Inflation Targeting under Fiscal Fragility

Aloísio Araujo[1,2], Rafael Santos[1], Paulo de Carvalho Lins[3], Serge de Valk[1*]

[1] EPGE/FGV, [2] IMPA, [3] University of Rochester

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Abstract

Indebted policymakers have a limited budget and are subject to inflationary shocks forcing them to either (i) increase interest rate to have inflation on the pre-announced target or (ii) accept higher inflation. We model the inter-temporal trade-off between fiscal and monetary policy when forward-looking, rational, and fully informed agents finance public deficits. We show that a high public-debt level opens the doors to adverse expectations, pressuring nominal interest rates and leading to target-coordination failure. Our parsimonious model with a single confidence shock supports the policy actions observed in the aftermath of the 2002 Brazilian crisis characterised by inflation expectations overshooting the target. First, a higher target level to restore coordination. Second, both a gradual reduction of debt-to-GDP, to improve fiscal strength, and an increase of the share of the public debt with pre-fixed interest rates as opposed to indexed debt. Finally, we find empirical evidence of higher debt levels and lower inflation targets increasing both the probability to overshoot the inflation target and increasing the size of the deviation from the target.

Keywords: Monetary Policy, Fiscal Policy, Debt Policy
JEL - Classification: E52, E62, E63

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1 Introduction

The inflation targeting regime has been supporting central banks in their task of coordinating market expectations towards price stability. However, occasionally, inflation expectations may lose their anchor and diverge from the central bank's target. As an example, in 2002 and 2003, Brazilian policymakers faced inflationary pressures when it became clear that the presidential candidate, Lula da Silva, would win. The perception was that Lula's victory could come with the implementation of a complete new policy-framework. Inflation expectations overshoot the target's upper-bounds at all horizons relevant to the central bank, something that we interpret as a target confidence crisis. In this paper, we focus on how fiscal fragility, marked by high public-debt service, limited tax room, and confidence crises, poses challenges to the inflation target regime.

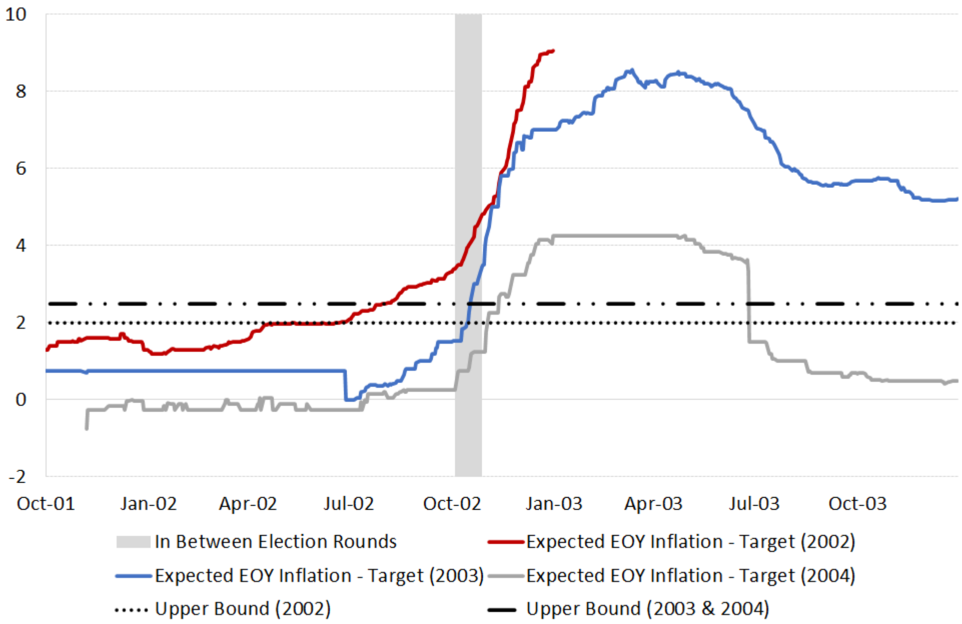

Figure 1: Expectation Shock

In response to rising inflation expectations, the outgoing and new administrations took several steps: i) in order to coordinate inflation expectations in the short run, they increased the target for 2003 in an extra meeting held on 6/27/2002 and unofficially again on 1/21/2003. ii) In 2003, responsible macroeconomic policies were sustained by public debt reduction, and iii) changes in the debt mix away from indexed bonds. Ultimately inflation expectations converged back to the target.

We model a fiscal and monetary policymaker that may lose credibility and overshoot its target when it becomes fiscally challenging to maintain spending. The framework closely follows Cole & Kehoe (1996, 2000), but is cast in a monetary policy setting instead of focusing on international debt crises. In our model, we consider a closed economy with two types of agents that form rational expectations with complete information: an altruistic policymaker and private-agents. The policymaker acts jointly as a fiscal authority and as a central bank that targets inflation. Given it is imperfectly committed, it might decide to deviate from the pre-announced inflation target. Such decision is the result of a trade-off between inflating public debt away – to make fiscal room for spending – and keeping inflation on target – to avoid the economic costs caused by deviating. We find that target failures may happen within the fiscal fragility zone – marked by high debt and limited fiscal room for public expenditures. In the said zone, monetary and fiscal authorities are restricted by private-agents rational expectations and exposed to the loss of confidence in their commitment to the target. The result is self-fulfilling target failures.

The policy decisions made in Brazil in 2002-2003 closely mirror the policy prescriptions of our model. We find that increasing the inflation target can help coordinate expectations away from the fiscal fragility zone in the short run. In the longer run, running down debt and changing the debt mix towards pre-fixed interest rate bonds help sustain the credibility of the central bank.

The fiscal limits on possible monetary policy achievements and the interdependence between fiscal discipline and price stability is a similar message as in Sargent & Wallace (1981), Leeper (1991), Sims (1994), Woodford (1995), Leeper & Leith (2016), and Cochrane (2017). Kwon *et al.* (2008) have found empirical evidence of inflation and debt growth being more positively correlated in countries with already high debt levels.

The importance of the target in helping coordinate expectations as been discussed in Araujo *et al.* (2016). They argue that a higher target can be beneficial when modelling the central bank as imperfectly committed, as opposed to a perfect commitment framework.

Finally, Fischer (1983) argues that indexation increases the impact on the price level of any inflationary shock and found that the inflationary impact of the 1974 oil shock was significantly stronger in countries that had adopted bond indexation.

To the best of our knowledge, the literature on inflation targeting and confidence crises is relatively thin, although very relevant to countries facing structural fragilities. In canonical NK-DSGE models, such as Christiano *et al.* (2005) and Smets & Wouters (2007), central banks have an important role as they help reduce macroeconomic volatility. However, central banks are always committed to the inflation target through a Taylor type of monetary policy rule and fiscal authorities are always responsible and never default. Recently, Arellano *et al.* (2019) have proposed to enrich such models by building on the structure of a NK-DSGE while allowing for sovereign default. Nevertheless, the central bank remains committed to the target within their framework. We hope to contribute to the literature by modeling an inflation targeting central bank that can be exposed to inflation target confidence crises.

Our model is anchored in the literature at the intersection of multiple equilibria with fiscal and monetary policy. We closely follow models used in the literature on confidence crises in debt markets as in Cole & Kehoe (1996, 2000) and Calvo (1988). Although we have chosen perfect information, their exist a large literature on the possibility of multiple equilibria in imperfect information settings such as Morris & Shin (2002) and Angeletos & Werning (2006). Still in an imperfect information framework, Araujo *et al.* (2016) model a trade-off between low and credible targets and find that imperfect commitment shifts the optimal inflation target upwards compared with a perfect commitment setting. A result quite similar to ours.

In section 2 we set out the model and derive the recursive form defining the equilibrium. Moving to section 3, we specify functional forms and parameter values in a quantitative analysis in order to match the situation in Brazil in 2002. We then move to analyze the results from our model. In section 4, we analyze the 2002 confidence crisis in Brazil and the subsequent policy responses. Finally, the last section presents concluding remarks.

2 Model

We consider a closed economy with two types of agents living infinite periods and forming rational expectations with complete information: a policymaker and private-agents. The policymaker acts as a fiscal and monetary authority, choosing current inflation and one-period debt to finance itself. It may choose to deviate from the target. The private agents receive a stream of fixed endowments and choose how much debt to hold and form expectations about next-period inflation taking into account an exogenous announced inflation target and the current debt level. When multiple equilibria happens, a sunspot variable determines the equilibrium.

2.1 Basic Setup

Private-agents

We assume a continuum of infinitely lived private-agents that choose consumption and savings to maximize their expected utility:

$$\max_{c_t, d_{t+1}} \; \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \tag{1}$$

where c_t is the private-agents' consumption in period t, g_t is government spending on public goods, and β is the inter-temporal discount rate. The usual non-negative consumption restriction applies. We assume that $0 < \beta < 1$ and that $u()$ is continuously differentiable, concave in g and linear in c, monotonically increasing, and satisfying the condition $\lim_{g \to 0^+} u(g) = +\infty$.

Each period, private agents receive a deterministic endowment e, which is taxed at a constant rate τ by the government, and receive payments on their bond holdings. The private-agents' budget constraint is:

$$c_t + d_{t+1} \leq (1 + r_t)d_t + \alpha_t(1 - \tau)e \tag{2}$$

where d_{t+1} are one-period bonds bought in t and d_t are the previous period bond holdings paying the interest rate $(1+r_t)$. The productivity factor α_t should be understood as a reduced form capturing the cost of deviating from the inflation target on economic activity. It will be properly defined further, but in the quantitative part the penalty function $\alpha(.)$ will be a function of its previous value α_{-1}, the inflation target π^a, and current inflation π. As usual a no-Ponzi condition is imposed on debt holdings. Given the linear utility in c, we can define the ex-post real interest rate as:

$$r_t = \frac{1 + \pi_t^e}{1 + \pi_t} \frac{1}{\beta} - 1 \tag{3}$$

where $\pi_t^e = \mathbb{E}(\pi_t | t - 1)$ is the expected inflation for period t that private agents formed in period $t - 1$ and where π_t is the current period inflation. When $\pi_t = \pi_t^e$, the ex-post real interest rate is equal to the inverse of the inter-temporal discount rate, $1/\beta$, minus one. An inflationary surprise, defined by $\pi_t > \pi_t^e$, reduces the ex-post real interest rate and consequently the payments the policymaker should make on its debt. Such a partial default on debt gives additional fiscal room for government spending.

Policymaker

We assume a benevolent policymaker that chooses both fiscal and monetary policies to maximize private agents' utility. As a monetary authority, the policymaker chooses the inflation rate π_t and, as a fiscal authority, next period's debt D_{t+1}:

$$\max_{\pi_t, D_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \tag{4}$$

subject to the budget constraint:

$$g_t + (1 + r_t)D_t \leq D_{t+1} + \alpha_t \tau e \tag{5}$$

where D_t is last period's debt. Each period, non-negative spending g_t and the repayments on the previous period obligations are financed through taxes τe and the issuance of new debt D_{t+1}. If the monetary authority deviates from inflation target π_a, the economy suffers a permanent negative shock to the endowments through α_t such that a deviation from the target, $\pi_t > \pi_a$, implies $0 < \alpha_t < 1$ and $\frac{\partial \alpha_t}{\partial \pi_t} < 0$. The first derivative implies that the higher the deviations the higher the cost to the economy. The penalty assumption for partial default on domestic currency debt resembles the Cole & Kehoe (2000) fixed penalty in the case of default on external debt.

In each period, the policymaker can satisfy the budget constraint by: i) adjusting expenditures, ii) issuing new debt D_{t+1}; iii) partially defaulting on debt through an inflationary surprise ($\pi_t > \pi_t^e$) and rolling-over the remaining debt. Rational expectations govern the strategic interactions between the policymaker and private agents. As in Cole & Kehoe (2000) self fulfilling multiple equilibria may occur. For some debt levels the best response from the policymaker's perspective depends on the expectations of private agents. If they expect a deviation from the target, the best response will be to deviate. We consider an exogenous sunspot variable ζ_t to determine the selection of the equilibrium.

We assume that private-agents behave competitively in making their choice of d_{t+1} and that they do not think that their individual actions affect next-period's aggregate state. We therefore distinguish between the individual decision d_{t+1} and the aggregate value D_{t+1}. In equilibrium, because all consumers are identical and start with $d_0 = D_0$, d_{t+1} matches D_{t+1} for any t, making debt markets clear.

Timing

At the beginning of each period, the policymaker implements the actual inflation rate and chooses how much debt to sell. Thereafter, private agents form their expectations about next-period's inflation and decide how much debt to hold. Finally, next-period's uncertainty is resolved through the realization of the sunspot variable.

1^{st}: Policymaker chooses actual inflation, π_t;

2^{nd}: Policymaker chooses next debt-level, D_{t+1};

3^{rd}: Private-agents form their next-period inflation expectation, π_{t+1}^e, and choose the amount of next-period debt, d_{t+1}, to hold;

4^{th}: The sunspot variable, ζ_{t+1}, is realized.

Discretionary Inflation

In each period, the policymaker may choose to deviate from the inflation target π^a. Private agents understand this when forming their expectations π_t^e. The discretionary inflation chosen by the policymaker is the result of a trade-off between potential short-term gains against long-term loses due to the costs of deviating from the inflation target. Let π^D be the endogenous level of discretionary inflation chosen in T[1] and subject to the No-Ponzi condition, $D_t \leq D$,

[1] We will furthermore impose $\pi^D \geq 0$ since deflation is not the focus of this paper.

and the policymaker's loss of credibility after choosing to deviate captured by the private agent's expectations $\pi_t^e = \pi^D \ \forall t > T$. The discretionary inflation problem can then be written as:

$$\pi^D = \underset{\pi}{\text{argmax}} \quad \underbrace{u(c_T, g_T)}_{\text{possibly increasing in } \pi} \quad + \underbrace{\frac{\beta}{1-\beta} u(c, g)}_{\text{decreasing in } \pi}$$

subject to

$$\begin{aligned} g_T &= D\left(1 - \frac{1+\pi_T^e}{1+\pi}\frac{1}{\beta}\right) + \alpha(\pi)\tau e \\ g &= D\left(1 - \frac{1}{\beta}\right) + \alpha(\pi)\tau e \\ c_T &= \left(\frac{1+\pi_T^e}{1+\pi}\frac{1}{\beta} - 1\right) D + \alpha(\pi)(1-\tau)e \\ c &= \left(\frac{1}{\beta} - 1\right) D + \alpha(\pi)(1-\tau)e \end{aligned} \tag{6}$$

Given rational expectations, in equilibrium, π^D is optimal given π^e and vice versa. We later numerically solve this problem by writing it as as a fixed point.

2.2 Recursive Equilibrium

We define a recursive equilibrium where the policymaker and private-agents choose their actions sequentially. At the beginning of each period, the aggregate state $s = (D, \pi^e, \zeta, \alpha_{-1})$ is public since the aggregate debt D, the expected inflation π^e for the current period, the realization of the sunspot variable ζ, and the past penalty α_{-1} have all been determined in the previous period. The policy choices (π, D'), the expected inflation $(\pi^{e\prime})$ for next period, the individual debt holdings d' for next period, jointly with s, determine the equilibrium. We denote by $\pi(.)$ and $D(.)$ the inflation and debt policy functions, and by $\pi^e(.)$ the inflation expectation-function, all yet to be defined.

To define a recursive equilibrium, we work the timing of actions in each period backward. We start the definition of a recursive equilibrium with private-agents as they move last. When forming expectation $\pi^{e\prime}$ at the end of any period, private agents know all the parameters of the economy, his individual public debt holding d, the aggregate state s, the policymaker's offering of new debt D', the current period inflation π, and the policymaker´s optimal policy functions. The private agent's value function is defined by the following functional equation:

$$
\begin{aligned}
V^{pa}(s,d,\pi,D') = &\max_{c,d'} u(c,g) + \beta \mathbb{E} V^{pa}(s',d',\pi',D'') \\
&\text{subject to} \\
&\qquad c + d' \leq \frac{1+\pi^e}{1+\pi}\frac{1}{\beta} d + \alpha(\pi,\pi^a,\alpha_{-1})(1-\tau)e \\
&\qquad s' = \Big(D', \pi^e(s,d,\pi,D'), \alpha(\pi,\pi^a,\alpha_{-1}), \zeta'\Big) \\
&\qquad \pi' = \pi(s') \\
&\qquad D'' = D(s') \\
&\qquad c \geq 0 \\
&\qquad d' \geq -A
\end{aligned}
\tag{7}
$$

in which $A > 0$, rules out Ponzi schemes in favor of private-agents but does not bind in equilibrium for any positive initial debt condition. The penalty function $\alpha(.)$ is a function of its previous value α_{-1}, the inflation target π^a, and current inflation π.

Each period, after the policymaker decided on how much debt it offered D' and defined the period's inflationπ, private agents decide on how much debt to hold. Let $d'(s,d,\pi,D')$ their debt policy function. When forming inflation expectations, private agents determine the nominal interest rate for the next period. In the absence of multiple equilibria, π is perfectly anticipated and the real return is always $1/\beta$. However, when multiple equilibria are possible, private-agents do not know the realization of the sunspot variable when forming expectations and what the policymaker will opt to do.

When forming inflation expectations private agents look at what what the policymaker could do next period. Their expectations can be written as $\pi^e(s,d,\pi,D') = \mathbb{E}\pi(s')$. When forming expectations, the set $(D', \pi^{e\prime}, \alpha) \in s'$ is known to private agents. Hence, the only unknown variable on which private-agents form their expectations is the realization of the sunspot variable ζ'. Integrating out the sunspot variable, whose distribution is commonly known, we obtain:

$$
\mathbb{E}\pi(s') = \begin{cases} f \times \pi^D(D', \pi^{e\prime}, \alpha) + (1-f) \times \pi^a & \text{if multiple eq.} \\ \pi^D(D', \pi^{e\prime}, \alpha) & \text{if deviating unique eq.} \\ \pi^a & \text{if not deviating unique eq.} \end{cases}
\tag{8}
$$

where f is the exogenous probability of the adverse equilibrium occurring.

The policymaker chooses, at the beginning of the period, inflation π and debt issuance D' given the state s. The policymaker knows that next period's debt level affects the private-agents' inflation expectations and will resolve the following problem:

$$V^p(s) = \max_{\pi, D'} u\Big(c(s,d,\pi,D'), g\Big) + \beta \mathbb{E} V^p(s')$$

subject to

$$g + \frac{1+\pi^e}{1+\pi}\frac{1}{\beta} D \leq D' + \alpha(\pi, \pi^a, \alpha_{-1})\tau e \tag{9}$$

$$s' = (D', \pi^e(s,d,\pi,D'), \alpha(\pi, \pi^a, \alpha_{-1}), \zeta')$$

$$g \geq 0$$

We can now define a recursive equilibrium for our model economy. An equilibrium is a list of value functions V for the representative private-agent, V^{pa}, and for the representative policymaker, V^p; functions $c()$ and $d'()$ for the private-agents' consumption and saving decisions; functions $\pi()$ and $D'()$ for the policymaker's inflation and debt decisions; an inflation expectation function $\pi^e()$; and an equation of motion for the aggregate debt level D' such that the following holds:

- Given D' and π, V^{pa} is the value function for the solution to the representative private-agent's problem, and c, d' and $\pi^{e\prime}$ are the maximizing choices when $d' = D'$;
- Given π^e, V^p is the value function for the solution to the policymaker's problem, and both D' and π are the maximizing choices;
- $D'(s)$ equals $d'(s,d,\pi,D')$.

Our definition of an equilibrium is similar to Cole & Kehoe (1996) and Cole & Kehoe (2000) and is restricted to a Markov equilibrium. Hence, the agents' future conditional plans can be derived from their policy functions.

2.3 The Fiscal Fragility Zone

In this section, we will show that the ability of the policymaker to effectively target inflation is restricted by debt levels. Assuming inflation has always been on target, then 3 different scenarios can be drawn[2]. When conditioning the policymaker's value function on private-agents' expectations and on its own choice to deliver or not the target, we define the following 3 scenarios:

1. $V^p(D|\pi = \pi^a, \pi^e = \pi^D) \geq V^p(D|\pi = \pi^D, \pi^e = \pi^D) \rightarrow \pi^e = \pi = \pi^a$

2. $\pi \in \{\pi^a, \pi^D\}$ depends on the sunspot

[2] Given the simplifying assumptions we made, a past deviation implies a loss of credibility in all subsequent periods.

3. $V^p(D|\pi = \pi^a, \pi^e = \pi^a) \leq V^p(D|\pi = \pi^D, \pi^e = \pi^a) \rightarrow \pi^e = \pi = \pi^D$

In the first case, the policymaker finds it better to always keep inflation on target, even when private agents think it will not. Consequently, there is only one equilibrium possible where private-agents have faith in the policymaker delivering on-target inflation. Conditional on (π^e, α_{-1}) and given that the sunspot ζ is simply disregarded, as only one equilibrium is possible, the only important variable defining the policymaker´s value function V^p is the debt level D. The same holds for the third case when the only equilibrium is the policymaker always deviating from the inflation target.

The more interesting scenario is when multiple equilibria, akin to self-fulfilling target failures, are possible. If private-agents believe the target will be delivered, then the policymaker will indeed prefer to do so. On the contrary, in the face of adverse expectations the policymaker chooses to deviate. The interval of debt $\left(\underline{\mathrm{D}}\ \overline{\mathrm{D}}\right)$, for which there exist multiple equilibria, is what we call the fiscal fragility zone. In this zone, private agents have doubts about the commitment of the monetary authority to the target. For debt levels under $\underline{\mathrm{D}}$ the target is perfectly supported.

2.4 Inflation-Indexed Debt

It is not unusual for governments to issue inflation indexed bonds. We will here look at the implications of changing the nature of the bonds. To achieve such indexed bonds within the framework of our model, we change the actions' timing in order to give private agents all the needed information to perfectly anticipate the policymaker's decisions. By allowing private agents to know the realization of the sunspot variable when forming their inflation expectations, bonds will simply pay real interest rate $\frac{1}{\beta}$ in all states of nature.

1^{st}: Policymaker chooses actual inflation, π_t;

2^{nd}: Policymaker chooses next debt-level, D_{t+1};

3^{rd}: Next-period sunspot variable, ζ_{t+1}, is realized.

4^{th}: Private-agents form next-period inflation expectation, π^e_{t+1}, and choose the amount of next-period debt d_{t+1} to hold;

With this new timing, private agents' information sets are given by $(s, d, \pi, D', \zeta') = s'$. Inflation expectations π^e, given information set s', will be such that $\pi^e(s') = \pi(s')$ which is the policymaker's next period inflation choice. As the policymaker's choices are anticipated, it is no longer possible to transfer resources from the private-agents to itself in the event of a bad sunspot.

3 Quantitative Analysis

This section presents a numerical example whose parameters have been chosen to match the situation of Brazil in late 2002. In a later section we use the model to help us interpret events from that period. We show that the crisis zone for our stylized model is fairly large, and we characterize optimal government policy when target failures may happen under adverse expectation conditions.

3.1 Functional Forms

We first start by defining the functional forms of the private agent's utility function and the penalty function for deviating from the inflation target. The private agent's utility is a weighted average of a linear consumption and logarithmic government spending term, quite similar to what can be found in Cole & Kehoe (2000). The weights are defined by the parameter $\rho \in (0, 1)$ that can be interpreted as some relative preference for consumption:

$$u(c_t, g_t) = \rho c_t + (1 - \rho)log(g_t) \tag{10}$$

The penalty function for deviating is continuous in order to mirror observations made in the literature on the impacts of inflation on GDP. More specifically, Barro (1995) and Sarel (1996) found negative impact of high inflation levels on economic growth and Boyd *et al.* (2001) on the financial sector. We furthermore define a limit for the penalty, captured by the parameter κ, so that the resources of the economy will be larger than $(1 - \kappa)e$ as $\lim_{\pi \to \infty} \alpha(\pi, .) = (1 - \kappa)$. The penalty α_t is defined as:

$$\alpha_t = \begin{cases} 1 & \text{if } \alpha_{t-1} = 1, \pi_t = \pi_a \\ (1 - \kappa) + \kappa e^{-(c_0 + c_1(\pi_t - \pi_a)^2)} & \text{if } \alpha_{t-1} = 1, \pi_t \neq \pi_a \\ \alpha_{t-1} & \text{otherwise} \end{cases} \tag{11}$$

Given the perpetuity assumption we made, the penalty is conditional on its previous value. Once the policymaker deviates the endowments will suffer a reduction in all future periods.

3.2 Calibration

Our model is calibrated on yearly data in order to match the usual time-frame targeted by central banks. For the discount factor, we use the historical average of the ex-post real interest rate for the period 1996-2019 [3]. The 2002 general government revenue over GDP is used as a proxy for the imposition rate on the economy's endowments. The exogenous crisis

[3]Using inflation indexed bonds, such as NTN-C or NTN-B, around 2002 would give similar results.

probability is calibrated on the country risk, captured by the EMBI + Brazil, around election time. The endowments e were chosen to represent a relatively poor government looking to increase spending. For the baseline exercises, we choose the neutral value of parameter ρ. The remaining parameters referring to the penalty function we chose in order to obtain a crisis zone around 70% of debt, matching gross debt levels in 2002, and reasonable levels of discretionary inflation π^D.

Table 1: Parameters of the Baseline Model

Parameter	Value	Meaning	Calibration
β	.915	Discount factor	Ex-post 1996-2019 real interest rate
τ	35%	Tax rate	General gov. revenue in % of GDP
π_a	3.5%	Inflation target	2002 BCB target
f	20%	Crisis prob.	EMBI+ Brazil on 10/2002
e	1.5	Endowment	Poor gov.
ρ	.5	Pref. for consumpt.	Neutral value
κ	20%	Limit to TFP cost	Crisis zone at a 70% debt level
c_0	.05	Fixed cost	Crisis zone at a 70% debt level
c_1	.5	Variable cost	Crisis zone at a 70% debt level

With our calibration we obtain a fiscal fragility zone spanning debt levels from 70 to 175% of GDP. Discretionary inflation $\pi^D \in [4, 20]$ for the considered debt levels. The policy-maker's ability to keep nominal interest rate low is restricted by private agents' expectations. Consequently, in the fiscal fragility zone, as the private agents take into consideration the additional uncertainty, nominal interest rates are pressured upwards (see figure 2).

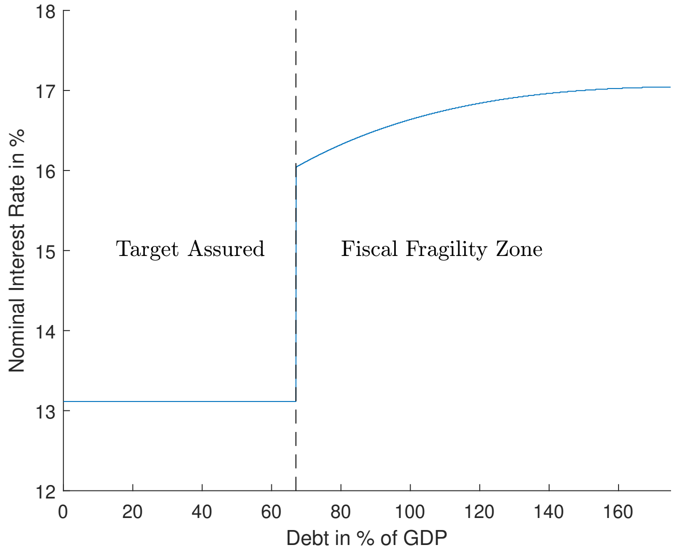

Figure 2: Nominal Interest Rates and the Fiscal Fragility Zone

3.3 Results

Optimal Fiscal Policy

The policymaker's optimal debt path depends upon the initial value of its debt stock. Outside the fiscal fragility zone, it prefers to maintain debt levels constant. Within the fiscal fragility zone, it might either: i) choose fiscal responsibility and run down its debt to avoid the costs of an adverse equilibrium; ii) maintain debt levels constant; or iii) increase its debt in order to maintain a given spending level (see figure 3). Those results are similar to Cole & Kehoe (2000).

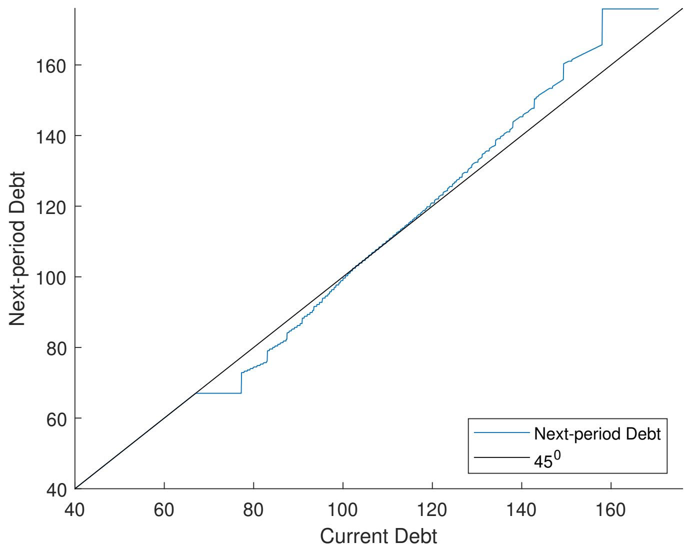

Figure 3: Debt Policy Function

The policymaker chooses a fiscally responsible debt path in order to avoid the expected endowment loss from deviating from the inflation target in the eventuality of adverse inflation expectation. Nevertheless, the fiscal room available to the policymaker may shrink to an extend where it becomes more interesting to run-up debt in order to maintain spending. Eventually, it will be exposed to an adverse expectations equilibrium and loose credibility. By opting to run up debt the policymaker will ultimately fail to give the needed fiscal support to the inflation target.

Coordinating Expectations Through the Target

The inflation target can help coordinating private agents' expectations. First, the cost of deviating depends on the target level. Second, private agents use the inflation target to form expectations in the fiscal fragility zone. The target, therefore, functions as a nominal anchor for expectations.

Let us now look at what happens to the fiscal fragility zone's lower bound $\underline{\mathrm{D}}$ defined by $V^p(\underline{\mathrm{D}}|\pi = \pi^a, \pi^e = \pi^D) = V^p(\underline{\mathrm{D}}|\pi = \pi^D, \pi^e = \pi^D)$. For a higher target the policymaker will increase discretionary inflation less than one on one with the inflation target. This increases $V^p(D|\pi = \pi^a, \pi^e = \pi^D)$ for each D as the cost of keeping on target decreases. Although $V^p(D|\pi = \pi^D, \pi^e = \pi^D)$ will also increase, given the smaller deviation costs, it will be of a lesser magnitude. Consequently, a target increase will make the lower bound $\underline{\mathrm{D}}$ shift upwards as can be seen in figure 4.

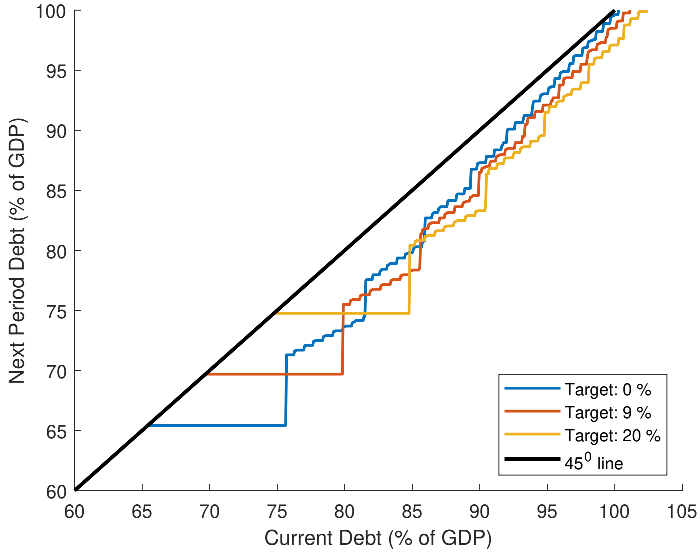

Figure 4: Inflation Target and the Fiscal Fragility Zone

Inflation-Indexed Debt

Under the previous timing, the policymaker was able to partially default on its debt when faced with adverse conditions in the fiscal fragility zone. However, now that it is unable to decrease debt servicing costs through the use of inflation, the policymaker can no longer increase spending. Marginal benefits of deviating will therefore only be reduced to the extend that the high inflation penalizes the economy. The absence of inflationary transfers inevitably increases the optimal inflation level at which the marginal costs and benefits of deviating cancel out. The discretionary inflation under both timings, for our calibrated model, is illustrated in figure 5.

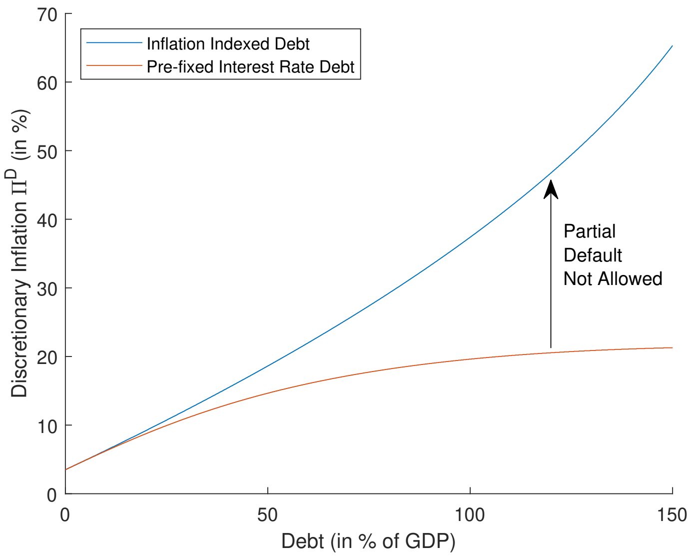

Figure 5: Discretionary Inflation π^D
Under Inflation-Indexed Debt

The higher discretionary inflation resulting from this timing change impacts the limits of the fiscal fragility zone. More specifically, it shifts the lower-bound of the fiscal fragility zone downwards. Higher discretionary inflation impacts both the left- and right-hand sides of the inequality $V^p(D|\pi = \pi^a, \pi^e = \pi^D) \geq V^p(D|\pi = \pi^D, \pi^e = \pi^D)$ that defines the debt levels at which the target is assured. Let us look at what happens to both the left- and right-hand side.

With regards to the left-hand side, the higher discretionary inflation implies higher fiscal costs for keeping inflation on target. The higher debt servicing costs associated will make it less appealing to keep inflation on target. Given that the marginal utility for spending is higher than consumption $V^p(D|\pi = \pi^a, \pi^e = \pi^D)$ would decrease as staying on target would imply a higher transfer of resources to private agents.

With regards to the right-hand side, the penalty associated with a larger discretionary inflation will depress the economy further than under the previous timing. $V^p(D|\pi = \pi^D, \pi^e = \pi^D)$ will therefore also decrease.

In our calibrated model the left-hand side effect is dominating. Consequently the debt levels sustaining the target with certainty will be reduced, shifting the fiscal fragility's lower bound downwards. This result might be surprising as tying hands, in this case through the use of inflation indexed bonds, is usually associated with a higher commitment ability.

Preference for Spending

Suppose that for some reason the preferences of the policymaker change and instead give more weight to public spending. This is captured by a reduction in the parameter ρ. Fixing initial debt, an increase in the preference for spending may change the policymaker's credibility as it can be pushed into the fiscal fragility zone. In the case of pre-fixed interest rate debt, increasing the preference for spending may result in private agents adjusting their expected inflation as the policymaker enters the fiscal fragility zone. Since discretionary inflation is an increasing function in the preference for public spending, expected inflation will keep on increasing thereafter as can be seen in figure 6 for a 100% debt to GDP level. Hence, a shock to ρ could starkly increase expected inflation as the policymaker moves into fiscal fragility territory.

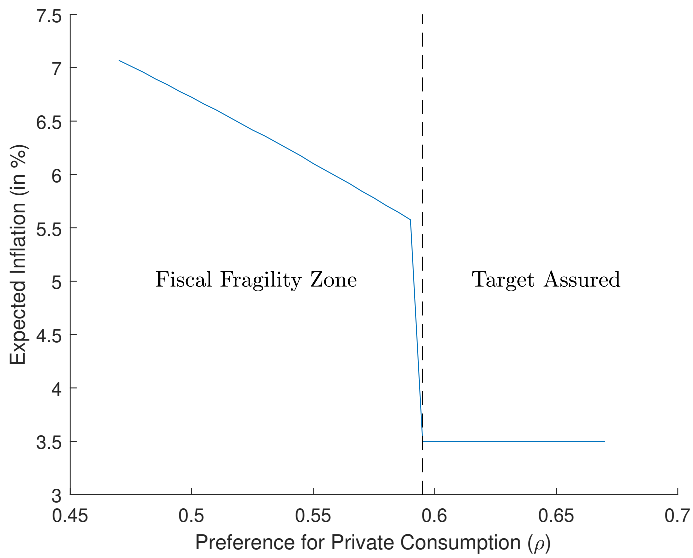

Figure 6: Inflation Expectations and Private Consumption Preferences

With respect to optimal fiscal policy, a policymaker that more highly values private consumption is less likely to run-up debt and loose the fiscal support for its inflation target. The parameter ρ captures this relative preference. In figure 7 it can be seen that the debt level up to which the policymaker remains fiscally responsible increases with respect to its preference for private consumption.

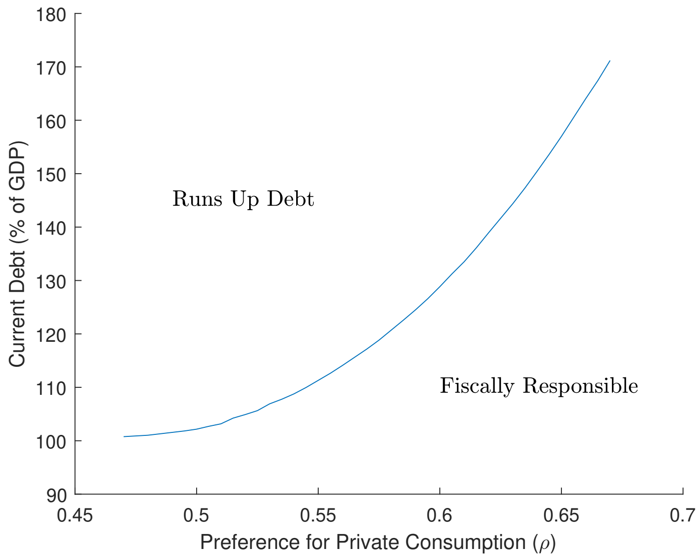

Figure 7: Debt Policy Function Inflection Point

4 Historical Event

Our model allows us to explain the dynamics observed during the 2002 Brazilian presidential election. More specifically, a loss of confidence in the commitment of the central bank to keep inflation on target while apparently no significant changes in fundamentals were observed. This loss of confidence resulted in the upwards shift in expected inflation for all horizons tracked by the central bank through a weekly survey. With the presidential election, agents changed their expectations about the government's preferences with regards to public spending as the candidate most likely to win run on a platform of increasing social spending. Something that could be interpreted, from the standpoint of our model, as a shock to the preference parameter ρ. This in turn can be sufficient to make the economy fall into the fiscal fragility zone, increasing expected inflation as a partial default through inflation can no longer be excluded.

Fiscal Policy

After the 2002 election gross public debt was gradually reduced. As can be seen in table 2, gross debt went down from almost 80% of GDP in 2002 to close to 70% in 2004. Furthermore, the government continued to run primary surpluses to meet its debt obligations, a signal of fiscal responsibility. From the perspective of our model, such fiscal policy is compatible with the policymaker trying to exit the fiscal fragility zone and give the needed fiscal support to its inflation target.

	Gross Debt (% of GDP)	Primary Result (% of GDP)
2000	66.7	1.73
2001	70.2	1.69
2002	79.8	2.16
2003	74.7	2.28
2004	70.6	2.70
2005	69.2	2.60

Table 2: Fiscal Policy

Inflation Target

Before the October elections, the 2003 target was exceptionally revised upwards from a previously announced 3.25 to 4%. Similarly, the upper and lower bounds widened from -/+ 2 to 2.5%. A year later, on the 25 of June 2003, the 2004 target was also increased by an additional 1.75 points to 5.5% (see table 3). The decision to increase the target in the short run makes sense from the perspective of our model, as it helps to coordinate and anchor the expectations of private agents around the new target.

Year	Date When Set	Target	Bounds
2002	28/6/2000	3.5	2
2003	28/6/2001	3.25	2
	27/6/2002	4	2.5
2004	27/6/2002	3.75	2.5
	25/6/2003	5.5	2.5
2005	25/6/2003	4.5	2.5

Table 3: Inflation Targets

Debt Management

The lion share of Brazilian public securities used some sort of indexer to readjust what should be paid to bondholders. More specifically inflation, exchange rates, and the overnight interbank lending rate on operations using public securities as collateral (Selic) are the main ones. On the one hand, debt indexed to inflation and exchange rates made up 56% of outstanding debt in 2002 and were gradually reduced to 32.5% in 2005. On the other hand, pre-fixed securities and those indexed to the Selic made up close to 44% in 2002 and increased to over 67% of outstanding debt in 2005. The latter type of debt is thought of as debt on which partial defaults are possible and referred to in our model as pre-fixed interest rate bonds. Our model suggested a debt management policy compatible with what is described in table 4.

	Pre -fixed	Selic	Debt Allowing Partial Default	Exchange rate	Price Index	Other	Indexed Debt
	A	B	A+B	C	D	E	C+D+E
2002	1.5	42.4	43.9	45.8	8.8	1.4	56
2003	9.5	46.5	56	32.4	10.3	1.4	44.1
2004	16.1	45.7	61.8	24.2	11.9	2.1	38.2
2005	23.6	43.9	67.5	17.6	13.1	1.8	32.5

Table 4: Debt Type (in % of Debt)

5 Empirical Results

In this section we ask whether there is empirical evidence for the predictions made based on our model. Using the calibrated model we concluded that i) the size of the deviation could be reduced by increasing the target and reducing debt and ii) the probability to overshoot the target would increase with debt and decrease with higher target levels.

The first order condition of the discretionary inflation problem, from equation 6, relates the deviation of inflation from the target, $\pi_{i,t} - \pi^A_{i,t}$, to observable and latent variables for each country i[4]. We estimate the following linear model to capture the relationship:

$$\begin{aligned} \pi_{i,t} - \pi^A_{i,t} = {} & \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \text{target}_{i,t} \\ & + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \quad t = 1, ..., T \end{aligned} \tag{12}$$

where the idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1}, ..., X_{i,T}, c_i) = 0, t = 1, ..., T$ with $X_{i,t}$ being a vector of the observable regressors at time t and for country i. The unobserved variable c_i captures the time-constant individual heterogeneity between countries. We use a fixed effect estimator as it seems reasonable to assume that the unobserved characteristics of each country are related to their choices of debt, revenue and inflation target. Hence we cannot assume that $\mathbb{E}(X_{i,t}c_i) = 0$ [5].

[4]For further details see the appendix.

[5]A Haussman test between a fixed and random effect estimator similarly suggests the use of the former.

We expect the coefficients on revenue to be negative and on debt to be positive. More specifically higher revenues increase the fiscal room available to the policymaker and decreases the potential benefits from using discretionary inflation. Increasing debt has the opposite effect and will therefore be associated with higher deviations from the target. Based on our calibrated model we furthermore expect the coefficient on the inflation target to be negative. The variables and parameters of the model are mapped into both observed series and latent variables as can be seen in table 7. The dataset includes 20 countries with at least 15 years of inflation targeting [6] covering the period between 2000 and 2019. Over the period 2010/2019 emerging markets had on average higher targets and inflation then developed countries. Furthermore, they also had on average the highest deviations from the target.

Country	Debt	Revenue	Target	CPI	Max Deviation
Brazil	72	33	4.5	5.8	6.2
Chile	17	23	3.0	3.1	1.7
Colombia	44	29	3.0	3.9	3.8
Indonesia	26	16	4.3	4.7	3.9
Mexico	49	24	3.0	4.0	3.8
Peru	24	21	2.0	2.9	2.7
Philippines	43	19	3.6	3.0	2.1
Poland	52	39	2.5	1.7	2.1
South Africa	48	28	4.5	5.1	2.2
Thailand	42	21	2.1	1.5	1.9
Turkey	32	32	5.2	10.0	15.3
Sub-total	**41**	**26**	**3.4**	**4.1**	**4.2**

Table 5: Descriptive Statistics for EM (2010/2019)

[6]The countries in the sample are New Zealand, Canada, United Kingdom, Sweden, Australia, Israel, Czech Republic, Poland, Brazil, Chile, Colombia, South Africa, Thailand, Mexico, Norway, Iceland, Peru, Philippines, Indonesia, Romania, and Turkey.

Country	Debt	Revenue	Target	CPI	Max Deviation
Australia	34	34	2.5	2.1	0.5
Canada	87	39	2	1.8	0.7
Czech Republic	38	41	2	1.8	1.2
Iceland	66	43	2.5	2.8	2.8
Israel	65	37	2	0.9	0.7
New Zealand	33	37	1.5	1.6	2.4
Norway	36	55	2.5	2.1	1.0
Sweden	40	49	2	1.2	0.2
United Kingdom	84	36	2	2.2	2.6
Sub-total	**54**	**41**	**2.1**	**1.8**	**1.3**

Table 6: Descriptive Statistics for Developed Countries (2010/2019)

Model	Serie	Expected Impact	Source
D	Gross debt (%GDP)	⇑	Fiscal Monitor (IMF)
τe	Revenue (%GDP)	⇓	Fiscal Monitor (IMF)
π^A	Inflation Target	⇓	Central Banks
e, f, c_1, c_2	Country Dummy	-	-

Table 7: Model Predictions

All the coefficients, with the exception of revenues, are of the expected sign. It is possible for revenues to capture the effects of the parameter ρ associated with the preference for public spending. The more the policymaker prefers public spending over private consumption, the more likely it is to try to inflate debt away. We also verified if dropping countries that overly impact the coefficients would change the results, in this case Turkey [7]. The results are not substantially altered, with the coefficient on debt and the inflation target remaining statistically significant. The results are robust to changes in the accounting unit used for debt and revenues. Using log of debt and revenues, converted using a PPP exchange rate, does not change the signs of the coefficients and the significance of the coefficients on the inflation target.

As noted by Ha *et al.* (2019) domestic inflation has been influenced a great deal by global shocks after 2001. In our sample the 2008-2009 period stands out as many countries

[7]We motivate estimating the model without Turkey based on the high Cook's distance of observation points from that country.

overshot their inflation targets after the great financial crisis. In order to account for global co-movements in inflation we also include time dummies [8]. The results are summarized in table 8.

	Model 1	Model 2	Model 3	Model 4
Revenue	0.171**	0.098	0.201***	0.132*
	(0.076)	(0.076)	(0.072)	(0.070)
Debt	0.069*	0.074**	0.084**	0.105***
	(0.035)	(0.034)	(0.034)	(0.032)
Target	-0.403***	-0.458***	-0.536***	-0.818***
	(0.063)	(0.062)	(0.157)	(0.157)
Revenue:Debt	-0.002*	-0.002*	-0.002**	-0.002***
	(0.001)	(0.001)	(0.001)	(0.001)
R^2	0.290	0.408	0.207	0.378
Dummies	Country	Country&Time	Country	Country&Time
Sample	All	All	Ex. Turkey	Ex. Turkey
Num. obs.	382	382	364	364
Note:			*p<0.1; **p<0.05; ***p<0.01	

Table 8: Deviation from the Inflation Target

Let us now assume that overshooting the target's upper bound is defined by the following linear relationship:

$$\begin{aligned} I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} &= \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \text{target}_{i,t} \\ &+ \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t}, \quad t = 1, ..., T \end{aligned} \tag{13}$$

where $\overline{\pi}^A_{i,t}$ is the upper bound of the inflation target [9]. The indicator $I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1$ when inflation $\pi_{i,t}$ overshoots the upper bound of the inflation target $\overline{\pi}^A_{i,t}$. The idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1}, ..., X_{i,T}, c_i) = 0, t = 1, ..., T$. The probability of overshooting the target will then be given by:

$$Pr(I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1|X_{i,t}, c_i) = \frac{1}{1 + e^{-X'_{i,t}\beta - c_i}}, \quad t = 1, ..., T \tag{14}$$

[8]Using the first principal component of the yearly inflation panel, more in the spirit of Ha *et al.* (2019), and letting the impact be country specific does not change the results much.

[9]We used a 1.5% tolerance bound for central banks only using target points

The coefficients continue with the same signs. However, the coefficients on debt are no longer relevant in any of the tested settings. Dropping Turkey from our base did not change any of the signs, however the coefficient on the target level was no longer significant in the setting without time dummies. The results are summarized in table 9.

	Model 1	Model 2	Model 3	Model 4
Revenue	0.140	0.099	0.131	0.117
	(0.090)	(0.104)	(0.091)	(0.106)
Debt	0.033	0.050	0.023	0.051
	(0.044)	(0.052)	(0.043)	(0.052)
Target	-0.619**	-1.220***	-0.331	-0.913**
	(0.263)	(0.372)	(0.275)	(0.381)
Revenue:Debt	-0.001	-0.001	-0.001	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)
Dummies	Country	Country&Time	Country	Country&Time
Sample	All	All	Ex. Turkey	Ex. Turkey
Num. obs.	338	338	320	320
Note:			*p<0.1; **p<0.05; ***p<0.01	

Table 9: Probability of Overshooting the Target

6 Conclusion

Indebted policymakers have a limited budget and are subject to inflationary shocks forcing them either (i) to accept a higher interest rate to have inflation on the pre-announced target (ii) to accept higher inflation. In the fiscal fragility zone – characterized by high debt and limited fiscal room – the central bank chooses to deviate in the face of private-agents' adverse expectations. In order to avoid such target failures and the resulting economic costs, the policymaker might choose to run down debt to a level where the target can be assured. However, debt levels can be such that increasing them, in order to preserve government spending, is the optimal policy choice.

For central bankers that might face doubts about their credibility to sustain the inflation target, we suggest a set of tools based on our model that has been successfully implemented in Brazil during a confidence crisis in 2002. We furthermore find that most of our results seem to be consistent with data from a panel of inflation targeting countries.

Coordination with fiscal authorities to ensure credible monetary policy is already a well-established result. In our model, high debt levels may lead the policymaker to become vulnerable to adverse shocks. Decreasing debt can be the optimal policy for the policymaker looking for a more solid fiscal support for its monetary policy. More novel suggestions are that i) increasing the inflation target can help coordinate private-agents' expectations, and ii) increasing the share of nominal debt, as opposed to inflation indexed, allows the policymaker to partially default and prevent higher inflation levels during an adverse scenario.

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A Empirical Results

A.1 Deviations from Target

The first order condition of the discretionary inflation problem (equation 6) is given by:

$$\frac{1+\pi_T^e}{(1+\pi_T^D)^2}\frac{1}{\beta}D(u_g^T-1)+\frac{1}{1-\beta}\left[\alpha'(1-\tau)e+u_g\alpha'\tau e\right]=0$$

where u_g^T is the marginal utility of government spending evaluated at time T, u_g the steady state equivalent, and α' the marginal penalty for deviation $\pi_T^D-\pi^A$. The equation can be rewritten as:

$$D\left(u_g^T-1\right)\left(\frac{1+\pi_T^e}{(1+\pi^D)^2}\right)=-\frac{\beta}{1-\beta}\tau e\alpha'\left[\frac{1-\tau}{\tau}+u_g\right]$$

and taking logs we obtain:

$$d+\log(u_g^T-1)+\log(1+\pi_T^e)-2\log(1+\pi^D)=c+\log(\tau e)+\log(-\alpha')$$

where $c=\log(\frac{\beta}{1-\beta})+\log(\frac{1-\tau}{\tau}+u_g)$ is a constant and $d=\log(D)$. From the hypotheses in our model $u_g^T>1$ and $\alpha'<0$. Using the approximation for $\log(1+x)\simeq x$ for small x and replacing expectations π_T^e:

$$\pi^D-\pi^A=-\frac{c}{1+f}-\frac{\log(\tau e)}{1+f}+\frac{d}{1+f}-\frac{\pi^A}{1+f}-\frac{\log(-\alpha')-\log(u_g^T-1)}{1+f}$$

The final term is tricky and may influence the sign on the explanatory variables when estimating the following equation for country i:

$$\begin{aligned}\pi_{i,t}-\pi_{i,t}^A=\;&\beta_{0,i}+\beta_1\text{revenue}_{i,t}+\beta_2\text{debt}_{i,t}+\beta_3\text{target}_{i,t}\\&+\beta_4\text{revenue}_{i,t}*\text{debt}_{i,t}+u_{i,t}\end{aligned}$$

where the interaction term between revenue and public debt is meant to capture the dynamics of the marginal utility of government spending at time t. The idiosyncratic error term $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1},...,X_{i,T},c_i)=0, t=1,...,T$. Coefficient $\beta_{0,i}$ captures a country fixed effect while all other coefficients are common to all countries. Based on the calibrated model, we expect the coefficients on revenue and the inflation target to be negative, while the coefficient on debt to be positive.