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Thiago Sevilhano Martinez

Essays on Longevity, Welfare, and Economic Growth

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Tese para obtenção do grau de doutor em economia apresentada à Escola de Pós-Graduação em Economia.

Orientador: Aloisio Pessoa de Araujo.

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Thiago Sevilhano Martinez

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À minha amada Elisabete.

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Resumo

Essa tese é composta por dois artigos sobre as relações entre longevidade, bem-estar e crescimento econômico. O primeiro artigo propõe uma nova medida equivalente de produtividade total dos fatores (PTF) para calcular os efeitos de bem-estar de melhorias na expectativa de vida e em anos de escolaridade. Tal medida usa um modelo de gerações sobrepostas para calcular a taxa de crescimento da PTF que seria necessária para manter o bem-estar atual se a educação e a longevidade não tivessem melhorado nas últimas décadas. Comparamos os desempenhos relativos de 34 países em PTF e nesta medida de bem-estar para os anos 1960-2014 e 1987-2014, com foco em países latino-americanos. Constatamos que, apesar das baixas taxas de crescimento da PTF na América Latina, de acordo com essa medida a região apresentou maior convergência de bem-estar em relação aos EUA.

A expectativa de vida aumentou em todo o mundo nas últimas décadas, com ganhos mais acentuados em países mais pobres do que nos países mais ricos. O progresso tecnológico na medicina está relacionado a essa melhora na longevidade e ao aumento dos gastos com saúde como proporção do PIB. Mas diante das dificuldades de medição da produtividade no setor saúde, como podemos avaliar a importância relativa da tecnologia e das preferências para determinar a longevidade e os gastos com saúde em países com estágios de desenvolvimento heterogêneos? Baseado em dois fatos estilizados sobre o envelhecimento humano, a curva Gompertz e a correlação Strehler-Mildvan, o segundo artigo propõe uma função de produção de longevidade na qual os ganhos de produtividade são identificados a partir de mudanças paramétricas nessas regularidades empíricas. Construímos um modelo de gerações sobrepostas com transformação estrutural relacionada ao setor saúde e simulamos trajetórias ótimas de expectativa de vida aos 30 anos e gastos em saúde como proporção do PIB para Brasil, França e EUA. Constatamos que o crescimento da produtividade na produção de longevidade explica de 70% a 80% dos ganhos de expectativa de vida em todos os casos e que o aumento dos gastos com saúde é mais relevante para o Brasil do que para os EUA e a França. Também constatamos que o crescimento dos gastos com saúde como proporção do PIB é determinado principalmente por preferências e que, com preferências calibradas para a França, os gastos com saúde são subestimados para o Brasil e superestimados para os EUA.

Palavras-chave: Longevidade, Bem-Estar, Crescimento Econômico, Transformação Estrutural

Abstract

This thesis is composed of two papers on the relations between longevity, welfare and economic growth. The first paper proposes a new total factor productivity (TFP)-equivalent measure to calculate the welfare effects of improvements in life expectancy and years of schooling. It uses an overlapping generations model to compute the TFP growth rate that would be necessary to maintain current welfare if education and longevity had not improved in recent decades. We compare the relative performances of 34 countries in TFP and in this welfare measure for the years 1960-2014 and 1987-2014, with a focus on Latin American countries. We find that, despite low TFP growth rates in Latin America, according to this measure the region showed greater convergence in welfare as compared to the US.

Life expectancy has increased around the world in recent decades, with more pronounced gains in poorer than in richer countries. Technological progress in medicine is related to this improvement in longevity and to the rise in health spending as a share of GDP. But faced with the difficulties in measuring productivity in the health sector, how can we assess the relative importance of technology and preferences in shaping longevity and health expenditures for countries at heterogeneous stages of development? Based on two stylized facts on human aging, the Gompertz curve and the Strehler-Mildvan correlation, our second paper proposes a longevity production function in which productivity gains are identified from parametric changes in these empirical regularities. We construct an overlapping generations model with structural transformation related to the health sector and we simulate optimal trajectories of life expectancy at age 30 and health expenditures as a share of GDP for Brazil, France, and the US. We find that productivity growth in longevity production explains 70% to 80% of life expectancy gains in all cases, and the rise in health expenditures are more relevant for Brazil than for the US and France. We also find that the increase in health expenditures as a share of GDP is driven by preferences, and preferences calibrated for France underestimate health spending in Brazil and overestimate it for the US.

Keywords: Longevity, Welfare, Economic Growth, Structural Transformation

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Introduction

Since Cass (1965) and Koopmans (1965) revival of Ramsey's (1928) contribution, optimal choices in growth economics are those that maximize intertemporal utility. In Ramsey's approach, the optimal path maximizes welfare, which is different from maximizing economic growth. The possible conflict between growth and welfare arises as capital accumulation promotes growth and future consumption, but at the expense of current consumption.

Our research is motivated by another possible tradeoff between growth and welfare, as individuals in developing countries may optimally prefer a trajectory of lower GDP growth with increased expenditure on health and education. For Brazil and other Latin American countries, this tradeoff could explain part of the poor performance in growth and productivity in recent decades, as a large part of the rise in taxes and government expenditures in this period is due to increased provision of these services.

The two essays that compose this thesis discuss the relations between health, education, and welfare using overlapping generations models of economic growth.

We associate health to lower death probabilities. The tradeoff underlying the decision between consumption and health expenditures corresponds to trading consumption today for additional periods of consumption in the future due to an increased lifespan, or in the words of Hall and Jones (2007), consumption at the intensive margin for consumption at the extensive margin. Rosen (1988) presents simple state-dependent preferences to deal with decisions regarding life-risk valuation, where it is necessary to compare utility in the state of being alive versus being dead. He shows that the willingness to pay to reduce the probability of death increases with income and age. Hall and Jones (2007) use these non-homothetic preferences to explain the rise of the share of health spending in GDP in the United States. We use these preferences in the models of both essays of this thesis to evaluate the welfare implications of increased life expectancy.

Education is considered in its traditional role as human capital in growth models, through the effects of more years of schooling on labor productivity, as in Bils and Klenow (2000). In addition, in our essay on health production we also take into account the direct role of education in fostering life expectancy, which is stressed in microeconomic studies in health economics, though uncommon in macroeconomic growth models with endogenous mortality.

The first essay in this thesis, "Productivity and welfare", deals with the relations between total factor productivity (TFP) growth and welfare improvement in a dynamic framework. The empirical literature on development accounting emphasizes the leading role of TFP in determining income differences between countries (Caselli, 2005, 2016). Per capita income growth in most Latin American countries has been deceptive since the 1980s, and this literature points to low TFP growth as a major driver of this (Restuccia, 2013). This lack of convergence to developed countries in relative income and TFP, however, may be eclipsing other sources of welfare convergence. Health and education production are economic activities which are hard to measure in GDP (Diewert, 2018), and their welfare consequences as a result of longer lifetimes and a delayed increase in labor productivity may be poorly reflected in past income and TFP growth.

In this paper, we propose a model-based method to evaluate the welfare effects of improvements in life expectancy and schooling in terms of welfare equivalence to TFP growth. This approach for welfare measurement builds on Becker et al. (2005) and Jones and Klenow (2016), who use consumption-equivalent welfare measures in models with Rosen's (1988) preferences to compute the additional consumption flow that would be necessary to maintain the level of utility if life expectancy had not risen in the last decades. In these papers, the consumption stream is exogenous, there is no production, and all compensations are related to the properties of the utility function. Our paper adds to this literature by including schooling in the analysis, and as its effect on utility is indirect through higher labor productivity, we must model the production side of the economy. Thus, TFP takes the place of consumption as the exogenous variable used for the welfare equivalent compensations. We model a perpetual youth overlapping generations economy as in Blanchard (1985), but extend this model by including the preferences for life-risk valuation given by Rosen (1988) and human capital. The TFP-equivalent is computed for 34 countries for the years 1960, 1987, and 2014. We focus on the results for the Latin American countries in our sample. We find that there is greater convergence in this welfare TFP-equivalent than in the usual TFP for Latin American countries.

In the second essay of this thesis, we evaluate the relative role of TFP growth versus pref-

erences in shaping both the increase in life expectancy and the rise of health expenditures as a share of GDP in recent decades for countries at different stages of development. However, the problems related to measuring productivity in health services pose a major obstacle in assessing this role.

Using two stylized facts on human aging, the Gompertz curve and the Strehler-Mildvan correlation, we propose a longevity production function in which total factor productivity (TFP) gains are identified from parametric changes in these empirical regularities. We estimate this longevity production function in a panel with 96 countries for the years 1960 to 2010. The longevity production parameters are used in a numerical model of growth with endogenous mortality and health-related structural transformation to simulate optimal trajectories of life expectancy at age 30 and the share of health expenditures share in GDP for Brazil in comparison with two developed countries, France and the US. We find that productivity growth explains most of the life expectancy gains in all cases, but the increase in health inputs played a greater role in Brazil than in the US and France. However, we also find that preferences are the main determinant of the increase in the share of health expenditure in GDP, and health spending in Brazil is underestimated in simulations with preferences calibrated for either France or the US.

Chapter 1

Productivity and Welfare

1.1 Introduction

Relative income as measured by gross domestic product (GDP) is widely used to compare the economic performance of countries. The prevailing view is that factors of production account for just a part of per capita income differences, and that the role of total factor productivity (TFP) gaps is very significant, explaining about half of the income differences (e.g. Caselli, 2005, 2016). However, GDP and productivity differences can underestimate welfare convergence, when economic activities which are hard to measure become increasingly more relevant over time. Health and education are two of these sectors in which measuring prices, quantities, and productivity is difficult (Diewert, 2018), and their GDP shares have been rising over the last decades.

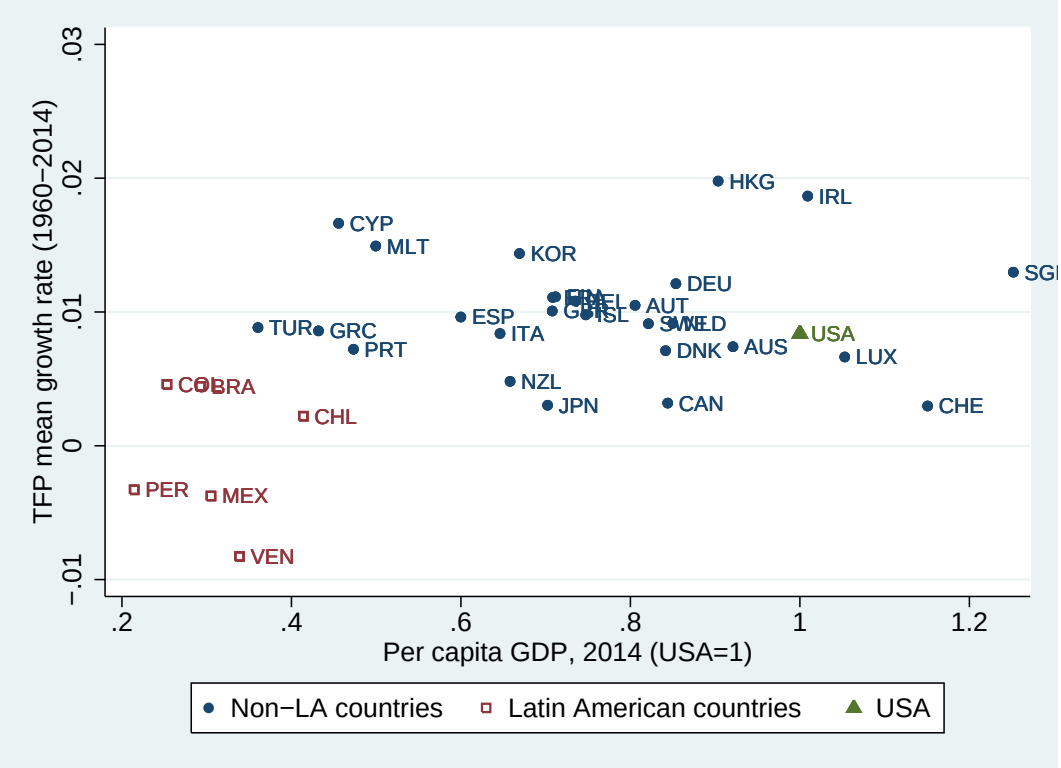
In this paper, we use an indirect method to evaluate the welfare effects of improvements longevity and education without modeling their production. We propose a productivity-equivalent welfare measure that converts differences in life expectancy at birth and mean years of schooling between countries and years to changes in TFP. This measure compares welfare for newborn agents in two overlapping generations economies, by computing the TFP level that would maintain the same lifetime expected utility in the second economy, but with the life expectancy and schooling observed in the first economy.

This "TFP-equivalent" measure can be used to compare welfare between two countries or for the same country in two periods. We compute the measure for 34 countries with all required data available in the Penn World Table 9.0 for years 1960, 1987, and 2014. Our focus is on the poor performances in Latin American countries in recent decades, which is emphasized by the

development accounting literature, as in (Restuccia, 2013).

Figure 1.1 plots mean TFP growth rates from 1960 to 2014 versus relative per capita GDP in 2014, and shows the poor relative performance of the Latin American countries in our sample, all of them with TFP growth rates smaller than the US economy.

Figure 1.1: *Total factor productivity (1960-2014)*



Notes: PPP-corrected real GDP data from PWT 9.0, series "rgdpna". TFP is computed as a residual using a Cobb-Douglas production function with physical capital coefficient of $\alpha = 0.37$. See the section on calibration for more details.

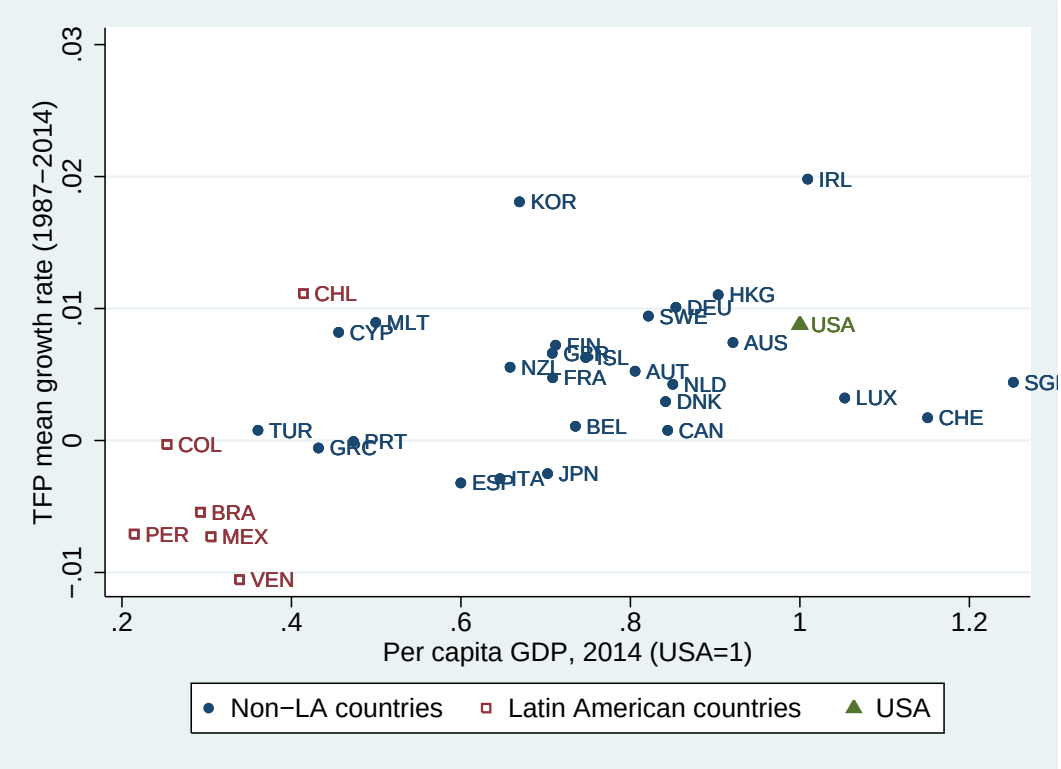
Latin American performance is generally worse for the period 1987-2014, as shown in Figure 1.2, with the exception of the Chilean economy.

Comparing our new TFP-equivalent for the countries in the sample, the main result we obtain is that there is greater convergence with this measure.

Our paper builds on the value of life analysis of Rosen (1988), which proposes preferences for evaluating the propensity to pay for marginal increases in life expectancy. As emphasized by Hall and Jones (2007), these preferences represent health as a superior good, implying a rising share of health spending in GDP. As societies grow richer, marginal utility of consumption falls relative to marginal utility of health spending, because additional years of life allow for extra consumption periods.

The implications of these preferences on welfare and growth are considered by Becker et al.

Figure 1.2: *Total factor productivity (1987-2014)*



Notes: PPP-corrected real GDP data from PWT 9.0. See Figure 1.1.

(2005), Hall and Jones (2007), Jones (2016), and Jones and Klenow (2016). Becker et al. (2005) compute an income-equivalent welfare measure for gains in life expectancy and find that growth in this "full income" is faster for poorer countries. Hall and Jones (2007) combine Rosen (1988) non-homothetic preferences for longevity to a growth model to justify the rising health share of GDP. Jones (2016) extends this reasoning to an endogenous growth framework, using it to explain the increasing participation of health-related research in total R&D composition. Jones and Klenow (2016) propose an alternative consumption-equivalent welfare measure that includes inequality and leisure in addition to longevity and consumption. They find that inequality in welfare between countries is stronger than inequality in income, but welfare growth rates are considerably greater in poorer countries, and that most of the boost in welfare growth as compared to usual income growth results from rising life expectancy.

Our approach is closer to Becker et al. (2005) and Jones and Klenow (2016), and we present two main contributions in relation to their work. First, we use a productivity-equivalent instead of a consumption-equivalent welfare measure, which favors a more direct comparison to the development accounting literature results. Second, we include education-related human capital in our utility compensation. These two differences imply a major change in methodology, since

in both Becker et al. (2005) and Jones and Klenow (2016) all welfare compensation is made in the utility function, making it unnecessary to model production, and both papers assume a single agent consuming an exogenous income stream. In our paper, as we want welfare compensation included in productivity, and as education effects on utility are indirect, being obtained through higher production associated to increased human capital, we use a perpetual young overlapping generations model similar to Blanchard (1985), with preferences compatible to the value of life approach of Rosen (1988).

This paper is also related to other debates in the previous literature. Prescott (1998) shows that adding intangible capital to a neoclassical model, whether it is produced by the same technology as other goods or in a human capital sector similar to Lucas (1988), is far from sufficient to explain the large differences in international incomes observed in data, and a TFP theory is needed. Developing accounting literature also points to TFP as a major reason behind differences in GDP per worker, which contributes to roughly half the gap between the US and other countries according to Caselli (2016). Our research is not opposed to these results, but it points out that income and productivity differences may not be enough to evaluate convergence in life standards.

Considering technological progress in representative agents' frameworks, Basu and Fernald (2002), Hulten and Schreyer (2010), and Basu et al. (2016) argue in favor of using changes in TFP to measure welfare gains. Basu et al. (2016) show that for a broad range of infinitely lived representative consumer models, welfare is summarized to a first order by initial assets and by the present value of discounted TFP residuals. We show that an analogous result is true for the steady-state equilibrium of our OLG model, as long as we keep health and education endowments constant. However, as our empirical analysis implies, TFP growth can underestimate the welfare effects of improvements in health and education.

The paper is outlined as follows: Section 2 presents our perpetual youth OLG model. Section 3 discusses the relation between productivity and the value of life, as well as the definition of the TFP-equivalent. Section 4 shows the calibration of the model to the US economy from 1960 to 2014 and the algorithm to compute the welfare TFP-equivalent for all of the countries. Section 5 presents the empirical results and section 6 summarizes the conclusions.

1.2 Model

1.2.1 Environment

We follow the perpetual youth model of Blanchard (1985), which allows great tractability by assuming an age-independent death probability, and extend it to evaluate the welfare effects of exogenous changes in mortality, as in the value of life approach of Rosen (1988).[1]

Consider a continuous-time overlapping generations model in which all agents have the same death rates and education. Individuals face a death hazard rate $\eta > 0$, that is constant throughout their lifetimes.

As in Rosen (1988), there are two states of nature, being alive or dead, and utility is state-dependent. Instantaneous utility depends on consumption only if the individual is alive, and in the state of nonsurvival it equals a constant $-\bar{u}$. The utility in the nonsurvival state is normalized to zero by summing $\bar{u}$ to instantaneous utility in both states. Assuming logarithmic dependence on consumption, the normalized instantaneous utility in time t of an individual born in time τ is:

$$u(c(t|\tau)) = \bar{u} + \log(c(t|\tau)) \tag{1.1}$$

For this same individual born in time τ, let $f(T|\tau)$ denote the probability density of dying in time T, and $F(T|\tau)$ the cumulative density of dying before time T. Thus, the expected lifetime utility of this individual is

$$\begin{aligned} V(\tau) &= \int_{\tau}^{\infty} f(T|\tau) \int_{\tau}^{T} e^{-\rho(t-\tau)} u(c(t|\tau)) dt dT \\ &= \int_{\tau}^{\infty} (1 - F(T|\tau)) e^{-\rho(t-\tau)} u(c(t|\tau)) dt \end{aligned}$$

by changing the order of integration.

Define the *survival function* $S(t|\tau)$ as the probability of surviving to time t for an individual

[1] Our notation is closer to the perpetual youth model presented by Acemoglu (2008).

born in τ, and the *death hazard rate* $\eta(t|\tau)$ as:

$$S(t|\tau) \equiv 1 - F(t|\tau) = \int_t^\infty f(T|\tau)dT \tag{1.2}$$

$$\begin{aligned}\eta(t|\tau) &\equiv \lim_{\upsilon \to 0} \frac{Pr(t \leq T < t + \upsilon | T \geq t, t > \tau)}{\upsilon} \\ &= \frac{f(t|\tau)}{S(t|\tau)} = -\frac{d}{dt} \ln S(t|\tau)\end{aligned} \tag{1.3}$$

so that

$$S(t|\tau) = e^{-\int_\tau^t \eta(s|\tau)ds} \tag{1.4}$$

Then by our assumption of a constant death hazard rate $\eta(t|\tau) = \eta$, the expected lifetime utility of this individual born in time τ is

$$V(\tau) = \int_\tau^\infty e^{-\rho(t-\tau)} S(t|\tau) u(c(t|\tau))dt \tag{1.5}$$

$$S(t|\tau) = e^{-\eta(t-\tau)} \tag{1.6}$$

Individuals maximize lifetime utility (1.5) and do not have any heirs to leave bequests. As in Yaari (1965) and Blanchard (1985), markets are complete and the agents buy annuities that continuously pay a premium in exchange for their wealth in the event they die. The insurance premium must equal the death hazard rate η, as the providers of annuities make zero profits because the market is competitive.

All agents are born with an human capital endowment h, which remains the same throughout their lifetime, and zero assets. During their lifetime, they accumulate assets according to

$$\dot{\omega}(t|\tau) = (r(t) + \eta)\omega(t|\tau) + w(t)\ell h - c(t|\tau) \tag{1.7}$$

$$\omega(\tau|\tau) = 0 \tag{1.8}$$

where $\omega(t|\tau)$ is total assets in t of an individual born in τ, $\dot{\omega}(t|\tau) \equiv d\omega(t|\tau)/dt$, $r(t)$ is the interest rate, η is the premium payment from annuities, ℓ is a scaling factor for labor supply, and $w(t)$ is the current wage from labor, which is supplied inelastically.

The labor supply scaling factor ℓ is a constant that will be used in the numerical simulations to account for differences between countries and over years in average hours worked and the ratio

of employed people over total population.

Production uses physical capital and labor as inputs in a Cobb-Douglas technology. Productivity grows exogenously at a constant rate γ. Then if $y(t)$ is per capita production and $k(t)$ is per capita physical capital,

$$y(t) = A(t)k(t)^{\alpha}(\ell h)^{1-\alpha} \tag{1.9}$$

$$\frac{\dot{A}(t)}{A(t)} = \gamma \tag{1.10}$$

for $0 < \alpha < 1$.

Profit maximization of competitive firms determines factor prices

$$w(t) = (1-\alpha)A(t)\left(\frac{k(t)}{\ell h}\right)^{\alpha} \tag{1.11}$$

$$r(t) + \delta = \alpha A(t)\left(\frac{\ell h}{k(t)}\right)^{1-\alpha} \tag{1.12}$$

for positive physical capital depreciation δ.

At each instant, new agents arrive at a constant birth rate n over current population $L(t)$. Total population is determined by aggregation over all generations. Thus, demographics is given by

$$L(t) = \int_{-\infty}^{t} L(t|\tau)d\tau \tag{1.13}$$

$$L(t|\tau) = S(t|\tau)L(\tau|\tau) \qquad \text{and} \tag{1.14}$$

$$L(\tau|\tau) = nL(\tau) \tag{1.15}$$

so that total population evolves according to

$$\dot{L}(t) = (n-\eta)L(t) \tag{1.16}$$

Per capita average consumption and wealth arise from aggregation, and per capita physical

capital equals average wealth:

$$c(t) = \frac{\int_{-\infty}^{t} c(t|\tau)L(t|\tau)d\tau}{L(t)} \tag{1.17}$$

$$\omega(t) = \frac{\int_{-\infty}^{t} \omega(t|\tau)L(t|\tau)d\tau}{L(t)} \tag{1.18}$$

$$k(t) = \omega(t) \tag{1.19}$$

The law of movement of per capita physical capital is

$$\dot{k}(t) = y(t) - c(t) - (\delta + n - \eta)k(t) \tag{1.20}$$

1.2.2 Competitive equilibrium

Definition. A *competitive equilibrium* consists of paths of per capita physical capital, wages, and interest rates $[k(t), w(t), r(t)]_{t=0}^{\infty}$, and paths of per capita consumption for each generation $[c(t|\tau)]_{t=0,t\geq\tau}^{\infty}$, such that:

i) Given prices $[w(t), r(t)]_{t=0}^{\infty}$, households maximize lifetime expected utility (1.5), subject to assets accumulation constraint (1.7);

ii) Firms maximize profits given prices, and market clearing determines the path of prices according to (1.11) and (1.12);

iii) The law of movement of per capita physical capital is given by (1.20).

Equilibrium trajectories for aggregate variables and prices are fully determined without reference for cohort-specific variables.

Propositon 1. *Let* $\tilde{k}(t) \equiv \frac{k(t)}{A(t)^{\frac{1}{1-\alpha}}\ell h}$, $\tilde{c}(t) \equiv \frac{c(t)}{A(t)^{\frac{1}{1-\alpha}}\ell h}$, *and* $g \equiv \frac{\gamma}{1-\alpha}$. *Then the equilibrium path for aggregate variables is characterized by the transition equations*

$$\frac{d\tilde{k}(t)}{dt} = \tilde{k}(t)^{\alpha} - \tilde{c}(t) - (g + \delta + n - \eta)\tilde{k}(t) \tag{1.21}$$

$$\frac{d\tilde{c}(t)}{dt} = \left[\alpha\tilde{k}(t)^{\alpha-1} - (g + \delta + \rho)\right]\tilde{c}(t) - n(\rho + \eta)\tilde{k}(t) \tag{1.22}$$

plus an initial $\tilde{k}(0) > 0$ *and a steady state* $\tilde{k}^*$ *normalized capital. There is a unique saddle-path stable steady state equilibrium* $(\tilde{k}^*, \tilde{c}^*)$, *which solves* $\frac{d\tilde{k}(t)}{dt} = \frac{d\tilde{c}(t)}{dt} = 0$.

Aggregate variables determine prices, and cohort-specific consumption solves

$$\frac{\dot{c}(t|\tau)}{c(t|\tau)} = r(t) - \rho \tag{1.23}$$

$$c(\tau|\tau) = (\rho + \eta)W(\tau)\ell h \tag{1.24}$$

$$for \quad W(\tau) \equiv \int_{\tau}^{\infty} e^{-\int_{\tau}^{s}(r(\upsilon)+\eta)d\upsilon} w(s)ds$$

Proof. See Appendix 1.A.

1.3 Human capital and productivity-equivalent welfare

1.3.1 Productivity and welfare

Now we turn to welfare and value of life considerations. As households' choices change with cohorts, there is no representative agent, and we must choose a welfare criterion. We follow the common practice in the literature based on the value of life approach in Rosen (1988) and associate welfare in time t to the lifetime expected utility $V(t)$ of an agent born in t, as in Becker et al. (2005) and Jones and Klenow (2016).

Jones and Klenow (2016) associate this welfare criterion to the veil of ignorance concept from Rawls (1971). By this principle, normative judgments are promoted from the viewpoint of an individual that ranks alternative societies without any prior knowledge of which preferences, talent or social position she will draw from the available positions in that society. Jones and Klenow (2016) suggest a version of this principle in which individuals behind the veil of ignorance are aware of their preferences, and they compare societies that differ in life expectancy, leisure time, and the mean and variance of consumption streams. Here we assume the same methodological principle, but considering that the individual behind the veil of ignorance compares societies that are heterogeneous regarding life expectancy and years of schooling.

Basu and Fernald (2002) argue that, in a representative agent economy and under very general conditions, productivity growth measured by Solow's residual is the correct measure of welfare improvement. They claim that to be true even under imperfect competition or non-constant returns to scale. In these cases, aggregate productivity mismeasures technological change, yet growth in a modified Solow's residual still represents welfare change.

Basu et al. (2016) show that, to a first-order approximation around the steady-state, the intertemporal utility for a representative consumer reflects the present discounted value of produc-

tivity residuals plus the initial stock of assets, under broad assumptions for production function and market competitiveness. They conclude that productivity growth and initial assets together comprise a sufficient statistic for households' welfare.

The next proposition shows that a similar result holds for our OLG economy.

Propositon 2. *Define welfare at time τ as the lifetime expected utility of a household born in τ. Then at the steady-state equilibrium, for $\tilde{k}^*$ and g defined as in Proposition 1, welfare is equal to*

$$V^*(\tau) = \frac{1}{(\rho+\eta)}\left[\bar{u} + \log c^*(\tau|\tau) + \frac{r^*-\rho}{(\rho+\eta)}\right] \tag{1.25}$$

where

$$c^*(\tau|\tau) = \frac{(\rho+\eta)w^*(\tau)\ell h}{r^*+\eta-g} \tag{1.26}$$

$$r^* = \alpha\tilde{k}^{*\alpha-1} - \delta \tag{1.27}$$

$$w^*(\tau) = (1-\alpha)\tilde{k}^{*\alpha}A(\tau)^{\frac{1}{1-\alpha}} \tag{1.28}$$

Furthermore, at the steady-state equilibrium and conditioned on the death rate and education endowments, productivity growth rate γ determines welfare improvement:

$$\frac{dV^*(\tau)}{d\tau} = \frac{\gamma}{(1-\alpha)(\rho+\eta)} \tag{1.29}$$

Proof. See Appendix 1.A.

1.3.2 Welfare TFP-equivalent

We have stated that there is an immediate relationship between productivity growth and welfare for a newborn household in the steady-state of our standard OLG economy, assuming health and education as fixed. Now we propose a new productivity-equivalent welfare measure for changes in human capital endowments, which builds on the consumption-equivalence tradition of Lucas (1987, 2003), Becker et al. (2005), and Jones and Klenow (2016).

These authors use models of expected intertemporal utility under exogenous consumption flows to perform welfare evaluations by calculating consumption-equivalent changes in utility. Lucas (1987, 2003) computes the compensation in the consumption level of a risk-averse household that is equivalent to eliminating consumption variance. Becker et al. (2005) find the consumption-

equivalent in intertemporal utility for the rise in life expectancy in many countries between 1960 and 2000. Jones and Klenow (2016) calculate consumption compensation to compare welfare in different countries taking into account leisure, life expectancy, and consumption inequality.

We evaluate the welfare effects of human capital improvements by computing the productivity-equivalent change in expected lifetime utility of a newborn household. Using our OLG model, we calculate the productivity compensation that is equivalent in welfare terms to the gains in life expectancy and years of schooling observed for each country.

Our approach differs from the literature in that we compute productivity-equivalent welfare in an overlapping generation's general equilibrium model instead of consumption-equivalent for a single household that just consumes its endowments. By proceeding this way, we are able to compute welfare equivalents for changes in human capital, whose effects on utility are indirect. Consumption is endogenous and total factor productivity is the exogenous variable that is compensated for by changes in human capital and mortality.

Definition. Let $V\left(A(\tau), k(\tau), h, \eta, n, \ell\right)$ denote the lifetime expected utility of a newborn individual in time τ, as a function of initial productivity $A(\tau)$, initial physical capital $k(\tau)$, human capital h, death rate η, population birth rate n, and the labor supply scaling factor ℓ.

Now consider another newborn individual that lives in a reference economy with the same parameters, birth rate, and initial physical capital, but different death rate and human capital endowments (h_0, η_0).

Then the *welfare TFP-equivalent* is defined as the adjusted productivity $\lambda A(\tau)$ that makes the newborn from the reference economy indifferent to possessing the health and education endowments of the first economy:

$$V\left(\lambda A(\tau), k(\tau), h_0, \eta_0, n, \ell\right) = V\left(A(\tau), k(\tau), h, \eta, n, \ell\right) \tag{1.30}$$

In our empirical exercise, we find the welfare-equivalent productivity change that accounts for the gains in longevity and schooling endowments, from 1960 to 2014 and from 1987 to 2014, for each country in our dataset. We take the US economy in 2014 as our reference to define (h_0, η_0), such that $\lambda^{US}_{2014} = 1$, and for each country we compute λ for the years 1960, 1987, and 2014.

1.4 Calibration

1.4.1 Common parameters: US 1960-2014

We first calibrate the OLG model in steady-state equilibrium to match data moments for the US economy from 1960 to 2014. Table 1.1 summarizes the calibration.

Table 1.1: *Calibration, US 1960-2014*

Parameter	Value	Description	Data targets
α	0.3742	Capital share	Average labor share
γ	0.0126	Productivity growth rate	Average per capita GDP growth
δ	0.0677	Physical capital depreciation	$r^* = 0.05$ and average GDP/capital ratio
ρ	0.0263	Rate of time preference	Average consumption /capital ratio, birth rate, and average life expectancy at birth

Data sources: World Bank's World Development Indicators for life expectancy at birth and Penn World Table 9.0 for the remaining.

Data moments for calibration are from Penn World Table 9.0, unless explicitly mentioned otherwise. The parameters common to all countries $(\alpha, \gamma, \delta, \rho)$ are calibrated using data for the US from 1960 to 2014.

The capital share parameter α is calibrated based on the mean labor share in the US from 1960 to 2014.

Productivity growth rate γ matches the mean GDP per capita growth rate in the US, $g = 0.0202$, considering that normalized GDP per capita $\tilde{y} \equiv \frac{y}{A^{\frac{1}{1-\alpha}} \ell h}$ is constant in steady-state:

$$\gamma = (1-\alpha)g$$
$$g = \frac{1}{2014 - 1960} (\log y_{2014} - \log y_{1960})$$

where $y = Y/L$ is GDP per capita.

We use Penn World Table (PWT) version 9.0 data for real GDP, which will also be used in the next section to compute the TFP-equivalent for the other countries in our sample. Real GDP variable is *rgdpna*, the purchasing power parity (PPP) corrected series that relies on 2011 benchmark price measurements from the International Comparison Program (ICP) and extrapolates to other years using national accounts data.

We prefer *rgdpna* instead of other GDP series that keep data from multiple ICP benchmarks,

which were added in version 8.0 from the PWT (Feenstra et al., 2015) to deal with the criticism of Johnson et al. (2013) and others to the major revisions between PWT versions. In these other series, all benchmark ICP measurements are considered (1970, 1975, 1980, 1985, 1996, 2005, and 2011, with different available years according to the country), and national accounts data are used to interpolate between these years and extrapolate for previous and posterior years. However, as we compared the series for Latin American countries, we noted that in these multiple ICP series there are some wide variations in GDP in short time intervals which differ markedly from national accounts data for the same periods, in particular for the years of hyperinflation in the 1990's and in the commodities price boom of the 2000's. Thus, we chose the *rgdpna* series because it relies only on the 2011 edition of ICP benchmark PPP corrections that according to Deaton and Aten (2017) improved the methodology in comparison to previous ICP editions. Pinkovskiy and Sala-i Martin (2016) also recommend this GDP series instead of the others in the Penn World Table after having compared them to an independent measure of economic activity based on night-time lights from satellite photos.

As the death hazard rate η is constant in our model, life expectancy at birth equals $\frac{1}{\eta}$. Therefore, we use the reciprocal of life expectancy at birth according to the World Bank's World Development Indicators (WDI) to match η for each country in the years 1960, 1987, and 2014. Before this, we need a death rate value to calibrate the remaining common parameters to all countries. We use $\bar{\eta} = 0.0134$, which is the reciprocal of 74.46, the mean life expectancy at birth for the US from 1960 to 2014.

McGrattan and Prescott (2003) compute mean returns in the US for physical capital and equities as being close to 4% and 5% respectively over the period 1880-2002. Then we target $r = 0.05$ to calibrate $\delta = 0.0677$, using the mean output-capital ratio $y/k = 0.3145$ and firms' demand for capital condition $r + \delta = \alpha y/k$.

The mean population birth rate $\bar{n} = 0.0233$ for the US matches

$$\bar{n} = \frac{1}{2014 - 1960} \left(\log L_{2014} - \log L_{1960} \right) + \bar{\eta}$$

Then, targeting the mean consumption-capital ratio $c/k = 0.2623$ for the US in 1960-2014 and using the law of movement of aggregate consumption (1.22) in steady-state, we get $\rho = 0.0263$

from

$$\rho = \frac{1}{1+\frac{\bar{n}}{c/k}}\left[\alpha\frac{y}{k} - \left(g+\delta+\bar{\eta}\frac{\bar{n}}{c/k}\right)\right] \tag{1.31}$$

1.4.2 The value of a statistical life

Calibration of $\bar{u}$ builds on estimations of the value of a statistical life, which is usually inferred from studies on wage differentials of occupations with higher risks to life from fatal accidents at work, as in Thaler and Rosen (1976). As exemplified by Murphy and Topel (2006), if workers require a $500 annual wage premium for a job that implies an additional risk of one death in 10,000 workers, the value of a statistical life is VSL = $500 x 10,000 = $5 million.

A survey by Viscusi and Aldy (2003) of this literature suggests a range of $5.5 to $7.5 million to the VSL. As our default value, we follow Murphy and Topel (2006) by assuming VSL = $6.3 million, the estimate of the VSL used for cost-benefit analysis by the US environmental protection agency in 1999. In our robustness exercises, we also test the Viscusi and Aldy (2003) extremes for the VSL.

The next proposition shows how to connect $\bar{u}$ to the VSL in our model.

Propositon 3. *Consider an economy at the steady-state equilibrium of our model. Then the VSL for individuals born in time τ and the utility parameter $\bar{u}$ are related by*

$$\bar{u} = (\rho+\eta)\frac{VSL(\tau)}{c^*(\tau|\tau)} - \log c^*(\tau|\tau) - \frac{r^*-\rho}{\rho+\eta} \tag{1.32}$$

for steady state $c^*(\tau|\tau)$ and r^* using values for the US determined by equations (1.26), (1.27), *and* (1.28) *in Proposition 2.*

Proof. See Appendix 1.A.

We target VSL(1999) = $6.3 million for individuals born in 1999 in the US, supposing the economy is in steady-state equilibrium. Parameters $(\alpha, \gamma, \delta, \rho)$ assume the values already defined. We set $\eta = 0.0131$ to match life expectancy at birth in 1999, which is 76.58 years.

With equations (1.26), (1.27), and (1.28), we compute $c^*(\tau|\tau)$ and r^* from the model.

Total factor productivity $A(1999)$ is computed as a residual from our Cobb-Douglas production function (1.9), using 1999 values for GDP per capita y and physical capital per capita k in addition to h and ℓ. Physical capital is obtained from the capital/GDP ratio computed with the

variables *rkna* and *rgdpna* from PWT 9.1.

Human capital h is the human capital index hc from PWT 9.0, which is calculated using rates of returns on education from Psacharopoulos (1994) and data on average years of schooling from Barro and Lee (2013) and Cohen and Leker (2014). The labor supply scaling factor ℓ is the product of two ratios, the number of employed people over the entire population and current average hours worked relative to the US in 2014.

Using these values, we solve for the steady-state equilibrium and find the constant normalized capital $\tilde{k}^*$. Then by Proposition 3, $\bar{u} = 5.73$.

1.4.3 Computing the TFP-equivalent

Next we compute the welfare TFP-equivalent for 34 countries for the years 1960, 1987, and 2014 assuming that the parameters $(\alpha, \gamma, \delta, \rho,)$ are the same as those calibrated for the US economy. The countries in the sample are those with all the needed variables available in PWT 9.0 for these years.[2] The algorithm to compute the TFP-equivalent for each country in year $\tau \in \{1960, 1987, 2014\}$ is:

1. Compute initial productivity $A(\tau)$ as a residual using the production function (1.9) and the year τ values of y, k, h and ℓ for the correspondent variables in PWT 9.0 indicated in the previous section.

2. Find the steady-state solution $(\tilde{k}^*, \tilde{c}^*)$ using the death rate $\eta = \frac{1}{\text{life expectancy}_\tau}$ and the birth rate $n = \frac{\log L_{2014} - \log L_{1960}}{2014-1960} - \eta$.

3. Calculate initial normalized capital $\tilde{k}(\tau)$ with $A(\tau)$ and year τ values of k and h, then use equations (1.21) and (1.22) to compute transition dynamics for $\tilde{k}$ and $\tilde{c}$ by the time-elimination method of Mulligan and Sala-i Martin (1991, 1993).

4. Find initial consumption for the newborn agent $c(\tau|\tau)$, using the relation $c(t|t) = c(t) - (\rho+\eta)k(t)$ and the initial values $(k(\tau), c(\tau))$ related to $(\tilde{k}(\tau), \tilde{c}(\tau))$.[3] Next, use the equation

[2] The list of countries, by World Bank regions, is - East Asia & Pacific: Australia, China/Hong Kong SAR, Japan, New Zealand, Republic of Korea, and Singapore; Europe & Central Asia: Austria, Belgium, Cyprus, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, Switzerland, Turkey, and United Kingdom; Latin America & Caribbean: Brazil, Chile, Colombia, Mexico, Peru, and Venezuela; Middle East & North Africa: Malta; North America: Canada and the United States. We have excluded Argentina and Norway from our database. Norway since its per capita GDP in 2014 is an outlier, about 60% higher than the US, and Argentina because it has not participated in the 2011 ICP round, and its PPP data is extrapolated from 2005, what can distort the results for methodological reasons. See Deaton and Aten (2017) for an analysis of differences between the 2005 and 2011 rounds of ICP.

[3] This relation results from the consumption out of wealth equations $c(t) = (\rho+\eta)[k(t) + W(t)\ell h]$ and $c(t|t) = (\rho+\eta)W(t)\ell h$.

(1.23) to calculate the trajectory of $c(t|\tau)$ for $T = 200$ maximum years of life.

5. Compute $V(\tau) = \int_{\tau}^{\tau+T} e^{-(\rho+\eta)(t-\tau)}(\bar{u} + \log(c(t|\tau)))dt$ using numerical integration.

6. Guess $\lambda = 1$.

7. Repeat steps 2 to 5 replacing η and h for the corresponding reference US 2014 values, $\eta_0 = 0.0127$ and $h_0 = 3.7387$, and $\lambda A(\tau)$ instead of $A(\tau)$. Call $V_\lambda(\tau)$ the new value for lifetime expected utility.

8. If $|V_\lambda(\tau) - V(\tau)| > 0.01$, redefine $\lambda = \lambda - \frac{|V_\lambda(\tau) - V(\tau)|}{V(\tau)}$ and return to step 7.

9. If $|V_\lambda(\tau) - V(\tau)| \leq 0.01$, stop. Then $\lambda A(\tau)$ is the resulting TFP-equivalent.

1.5 Results

Table 1.2 presents our main findings. For the whole 1960-2014 period, average TFP growth for the group of other countries was similar to that of the US, but for the Latin American countries TFP growth was lower than for the US. However, Latin America's relative performance is improved in our welfare TFP-equivalent measure, with a growth rate higher than the US and closer to the other countries.

Table 1.2: *Average growth rates, productivity (TFP) and welfare (TFP-equivalent)*

	TFP	TFP-equivalent
1960-2014		
US	0.84	1.67
Latin America	0.08	1.82
Other countries	0.88	2.17
1987-2014		
US	0.88	1.51
Latin America	-0.52	1.13
Other countries	0.35	1.48

Notes: Average TFP growth and TFP-equivalent welfare growth calculated using the calibration and data presented in the previous section. Growth rates for the groups are means weighted by population in 2014.

These results are stronger for the second half of this period, the years 1987-2014. Current TFP growth is higher in the US than for the other countries, and it is negative in Latin America.

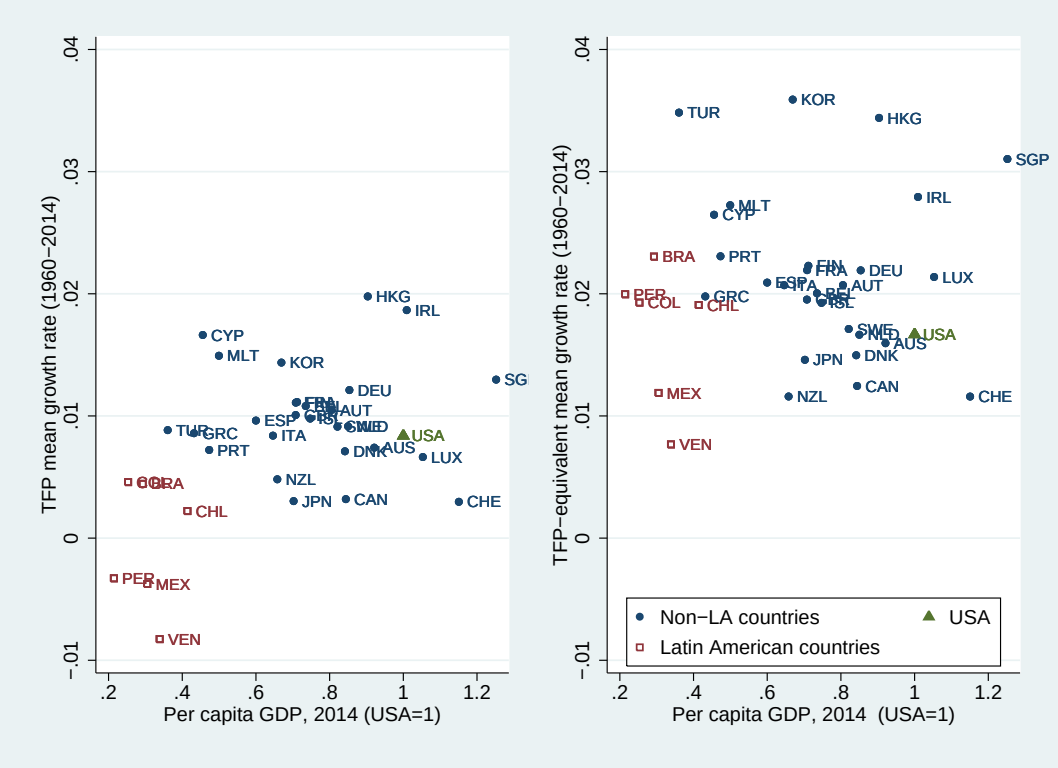
For welfare growth in TFP-equivalents the order is the same, but the growth rates are much closer to each other.

These numbers point to higher convergence in welfare, despite the poor performance for TFP growth in Latin America.

The next figures present disaggregated results by country, and we focus our comments on comparing Latin American countries to the US. Figure 1.3 plots both TFP and TFP-equivalent mean growth rates from 1960 to 2014 against per capita GDP relative to the US economy in 2014. Latin American countries and the US are highlighted from the other countries. The TFP plot just repeats Figure 1.1 for ease of comparison to the welfare TFP-equivalent.

By definition, as both years of schooling and life expectancy at birth have risen throughout this period in every country, the welfare TFP-equivalent growth is faster than TFP growth for all of them. The result that must be emphasized is that all the Latin American countries in our sample present slower mean TFP growth rates than the US economy but, except for Mexico and Venezuela, growth in TFP-equivalent is faster than in the US.

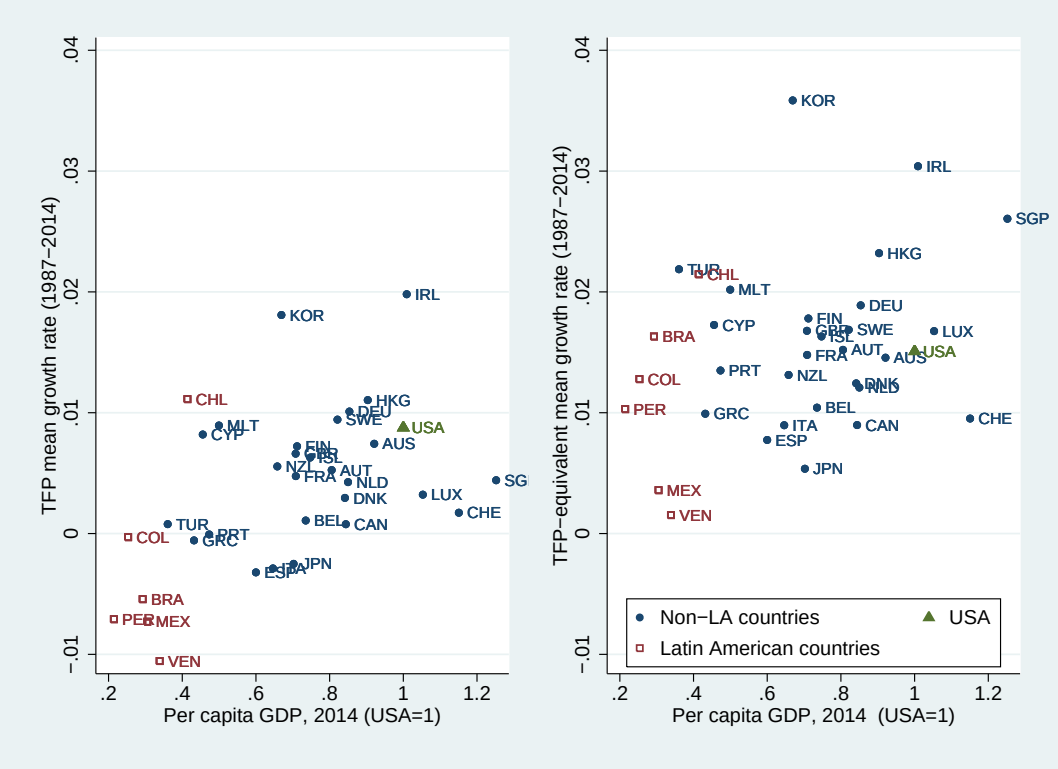
Figure 1.3: *TFP x TFP-equivalent (1960-2014)*



Notes: Panel a) reproduces Figure 1.1, comparing relative GDP in 2014 to mean TFP growth since 1960; Panel b) compares relative GDP in 2014 to mean TFP-equivalent growth since 1960. Data: PWT 9.0 and World Bank WDI, see previous section.

Figure 1.4 replicates the previous figure in a shorter time span, 1987 to 2014. From the TFP plot, we can see that only Chile presented positive mean TFP growth, which was higher than US productivity growth over these years. In welfare TFP-equivalents, growth for Brazil is also higher than in the US, and in Colombia and Peru it is close. TFP-equivalent growth is only far from that of the US in Mexico and Venezuela.

Figure 1.4: *TFP x TFP-equivalent (1987-2014)*

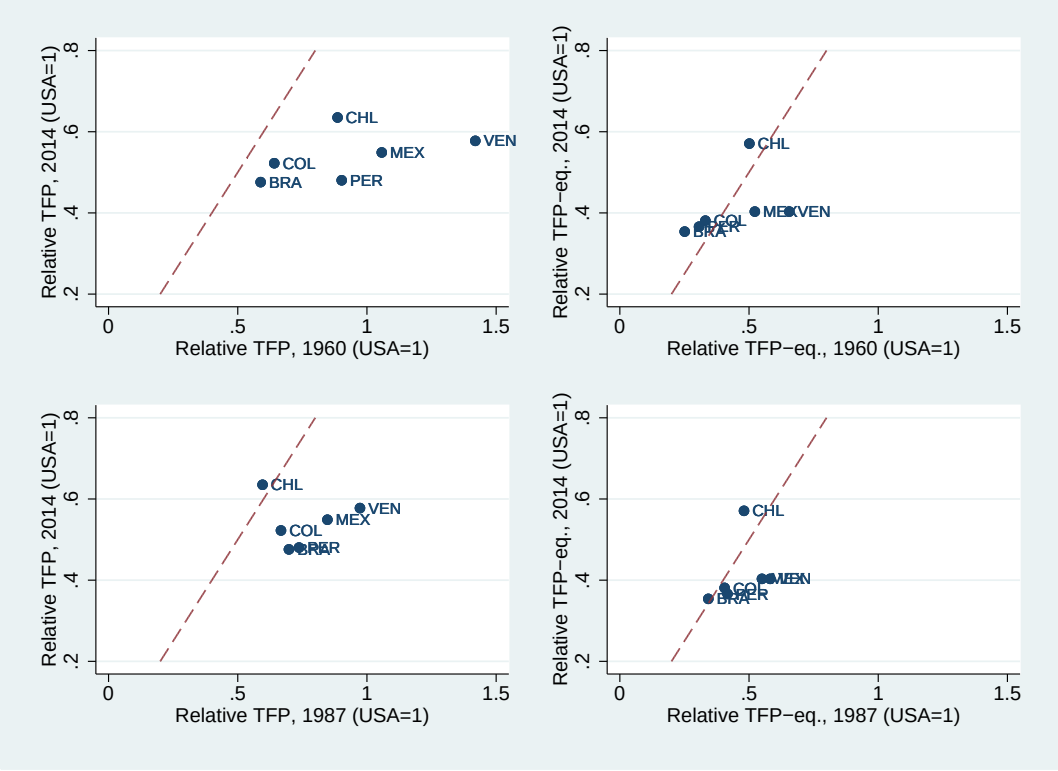


Notes: Panel a) reproduces Figure 1.2, comparing relative GDP in 2014 to mean TFP growth since 1987; Panel b) compares relative GDP in 2014 to mean TFP-equivalent growth since 1987. Data: PWT 9.0 and World Bank WDI, see previous section.

These results are presented from another perspective in Figure 1.5, which compares TFP and TFP-equivalent levels for the Latin American countries in our sample relative to the US. In each panel, countries at the left side of the 45º line had greater TFP or TFP-equivalent growth in the period under consideration. As is summarized by the figure, there is greater convergence between Latin American countries and the US in welfare than in TFP.

Finally, Table 1.3 reports some robustness checks for the US and the Latin American countries in our sample for welfare TFP-equivalent growth rates under alternative parameters, covering the period 1960-2014. In each alternative scenario, we test alternative targets to the physical capital share of GDP, the interest rate r, or the Value of a Statistical Life (VSL). Parameters

Figure 1.5: *Relative TFP and TFP-equivalent (1960 x 2014, 1987 x 2014)*



Notes: The panels in this figure compare levels of TFP (left side) or TFP-equivalent (right side), in 2014 versus 1960 (upper panels) or 1987 (lower panels), for Latin American countries. Data: PWT 9.0 and World Bank WDI, see previous section.

$\bar{u}$, δ, α, and ρ are calibrated again according to the corresponding changes to the targets.

Table 1.3: *Robustness checks, average TFP-equivalent growth, 1960-2014*

Baseline calibration								
Targets			*Parameters*				*1960-2014*	
k share	r	VSL	$\bar{u}$	α	δ	ρ	US	LA
0.626	0.05	6.3	5.73	0.374	0.068	0.026	1.67	1.82

Alternative scenarios, changes from baseline			
		1960-2014	
Targets	*Parameters*	US	LA
VSL = 5.5	$\bar{u} = 4.52$	1.60	1.65
VSL = 7.5	$\bar{u} = 7.54$	1.76	2.08
r = 0.04	$\bar{u} = 3.40, \delta = 0.078, \rho = 0.017$	1.64	1.68
k share = 0.67	$\bar{u} = 5.03, \alpha = 0.33, \delta = 0.054$	1.73	1.86

Notes: First line is the default calibration from Tables 1.2 and 1.1. The targets are capital share of GDP *kshare*, interest rate r, and the Value of a Statistical Life (VSL). Column "LA" shows the population-weighted means for the Latin American countries Brazil, Chile, Colombia, Mexico, Peru, and Venezuela.

In Table 1.3, the first line shows the baseline calibration. In the second and the third lines we change the VSL to the extremes of the range found in the survey by Viscusi and Aldy (2003),

$5.5 to $7.5 million dollars. Line four tests a yearly interest rate of 4%. Line five tests a higher capital share of GDP.

In all of the simulations, welfare TFP-equivalent growth is higher for this group of Latin American countries than for the US. In the scenarios that reduce the VSL target to 5.5 and the interest rate to 0.04, the TFP-equivalent growth rates for the US and Latin American countries are close. These are scenarios with the lowest values for the Rosen (1988) utility parameter $\bar{u}$, which reduces the welfare value of improvements in longevity. Nonetheless, our results do not change qualitatively.

1.6 Conclusions

This paper presented a new productivity-equivalent welfare measure to account for the value of increases in life expectancy and schooling in an overlapping generations perpetual youth economy. We calculated this measure for 34 countries for the periods 1960-2014 and 1987-2014, and compared it to TFP performance for this period, with a focus on six Latin American countries in our sample.

Our main result is that there is greater convergence for Latin American countries in relation to the US for this TFP-equivalent measure than for TFP.

There is heterogeneity among Latin American countries, as some of these countries have performed much better than others, but for most of them welfare TFP-equivalent growth was faster than in the US for the period 1960-2014, and for all of them the relative performance in this welfare measure was greater than that in TFP. This result was robust to changes in the targets for model calibration.

Appendix

1.A Proofs of propositions

Proof of Proposition 1.

A newborn consumer in time τ maximizes expected lifetime utility in equation (1.5) subject to prices $[w(t), r(t)]_{t=\tau}^{\infty}$, survival probability (1.6), assets accumulation constraint (1.7), initial assets $\omega(\tau|\tau) = 0$, and a non-Ponzi condition on assets accumulation.

Although the survival function is simple enough so that we could just add the constant death hazard rate η to the discount rate ρ, we will define the survival probability $S(t|\tau)$ as a state variable in the newborn dynamic optimization problem. By proceeding this way, we can use its associated costate variable q together with the costate variable μ for the assets variable $\omega(t|\tau)$ to calibrate the propensity to pay for increases in life expectancy, as we show in Proposition 3.[4] Then, by equation (1.6), the law of movement and the initial value for $S(t|\tau)$ are

$$\dot{S}(t|\tau) = -\eta S(t|\tau) \tag{1.A.1}$$

$$S(\tau|\tau) = 1 \tag{1.A.2}$$

The current-value Hamiltonian is

$$\mathcal{H} = S(t|\tau)u(c(t|\tau)) + \mu(t|\tau)\left[(r(t)+\eta)\omega(t|\tau) + w(t)\ell h - c(t|\tau)\right] - q(t|\tau)\eta S(t|\tau) \tag{1.A.3}$$

where the control variable is $c(t|\tau) \geq 0$ and the state variables are $\omega(t|\tau) \in \mathbb{R}$ and $S(t|\tau) > 0$. From the maximum principle, the necessary conditions for an interior solution are[5]

[4] Here we follow Jones (2016) in defining the conditional survival probability as a state variable. However, this approach of using the costate variable q to derive the value of a statistical life in Proposition 3 and calibrate the parameter $\bar{u}$ is new to our paper.

[5] For sufficiency, note that the Hamiltonian (1.A.3) is concave in (c, ω, S) and that $\lim_{t\to\infty} \mu(t|\tau)e^{-\rho(t-\tau)}\omega(t|\tau) \geq 0$ and $\lim_{t\to\infty} q(t|\tau)e^{-\rho(t-\tau)}S(t|\tau) \geq 0$ for any feasible solution (see theorem 7.14 in Acemoglu, 2008).

First order conditions:

$$u'(c(t|\tau)) = \frac{\mu(t|\tau)}{S(t|\tau)} \tag{1.A.4}$$

$$\dot{\mu}(t|\tau) = (\rho - \eta - r(t))\mu(t|\tau) \tag{1.A.5}$$

$$\dot{q}(t|\tau) = (\rho + \eta)q(t|\tau) - u(c(t|\tau)) \tag{1.A.6}$$

Transversality conditions:

$$\lim_{t\to\infty} \mu(t|\tau)e^{-\rho(t-\tau)}\omega(t|\tau) = 0 \tag{1.A.7}$$

$$\lim_{t\to\infty} q(t|\tau)e^{-\rho(t-\tau)}S(t|\tau) = 0 \tag{1.A.8}$$

Substitute the definition of the utility function $u(c) = \bar{u}+\log(c)$ and manipulate the first order conditions (1.A.4) and (1.A.5) to find the cohort-specific consumption Euler equation (1.23):

$$\frac{\dot{c}(t|\tau)}{c(t|\tau)} = r(t) - \rho$$

Integrate and rearrange the cohort-specific Euler equation (1.23) and first order condition (1.A.5), both evaluated at time $\upsilon \in [s,t]$, for $t \geq \tau$,

$$c(s|\tau) = c(t|\tau)e^{\int_t^s [r(\upsilon)-\rho]d\upsilon} \tag{1.A.9}$$

$$\mu(s|\tau) = \mu(t|\tau)e^{-\int_t^s [r(\upsilon)+\eta-\rho]d\upsilon} \tag{1.A.10}$$

Evaluate assets accumulation equation (1.7) at time s, multiply by $e^{-\int_t^s (r(\upsilon)+\eta)d\upsilon}$, and integrate in $[t,T]$ to find

$$\omega(T|\tau)e^{-\int_t^T (r(\upsilon)+\eta)d\upsilon} = \omega(t|\tau) + \int_t^T e^{-\int_t^s (r(\upsilon)+\eta)d\upsilon}(w(s)\ell h - c(s|\tau))ds \tag{1.A.11}$$

On the right hand side of (1.A.11), substitute (1.A.9). On the left hand side of (1.A.11), use that $e^{-\int_t^T [r(\upsilon)+\eta]d\upsilon} = \frac{\mu(T|\tau)e^{-\rho(T-\tau)}}{\mu(t|\tau)e^{-\rho(t-\tau)}}$ from (1.A.10) evaluated at T instead of s, and note that $\mu(t|\tau)e^{-\rho(t-\tau)} > 0$. Then taking the limit of (1.A.11) as $T \to \infty$ and using the transversality

condition (1.A.7), we can express the cohort-specific consumption as a function of total wealth

$$c(t|\tau) = (\rho + \eta)\left[\omega(t|\tau) + W(t)\ell h\right] \tag{1.A.12}$$
$$\text{for} \quad W(t) \equiv \int_t^\infty e^{-\int_t^s (r(\upsilon)+\eta)d\upsilon} w(s)ds$$

which results in equation (1.24) for $t = \tau$, as $\omega(\tau|\tau) = 0$.

Now we turn to aggregate consumption per worker. From definition (1.17), total consumption is

$$c(t)L(t) = \int_{-\infty}^t c(t|\tau)L(t|\tau)d\tau \tag{1.A.13}$$

which implies by taking logs and differentiating in t

$$\frac{\dot{c}(t)}{c(t)} + \frac{\dot{L}(t)}{L(t)} = \frac{1}{c(t)L(t)}\left[\int_{-\infty}^t [\dot{c}(t|\tau)L(t|\tau) + c(t|\tau)\dot{L}(t|\tau)]d\tau + c(t|t)L(t|t)\right] \tag{1.A.14}$$

Then combining equations (1.14), (1.15), (1.16), and (1.23) to (1.A.14), we find the aggregate consumption Euler equation

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho - n\left[\frac{c(t) - c(t|t)}{c(t)}\right] \tag{1.A.15}$$

where the last term is due to the newborn agents consuming less than the aggregate mean consumption.

Substitute (1.A.12), (1.18), and (1.19) in definition (1.17) to express aggregate per capita consumption as a function of per capita wealth

$$c(t) = (\rho + \eta)\left[k(t) + W(t)\ell h\right] \tag{1.A.16}$$

Combining equations (1.12), (1.A.12) and (1.A.16) to (1.A.15), and definition (1.9) to (1.20), the laws of movement of aggregate per capita consumption and per capital physical capital can be written as

$$\dot{k}(t) = A(t)k(t)^\alpha(\ell h)^{1-\alpha} - c(t) - (\delta + n - \eta)k(t) \tag{1.A.17}$$
$$\dot{c}(t) = \left[\alpha A(t)\left(\frac{\ell h}{k(t)}\right)^{1-\alpha} - \delta - \rho\right]c(t) - n(\rho + \eta)k(t) \tag{1.A.18}$$

Defining $\tilde{k}(t) \equiv \frac{k(t)}{A(t)^{\frac{1}{1-\alpha}}\ell h}$, $\tilde{c}(t) \equiv \frac{c(t)}{A(t)^{\frac{1}{1-\alpha}}\ell h}$, and $g \equiv \frac{\gamma}{1-\alpha}$, it is straightforward to show that (1.A.17) and (1.A.18) imply equations (1.21) and (1.22)

$$\frac{d\tilde{k}(t)}{dt} = \tilde{k}(t)^{\alpha} - \tilde{c}(t) - (g + \delta + n - \eta)\tilde{k}(t)$$
$$\frac{d\tilde{c}(t)}{dt} = \left[\alpha\tilde{k}(t)^{\alpha-1} - (g + \delta + \rho)\right]\tilde{c}(t) - n(\rho + \eta)\tilde{k}(t)$$

To find steady state equilibrium allocation $(\tilde{k}^*, \tilde{c}^*)$, set $\frac{d\tilde{k}(t)}{dt} = \frac{d\tilde{c}(t)}{dt} = 0$ and solve the system for $\tilde{k} > 0$. Two solutions arise, but only one is saddle-path stable.

Proof of Proposition 2.

Consider a newborn individual at time τ in an economy with current normalized capital equal to $\tilde{k}(\tau) = \tilde{k}^*$, the steady state equilibrium value.

From (1.1) and (1.23), utility flow along the steady state equilibrium is

$$u^*(c(t|\tau)) = \bar{u} + (r^* - \rho)(t - \tau) + \log(c(\tau|\tau)) \tag{1.A.19}$$

Then combining definitions (1.5), (1.6), and (1.A.19) and integrating, the expected lifetime utility is given by equation (1.25)

$$V^*(\tau) = \frac{1}{(\rho + \eta)}\left[\bar{u} + \log c^*(\tau|\tau) + \frac{r^* - \rho}{(\rho + \eta)}\right]$$

Just substitute normalized capital $\tilde{k}^*$ in (1.9), (1.11), and (1.12) to find that factor prices in steady state equilibrium are given by (1.28) and (1.27)

$$w^*(\tau) = (1 - \alpha)\tilde{k}^{*\alpha}A(\tau)^{\frac{1}{1-\alpha}}$$
$$r^* = \alpha\tilde{k}^{*\alpha-1} - \delta$$

As productivity $A(\tau)$ grows at a constant rate γ, and $g = \frac{\gamma}{1-\alpha}$,

$$w^*(t) = w^*(\tau)e^{g(t-\tau)} \tag{1.A.20}$$

for any time $t > \tau$.

Using equation (1.A.20), the discounted present value of total future wages is

$$\begin{aligned} W^*(\tau) &= \int_\tau^\infty e^{-\int_\tau^t (r^*+\eta)dv} w^*(t)dt \\ &= w^*(\tau)\int_\tau^\infty e^{-(r^*+\eta-g)(t-\tau)}dt \\ &= \frac{w^*(\tau)}{r^*+\eta-g} \end{aligned} \tag{1.A.21}$$

and substituting (1.A.21) in equation (1.24) results in (1.26)

$$c^*(\tau|\tau) = \frac{(\rho+\eta)w^*(\tau)\ell h}{r^*+\eta-g}$$

Now differentiate equation (1.25) in τ and use equation (1.28) to find (1.29)

$$\begin{aligned} \frac{dV^*(\tau)}{d\tau} &= \frac{1}{(\rho+\eta)}\frac{d\log c^*(\tau|\tau)}{d\tau} \\ &= \frac{1}{(\rho+\eta)}\frac{d\log w^*(\tau)}{d\tau} \\ &= \frac{\gamma}{(1-\alpha)(\rho+\eta)} \end{aligned}$$

Proof of Proposition 3. Recall from Proposition 1 the first order condition (1.A.6) and the transversality condition (1.A.8), both related to the survival probability $S(t|\tau)$:

$$\begin{aligned} \dot{q}(t|\tau) &= (\rho+\eta)q(t|\tau) - u(c(t|\tau)) \\ \lim_{t\to\infty} q(t|\tau)e^{-\rho(t-\tau)}S(t|\tau) &= 0 \end{aligned}$$

Rearrange equation (1.A.6) evaluated at time $s \geq t \geq \tau$, multiply it by $e^{-(\rho+\eta)(s-t)}$ and integrate in $[t,T]$:

$$-\int_t^T \left[\dot{q}(s|\tau) - q(s|\tau)(\rho+\eta)\right] e^{-(\rho+\eta)(s-t)}ds = \int_t^T e^{-(\rho+\eta)(s-t)}u(c(s|t))ds$$

or

$$q(t|\tau) - q(T|\tau)e^{-\rho(T-t)}S(T|\tau) = \int_t^T e^{-\rho(s-t)}S(s|\tau)u(c(s|\tau))ds$$

Taking the limit as $T \to \infty$ and using the transversality condition (1.A.8) implies

$$q(t|\tau) = \int_t^{\infty} e^{-\rho(s-t)} S(s|\tau) u(c(s|\tau)) ds \tag{1.A.22}$$

Equation (1.A.22) shows that the costate variable $q(t|\tau)$ equals the expected discounted utility for an individual born in τ and alive in t. Evaluating at $t = \tau$, we find that the costate $q(\tau|\tau)$ of a newborn agent equals lifetime expected utility:

$$q(\tau|\tau) = V(\tau) \tag{1.A.23}$$

The value of a statistical life $VSL(\tau)$, which measures the marginal disposition to trade wealth for survival probability, can be written as the reason between the respective costate variables in the Hamiltonian

$$VSL(\tau) = \frac{q(\tau|\tau)}{\mu(\tau|\tau)} = c(\tau|\tau) V(\tau) \tag{1.A.24}$$

Finally, for a newborn individual in an economy at the steady state equilibrium, we can substitute equation (1.25) in (1.A.24) and isolate $\bar{u}$ to find (1.32)

$$\bar{u} = (\rho + \eta) \frac{VSL(\tau)}{c^*(\tau|\tau)} - \log c^*(\tau|\tau) - \frac{r^* - \rho}{\rho + \eta}$$

Chapter 2

Life, Early Health Expenditures, and Structural Transformation

2.1 Introduction

Life expectancy at birth has been rising steadily around the world in recent decades, from a world mean of 52.6 years in 1960 to 72.6 years in 2018, according to the World Bank's World Development Indicators (WDI) database. This tendency was widespread, but more pronounced for latecomer countries, as the average increase in life expectancy in the period was 24 years for low and middle income countries, twice the average increase of 12 years for high income countries. Despite the still prevailing gaps in life expectancy between developing and developed countries,the convergence in welfare measures that combine both longevity and income differences was markedly greater than in per capita GDP alone (Becker et al., 2005; Jones and Klenow, 2016). There is a wide range of possible determinants of the falling mortality rate (Cutler et al., 2006), which varies across the countries and involves particular institutional issues.

In this paper, we ask how this trend is related to the rise in the share of health expenditures of GDP, and to what extent these historical and cross-country differences in both health spending and longevity can be explained by economic processes of structural transformation without resorting to institutional differences.

We use a model of overlapping generations, endogenous mortality, and growth with health-related structural transformation to compare optimal trajectories of life expectancy and the share of health spending in Brazil, a middle income developing country, with France and the United States, two developed countries with heterogeneous profiles regarding life expectancy, income,

and healthcare institutional settings. This paper makes two main contributions. The first is in elucidating the technological aspects behind the structural transformation related to the health sector, as opposed to the role of preferences. As a way to deal with problems in measuring real output and productivity growth in health services, we propose a new approach to infer total factor productivity gains (TFP) from two stylized facts on human aging, the Gompertz law and the Strehler-Mildvan correlation. The second main contribution is in comparing health-related structural change for heterogeneous countries in terms of economic development.

Structural transformation is the process of changing the relative participation of economic sectors in consumption and production, usually studied in the major sectors of agriculture, manufacturing and services (Herrendorf et al., 2014). In this paper, the structural transformation in question is the rising share of health expenditures, as well as the corresponding changes in the sectoral composition of the economy. One of the economic forces that drives structural transformation are non-homothetic preferences, which were used to explain the trend of increasing health spending in the USA by Hall and Jones (2007). The authors incorporate the preferences for evaluating life-threatening decisions proposed by Rosen (1988), which imply that the demand for goods that reduce mortality grows more than proportionally with income. Our paper assumes these same preferences. The death rate increases as the agents get older, but it can be mitigated by using health services and medical goods. Health spending grows more than proportionally with income, since agents become more willing to pay for a fall in their probability of dying the higher their level of consumption.

The other major driver of structural change operates through the heterogeneity in technological characteristics of the health and non-health sectors: differentials between sectors in TFP growth rates and in the intensity in the use of factors of production and intermediate inputs. Regarding the intensity in the use of capital and labor, Acemoglu and Guerrieri (2008) show in a two-sector model that capital deepening increases the relative output growth rate of the capital-intensive sector, but displaces workers to the labor-intensive sector due to the effects on relative prices. In our model, to remain consistent with the empirical evidence on labor shares, the health services sector is more labor intensive, while non-health production is more intensive in physical capital. The stock of physical capital is accumulated endogenously to the model, but there is also an exogenous trend of human capital growth, reflecting the rise in schooling.

Regarding the structural transformation caused by different productivity growth rates by sector, Baumol (1967), and more recently Ngai and Pissarides (2007), highlight that the higher

productivity growth in manufacturing, combined with a price inelastic demand in services, increases the relative share of services in the economy and reduces aggregate productivity growth. To verify the relevance of this mechanism in the rise of the share of health services in GDP, however, we are faced with an obstacle related to measuring productivity gains in this sector.

The healthcare sector, which comprises the majority of health expenditures, is usually depicted as an economic activity with stagnant or even declining productivity. However, major advances in medical technologies have taken place in recent decades. Cutler et al. (2006) call 'the era of big medicine' the period since the 1930s, characterized by a sharp drop in mortality rates in developed countries due to advances in medicine. Particularly since the 1960s, expensive and intensive personal medical treatments have reduced mortality from chronic conditions such as heart disease.

There is an ongoing debate (see Sheiner and Malinovskaya, 2016, for a review) about whether the lack of quality correction for product measurement underestimates productivity growth in health services. If a medical appointment today is more expensive than fifty years ago, this is typically measured as a higher price for the same good, notwithstanding the fact that today this same appointment has a greater chance of prolonging the patient's life. So there is a hidden productivity gain in the increase in health spending, if we consider that a medical appointment is in fact an input to produce what really matters to the consumers, a reduction in their probability of death. Diewert (2018) also points to the weaknesses in current measures of productivity in health services and suggests metrics that consider the success of medical treatments.

To deal with this problem, we propose a methodology to infer productivity gains and a functional form for longevity technology based on two empirical regularities on human aging, the Gompertz curve and the Strehler-Mildvan (SM) correlation. The Gompertz curve is the log-linear relationship between age and the death rate that accurately describes human mortality for individuals from 30 years of age onwards. The SM correlation is the negative correlation between the parameters of the Gompertz curve observed when comparing different populations. In population groups with lower mortality rates at age 30, there is a more accelerated increase in death rates with age, so that mortality rates converge for groups around age 100. We observe the SM correlation for 33 countries in 1960 and 2010 and we propose to interpret it as a technological frontier of medical advances, whose shifts reflect gains in TFP of the longevity technology. These regularities are widely known in population and biological studies, as presented by Gavrilov and Gavrilova (2005) and Strulik and Vollmer (2013), but to the best of our knowledge they have

never been used to infer technological gains in health production.

In our model, there are two stages in longevity production, as consumers buy medical goods and health services produced on the market and use them as intermediate inputs in a household production process that reduces their probability of death. Schooling as their own human capital is also an input to this household production. We show that the equilibrium allocation does not depend on how we identify TFP gains in longevity production as arising from the market or the household stages of production. Moreover, we write the optimal choices as functions of the parameters of this single stage longevity production function, and we compute equilibrium allocations without referring to the price of health services. Hence, in our numerical exercises we do not need to rely on the available price indexes for health services to define productivity indexes for this sector, as they are not necessary to numerically solve the model if we know the longevity production function.

Next, we estimate the parameters of this longevity production function in a panel data regression with data on mortality, physician's density, and years of schooling for 96 countries in five-year intervals from 1960 to 2010, for five-year age classes from 30 to 94 years. We use these parameters and calibrate the model to reproduce initial mortality in 1960 and the health spending trajectory for France from 1960 to 2015. We then simulate the model for the US and Brazil with most of the parameters kept the same as France, and perform counterfactual analysis and decompositions to compare health spending and life expectancy trajectories for the three countries.

Our findings largely show that TFP in longevity production is the main determinant of changes in life expectancy at age 30, explaining 70% to 80% of the differences in history and between the countries. As for the remaining differences in longevity, changes in health inputs are more relevant in Brazil than in France and the US, corresponding, in a historical decomposition, to 19%, versus 13% and 10%, respectively, of the rise in longevity between 1960 and 2015. Health expenditures as a share of GDP, however, are mostly determined by preferences for longevity improvements, as in Hall and Jones (2007).

This paper is structured as follows. In section 2, we introduce empirical facts on human aging and the longevity production function. In section 3, we present our model of growth with endogenous mortality and structural transformation. In section 4, we report the panel data estimates of longevity production. In section 5, we discuss model calibration for France, Brazil, and the US. Section 6 shows the results and section 7 summarizes the conclusions and suggestions

for further research.

2.2 Human aging

In this section, we first present two stylized facts on human aging, the Gompertz curve and the Strehler-Mildvan (SM) correlation, and some related theoretical formulations from the research on the biology of human longevity. Our presentation draws on previous work from Holger Strulik and his coauthors (Strulik, 2010; Strulik and Vollmer, 2013; Dalgaard and Strulik, 2014, 2017), who introduced this literature to economists, and on Gavrilov and Gavrilova (1991, 2005, 2019a).

Next, we introduce our approach to identify total factor productivity growth in health production from changes in the SM correlation, as well as our longevity production function, that reproduces these empirical regularities.

2.2.1 Stylized facts on human aging

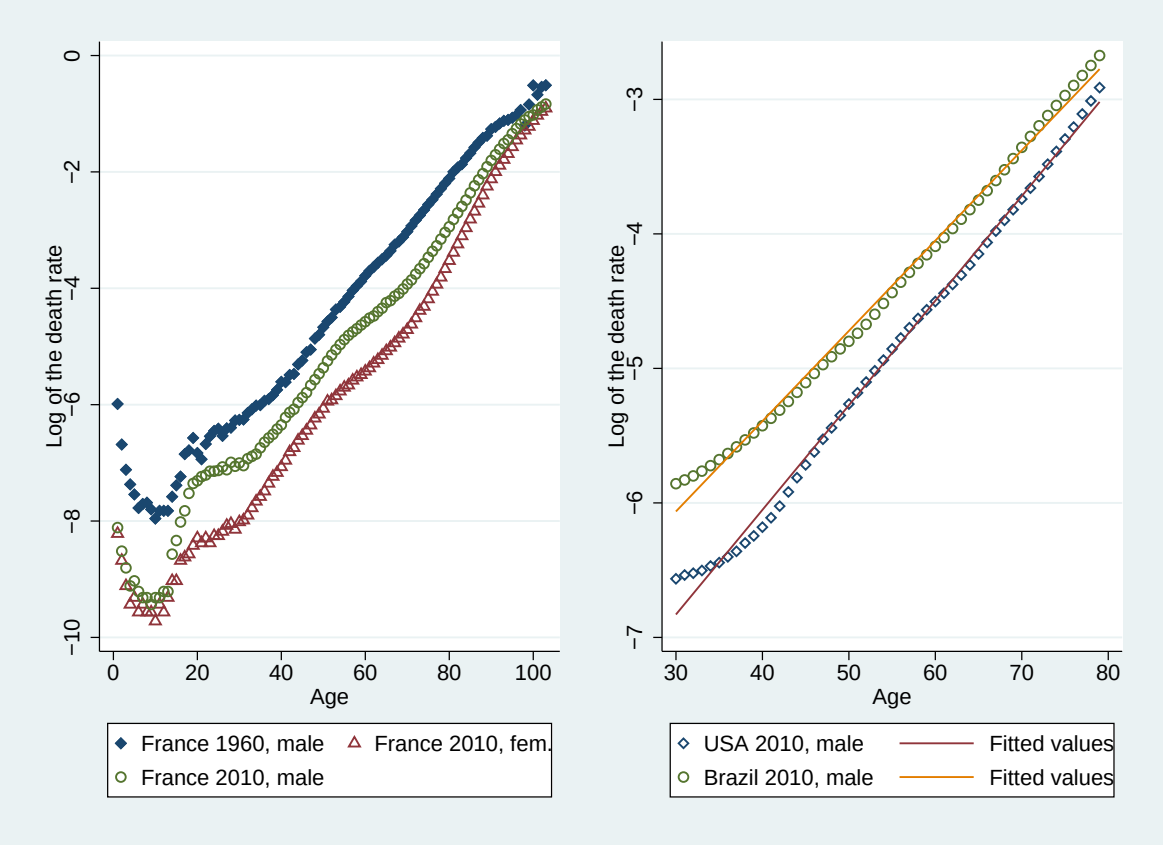
According to Gavrilov and Gavrilova (2005), the definition of aging in reliability theory is as appropriate for biological aging as it is for mechanical devices. Aging is the increase in the risk of failure of a system with time, or in longevity studies, the rise in the death rate of organisms as survival time increases. Gavrilov and Gavrilova (2005) claim that any successful theory of human aging needs to explain three mortality laws on human aging, the Gompertz law, the compensation law of mortality or Strehler-Mildvan correlation, and the late-life mortality plateau.

Panel a) in Figure 2.1 illustrates the general pattern of human aging with some mortality data for France in 1960 and 2010. Gompertz (1825) first noted that human mortality rates increase log-linearly with age from around 30 years old. This empirical regularity is the so-called Gompertz law, which is observed for males and females in different human societies. Panel b) in Figure 2.1 presents the Gompertz curve for males in Brazil and the US in 2010.

Let a denote years of age minus 30, and $\lambda(a)$ the *death hazard rate* or *force of mortality*, defined for each age in a continuous time framework as:

$$\lambda(a) \equiv \lim_{h \to 0} \frac{Pr(a \leq a_{\text{death}} < a + h | a_{\text{death}} \geq a)}{h} \tag{2.1}$$

Figure 2.1: *Human mortality: a) France 1960 and 2010; b) Gompertz Law, US and Brazil 2010*



Data: Human Life-Table Database (HLD)

So, for $a \geq 0$, the Gompertz curve is stated as:

$$\lambda(a) = Be^{\gamma a} \qquad \text{or} \qquad \log \lambda(a) = \log B + \gamma a \tag{2.2}$$

where B and γ are parameters that depend on environment or population groups. B is the base mortality parameter, which equals the death rate at $a = 0$, and γ is the aging speed parameter, that measures how fast the death rate rises as individuals get older.

Although the aging pattern expressed by the Gompertz law is common to all mankind, the parameters of equation (2.2) vary over time and across countries. Moreover, extensive evidence shows that there is an inverse relationship between these parameters: the lower the death rate in 30 years olds B, the greater the slope of the Gompertz curve, given by the aging speed parameter γ.

Despite the life expectancy gains in recent human history, the mortality rate for nonagenarians and centenarians remains high and is comparable to that of decades ago. In populations with lower base death rates, there is an accelerated increase in mortality rates over the years, so

that around the age of 100 there is an equalization with populations of higher base mortality rates. Thus, when we compare the Gompertz curves of two population groups, typically in the group with higher life expectancy at age 30 the base mortality is lower and the aging speed is higher. For instance, this pattern is evident when comparing females to males in the same country (Figure 2.1 panel a) or a developed to a developing country (Figure 2.1 panel b).

The relationship between the parameters of the Gompertz curve is called the Strehler-Mildvan correlation, after Strehler and Mildvan (1960), who noted the negative linear correlation between $\log B$ and γ for a group of 32 populations around the world with data for years around 1950.

Consider Φ and $\mathcal{A}$ as two positive parameters that are common to human populations, and B, γ the Gompertz law parameters, which are population-specific. The SM correlation can therefore be written as:

$$\log B = \log \Phi - \mathcal{A}\gamma \tag{2.3}$$

Combining equations (2.2) and (2.3), the Gompertz law in terms of the population-invariant parameters Φ and $\mathcal{A}$ is

$$\lambda = \Phi e^{\gamma(a-\mathcal{A})} \tag{2.4}$$

Strehler and Mildvan (1960) also propose a model of aging, in which an organism stays alive as long as its 'vitality' level is greater than zero. Vitality decays deterministically with age, and it reaches zero as $a = \mathcal{A}$, so that $\mathcal{A}$ is the maximum age attainable.

Gavrilov and Gavrilova (1991, 2005) argue that estimates of the SM correlation are biased because they do not consider the Makeham (1860) amendment to the Gompertz curve, which adds an age-independent constant to equation (2.2) in order to account for extrinsic causes of mortality. They name the version of SM correlation with this correction as the compensation law of mortality. Using more than 200 life tables from different countries, they estimate a value of 95 for parameter $\mathcal{A}$, and interpret it as the human life span in years. Strulik and Vollmer (2013) estimate this parameter for developed countries in different periods and they conclude that it has increased since the mid-twentieth century. For a group of 26 developed countries in 1975-1999, they find $\mathcal{A} = 96.5$, taking the mean for females and males.

The definition of $\mathcal{A}$ as the human life span parameter does not assume it as an upper bound of

human longevity, but that it is related to the end of the Gompertz law period of life. Regarding extreme old age, historical records show that the force of mortality increases at a slower pace than predicted by the Gompertz law and eventually becomes constant, so that late nonagenarians and centenarians apparently stop aging, the phenomenon of *negligible senescence* (Gavrilov and Gavrilova, 2005; Strulik, 2010). Nevertheless, Gavrilov and Gavrilova (2019b) claim that according to recent studies this mortality deceleration and mortality plateau in late-life may be spurious, resulting from a combination of misreporting of age by the elderly and a very small number of people surviving to extreme ages. Furthermore, more recent and reliable data on the US shows that the mortality rate continues to rise in a Gompertz pattern even at extreme old ages, which Gavrilov and Gavrilova (2019c) call "Gompertzialization" of old-age mortality.

As a way to explain these empirical regularities, Gavrilov and Gavrilova (1991, 2005) propose a theory of biological aging based on reliability theory, that was originally developed to study system failures in engineering applications. This approach models the aging of systems composed of non-aging elements, or how the increase in the failure rate of the system results from the linkages between parts with constant failure rates. To produce a rich aging pattern that generates the compensation law of mortality, the Gompertz law stage of life and a late-life deceleration of mortality, their model combines different types of connections between components and an unequal initial distribution of redundant parts between parallel blocks (Gavrilov and Gavrilova, 2005). Strulik (2010) and Strulik and Vollmer (2013) present another bio-gerontology approach to explain these facts on human aging, based on the accumulation of physiological deficits, and Dalgaard and Strulik (2014, 2017) propose economic models that build on this theory.

In this paper, we consider the Gompertz law and the SM correlation as established stylized facts on human aging and assume a functional form for longevity production that reproduces them. Therefore, any biological theory that explains why the Gompertz curve and the SM correlation hold should, in principle, be compatible with our model. As we are not interested in late-life mortality in our study, we adhere to the usual assumption in numerical economic models of a maximum lifetime of 100 years and assume that this is the value for the human lifespan parameter $\mathcal{A}$.

2.2.2 Technological progress and SM correlation parameters

The SM correlation parameters Φ and $\mathcal{A}$ were interpreted as species-invariant for studies that relied mostly on data from the first half of the 20th century, such as Gavrilov and Gavrilova

(1991), which uses data from before the 1960s. However, this view has been refuted by mortality trends since the second half of the 20th century. Estimated Gompertz curve parameters for some countries depicted that both base mortality B and aging speed γ have been lower in recent decades, which would not be possible if the SM correlation parameters Φ and $\mathcal{A}$ were stable; direct estimations of the SM correlation parameters have also shown that they have changed since the middle of the century (Yashin et al., 2001; Strulik and Vollmer, 2013).

Nevertheless, the instability of the SM correlation in the last decades does not mean that this relation is irrelevant, as in this same period major technological innovations in medicine played a determining role in mortality reductions (Cutler et al., 2006) and this may have changed these parameters related to human aging.

As a way to highlight the long-run differences in the SM correlation, we compare the curve as computed around two years separated by a 50 year timespan, 1960 and 2010. We estimate Gompertz law parameters B and γ for 33 countries with some life-tables available for both 1958-1962 and 2008-2012 periods in the Human Life-Table Database (HLD). Our sample in each country/year comprises males from 30 to 90 years old. Results for females are similar.

We use the HLD abridged life tables with five-year age intervals, which is also assumed later as the period length in the calibration of our model. The Gompertz curve in equation (2.2) is a continuous time model of aging. As stated by the survival analysis literature (e.g. Jenkins, 2005), grouping a continuous time Gompertz model in discrete time intervals of ages a under the assumption of constant mortality in each interval results in

$$\text{cloglog}(\eta) = \log B + \gamma a \tag{2.5}$$

where $\text{cloglog}(\eta)$ is the *complimentary log-log link* of the discrete time death hazard rate η

$$\text{cloglog}(\eta) \equiv \log(-\log(1-\eta)) \tag{2.6}$$

$$\eta \equiv P(a_{\text{death}} = a | a_{\text{death}} \geq a) \tag{2.7}$$

By estimating the discrete time Gompertz curve in equation (2.5) twice for each country, for the years around 1960 and 2010, we find 66 pairs (B, γ) of parameters. Figure 2.2 presents our estimations of these Gompertz law parameters and a linear fit for each period, which corresponds to the SM correlation for the years 1960 and 2010.[1]

[1] In the Gompertz curve estimates in Figure 2.2 there is no specific treatment for dealing with extrinsic mortality,

Figure 2.2: *Strehler-Mildvan correlation, 1960 and 2010*

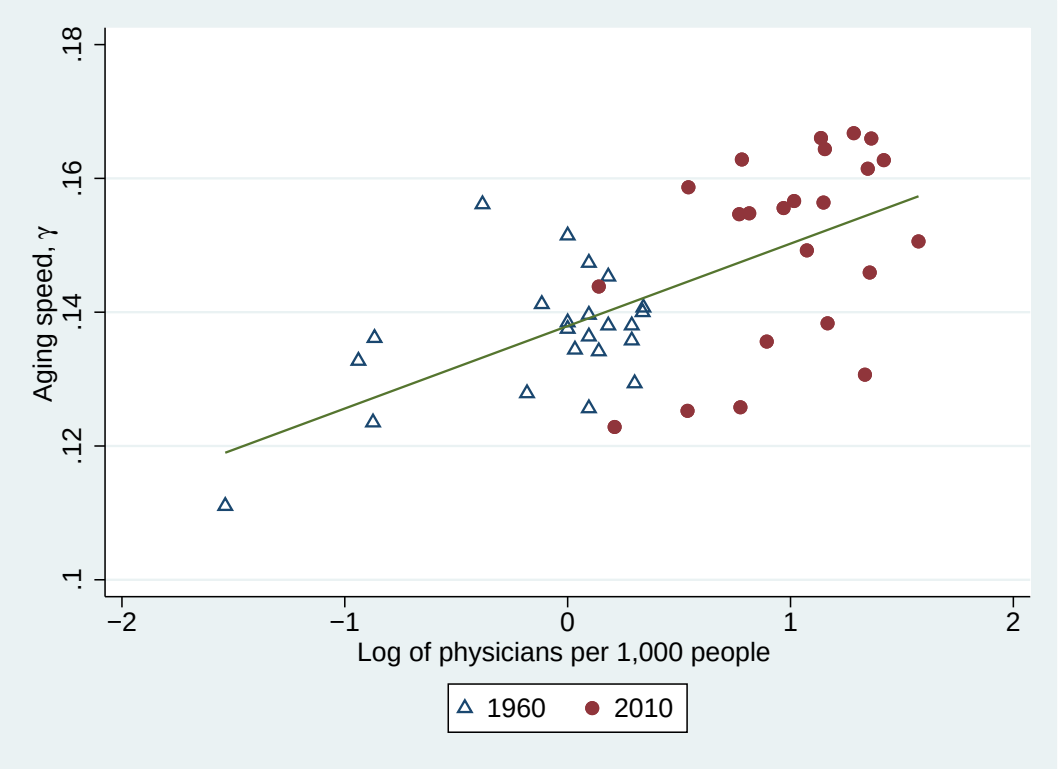
Notes: Gompertz curve parameters for males aged 30 to 90 years old, for 33 countries with life tables for both 1958-1962 and 2008-2012 periods in the Human Life-Table Database (HLD). Countries: Argentina, Australia, Austria, Belgium, Bulgaria, Canada, Costa Rica, Cyprus, Czech Republic, Denmark, Estonia, Finland, France Iceland, Ireland, Italy, Japan, Luxembourg, Malta, Mauritius, Netherlands, New Zealand, Norway, Poland, Portugal, Singapore, Slovenia, Spain, Sweden, Switzerland, Taiwan, Trinidad and Tobago, and the United States

For the 23 of these countries which also have data on physicians' density for the years 1960 and 2010 in the World Bank's WDI, we plot the log of physicians' density versus aging speed in 1960 and 2015. Figure 2.3 illustrates the correlation between health spending and the aging speed γ.

We interpret the SM correlation as a technology frontier for health production, and this curve displacement in Figure 2.2 as a result of total factor productivity (TFP) change in health production. At each moment, the SM correlation is determined by the state-of-the-art in medicine, and differences in health inputs result in movements along the SM curve, reducing the base mortality B but increasing the aging speed γ (see Figure 2.3) since mortality near the maximum lifespan is given. Over the years, productivity growth reduces the intercept of the SM correlation, Φ in equation (2.4), which reduces base mortality without increasing aging speed as late-life mortality

such as a Makeham age-independent mortality term. In our panel regression estimation, we deal with this issue by removing deaths by external causes from total mortality using the World Health Organization mortality database, as explained in the econometric section of the paper.

Figure 2.3: *Physicians' density versus aging speed, 1960 and 2010*



Notes: physicians by 1,000 people from World Bank development indicators, 23 countries, Gompertz curve aging speed parameter γ from Figure 2.2. Countries: Argentina, Australia, Austria, Belgium, Bulgaria, Canada, Costa Rica, Cyprus, Iceland, Japan, Luxembourg, Malta, Mauritius, Netherlands, New Zealand, Norway, Poland, Portugal, Singapore, Sweden, Switzerland, Trinidad and Tobago, and the United States.

is lower. The parallel displacement of this curve is compatible with an assumed constant human lifespan $\mathcal{A}$, since the slope of the SM correlation is equal to the absolute value of this parameter.

Let X denote a broad measure of health inputs, including visits to physicians and the effects of schooling on mortality. The technology that produces reductions in death probabilities operates according to:

$$\text{cloglog}(\eta) = \log \Phi_t + \gamma(a - \mathcal{A}) \tag{2.8}$$

$$\gamma = \gamma_0 + \gamma_1 \log X \tag{2.9}$$

for $0 < \gamma_0, \gamma_1 < 1$.

Equation (2.8) is the discrete time version of equation (2.4), and equation (2.9) represents the correlation in Figure 2.3. In this technology, higher TFP reduces Φ_t, and health inputs cause an increase in the aging parameter γ, which reduces the death rate η as age a is lower than

maximum life span $\mathcal{A}$.

Combining equations (2.8) and (2.9) results in a decreasing returns to scale Cobb-Douglas production function:

$$\vartheta = DX^{\gamma_1(\mathcal{A}-a)} \tag{2.10}$$

were

$$D = {\Phi_t}^{-1} e^{\gamma_0(\mathcal{A}-a)}$$

$$\text{cloglog}(\eta) = -\log \vartheta$$

We call this expression the *longevity production function*. As in Jones (2016), other sources of utility provided by the health sector but not related to longevity are ignored. The model in the next section differentiates market and household stages of longevity production, and we will call health sector the market stage of production from now on. The measure of health status ϑ that is the output of this production process is named *vitality* after Strehler and Mildvan (1960), who interpreted the empirical regularities on human aging as a process of decaying vitality. In our model, vitality is inversely proportional to the death rate η.

In equation (2.10), longevity inputs X are a mix of own human capital and production factors allocated to health services. The aging process increases death rates by reducing the longevity TFP D and the degree of returns to scale $\gamma_1(\mathcal{A}-a)$. Over time, technical progress causes an increase in TFP D as Φ_t is reduced.

It is interesting to note that the functional form (2.10) resembles the health status production function given by Hall and Jones (2007), as they assume age-specific parameters which are equivalent to $D_a = e^{\gamma_0(\mathcal{A}-a)}$ and $\gamma_a = \gamma_1(\mathcal{A}-a)$. However they estimate these two parameters D_a, γ_a for each age class, which would correspond to 26 parameters with our 30-34 to 90-94 age classes. The functional form (2.10) is considerably more parsimonious, since it demands only the maximum lifespan $\mathcal{A}$ and the two parameters γ_0 and γ_1. In Appendix 2.A, we compare the longevity production function (2.10) to other models of health production in the literature

2.3 Model

Our framework is an overlapping generations model with two nonstandard features. The first is the longevity production function, that builds on the evidence on human aging provided in the previous section and has two stages of production, market and household. Two health-related outputs are produced by firms in the market stage: medical goods and healthcare services. Consumers use both of them in addition to their own human capital as inputs in the household stage of longevity production, and the output of this household production process is inversely related to their death probability.[2]

The second feature is the modeling of growth with structural transformation in an overlapping generations economy. There are two sectors of market production in the model, health services and a sector that produces a general purpose good, and production in these sectors is related through input-output linkages. Structural change here refers to the rise in the share of health services in GDP, due to non-homothetic preferences as in Hall and Jones (2007) and to supply-side differences between the sectors, in productivity growth rates and capital intensity.

2.3.1 Consumers

The economy is populated by agents that are heterogeneous only in age. As we are interested in the period of life that refers to the Gompertz law, we assume that individuals are 30 years old at the initial age $a = 0$, they face a death probability that rises as they get older, and the maximum attainable age is $a = \mathcal{A}$ years, corresponding to 100 years old or $\mathcal{A} = 14$ as the variable a measures age in five-year intervals. Agents are workers up until 65 years old, or $a_R = 7$, and retirees after this age.

In each period, individuals maximize utility by choosing consumption c, medical goods m, healthcare services z, and next period assets ω'. They know their age a, assets ω, and the aggregate state of the economy $s = \left(t, K, H, \{\omega(a), N(a), ed(a,t), \bar{\eta}(a,t)\}_{a<\mathcal{A}}\right)$, which includes current time t, stocks of physical and aggregate human capital per capita (K, H), and age distributions for assets $\omega(a)$, population $N(a)$, years of schooling by cohort $ed(a,t)$, and death rates from external causes by cohort $\bar{\eta}(a,t)$.[3]

[2] This characterization of health as the output of a household production function that uses the medical care produced in the market and the consumer's educational capital as inputs was first proposed by Grossman (1972a,b). See Appendix 2.A for a comparison of our approach to Grossman's health capital model.

[3] In the recursive problem we only use time indexes for variables that are determined exogenously to the model.

As in Rosen (1988) and Hall and Jones (2007), the period-by-period utility function is

$$u(c) = \bar{u} + \frac{c^{1-\sigma}}{1-\sigma} \tag{2.11}$$

where $\bar{u}$ is a constant related to the value of being alive. Rosen (1988) claims that preferences are inherently state-dependent in problems regarding life-risk valuation, since the agent must compare his utility in the state of being dead versus being alive. If the utility flow of a dead individual is a constant $-\bar{u}$, normalizing to zero the utility in this state of nature requires summing the constant $\bar{u}$ to the period-by-period utility of alive agents, as in equation (2.11). These preferences imply that the willingness to pay for reduced mortality rates increases with the consumption level c and age, as shown by Rosen (1988). Hall and Jones (2007) use this utility to explain the rising share in GDP of health expenditures.

Workers' labor supply is inelastic, and labor income depends on human capital h and experience χ, that yields from years of schooling and work experience in a Mincerian way:

$$\log h = \psi_0 ed(a,t) \tag{2.12}$$

$$\log \chi = \psi_1 a - \psi_2 a^2 \tag{2.13}$$

where ψ_0 is the returns to schooling parameter, ψ_1 and $\psi_2 > 0$ are the returns to experience parameters, and $ed(a,t)$ denotes years of schooling, which is exogenous to our model and fixed at the agents' birth year $t-a$.

The probability that the consumer will not live to the next period as a result of health issues η is inversely related to vitality ϑ, which is determined by medical goods m, health services z, own human capital h, current time t, and age a according to a functional form coherent to the empirical evidence on human aging presented in the previous section:

$$\vartheta = \Gamma(a,t)\left[(z^{\varepsilon}m^{1-\varepsilon})^{\mu}h^{1-\mu}\right]^{\gamma_1(\mathcal{A}-a)} \tag{2.14}$$

At the initial age $a = 0$, consumers have no assets, $\omega = 0$. Markets are complete and agents can borrow up to their natural debt limit $\underline{\omega}(a,s)$. In all periods they get payments from an annuity, that are proportional to their assets and depend on the equilibrium total death rate for their age, η^*. These annuities are supplied by insurance firms that receive the consumer's assets at death. Zero profits for insurers imply that the annuity's periodic payment to a consumer with

assets ω and a years old is equal to $\frac{\eta^*(a,s)}{1-\eta^*(a,s)}\omega$.

The consumer's problem is:

$$V(a,\omega,s) = \max_{\omega',c,m,z} \left\{u(c) + \bar{\beta}V(a',\omega',s')\right\} \tag{2.15}$$

subject to

$$\omega' = R\omega + 1_{a<a_R} w(s)h\chi - c - m - p(s)z \tag{2.16}$$

$$R = 1 + r(s) + \frac{\eta^*(a,s)}{1-\eta^*(a,s)} \tag{2.17}$$

$$\eta = 1 - e^{-\frac{1}{\vartheta}} \tag{2.18}$$

$$\bar{\beta} = \beta\left(1 - \eta - \bar{\eta}(a,s)\right) \tag{2.19}$$

$$s' = F(s) \tag{2.20}$$

$$\omega' \geq \underline{\omega}(a,s) \tag{2.21}$$

$$c,m,z \geq 0 \tag{2.22}$$

where a prime indicates next period variables, $r(s)$ is the interest rate, $w(s)$ is the wage rate, and $p(s)$ is the price of health services. As a simplifying assumption, we assume that the numeraire good is used both for consumption c and as a medical input m. In equation (2.16), $1_{a<a_R}$ is the indicator function for workers, which are under the age of retirement a_R. Equation (2.20) states the law of motion for the aggregate state $F(s)$, which is known by the agents. Equation (2.18), that relates the death probability to the vitality level, was derived in the last section with equation (2.10).

2.3.2 Firms

There are two sectors in the economy, $j = y, z$. Sector y produces a general purpose good used for consumption, medical goods, investment in physical capital, and intermediate inputs for both sectors. Sector z supplies health services.

As in Kehoe et al. (2018), firms produce gross output Y or Z combining value added Q_j and intermediate inputs I_j. In both sectors, competitive firms use a constant returns to scale

Cobb-Douglas production function.

$$Y = \Lambda_y Q_y^{\theta_y} I_y^{1-\theta_y} \qquad \text{and} \qquad Z = \Lambda_z Q_z^{\theta_z} I_z^{1-\theta_z} \tag{2.23}$$

$$Q_y = K_y^{\alpha_y} (\Omega_y(t) H_y)^{1-\alpha_y} \qquad \text{and} \qquad Q_z = K_z^{\alpha_z} (\Omega_z(t) H_z)^{1-\alpha_z} \tag{2.24}$$

where $\Omega_j(t)$ is labor-augmenting productivity, K_j is physical capital, H_j is human capital including both schooling and experience, and Λ_j, θ_j, and α_j are sector-specific parameters. All variables are set in per capita units.

Productivity increases at the labor-augmenting technical change rate g_j in each sector:

$$\Omega_j(t+1) = e^{g_j} \Omega_j(t) \qquad \text{for} \qquad \Omega_j(0) = 1 \tag{2.25}$$

2.3.3 Equilibrium

A *competitive recursive equilibrium* consists of a value function V, policy functions for consumption c, medical goods m, health services z, and next period assets ω', price functions p, w, r, and a law of motion F for aggregate states such that:

i For given prices, the value function and policy functions solve the consumer's problem.

ii Prices satisfy profits' maximization by firms.

iii There is market clearing for both goods and factors of production.

iv The law of motion for aggregate states s is induced by consumers and firms' solutions, exogenous productivity growth, and demography.

Let N denote total population, $n \equiv \frac{N_a}{N}$ the population weights by age, and δ the depreciation rate for physical capital, then market clearing for outputs and factors requires that for each t:

$$Y = \sum_{a<\mathcal{A}} n(c+m) + I_y + I_z + \frac{N'}{N} K' - (1-\delta)K \tag{2.26}$$

$$Z = \sum_{a<\mathcal{A}} nz, \qquad K = \sum_{a<\mathcal{A}} n\omega, \qquad H = \sum_{a<a_R} nh\chi$$

where c, m, z, ω, h and χ are age-specific variables that are either outcomes of the consumer's policy functions or originate from the human capital equations (2.12) and (2.13).

2.3.4 Productivity and measurement in longevity production

We model survival probability as the outcome of a household longevity production function, which employs health services z and medical goods m produced by firms as intermediate inputs. When consumers choose their levels of health expenditures, they know the relative price p and the quantities z and m that they buy.

Nevertheless, to bring our model to the data, we have to deal with the problems related to measuring of healthcare services production, as discussed by Sheiner and Malinovskaya (2016) and Diewert (2018).[4]

In terms of the variables of our model, we can infer from data the nominal expenditure in health services pZ, however price p and quantity Z indexes are disputable. Hence, for a given vector of age-specific death rates $\{\eta_a\}_{a=0}^{\mathcal{A}-1}$ and factor inputs (K_z, H_z) used in health services, we are unable to distinguish health services productivity $\Omega_z(t)$ from productivity growth in household vitality production $\Gamma(t,a)$.

The next proposition shows that in equilibrium we can write longevity production as a single stage production function, in which vitality or survival probability is determined by physical and human capital employed in health services. This is relevant since it allows us to compare the reduced form longevity production function in proposition 1 directly to the empirical evidence in human aging discussed in the previous section. We do not need to rely on the available price deflators and quantity indexes for healthcare services production, nor assume alternative ones, to calibrate our model and find optimal choices for survival probabilities, allocation of factor inputs between sectors, and even the share of health services in GDP.

This proposition implies that, for any given trajectory for longevity TFP $D(t,a)$ and productivity $\Omega_y(t)$ in sector y, the same optimal allocation is consistent with alternative identification to health services prices p and productivity $\Omega_z(t)$, and $\Gamma(t,a)$ is computed as a residual. Different approaches to define price and quantity indexes in health services change GDP accounting and measured aggregate productivity, but do not change optimal choices, welfare, and GDP shares.

Propositon 1. *At the competitive recursive equilibrium, we can write optimal choices as functions of the interest rate r_t and the GDP share of the non-health sector ν, instead of prices r, w, p. Furthermore, in each period t, longevity production as a function of factor inputs in*

[4]We have assumed that the numeraire good can be converted into units of medical goods without costs, as healthcare services are the main focus of concerns regarding measurement issues in health production.

health services K_z and H_z, relative health spending by age ξ, and own human capital h equals:

$$\vartheta = D(a,t)\left[(\xi x)^{\mu} h^{1-\mu}\right]^{\gamma_1(\mathcal{A}-a)} \tag{2.27}$$

where

$$\begin{aligned}
\xi &\equiv \frac{z}{Z} \\
x &\equiv {K_z}^{\lambda} {H_z}^{1-\lambda} \\
\lambda &\equiv \varepsilon\theta_z\alpha_z + (1-\varepsilon\theta_z)\alpha_y \\
D(a,t) &\equiv \Gamma(a,t)\left[\Psi\Omega_z(t)^{\varepsilon\theta_z(1-\alpha_z)}\Omega_y(t)^{(1-\varepsilon\theta_z)(1-\alpha_y)}\right]^{\mu\gamma_1(\mathcal{A}-a)} \\
\Psi &\equiv \left(\frac{1-\varepsilon}{\varepsilon}\right)^{(1-\varepsilon)}\left[\frac{\theta_y}{\theta_z}\left(\frac{\alpha_y}{\alpha_z}\right)^{\alpha_y}\left(\frac{1-\alpha_y}{1-\alpha_z}\right)^{1-\alpha_y}\right]^{1-\varepsilon\theta_z}\Theta_y^{1-\varepsilon\theta_z}\Theta_z^{\varepsilon\theta_z} \\
\Theta_j &\equiv \left[\Lambda_j(1-\theta_j)^{1-\theta_j}\right]^{\frac{1}{\theta_j}}, \qquad \textit{for } j = y, z
\end{aligned}$$

Proof. See Appendix 2.B.

We combine (2.27) with the expression for longevity TFP $D(t,a)$ in equation (2.10) to find an expression of the longevity production function which we can estimate from data on human aging:

$$\vartheta = e^{\phi_0 + g_\Phi t + \gamma_0(\mathcal{A}-a)}\left[(\xi x)^{\mu} h^{1-\mu}\right]^{\gamma_1(\mathcal{A}-a)} \tag{2.28}$$

where $\log\Phi = -\phi_0 - g_\Phi t$ is the log of the intercept of the SM correlation, which is inversely related to the TFP growth rate in longevity production g_Φ and to the initial level of this productivity ϕ_0.

Thus, if we use data to determine the reduced form productivity $D(t,a) = e^{\phi_0 + g_\Phi t + \gamma_0(\mathcal{A}-a)}$, we can use it to numerically solve our model without specifying productivity in health services $\Omega_z(t)$ and productivity in household vitality production $\Gamma(t,a)$. This is possible given that we can find the solutions for consumers' and firms' problems as functions of the interest rate and the GDP shares of the two sectors, replacing the interest rate, the wage rate and the price of health services.

2.4 Longevity production function estimation

We estimate the parameters of the longevity production function (2.28) in a panel data regression with mortality data for 96 countries, in five-year intervals from 1960 to 2010, comprising 11 periods. For each country, data is segmented by sex and by up to 13 age classes of five years, from 30 to 94 years, corresponding to the Gompertz law period of life. As data is missing for many country/year/age combinations, we have an unbalanced panel with 10,040 observations.[5] We chose the Hausman-Taylor instrumental variables estimator as our preferred specification.

2.4.1 Variables and data

Data on the number of deaths by sex, age group, country and year is taken from the World Health Organization (WHO) mortality database. This source presents deaths disaggregated by causes, according to different editions of the International Classification of Diseases (ICD 7 to 10).

Our estimation of the longevity production function relies on the original Gompertz law, instead of the Gompertz-Makeham version that includes a constant to account for extrinsic causes of mortality as in Gavrilov and Gavrilova (1991) and Strulik and Vollmer (2013). To avoid a biased estimation of the Strehler-Mildvan correlation, we remove from total deaths those motivated by external causes, such as homicides, suicide and accidents. Data on deaths by external causes is used in model calibration to account for the exogenous variable $\bar{\eta}$ of extrinsic death rate. By proceeding in this way, we keep a linear functional form in our regressions, which is the same procedure as carried out by Hall and Jones (2007) in estimating mortality regressions in the US.[6]

Population data in the WHO mortality database is only available for some of the countries and years, thus we employ a uniform procedure for the entire sample, using only population data from the United Nations (UN) world population prospects. Death rates η are computed

[5] There are two main determinants of data availability. The first is the number of periods covered for each country, which ranges from 3 to 11 due to missing mortality or physicians' density data in some years. The second is the lack of mortality data for some age classes, as the maximum age with data available varies by country and year. Most of the country/year pairs have data for at least the age class of 80-85 years, but for a few of them the maximum class is 65-69 or 70-74 years, and classes 85-89 and 90-94 are missing for most of the countries before 2000.

[6] Extrinsic causes of mortality include external causes such as accidents and homicides, but also other causes related to the environment such as acute infections (Gavrilov and Gavrilova, 2005). We ignore this distinction here, as deaths from external causes are our best available proxy for extrinsic mortality. Furthermore, as we observe in our sample, the Makeham approach of assuming an age-independent rate for extrinsic mortality is also an imperfect approximation, because the death rate by external causes typically rises at an exponential rate in the population over 60.

combining the net deaths excluding those from external causes from WHO with UN population data.[7]

We build the human capital variable h from the Barro and Lee (2013) dataset on mean years of schooling by five-year age categories, separated for females and males, from 1960 to 2010. As in the model, we calculate human capital through the Mincerian approach, $h = e^{\psi_0 ed}$, considering a typical value for returns to schooling of $\psi_0 = 0.10$ per year, and ed as years of schooling. By using this variable instead of just years of schooling in the regressions, we can interpret the estimated coefficient directly in terms of the human capital variable of the model.

As our health inputs variable x, we use physicians per 1,000 people from the World Bank development indicators. This is our only variable of factor inputs for health production. First, because of the lack of an internationally comparable time series for equipment in health production. There is a series of hospital beds per 1,000 people in the World Bank development indicators database that in principle could be used as a proxy for physical capital. But for many countries, including Brazil, France, the United States, and considering the world mean, this series shows a trend of falling hospital beds per capita throughout the years. For this reason, we reject this series as a reliable proxy for physical capital in the health sector, as it is in opposition to the evidence of rising health spending and physicians per capita. Second, physicians per 1,000 people is a reasonable proxy for human capital resources allocated to health production, but we can reason that it is also related in some sense to physical capital, as health services in countries that use more medical equipment arguably demand relatively more physicians and other highly skilled workers compared to lower skilled health workers.

2.4.2 Regression specification and results

In the model, the time of death is uncertain, but as the agents of each cohort are homogeneous, the share of deaths by cohort is deterministic in equilibrium. In order to estimate the longevity production function (2.28), we assume that the log of the Φ component of TFP in longevity production, $\phi_{it} = -\log \Phi_{it}$, contains a stochastic disturbance term and is decomposed as:

[7] To compute death rates, we consider the definitions of mortality rates from the abridged life tables of the UN world population prospects. The age-specific central death rate is $m_a = \text{deaths}_a/\text{population}_a$, where deaths_a is the total number of deaths in the five-year period and population_a is population in the middle of the period. We assume that the average number of years lived by those who die at any time interval of 5 years is half of the interval, 2.5 years, at all ages. The death rate is the probability of dying in the next interval, computed as $\eta_a = \frac{1}{\left(1+\frac{1}{5}\left(\frac{1}{m_a}-2.5\right)\right)}$

$$\phi_{it} = \phi_0 + \phi_j + \phi_m + \phi_i + g_\Phi t + \varepsilon_{it} \tag{2.29}$$

where ε_{it} is an idiosyncratic zero mean and constant variance iid error. The individual index i comprises permanent country j, sex m, and age a differences that are not explained by the other parameters. Country ϕ_j and sex ϕ_m specific effects are fixed, while the individual effect ϕ_i will be assumed as exogenously fixed or randomly drawn from a distribution depending on econometric specification.

Therefore, the equation to be estimated is:

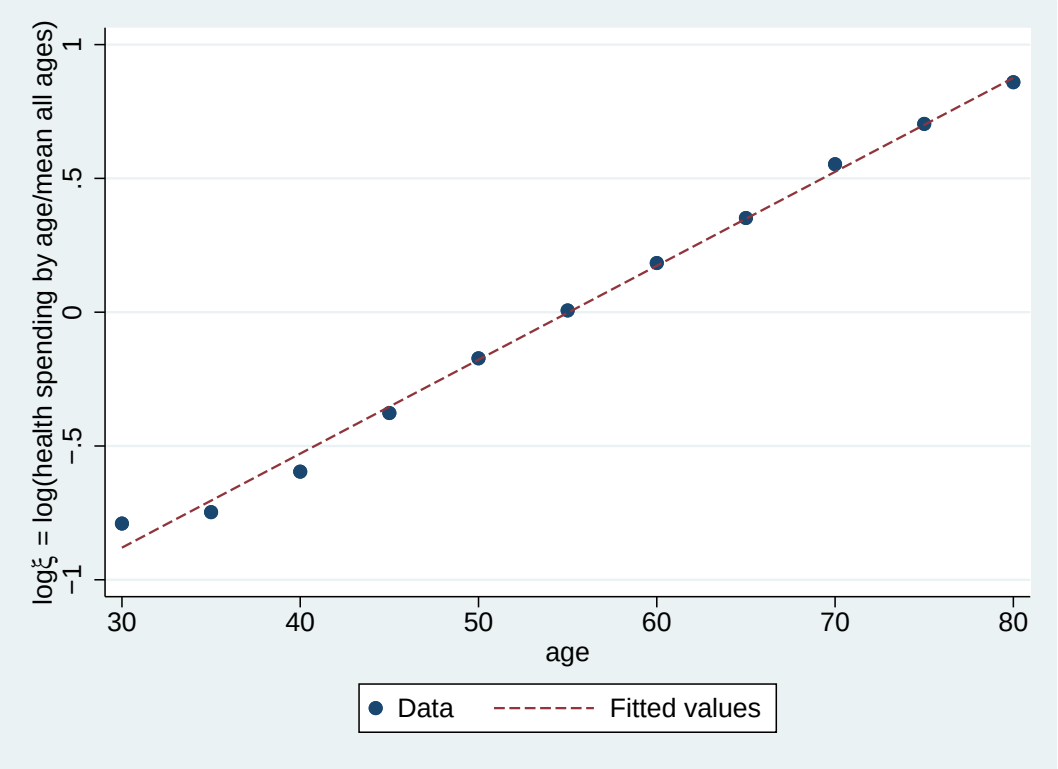
$$\begin{aligned} v_{it} &= \phi_0 + g_\Phi t + \gamma_0 \bar{a} + \gamma_1 \bar{a}[\mu(\log x_t + \log \xi_{it}) + (1-\mu)\log h_{it}] + e_{it} \\ e_{it} &= \phi_j + \phi_m + \phi_i + \varepsilon_{it} \\ &\quad for \\ \bar{a} &\equiv a - \mathcal{A} \\ v_{it} &\equiv -\text{cloglog}(\eta_{it}) \end{aligned} \tag{2.30}$$

where vitality in log scale $v = \log \vartheta$, the dependent variable in the regression, is defined as minus the complimentary log-log of the death rates, or $v_{it} = -\log(-\log(1-\eta_{it}))$, and $\bar{a}$ is the age deviation from the maximum lifespan $\mathcal{A}$.

There is an omitted variable problem regarding the lack of data on the health expenditure profile by age and sex, which is necessary to construct the variable ξ_{it} in equation (2.30). These data are available only for a few countries and years, and the methodologies for building them are not necessarily compatible between the countries. To overcome this problem, let's assume that changes in the age spending profile over the years can be disregarded. That is, we assume that $\xi_{it} = \xi_a$, the deviations in health expenditures in relation to the population average are different between individuals in the age dimension, but do not vary over time. We also assume that the differences by country and sex are absorbed in the specific effects ϕ_j , ϕ_m and ϕ_i. Under these assumptions, we use the distribution of health expenditure by age for France in 2010, calculated by Gastaldi-Ménager et al. (2016) as a reference. Thus, coherently to the evidence shown in Figure 2.4, $\log \xi_a$ is considered to be linear in age.[8]

[8] We thank the authors for providing us with the health expenditure data underlying the results in their paper.

Figure 2.4: *Health expenditure age profile, France 2010*



Data: Gastaldi-Ménager et al. (2016)

Thus, the parameters of the longevity production function are inferred from the following panel data regression:

$$v_{it} = \beta_0 + \beta_1 t + \beta_2 \bar{a} + \beta_3 \bar{a}^2 + \beta_4 \bar{a} \log x_t + \beta_5 \bar{a} \log h_{it} + e_{it} \tag{2.31}$$

$$e_{it} = \phi_j + \phi_m + \phi_i + \varepsilon_{it}$$

We estimate equation (2.31) using three panel regression estimators, fixed effects, random effects and the Hausman and Taylor (1981) estimator. In all the estimated regressions, we assume that the regressors are not correlated with e_{it}. In the fixed effects specification, effect ϕ_i is a parameter exogenously fixed for each individual, and possibly correlated with the regressors, whose coefficients are consistently estimated by demeaning the variables. However, this transformation eliminates the time-invariant covariates from the regression, some of which are relevant to the calibration of our model. In the random effects estimator, it is assumed that ϕ_i is drawn from a normal distribution and that the covariates are not correlated with ϕ_i, so that the equation is estimated in levels and we maintain the coefficients of time-invariant variables. However, if any regressor is correlated with ϕ_i, the estimates are not consistent. The Hausman-Taylor esti-

mator is a particular random effects estimator, in which ϕ_i can be correlated with some of the covariates, in our case with physicians' density x_t and own human capital h_{it}, because it uses the regressors that are assumed uncorrelated with ϕ_i as internal instruments to obtain consistent estimates. Table 2.1 reports the results of the estimations.

Table 2.1: *Longevity production estimates*

	(1) FE	(2) RE	(3) HT
t	0.0565***	0.0501***	0.0565***
	(0.00159)	(0.00151)	(0.00158)
$\bar{a}$		0.462***	0.458***
		(0.0146)	(0.0160)
$\bar{a}^2$		-0.000944	-0.000343
		(0.000769)	(0.000841)
$\bar{a} \log x_t$	0.00708***	0.0105***	0.00708***
	(0.000884)	(0.000856)	(0.000880)
$\bar{a} \log h_{it}$	0.0182***	0.0224***	0.0182***
	(0.00219)	(0.00207)	(0.00218)
ϕ_{USA}			-0.249**
			(0.0759)
ϕ_{Brazil}			-0.432***
			(0.0766)
$\phi_{females}$		0.476***	0.478***
		(0.0111)	(0.0121)
Observations	10040	10040	10040

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: FE, RE, and HT refer to fixed effects, random effects, and Hausman-Taylor estimator, respectively. The panel is unbalanced, with 96 countries, 13 five-year age classes (30 to 94 years), 11 five years periods (1960 to 2010), and country and sex fixed effects. France is used as the reference country, and country fixed effects not used in the calibration are omitted.

To test for the consistency of the random effects and Hausman-Taylor estimations, we perform Hausman tests to compare them to the fixed effects estimates, which are consistent under the assumption of exogeneity in relation to the idiosyncratic error e_{it}. For the random effects estimators, the null hypothesis that the coefficients are not systematically different from the coefficients estimated by fixed effects is rejected at any reasonable significance level, with a p-value of 0.0000, which indicates a strong violation of the assumption of regressors not correlated with the

individual effect ϕ_i. In the Hausman-Taylor case, this null hypothesis is not rejected at any usual significance level, with a p-value of 0.9998, thus the use of this specification is recommended.

Regarding the results in Table 2.1, the only coefficients not statistically significant at a 5% confidence interval are some of the country fixed effects not shown in the table and β_2 for the squared age deviation term $\bar{a}^2$. This term originates from the assumed linear functional form $\log \xi_a = \zeta_0 + \zeta_1 \bar{a}$, as $\beta_2 = \zeta_1 \gamma_1 \mu$. The fact that β_2 is non-significant is indicative that the effects of the omission of ξ_a in the regression are of a second order magnitude. Regardless, as the parameter γ_0 is affected by the identification of ζ_0 because $\gamma_0 = \beta_1 - \zeta_0 \gamma_1 \mu$, we are using the estimated β_1 only as an initial guess for calibrating the parameter γ_0. The estimated constant β_0 for the regression is not used in the calibration of the parameter ϕ_0, as it includes scaling effects of the variable x_t of health inputs, and the female dummy is only a control as our macroeconomic model does not distinguish between sexes. The corresponding estimated values for the parameters of the longevity production function (2.30) using the estimates of the Hausman-Taylor specification are presented in Table 2.2.

2.5 Calibration

This section presents the calibration of the model to the three countries being compared, France, the United States, and Brazil. We run the model separately for each country, but keep most of the parameters the same between the simulations. We chose France as the reference country for the calibration of common parameters, as in relation to the other countries it has a higher life expectancy, and its position is intermediate in other features such as per capita income and share of health expenditure in GDP. In terms of institutional characteristics, France is a developed country as is the US, and supports a universal public health system as in Brazil.

Since the first year of the model's simulation is 1960, we exogenously set or calibrate in equilibrium the parameters to reproduce data from 1960 to 2015, taking as given some parameters from the longevity production function estimation presented in the previous section. Table 2.2 summarizes the parameter values which are common to the three countries, and Table 2.3 the parameters that are specific for each country.

Table 2.2: *Calibration, common parameters*

Parameter	Value	Target
Longevity production:		
g_Φ	0.0565	Longevity production estimation
μ	0.28	Longevity production estimation
γ_1	0.025	Longevity production estimation
γ_0	0.501	[EQ] Gompertz curve slope, France 1960 (WHO)
$\mathcal{A}$	14	Maximum age = 100 years, minimum age = 30 years, $\mathcal{A} = (age_{MAX} - age_{MIN})/5$
ε	0.79	Share of medical goods in total health expenditures = 0.21, France 1970-2017 (OECD)
α_z	0.25	Capital share in health services, France 1995-2011 (WIOD)
θ_z	0.76	Value added share in gross output for health services, France 1995-2011 (WIOD)
Non-health production:		
α_y	0.38	Capital share in non-health sector, France 1995-2011 (WIOD)
θ_y	0.48	Value added share in gross output for non-health sector, France 1995-2011 (WIOD)
Physical and human capital:		
δ	0.18	Physical capital depreciation rate = 0.033 per year, France 1960-2017 (PWT 9.1)
ψ_0	0.10	Mincerian returns to schooling, literature (e.g. Lee and Lee, 2016)
ψ_1, ψ_2	0.047, $6.3\text{x}10^{-4}$	[EQ] Returns to experience, France 1993-2001 experience-wage profiles data from Lagakos et al. (2018) matches France 2000 in equilibrium
Households and preferences:		
β	0.92	[EQ] Average annual capital/GDP ratio = 2.77 for US 1990-1995, Kehoe et al. (2018)
σ	3.5	[EQ] Change in health expenditures share in GDP, France 1970-2015 (OECD)
$\bar{u}$	$1.3\text{x}10^{-3}$	[EQ] Average health expenditures share in GDP, France 1970-2015 (OECD)

Notes: [EQ] parameters are calibrated to match the targets in equilibrium.

2.5.1 Households and preferences parameters

We assume that consumers enter the economy at age 30 and live up to a maximum of 100 years old, as it is our intention to reproduce the Gompertz law period of life. Agents retire from work at age 65, in all countries. Each period in the model equals five years, thus all data and parameters in Tables 2.2 and 2.3 refer to this periodicity. The age variable a in the model is then normalized as $a = (\text{age} - 30)/5$, and parameter $\mathcal{A} = 14$ in the longevity production function corresponds to the maximum attainable age for a.

We set both the curvature parameter of the utility function σ and the value of life parameter

Table 2.3: *Calibration, country-specific parameters*

Parameter	Value by country			Target
	France	US	Brazil	
ϕ_0	-1.53	-1.78	-1.96	US, Brazil: longevity production estimation. France: [EQ] Gompertz curve intercept, 1960 (WHO)
A_y	16.4	16.7	14.4	[EQ] Relative per capita GDP 2010, France = 100 (PWT 9.1)
g_y	0.033	0.066	0.057	[EQ] Per capita GDP growth 1960-2015 (PWT 9.1)
K_0	15.6	27.1	3.1	[EQ] Per capita physical capital growth 1960-2015 (PWT 9.1)

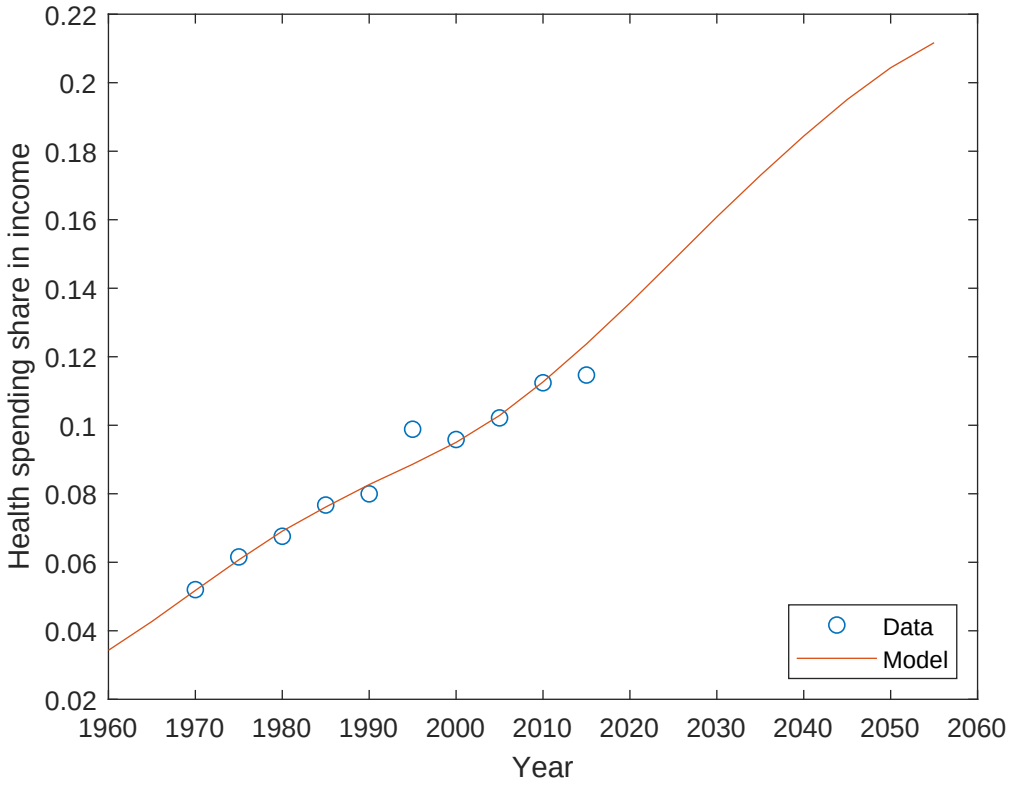
Notes: [EQ] parameters are calibrated to match the targets in equilibrium.

$\bar{u}$ to reproduce the history of total current health expenditures as a share of GDP for France in 1970-2015, according to OECD dataset in health expenditure and financing. Figure 2.5 presents for France this data and the model's projections on health spending as a share of GDP over a 100-year timespan, from 1960 to 2055. In our simulations, and as emphasized by Hall and Jones (2007) and Jones (2016), parameter σ is a main determinant of the slope of the optimal health expenditure curve, so we calibrate it to 3.5 to match the corresponding slope in the data. Parameter $\bar{u}$ is particularly influential in the intercept of this curve, as shown in our robustness exercises that consider the US and Brazil as references, then we set it to 0.0013 to force the average health share in GDP in the years 1970 to 2015 for the French calibration of the model to replicate this average in the data.[9]

The discount rate β is the only parameter common to all the countries that is targeted to the US economy instead of France. Under our choice of $\beta = 0.92$, which corresponds to an annual discount rate of 0.983, the mean capital/GDP ratio of the years 1990 and 1995 in the model calibrated for the United States equals 0.5543 in five-year intervals. On an annual basis, this rate is 2.771, which almost pairs the 2.779 capital/GDP ratio computed by Kehoe et al. (2018) for the US in 1992 using the method proposed by Backus et al. (2008) for computing physical capital series. Backus et al. (2008) notice that capital/output ratios vary substantially even between developed countries, which imply differences in the marginal product of capital that are not easily explained by a standard neoclassical model, and they look at the role of international

[9]This value calibrated for $\bar{u}$ may seem very different from the value of this parameter in other papers in the literature that use these preferences, as Hall and Jones (2007) that set it to 26 in the baseline calibration. However, as $\bar{u}$ is directly related to the value of life as measured in consumption units, it changes according to the reference value assumed for GDP per capita, which is 100 for France in 2010 in our default calibration. By testing some alternative reference GDP values, our calibration algorithm finds very similar equilibrium trajectories for France under different $\bar{u}$. Some of these alternative (GDP_{REF},$\bar{u}$) pairs are: (150,0.00049), (10,0.43), and (2.5,14).

Figure 2.5: *Health expenditure GDP share, model versus data, France*



Data: OECD dataset in health expenditure and financing.

differences in taxation to justify this. As the aim of this text is not to provide any particular explanation on this issue in our model, we target just one observation in the capital/GDP ratio for the US, and use it to set the parameter β, which is common to all the countries.

2.5.2 Longevity production parameters

We set most of the parameters for longevity production by using the econometric evidence presented in the previous section. Parameter $g_\Phi = 0.0565$ is the TFP growth in longevity production, which is given by the coefficient of the time variable in the longevity production regression $g_\Phi = \beta_3$, for a five-year period and assumed to be the same in all the countries. On an annual basis, this TFP growth rate corresponds to 1.13% per year. Parameter $\gamma_1 = 0.025$, which determines the degree of returns to scale in longevity production when multiplied by $(a - \mathcal{A})$, is the sum of health inputs and own human capital coefficients in the regression from the previous section, $\gamma_1 = \beta_4 + \beta_5$. Parameter $\mu = 0.28$ is the elasticity of health inputs in longevity production, while $1 - \mu = 0.72$ is the elasticity of own human capital, controlled by

the degree of returns to scale, or $\mu = \beta_4/\gamma_1$.

Parameter ε is set to 0.79, so that $1-\varepsilon$ equals the average share of medical goods in total current expenditure on health for France from 1970 to 2017, according to the OECD dataset on health expenditure and financing.

We calibrate two parameters using the Gompertz curve for France in 1960, γ_0 and ϕ_0. As discussed in the previous section, our panel regression exercise provides an imprecise estimation of parameter γ_0, which measures the effect of aging on longevity production TFP. We use the age coefficient in the panel regression, $\beta_2 = 0.458$, as an initial guess for calibrating γ_0. Thus, we set $\gamma_0 = 0.501$ by targeting, in equilibrium, the slope of the Gompertz curve for France in 1960, with mortality data for ages up to 80 years old and excluding mortality by external causes.

Parameter ϕ_0 determines longevity production TFP in the initial year and is specific for each country. For France, we calibrate it in equilibrium to match the intercept of the Gompertz curve in 1960 (excluding mortality by external causes), resulting in $\phi_0 = -1.53$. We show the data and the model projected Gompertz curve for France in 1960 in Figure 2.8 in the results section. For the other countries, we use the country fixed effect of the panel regression to compute the deviation of parameter ϕ_0 from the value of this parameter as calibrated for France. This ϕ_0 parameter determines initial TFP in health production, but as we assume that TFP growth in longevity production is equal in all the three countries, differences in ϕ_0 imply permanent TFP gaps. Thus, as shown in Table 2.3, longevity production TFP is lower in the US than in France, and lower in Brazil than in the two developed countries. For Brazil and the United States we also perform simulations with alternative ϕ_0 parameter values, corresponding to the bounds of the 95% confidence interval for the respective country's fixed effects in the longevity production regression.

2.5.3 Production, physical capital, and human capital parameters

For both sectors, we set the production function elasticities to match data for French 1995-2011 input-output tables in the World Input-Output Database (WIOD, see Timmer et al., 2015). Sector z in our model corresponds to *Health and Social Work* in WIOD, and sector y to the aggregation of all the remaining sectors. We set $\theta_y = 0.48$ and $\theta_z = 0.76$ to match the respective average value added shares in gross output for each sector, and $\alpha_y = 0.38$, $\alpha_z = 0.25$ after their average capital shares in value added.

Parameter A_y is TFP in sector y in year 2010, $Y_{2010} = A_y {K_{y,2010}}^{\alpha_y} {H_{y,2010}}^{(1-\alpha_y)}$, which is

the year used to calibrate this sector's TFP levels. It is uniquely related to the productivity parameter Λ_y by $A_y = \Theta_y e^{(1-\alpha_y)g_y t_{2010}}$ and the expression for Θ_y as a function of Λ_y in equation (2.27). We chose A_y for each country to reproduce in equilibrium the relative per capita GDP in 2010 from version 9.1 of the Penn World Tables (PWT), assuming that it is equal to 100 for France. Data for GDP per capita from PWT 9.1 corresponds to the ratio of the variable *rgdpna* accumulated over five years to population *pop*.[10]

For the initial stock of physical capital per capita in 1960, we set K_0 for each country such that per capita physical capital growth from 1960 to 2015 in the model equilibrium equals the corresponding change in the data, using the PWT 9.1 series of physical capital services *rkna*. This physical capital series is compatible with the *rgdpna* series which is used for productivity analysis in PWT 9.1. It is equal to 1 for all countries in 2011, as it does not rely on assumptions to guess an initial stock of capital which is comparable between the countries. We set K_0 to target the change in physical capital per capita, instead of levels series, that are harder to compute for international comparisons. The only physical capital level data that we use is the Kehoe et al. (2018) and Backus et al. (2008) capital/GDP ratio in US 1992 for calibrating β.

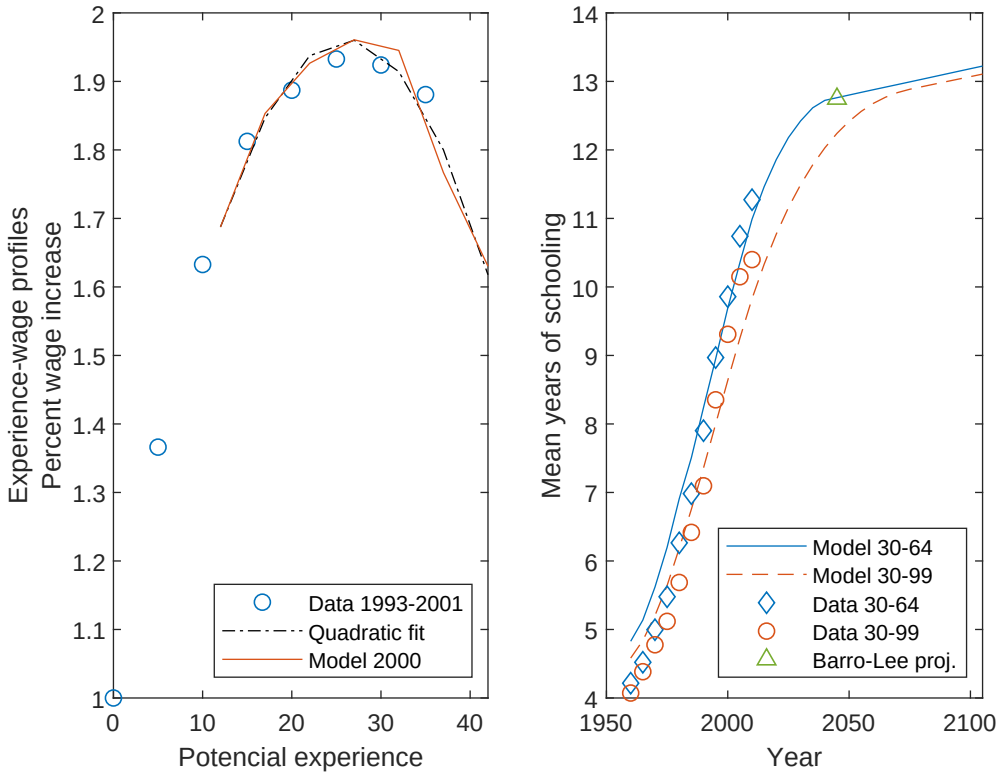
We set the physical capital five-year depreciation rate to $\delta = 0.18$, to match the average depreciation rate for France during the period 1960-2015 in the PWT 9.1 series, or 0.033 on an annual basis. This series is computed considering asset-specific depreciation rates, and it is compatible to the *rkna* physical capital series that we use.

To compute human capital stocks, we assume $\psi_0 = 0.10$, which is a standard value for returns per year of schooling in Mincer regressions, and a common value assumed in studies that calculate aggregate human capital stocks such as Bils and Klenow (2000) and Lee and Lee (2016). We calibrate returns to experience parameters to $\psi_1 = 0.047$ and $\psi_2 = 0.00063$, such that the wage-expenditures profile for France 2000 in the model equilibrium targets the corresponding data for France 1993-2001 as presented by Lagakos et al. (2018), see panel a) of Figure 2.6. These values calibrated for ψ_1 and ψ_2 are close to the mean values of Mincer regression estimates for these

[10] The current set of alternative GDP variables in the PWT was introduced in version 8.0 (Feenstra et al., 2015), and GDP growth for each country may be very different as we choose this *rgdpna* series, which is based on a single purchasing power parity-converted (PPP) benchmark correction, or any of the series based on multiple PPP benchmarks, such as *rdgpo* and *cdgpo*. The *rgdpna* series uses the 2011 edition of the International Comparison Program (ICP) price surveys to compute the PPP-adjusted relative GDP for this year, and national accounts data from each country for extrapolating to other years. We prefer this series, as Deaton and Aten (2017) argue in favor of the methodological improvements in ICP 2011 for measuring relative income in comparison to previous ICP versions, and Pinkovskiy and Sala-i Martin (2016) recommend using the *rgdpna* series rather than the series corrected for multiple PPP or market exchange rates after comparing them to an independent measure of economic activity, night-time lights from satellite photos. As we assume a five-year interval in our model, we use relative GDP in 2010 instead of 2011 as the reference for calibrating TFP level in the non-health production sector y.

coefficients reported by Bils and Klenow (2000) in a sample of 52 countries, 0.052 and 0.00071.

Figure 2.6: *Human capital, France: a) Returns to experience and b) Years of schooling*



Data: Lagakos et al. (2018) experience-wage profiles,
Barro and Lee (2013, 2015) schooling data and projections.

With these parameters, we find human capital by each cohort and year using the Barro-Lee educational attainment dataset for France, the US, and Brazil. For the years 1960 to 2010, we use Barro and Lee (2013) data on mean years of schooling by age, which is available in five-year categories from 30 to 74 years old and in an open interval 75+. In our model, we assume that schooling is constant throughout the agent's lifetime, but in the Barro-Lee dataset schooling for any cohort changes according to the year. These changes are in part due to the differences in sampling o the years, but are also due to aging selection, as more educated people live longer and this raises the mean years of schooling of the cohort as it gets older. To maintain the model assumption that each cohort has a fixed number of years of schooling, we take a weighted mean of years of schooling for all the available observations of each cohort, with weights proportional to the cohort size in each year.

For the years after 2010, we apply a linear projection to schooling in the younger cohort, which must respect an imposed value for mean years of schooling for ages 30 to 69 years in 2045,

using population projections by age from the United Nations. This value is the mean years of schooling projection for ages 25 to 64 years in 2040 from Barro and Lee (2015). We impose a maximum of 16 years of schooling, but none of the countries reach more than 14 years of schooling in our projections. In panel b) of Figure 2.6, we show for France the resulting mean years of schooling projected for the simulated model, for workers and the entire population, and we compare it to the Barro-Lee data and projections of mean years of schooling.

Parameter g_y in our model determines TFP growth rate in sector y, which equals $(1-\alpha_y)g_y$. We target per capita GDP growth from 1960 to 2015 to calibrate g_y for each country, according to the *rgpna* series from PWT 9.1, which preserves the original GDP growth series from the national accounts of each country. As the growth rates of physical and human capital were determined by the data and procedures described above, parameter g_y in fact is given as the residual from GDP growth.

Table 2.4: *Simulated mean growth rates, 1960-2015 (% per year)*

Growth rates	France	US	Brazil
GDP per capita	2.17	2.04	2.32
Physical capital per capita	2.94	2.44	2.82
Human capital per capita	1.03	0.46	0.88
TFP, non-health production	0.40	0.82	0.71
TFP, longevity production	1.13	1.13	1.13

Notes: GDP and physical capital growth rates from PWT 9.1 targeted in baseline calibration, series *rgdpna* and *rkna*. Schooling data from Barro and Lee (2013). TFP in non-health production by residual, estimated TFP in longevity production.

Table 2.4 summarizes the growth rates of factors of production and sector-specific TFP growth rates implied by the model, on an annual basis. France presents the highest growth rates both in physical capital and human capital, and as the differences between the countries in per capita GDP growth rates are relatively small, TFP growth residual in sector y is markedly lower in France than in the other countries. The change in mean years of schooling for the working population implicit in the model, which rises from 4.8 to 11 years in France versus 10.2 to 13.5 in the US and 2.7 to 7.3 in Brazil, explains much of of this difference. We also note that for the three countries TFP growth in non-health production is lower than TFP growth for longevity production

2.5.4 Population and asset distribution

Initial distribution of population by age follows the United Nations population projections for each country in 1960. Population size is normalized to one in 1960, and changes in the other years according to the endogenous death rates and the exogenous flow of births at age 30 in each period. Using the UN population projections, we calculate 'birth' rates as the ratio between the current population aged 30-34 years and the population aged 30 to 49 years in the previous five-year period, from 1960 to 2100, which is the last year with UN projections. We smooth these birth rates by adopting a moving average of five periods (25 years), assume a constant birth rate after 2100, and apply the resulting series of birth rates sequentially to the population aged 30 to 49 years in the model.

Regarding asset distribution in 1960, we assume that the share of total assets increases linearly with age until the age of retirement of 65 years, which is a recurring pattern in the simulations for the other years considering alternative assumptions on the initial distribution. For 65+ years, we assume a smooth consumption profile, as in the consumption profile computed for France 1979 by Albis et al. (2016, 2017).

The series of death rate from external causes $\bar{\eta}$ for each country and age is obtained from the procedures described in the econometric section. To complete the missing data for ages 85-94 before 2000 in the three countries, we use linear projections of $\log \bar{\eta}$ from younger ages. As the Brazilian series begin in 1980, we apply the 1980 rates to earlier years. For the years after of 2015, we assume that the rate of mortality by external causes is the same as 2015.

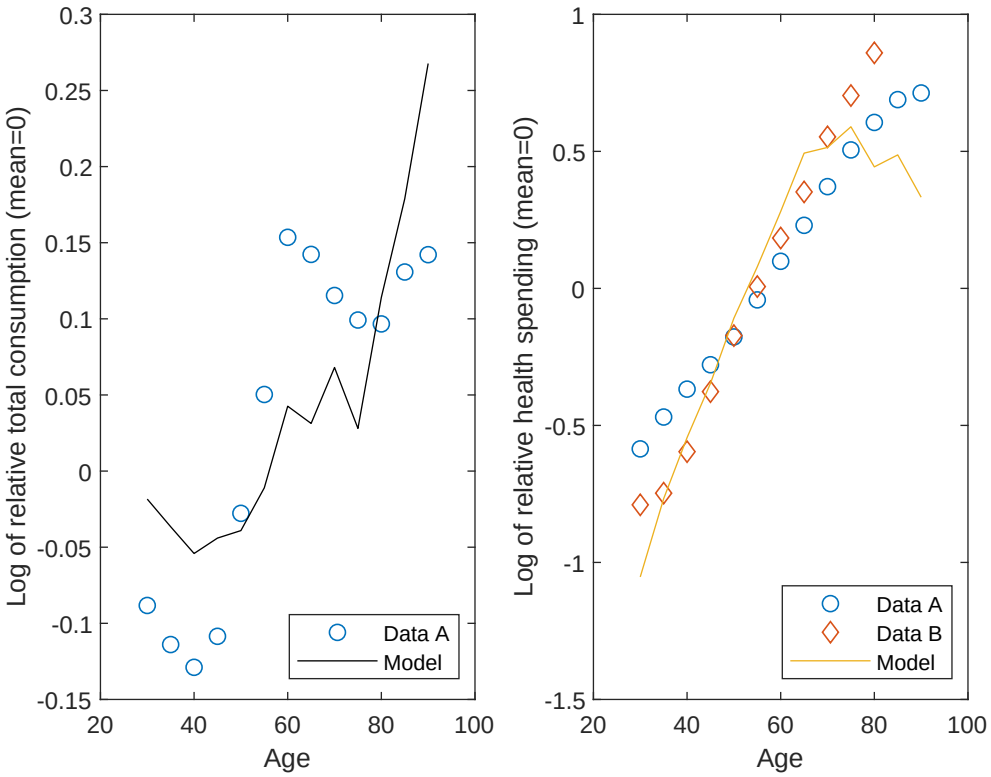
2.6 Results

2.6.1 Model fit to non-targeted data: France

As a way to evaluate the goodness of fit of our benchmark calibration, we compare some of our results to empirical evidence that was not used for calibrating the model for France. Figure 2.7 presents the data and the model fit to age profiles for total consumption and health expenditures for France in 2010 . We show estimates of expenditure profiles for France in 2010 from two sources, Gastaldi-Ménager et al. (2016) and Albis et al. (2016, 2017).

In panel a) of Figure 2.7, the fit of total consumption profile comparing to Albis et al. (2016, 2017) data overestimates the share of the youngest and the oldest ages, and underestimates ages

Figure 2.7: *Expenditure age profiles, model versus data, France 2010: a) Total consumption and b) Health spending*



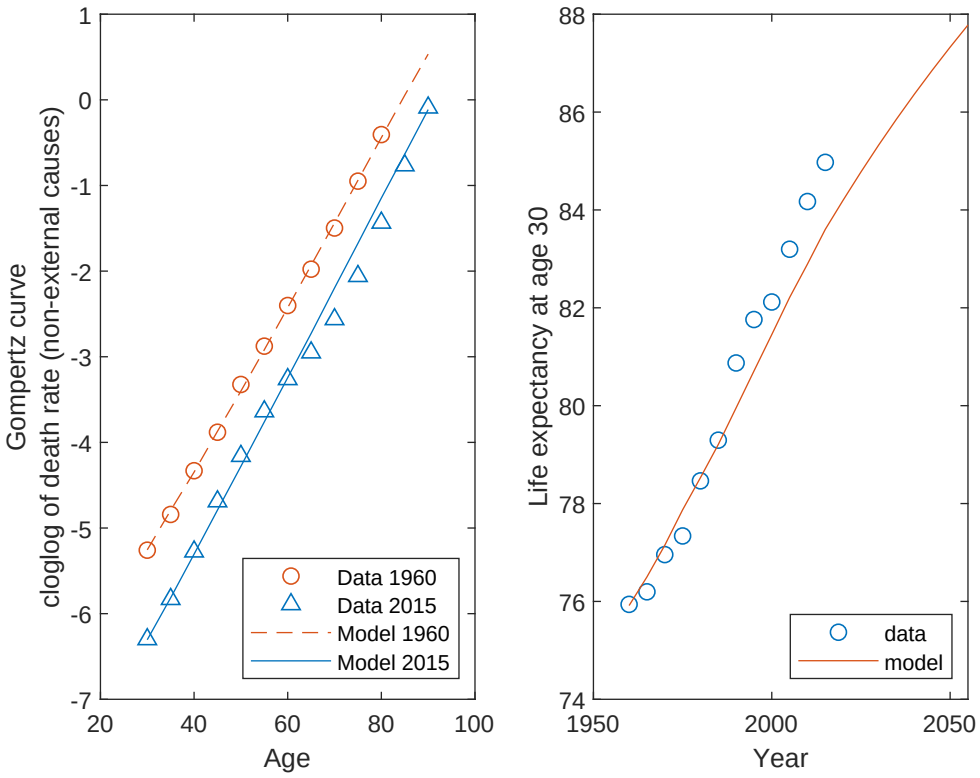
Sources: Data A - Albis et al. (2016, 2017), Data B - Gastaldi-Ménager et al. (2016).

60-75. In panel b), which is the most relevant for our purposes, the model's projected health expenditure age profile is fairly close to Gastaldi-Ménager et al. (2016) data, except in the oldest ages when the model predicts a smooth reduction in health spending that is not observed in the data.

Next, we compare our model results to data on death rates for France. The first panel in Figure 2.8 compares the Gompertz curve excluding deaths by external causes computed with data from the World Health Organization (WHO) mortality database to the corresponding Gompertz curve projected by the model, for the years 1960 and 2015. We targeted the intercept and slope of the French Gompertz in 1960 curve to calibrate the model. However, we did not target the Gompertz curve for the other years, and the model provides a good fit to the 2015 Gompertz curve. The model in 2015 slightly overestimates mortality for ages between 65 and 80 years, but the fit is accurate for the other ages.

The second panel of Figure 2.8 shows life expectancy at age 30 for France from 1960 to 2015, as computed with model projections and data from the WHO mortality database. Both the

Figure 2.8: *Mortality trajectories, model versus data, France*



Data: number of deaths from WHO mortality database and population from UN world population prospects. Gompertz curve parameters for France in 1960 are used in model calibration, but the Gompertz curve for 2015 and other years is not.

model and the data series take into account deaths by external causes, which are the same for both.Model fit for life expectancy in the initial year is driven by our calibration to replicate the Gompertz curve in 1960. For the remaining years, model predictions provide a good fit to the data for years before 1990, but since then the model underestimates the rise in life expectancy. Taking the entire 1960-2015 period into consideration, the model explains most of the increase in life expectancy at age 30, which equals 7.7 years in the model and 9 years in the data.

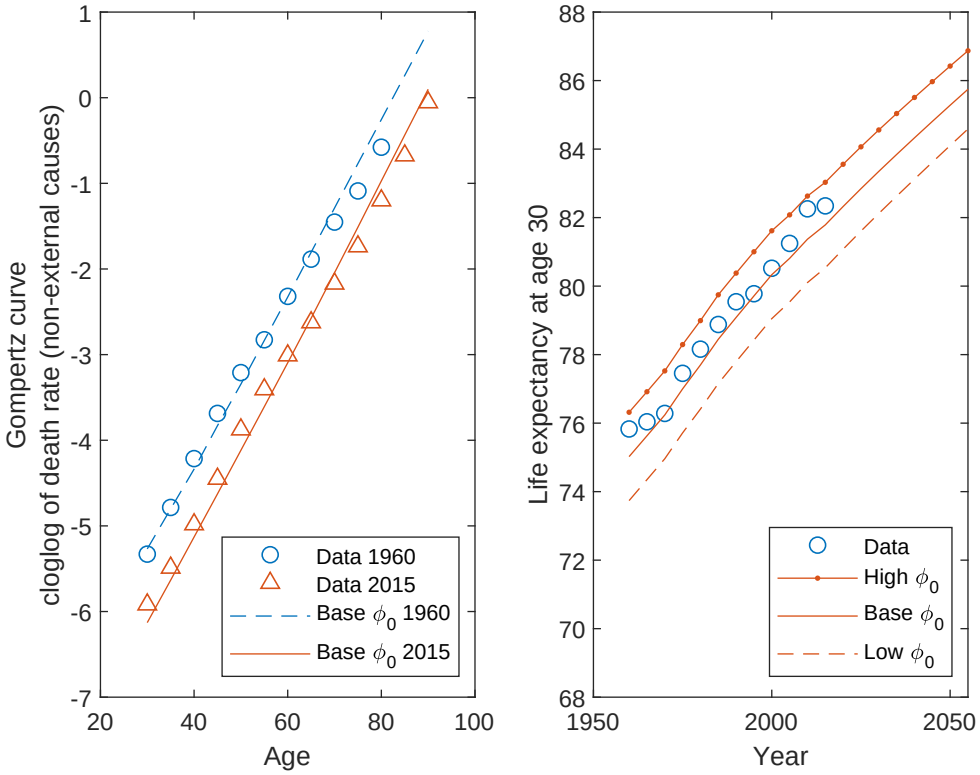
2.6.2 Model fit to non-targeted data: US and Brazil

As most of the model parameters were calibrated to France or taken from the panel regression, comparing the model's performance in matching non-targeted data for the United States and Brazil helps us to understand the forces that promote health related structural transformation in countries with different institutional backgrounds and stages of development.

In each figure for the US and Brazil, we present results for three model calibrations, that

differ in the value assumed for parameter ϕ_0, the log of initial TFP in health production. These ϕ_0 values are the center and the bounds of the 95% confidence interval for country fixed effects in our longevity production regression.

Figure 2.9: *Mortality trajectories, model versus data, US*



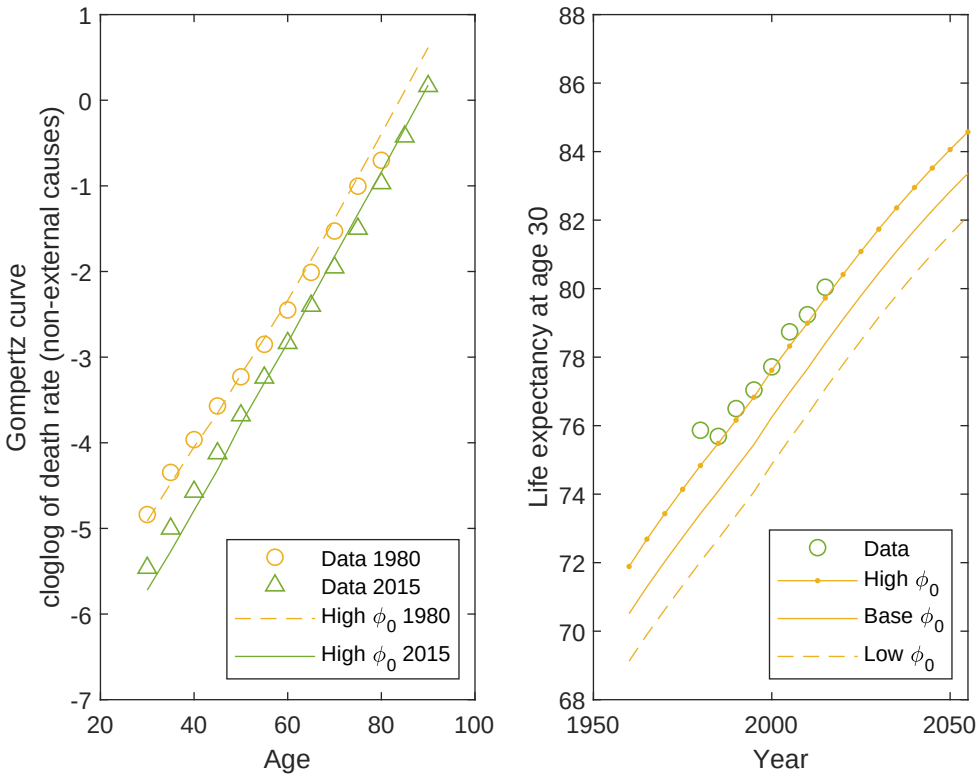
Data: number of deaths from WHO mortality database and population from UN world population prospects. Life expectancy curves for the center and the bounds of the country fixed effect confidence interval, representing low, base, and high longevity TFP ϕ_0. Gompertz curve for the base TFP value.

Figure 2.9 compares non-targeted data to model predictions of mortality rates for the United States in different years. The first panel shows the Gompertz curve, excluding external of causes mortality, for the years 1960 and 2015. None of these curves were targeted in the calibration, but the model closely replicates both curves for the US. In the second panel, we plot life expectancy at age 30 in the US from 1960 to 2015, and the model provides a very good fit to the data. The total improvement in life expectancy at age 30 equals 6.8 years in the model calibration for base ϕ_0, which almost pairs the increase of 6.5 years in the data.

We provide an analogous comparison for Brazil regarding goodness of fit for mortality in Figure 2.10. Different from the US and France, the earliest year with available data for Brazil in the WHO mortality database is not 1960, but rather 1980. The first panel of Figure 2.10 shows

the Gompertz curve for Brazil in 1980 and 2015, plotted for the high TFP value of ϕ_0, which is the best fit. The second panel of Figure 2.10 presents life expectancy at age 30 in Brazil from 1980 to 2015 for the data and since 1960 for the model.As seen in both figures, model projections overestimate mortality in Brazil, but are still relatively close to the upper bound of high TFP ϕ_0. Besides this, the model's predicted changes in life expectancy since 1980 are reasonably accurate, an improvement of 4.9 years under high ϕ_0 compared to an increase to 4.2 years in the data.

Figure 2.10: *Mortality trajectories, model versus data, Brazil*

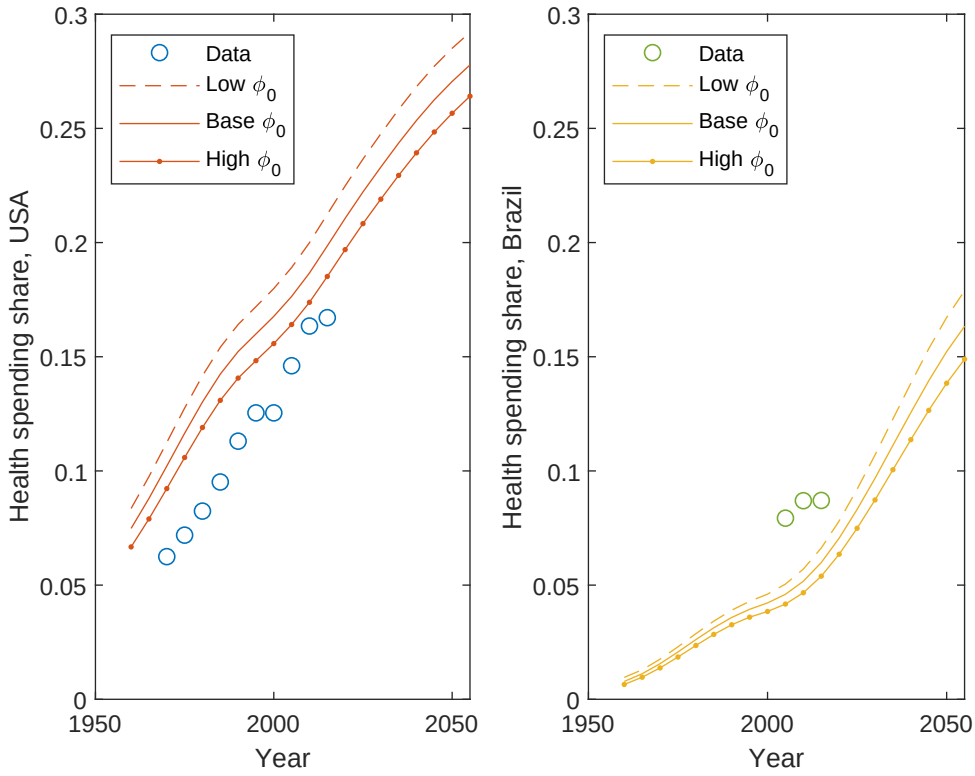


Data: Life expectancy curves for the center and the bounds of the country fixed effect confidence interval, representing low, base, and high longevity TFP ϕ_0. Gompertz curve for the high TFP value.

In Figure 2.11, we plot health expenditure as a share of GDP in model predictions and in the data for the US and Brazil. We targeted this same data for France to calibrate preference parameters, but health expenditure data for the US and Brazil were not targeted. The first panel of Figure 2.11 compares data for the US from the OECD dataset on health expenditure and financing since 1970 to model projections from 1960 to 2055. The second panel is analogous for Brazil, but over a shorter data timespan from 2005 to 2015, due to more restricted data availability.

The United States is the country with the highest share of health spending in the world, but

Figure 2.11: *Health expenditure GDP share, model versus data, US and Brazil*



Data: OECD dataset in health expenditure and financing.

under the preferences calibrated for France, the model predicts an even higher share for the US. The slope of the optimal health spending curve, however, is similar in the model and in the data, so that the model explains most of the increase in health share in the US since 1970, 9.6% in the model versus 10.5% in the data for base ϕ_0.

For Brazil, there is not enough available data to compare the model's predictions to the historical change in health share. The health share of income predicted by the model in the years 2005-2015, between 4.2% and 6.6%, is considerably lower than the range in the data, from 7.9% to 8.7%.

Figure 2.11 also illustrates how the agent's optimal choices of health spending varies with longevity TFP. The highest TFP value in each period is associated to the lowest health spending share, so that the income effect prevails as the consumers choose to reduce health expenditures when it is most effective.

2.6.3 Health share decomposition

To investigate the role of technology and preferences in shaping structural change related to the health sector, we perform counterfactual simulations in our model. We begin decomposing the change in the health share of income for each country from 1960 to 2015 in the baseline model scenario.[11] Our decomposition distinguishes three determinants of structural transformation emphasized by the literature (Herrendorf et al., 2014), non-homothetic preferences and sector heterogeneity in TFP growth and in capital intensity. We also consider heterogeneity in intermediate inputs shares as they amplify the effects of capital intensity heterogeneity.

To compute the contribution of sector heterogeneity in TFP growth to structural transformation, we perform a counterfactual simulation without the sector gap in productivity growth, for each country. We make the TFP growth in health production g_Φ equal to TFP growth in non-health production, $(1-\alpha_y)g_y$, and we calculate the contribution to structural change as the difference between the change in the health share in GDP from 1960 to 2015 in the baseline scenario and the same change in this counterfactual simulation. The contribution of sector heterogeneity in capital and intermediate inputs intensity is computed by a counterfactual simulation with $\alpha_z = \alpha_y$ and $\theta_z = \theta_y$, and then calculating the contribution by the same procedure used for sector heterogeneity in TFP growth rates. Finally, we calculate the contribution of non-homothetic preferences as the residual, the total change in the health share minus the contribution from the two sources of sector heterogeneity.

Table 2.5 summarizes the results of this procedure, presenting a decomposition of the change in health share in percentage points and as proportions of the total variation for each country. The main result is that sector heterogeneity and non-homothetic preferences drive structural transformation related to health production in opposite directions, and we observe a substantial increase in the health share for the three countries because the effect of non-homothetic preferences in health spending is considerably stronger. The effect of the sector differences on capital and intermediate inputs elasticities is small in comparison to the other effects.

2.6.4 Life expectancy decomposition

In Tables 2.6, 2.7, and 2.9 we show the results of decompositions for differences in life expectancy at age 30 between the years and the countries. For each exercise, we compare life expectancy in

[11] In Tables 2.5 and 2.6, we do not present the results for the US and Brazil simulations with alternative ϕ_0 values because they are almost identical to the baseline scenario.

Table 2.5: *Change in health share, 1960-2015 decomposition*

	France		US		Brazil	
	pp	%	pp	%	pp	%
Total change (model)	8.9	100	12.4	100	5.2	100
Sector heterogeneity:						
Productivity growth	-3.2	-36	-2.2	-18	-0.5	-9
Capital and interm. inputs intensity	-0.2	-2	-0.9	-7	0.3	5
Residual: non-homothetic preferences	12.4	138	15.4	125	5.4	104

Notes: Decomposition in percentage points (pp) and share of total change (%).

two country-year pairs, decomposing the difference in determinant factors by computing how life expectancy would differ in a certain country/year if it was equal to the reference country/year in that factor. The factors that determine differences in life expectancy are mortality by external causes and the components of the longevity production function: TFP in health/longevity production, factor inputs in health services, and own human capital.

Table 2.6 presents the decomposition of changes in life expectancy from 1960 to 2015 for each country, as computed in the baseline scenario of the model. TFP growth in health production is the main determinant of life expectancy improvement, explaining at least 70% of the change for the three countries, or 5.5 to 5.8 additional years.

Table 2.6: *Change in life expectancy at age 30, 1960-2015 decomposition*

	France		US		Brazil	
	years	%	years	%	years	%
Total change (model)	7.7	100	6.8	100	7.9	100
Longevity production TFP	5.5	72	5.5	81	5.8	74
Health inputs	1	13	0.7	10	1.5	19
Own human capital	0.7	9	0.6	8	0.5	7
Mortality from external causes	0.5	6	0	0	0	1

Notes: Decomposition in years and share of total change (%).

The increase in factors of production allocated to health services, physical and human capital, are responsible for 10% to 20% of the rise in life expectancy. Brazil is the country in which this determinant is most relevant, explaining 19% of the total change or 1.5 years of life, while in France and in the US this contribution equals 1 and 0.7 years respectively. As the effects of health spending in life expectancy operate through this channel, we note that the rising share of health expenditures in income is relevant when explaining the increase in life expectancy, but much less than technological progress.

The remaining causes are each responsible for less than 10% of total life expectancy improve-

ment. Reduction in mortality by external causes is only relevant for France.[12]

In Table 2.7, we compare life expectancy at age 30 in the US and Brazil to France in 2015, and we decompose the difference in the same determinants from Table 5. For this decomposition we present alternative results with the longevity TFP bounds for ϕ_0, whose results were not significantly different from the baseline scenario for the historical decomposition in Table 5, but are different enough to carry out a comparison between the levels in 2015. The first line shows the difference of life expectancy in the data between each country and France in 2015. Both countries have lower life expectancy at age 30 than France, about five years less for Brazil and half this difference for the US, and these differences are explained by the model by varying the health TFP parameter ϕ_0 inside the confidence interval bounds.

Table 2.7: *Differences in life expectancy at age 30 versus France, year 2015 decomposition*

	US				Brazil			
			ϕ_0 bounds				ϕ_0 bounds	
	yr	%	yr	yr	yr	%	yr	yr
Total difference (data)	-2.6				-4.9			
Total difference (model)	-1.8	100	-3.1	-0.6	-5.2	100	-6.5	-3.9
Longevity production TFP	-2.1	117	-3.4	-0.8	-3.9	75	-5.3	-2.5
Health inputs	0.3	-17	0.3	0.3	-0.4	8	-0.3	-0.4
Own human capital	0.4	-20	0.4	0.4	-0.4	8	-0.4	-0.4
Mortality from external causes	-0.4	21	-0.4	-0.4	-0.5	10	-0.5	-0.5

Notes: Decomposition in years (yr) and share of total change (%). Includes results for calibration using alternative health production TFP in relation to France, ϕ_0 upper and lower bounds.

For the US, some determinants act in opposing directions. Higher health spending and years of schooling result in a higher life expectancy in the US than in France, but this is counterbalanced in part by a higher mortality from external causes and largely by a lower TFP in longevity production, which reduces life expectancy in the US by two years when compared to France. For Brazil, all the causes reduce life expectancy in relation to France, but the lower health production TFP is the main factor, responsible for 75% of the difference or about four years.

2.6.5 Brazil 2010 vs France 1960

A relevant question when comparing health-related structural change for countries at different stages of development is whether poorer countries repeat the same historical pattern as currently developed countries or perform different trajectories. To address this question, we compare

[12] It is worth noting here our assumption that the death rates by external causes of mortality were constant in Brazil from 1960 to 1980.

France in 1960 to Brazil in 2010, which is the first year (assuming five-year intervals) in our baseline simulation with a higher per capita income in Brazil than in France 1960.

Table 2.8: *Differences in life expectancy and health share, France 1960 vs Brazil 2010 (first year with per capita GDP higher than France in 1960 in model projection)*

	Brazil 2010			
		Model calibration		
	Data	Base	Low ϕ_0	High ϕ_0
Health share in GDP (%)	8.7	5.2	5.7	4.7
Difference to France 1960 (%)	5.3	1.7	2.3	1.2
Life expectancy at age 30 (years)	79.2	77.7	76.3	79
Difference to France 1960 (years)	3.3	1.7	0.4	3.1

Notes: Includes results for calibration using alternative health production TFP in relation to France, ϕ_0 upper and lower bounds.

Table 2.8 summarizes the differences in life expectancy and health spending from Brazil 2010 to France 1960 in the data and according to three model scenarios. For the health share in aggregate expenditure, comparing the data for Brazil 2010 to model simulation for France in 1960, Brazil allocates a considerable higher share of income to health spending, with more than 5 percentage points.[13]

Table 2.9: *Decomposition for differences in life expectancy at age 30, France 1960 x Brazil 2010 (first year with per capita GDP higher than France in 1960 in model projection)*

	Brazil 2010 - France 1960			
			ϕ_0 bounds	
	years	%	years	years
Total difference (data)	3.3			
Total difference (model)	1.7	100	0.4	3.1
Longevity production TFP	1.2	71	-0.2	2.6
Health inputs	0.3	19	0.4	0.3
Own human capital	0.1	5	0.1	0.1
Mortality from external causes	0.1	4	0.1	0.1

Notes: Decomposition in years and share of total change (%). Includes results for calibration using alternative health production TFP in relation to France, ϕ_0 upper and lower bounds.

In the three scenarios considered, the model with preferences calibrated for France explains less than half of this difference. Even so, the model still predicts 1.2 to 2.3 more points of health spending for Brazil 2010.

[13] Model simulation for the health spending share of GDP in France 1960 is 3.4%, which is compatible with this share in the first year with available data, 1970, when it is 5.2% both in the model and the data.

Life expectancy at age 30 for Brazil 2010 is 3.3 years higher than in France 1960 according to the data. The model reproduces about half of this difference in the baseline simulation, but explains most of it in the counterfactual simulation with higher health production TFP for Brazil.

We decompose this difference in life expectancy at age 30 from Brazil 2010 to France 1960 (see Table 2.9). As in the decompositions from Tables 2.6 and 2.7, health production TFP is the main cause of the differences in life expectancy. In the baseline simulation, all the determinants contribute to a higher life expectancy in Brazil 2010 than in France 1960, but health production TFP alone makes up 70% of it. In the alternative simulations, the low health TFP value makes it lower for Brazil 2010 than for France 1960, with the difference in life expectancy almost reaching zero, while the high health TFP value takes the model close to the difference observed in the data.

2.7 Conclusion

This paper proposes a model of longevity production, overlapping generations, and structural transformation that builds on empirical regularities of human aging to characterize technical progress in health production. We estimate the longevity production function in a panel with 96 countries for the years 1960 to 2010, and the parameters are used to calibrate the model for France and compare it to simulated trajectories for the health expenditure share of GDP and life expectancy at age 30 for the United States and Brazil.

The model reproduces approximate trajectories of increases in life expectancy for the three countries. The decompositions of differences in life expectancy, over time and between the countries, show that total factor productivity in longevity production is the main determinant of these differences. From a historical perspective, 70% to 80% of the rise in life expectancy from 1960 to 2015 is explained by health-related TFP growth, the parallel displacement of the SM correlation. This same percentage is obtained comparing the gap in life expectancy in Brazil to France. For the US gap in comparison to France, lower longevity TFP dominates two effects in the opposed direction, higher health spending and human capital associated to schooling.

As to the effects of health inputs in life expectancy, they are more relevant for Brazil, where they explain 1.5 years or 19% of the rise in life expectancy from 1960 to 2015 according to the model. In the US and France, health inputs are responsible for 0.7 and 1 year of increase in life expectancy, or 10% and 13% of the total increase, respectively.

The trajectories of health expenditure shares in GDP, however, are mostly determined by preferences. In fact, in all the historical decompositions the effect of longevity TFP growth is contrary to the observed trend, pushing toward a reduction in the share of health spending in GDP, as the estimated technical progress rate in health production is higher than the calibrated TFP growth in non-health production for the three countries. The rising value of life effect produced by non-homothetic preferences, noted by Hall and Jones (2007), largely dominates other effects. Moreover, the French calibration for the value of life overestimates health spending in the US and underestimates it in Brazil. The difference from the data is smaller for the US, but it is interesting to note that according to this model, health spending in the US would be even higher with "French preferences", with a health share at around 20% of GDP in 2015.

For Brazil, this numerical simulation predicts part of the share of health spending in GDP. This result also prevails in the exercise in which we compare France 1960 to Brazil 2010, whose per capita GDP levels are similar according to the model's prediction. In the three simulations, the model explains less than half of the difference in health expenditure, which is about 5% greater for Brazil 2010. Even so, the model predicts a higher health share than for France 1960, so that it is optimal for Brazil to allocate a higher share of resources to the health sector than France did with the same income, even under "French preferences".

These results raise a relevant question for further research: if most of the longevity gains are due to technological progress, and considering that the assumption of common TFP growth rate to all countries performs well when explaining the long-run mortality trends, why do the countries raise their health expenditures so incisively and what explains the differences between them if we do not allow for differences in preference parameters?

There is no doubt that part of the answer relies on institutional differences between the countries. Access to health services is a primary candidate, as in our functional form the estimated returns to scale in longevity production are low, and declining with age. Hence, countries with more equal access to health services should present better longevity performance under the same mean health spending than countries with greater inequality. This hypothesis seems promising in explaining the differences between the US and France. But how about Brazil, which, like France, supports a universal public health system as France does? Brazilians in the upper 25% percentile of income adhere to private health insurance because of problems in quality provision, such as congestion of public services. Is quality heterogeneity in health provision in Brazil enough to explain these differences?

There are also some relevant extensions of our model that require further research and could improve our understanding of the linkages between health expenditures and longevity. First, by introducing a channel through which countries need to pay to incorporate technical progress. We have assumed disembodied technical progress in both sectors, to remain closer to the standard neoclassical growth model. Although it is reasonable however to assume that many of the technologies in the health sector are public goods, non-rival and non-excludable, such as new treatments that any physician in the world could learn at a low cost, it is clear that a relevant part of these technologies is incorporated in machines and physical capital in general. Combining our human aging framework for health production to a model with embodied technical progress in physical capital, as in Greenwood et al. (1997), would introduce such a channel.

Second, as argued by Jones (2016), the non-homothetic preferences for longevity production in an endogenous growth model can explain the increase in health-related research in total R&D expenditures observed in the data. This provides a reason for our econometric result that the technical progress rate in longevity production is higher than TFP growth in non-health production. Therefore, by adding endogenous growth to our analysis we may find a stronger connection between the preferences-driven process of rising health spending and the productivity-driven increase in life expectancy.

Appendix

2.A Relation to the literature on health production

In this appendix, we compare the longevity production function proposed in this paper to some alternative health production functions, namely those outlined by Grossman (1972a,b), Hall and Jones (2007), and Dalgaard and Strulik (2014). Throughout the exposition, we use our own notation specific to this paper rather than the original notations used by the authors.

2.A.1 Grossman's health capital

Grossman (1972a,b) proposed the first model of demand for health based on human capital theory, which is still considered today as the canonical model of this area of study (Galama and Van Kippersluis, 2019). In the model proposed by Grossman (1972a,b, 2000), consumers demand the commodity "good health". This commodity is represented by a durable stock named *health capital*, which can be broadly defined as related both to longevity and to days free of disease. Health capital yields a flow of healthy time that is demanded for consumption purposes and as an investment, since poor health is a source of disutility and as well as reduces the available time endowment. The initial level of health capital depreciates with age, and the consumer dies when health capital falls below a certain minimum survival level.

Longevity is endogenous, since the consumer can invest to increase health capital.[14] Consumers invest in health capital according to a household production function that uses market inputs as medical care and the consumer's time, as in the household production framework developed by Becker (1965). The efficiency of household health production depends on the consumer's knowledge or education-related human capital, which improves productivity both in market and non-market activities.

[14] In fact, the original Grossman (1972a,b) model lacks an explicit condition for the endogenous determination of the optimal choice for the length of life, which is provided by Ehrlich and Chuma (1990).

Grossman (1972a,b, 2000) assumes a health production function that is similar to an equation of physical capital accumulation:

$$H_{t+1} - H_t = I_t(x_t, i_t; E) - \Delta_t(H_t) \tag{2.A.1}$$

$$\Delta_t(H_t) = \delta_t H_t \tag{2.A.2}$$

$$H_0 \quad \text{constant} \tag{2.A.3}$$

where the index t refers both to time and age, H_t is health capital, $\Delta_t(H_t)$ is the depreciation of health capital, and I_t is the investment function, which depends on medical care and other market inputs summarized in x_t, on time allocated to household health production i_t, and on knowledge capital E that is assumed constant. Depreciation of health capital is given by an exogenous age-dependent depreciation rate δ_t multiplied by the stock of health capital. The flow of healthy days is proportional to H_t, and the consumer is alive as long as $H_t \geq H_{min}$. The health investment function I_t has constant returns to scale in Grossman (1972a,b), but Ehrlich and Chuma (1990) argue, using a continuous time version of the model, that optimal choices are indeterminate without decreasing returns to scale in I_t.

Recent papers that build on Grossman's model of demand for health include Scholz and Seshadri (2016), Jung and Tran (2016), Jung et al. (2017), and Galama and Van Kippersluis (2019). Scholz and Seshadri (2016) assume that health production is given by $I_t(x_t, i_t) = D_t({x_t}^{\varepsilon} {i_t}^{1-\varepsilon})^{\gamma}$, for $0 < \gamma, \varepsilon < 1$, where the health technology productivity term D_t grows at 2 percent per year by assumption. The health depreciation rate δ is assumed constant, length of life however is stochastic since the stock of health is submitted to an additive age-dependent health shock ϵ_t, such that $\Delta_t(H_t) = \delta H_t - \epsilon_t$. Jung and Tran (2016) and Jung et al. (2017) assume that $I_t(x_t) = D_t {x_t}^{\gamma}$ and $\Delta_t(H_t) = \delta_t H_t - \epsilon_t$, which differs from Scholz and Seshadri (2016) since they ignore the role of time inputs and assume an age-dependent depreciation rate δ_t. Galama and Van Kippersluis (2019) extend Grossman's framework by including several behaviors that affect the health gradient by socio-economic status in a life cycle model. In their numerical exercise, $I_t(x_t) = D{x_t}^{\gamma}$ and $\Delta_t(H_t) = \delta_t {H_t}^{\nu}$, with a functional form for the depreciation rate given by $\delta_t = e^{b_0 + b_1 t}$. Parameter ν may account for different relationships between $\Delta_t(H_t)$ and health capital, with $\nu = 1$ representing Grossman's case and $\nu < 0$ a case with faster health deterioration for the unhealthy individuals, as in Dalgaard and Strulik (2014).

As in Grossman's approach, in this paper we also assume a household production function

for health status that uses the consumer's education-related human capital and market inputs as medical goods and healthcare services. Unlike his approach, time inputs are not modeled, and we ignore the demand for health for purposes unrelated to length of life determination. Another difference between the frameworks is that the original Grossman (1972a,b, 2000) model is deterministic, while the longevity production model on this paper is stochastic, as vitality is inversely proportional to the consumer's death rate. Grossman (2000) reviews some stochastic versions of his model proposed by other authors, which add a random shock to health capital. The same approach is later followed by Scholz and Seshadri (2016), Jung and Tran (2016), and Jung et al. (2017).

This paper proposes a functional form for longevity production that is coherent to stylized facts on human aging, while Grossman's health production equation (2.A.1) is defined by an analogy to physical capital accumulation.

Although we have not defined longevity production in terms of a durable stock as health capital in our exposition, we can do so for ease of comparison to Grossman's model. Let L_{at} denote the *longevity capital* of a consumer at age a in time t, defined in terms of vitality ϑ as

$$L_{at} \equiv \vartheta_{a-1,t-1} \qquad \text{or} \qquad L' = \vartheta \tag{2.A.4}$$

Thus, using equation (2.10), the corresponding equations to (2.A.1), (2.A.2), and (2.A.3) in this paper are, in the notation of the recursive problem:

$$L' - L = -\delta^L L \tag{2.A.5}$$

$$\delta^L = 1 - \frac{DX^{\gamma_1(\mathcal{A}-a)}}{L} \tag{2.A.6}$$

$$L_0{}' = D_0 X_0{}^{\gamma_1 \mathcal{A}} \tag{2.A.7}$$

where the index 0 corresponds to initial age $a = 0$ or 30 years old.

In Grossman's model, the initial stock of health capital H_0 and the depreciation rate δ_t are both exogenous, and the consumer accumulates health capital according to a health investment function I_t to delay the time of death. In our model, the consumer chooses the initial stock of longevity capital $L_0{}'$ and the depreciation rate δ^L at each period, both of which are determined in a similar way to Grossman's health investment function, and there is no additive accumulation of longevity capital.

The compensation law of mortality or SM correlation determines that lower base mortality at age 30 implies higher aging speed. Thus, a larger initial stock of longevity capital results in a higher depreciation rate. This is expressed in equation (2.A.6) by $\frac{\partial \delta^L}{\partial L} > 0$, such that the depreciation rate increases together with the current longevity capital stock.

2.A.2 Hall and Jones's health status

For an individual at age a in time t, Hall and Jones (2007) define overall health status $\tilde{\vartheta}_{at}$ as the reciprocal of the death rate, which is composed of an external causes component $\bar{\eta}_{at}$ and a non-external causes component η_{at}, such that $\tilde{\vartheta}_{at} = \frac{1}{\bar{\eta}_{at}+\eta_{at}}$. They assume that health status associated to the death rate by non-external causes $\vartheta_{at} = \frac{1}{\eta_{at}}$ is given by a Cobb-Douglas production function:

$$\vartheta_{at} = D_a(\phi_t x_{at} \omega_{at})^{\gamma_a} \tag{2.A.8}$$

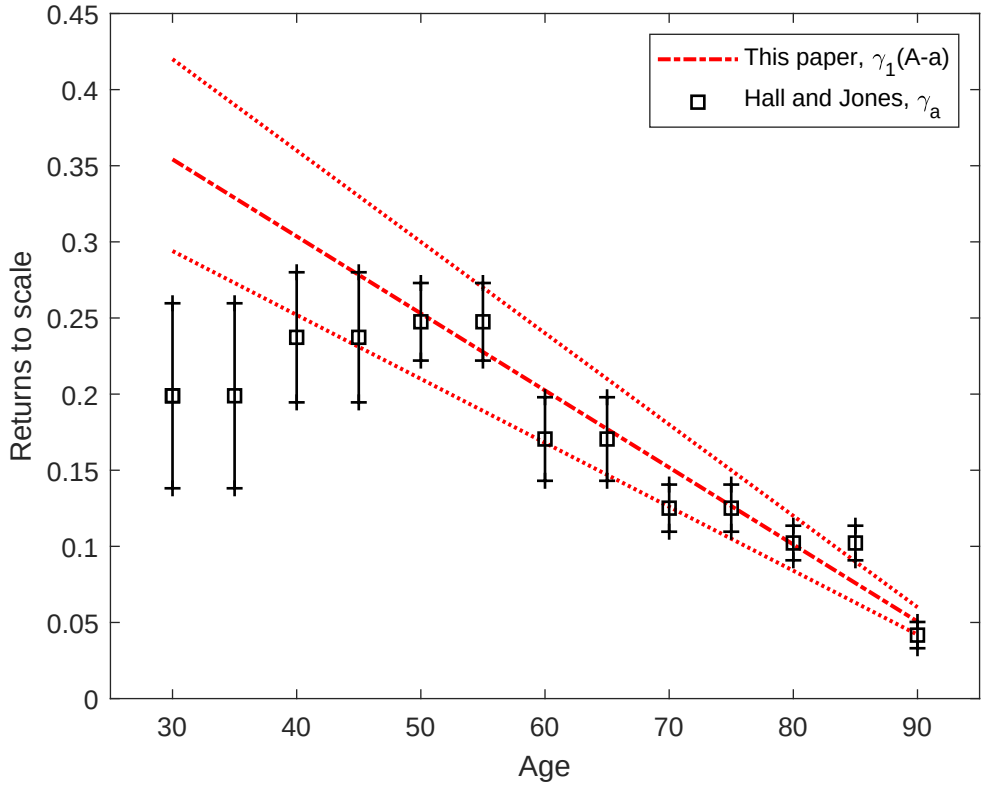
where ϕ_t is the efficiency of one unit of output allocated to healthcare, which is a measure of relative technological progress in health services over output production, x_{at} is health spending, and ω_{at} represents the effects of all other determinants of mortality such as education and pollution.

In order to identify D_a and γ_a, Hall and Jones (2007) estimate these parameters conditionally on assumed trends for ϕ_t and ω_{at}. Their baseline estimation assumes that technological progress in the health sector is equal to the general trend of goods production, or $\phi_t = 1$, and an alternative estimation considers that productivity growth in healthcare production is one percent per year faster. For ω_{at}, they refer to the empirical literature on determinants of mortality and suppose that 1/3 of the declining trend in mortality is not explained by technological progress or rising health spending in the baseline estimation, or 1/2 in a robustness check estimation. They apply GMM to estimate D_a and γ_a by age class using the assumed trends as instruments, with data for the US from 1950 to 2000 in five-year periods, and 20 age classes from 0-4 to 95-99 years.

Notwithstanding the differences in econometric methods and data, when comparing our paper to Hall and Jones (2007) the estimated returns to scale for longevity production are strikingly similar, as shown in Figure 2.12 for their baseline estimation. Our estimated confidence intervals are higher for ages under 40 years, but overlap with Hall and Jones's estimates for the remaining ages. Regardless of the estimates of returns to scale, the longevity production function in equation (2.10) is more parsimonious, as the parametric forms $\gamma_a = \gamma_1(\mathcal{A} - a)$ and $D_a = e^{\gamma_0(\mathcal{A}-a)}$ greatly

reduce the number of estimated parameters.

Figure 2.12: *Estimated returns to scale by age in longevity production*



Notes: "Hall and Jones" are the age-specific returns to scale parameters γ_a with 95% confidence intervals estimated by Hall and Jones (2007) using data for the US from 1950 to 2000, in their baseline estimation. "This paper" is the $\gamma_a = \gamma_1(\mathcal{A} - a)$ line associated to the γ_1 estimated in our paper by a panel regression with data for 96 countries from 1960 to 2010. The dotted lines represent the corresponding bounds of the 95% confidence interval for γ_1.

Another difference arises when comparing equation (2.10) to Hall and Jones's health status production in (2.A.8), as they assume that all productivity growth in health production is submitted to the age-specific decreasing returns, or ${\phi_t}^{\gamma_a}$, while our functional form allows for technical progress in the term $D_{at} = {\Phi_t}^{-1} e^{\gamma_0(\mathcal{A}-a)}$. This is not an arbitrary choice, since it conforms to the empirical long-run reduction in the intercept parameter Φ of the Strehler-Mildvan correlation, which we represent as productivity growth g_Φ in $\Phi_t = e^{-\phi_0 - g_\Phi t}$.

2.A.3 Dalgaard and Strulik's health deficits

Strulik (2010), Strulik and Vollmer (2013), and Dalgaard and Strulik (2014) introduced contemporaneous bio-gerontology research on human aging into the economics literature. The survey carried out by Strulik (2010) presents the empirical regularities on human aging, as well as some

related biological theories of aging. Strulik and Vollmer (2013) estimate Gompertz-Makeham curves and the SM correlation using life-table data for developed countries over 25-year time intervals, from 1900 to 1999 for 12 countries and since 1950 for 26 countries. They find that the SM correlation parameters changed in the second half of the twentieth century, with values depending on the group of countries considered.

In the exploratory analysis in the "human aging" section of this paper, we also estimate Gompertz curves and the SM correlation, but using a sample of 33 countries that includes some developing economies. The main difference to their exercise, however, is that we observe the pattern of long-run changes in the SM correlation parameters by comparing Gompertz curve estimates in the years 1958-1962 and 2008-2010; two pictures separated by a 50-year window, rather than estimating these parameters for larger 25-year intervals, namely 1950-1974 and 1975-1999 as Strulik and Vollmer (2013) do. After this, in a panel regression with 96 countries for the entire 1960-2010 period, we estimate the parameters that determine the changes in the SM correlation and the Gompertz curves, interpreting them in terms of a longevity production function.

The health model developed by Dalgaard and Strulik (2014), used in other papers such as Dalgaard and Strulik (2017) and Böhm et al. (2020), takes a different route. This model of health production builds on the biological theory discussed in Mitnitski et al. (2002), which understands aging as the increased frailty of human bodies that results from the accumulation of health deficits. The main variable in this theory is the stock of health deficits $D(t)$. Dalgaard and Strulik (2014) propose a continuous time model of aging with individuals that invest in health inputs to delay the accumulation of health deficits and their time of death, which is deterministically chosen in the model. The authors show that, defining health capital $H(t)$ as the difference between an initial health stock H_0 and the stock of deficits, $H(t) = H_0 - D(t)$, their model is related to a continuous time version of Grossman's model by:

$$\dot{H}(t) = I(\mu, x(t)) - \mu(H_0 - H(t)) \tag{2.A.9}$$

where μ is the force of mortality, a constant that determines the speed of the aging process.

Comparing Dalgaard and Strulik's (2014) model to ours, we note that despite the fact that both models are based on the bio-gerontology research on human aging, the resulting functional forms are not the same. Our framework does not rely on any particular biological theory of human

aging, but rather on the direct reproduction of the stylized facts of the Gompertz law and the SM correlation. Further research is needed to better understand the origins of the differences and the respective empirical consequences. One difference is the health investment function $I(t)$, which is assumed by Dalgaard and Strulik (2014) as an additive to the stock of deficits (or health capital, in equation (2.A.9)). In our model, the determinants of health investments affect the depreciation rate of longevity capital, and there is no additive investment function.

Another relevant difference regards the depreciation of health capital. As stressed by Dalgaard and Strulik (2014, 2015), the original version of Grossman's model with constant depreciation rate δ implies that the depreciation of health capital is higher for healthier individuals as $\Delta(t) = \delta H(t)$, so that health capital losses are higher for younger individuals than for the older ones. They argue that there is a drawback to assuming that depreciation rates rise with age, as these rates are exogenous to the model or build from arbitrary functional forms.

In Dalgaard and Strulik's (2014) health deficits model, however, depreciation is higher for the unhealthier individuals, as $\Delta(t) = \mu(H_0 - H(t))$, which they defend as more intuitive. The model proposed in our paper offers a new answer to this problem, as the depreciation rate of longevity capital rises with age, but not in an arbitrary form, since it is driven by the Gompertz law and the SM correlation. Moreover, when comparing two individuals of the same age, health depreciation is higher for the healthier individual. This is a consequence of the compensation law of mortality, as late-life mortality convergence requires that the aging speed is higher for populations with lower current mortality.

2.B Proof of Proposition 1

2.B.1 Market production

Firms operate using the production functions (2.23) and (2.24). For ease of notation, let $Y_y = Y$ denote gross output in the non-health sector and $Y_z = Z$ gross output in the health services sector, such that for $j = \{y, z\}$,

$$Y_j = \Lambda_j {Q_j}^{\theta_j} {I_j}^{1-\theta_j} \tag{2.B.1}$$

$$Q_j = {K_j}^{\alpha_j} (\Omega_j(t) H_j)^{1-\alpha_j} \tag{2.B.2}$$

If p_j is the *gross output price* for the sector j, profits maximization imply

$$p_j Y_j - p_y I_j - (r+\delta)K_j - wH_j = 0 \tag{2.B.3}$$

Then as $p_j Y_j - p_y I_j$ is the *nominal value added* of sector j, by defining P_j as the *value added price* index of sector j, we can write

$$P_j Q_j = (r+\delta)K_j + wH_j \tag{2.B.4}$$

$$P_j \equiv \frac{p_j Y_j - p_y I_j}{Q_j} \tag{2.B.5}$$

The first order conditions for intermediate inputs, physical capital, and human capital are

$$I_j = (1-\theta_j)\frac{p_j Y_j}{p_y} \tag{2.B.6}$$

$$r = \alpha_j \theta_j \frac{p_j Y_j}{K_j} - \delta \tag{2.B.7}$$

$$w = (1-\alpha_j)\theta_j \frac{p_j Y_j}{H_j} \tag{2.B.8}$$

Substituting the first order condition for intermediate inputs (2.B.6) in the gross output production function (2.B.1),

$$Y_j = \Theta_j \left(\frac{p_j}{p_y}\right)^{\frac{1-\theta_j}{\theta_j}} Q_j \tag{2.B.9}$$

for

$$\Theta \equiv [\Lambda_j (1-\theta_j)^{1-\theta_j}]^{\frac{1}{\theta_j}}$$

and the nominal value added of each sector is equal to:

$$P_j Q_j = \theta_j p_j Y_j \tag{2.B.10}$$

The gross output of sector y is defined as the numeraire good, $p_y = 1$, and to simplify the notation we also define $p = p_z$. Combining the first order conditions for physical capital (2.B.7) of the

two sectors to (2.B.9), (2.B.1), and (2.B.2), we find that the relative price of health services p is:

$$p = \left[\frac{\theta_y \Theta_y \alpha_y}{\theta_z \Theta_z \alpha_z} \left(\frac{\Omega_y(t) H_y}{K_y} \right)^{1-\alpha_y} \left(\frac{K_z}{\Omega_z(t) H_z} \right)^{1-\alpha_z} \right]^{\theta_z} \tag{2.B.11}$$

Together with the first order conditions for human capital (2.B.8) in the two sectors, equation (2.B.11) determines that the ratios between physical capital and human capital in the two sectors are proportional to each the other:

$$\frac{K_y}{H_y} = \frac{\alpha_y(1-\alpha_z)}{\alpha_z(1-\alpha_y)} \frac{K_z}{H_z} \tag{2.B.12}$$

Substituting (2.B.12) back into (2.B.11) results in an expression for the relative price of health services that only varies with TFP in the two sectors $\Omega_y(t), \Omega_z(t)$ and factor inputs allocated to healthcare services K_z, H_z:

$$p = \left[\frac{\theta_y \Theta_y}{\theta_z \Theta_z} \left(\frac{\alpha_y}{\alpha_z} \right)^{\alpha_y} \left(\frac{1-\alpha_y}{1-\alpha_z} \right)^{1-\alpha_y} \frac{\Omega_y(t)^{1-\alpha_y}}{\Omega_z(t)^{1-\alpha_z}} \left(\frac{K_z}{H_z} \right)^{\alpha_y - \alpha_z} \right]^{\theta_z} \tag{2.B.13}$$

The aggregated GDP of this economy is the sum of nominal value added from the two sectors, $GDP = P_y Q_y + P_z Q_z$, then from equations (2.B.2), (2.B.9), (2.B.10), and (2.B.13) it equals:[15]

$$GDP = \theta_y Y + \theta_z pZ \tag{2.B.14}$$

$$Y = \Theta_y {K_y}^{\alpha_y} \left(\Omega_y(t) H_y \right)^{1-\alpha_y} \tag{2.B.15}$$

$$pZ = \frac{\theta_y \Theta_y}{\theta_z} \left(\frac{\alpha_y}{\alpha_z} \right)^{\alpha_y} \left(\frac{1-\alpha_y}{1-\alpha_z} \right)^{1-\alpha_y} {K_z}^{\alpha_y} \left(\Omega_y(t) H_z \right)^{1-\alpha_y} \tag{2.B.16}$$

It is relevant to stress that using equation (2.B.16), we can determine the equilibrium nominal value added for health services $\theta_z pZ$ even if we do not know the TFP for market production of health services, which is determined by $\Omega_z(t)$.

2.B.2 Households and longevity production

Consumers solve the recursive utility maximization problem by choosing current consumption c, medical goods m, health services z, and next period assets ω' under the definitions and constraints

[15] Real GDP is, in fact, the nominal GDP deflated by a price index for aggregate value added. Nevertheless, we have tested for some empirical alternatives to the price index of health services, and for all of them the corresponding measured inflation in the aggregate price index, induced by the change of relative prices associated to structural transformation, was negligible. Thus, for model calibration we use GDP computed by equation (2.B.14) to target real GDP in the data.

stated in equations (2.11) to (2.22).

The first order condition for next period assets and the budget constraint imply the consumption Euler equation:

$$\frac{c'}{c} = (\bar{\beta}R')^{\frac{1}{\sigma}} \tag{2.B.17}$$

Consumers in the last period of life $a = \mathcal{A} - 1$ choose $z = m = 0$. For all the other ages, first order conditions for health services and medical goods result in

$$z = \frac{\bar{\beta}(1-\eta)V'\mu\gamma_1(\mathcal{A}-a)c^{\sigma}}{\vartheta}\frac{\varepsilon}{p} \tag{2.B.18}$$

$$m = \frac{\bar{\beta}(1-\eta)V'\mu\gamma_1(\mathcal{A}-a)c^{\sigma}}{\vartheta}(1-\varepsilon) \tag{2.B.19}$$

where V' is the next period value function.

Combining (2.B.18) and (2.B.19), optimal choices of health services and medical goods are connected by

$$m = \frac{(1-\varepsilon)}{\varepsilon}pz \tag{2.B.20}$$

then we substitute the variable m in equation (2.14) for the expression in (2.B.20) to find:

$$\vartheta = \Gamma(a,t)\left[\left(\frac{1-\varepsilon}{\varepsilon}\right)^{\mu(1-\varepsilon)}(p^{1-\varepsilon}z)^{\mu}h^{1-\mu}\right]^{\gamma_1(\mathcal{A}-a)} \tag{2.B.21}$$

We define the relative health spending by age ξ as the ratio between health spending at age a and the mean health spending per capita for all ages. As optimal spending in medical goods is proportional to expenditures in health services by (2.B.20), we can write ξ as health services for age z over the gross output of health services per capita Z:

$$\xi = \frac{z}{Z} \tag{2.B.22}$$

Thus, substituting $z = \xi Z$ in (2.B.21), the term $p^{1-\varepsilon}z = \xi p^{-\varepsilon}pZ$ is expanded using equations (2.B.13) and (2.B.16) for p and pZ. Defining the health services inputs index $x = {K_z}^{\lambda}{H_z}^{1-\lambda}$, for $\lambda \equiv \varepsilon\theta_z\alpha_z + (1-\varepsilon\theta_z)\alpha_y$, after some manipulations we find the single stage longevity production function in equation (2.27).

2.B.3 Computing the equilibrium

Models of growth with structural transformation are characterized by unbalanced growth trajectories. For structural change models with explicit solutions to the long-run balanced growth path, it is possible to find these solutions and simulate transition dynamics from known initial conditions. Since this is not the case in our paper, as in Kehoe et al. (2018) we must solve numerically for the balanced growth path and transition dynamics simultaneously. Kehoe et al. (2018) simulate trajectories from the period of interest up to 100 years in the future, requiring that the model converges to a balanced growth trajectory at the end of this timespan. They assume that after 50 years the exogenous variables and the preferences and production parameters that promote structural transformation gradually converge to values that bring the economy to balanced growth.

In our model of structural transformation with overlapping generations, the procedure to compute equilibrium trajectories is similar. We simulate the model for 40 periods, which correspond to 200 years in the data. But instead of changing model parameters after some years, we assume that the interest rate and GDP shares of each sector are constant after the last simulated period. As we solve the recursive utility maximization problem of each cohort through backwards value function iteration, starting from the last period of life attainable, this assumption imposes a terminal condition for the generations that are still alive after the last period. In fact, we assume that our general equilibrium model works as a partial equilibrium model for these generations after period 40. However, we do not assume specific values for the sectoral GDP shares and the interest rate, we merely require that they maintain their values for the period 40 as computed in equilibrium.

We guess sequences of 40 periods for the health services share in value added $1-\nu$, the interest rate r, and the aggregate stock of human capital H. We do not need to guess the price p and productivity Ω_z of health services, as long as we know the productivity of the longevity production function $D(a,t)$ and productivity in non-health production $\Omega_y(t)$.

For computing the numerical solutions, we detrend some variables by the productivity in the non-health sector $\Omega_y(t) = e^{g_y t}$; detrended variables are designated by a tilde over the variable, as in $\tilde{K} = \frac{K}{\Omega_y(t)}$.

Let $\nu = \frac{P_y Q_y}{P_y Q_y + P_z Q_z}$ denote the value added share of the non-health sector, and the non-health shares of factor inputs be denoted by $\kappa = \frac{K_y}{K}$ and $\ell = \frac{H_y}{H}$.

Then using the optimal conditions above, the relevant aggregate variables conditional on the guesses for ν, r, and H are:

$$\kappa = \left[1 + \frac{\alpha_z}{\alpha_y}\left(\frac{1-\nu}{\nu}\right)\right]^{-1} \tag{2.B.23}$$

$$\ell = \left[1 + \left(\frac{1-\alpha_z}{1-\alpha_y}\right)\left(\frac{1-\nu}{\nu}\right)\right]^{-1} \tag{2.B.24}$$

$$\tilde{K} = \frac{\ell H}{\kappa}\left(\frac{\alpha_y \theta_y \Theta_y}{r+\delta}\right)^{\frac{1}{1-\alpha_y}} \tag{2.B.25}$$

$$\tilde{Y} = \Theta_y (\kappa \tilde{K})^{\alpha_y} (\ell H)^{1-\alpha_y} \tag{2.B.26}$$

$$\tilde{w} = (1-\alpha_y)\theta_y \frac{\tilde{Y}}{\ell H} \tag{2.B.27}$$

$$\widetilde{pZ} = \Theta_y \frac{\theta_y}{\theta_z}\left(\frac{\alpha_y}{\alpha_z}\right)^{\alpha_y}\left(\frac{1-\alpha_y}{1-\alpha_z}\right)^{1-\alpha_y}[(1-\kappa)\tilde{K}]^{\alpha_y}[(1-\ell)H]^{1-\alpha_y} \tag{2.B.28}$$

$$x = [(1-\kappa)\tilde{K}\Omega_y(t)]^{\lambda}[(1-\ell)H]^{1-\lambda} \tag{2.B.29}$$

We compute the policy functions through a recursive algorithm, by iterating backwards from the last period of life, for all the cohorts from the last period to the first. Our algorithm takes the aggregate variables as given, and uses endogenous gridpoints (Carroll, 2006), generated from a predetermined grid of assets for the last year of life and the following equations derived from the first order conditions:

$$eef = \frac{(e^{g_y}\tilde{c}')^{\sigma}}{R'} \tag{2.B.30}$$

$$\xi = \left[\frac{(eef + \bar{\beta}(1-\eta)\tilde{c}^{\sigma})V'\varepsilon\mu\gamma_1(\mathcal{A}-a)}{\widetilde{pZ}\Omega_y(t)^{1-\sigma}D(a,t)(x^{\mu}h^{1-\mu})^{\gamma_1(\mathcal{A}-a)}}\right]^{\frac{1}{1+\mu\gamma_1(\mathcal{A}-a)}} \tag{2.B.31}$$

$$\vartheta = D(a,t)[(\xi x)^{\mu}h^{1-\mu}]^{\gamma_1(\mathcal{A}-a)} \tag{2.B.32}$$

$$\eta = 1 - e^{-\frac{1}{\vartheta}} \tag{2.B.33}$$

$$\tilde{c} = \left[\frac{eef}{\beta(1-\eta-\bar{\eta})}\right]^{\frac{1}{\sigma}} \tag{2.B.34}$$

$$V = u(\Omega_y(t)\tilde{c}) + \beta(1-\eta-\bar{\eta})V' \tag{2.B.35}$$

$$R = 1 + r + \frac{\eta+\bar{\eta}}{(1-\eta-\bar{\eta})} \tag{2.B.36}$$

$$\tilde{\omega} = \frac{1}{R}\left[e^{g_y}\tilde{\omega}' + \tilde{c} + \frac{\xi\widetilde{pZ}}{\varepsilon} - 1_{a<a_R}\tilde{w}h\chi\right] \tag{2.B.37}$$

After the household's optimal choices are aggregated and compared to market clearing condi-

tions, we guess new values for the series ν, r, and H. This algorithm is repeated until the market clearing errors are lower than a tolerable bound.

2.3 Robustness checks

As a way to assess the sensibility of the model to alternative parametrizations, Table 2.10 presents robustness checks for some of the results for France. For each of the estimated longevity production parameters μ, γ_1, and g_Φ, we perform alternative simulations that use the upper and lower bounds of the respective 95% confidence intervals and maintain default values for the other parameters.

Table 2.10: *Robustness checks for France, parameters μ, γ_1, and g_Φ*

		μ		γ_1		g_Φ	
	Base	Low	High	Low	High	Low	High
Parameter values		0.20	0.35	0.021	0.030	0.053	0.060
Health share in GDP (%)							
Year 2015	12.4	10.6	13.7	11.6	13.0	12.7	12.1
Difference to 1960	9.0	8.0	9.5	8.7	9.1	9.2	8.7
Life expectancy at age 30 (years)							
Year 2015	83.6	83.6	83.6	83.4	83.8	83.3	83.9
Difference to 1960	7.7	7.5	7.9	7.5	7.9	7.4	8.0

Notes: The simulated baseline scenario uses the point estimates $\mu = 0.28, \gamma_1 = 0.0253$, and $g_\Phi = 0.0565$ from the panel regression. Parameter values "Low" and "High" are the bounds of the estimated 95% confidence intervals for each parameter, and the results in each column correspond to alternative simulations that use these bounds and maintain the default values for the other parameters.

For the health share of GDP, the parameter that most affects the results is μ, the elasticity of health inputs in longevity production, controlled by the degree of returns to scale. By changing this parameter on the confidence interval, the share of health spending in GDP for France in 2015 oscillates by an amplitude of three percentage points. The rise in this share from 1960 to 2015 varies from 8 to 9.5 percentage points. Parameter μ is related to the relative importance in longevity production of health inputs in opposition to the own human capital h, which has a corresponding elasticity of $1 - \mu$. The robustness check for this parameter indicates that health spending is higher when medical goods and healthcare services are relatively more important than education for longevity production.

Parameter γ_1 measures the degree of returns to scale in longevity production when multiplied by $(\mathcal{A} - a)$, and larger values of this parameter promote higher health expenditures.

Among these three parameters, the TFP growth rate in longevity production g_Φ is the least

influential on health expenditures considering variations in the 95% confidence interval. As it varies from 5.3% to 6.0% in the five-year period, or 1.06% to 1.20% per year on an annual scale, the share of health expenditure in GDP changes by 0.6 percentage points. Higher productivity growth in longevity production reduces health spending, albeit by a small amount.

Changes in the results for life expectancy at age 30 resulting from these parametric variations are minor, in a range of less than one year. The order of influence of the three parameters is reversed, as g_Φ is the most relevant, but the differences in influence are very small.

Therefore, the simulations with these alternative parameter values indicate that the paper's results regarding predicted trends for life expectancy at age 30 and the share of health spending in GDP are robust to parametric changes within the bounds of the estimated confidence intervals in the longevity production panel regression.

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