This lowers welfare and makes the characterization of optimal policies rather cumbersome if one is not willing to make strong assumptions on endogenous variables.

In this section, we offer a brief but rigorous discussion of tax perturbations when the planner does not have the independent tools to hold employment at the desired levels when income taxes are changed. As previously mentioned for the problem to remain tractable, we restrict our analysis to the case in which the map $\lambda : [0,1) \to R_{++}$ is of the form $\lambda(p) = 1/p - 1$ and assume the income tax that solves the planner's program is progressive. Under these assumptions, the impact of local perturbations remains local and can be done without apologies, as we shall prove. The appeal of progressive taxes cannot be exaggerated, and, in fact, many tax systems around the world are designed to be progressive. Of course, progressivity should not be an assumption but a result. Yet, our goal here is not to derive an optimal schedule but to highlight the consequences of not controlling p. The implicit assumption for our exercise is, therefore, that progressivity is optimal for a standard tax system, given the set of primitives and Pareto weights that define the planner's program.[29]

[28]The details of the numerical implementation is presented in E

[29]Whether progressivity is in fact optimal when we consider values for the parameters that characterize the primitives that are disciplined by real-world data, is something that we shall not dwell on.

It will be convenient to represent the tax system using, $T(c)$ the net amount of taxes paid by someone whose consumption is c, i.e., someone whose earnings are $y = c+T(c)$. Clearly, a tax system is progressive if and only if the marginal tax rate (whenever well defined) is non-decreasing, that is, the mapping $c \to T(c)$ is convex.

We first show that if the worker faces a progressive tax rate then his optimum is given by his first-order condition.[30] To prove it, we take a progressive tax system T and consider an optimal solution (not necessarily unique) of the worker's problem, (c^*, p^*), under this system. We then consider a first-order approximation $\hat{T}$ to the tax system around the optimal consumption c^* and consider the worker's problem under this affine tax schedule $\hat{T}$. For each employment probability, p, we calculate the optimal consumption level, $c\big(p|\hat{T}\big)$, under this fictitious tax system. This enables us to write the worker's problem as a function, $W\big(p|\hat{T}\big)$, of this employment probability. We check that this function is strictly concave, attaining a maximum at p^*, leading to the allocation $(c(p^*|\hat{T}), p^*) = (c^*, p^*)$. Since $\hat{T}$ is a first-order approximation of the progressive tax rate, T, we have $\hat{T}(c) \leq T(c)$ for every c, with equality at c^*. This finally enables us to conclude that (c^*, p^*) is the optimal solution. Moreover, we also establish that optimality is guaranteed by the worker's first-order condition.

Lemma 2. *Assume that the planner offers a progressive tax rate. Then the agent's first-order condition is necessary and sufficient for optimality.*

For an arbitrary progressive tax scheme, we can use the first-order conditions points at which the marginal tax rate is continuous (and C^2) to find out how optimal consumption and unemployment probability change with θ. We start by analyzing the relationship between the unemployment rate and productivity. In Appendix F we show that $p(\theta)$ is always decreasing in θ: more productive types always face a lower unemployment rate.[31] Since the government is not taxing the employment rate, this is true even at points at which the marginal tax rate may be discontinuous and there is pooling at the correspondent consumption level. The relationship between consumption and productivity is somewhat more involved when the worker faces a progressive tax rate. However, under the regularity assumption A, below, more productive workers always consume more.

Assumption A: $(\eta')^2 \geq \eta''\eta/2$.

Remark 2. *Assumption A holds, for instance, for any power function.*

Remark 3. *Since any progressive tax schedule can be approximated by a sequence of C^2 tax schedules and the optimal solution is upper hemi-continuous, then the employment probability is always increasing in productivity, and, under assumption A, so is c. This is verified in the proof of Proposition 9.*

[30] The function T is assumed to be convex, but not necessarily differentiable. Therefore, the first-order condition has to hold for an element of the sub-differential ∂T (nonempty due to convexity) evaluated at the optimal solution.

[31] This is also consistent with the empirical evidence we use in the numerical assessments of Section 4.1.

Lemma 2 helps us characterize the set of optimal solutions if the planner's objective leads to progressive schedules. Our first result delivers a set of necessary conditions that the tax rates for such progressive systems must satisfy. As a corollary, if the optimal tax solution is indeed progressive among all tax rates, then the necessary conditions that we display below hold for any optimal tax system.

The set of progressive tax systems is quite large. We focus here on the case in which the optimal tax system is C^1 and piecewise C^2.[32] We call such tax systems regular. Proposition 9 presents necessary conditions that any regular tax system must satisfy. To present this result, we remark that any optimal allocation maximizes the government revenue subject to delivering an expected utility, U, to the worker. This leads to the existence of a Lagrangian multiplier λ relative to the last constraint.

Proposition 9. *Consider an optimal regular progressive tax schedule.*

i) *If $T''(c(\theta_1)) > 0$ around the neighborhood of the optimal consumption level $c(\theta_1)$ of type θ_1, then*

$$
\begin{aligned}
0 = -f(\theta)\left[p(\theta)T'(c(\theta_1))\frac{\kappa_c}{\left(c'(\theta_1)\right)^2} + \left(T(c(\theta)) - C(\underline{u})\right)\frac{\kappa_p}{\left(c'(\theta_1)\right)^2}\right] \\
+ \int_{\underline{\theta}}^{\theta_1} p(\theta)f(\theta)d\theta - \lambda\int_{\underline{\theta}}^{\theta_1}\theta p(\theta)\eta'(z(\theta))f(\theta)d\theta \\
+ \int_{\underline{\theta}}^{\theta_1}\left[p(\theta)T'(c(\theta))\frac{\partial c(\theta)}{\partial T} + \left(T(c(\theta)) - C(\underline{u})\right)\frac{\partial p(\theta)}{\partial T}\right]f(\theta)d\theta,
\end{aligned}
$$

where the explicit formulas for $\kappa_p \in \mathbb{R}$, $\kappa_c > 0$, $\partial c(\theta)/\partial T < 0$ and $\partial p(\theta)/\partial T < 0$ are found in Appendix F, equations (F.9) and (F.10), respectively.

ii) *If $p(\bar{\theta}) > 0$ (there is no exclusion) the the following condition must be satisfied:*

$$
\begin{aligned}
0 = \int_{\underline{\theta}}^{\bar{\theta}} p(\theta)f(\theta)d\theta - \lambda\int_{\underline{\theta}}^{\bar{\theta}}\theta p(\theta)\eta'(\theta)f(\theta)d\theta \\
+ \int_{\underline{\theta}}^{\bar{\theta}}\left[p(\theta)T'(c(\theta))\frac{\partial c(\theta)}{\partial T} + \left(T(c(\theta)) - C(\underline{u})\right)\frac{\partial p(\theta)}{\partial T}\right]f(\theta)d\theta.
\end{aligned}
$$

iii) *The following condition is necessary for the optimality of the unemployment insurance*

[32] It is possible to perform a similar exercise to obtain necessary conditions for the case in which the tax rate is piecewise linear with finitely many brackets. But general optimality conditions seem quite difficult.

benefit:

$$0 = -\int_{\underline{\theta}}^{\theta_1} (1 - p(\theta))\, u'\left(c(\theta)\right)^{-1} f(\theta) d\theta + \lambda \int_{\underline{\theta}}^{\bar{\theta}} \left(1 - p\left(\theta\right)\right) f(\theta) d\theta$$
$$+ \int_{\underline{\theta}}^{\bar{\theta}} \left[p(\theta) T'\left(c(\theta)\right) \frac{\partial c(\theta)}{\partial \underline{u}} + \left(T\left(c(\theta)\right) - C\left(\underline{u}\right)\right) \frac{\partial p(\theta)}{\partial \underline{u}} \right] f(\theta) d\theta,$$

where $\partial c(\theta)/\partial \underline{u} > 0$ *and* $\partial p(\theta)/\partial \underline{u} < 0$, *with the explicit formulas provided in the Appendix F, equation (F.11).*

Item i) takes a consumption level at which marginal tax rates increases. Around such points, it is possible to perturb the marginal tax rate preserving convexity.[33] For concreteness, consider a perturbation that locally increases the marginal tax rate – Figure F.2, in the online appendix – for some type θ_1. This change will de-incentivize consumption and production for that type, leading to a fiscal loss of $p(\theta) T'\left(c(\theta_1)\right)$ times the coefficient, $-\kappa_c / \left(c'\left(\theta_1\right)\right)^2 < 0$, of consumption decay. Since workers trade-off income and employment probability, the effect on the job probability for that type cannot *a priori* be signed. The fiscal effect from that type is given by the first line of item i).

Such perturbation increases taxes for all types which are more productive than θ, leading to a direct fiscal effect displayed in the second line and also on an indirect effect in the third line as an increase in taxes will reduce production, $\partial c(\tilde{\theta})/\partial T < 0$ and employment $\partial p(\tilde{\theta})/\partial T < 0$. Finally, these tax increases make every worker more productive than type θ_1 worse-off and this is accounted in the second line of the formula,

$$0 = -f(\theta) \left[p(\theta) T'\left(c(\theta_1)\right) \frac{\kappa_c}{\left(c'\left(\theta_1\right)\right)^2} + \left(T\left(c(\theta)\right) - C\left(\underline{u}\right)\right) \frac{\kappa_p}{\left(c'\left(\theta_1\right)\right)^2} \right]$$
$$+ \int_{\underline{\theta}}^{\theta_1} p(\theta) f(\theta) d\theta + \int_{\underline{\theta}}^{\theta_1} \left[p(\theta) T'\left(c(\theta)\right) \frac{\partial c(\theta)}{\partial T} + \left(T\left(c(\theta)\right) - C\left(\underline{u}\right)\right) \frac{\partial p(\theta)}{\partial T} \right] f(\theta) d\theta$$
$$- \lambda \int_{\underline{\theta}}^{\theta_1} \theta p\left(\theta\right) \eta'\left(\theta\right) f(\theta) d\theta.$$

The second perturbation (item ii) considers changes in lump-sum taxes for every worker. The first and the second lines display its fiscal effects, while the third line represents its cost in terms of the worker's utility.

Finally, the last perturbation (item iii) considers a change in the level of unemployment insurance. The first line yields its direct fiscal cost. The second line considers the behavior led by this change. As expected, workers choose to find jobs with smaller probability $\partial p(\tilde{\theta})/\partial u < 0$ but increase the consumption whenever employed, $\partial c(\tilde{\theta})/\partial u > 0$. Finally, an

[33] It, therefore, avoids the small bunching and jumping effects induced by Saez's (2001) perturbation.

increase in unemployment insurance makes workers better-off and this is accounted in the second line of

$$0=-\int_{\underline{\theta}}^{\theta_1}(1-p(\theta))\,u'(c(\theta))^{-1}f(\theta)d\theta+\lambda\int_{\underline{\theta}}^{\bar{\theta}}(1-p(\theta))\,f(\theta)d\theta$$
$$+\int_{\underline{\theta}}^{\bar{\theta}}\left[p(\theta)T'(c(\theta))\frac{\partial c(\theta)}{\partial\underline{u}}+(T(c(\theta))-C(\underline{u}))\frac{\partial p(\theta)}{\partial\underline{u}}\right]f(\theta)d\theta$$

6 Conclusion

If labor markets display frictions, the analysis of labor income taxes must not only take its distributive role into account but also its impact on employment. Fiscal externalities, if not corrected with appropriate policy instruments may lead to the sub-optimal entry of firms, as shown by Geromichalos (2015).

A co-monotonicity condition between output and employment is shown to provide a simple empirical check for the presence of inefficiencies in labor market policies. Because this is only a necessary condition we offer an alternative test which relies on the behavior of wedges that arise in constrained efficient allocations: the government should always distort matching probabilities and effort in the same direction.

We also devise a simple procedure for calculating a lower bound for the losses associated with any inefficient policy. We find them to be far from trivial in the U.S.'s case. For our preferred calibration, it is possible to increase the U.S. government revenues by 2.17% while preserving everyone's utility. Our procedure also allows us to calculate the fiscal externalities in the current system and explore the role played by progressivity.

We have assumed that firms have no market power. An interesting alternative is to assume that firms have some degree of market power and labor decisions are decided through some bargaining process as in Lehmann et al. (2006). To make the problem relevant for the redistribution/insurance analysis that is the focus of our work the transferable utility approach that makes their model tractable must be relaxed. We leave this interesting extension for future work.

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A Proofs

Lemma 3. *If an allocation is constrained efficient, there can be no idle resources.*

Proof Just note that any idle resources can be returned in an incentive-compatible way by making $\hat{u}(\theta) = u(\theta) + \delta$ for all θ and $\hat{\underline{u}} = \underline{u} + \delta$. □

Proof of Lemma 1 Let $\underline{u} := \sup_{\tilde{\theta}} \underline{u}(\tilde{\theta})$ and suppose that we can find θ such that $\underline{u}(\theta) < \underline{u}$.

For all θ,

$$p(\theta)\left(u(\theta) - \theta h(\theta)\right) + (1 - p(\theta))\underline{u}(\theta) \geq \underline{u},$$

which implies that $\underline{u}(\theta) < \underline{u} < u(\theta)$. Because $C(\cdot)$ is convex, one can construct a less costly allocation by decreasing $u(\theta)$ by $(1 - p(\theta))\,\varepsilon$ and increasing $\underline{u}(\theta)$ by $p(\theta)\varepsilon$ for small enough ε. If this change is possible in a positive measure set of types $\widetilde{\Theta} \subset \Theta$, then it saves a positive amount of resources, which then can be redistributed by increasing the resultant utilities $u^*(\theta)$ and $\underline{u}^*(\theta)$ uniformly by some $\chi > 0$. □

Proof of Proposition 1 Assume that $\underline{u} < u(\hat{\theta}) - \hat{\theta}h(\hat{\theta})$ and notice that this implies that $\hat{\theta} = \bar{\theta}$. Because $w(\theta)$ is decreasing, we can find $\varepsilon > 0$ such that $\underline{u}+\varepsilon < u(\theta)$ and $w(\theta) > \underline{u}+\varepsilon$ for every θ. Then define a new allocation by increasing the unemployment utility by ϵ, $\tilde{\underline{u}} = \underline{u} + \varepsilon$ and, for every θ, $\tilde{u}(\theta) = u(\theta) + \eta(\theta)$ for $\eta(\theta) = (1 - p(\theta))\epsilon/p(\theta)$. Note that

$$p(\theta)\left[\tilde{u}(\theta) - \hat{\theta}h(\theta)\right] - (1 - p(\theta))\tilde{\underline{u}} = p(\theta)\left[u(\theta) - \hat{\theta}h(\theta)\right] - (1 - p(\theta))\underline{u},$$

for all θ, $\hat{\theta}$, showing that the new allocation not only is incentive compatible but also preserves the expected utility of all agents. Finally, because of the concavity of $C(\cdot)$,

$$p(\theta)C\left(u(\theta)-\frac{1-p(\theta)}{p(\theta)}\epsilon\right)+(1-p(\theta))C\left(\underline{u}+\epsilon\right)-\left[p(\theta)C\left(u(\theta)\right)+(1-p(\theta))C\left(\underline{u}\right)\right]<0.$$

A contradiction. $\square$

Proof of Proposition 2 Necessary conditions for program $\mathcal{P}^{EF}$ include

$$p(\theta)\left[N'(h(\theta))-\theta C'(u(\theta))\right]f(\theta)=-\mu(\theta)p(\theta), \tag{A.1}$$

$$-\dot{\mu}(\theta)=C'(u(\theta))f(\theta)-\alpha(\theta).,\tag{A.2}$$

$\alpha(\theta)\geq 0$, and $\alpha(\theta)[w(\theta)-\varpi(\theta)]=0$ for all θ. Define the normalized weight $\tilde{\alpha}(\theta)=\alpha(\theta)/f(\theta)$, then integrate (A.2) to obtain

$$\mu(\theta)=\int_{\underline{\theta}}^{\theta}\left[\tilde{\alpha}(t)-C'(u(t))\right]f(t)dt,$$

using the free boundary condition $\mu(\underline{\theta})=0$. Noting that (A.1) can be rewritten as

$$\left[1-\frac{\theta C'(u(\theta))}{N'(h(\theta))}\right]\frac{N'(h(\theta))}{\theta C'(u(\theta))}=-\frac{\mu(\theta)}{\theta C'(u(\theta))f(\theta)},$$

just use the definition of $\tau_n(\theta)$ to obtain (2.6). $\square$

Proof of Corollary 1 Re-write (A.1) as

$$\tau_n(\theta)N'(h(\theta))f(\theta)=\mu(\theta),$$

and, by assuming τ_n to be sufficiently smooth, differentiate this expression to obtain

$$\dot{\tau}_n(\theta)N'(h(\theta))f(\theta)+\tau_n(\theta)N''(h(\theta))\dot{h}(\theta)f(\theta)+\tau_n(\theta)N'(h(\theta))\dot{f}(\theta)=-\dot{\mu}(\theta)\leq C'(u(\theta))f(\theta).$$

For $\tau_n>0$,

$$\dot{\tau}_n(\theta)\theta+\tau_n(\theta)\frac{N''(h(\theta))}{N'(h(\theta))}\dot{h}(\theta)\theta+\tau_n(\theta)\frac{\dot{f}(\theta)\theta}{f(\theta)}\leq\frac{\theta C'(u(\theta))}{N'(h(\theta))}=1-\tau_n(\theta).$$

$\square$

Proof of Proposition 3 The first-order condition with respect to $p(\theta)$ is

$$\Big\{N(h(\theta)) - C(u(\theta)) + C(\underline{u}) + C'(u(\theta))\left[u(\theta) - \theta h(\theta) - \underline{u}\right] \\ - \kappa\vartheta'(p(\theta))\Big\} f(\theta) = \mu(\theta)h(\theta) \quad \text{(A.3)}$$

The first-order condition with respect to $h(\theta)$ is

$$\left[N'(h(\theta)) - \theta C'(u(\theta))\right] f(\theta) = \mu(\theta),$$

which, after multiplying by $h(\theta)$, becomes

$$\left[N'(h(\theta))h(\theta) - \theta h(\theta)C'(u(\theta))\right] f(\theta) = \mu(\theta)h(\theta). \quad \text{(A.4)}$$

Substituting (A.4) in (A.3), we get

$$N(h(\theta)) - C(u(\theta)) + C(\underline{u}) + C'(u(\theta))\left[u(\theta) - \theta h(\theta) - \underline{u}\right] \\ - \kappa\vartheta'(p(\theta)) = N'(h(\theta))h(\theta) - \theta h(\theta)C'(u(\theta)),$$

which simplifies to

$$N(h(\theta)) - N'(h(\theta))h(\theta) - C(u(\theta)) + C(\underline{u}) + C'(u(\theta))\left[u(\theta) - \underline{u}\right] = \kappa\vartheta'(p(\theta)). \quad \text{(A.5)}$$

Differentiating (A.12) with respect to θ yields

$$N'(h(\theta))\dot{h}(\theta) - N'(h(\theta))\dot{h}(\theta) - N''(h(\theta))\dot{h}(\theta)h(\theta) - C'(u(\theta))\dot{u}(\theta)+ \\ C'(u(\theta))\dot{u}(\theta) + C''(u(\theta))\left[u(\theta) - \underline{u}\right]\dot{u}(\theta) = \kappa\vartheta''(p(\theta))\dot{p}(\theta),$$

which simplifies to

$$-N''(h(\theta))h(\theta)\dot{h}(\theta) + C''(u(\theta))\left[u(\theta) - \underline{u}\right]\dot{u}(\theta) = \kappa\vartheta''(p(\theta))\dot{p}(\theta).$$

The agent's first-order condition

$$\dot{u}(\theta) = \theta\dot{h}(\theta) - \frac{\dot{p}(\theta)}{p(\theta)}\left[u(\theta) - \theta h(\theta) - \underline{u}\right] \quad \text{(A.6)}$$

allows us to write

$$
\begin{aligned}
&- N''(h(\theta))h(\theta)\dot{h}(\theta) + C''(u(\theta))\left[u(\theta) - \underline{u}\right]\Bigg\{\theta\dot{h}(\theta) - \\
&\qquad\qquad \frac{\dot{p}(\theta)}{p(\theta)}\left[u(\theta) - \theta h(\theta) - \underline{u}\right]\Bigg\} = \kappa\vartheta''(p(\theta))\dot{p}(\theta),
\end{aligned}
$$

which rearranging yields

$$
\begin{aligned}
&\left\{C''(u(\theta))\left[u(\theta) - \underline{u}\right]\theta - N''(h(\theta))h(\theta)\right\}\dot{h}(\theta) \\
&\qquad = \left\{\kappa\vartheta''(p(\theta))p(\theta) + C''(u(\theta))\left[u(\theta) - \underline{u}\right]\left[u(\theta) - \theta h(\theta) - \underline{u}\right]\right\}\frac{\dot{p}(\theta)}{p(\theta)}.
\end{aligned}
$$

$p(\theta)$ and $h(\theta)$ are, therefore, co-monotone.

The agent's second-order condition $\dot{p}(\theta)h(\theta) + p(\theta)\dot{h}(\theta) \leq 0$ leads to $\dot{p}(\theta) \leq 0$ and $\dot{h}(\theta) \leq 0$. This completes the proof. □

Proof of Proposition 4 First, note that the sign of the labor wedge, $\tau^n(\theta)$, is pinned down by the sign of $\mu(\theta)$ through the first-order condition with respect to $h(\theta)$,

$$
N'(h(\theta)) - \theta C'(u(\theta)) = \frac{\mu(\theta)}{f(\theta)}.
$$

The same is true for the vacancy wedge sign, $\tau^p(\theta)$, which is determined by the first-order condition with respect to $p(\theta)$,

$$
N\left(h(\theta)\right) - C\left(u(\theta)\right) + C(\underline{u}) + C'\left(u(\theta)\right)\frac{w(\theta) - \underline{u}}{p(\theta)} - \kappa\vartheta'(p(\theta)) = \frac{h(\theta)\mu(\theta)}{f(\theta)}.
$$

Hence, the signs of both wedges coincide. □

Proof of Proposition 5 Both effort and vacancy wedges are pinned down by the sign of μ. Hence, to prove (i), it suffices to note that this is a free boundary program, which implies $\mu(\underline{\theta}) = 0$. For (ii), we need to derive the sign of $\mu(\bar{\theta})$. The planner's dual problem may be written as the following optimal control program,

$$
\begin{aligned}
\max \int_{\underline{\theta}}^{\bar{\theta}} &\left\{p(\theta)\left[N\left(h(\theta)\right) - C\left(\frac{w(\theta) - \underline{u}}{p(\theta)} + \theta h(\theta) + \underline{u}\right)\right]\right. \\
&\qquad\qquad \left. - (1 - p(\theta))C(\underline{u}) - \frac{\kappa}{\lambda\left(p(\theta)\right)}\right\} f(\theta)d\theta,
\end{aligned}
$$

subject to

$$\int_{\underline{\theta}}^{\overline{\theta}} a(\theta)w(\theta)f(\theta)d\theta \geq A,$$

$$\dot{w}(\theta) = -p(\theta)h(\theta),$$

$$w(\bar{\theta}) = \underline{u},$$

and

$$p(\theta)h(\theta) \text{ decreasing.}$$

Here, $h(\theta)$ and $p(\theta)$ are the controls and $w(\theta)$ is the state variable.

We will restrict our attention to C^2 solutions, which satisfy the monotonicity condition. Thus, we can write the Lagrangian as

$$V(A, \underline{u}) = \max \int_{\underline{\theta}}^{\overline{\theta}} \left\{ p(\theta) \left[N\left(h(\theta)\right) - C\left(\frac{w(\theta) - \underline{u}}{p(\theta)} + \theta h(\theta) + \underline{u}\right)\right] - (1 - p(\theta))C(\underline{u}) - \kappa\vartheta(p(\theta)) + \psi\left[a(\theta)w(\theta) - A\right] \right\} f(\theta)d\theta - \mu(\theta)p(\theta)h(\theta).$$

Ignoring bunching, the first-order conditions are

$$p(\theta)\left[N'(h(\theta)) - \theta C'(u(\theta))\right] f(\theta) = \mu(\theta)p(\theta), \tag{A.7}$$

$$\left[N\left(h(\theta)\right) - C\left(u(\theta)\right) + C(\underline{u}) + C'\left(u(\theta)\right)\frac{w(\theta) - \underline{u}}{p(\theta)} - \kappa\vartheta'(p(\theta))\right] f(\theta) \;=\; \mu(\theta)h(\theta), \tag{A.8}$$

and

$$-\dot{\mu}(\theta) = -C'(u(\theta))f(\theta) + \psi a(\theta)f(\theta). \tag{A.9}$$

To sign the labor wedge in (A.7), we must assess the sign of $\mu(\theta)$. Integrating $-\dot{\mu}(\theta) = -C'(u(\theta))f(\theta) + \psi a(\theta)f(\theta)$ from $\underline{\theta}$ to $\bar{\theta}$, we obtain

$$\mu(\theta) = -\int_{\underline{\theta}}^{\theta} \left[\psi a(t) - C'(u(t))\right] f(t)dt, \tag{A.10}$$

using $\mu(\underline{\theta}) = 0$.

Recall that, because $w(\bar{\theta}) = \underline{u}$, we do not necessarily have $\mu(\bar{\theta}) = 0$. We want to know the sign of $\mu(\bar{\theta})$. Notice that the allocation $(u^*(\theta) + x, \underline{u} + x, p(\theta), h(\theta))$ is always feasible for $|x| < \varepsilon$ for some $\varepsilon > 0$.

Therefore, we obtain

$$-\frac{\partial V(A,\underline{u})}{\partial A} = \psi = \int_{\underline{\theta}}^{\overline{\theta}} \left[p(\theta)C'(u(\theta)) + (1-p(\theta))C'(\underline{u})\right] f(\theta)d\theta.$$

replacing the expression above in (A.10) and using (A.7) we obtain (2.8). Finally, evaluating (A.10) at $\bar{\theta}$ yields

$$\mu(\bar{\theta}) = -\int_{\underline{\theta}}^{\overline{\theta}} \left[p(\theta)C'(u(\theta)) + (1-p(\theta))C'(\underline{u})\right] f(\theta)d\theta + \int_{\underline{\theta}}^{\overline{\theta}} C'(u(\theta))f(\theta)d\theta$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} (1-p(\theta))\left(C'(u(\theta)) - C'(\underline{u})\right) f(\theta)d\theta > 0.$$

The least productive agent faces a positive marginal tax rate. □

Proof of Proposition 6 In the Rawlsian program, the co-state equation becomes

$$-\dot{\mu}(\theta) = -C'(u(\theta))f(\theta) \quad \forall\theta, \tag{A.11}$$

whereas the optimality condition for the free boundary constraint is $\mu(\underline{\theta}) = 0$. Hence, $\mu(\theta) \leq 0 \; \forall \, \theta > \underline{\theta}$.

The result is then immediate from the first-order condition for $h(\theta)$. □

Proof of Proposition 7 Differentiating the zero profit condition,

$$N(h(\theta)) - \chi(u(\theta)) = \frac{\kappa}{\lambda(p(\theta))p(\theta)},$$

we obtain

$$N'(h(\theta))\dot{h}(\theta) - \chi'(u(\theta))\dot{u}(\theta) = -\frac{\kappa[\lambda'(p(\theta))p(\theta) + \lambda(p(\theta))]\dot{p}(\theta)}{[\lambda(p(\theta))p(\theta)]^2}.$$

From the firms' first-order conditions,

$$N'(h(\theta))\left[\dot{h}(\theta) - \frac{\dot{u}(\theta)}{\theta}\right] = -\frac{\kappa\left[\lambda'(p(\theta))p(\theta) + \lambda(p(\theta))\right]\dot{p}(\theta)}{[\lambda(p(\theta))p(\theta)]^2}.$$

Now, using the agents' envelope and the definition of $w(\theta)$,

$$N'(h(\theta))\left\{\frac{\dot{p}(\theta)}{\theta p(\theta)}\left[u(\theta) - \theta h(\theta) - \underline{u}\right]\right\} = -\frac{\kappa\left[\lambda'(p(\theta))p(\theta) + \lambda(p(\theta))\right]\dot{p}(\theta)}{[\lambda(p(\theta))p(\theta)]^2}$$

must hold for all agents.

But we know from Claim 1 that there is at least one type $\hat{\theta}$ such that $\underline{u} = \big(u(\hat{\theta}) - \hat{\theta}h(\hat{\theta})\big)$, which implies

$$\frac{\kappa\left[\lambda'(p(\hat{\theta}))p(\hat{\theta}) + \lambda(p(\hat{\theta}))\right]\dot{p}(\hat{\theta})}{\left[\lambda(p(\hat{\theta}))p(\hat{\theta})\right]^2} = 0.$$

This condition cannot be satisfied for any $p(\theta) < 1$. □

Proof of Proposition 8 **Part i)** First note that since both $N(h)$ is strictly monotonic in h and $\chi(u)$ is strictly monotonic in u, then the firm offer corresponds to an offer of a pair (h, u).

Let $(p(\theta), h(\theta), c(\theta))\}$ denote θ-type worker's choice. Then, define the set

$$A := \left\{(h, u)\Big| \quad \exists\theta; (p(\theta), h(\theta), u(\theta)) = (p(\theta), h, u)\right\}$$

For $(h, u) \in A$ let $\mathrm{p}(h, u)$ be such that $(\mathrm{p}(h, u), h, u) = (p(\theta), h(\theta), u(\theta))$ for some θ.

We define $\mathcal{T}(h, u) = K(h, u)$ if $(h, u) \notin A$, where $K(h, u)$ satisfies

$$[N(h) - C(u) - K(h, u)] < 0.$$

If $(h, u) \in A$ we let $\mathcal{T}(h, u)$ be given by

$$[N(h) - C(u) - \mathcal{T}(h, u)] = \frac{\kappa}{p(h, u)\,\lambda(p(h, u))}.$$

Since there is free entry of firms and $[N(h) - C(u) - \mathcal{T}(h, u)] > 0$, a firm offering (h, u) induces $p(h, u)$ and breaks even. Moreover, a firm offering $\mathcal{T}(h, u) \notin A$ could not make a profit. These conditions guarantee implementability.

Part ii) We consider an allocation in which $\theta \to p(\theta)$ is strictly decreasing and $\theta \to (p(\theta), h(\theta), u(\theta))$ is strictly positive and differentiable around a neighborhood of some interior type θ. The firm must break-even for every type θ. Hence,

$$\big[N(h(\theta)) - \chi(u(\theta))) - \mathcal{T}(N(h(\theta)) - \chi(u(\theta)))\big] = \frac{\kappa}{p(\theta)\,\lambda(p(\theta))}. \tag{A.12}$$

Since the L.H.S. of (A.12) is strictly positive, the firm benefits from a decrease in $p(\theta)$. Therefore, it must maximize

$$\max u(\theta) - \theta h(\theta)$$

s.t.,

$$N(h(\theta)) - \chi(u(\theta))) = K$$

implying

$$\theta N'(h(\theta)) = \chi'(u(\theta)). \tag{A.13}$$

Moreover, if

$$\begin{aligned}\frac{d}{d\theta}\left[N\left(h(\theta)\right)-\chi(u\left(\theta\right))\right] &= \theta N'\left(h(\theta)\right)\dot{h}\left(\theta\right)-\chi'(u\left(\theta\right))\dot{u}\left(\theta\right)\\ &= N'\left(h(\theta)\right)\left[\theta\dot{h}\left(\theta\right)-\dot{u}\left(\theta\right)\right]<0,\end{aligned}$$

then gross profits are strictly decreasing in θ. Using this and observing that the R.H.S. of (A.12) above is differentiable and hence so is the L.H.S., we conclude that $\dot{p}(\theta)<0$ implies $1-\mathcal{T}'>0$. In summary, the firm must be maximizing profits subject to delivering utility $w(\theta)=p(\theta)\left[u(\theta)-\theta h(\theta)-\underline{u}\right]$ to type θ.This implies that the following second order condition must hold,

$$N''\left(h(\theta)\right)-\theta^2\chi''(u\left(\theta\right))\leq 0. \tag{A.14}$$

Recall that (F.3) must hold in a neighborhood of θ. Since both sides of (F.3) are differentiable, one obtains

$$N'\left(h(\theta)\right)+\theta N''\left(h(\theta)\right)\dot{h}(\theta)=\chi''(u\left(\theta\right))\dot{u}(\theta).$$

The second order condition (A.14) hence reads

$$N''\left(h(\theta)\right)-\theta^2\left[\frac{N'\left(h(\theta)\right)+\theta N''\left(h(\theta)\right)\dot{h}(\theta)}{\dot{u}(\theta)}\right]\leq 0.$$

Notice that incentive compatibility implies

$$\theta\dot{h}\left(\theta\right)-\dot{u}\left(\theta\right)=\frac{\dot{p}(\theta)}{p(\theta)}\left[u(\theta)-\theta h(\theta)-\underline{u}\right]$$

Therefore, by having $\dot{h}\left(\theta\right)<0, \dot{u}\left(\theta\right)<0$, $\theta\dot{h}\left(\theta\right)-\dot{u}\left(\theta\right)$ negative but close to zero, and

$$\theta^2\left[\frac{N'\left(h(\theta)\right)}{-\dot{u}(\theta)}\right]>-N''\left(h(\theta)\right)$$

$$\theta^2\left[\frac{N'\left(h(\theta)\right)}{-N''\left(h(\theta)\right)}\right]>-\dot{u}(\theta)$$

we can violate the second order condition. This implies that the allocation cannot be implemented. □

B Figures

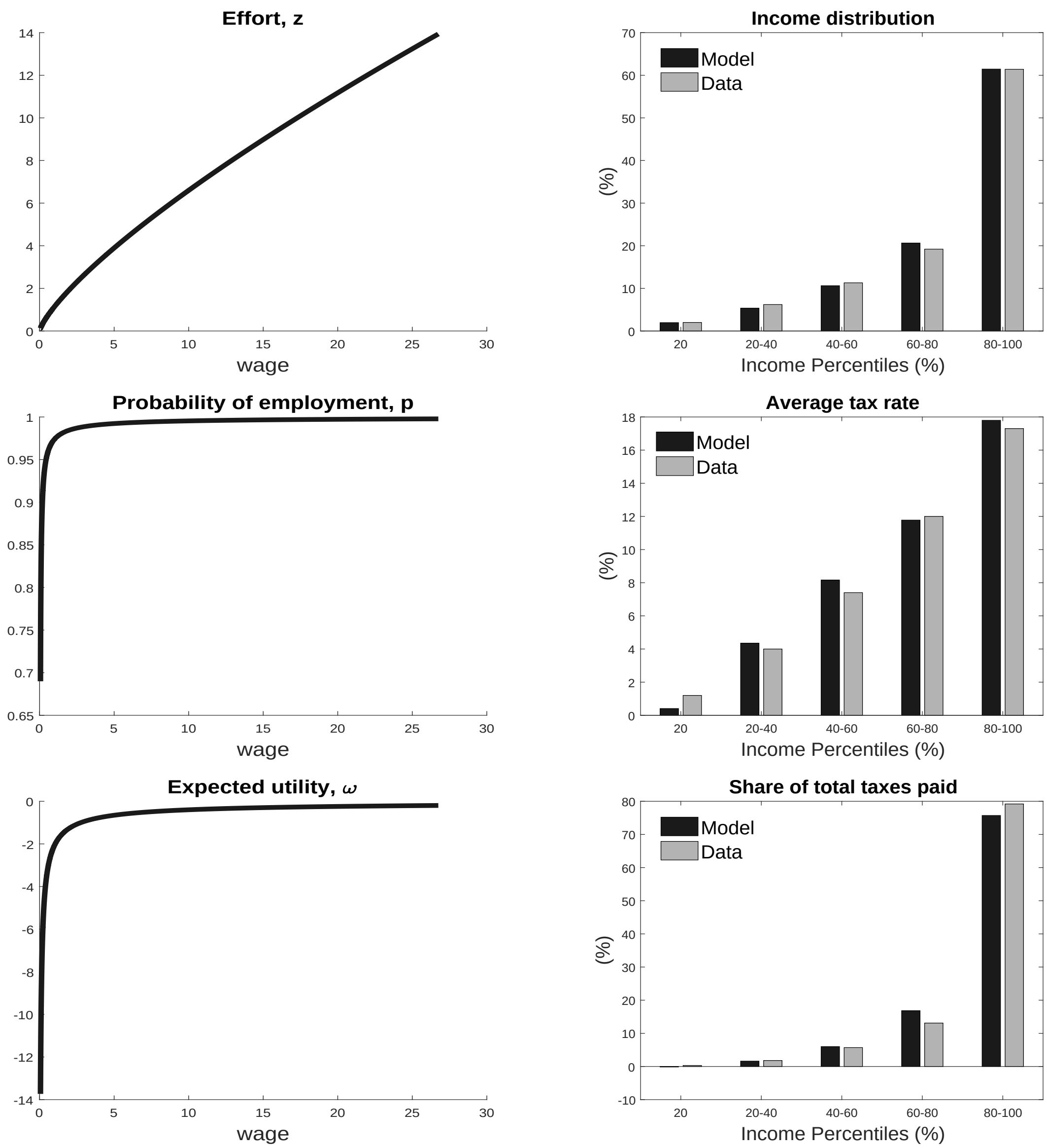

Figure 1: **Calibrated economy** The panels on the left side of the figure display the behavior of z, p, and w for the calibrated economy. The panels on the right side compare the distribution of income and taxes in the model and in the data.

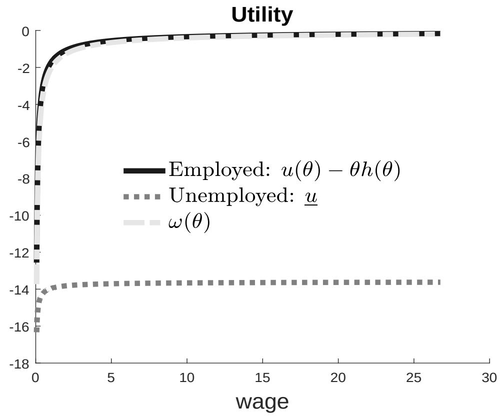

Figure 2: **Expected utility decomposition: Benchmark** The figure displays the utility conditional on being employed, the unemployment utility, and the expected utility of each type at the benchmark.

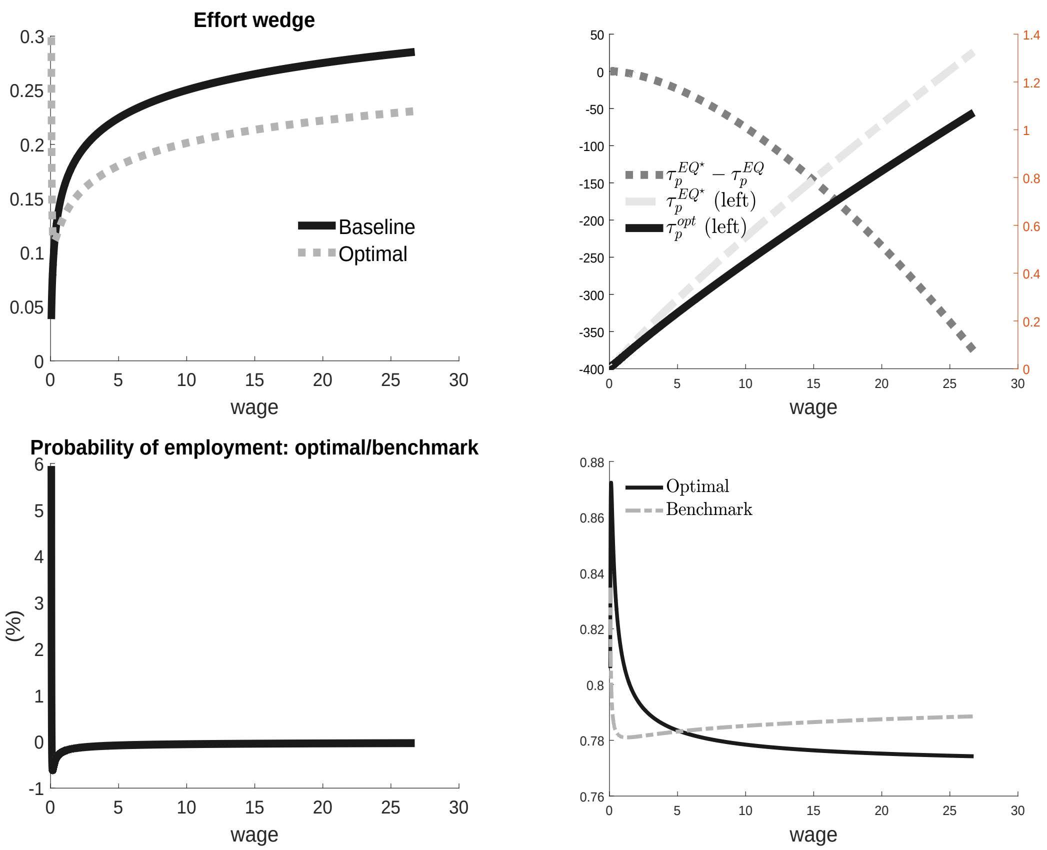

Figure 3: **Optimal allocation** For both the optimal and the baseline allocations, the top panels display effort (left) and vacancy (right) wedges as defined in (1.3) and (1.4), respectively. The top right panel also displays the counterfactual wedge τ_p^{EQ*}. The bottom panels compare the probability of employment (left) and the hourly wage (right) for the baseline and the optimal policies.

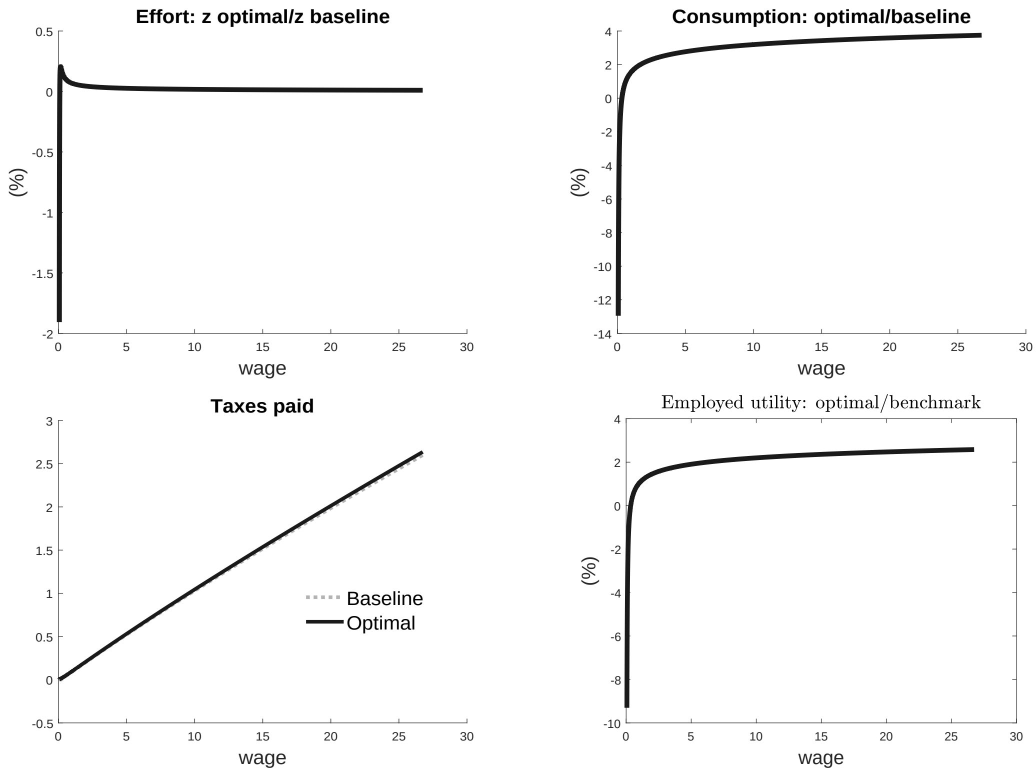

Figure 4: **Optimal allocation** The figure displays the percentage variation between the baseline and the optimal output (top left) consumption (top right) and the taxes paid (bottom left). The bottom-right panel displays utility conditional on working at the benchmark and at the optimum.

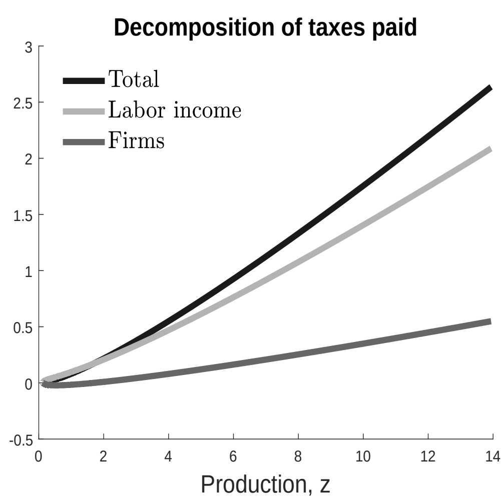

Figure 5: **Optimal Taxes** The figure displays the decomposition of total taxes into labor income taxes and taxes/subsidies to vacancy creation. Marginal tax rates pin down the growth of labor income taxes, but not the level. In the decomposition displayed above we have fixed average labor income taxes at the current level observed in the U.S..

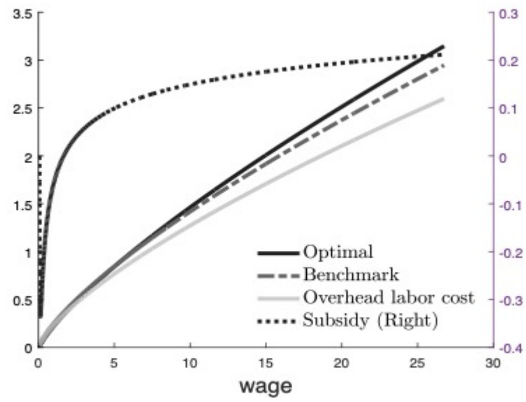

Figure 6: **Subsidies on Vacancy Creation** The figure displays gross profits, $z-y$, at the benchmark and the optimum, and overhead labor costs, $\kappa/(\lambda(p)p)$, at the optimum. Total difference between the two represent subsidies on vacancy creation, explicitly calculated as a proportion of overhead labor costs (right vertical axis).

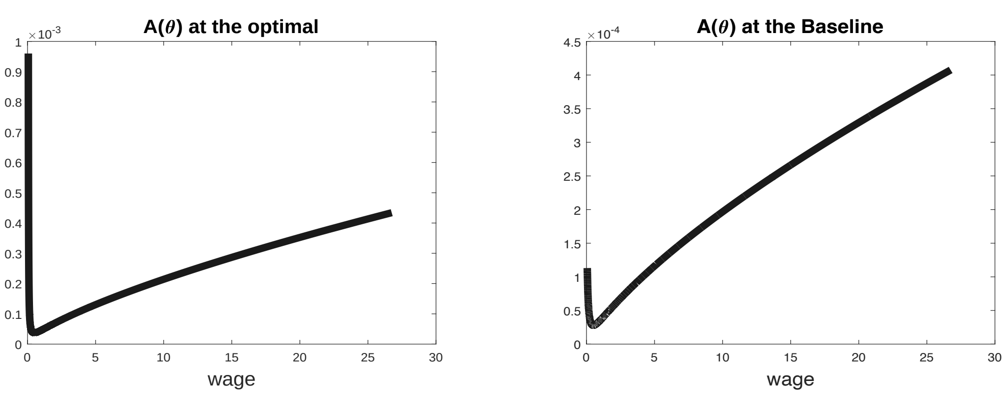

Figure 7: **Implicit Welfare Weights** The figure displays the optimal accumulated weight $\Lambda(\theta)$ using the productivity $w = \theta^{-1-\gamma}$ in the horizontal axis to keep the representation aligned with the other figures.

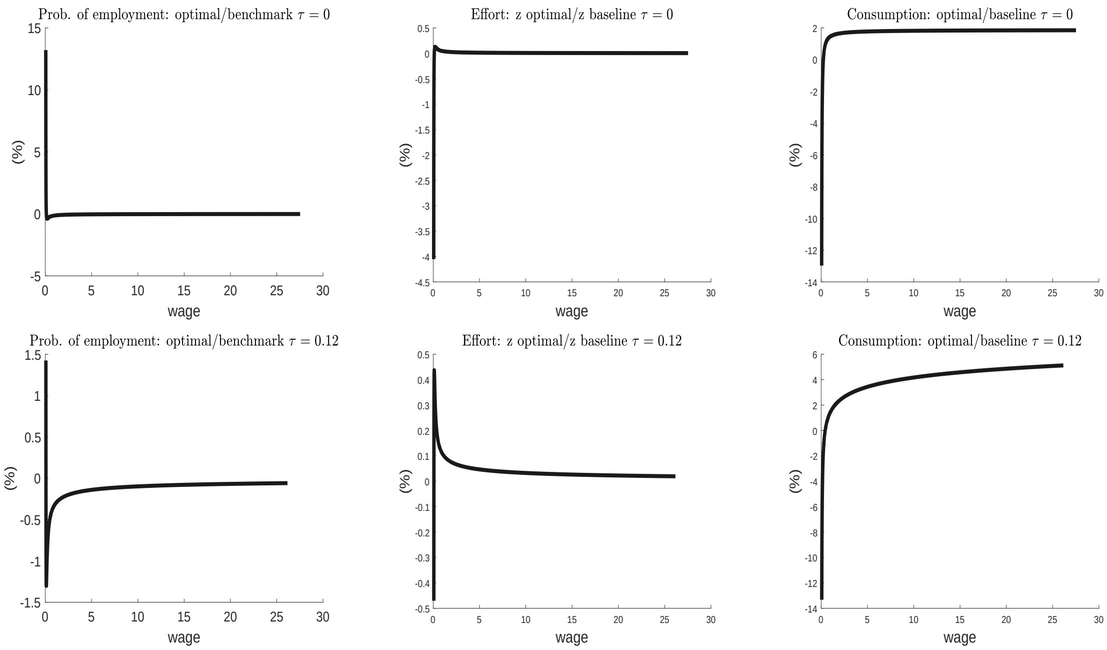

Figure 8: **Sensitivity** The figure displays the percentage variation between the baseline and the optimal probability of working (left), output (middle), and consumption (right) for $\tau = 0.00$ and $\tau = 0.12$.

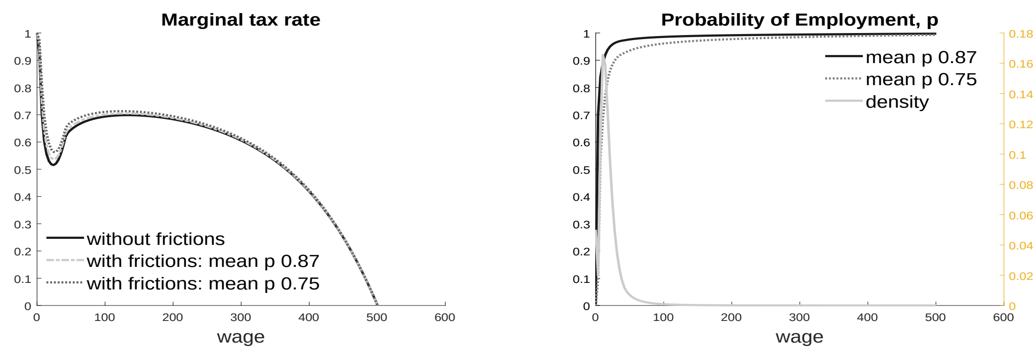

Figure 9: **Optimal Tax Schedule** The figure displays the marginal tax rates (left panel) for a planner with a Utilitarian objective. To see how frictions affect the marginal tax rates, we change the vacancy posting costs, κ, to generate different levels of employment rate and compare the result with the case without frictions.

Online Appendix - Not for publication

C Derivations for Quantitative Assessment

C.1 Deriving expression (4.1)

Using our preferred parametrization (and returning to the primal variables), it is then the case that an equilibrium allocation solves

$$\max_{p,z,y} p\left[\frac{1}{1-\sigma}(\zeta y^{1-\tau})^{1-\sigma} - \frac{\theta}{1+\gamma}z^{1+\gamma} - \underline{u}\right]$$

subject to

$$z - y \geq \frac{\kappa}{1-p}. \tag{C.1}$$

The first-order condition for this problem is

$$p(1-\tau)\zeta^{1-\sigma}y^{(1-\tau)(1-\sigma)-1} = \alpha(\theta), \tag{C.2}$$

with respect to y,

$$\theta p z^{\gamma} = \alpha(\theta), \tag{C.3}$$

with respect to z, and

$$\frac{1}{1-\sigma}(\zeta y^{1-\tau})^{1-\sigma} - \frac{\theta}{1+\gamma}z^{1+\gamma} - \underline{u} = \alpha(\theta)\frac{\kappa}{(1-p)^2} \tag{C.4}$$

with respect to p.

Using (C.2) and (C.3), we get

$$\left[(1-\tau)\zeta^{1-\sigma}y^{(1-\tau)(1-\sigma)-1}\right]^{\frac{1}{\gamma}}\theta^{\frac{1}{-\gamma}} = z, \tag{C.5}$$

which shows that no extra distortion, beyond that caused by labor income taxes, is introduced by the firm.

Multiplying (C.2) by $y/((1-\tau)(1-\sigma))$ and (C.3) by $z/(1+\gamma)$ and adding the two yields

$$p\left[\frac{(\zeta y^{1-\tau})^{1-\sigma}}{1-\sigma} - \theta\frac{z^{1+\gamma}}{1+\gamma} - \bar{u}\right] = \frac{\alpha(\theta)y}{(1-\tau)(1-\sigma)} - \frac{\alpha(\theta)z}{1+\gamma} - p\bar{u},$$

which, using (C.4), can be written as

$$\alpha(\theta)\frac{p\kappa}{(1-p)^2} = \frac{\alpha(\theta)y}{(1-\tau)(1-\sigma)} - \frac{\alpha(\theta)z}{1+\gamma} - p\bar{u}.$$

Next, using the fact that constraint (C.1) is active at the optimum, we have

$$(z-y)^2 = \frac{\kappa^2}{(1-p)^2}, \quad \text{and} \quad p = 1 - \frac{\kappa}{z-y},$$

which can be used to obtain

$$\bar{u} = \theta z^\gamma \left[\frac{1-(1-\tau)(1-\sigma)}{(1-\tau)(1-\sigma)} y + \frac{\gamma}{1+\gamma} z - \frac{z^2}{\kappa} + \frac{2zy}{\kappa} - \frac{y^2}{\kappa} \right].$$

Let $A(y) := (1-\tau)\zeta^{1-\sigma} y^{(1-\tau)(1-\sigma)-1}$, and then we can finally write

$$-\frac{\bar{u}}{A(y)} + \frac{1-(1-\tau)(1-\sigma)}{(1-\tau)(1-\sigma)} y - \frac{y^2}{\kappa} = -\left[\frac{\gamma}{1+\gamma} A(y)^{\frac{1}{\gamma}} + \frac{2A(y)^{\frac{1}{\gamma}} y}{\kappa} \right] \theta^{-\frac{1}{\gamma}} + \frac{A(y)^{\frac{2}{\gamma}}}{\kappa} \theta^{-\frac{2}{\gamma}}.$$

C.2 Deriving the Expression for $\Lambda(\theta)$

The co-state equation associated with program P^{EF} is

$$-C'(u(\theta)) f(\theta) + \alpha(\theta) = \dot{\mu}(\theta) \tag{C.6}$$

for $\alpha(\theta)$ the multiplier attached to constraint $w(\theta) - \varpi(\theta) \geq 0$.

Beyond the non-negativity constraint, $\tilde{\alpha}(\theta) \geq 0$ for the multiplier, $\alpha(\cdot)$, the complementary slackness condition, $\alpha(\theta)\left[w(\theta) - \varpi(\theta)\right] = 0$ must be satisfied at all θ.

Integrating (C.6), and using $\mu(\underline{\theta}) = 0$,

$$\int_{\underline{\theta}}^{\theta} \left[\alpha(s) - C'(u(s)) f(s)\right] ds = \mu(\theta)$$

we define

$$\Lambda(\theta) = \int_{\underline{\theta}}^{\theta} \alpha(s) ds = \mu(\theta) + \int_{\underline{\theta}}^{\theta} C'(u(s)) f(s) ds,$$

which can be inspected using

$$N'(h(\theta)) \tau^n(\theta) f(\theta) = \mu(\theta), \tag{C.7}$$

Indeed, using (C.7), we write $\Lambda(\theta)$ as

$$\Lambda(\theta) = N'(h(\theta)) \tau^n(\theta) f(\theta) + \int_{\underline{\theta}}^{\theta} C'(u(s)) f(s) ds.$$

D Elasticities for Optimal Tax Formulae

To use a tax perturbation approach in our setting, we must define the appropriate elasticities. The relevant elasticities are the ones that are derived when we consider the impact of a reform where the government perturbs the labor income tax system but adjust the rest of the system in such a way as to keep $(p(\theta))_\theta$ fixed. As the candidate optimal tax, $T(\cdot)$, is replaced by $T_\mu(\cdot) = T(\cdot) + \mu H(\cdot)$, workers change their behavior for the same labor contract (z, y). Holding the contract fixed would then lead to a change in vacancy creation by firms. What we assume here is that the planner uses its additional policy instruments to induce a reaction by the firm that keeps p fixed.

The agent's effort choice is the solution to

$$\max_y \varphi(y - T(y)) - h\left(\frac{y + \phi(p)}{w}\right),$$

where $\phi(p)$ is the cost for the firm of creating the efficient level of vacancies taking into account the taxes or subsidies for vacancy creation.

The first-order condition is

$$\varphi'(y - T(y))\left[1 - T'(y)\right] - \eta'\left(\frac{y + \phi(p)}{w}\right)\frac{1}{w} = 0$$

Define

$$\begin{aligned}
\Delta &= \varphi''(c)\left[1 - T'(y)\right]^2 - \varphi'(c)T''(y) - \eta''(n)\frac{1}{w^2} \\
&= \frac{\varphi'(c)\left[1 - T'(y)\right]}{y}\left\{\frac{\varphi''(c)}{\varphi'(c)}\left[1 - T'(y)\right]y - \frac{T''(y)y}{1 - T'(y)} - \frac{\eta''(n)}{\varphi'(c)}\frac{y}{w^2}\frac{1}{1 - T'(y)}\right\} \\
&= \frac{\varphi'(c)\left[1 - T'(y)\right]}{y}\left\{\frac{\varphi''(c)c}{\varphi'(c)}\frac{\left[1 - T'(y)\right]y}{c} - \frac{T''(y)y}{1 - T'(y)} - \frac{\eta''(n)\,n}{\eta'(n)}\frac{y}{z}\right\} \\
&= -\frac{\varphi'(c)\left[1 - T'(y)\right]}{y}\left\{\sigma(c)\pi_0(y) + \pi_1(y) + \gamma(n)\frac{y}{z}\right\},
\end{aligned}$$

where $\sigma(c) = \varphi''(c)c/\varphi'(c)$, $\gamma(n) = \eta''(n)n/\eta'(n)$, $\pi_0(y) = [1 - T'(y)]/(1 - T(y)/y)$ and $\pi_1(y) = T''(y)y/(1 - T'(y))$.

$$\begin{aligned}
\Delta_{1-\tau} &= \varphi''(c)\left[1 - T'(y)\right]y + \varphi'(c) \\
&= \varphi'(c)\left\{\frac{\varphi''(c)c}{\varphi'(c)}\frac{\left[1 - T'(y)\right]y}{c} + 1\right\} \\
&= \varphi'(c)\left\{1 - \sigma(c)\pi_0(y)\right\}
\end{aligned}$$

$$\begin{aligned}\Delta_B &= \varphi''(c)\left[1-T'(y)\right] \\ &= \frac{\varphi'(c)}{y}\frac{\varphi''(c)c}{\varphi'(c)}\frac{\left[1-T'(y)\right]y}{c} = -\frac{\varphi'(c)}{y}\sigma(c)\pi_0(y)\end{aligned}$$

$$\begin{aligned}\Delta_w &= -\eta''\left(n\right)\frac{z}{w^3} - \eta'\left(n\right)\frac{1}{w^2} \\ &= -\frac{\varphi'(c)}{w}\left\{\frac{\eta''\left(n\right)}{\varphi'(c)}\frac{z}{w^2} + \frac{\eta'\left(n\right)}{\varphi'(c)w}\right\} \\ &= -\frac{\varphi'(c)}{w}\left\{\frac{\eta''\left(n\right)n}{\eta'\left(n\right)}\frac{\eta'\left(n\right)}{\varphi'(c)w} + \frac{\eta'\left(n\right)}{\varphi'(c)w}\right\} \\ &= -\frac{\varphi'(c)}{w}\left[1-T'(y)\right]\left\{\gamma(n)+1\right\}\end{aligned}$$

This allows us to define

$$\varepsilon = -\frac{\Delta_{1-\tau}}{\Delta}\frac{1-T'(y)}{y}, \nu = -\frac{\Delta_B}{\Delta}\left[1-T'(y)\right], \zeta = -\frac{\Delta_w}{\Delta}\frac{w}{y}, \varepsilon^c = \varepsilon - \nu,$$

where

$$\varepsilon = \frac{1-\sigma(c)\pi_0(y)}{\sigma(c)\pi_0(y)+\pi_1(y)+\gamma(n)\frac{y}{z}}$$

is the uncompensated elasticity of taxable income,

$$\nu = \frac{-\sigma(c)\pi_0(y)}{\sigma(c)\pi_0(y)+\pi_1(y)+\gamma(n)\frac{y}{z}}$$

is the income elasticity of taxable income,

$$\varepsilon^c = \frac{1}{\sigma(c)\pi_0(y)+\pi_1(y)+\gamma(n)\frac{y}{z}}$$

is the compensated elasticity of taxable income, and

$$\zeta = \frac{1+\gamma(n)}{\sigma(c)\pi_0(y)+\pi_1(y)+\gamma(n)\frac{y}{z}}$$

is the cross-sectional elasticity of taxable income.

E Computation of the optimal tax schedule

We follow the calibration of Mankiw et al. (2009). In the baseline calibration, they use CPS data to parameterize the wage distribution using a lognormal approximation with mean

and standard deviation equal to 2.757 and 0.5611, respectively. Then, this continuous wage distribution is discretized as follows. First, the grid for wages $w_i = \{w_1, ..., w_N\}$ is uniform, ranging from 0.01 to 500.51 with the distance between points equal to $\Delta = 3.5$. Then, the discrete probability mass function $\pi(w_i)$ is computed using $F\left(w_i + \frac{\Delta}{2}\right) - F\left(w_i - \frac{\Delta}{2}\right)$. In addition, each agent has the same additively separable preferences over consumption and labor, where the utility function is given by

$$u(c,n) = \frac{c^{1-\sigma} - 1}{1-\sigma} - \alpha\frac{n^\gamma}{\gamma}$$

where $\sigma = 1.5$, $\alpha = 2.55$ and $\gamma = 3$.

The fixed-point algorithm proceeds according to the following steps.

1. Guess a tax schedule, a government transfer and a profile for $p(w_i)$

2. Compute the optimal labor supply and the associated consumption

$$p(w_i)\left[\frac{c_i^{1-\sigma}}{1-\sigma} - \alpha\frac{h_i^{1+\gamma}}{1+\gamma}\right] + (1 - p(w_i))\bar{u} \quad \text{where} \quad c_i = w_i h_i \tag{E.1}$$

3. Update the tax schedule using

$$\begin{aligned}\frac{T'(y)}{1-T'(y)} = (1+\gamma)\left[\frac{u_c(c(w_i))}{w_i\pi(w_i)}\right]\Bigg[\sum_{w_i}^{w_N}\frac{\pi(w_i)}{u_c(c(w_i))} - (1-\Pi(w_i))\sum_{w_1}^{w_N}\frac{\pi(w_i)}{u_c(c(w_i))} \\ +(1-\Pi(w_i))\sum_{w_1}^{w_N}\left(\frac{\pi(w_i)}{u_c(c(w_i))} - \frac{\pi(w_1)}{u_c(c(w_1))}\right)(1-p(w_i))\Bigg]\end{aligned} \tag{E.2}$$

4. Compute each agent's tax liability at the new schedule

5. update $p(w_i)$ using

$$N\left(h(\theta)\right) - C\left(u(\theta)\right) + C(\underline{u}) + C'\left(u(\theta)\right)\left[u(\theta) - \underline{u}\right] - \kappa\vartheta'(p(\theta)) = N'(h(\theta))h(\theta). \tag{E.3}$$

or

$$p = 1 - \left[\frac{\kappa}{N\left(h(\theta)\right) - C\left(u(\theta)\right) + C(\underline{u}) + C'\left(u(\theta)\right)\left[u(\theta) - \underline{u}\right] - N'(h(\theta))h(\theta)}\right]^{1/2} \tag{E.4}$$

6. aggregate government revenue and update transfers

7. Check the convergence and go to step 1

F Proofs for Section 5.

F.1 Proof of Lemma 2

Proof of Lemma 2
First, take $\alpha, \beta \in \mathbb{R}$ and consider the problem

$$W(p \mid \alpha, \beta) := \max_c p \left[\varphi(c) - \theta\eta\left(c + \alpha + \beta c + \frac{\kappa}{1-p}\right) - \underline{u}\right] + \underline{u}.$$

We first argue that this problem is strictly concave. Fix a particular p and define implicitly $c(p)$ by the following optimality condition:

$$\varphi'(c(p)) - \theta(1+\beta)\eta''\left(c(p) + \alpha + \beta c + \frac{\kappa}{1-p}\right) = 0.$$

This yields:

$$c'(p) = \left(\frac{\theta(1+\beta)\eta'}{\varphi'' - \theta(1+\beta)^2\eta''}\right)\left(\frac{\kappa}{(1-p)^2}\right) < 0. \tag{F.1}$$

The following inequality will be used below:

$$c'(p) > -\left(\frac{\kappa}{(1+\beta)(1-p)^2}\right) \tag{F.2}$$

Using the envelope theorem we get:

$$\frac{dW(p \mid \alpha, \beta)}{dp} = \left[\varphi(c) - \theta\eta\left(c + \alpha + \beta c + \frac{\kappa}{1-p}\right) - \underline{u} - \theta p\eta'\left(\frac{\kappa}{(1-p)^2}\right)\right]$$

and, using (F.2) we obtain

$$\begin{aligned}\frac{d^2W(p \mid \alpha, \beta)}{dp^2} &= -2\theta\eta'\left(\frac{\kappa}{(1-p)^2}\right) - \theta p\eta'\left(\frac{\kappa}{(1-p)^2}\right)^2 - \theta p\eta''\left(\frac{2\kappa}{(1-p)^3}\right) \\ &\quad - \theta p\eta''\left(\frac{\kappa}{(1-p)^2}\right)\left[\frac{\kappa}{(1-p)^2} + (1+\beta)c'(p)\right] \\ &< -2\theta\eta'\left(\frac{\kappa}{(1-p)^2}\right) - \theta p\eta''\left(\frac{\kappa}{(1-p)^2}\right)^2 - \theta p\eta'\left(\frac{2\kappa}{(1-p)^3}\right) < 0.\end{aligned}$$

Next, consider the problem of a worker who faces an increasing and weakly convex schedule $c \to T(c)$. Let $W(p \mid T)$ be his value function when he chooses employment probability p. Suppose, towards a contradiction, that the problem admits two solutions, (c_1, p_1), and (c_2, p_2):

$$W(p_1 \mid T) = W(p_2 \mid T). \tag{F.3}$$

First, observe that $p_1 = p_2$ implies that $c_1 = c_2$. Henceforth, assume that $p_1 < p_2$ and recall that (F.1) implies $c(p_1) > c(p_2)$. Since T is convex, its subdifferential ∂T at $c(p_k)$ (for $k = 1, 2$) is a nonempty set. Moreover, there exists $T'_k \in \partial T(c(p_k))$ for $k = 1, 2$ such that the worker's first-order condition at (c_k, p_k) reads

$$\varphi'(c_1) - \theta\,(1 + T'_k)\,\eta'\left(c_1 + T(c_1) + \frac{\kappa}{1-p}\right) = 0. \tag{F.4}$$

Since T is convex, $c(p_1) > c(p_2)$ implies $T'_2 \leq T'_1$. Now, consider the worker's s problem under the following affine tax $\hat{T}$ defined by

$$\hat{T}(\tilde{c}) := T(c(p_1)) + T'_1(\tilde{c} - c(p_1))\,.$$

Write $W(p \mid \hat{T})$ for the value function when the worker chooses employment p under this tax. By construction,

$$W(p_1 \mid \hat{T}) = W(p_1 \mid T). \tag{F.5}$$

In light of our findings from the first part of this proof, this problem is strictly concave and it is solved by $(c(p_1), p_1)$ satisfying the first-order condition indicated by (F.4). Therefore,

$$W(p_1 \mid \hat{T}) > W(p_2 \mid \hat{T}). \tag{F.6}$$

Next, notice that since $T, \hat{T}(c(p_1)) = T(c(p_1))$ is weakly convex, $\hat{T}(c(p_2)) \leq T(c(p_2))\,,$ implying

$$W(p_2 \mid \hat{T}) \geq W(p_2 \mid T). \tag{F.7}$$

Therefore, using (F.5), (F.5), (F.5) we obtain $W(p_2 \mid T) < W(p_1 \mid T)$, contradicting (A.12). □

F.2 Proof of Proposition 9

Proof of (i).

Perturbation 1: This perturbation takes a point $c(\theta_1)$ at which $T''(c(\theta_1)) > 0$ and considers a perturbation in which T' increases (decreases) and then decreases (increases) around this point. Let λ be the Lagreagean multiplier corresponding to the worker's utility. Forgetting

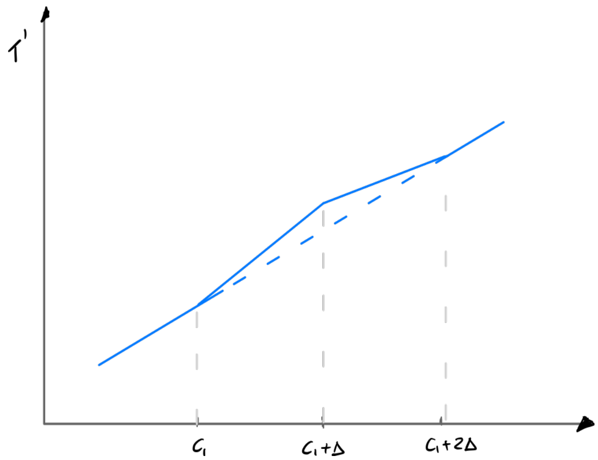

Figure 10: **Tax Perturbation** The figure displays the type of perturbation described in the text. The dotted line indicates the original schedule.

terms of higher order we may assume that $T'(c) = \alpha + \beta(c - c_1)$ for some subinterval $[c(\theta_1), c(\theta_1) + 2\Delta] := [c_1, c_1 + 2\Delta]$.

Consider the following perturbed tax schedule:

$$\tilde{T}'(c) = \alpha + \beta(c - c_1) + \varepsilon(c - c_1),$$

for all $c \in [c_1, c(\theta_1) + \Delta]$, and

$$\tilde{T}'(c) = \alpha + \beta(c - c_1) + \varepsilon\Delta - \varepsilon(c - c_1 - \Delta)$$

for all $c \in (c(\theta_1) + \Delta, c(\theta_1) + 2\Delta)$. We have $\tilde{T}'(c) = T'(c)$ elsewhere.

First notice that for $\theta \in (\theta_2, \theta_1)$ we have:

$$\begin{aligned} c(\theta) &= c(\theta_1) + \int_\theta^{\theta_1} c'(t)\, dt \\ p(\theta) &= p(\theta_1) + \int_\theta^{\theta_1} p'(t)\, dt \end{aligned}$$

and

$$\begin{aligned} \tilde{c}(\theta) &= c(\theta_1) + \int_\theta^{\theta_1} \tilde{c}'(t)\, dt \\ \tilde{p}(\theta) &= p(\theta_1) + \int_\theta^{\theta_1} \tilde{p}'(t)\, dt, \end{aligned}$$

where "$\sim$" is used here to specify the variables after the reform.

We can calculate $(c'(\theta), p'(\theta))$ through the following first-order conditions

$$p\left[\varphi'(c) - (1 + T'(c))\,\theta\eta'\left(c + T(c) + \frac{\kappa}{1-p}\right)\right] = 0,$$

and

$$\left[\varphi(c) - \theta\eta\left(c + T(c) + \frac{\kappa}{1-p}\right) - u\right] - \theta p\eta'\left(c + T(c) + \frac{\kappa}{1-p}\right)\frac{\kappa}{(1-p)^2} = 0.$$

The determinant of the system above is

$$\begin{aligned}\det H = p\left[\varphi'(c) - T'(c)\theta\eta' - (1+T'(c))^2\,\theta\eta'\right]\Bigg[&-2\theta\eta'\frac{\kappa}{(1-p)^2} - 2\theta p\eta''\frac{\kappa}{(1-p)^3} \\ &- \theta p\eta'\frac{\kappa^2}{(1-p)^4}\Bigg] + 2p\,(1+T'(c))^2\,\theta\eta''\left[\theta\eta'\frac{\kappa}{(1-p)^2} + 2\theta p\eta'\frac{\kappa}{(1-p)^3}\right] > 0.\end{aligned}$$

Straightforward algebra yields

$$c'(\theta) = \frac{1}{\det H}\left[p\,(1+T'(c))\,\theta\frac{\kappa}{(1-p)^2}\left[\left(-2\eta' - \frac{2p\eta'}{1-p}\right)\eta' + \eta''\eta\right]\right] < 0,$$

since we have made Assumption A, $2(\eta')^2 \geq \eta''\eta$.

We obtain

$$\begin{aligned}p'(\theta) = \frac{1}{\det H}\Bigg[p\left[\varphi'(c) - T'(c)\theta\eta' - (1+T'(c))^2\,\theta\eta'\right]\left(\eta + p\eta'\frac{\kappa}{(1-p)^2}\right) \\ - p^2\,(1+T'(c))^2\,\theta\eta''\eta\Bigg] < 0.\end{aligned}$$

Calculating the difference in the determinant for $c \in (c(\theta_1), c(\theta_1) + \Delta)$ **:**

$$\begin{aligned}\det\tilde{H}(c,p,\theta) &\simeq \det H(c,p,\theta) + p\varepsilon\theta\eta'\left(2\theta\eta'\frac{\kappa}{(1-p)^2} + 2\theta p\eta'\frac{\kappa}{(1-p)^3} + \theta p\eta''\frac{\kappa^2}{(1-p)^4}\right) \\ &: = \det H(c,p,\theta) + \varepsilon G(c,p,\theta),\end{aligned}$$

where

$$G(c,p,\theta) := p\theta\eta'\left(2\theta\eta'\frac{\kappa}{(1-p)^2} + 2\theta p\eta'\frac{\kappa}{(1-p)^3} + \theta p\eta''\frac{\kappa^2}{(1-p)^4}\right).$$

Calculating the difference in the determinant for $c \in (c(\theta_1) + \Delta, c(\theta_1) + 2\Delta)$:

$$\det \tilde{H}(c, p, \theta) \simeq \det H(c, p, \theta) - \varepsilon G(c, p, \theta).$$

Calculating comparative statics for $c \in (c(\theta_1), c(\theta_1) + \Delta)$.

$$\begin{aligned} \tilde{c}'(\theta) &\simeq c'(\theta) - \varepsilon \frac{G(c, p, \theta)}{\det H(c, p, \theta)} c'(\theta) := c'(\theta) + \kappa_c \varepsilon \\ \tilde{p}'(\theta) &\simeq p'(\theta) - \varepsilon \frac{G(c, p, \theta)}{\det H(c, p, \theta)} p'(\theta) - \varepsilon \frac{p\theta\eta' \left(\eta + p\eta' \frac{\kappa}{(1-p)^2}\right)}{\det H(c, p, \theta)} := p'(\theta) + \kappa_p \varepsilon, \end{aligned}$$

where $\kappa_c > 0$ and $\kappa_p \in \mathbb{R}$.
Calculating comparative statics for $c \in (c(\theta_1) + \Delta, c(\theta_1) + 2\Delta)$.

$$\begin{aligned} \tilde{c}'(\theta) &\simeq c'(\theta) - \kappa_c \varepsilon \\ \tilde{p}'(\theta) &\simeq p'(\theta) - \kappa_p \varepsilon \end{aligned}$$

Calculating the $\tilde{\theta} = \theta_1 - \Delta\tilde{\theta}$ **at which the worker consumes** $c(\theta_1) + \Delta$.

$$\tilde{c}(\theta_1) + \Delta \simeq c(\theta_1) - \left(c'\left(\tilde{\theta}\right) - \kappa_c \varepsilon\right) \tilde{\Delta}\theta$$

Hence, ignoring terms of second-order, we have

$$\tilde{\Delta}\theta \simeq \frac{\Delta}{|c'(\theta_1)|}.$$

The Fiscal effect from the Interval $\left(\theta_1 - 2\tilde{\Delta}\theta, \theta_1\right)$.

Notice that by the envelope theorem, we may neglect the effect on the welfare of types in $(\theta_1 - 2\Delta\theta, \theta_1)$. The effect on welfare of the government is of the order of:

$$\begin{aligned} &\frac{\partial W_G(\theta_1, c(\theta_1), p(\theta_1))}{\partial c} \varepsilon \left(\tilde{\Delta}\theta\right)^2 \kappa_c + \frac{\partial W_G(\theta_1, c(\theta_1), p(\theta_1))}{\partial p} \varepsilon \left(\tilde{\Delta}\theta\right)^2 \kappa_p \\ &- \simeq (\Delta)^2 \varepsilon \left[\frac{\partial W_G(\theta_1, c(\theta_1), p(\theta_1))}{\partial c} \frac{\kappa_c}{(c'(\theta_1))^2} + \frac{\partial W_G(\theta_1, c(\theta_1), p(\theta_1))}{\partial p} \frac{\kappa_p}{(c'(\theta_1))^2}\right]. \end{aligned}$$

We remark that

$$W_G(\theta, c(\theta), p(\theta)) = -f(\theta)p(\theta)(T(c(\theta)) - C(u)) + f(\theta)C(u)$$

Hence, the effect is

$$-(\Delta)^2\varepsilon\left[f(\theta_1)p(\theta_1)T'\left(c(\theta_1)\right)\frac{\kappa_c}{\left(c'\left(\theta_1\right)\right)^2}+f(\theta_1)\left(T\left(c(\theta_1)\right)-C\left(u\right)\right)\frac{\kappa_p}{\left(c'\left(\theta_1\right)\right)^2}\right]$$

Calculating the Fiscal Effect from the interval $\left(\underline{\theta},\theta_1-2\tilde{\Delta}\theta\right)\simeq\left(\underline{\theta},\theta_1\right).$
For that we use the fact that

$$\tilde{T}\left(c\left(\theta_1\right)+\Delta\right)=T\left(c\left(\theta_1\right)+\Delta\right)+\varepsilon\left(\Delta\right)^2.$$

This leads to a direct income-effect given by:

$$\int_{\underline{\theta}}^{\theta_1}\varepsilon\left(\Delta\right)^2p(\theta)f(\theta)d\theta.$$

The indirect fiscal effect from types $\left(\underline{\theta},\theta_1\right)$ is given by:

$$\varepsilon\left(\Delta\right)^2\int_{\underline{\theta}}^{\theta_1}\left[\frac{\partial W_G\left(\theta,c\left(\theta\right),p(\theta)\right)}{\partial c}\frac{\partial c}{\partial T}\left(\theta\right)+\frac{\partial W_G\left(\theta,c\left(\theta\right),p(\theta)\right)}{\partial p}\frac{\partial p}{\partial T}\left(\theta\right)\right]d\theta, \tag{F.8}$$

where

$$\frac{\partial c}{\partial T}\left(\theta\right)=-\frac{1}{\det H\left(c,p,\theta\right)}\frac{\kappa}{\left(1-p\right)^2}\theta^2p\left(1+T'(c)\right)\eta''\left(\eta'+2p\eta'\frac{\kappa}{1-p}\right)<0 \tag{F.9}$$

$$\begin{aligned}\frac{\partial p}{\partial T}\left(\theta\right)=\frac{1}{\det H\left(c,p,\theta\right)}\Bigg[&-p\left(1+T'(c)\right)^2\theta\eta''\theta\eta'\\&+\left(p\varphi''(c)-pT''(c)\theta\eta'\right)\left(\theta\eta'+\theta p\eta''\frac{\kappa}{\left(1-p\right)^2}\right)\Bigg]<0.\end{aligned} \tag{F.10}$$

Therefore, its effect on the government's budget is:

$$\varepsilon\left(\Delta\right)^2\int_{\underline{\theta}}^{\theta_1}\left[p(\theta)T'\left(c(\theta)\right)\frac{\partial c(\theta)}{\partial T}+\left(T\left(c(\theta)\right)-C\left(u\right)\right)\frac{\partial p(\theta)}{\partial T}\right]f(\theta)d\theta.$$

Calculating the utility $\left(\underline{\theta},\theta_1-2\tilde{\Delta}\theta\right)\simeq\left(\underline{\theta},\theta_1\right).$
Notice that the impact on the utility on type $\theta<\theta_1$ is of the order of $\theta p(\theta)\eta'\varepsilon\left(\Delta\right)^2$ and

hence the effect adds up to

$$-\lambda\varepsilon\left(\Delta\right)^{2}\int_{\underline{\theta}}^{\theta_1}\theta p\left(\theta\right)\eta'\left(\theta\right)f(\tilde{\theta})d\tilde{\theta}.$$

Finally, observe that the same policy could be made for $\varepsilon<0$. Hence the sum of all effects above must cancel, which implies i) from Proposition 9.

Proof of (ii).

We assume that $p\left(\bar{\theta}\right))>0$ and consider a perturbation in which we add or subtract a lump-sum ε to every employed worker's tax. This will lead to the following direct fiscal effect,

$$+\varepsilon\int_{\underline{\theta}}^{\bar{\theta}}p(\theta)f(\theta)d\theta,$$

the indirect fiscal effect,

$$\varepsilon\int_{\underline{\theta}}^{\bar{\theta}}\left[p(\theta)T'\left(c(\theta)\right)\frac{\partial c(\theta)}{\partial T}+\left(T\left(c(\theta)\right)-C\left(u\right)\right)\frac{\partial p(\theta)}{\partial T}\right]f(\theta)d\theta,$$

and, the following effect from worker's utility,

$$-\varepsilon\lambda\int_{\underline{\theta}}^{\bar{\theta}}\theta p\left(\theta\right)\eta'\left(\theta\right)f(\theta)d\theta.$$

Adding such terms and making the sum equal to zero leads to the desired result.

Proof of (iii).

Using straightforward algebra we get

$$\begin{pmatrix}\frac{\partial c}{\partial u}\left(\theta\right)\\ \frac{\partial p}{\partial u}\left(\theta\right)\end{pmatrix}=\frac{1}{\det H}\begin{pmatrix}p\left(1+T'(c)\right)\theta\eta''\frac{\kappa}{(1-p)^{2}}\\ p\left[\varphi''-T''(c)\theta\eta'-\left(1+T'(c)\right)^{2}\theta\eta''\right]\end{pmatrix}. \tag{F.11}$$

Therefore, we obtain a direct fiscal impact equal to

$$-\int_{\underline{\theta}}^{\theta_1}\left(1-p(\theta)\right)\varphi'\left(c(\theta)\right)^{-1}f(\theta)d\theta,$$

an indirect fiscal impact equal to

$$\int_{\underline{\theta}}^{\bar{\theta}} \left[p(\theta) T'\left(c(\theta)\right) \frac{\partial c(\theta)}{\partial u} + \left(T\left(c(\theta)\right) - C\left(u\right)\right) \frac{\partial p(\theta)}{\partial u} \right] f(\theta) d\theta.$$

Finally, an increase in the unemployment insurance leads to the following welfare from the worker's utility: from worker' s utility

$$\lambda \int_{\underline{\theta}}^{\bar{\theta}} \left(1 - p\left(\theta\right)\right) f(\theta) d\theta.$$

It follows that the sum of these three terms must be zero, which proves the Proposition.

G Sufficiency

Proposition 10. *For a given $\underline{u}$ the functions $(w^\star(\theta), p^\star(\theta), h^\star(\theta))$ satisfying (A.1), (A.2), and (A.3) maximize the planner's program, given $\underline{u}$.*

Proof. The Hamiltonian associated with the planner's program is

$$\mathcal{H} = \Big\{ p(\theta) \Big[N(h(\theta)) - C\left(\frac{w(\theta) - \underline{u}}{p(\theta)} + \theta h(\theta) - \underline{u} \right) \\ - \left(1 - p(\theta)\right) C\left(\underline{u}\right) - \kappa \zeta(p(\theta)) + \psi w(\theta) \Big] \Big\} f(\theta) - f(\theta) \mu(\theta) p(\theta) h(\theta)$$

Let, then,[34]

$$\hat{\mathcal{H}}(w, \mu(\theta), \theta) := \max_{p,h} \Big\{ p \Big[N(h) - C\left(\frac{w - \underline{u}}{p} + \theta h - \underline{u} \right) \\ - \left(1 - p\right) C\left(\underline{u}\right) - \kappa \zeta(p) + \psi w \Big] \Big\} f(\theta) - f(\theta) \mu(\theta) p h \quad \text{(G.1)}$$

To apply Arrow's sufficiency theorem — see Seiestad and Sydsæter (1987, p. 107) —, we have to show that $\hat{\mathcal{H}}(w, \mu(\theta), \theta)$ is concave in w for each θ.

Using the envelope theorem, the derivative of $\hat{\mathcal{H}}$ with respect to w reads

$$\frac{\partial \hat{\mathcal{H}}}{\partial w} = -C' \left(\frac{w - \underline{u}}{p(\theta)} + \theta h(\theta) - \underline{u} \right)$$

[34] We use the fact that $f(\theta) > 0$ for all θ to work with a slighlty modified definition of μ which is more convenient for our purposes.

Define $p(\theta, w)$ and $h(\theta, w)$ as solutions to the following optimality conditions,

$$p(\theta)\left[N'(h(\theta)) - \theta C'\left(u(\theta)\right) - \mu(\theta)\right] = 0, \tag{G.2}$$

and

$$\left[N(h(\theta)) - C\left(u(\theta)\right) + C\left(\underline{u}\right) + C'\left(u(\theta)\right)\frac{w - \underline{u}}{p(\theta)} - \kappa\zeta'(p(\theta))\right] - \mu(\theta)h(\theta) = 0. \tag{G.3}$$

In Lemma 4 we prove that no two pairs (p, h) and (p', h') can satisfy the first order conditions for the same $(w, \mu(\theta), \theta)$. Hence $p(\theta, w)$ and $h(\theta, w)$ solve

Next, define

$$u\left(\theta, w\right) := \frac{w - \underline{u}}{p(\theta, w)} + \theta h(\theta, w) - \underline{u}.$$

Concavity requires that

$$u_w\left(\theta, w\right) = \frac{d}{dw}\left[\frac{w - \underline{u}}{p(\theta, w)} + \theta h(\theta, w) - \underline{u}\right] \geqslant 0.$$

Now, assume towards a contradiction that

$$\frac{d}{dw}\left[\frac{w - \underline{u}}{p(\theta, w)} + \theta h(\theta, w) - \underline{u}\right] < 0.$$

From (G.2) we have

$$\frac{\partial h}{\partial w} = \frac{\theta C''\left(u(\theta)\right)u_w\left(\theta, w\right)}{N''(h(\theta))} > 0.$$

Next, note that

$$H\begin{bmatrix} \frac{\partial h}{\partial w} \\ \frac{\partial p}{\partial w} \end{bmatrix} = \begin{bmatrix} \theta C''\left(u(\theta)\right) \\ -C''\left(u(\theta)\right)\frac{w - \underline{u}}{p(\theta)^2} \end{bmatrix},$$

for

$$H = \begin{bmatrix} p(\theta)\left[N''^2 C''\left(u(\theta)\right)\right] & \theta C''\left(u(\theta)\right)\frac{w-\underline{u}}{p(\theta)} \\ \theta C''\left(u(\theta)\right)\frac{w-\underline{u}}{p(\theta)} & -C''\left(u(\theta)\right)\frac{(w-\underline{u})^2}{p(\theta)^3} - \kappa\zeta''(p(\theta)) \end{bmatrix}.$$

It is easy to verify that the Hessian above is negative definite with a strictly positive determinant. We can recover $\partial h/\partial w$ using

$$\begin{bmatrix} \frac{\partial h}{\partial w} \\ \frac{\partial p}{\partial w} \end{bmatrix} = H^{-1}\begin{bmatrix} \theta C''\left(u(\theta)\right) \\ -C''\left(u(\theta)\right)\frac{w-\underline{u}}{p(\theta)^2} \end{bmatrix}.$$

Since

$$H^{-1}=\frac{1}{\det H}\left[\begin{array}{cc} -C''\left(u(\theta)\right)\frac{(w-\underline{u})^2}{p(\theta)^3}-\kappa\zeta''(p(\theta)) & -\theta C''\left(u(\theta)\right)\frac{w-\underline{u}}{p(\theta)} \\ -\theta C''\left(u(\theta)\right)\frac{w-\underline{u}}{p(\theta)} & p(\theta)\left[N''^2 C''\left(u(\theta)\right)\right] \end{array}\right],$$

we have

$$\begin{aligned}\frac{\partial h}{\partial w} &= \left(-C''\left(u(\theta)\right)\frac{(w-\underline{u})^2}{p(\theta)^3}-\kappa\zeta''(p(\theta))\right)\theta C''\left(u(\theta)\right) \\ &\qquad + \theta C''\left(u(\theta)\right)\frac{w-\underline{u}}{p(\theta)}C''\left(u(\theta)\right)\frac{w-\underline{u}}{p(\theta)^2} \\ &\qquad = -\kappa\zeta''(p(\theta))\theta C''\left(u(\theta)\right)<0.\end{aligned}$$

A contradiction.

We conclude that, for each unemployment insurance level, the problem admits at most one C^2 solution. □

Lemma 4. *For any w, there is only one solution to the first-order condition.*

Proof. Suppose towards a contradiction that there are two solutions: $(h^2(\theta), p^2(\theta))$ and $(h^1(\theta), p^1(\theta))$. Assume w.l.o.g. that $h^2(\theta) > h^1(\theta)$ then, from

$$N'(h(\theta))-\theta C'\left(u(\theta)\right)-\mu(\theta)=0$$

we must have $p_2(\theta) > p_1(\theta)$. Moreover,

$$\begin{aligned}&\frac{\partial}{\partial p}\left[N'(h(\theta))-\theta C'\left(\frac{w-\underline{u}}{p(\theta)}+\theta h(\theta)-\underline{u}\right)-\mu(\theta)\right] \\ &\qquad = \frac{w-\underline{u}}{p(\theta)^2}\theta C''\left(\frac{w-\underline{u}}{p(\theta)}+\theta h(\theta)-\underline{u}\right)>0\end{aligned}$$

and

$$\begin{aligned}&\frac{\partial}{\partial h}\left[N'(h(\theta))-\theta C'\left(\frac{w-\underline{u}}{p(\theta)}+\theta h(\theta)-\underline{u}\right)-\mu(\theta)\right] \\ &\qquad = N''(h(\theta))-\theta^2 C'\left(\frac{w-\underline{u}}{p(\theta)}+\theta h(\theta)-\underline{u}\right)\end{aligned}$$

allow us to define

$$p'(h)=\frac{-N''(h(\theta))+\theta^2 C''\left(\frac{w-\underline{u}}{p(\theta)}+\theta h(\theta)-\underline{u}\right)}{\frac{w-\underline{u}}{p^2(\theta)}\theta C''\left(\frac{w-\underline{u}}{p(\theta)}+\theta h(\theta)-\underline{u}\right)}$$

and guarantees that there exists a strictly increasing C^1 function $p : [h_1(\theta), h_2(\theta)] \rightarrow [p_1(\theta), p_2(\theta)]$ with $p(h_1(\theta)) = p(\theta_1)$ and $p(h_2(\theta)) = p(\theta_2)$ such that

$$N'(h) - \theta C'\left(\frac{w-\underline{u}}{p(h)} + \theta h - \underline{u}\right) - \mu(\theta) = 0$$

for every $h \in [h_1(\theta), h_2(\theta)]$.

Notice also that

$$\begin{aligned} 0 &= N(h_2(\theta)) - C(u_2(\theta)) + C(\underline{u}) + C'(u_2(\theta))\frac{w-\underline{u}}{p_2(\theta)} - \kappa\zeta'(p_2(\theta)) - \mu(\theta)h_2(\theta) \\ &= N(h_1(\theta)) - C(u_1(\theta)) + C(\underline{u}) + C'(u_1(\theta))\frac{w-\underline{u}}{p_1(\theta)} - \kappa\zeta'(p_1(\theta)) - \mu(\theta)h_1(\theta). \end{aligned}$$

However, for every $h \in [h_1(\theta), h_2(\theta)]$ we have

$$\begin{aligned} &\frac{d}{dh}\left[N(h) - C(p(h), h) + C(\underline{u}) + C'(p(h), h)\frac{w-\underline{u}}{p(h)} - \kappa\zeta'(p(h)) - \mu(\theta)h\right] \\ &= C''(p(h), h)\,\theta\frac{w-\underline{u}}{p(h)} - \left[C''(u(\theta))\frac{(w-\underline{u})^2}{p(\theta)^3}\right. \\ &\left.+\kappa\zeta''(p(\theta))\right]\frac{-N''(h(\theta)) + \theta^2 C''\left(\frac{w-\underline{u}}{p(\theta)} + \theta h(\theta) - \underline{u}\right)}{\frac{w-\underline{u}}{p^2(\theta)}\theta C''\left(\frac{w-\underline{u}}{p(\theta)} + \theta h(\theta) - \underline{u}\right)} \\ &= \frac{-C''(u(\theta))\frac{(w-\underline{u})^2}{p(\theta)^3} - N''(h(\theta)) + \theta^2 C''\left(\frac{w-\underline{u}}{p(\theta)} + \theta h(\theta) - \underline{u}\right)}{\frac{w-\underline{u}}{p^2(\theta)}\theta C'\left(\frac{w-\underline{u}}{p(\theta)} + \theta h(\theta) - \underline{u}\right)} - \\ &\frac{\kappa\zeta''(p(\theta))\left[-N''(h(\theta)) + \theta^2 C''\left(\frac{w-\underline{u}}{p(\theta)} + \theta h(\theta) - \underline{u}\right)\right]}{\frac{w-\underline{u}}{p^2(\theta)}\theta C'\left(\frac{w-\underline{u}}{p(\theta)} + \theta h(\theta) - \underline{u}\right)} < 0. \end{aligned}$$

A contradiction. $\square$

H Auxiliary Results

Proposition 11. *Any constraint efficient allocation can be implemented whenever the government observes any one of the following pairs: (i) (c, z), (ii) (z, p), or (iii) (c, p).*

Proof of Proposition 11: (i) The government observes (c, z). For $z \in \left[\eta^{-1}\left(h^*(\bar{\theta})\right), \eta^{-1}\left(h^*(\underline{\theta})\right)\right]$ let $\theta(z)$ solve $\eta^{-1}(h^*(\theta)) = z$. For $z \notin \left[\eta^{-1}\left(h^*(\bar{\theta})\right), \eta^{-1}\left(h^*(\underline{\theta})\right)\right]$ the firm pays a tax $\mathcal{T}(c, z) = \infty$. Recall that because $h(\cdot)$ is monotonic, $\theta(z)$ is uniquely defined. Implicitly

define $T(z)$ by

$$\lambda\left(p^*(\theta)\right)p^*(\theta)\left[z-c\left(\theta\right)-T\left(z\right)\right]=\kappa. \tag{H.1}$$

For each $z\in\left[\eta^{-1}\left(h^*(\bar{\theta})\right),\eta^{-1}\left(h^*(\underline{\theta})\right)\right]$, the associated tax is defined by

$$\mathcal{T}\left(c,z\right)=\begin{cases} T\left(z\right) & \text{if} \quad c=c(\theta) \\ \infty, & \text{otherwise.}\end{cases}$$

Clearly, we can restrict attention to contracts $(c,z)\in\left\{\left(c^*\left(\theta\right),\eta^{-1}\left(h^*(\theta)\right)\right\}_{\theta\in\Theta}\right.$ (otherwise the firm pays $\mathcal{T}(c,z)=\infty$). But then the free-entry condition uniquely adjusts the probability $p(c,z)$ to satisfy (H.1). This proves the claim.

(ii) The government observes (z,p). Let $\theta(z,p)$ solve $\eta^{-1}\left(h^*(\theta)\right)=z$ and $p^*(\theta)=p$ (if we cannot find this type set $\mathcal{T}(z,p)=\infty$). Define $T(z,p)$ by

$$\lambda\left(p^*(\theta)\right)p^*(\theta)\left[z-c^*\left(\theta\right)-T\left(z,p\right)\right]=\kappa.$$

$$\mathcal{T}\left(z,p\right)=\begin{cases} T\left(z,p\right) & \text{if} \quad (z,p)\in\{(\eta^{-1}\left(h^*(\theta)\right),p^*(\theta)\}_{\theta\in\Theta} \\ & \infty \quad \text{otherwise.}\end{cases}$$

Notice that each contract $(c,z)\in\left\{\left(c^*\left(\theta\right),\eta^{-1}\left(h^*(\theta)\right)\right\}_{\theta\in\Theta}\right.$ exactly leads to the probability $p^*(\theta)$ and, hence, to zero profits. Now suppose that a firm offers a contract $(c,z)\notin\left\{\left(c^*\left(\theta\right),\eta\left(h^*(\theta)\right)\right\}_{\theta\in\Theta}\right.$. Let $\theta(z)$ solve $\eta^{-1}\left(h^*(\theta\left(z\right))\right)=z$.

If there is no $p\in(0,1)$ that makes this contract attractive to a type, then $p=0$, and, hence, the firm obtains negative profits. Hence, assume that there exists a type $\tilde{\theta}$ and $\tilde{p}\in(0,1)$ that is attracted to this contract. Let $\tilde{\theta}(c,z)$ be the smallest type among all the types that are attracted to this contract and would accept it with smallest probability. The queue of this contract is defined so that its probability satisfies

$$p(c,z)\left[u\left(c^*\left(\theta\right)\right)-\tilde{\theta}(c,z)\eta^{-1}\left(z\right)-\underline{u}\right]=w^*\left(\tilde{\theta}(c,z)\right)-\underline{u}.$$

Of course, $p(c,z)\neq p^*\left(\theta\left(z\right)\right)$, otherwise $(c,z)=\left(c^*\left(\theta\left(z\right)\right),\eta^{-1}\left(h^*(\theta\left(z\right))\right)\right)$ would be true. Hence, this contract would lead to a profit equal to $-\infty$.

(iii) The government observes (c,p). For each $(c,p)\in\left\{\left(c^*\left(\theta\right),p^*(\theta)\right\}_{\theta\in\Theta}\right.$, let $\theta(p)$ solve $p^*\left(\theta(p)\right)=p$. Define $T(c,p)$ by

$$\lambda\left(p^*(\theta(p))\right)p^*(\theta(p))\left[\eta\left(h^*\left(\theta(p)\right)\right)-c^*\left(\theta(p)\right)-T\left(c,p\right)\right]=\kappa.$$

Define $\mathcal{T}(c,p)$ by

$$\mathcal{T}\left(c,p\right)=\begin{cases} T\left(c,p\right) & \text{if} \quad (c,p)\in\{(c^*(\theta),p^*(\theta)\}_{\theta\in\Theta} \\ & \infty, \quad \text{otherwise.}\end{cases}$$

Every contract $\left(c^*\big(\tilde{\theta}\big), \eta^{-1}\big(h^*(\tilde{\theta})\big)\right) \in \left\{\left(c^*\big(\theta\big), \eta^{-1}\big(h^*(\theta)\big)\right\}_{\theta\in\Theta}$ clearly leads to the probability $p^*(\tilde{\theta})$ and, hence, to zero profits. Assume that the firm offers a contract $(c, z) \notin \left\{\left(c^*(\theta), \eta^{-1}(h^*(\theta))\right)\right\}_{\theta\in\Theta}$, and let $p(c, z)$ be the probability that a worker finds a job according to this contract. If $c = c^*(\theta(p(c, z)))$, then, by the worker's indifference condition, we must have $z = \eta^{-1}(h^*(\theta(p(c, z))))$ and, hence, $(c, z) \in \left\{\left(c^*(\theta), \eta^{-1}(h^*(\theta))\right)\right\}_{\theta\in\Theta}$. Otherwise, if $c \neq c^*(\theta(p(c, z)))$, then this contract leads to a profit equal to $-\infty$. $\square$

The statement of Proposition 11 uses c instead of y. Because consumption is y minus labor income taxes, which is observed by the government, the two are equivalent.

Proposition 12. *If a constrained efficient allocation is such that consumption is decreasing in θ then it can be implemented by a tax system such that, for all θ, $\theta\chi'(u(\theta)) = N'(h(\theta))$.*

Proof of Proposition 12 Consider an incentive-feasible allocation such that $u(\theta)$ is monotonic. In this case, define the tax $\mathscr{T}(u)$ through

$$\mathscr{T}'(u) = 1 - \frac{\theta C'(u)}{N'(h(u))}.$$

It remains to show that we can set taxes on firms so that firms obtain zero profits. We define

$$\tilde{\Theta}(u, h) := \{\theta : [u - \theta h - \underline{u}] > w(\theta)\}.$$

If $\tilde{\Theta}(u, h) \neq \emptyset$, define

$$p(h, u) := \min\left\{p : p[u - \theta h - \underline{u}] = w(\theta) \text{ for some } \theta \in \tilde{\Theta}(u, h)\right\}.$$

With this notation in hand, define taxes on firms, $T^f(h, u)$, through

$$\lambda(p(h, u))\, p(h, u)\left[h - u - \mathscr{T}(u) - T^f(h, u)\right] = \kappa,$$

if $\tilde{\Theta}(u, h) \neq \emptyset$, $T^f(h, u) := 0$ if $\tilde{\Theta}(u, h) = \emptyset$ and notice that this offer would never attract any worker. $\square$