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ESCOLA DE ECONOMIA DE SÃO PAULO

LIGIA DE OLIVEIRA RODRIGUES

ESSAYS ON LOW INTEREST RATES

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Dissertação apresentada à Escola de Economia de São Paulo da Fundação Getulio Vargas como requisito para obtenção do título de Mestre em Economia.

Área de Concentração: Macroeconomia

Orientador: Prof. Dr. Paulo Sérgio Tenani

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São Paulo, 4 de Dezembro de 2020:

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"One is reminded of the old joke about the centipede who was asked how he managed to coordinate his 100 legs: He started thinking about it and could never walk properly again."

Paul Krugman

ABSTRACT

Why are interest rates are so low in advanced economies? Are interest rates also low in emerging markets? This paper investigates real interest rates decline over the last decades, focusing on two hypotheses to explain such phenomenon: (i) convenience yield or the premium for safe and liquid assets, and (ii) secular stagnation. First, we apply a Bayesian VAR with common trends for the United States, the United Kingdom, Chile, Mexico, and Brazil to assess how much an increase in convenience yield can explain the natural interest rate drop from 1998 to 2019. Second, we simulate an overlapping generation model with secular stagnation features to demonstrate how these forces can justify change in the natural interest rate from 1970 to 2015 for the United States and the United Kingdom. Main findings indicate that demographic changes and productivity slowdown have been pressuring the natural interest rates down since the 1970s in United States and the United Kingdom. However, increase in the convenience yield, the premium investors are inclined to pay for safe and liquid assets, has also been putting downward pressure on the natural interest rates since the 1990s in the United States and the United Kingdom, and since 2008 in Chile and Mexico. For Brazil, results are inconclusive.

Keywords: natural rate of interest, r*, convenience yield, VAR with common trends, secular stagnation, overlapping generation model

RESUMO

Por que as taxas de juros estão tão baixas nas economias avançadas? E nas economias emergentes? Esta dissertação investiga o declínio das taxas de juros reais nas últimas décadas, focando em duas hipóteses para explicar tal fenômeno: (i) *convenience yield*, prêmio que investidores estão dispostos a pagar por ativos seguros e líquidos e (ii) estagnação secular. Primeiro, aplicamos um VAR Bayesiano com tendências comuns para Estados Unidos, Reino Unido, Chile, México e Brasil para avaliar se o aumento no convenience yield pode explicar a queda na taxa de juros natural de 1998 a 2019. Depois, simulamos um modelo intergeracional para demonstrar como os principais fatores da estagnação secular podem justificar a mudança na taxa de juros natural de 1970 a 2015 para os Estados Unidos e o Reino Unido. As principais descobertas indicam que as mudanças demográficas e a desaceleração da produtividade têm pressionado as taxas de juros naturais para baixo desde a década de 1970 nos Estados Unidos e no Reino Unido. No entanto, o aumento no *convenience yield*, os prêmios que investidores estão inclinados a pagar por ativos seguros e líquidos, também explica parte da queda das taxas de juros naturais desde a década de 1990 nos Estados Unidos e no Reino Unido, e desde 2008 no Chile e no México. Para o Brasil, os resultados são inconclusivos.

Palavras-chave: taxa de juros natural, r*, convenience yield, VAR com tendências comuns, estagnação secular, modelo intergeracional.

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SUMMARY

1 INTRODUCTION

After more than a decade of the worst financial crisis since the Great Depression, the world is still facing a sluggish global economic growth coupled with a very low interest rates in the main developed countries. A vast literature has been arising since the mid-2000s to shed light on the decline in the real interest thenceforth. As summed up by DeLong:

> Ben Bernanke, the former Federal Reserve chairman, says we have entered an age of a 'global savings glut'. Kenneth Rogoff of Harvard points to the emergence of global 'debt supercycles'. Princeton's Paul Krugman warns of the return of 'Depression economics'. And former Treasury Secretary Lawrence Summers calls for broad structural shifts in government policy to deal with 'secular stagnation'. All of these experts are expecting a future that will be very different than the second half of the 20th century, or even the so-far, not-so-good third millennium. But they are influenced by different inclinations – toward optimism or pessimism, toward cautious repairs or an abrupt break with policy as usual – to diagnose the malady and prescribe the treatment. (DELONG, 2015)

Throughout this paper, we focus on (i) the shortage of safe assets and (ii) the secular stagnation hypotheses to explain real interest rate drop in the last years. First, we estimate the natural interest rate trends in the United States, the United Kingdom, Chile, Mexico, and Brazil through two Bayesian VAR models based on Negro et al. (2017). Our main contributions are estimating the interest rates trends of the United Kingdom, Chile, Mexico, and Brazil, beyond the United Sates. Main findings indicate a rise in the premium that investors are inclined to pay for safe assets or government's treasuries, which has been pushing interest rates down since the 1990s in the United States and the United Kingdom, and since 2008 in Chile and Mexico.

Second, we use a Overlapping Generation Model with three periods to simulate the natural interest rate drop from 1970 to 2015, and to decompose the main secular stagnation features for the United Kingdom. Such model was developed by Eggertsson, Mehrotra and Robbins (2019) and applied originally to the United States. Our main contribution is to conclude that demographic changes and productivity slowdown are the main factors behind the interest rate drop in the United Kingdom for the 1970-2015 period, which is aligned with the main findings found by Eggertsson, Mehrotra and Robbins (2019) for the United States. Furthermore, there is evidence of secular trends also pressuring the real interest rates down since the 1970s in the United States and the United Kingdom.

This paper is composed of four main chapters, in addition to the introduction (1) and conclusion (6). In chapter 2, we briefly discuss safe assets and secular stagnation views on interest rates decline over the last decades. In chapters 3 and 4, we estimate the natural interest rate through two Bayesian VAR models to identify the relationship between the interest rate trends and the investor's preference for safe assets for the United States, United Kingdom,

Chile, Mexico, and Brazil. Finally, in chapter 5, we explain the Secular Stagnation Overlapping Generation Model and apply it to the United States and the United Kingdom.

2 A SHORT REVIEW OF THE SECULAR DEBATE

2.1 Shortage of Safe Assets

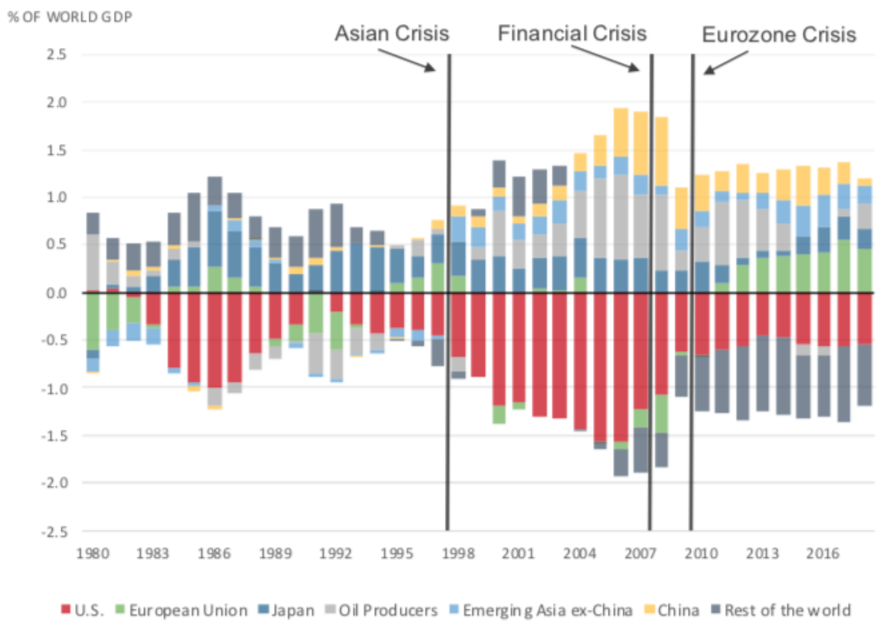
The hypothesis of a assets shortage resulting in low interest rates for advanced economies has risen from Bernanke et al. (2005). In March 2005, the governor of the Federal Reserve System states "that over the past decade a combination of diverse forces has created a significant increase in the global supply of saving – a global saving glut – which helps to explain both the increase in the U.S. current account deficit and the relatively low level of long-term real interest rates in the world today" (BERNANKE et al., 2005). After financial crises in the 1990s (mainly Asia crisis in 1997), emerging economies adopted a new strategy to become net exporters of financial capital through the accumulation of foreign exchange reserves. Thus, the developing world grew into a net supplier of funds to developed economies, pressuring down interest rates in rich countries, mainly in the United States.

Caballero and Farhi (2014) further developed Bernanke's explanation, observing that the shortage of safe assets indeed became more severe during the 2008 crisis. The authors believe that, without new financial innovation or change in agents' preferences, shortage of assets should worse over time, "pushing down real interest rates, putting pressure on the financial system, and straining monetary policy during contractions". Caballero, Farhi and Gourinchas (2015) modeled global imbalances in the developed world, including a "Zero Lower Bond" (ZLB) rigidity, reaching a similar conclusion in regards to interest rate and global imbalances correlation (Figure 1): safe asset scarcity pressures interest rates to low levels, pushing the global economy into a global safety trap at the ZLB.

In line with this literature that discuss the role of the safe assets, with an emphasis on the spread between Treasury and corporate bonds, Negro et al. (2017) construct a model that incorporates a convenience yield in the real interest rate estimation for the United States. In Negro et al. (2019), the authors expand the previous model to a global interest rate based on seven advanced economies (USA, UK, Germany, France, Canada, Italy, and Japan). They find the global real interest rate fell materially in the last three decades, after being roughly stable at almost 2 percent for more than a hundred years. The authors conclude the two most important drivers of this "secular decline" are "an increase in the premium that international investors are willing to pay to hold safe and liquid assets" and a "lower economic growth around the world".

In chapters 3 and 4, based on Negro et al. (2017), we analyze the trend of interest natural rate and its association with safe assets for the United States, the United Kingdom, Chile, Mexico, and Brazil. Results for advanced economies were similar to Negro et al. (2017) as expected, while findings for emerging economies were a new contribution to this literature.

Figure 1 – Current Account Balances



Source – Extracted from Caballero, Farhi and Gourinchas (2015): "The graph shows current account balances as a fraction of world GDP from 1980 to 2018. Source: World Economic Outlook (Oct. 2018), and Authors' calculations."

2.2 Secular Stagnation

In December 1938, Hansen employed the term 'secular stagnation' for the first time in a presidential address delivered at the American Economic Association: "This is the essence of secular stagnation - sick recoveries which die in their infancy and depressions which feed on themselves and leave a hard and seemingly immovable core of unemployment" (HANSEN, 1939). Still worried by the effects of the Great Depression as unemployment remained high, the economist related a slower growth population to a lower need of capital investment, which, in the absence of advance in technology, would maintain employment at insufficient levels.

Worth noting Summers comments on Hansen's idea:

> Hansen turned out to be completely wrong but completely wrong in a way that suggests that at some future point he could turn out to be right. What actually happened was that the Second World War came along, which brought about a vast increase in demand, and the government became a mass absorber of savings by running large deficits. [...] The postwar economy was aided by pent-up demand released by the end of wartime credit controls and rationing; the economy was also aided by a massive government project to build out suburbia. The economy was also importantly aided by an unexpected (and still partially unaccounted for) rise in fertility to nearly four children per woman, which created the Baby Boom. [...] My thesis is that Hansen was a couple of generations too early but that the issues he identified can be a chronic problem for capitalist economies. Moreover, they quite likely are a chronic problem for the economies of the industrialized world today. (SUMMERS, 2016)

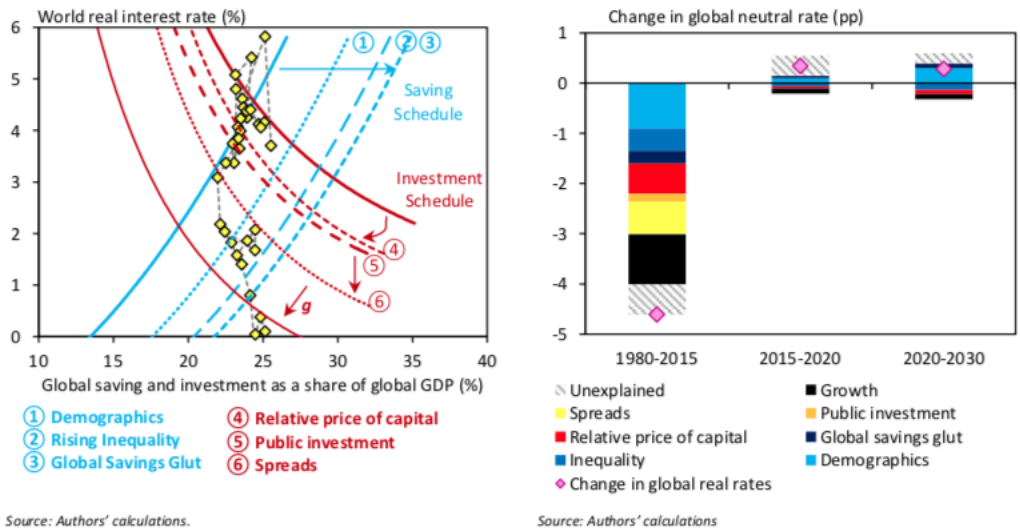
The term 'secular stagnation' is revived by Summers at the end of 2013, with a full speech describing its main propositions in February 2014 to the National Association for Business Economics. For Summers (2014), a secular stagnation is mainly defined by a mismatch in desired saving and investment that leads to a decline in the real interest rate, but, even in the presence of low rates, demand continues deficient with the economy running below its previous potential. Its main features can be summarized as: (i) lower demand for debt-financed investment mainly driven by less extensive in capital leading technology companies; (ii) declining rate of population growth and lower rate of technological progress in line with Hansen (1939); (iii) increase in inequality, which results in a overall higher propensity to save; (iv) decrease in the relative price of capital goods, that reduces the propensity to invest; (v) disinflation, as it leads to lower required pretax real interest rate; and (vi) global higher accumulation of central bank reserves that are considered safe assets, in particular treasuries issued by the United States, which is related to Bernanke et al. (2005). To sum up, Summers believes that are forces increasing savings propensity (changes in income distribution, reserves accumulation, higher retirement), and forces decreasing investment propensity (declining population growth, lower technology advances, drop in capital goods prices), both pressuring equilibrium real rates down.

Following this secular stagnation view, Rachel and Smith (2015) provide a savings-investment framework (Figure 2) to measure by how much each secular trend has been impacting interest rates in the last thirty years. Their main findings suggest that savings schedule has shifted due to demographic forces (90bps of the decline in interest rates), increase in inequality (45bps), and higher accumulation of reserves (25bps); on the other side, investment schedule changed owing to a decline in the price of capital (50bps), lower public investments (20bps), and higher spread between risk free rate and capital return (70bps). All these forces sum up to justify a decline of 300bps in global real rates. Lower growth trend accounts for 100bps of the fall in real rates and roughly 50bps are unexplained.

Another similar perspective is found in Rachel and Summers (2019), in which the authors estimate the natural rate of interest through the HLW model (HOLSTON; LAUBACH; WILLIAMS, 2017), finding a decline in the real interest rates for the OECD members (aggregated data for 36 countries) of 2 percentage points between 1990 and 2017. They further develop two general equilibrium models to decompose the decline in the natural interest rate into private sector forces (TFP growth, population growth, retirement, and inequality) and public policies (government debt, government policy, social security, old-age healthcare). Main results indicate the downward trend in interest rates is best explained by "changes in private sector saving and investment propensities", and that interest rates would have reduced even more to substantially negative levels in the last decades but for the rise in government debt and growth in social insurance programs, which “suggests substantial grounds for concern over secular stagnation”.

Alike findings were also reported by Eggertsson, Mehrotra and Robbins (2019), who construct an overlapping generation model ("OLG") to quantify the secular stagnation forces that

Figure 2 – Shifts in Desired Saving & Investment and Secular Drivers of Real Interest Rates



Source – Extracted from Rachel and Smith (2015)

have been lowering the natural interest rates in the last decades. They use a 56-period quantitative life cycle model for the United States to find a decline in natural rate from 2.55% in 1970 to -1.47% in 2015 (-4.02 percentage points). According to the model, such drop is explained mainly by decreases in fertility (43% of total change), mortality (43%), productivity's rate (44%), while increase in government debt (-49%) strongly counterbalanced the downward trends. In chapter 5, we re-simulated this OLG model for the United States, and also estimated the model for the United Kingdom, reaching similar findings for both countries.

3 LONG TERM TRENDS OF REAL INTEREST RATES

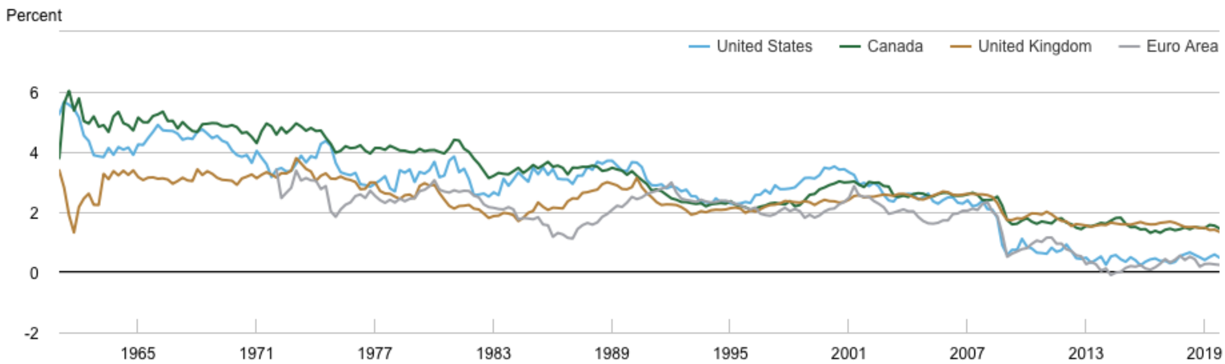
3.1 Estimating the R Star

In order to assess why interest rates are low, we need first to evaluate how interest rates have changed over the past decades. The Laubach and Williams (2003) model ("LW") has been extensively used for estimating the real interest rates or r star ("r*") for advanced economies. Initially conceived for the USA economy, the model was updated in 2017 to include Canada, the Euro Area, and the United Kingdom, being known as the HLW model (HOLSTON; LAUBACH; WILLIAMS, 2017).

Laubach and Williams (2003) define the natural rate of interest as "the real short-term interest rate consistent with output converging to potential, where potential is the level of output consistent with stable inflation". Worth noting the r* estimation also requires estimations of the potential output and the trend growth rate. The authors use a Kalman filter to the unobserved variables (the natural rate of interest, the level of potential output, and the trend growth rate), and a restricted VAR to extract trend-cycle decomposition for r*, trend growth rate, and output gap.

The website of the Federal Reserve Bank of New York [1] provides updated estimates of LW and HLW models quarterly. The HLW's model estimated from 1Q61 to 4Q19 by the New York FED indicates a downward trend in natural real rate since the 1970s for the USA, the UK, and Canada, with a remarkable drop in 2009 for the USA and the Euro Area (Figure 3).

Figure 3 – The Natural Rate of Interest based on HLW model



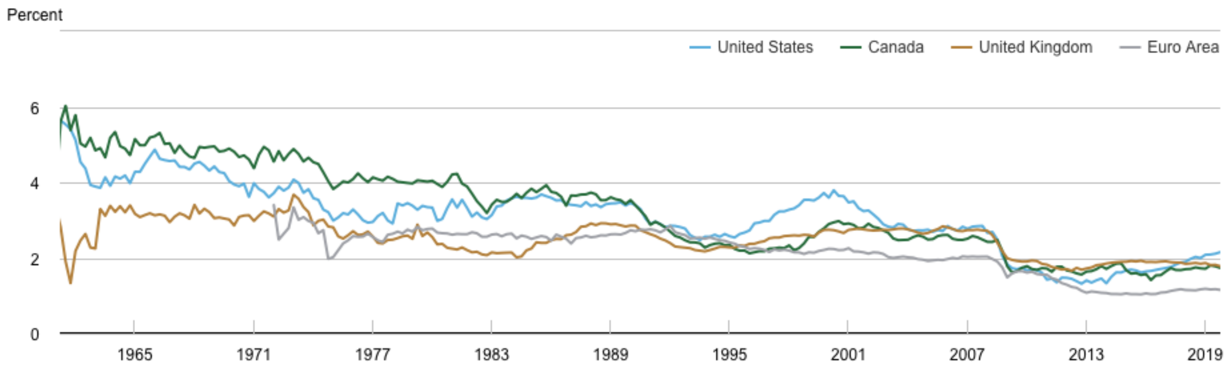
Source – New York FED

To estimate the path of the natural rate of interest, we use a Bayesian VAR ("BVAR") with common trends based on Negro et al. (2017). This approach is related to the LW and HLW models, but it has the advantage of not requiring potential output estimation. Indeed, the LW and HLW models can be viewed as a restricted version of the BVAR described in the next section.

[1] <https://www.newyorkfed.org/research/policy/rstar>

Additionally, since the LW and HLW models relate the natural interest rate and the potential GDP, a large part of the drop in r* over the past decades is necessary explained by a material fell in the estimated trend growth (Figure 4). Such limitation is not faced by the model developed by Negro et al. (2017).

Figure 4 – Trend Growth based on HLW model



Source – New York FED

3.2 Empirical Framework

Based on Negro et al. (2017), the Bayesian VAR to be estimated follows:

$$y_t = \Lambda \bar{y}_t + \tilde{y}_t \tag{3.1}$$

$$\bar{y}_t = \bar{y}_{t-1} + e_t \tag{3.2}$$

$$\Phi(L)\tilde{y}_t = \varepsilon_t \tag{3.3}$$

where y_t is a n x 1 vector of observed variables, decomposed into a q x 1 vector of trends $(\bar{y}_t)$, and a n x q vector of stationary elements $(\tilde{y}_t)$ through the measurement equation (3.1); y_t and $(\bar{y}_t)$ follows a random walk (3.2); and ε_t represents a VAR (3.3) with n x n matrices (Φ) and (q + n) x 1 shocks vector independently and identically distributed as a multivariate Gaussian distribution. (3.2) and (3.3) are transition equations.[2]

Following the BVAR model, Negro et al. (2017) assume the short-term interest rate presents a trend and a stationary component with τ being the maturity of the security expressed quarterly:

$$R_{\tau,t} = \bar{R}_{\tau,t} + \tilde{R}_{\tau,t} \tag{3.4}$$

Additionally, based on the Fisher equation, the trend in the nominal short term interest rate $(\bar{R}_{1,t})$ can be written as the trends in the real interest rate $(\bar{r}_t)$ and inflation $(\bar{\pi}_t)$:

$$\bar{R}_{1,t} = \bar{r}_t + \bar{\pi}_t \tag{3.5}$$

[2] BVAR is assumed to be stationary with 4 lags, the prior is a standard Minnesota (overall tightness equal to 0.2), and parameters are estimated by a Gibbs sampler.

As we substitute (3.6) in (3.4), we obtain for a nominal short-term interest rate ($\tau = 1$):

$$R_{1,t} = \bar{r}_t + \bar{\pi}_t + \tilde{R}_{1,t} \tag{3.6}$$

In order to distinguish changes in real rate ($\bar{r}_t$) and inflation ($\bar{\pi}_t$) trends, inflation (π_t) and inflation expectations (π_t^e) are included as below:

$$\pi_t = \bar{\pi}_t + \tilde{\pi}_t \tag{3.7}$$

$$\pi_t^e = \bar{\pi}_t + \tilde{\pi}_t^e \tag{3.8}$$

As expectation hypothesis do not necessarily hold, changes in the premium may explain movements in yields. To address this, it is added an exogenous component representing the trend in the nominal premium ($\bar{tp}_t$), along with the real interest rate ($\bar{r}_t$), inflation ($\bar{\pi}_t$) and a stationary component ($\tilde{R}_{80,t}$), to model the long term interest rate ($R_{80,t}$):

$$R_{80,t} = \bar{r}_t + \bar{\pi}_t + \bar{tp}_t + \tilde{R}_{80,t} \tag{3.9}$$

Finally, the long-run projections for the short-term interest rates are decomposed into the real interest rate trend, the inflation trend, and a stationary component:

$$R^e_{1,t} = \bar{r}_t + \bar{\pi}_t + \tilde{R}^e_{1,t} \tag{3.10}$$

The equations system (3.6),(3.7), (3.8), (3.9), and (3.10) corresponds to the BVAR (3.1), where variables follow the random walk described in (3.2), except for the stationary components that advance as (3.3). This is the BVAR Baseline Model, in which the trend in real interest rate is defined by the short-term and long-term interest rate, the inflation rate, and the expectations of interest and inflation rates.

3.3 Data

The model's main variables are: (i) the short-term interest rates of government securities, (ii) the long-term interest rates of government securities, (iii) the inflation rate, (iv) the long-term expectations for short-term inflation, and (v) the long-term expectations for the short-term yields on government treasuries.

For the United States, we updated data as described in Negro et al. (2017). Except for expectations, the annualized personal consumption expenditures (implicit price deflator) for inflation, the 3-month Treasury Bill rate for the short-term yield and the 20-year Treasury constant maturity rate for the long-term yield are available from the Federal Reserve Bank of St. Louis or FRED [3]. The long-term expectation for the short-term inflation and the long-term expectation

[3] <https://fred.stlouisfed.org>

for the 3-month Treasury Bills are collected from the Survey of Professional Forecasters [4]. The original paper estimated the VAR model from 1Q60 to 4Q16. We updated data to estimate the model from 1Q60 to 4Q19.

Main sources for constructing database for the United Kingdom from 1Q60 to 4Q19 are the Bank of England [5], the HM Treasury [6], and the OECD [7]. We obtained data for inflation, measured as the CPI, from the OECD. The short-term interest rate, calculated as the Treasury Bill Discount Rate, is available from BoE on "A Millennium of Macroeconomic Data for the UK". The long-term interest yield, defined as the government bonds maturing in 20 years, is also collected from the BoE (also "A Millennium of Macroeconomic Data for the UK" series). For the long-term inflation expectations, we use annually observations of the "A Millennium of Macroeconomic Data for the UK" of BoE from 1960 to 2003, and the quarterly market median forecast of annual CPI inflation from 2014 onward, also obtained from BoE. The long-term interest rate expectations are only available since 2012 for Official Bank Rate on "Forecasts For The UK Economy" from the HM Treasury.

Database for Brazil, Mexico and Chile are constructed from 1998 onward as early data is less reliable and estimation would be more susceptible to political and economic crises in the region, for instance, Latin America debt crisis in the 1980s and Asia crisis in the 1990s. Data sources for the three countries are the OECD and the local central Banks.

For Chile, CPI, the short-term and long-term interest rates are obtained from the OECD for the 1Q98 to 4Q19 period, only the long-term interest yield is available since 4Q04. The inflation and interest expectations are collected from the Central Bank of Chile [8], with inflation expectations since the 1Q03 and yield expectations since 4Q04.

Mexico's inflation and the long-term interest rate are also obtained from the OECD for, respectively, 1Q98-4Q19 and 4Q01-4Q19, while the short-term interest rate, the long-term expectations of inflation and yield are available from the Bank of Mexico [9] for, respectively, 1Q98-4Q19, 4Q01-4Q19, and 4Q01-4Q19.

For Brazil, inflation is also based on CPI obtained from the OECD, while the short-term interest, measured as Selic rate, and the expectations of interest rate and inflation are available from the Central Bank of Brazil [10]. The long-term interest rate, defined as the National Treasury Bonds - series F ("NTN-F"), is collected from the Brazilian Treasury [11]. There are quarterly observations available from 1Q98 to 4Q19 for CPI and Selic rate, and from 1Q02 to 4Q19 for NTN-F and expectations of interest rate. For inflation expectations, there are only yearly

4 <https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters>

5 <https://www.bankofengland.co.uk/boeapps/database/>

6 <https://www.gov.uk/government/statistics/database-of-forecasts-for-the-uk-economy>

7 <https://data.oecd.org>

8 <https://www.bcentral.cl/en/web/banco-central/areas/statistics>

9 <https://www.banxico.org.mx/SieInternet/>

10 <https://www3.bcb.gov.br/sgspub/localizarseries/localizarSeries.do?method=prepararTelaLocalizarSeries>

11 <https://sisweb.tesouro.gov.br/apex/f?p=2031:2:0::::>

observations from 1Q01 to 4Q19.

3.4 Results

3.4.1 Advanced Economies

The BVAR Baseline Model is a simple estimation based on only the short-term and long-term interest rates, inflation, and the expectations of inflation and interest rates. For developed economies, we follow Negro et al. (2017)'s strategy of treating the short-term rate as not observable since 4Q08 to avoid zero lower bound noise on the trend decomposition [12].

Table 1 – BVAR Baseline - Main Results for USA and UK

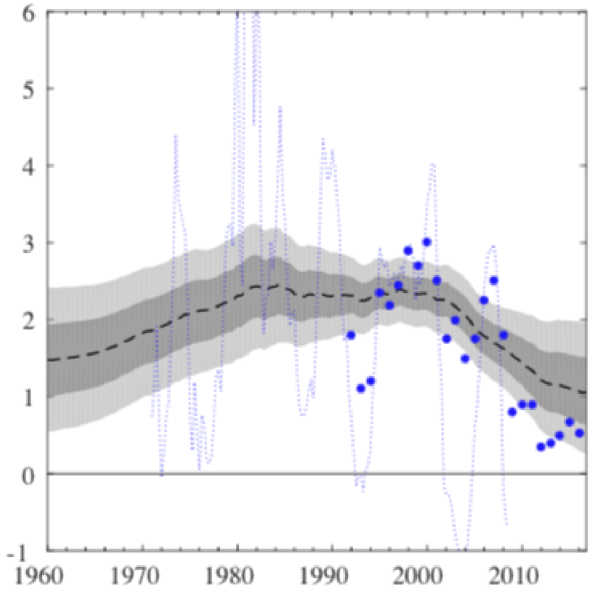
Change (1998Q1-2019Q4)	United States	United Kingdom
$\bar{r}_t$	-1.3938*	-0.5279
Posterior Coverage Intervals of 68%	(-1.7944,-0.9444)	(-1.0055,-0.0659
Posterior Coverage Intervals of 95%	(-2.1527,-0.4706)	(-1.4753,0.4173)

Note – * indicates the decline is significant as the 95% coverage intervals do not include zero.

We estimate the model from 1Q60 to 4Q19 for the United States (figure 6), reaching very similar findings compared to Negro et al. (2017) (figure 5). The figures below show the estimates of $\bar{r}_t$, where the dashed black line is the posterior median of $\bar{r}_t$ and the shaded areas represent the 68% and 95% posterior coverage intervals. In our estimates, $\bar{r}_t$ also slightly increases from the 1960s to the 1980s, remains relatively stable until the late 1990s, and then starts declining. Our results indicate a median decline in $\bar{r}_t$ from 1Q98 to 4Q19 of 1.39 percentage points (table 1), close to the drop in $\bar{r}_t$ from 1Q98 to 4Q16 of 1.29 percentage points obtained by Negro et al. (2017). Updating the original model with 12 quarters confirms the downward trend on $\bar{r}_t$, with a fell from 2.28% in 1Q98 to 0.94% in 4Q19 in our estimates compared to a decline from 2.36% in 1Q98 to 1.06% in 4Q16 in the aforementioned paper.

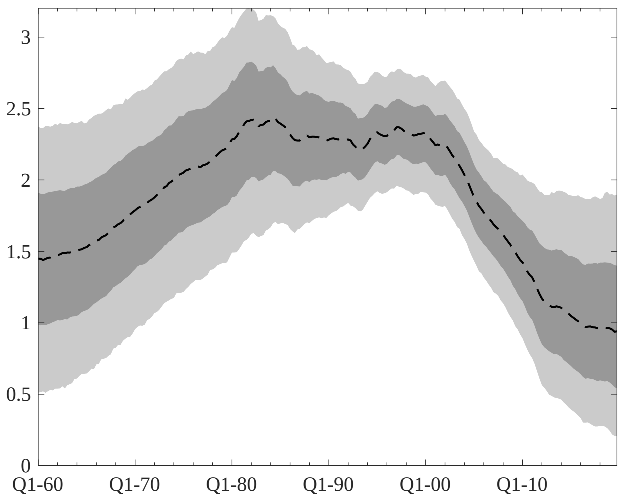
[12] The prior for $\sum_e$, "the variance covariance matrix of the innovations to the trends $\bar{y}_t$", [...] "is therefore diagonal with elements equal to 1/400", except for inflation trend, in which the authors use 1/200. In addition, the "prior for the variance $\sum_e$ is a very uninformative Inverse Wishart distribution centered at a diagonal matrix with elementary elements (except for inflation, for which the diagonal element is 2, and expectations, for which the variance is 0.5; these numbers reflect presample variances, except for expectations which are not available) [...] The initial conditions y_0 for the trend components $\bar{y}_t$ are set at presamples averages inflation, the real rate, and the term spread (2, 0.5, and 1 for $\bar{\pi}_0$, $\bar{r}_0$, and $\bar{tp}_0$, respectively)" (NEGRO et al., 2017). We keep the same rationale to estimating the model for United Kingdom.

Figure 5 – Trends and Observables - BVAR Baseline Model: Extracted from Negro et al. (2017)



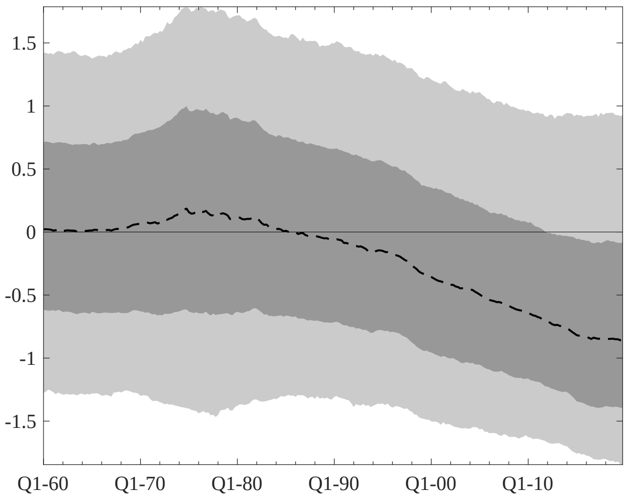
Note – The dotted blue line is the short-term interest rate minus the long-term inflation expectations ($\bar{R}_{1,t}$ - $\tilde{\pi}^e_t$), and the blue dots are the long-term expectation for the short-term rate minus the long-term inflation expectations ($\bar{R}^e{}_{1,t}$ - $\tilde{\pi}^e_t$).

Figure 6 – BVAR Baseline Model for USA



We apply the same BVAR Baseline Model to the United Kingdom over the sample 1Q60-4Q19. Estimates indicate $\bar{r}_t$ slightly rises until the 1980s, and then begins a gradual decline. The median drop in $\bar{r}_t$ from 1Q98 to 4Q19 is about 0.53 percentage points, but it is not significant as zero is within 95% coverage intervals.

Figure 7 – BVAR Baseline Model for UK



3.4.2 Emerging Economies

For emerging economies, sample period is shorter compared to the USA and the UK. We also use slightly higher variance for some parameters as developing countries observables present higher volatility [13].

Table 2 – BVAR Baseline - Main Results for Chile, Mexico and Brazil

Change (1998Q1-2019Q4)	**Chile**	**Mexico**	**Brazil**
$\bar{r}_t$	-2.4134*	-0.931*	-0.1671
Posterior Coverage Intervals of 68%	(-2.776,-2.0653)	(-1.3086,-0.5138)	(-0.646,0.2963)
Posterior Coverage Intervals 95%	(-3.1214,-1.7182)	(-1.6649,-0.1309)	(-1.1113,0.7287)

Note – * indicates the decline is significant as the 95% coverage intervals do not include zero.

Estimates for Chile and Mexico show a significant decline in $\bar{r}_t$ (zero is not within the 95% coverage intervals of each country) over the sample. Both $\bar{r}_t$ keep stable through the 2000s until mid-2010s, then begins dropping close to 2008. For Chile, $\bar{r}_t$ starts a sharp fall, reaching levels similar to advanced economies. For Mexico, $\bar{r}_t$ stops falling in 2013, and remains relatively stable with a small upward trend. Overall, the model indicates a median decline in the Chilean $\bar{r}_t$ of 2.41 percentage points, from 3.55% in 1Q98 to 1.13% in 4Q19, and a median fell in the Mexican $\bar{r}_t$ of 0.93 percentage points, from 3.41% in 1Q98 to 2.48% in 4Q19.

[13] The prior for the variance $\sum_e$ is also a uninformative Inverse Wishart distribution centered at a diagonal matrix with elementary elements, except for inflation and short-term rate, for which the diagonal element is 10 for both. The initial conditions $\underline{y_0}$ for the trend components $\bar{y}_t$ are set at averages inflation, the real rate, and the term spread (4, 3, and 1 for $\bar{\pi}_0$, $\bar{r}_0$, and $t\bar{p}_0$, respectively)

Figure 8 – BVAR Baseline Model for Chile

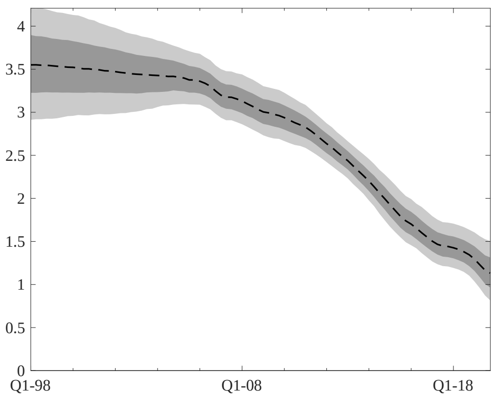
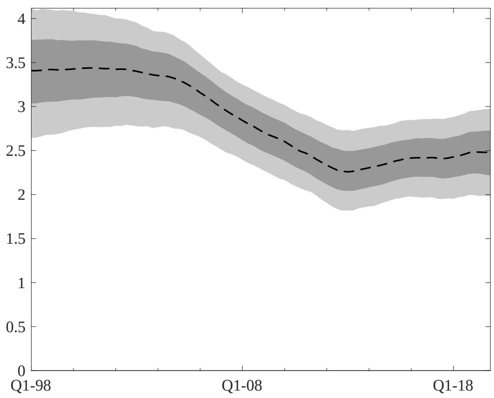
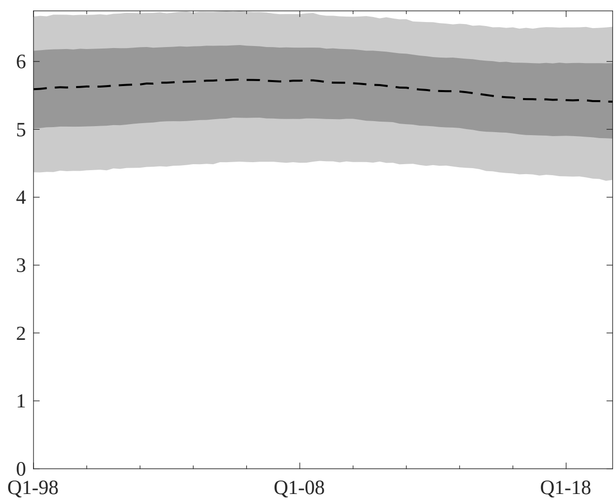


Figure 9 – BVAR Baseline Model for Mexico



Estimates do not capture any substantial change in $\bar{r}_t$ for Brazil, with a drop of only 0.17 percentage points (not significant). The model indicates the median $\bar{r}_t$ starts in 1Q98 at 5.59%, reaches 5.71% in 4Q08, then begins to decline gradually to 5.41% in 4Q19. High volatility of observable variables, driven by successive economic and political crisis, may be impairing the model accuracy.

Figure 10 – BVAR Baseline Model for Brazil



4 THE LIQUIDITY PREMIUM: IN SEARCH OF SAFE ASSETS

4.1 The Convenience Yield

Also in accordance with Negro et al. (2017), we extend the BVAR with common trends (described in section 3.2) to calculate the convenience yield share on interest rates. We intend to assess how much of the decline in real interest rate trends can be explained by the investors preference towards safe assets. Negro et al. (2017) follow the empirical strategy of Krishnamurthy and Vissing-Jorgensen (2012), which mainly consists of including investors preference for government securities over corporate bonds due to liquidity and safety characteristics.

From the Euler equation for investing in a short-term government security, it is defined the net real return (r_t):

$$E_t[(1+R_t)(1+CY_{t+1})M_{t+1}] = 1 \tag{4.1}$$

where CY_{t+1} is the convenience yield, the premium for the liquidity and safety of government treasuries over corporate securities, and M_{t+1} is a stochastic discount factor or the marginal substitution rate between consumption in period t and t+1.

Negro et al. (2017) assume the covariance between CY_t and M_t is stationary to simplify (4.1) to:

$$\bar{r}_t = \bar{m}_t - \bar{cy}_t \tag{4.2}$$

being $\bar{cy}_t$ = log(1 + CY_t) and $\bar{m}_t$ = -log$\bar{M}_t$. Then, we can substitute $\bar{r}_t$ with $\bar{m}_t$ - $\bar{cy}_t$ in (3.6), (3.9), and (3.10).

To identify the convenience yield ($\bar{cy}_t$), the authors have to obtain securities that present less safety and liquidity than treasuries. As a result, Negro et al. (2017) use the yield of corporate bonds ("bond") as below:

$$R_t^{bond} = \bar{m}_t - \lambda_{cy}^{bond}\bar{cy}_t + \bar{d}_t + \bar{pi}_t + \bar{tp}_t + \tilde{R}_t^{bond} \tag{4.3}$$

Some important details are: (i) $0 \leq \lambda_{cy}^{bond} \leq 1$, which indicates bonds are less liquid and safe than government treasuries, and (ii) $\bar{d}_t$ is the trends in corporate bonds default probability. Negro et al. (2017) point out there is no clear trend in the default probability over the period. Based on that and aligned with the authors, we will disregard $\bar{d}_t$.

Combining (4.3) and (3.9), we obtain that the difference between yields on corporate bonds and equivalent government securities maturities reflects the convenience yield trend:

$$\bar{R}_t^{bond} - \bar{R}_{80,t} = (1 - \lambda_{cy}^{bond})\bar{cy}_t \tag{4.4}$$

where λ_{cy}^{bond} is assumed to be zero, which means corporate bonds do not present any convenience yield.

Finally, the equations system (3.6),(3.7), (3.8), (3.9), (3.10), and (4.3) also represents a BVAR (3.1), where variables follow the random walk described in (3.2), except for the stationary components that evolves as (3.3). This is the BVAR Convenience Yield Model, in which we add the corporate bond yields to the baseline equations.

4.2 Data

Data is obtained as described in section 3.3 for the interest rate of government securities, the inflation rates, and the expectations for inflation and government yields.

As we include the corporate bond interest rates to estimate the convenience yield in the section above (4.1), we have to add a security that is known to be less safety and liquid than the respective government treasuries. Following Negro et al. (2017) and Krishnamurthy and Vissing-Jorgensen (2012), we use the Moody's Seasoned Baa Corporate Bond Yield series, available from FRED [1], for USA and UK models from 1Q60 to 4Q19. According to Krishnamurthy and Vissing-Jorgensen (2012), the "Baa index is constructed from a sample of long-maturity ($\geq$ 20 years) industrial and utility bonds (industrial only 2002 onward)".

For Brazil, Chile and Mexico, we believe a high yield series is more appropriate to measure corporate bond yields. Hence, we use the ICE BofA US High Yield Index Effective Yield, also obtained from FRED, from 1Q98 to 4Q19. According to FRED, the ICE US High Yield BofA Index

> tracks the performance of US dollar denominated below investment grade rated corporate debt publicly issued in the US domestic market. To qualify for inclusion in the index, securities must have a below investment grade rating (based on an average of Moody's, S&P, and Fitch) and an investment grade rated country of risk (based on an average of Moody's, S&P, and Fitch foreign currency long term sovereign debt ratings). Each security must have greater than 1 year of remaining maturity, a fixed coupon schedule, and a minimum amount outstanding of USD 100 million. [2]

We understand this High Yield Index is fair enough for Chile and Mexico, historically investment grade rated countries. Even for Brazil, a historically speculative grade rated country (only few years in the end of 2000s and the beginning of 2010s as investment grade rated), we believe the High Yield Index is appropriate because it can give some information about overall changes in interest yield changes. As written by Negro et al. (2017):

> We should also stress that our results focus on secular *changes* in the convenience yield, as opposed to its *level*. The level of Baa/Treasury spread may be affected by factors other than the safety and liquidity premiums (e.g, the average default probability of corporate bonds). The key identifying assumption we use is that secular changes in the spread primarily reflect secular changes in the convenience yield. (NEGRO et al., 2017)

[1] <https://fred.stlouisfed.org/series/DBAA>

[2] <https://fred.stlouisfed.org/series/BAMLH0A0HYM2EY>

4.3 Results

4.3.1 Advanced Economies

The BVAR Convenience Yield Model refines the BVAR Baseline Model including a term to capture the attractiveness of government treasuries vis-a-vis corporate bonds. Besides the short-term and long-term interest rates, the inflation rate, and the expectations of inflation and interest rates, the corporate bond yields are added to this model. We continue excluding the short-term rate since 4Q08 for advanced economies. [3]

Table 3 – BVAR Convenience Yield - Main Results for USA and UK

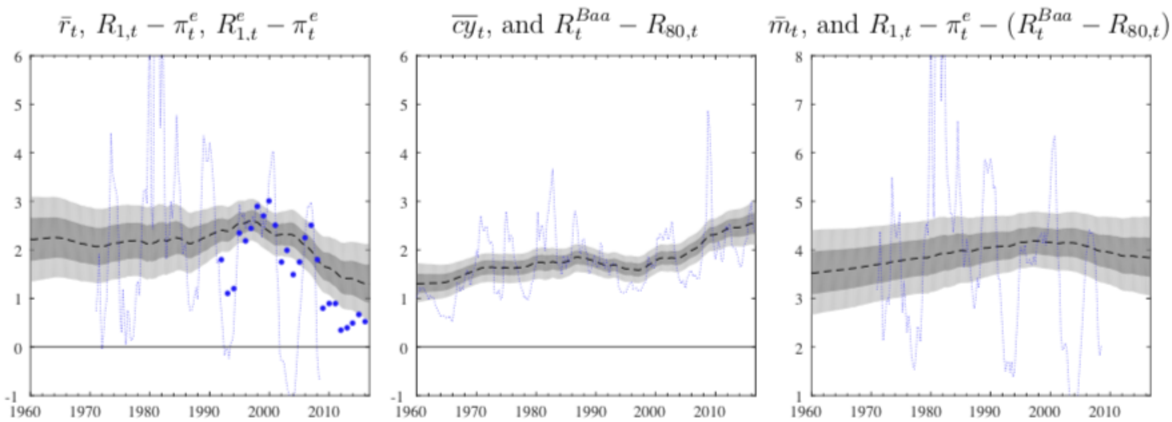
Change (1998Q1-2019Q4)	**United States**	**United Kingdom**
$\bar{r}_t$	-0.9253*	-0.8943
Posterior Coverage Intervals of 68%	(-1.3439,-0.5245)	(-1.4098,-0.3899)
Posterior Coverage Intervals of 95%	(-1.7237,-0.0787)	(-1.9166,0.1216)
$\bar{m}_t$	-0.2058	-0.2927
Posterior Coverage Intervals of 68%	(-0.5635,0.136)	(-0.6297,0.0484)
Posterior Coverage Intervals of 95%	(-0.8942,0.4759)	(-0.9681,0.3777)
$-\bar{cy}_t$	-0.7164*	-0.5965
Posterior Coverage Intervals of 68%	(-0.9474,-0.4881)	(-0.9793,-0.2344)
Posterior Coverage Intervals of 95%	(-1.1903,-0.2581)	(-1.338,0.1238)

Note – * indicates the decline is significant as the 95% coverage intervals do not include zero.

We estimate the model from 1Q60 to 4Q19 for the United States (figure 12), obtaining results aligned to Negro et al. (2017) (figure 11). The figures below show the estimates of $\bar{r}_t$, $\bar{m}_t$, and $\bar{cy}_t$, in which the dashed black line is the posterior median of each variable and the shaded areas represent the 68% and 95% posterior coverage intervals. Our estimates indicate a median decline in $\bar{r}_t$ from 1Q98 to 4Q19 of 0.92 percentage points for the USA (table 3), lower than baseline's results and the drop in $\bar{r}_t$ from 1Q98 to 4Q16 of 1.27 percentage points obtained by the cited paper. Also, in line with the original model, we find that the decline in $\bar{r}_t$ is explained mostly by an increase in $\bar{cy}_t$. We estimate convenience yield increases by 0.72 percentage points from 1Q98 to 4Q19 while Negro et al. (2017) obtained a rise of 0.92. The effect of $\bar{cy}_t$ seems to lose steam in the last 3 years, even so it still corresponds to 77% of the estimated decline in $\bar{r}_t$.

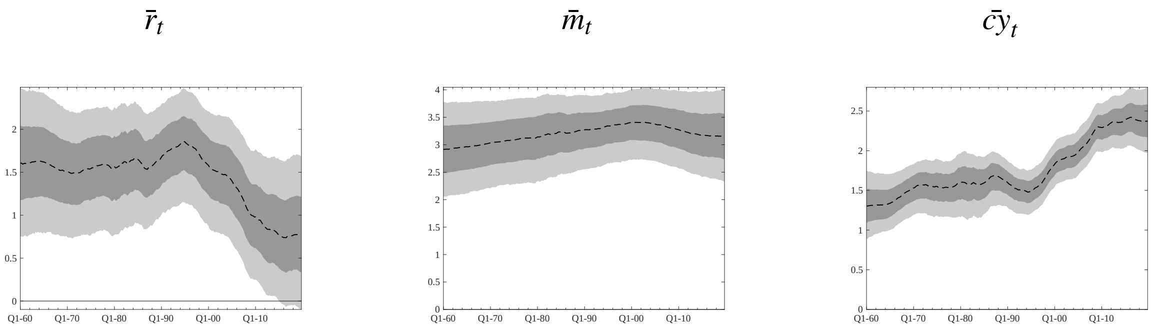
[3] "We use exactly the same priors [...] except that since we decompose the trend $\bar{r}_t$ into two components, $\bar{m}_t$ and $-\bar{cy}_t$, we center the corresponding diagonal value of $\sum_e$ to a number that is 1/2 the value chosen for $\bar{r}_t$" (NEGRO et al., 2017). We keep the same rationale for estimating the model for United Kingdom.

Figure 11 – Trends and Observables - BVAR Convenience Yield: Extracted from Negro et al. (2017)



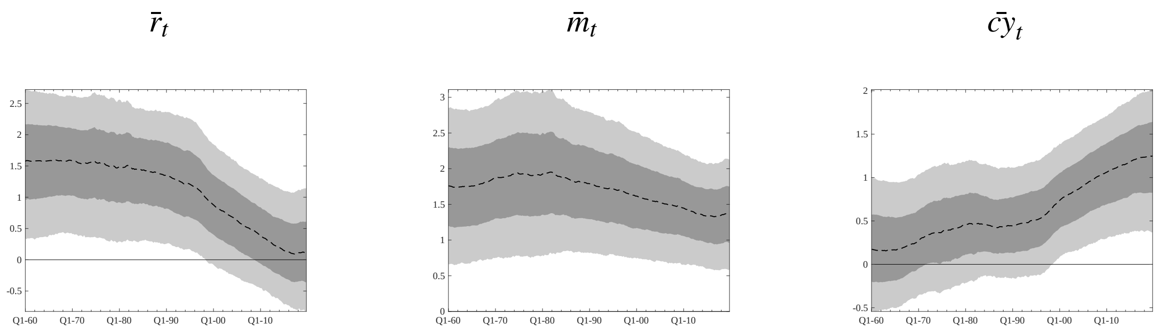
Note – Note: The left panel shows the $\bar{r}_t$, $\bar{R}_{1,t}$ - $\tilde{\pi}^e_t$ (dotted blue line), and $\bar{R}^e{}_{1,t}$ - $\tilde{\pi}^e_t$ (blue dots). The middle panel shows the $\bar{cy}_t$ and $\bar{R}^{bond}_t$ - $\bar{R}_{80,t}$ (dotted blue line). The right panel shows the $\bar{m}_t$ and $\bar{R}_{1,t}$ - $\tilde{\pi}^e_t$ - ($\bar{R}^{bond}_t$ - $\bar{R}_{80,t}$) (dotted blue line).

Figure 12 – BVAR Convenience Yield Model For USA



We apply the BVAR Convenience Yield Model to the United Kingdom over the sample 1Q60-4Q19. Results suggest a drop in median $\bar{r}_t$ of 0.89 percentage points, from 0.94% in 1Q98 to 0.05% in 4Q19. Again, the decline is not significant at 95%, since zero is within this coverage interval. Even so, it is worth highlighting $\bar{cy}_t$ also explains the majority (roughly 67%) of the fell verified in $\bar{r}_t$.

Figure 13 – BVAR Convenience Yield Model For UK



4.3.2 Emerging Economies

The BVAR Convenience Yield also seems to explain part of the $\bar{r}_t$ decline seen in Chile and Mexico mainly after 2008, but results for Brazil are more uncertain. On contrary to the advanced economies, $\bar{m}_t$ changes are also relevant for the emerging economies.

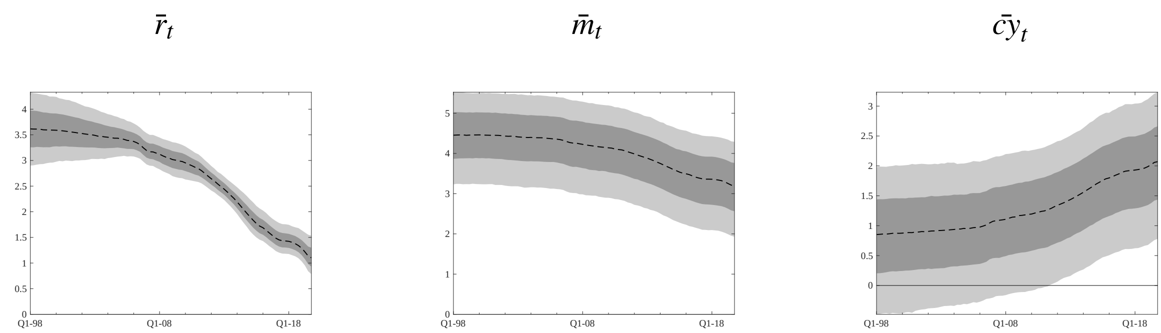
Table 4 – BVAR Convenience Yield - Main Results for Chile, Mexico and Brazil

Change (1998Q1-2019Q4)	**Chile**	**Mexico**	**Brazil**
$\bar{r}_t$	-2.4969*	-0.9473*	-0.2569
Posterior Coverage Intervals of 68%	(-2.876,-2.1246)	(-1.3577,-0.5272)	(-0.7379,0.2318)
Posterior Coverage Intervals 95%	(-3.2466,-1.7835)	(-1.7423,-0.1199)	(-1.2235,0.7193)
$\bar{m}_t$	-1.2742*	-0.5199	-0.217
Posterior Coverage Intervals of 68%	(-1.6132,-0.9449)	(-0.8439,-0.1955)	(-0.5722,0.1396)
Posterior Coverage Intervals 95%	(-1.9558,-0.6154)	(-1.1711,0.1167)	(-0.9095,0.4864)
$-\bar{cy}_t$	-1.2176*	-0.4248	-0.0398
Posterior Coverage Intervals of 68%	(-1.5431,-0.8947)	(-0.7357,-0.1061)	(-0.3737,0.3074)
Posterior Coverage Intervals 95%	(-1.8865,-0.5982)	(-1.0417,0.2096)	(-0.7304,0.6243)

Note – * indicates the decline is significant as the 95% coverage intervals do not include zero.

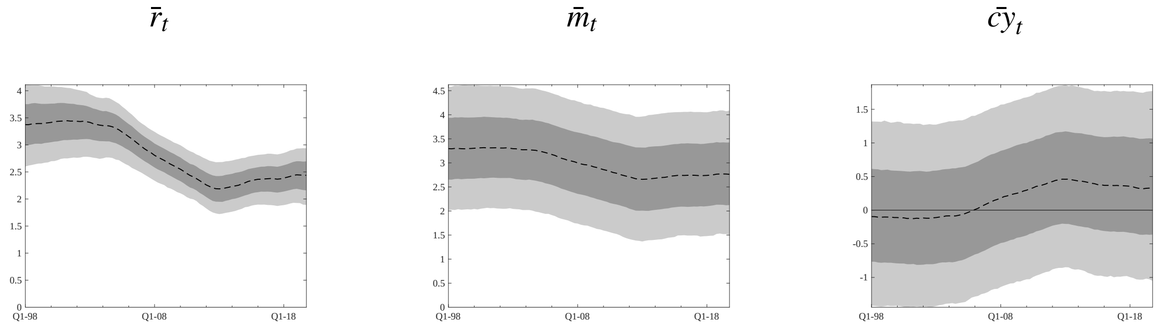
Estimates for Chile show a strong decline in $\bar{r}_t$ of 2.50 percentage points, from 3.62% in 1Q98 to 1.10% in 4Q19. Roughly 50% of this drop is driven by $\bar{cy}_t$, which increases by 1.21 percentage points over the period, and the remaining 50% by $\bar{m}_t$, which rises by 1.27 percentage points. Interesting to note that all these trends become stronger after 2008. All changes over this period are significant as 95% coverage intervals for each variable do not include zero.

Figure 14 – BVAR Convenience Yield Model For Chile



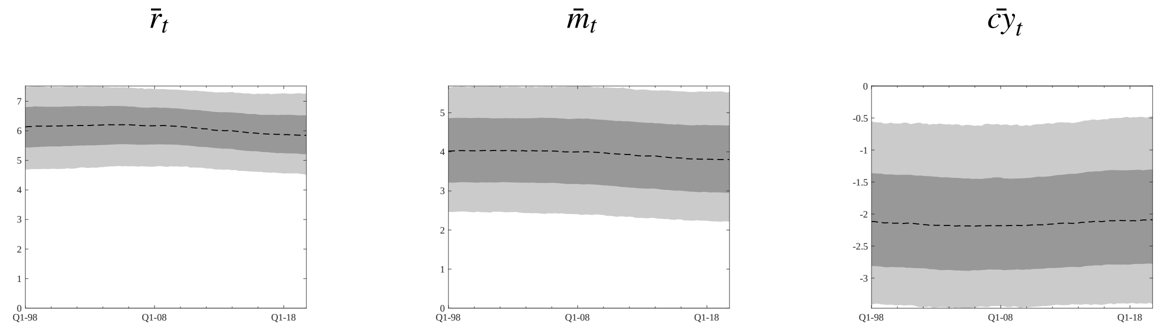
Mexico's results indicate also a sharp fall in the median $\bar{r}_t$ of 0.95 percentage points, from 3.37% in 1Q98 to 2.44% in 4Q19. Similar to Chile, $\bar{cy}_t$ is responsible for roughly 0.45% of the $\bar{r}_t$ drop as $\bar{cy}_t$ increases by 0.42 percentage points over the period. Regarding $\bar{m}_t$, the estimates drop by 0.52 percentage points from 1Q98 to 4Q19. The decline in $\bar{r}_t$ is significant at 95% coverage interval. Worth mentioning, $\bar{cy}_t$ is very close to zero until 2008.

Figure 15 – BVAR Convenience Yield Model For Mexico



As estimates do not show any substantial change in $\bar{r}_t$ for Brazil, the shifts in $\bar{m}_t$ and $\bar{cy}_t$ are also not significant. In line with what happened on the estimates of the BVAR Baseline Model for Brazil, the high variance of observable variables may be impairing the model's calibration. All in all, important to note $\bar{cy}_t$ not only seems to have no impact on Brazil's $\bar{r}_t$ but it is also negative, when for all other countries it is positive (after 2008 for Mexico).

Figure 16 – BVAR Convenience Yield Model For Brazil



5 THE SECULAR TRENDS

5.1 An Overlapping Generation Model for Secular Stagnation

To quantify the secular forces that have been pressuring the natural interest rates down as summarized by Summers (2014), we use an overlapping generation model ("OLG") as proposed by Eggertsson, Mehrotra and Robbins (2019). Our main goal is to decompose the share of each factor in interest rates drop over the last decades for the United Kingdom and compare them to the results obtained for the United States by Eggertsson, Mehrotra and Robbins (2019).

The authors developed a 56-period quantitative life cycle model to explain how demographic and technological changes since the 1970s affect interest rates. The OLG's model economy comprehends 3 periods: (1) young, where households are born and borrow from middle age to consume; (2) middle age, where households work, consume, have children and save for retirement; and (3) old, where households retire and fully consume their endowment. [1] Total population alive at any given time (N_t) is simple the the sum of individual ages ($N_{j,t}$). [2] The model's resolution comprises solving the problem of households, final goods firms, intermediate goods firms, and relative price of capital goods to reach a competitive and a stationary equilibrium at the end.[3]

5.1.1 Household

Utility is comprised of consumption ($C_{j,t+j-1}$) and bequests ($x_{J,t+J-1}$). Both are given by a constant elasticity of substitution (CES) utility function. Being β a discount rate and s^j the unconditional probability of reaching age j, a household born at time t with age j maximizes its expected utility for a lifetime as follows:

$$U_t = \Sigma_{j=26}^{J} s^j \beta^{j-1} \upsilon(C_{j,t+j-1}) + s^J \beta^{J-1} \mu \nu(x_{J,t+J-1}) \tag{5.1}$$

The household's flow budget constraint states that consumption ($c_{j,t}$), purchase or borrow of real assets ($a_{j+1,t+1}$), and bequests left ($x_{j,t}$) have to be equal wage [4], profits from firms ($\Pi_{j,t}$),

---

1 Households are supposed to begin middle age at 26 and to die at age J equal to 81 with a stochastically probability.

2 The rate of population growth (n) and population (N_t) are calculated in the steady state as:$n = \Gamma^{1/25} - 1$; $N = \Sigma_{j=26}^{J} N_j$; $N_{j+1} = s_j \frac{N_j}{1+n} for j \epsilon (27, J)$; where Γ represents the total fertility rate and s_j is the probability of surviving between age j and j + 1.

3 Household has the same schedule of exogenous labor capital (hc_t) that changes with age. Retirement is assumed to take place after 65 in the model, with no wage income after that, and labor supplied is inelastically. Labor supply is defined as: $L_t^s = \Sigma_{j=26}^{J} N_{j,t} hc_j$.

4 At full employment, wage is the specific labor productivity hcj discounted labor taxes $(1 - \tau^w)$.

return from real assets (r_t^k), and bequest received ($q_{j,t}$).[5] The complete budget constraint of a household of age j at time t, being ξ the price of asset and δ the depreciation rate of capital, is: [6]

$$c_{j,t} + \xi a_{j+1,t+1} + \Gamma_{26,t-j+26} x_{j,t} = (1-\tau^w) w_t hcj + \Pi_{j,t} + [r_t^k + \xi_t(1-\delta)]\left(a_{j,t} + q_{j,t} + \frac{1-s_j}{s_j} a_{j,t}\right) \quad (5.2)$$

There is also a borrowing constraint for the households that want to borrow equal to:

$$a_{j,t} \geq \frac{D_t}{1+r_t} \quad (5.3)$$

5.1.2 Final Goods Firms

The final goods firms modify the intermediate goods to sell them to the households. The final good is defined as the CES aggregate of final goods:

$$Y_t = \left[\int_0^1 y_t^f(i)^{\frac{\theta_t - 1}{\theta_t}} di \right]^{\frac{\theta_t}{\theta_t - 1}} \quad (5.4)$$

Monopolistically competitive, these firms set prices in each period and face the following demand curve:

$$y_t^f(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta_t}, \quad (5.5)$$

where θ_t is a time-varying shock.

The final good producers use a linear technology function to obtain the final goods ($y_t^f = y_t^m$), and the final firm goods set real prices and $y_t^f(i)$ to maximize the real profits subject to the production constraint:

$$\max \frac{p_t(i)}{P_t} y_t^f(i) - \frac{p_t^{int}}{P_t} y_t^f(i)$$
$$\text{subject to } y_t^f(i) = Y_t \frac{p_t(i)}{P_t}^{-\theta_t}$$

The optimal condition implies that the real price of the final good is a time-varying markup of the real price of intermediate good:

$$\frac{p_t(i)}{P_t} = \frac{\theta_t}{\theta_t - 1} \frac{p_t^{int}}{P_t} \quad (5.6)$$

5 The authors assume bequest received to be zero for all periods except age J-24, and bequest given to be zero for all periods except age J.

6 The authors also assume there is a bequest insurance market. Hence, all surviving children receives some bequest. The relationship between bequests given (age J) and bequests received (age k) at time t is equal to: $q_{k,t} = \frac{N_{J,t-1} x_{J,t-1} . \Gamma_{26,t-J+26}}{N_{k,t}}$.

Given all the intermediate goods have the same price, all the final goods firms set the same price ($p_t(i) = P_t$), which simplifies (5.6) to:

$$\frac{p^{int}}{P_t} = \frac{\theta_t - 1}{\theta_t} \tag{5.7}$$

At equilibrium, total profits (Π_t) will be equal to $\frac{Y}{\theta}$, and those total profits are totally returned to the households in line with wage income:

$$\frac{Y_t}{\theta_t} = \Sigma_{j=26}^{J} N_{j,t}\Pi_{j,t} \tag{5.8}$$

5.1.3 Intermediate Goods Firms

The perfectly competitive intermediate goods firms sell its products to the final goods firms at real price $\frac{p_t(int)}{P_t}$. These firms use a CES production function with a substitution elasticity (σ), labor (L_t) and capital (K_t) as

$$Y_t = \left(\alpha(A_{k,t}K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_{l,t}L_t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{5.9}$$

Considering the production function defined above, the intermediate goods firms maximize their real profits according to:

$$\Pi_t^{int} = \max \frac{p_t^{int}}{P_t} Y_t - w_t L_t - r_t^k K_t \tag{5.10}$$

With the first order conditions:

$$w_t = \frac{p_t^{int}}{P_t}(1-\alpha)A_{l,t}\left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}} \tag{5.11}$$

$$r_t^k = \frac{p_t^{int}}{P_t}\alpha A_{k,t}\left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}} \tag{5.12}$$

The relation between the real risk-free rate and the return on capital is given by a no-arbitrage condition:

$$1 + r_t = \frac{r_t^k + (1-\delta_t)\xi}{\xi_t - 1} \tag{5.13}$$

5.1.4 Price of Capital Goods

The authors assumed perfectly competitive firms in investment-specific production firms. These firms modify the final composite goods into capital goods, maximizing profit subject to the production function as:

$$\Pi^K = \xi_t * K_t - Y_t^K \tag{5.14}$$
$$K_t = z_t Y_t^K, \tag{5.15}$$

where ξ is the relative price of capital goods and z_t is the productivity of the capital-producing sector.[7].

5.1.5 Government

Finally, the government spends G_t and may issue debt limited to the following budget constraint:[8]

$$b_{g,t} = G_t + (1 + r_t)b_{g,t-1} - T_t \tag{5.16}$$

Worth noting tax bill is collected on labor income.

5.2 Data

We applied the same strategy of Eggertsson, Mehrotra and Robbins (2019) to obtain the main parameters for the OLG model calibrated for the United Kingdom. The parameters are obtained from three sources:

1. Statistical data we can directly observe for demographics and economics;
2. Estimates from literature, and;
3. Estimates made by the OLG model to match specific moments.

The first category of parameters includes annually UK data for the total factor productivity ("TFP"), the government debt, the total fertility rate ("TFR"), the survival rates, and the population pyramid.

The total factor productivity was obtained from the FRED [9] for 1948-2015, then we applied a HP filter to original data before using it. Also, we maintained the same adjustments made for US data: the initial level of TFP in 1970 is the average from 1948-1974 and the final value of TFP is the average from 1975-1993, with an optimistic series only changing the terminal level of TFP to its 1948-1974 average.

7 The aggregate capital stock follows the standard law of motion: $K_{t+1} = (1-\delta)K_t + \frac{I_t}{\xi_t}$, being δ the depreciation rate and I_t investment

8 We followed the authors choice of economizing "on notation by omitting real and nominal bonds as assets [...] so there is both a well-defined real interest rate r_t on a risk-free one-period real bond and a nominal interest rate i_t on nominal bonds" (EGGERTSSON; MEHROTRA; ROBBINS, 2019).

9 <https://fred.stlouisfed.org>

The UK government debt data was collected from Bank of England [10] on "A Millennium of Macroeconomic Data for the UK" from 1970 to 2015 (annually). We considered for the model the 5-year moving average of debt.

The fertility rate was available from Office for National Statistics [11]. We used the 5-year moving average of 0.5 x UK total fertility rate from 1945 to 2015.

The survival rates are used from 1970 to 2007 from age 25 to 80. The life table survivor was obtained from United Nations - World Population Prospects 2019 [12], where the survival rates are available at ages 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, and 80. Since figures for the whole period are pretty similar for US and UK, we maintained the same database for UK, that means, the authors interpolated survival rates by age from 25 to 80 for all the period.

The population data was also collected from from United Nations - World Population Prospects 2019 for 1970 and 2015. Population by age is available within age groups: 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70-74, 75-79, and 80-84. In order to achieve a population by each age from from 25 to 80, we divided population within each age bin for 5.

The second category of parameters is based on literature in line with Eggertsson, Mehrotra and Robbins (2019). We assumed these kinds of parameters to be overall similar for United States and United Kingdom.

Table 5 – OLG - Parameters From Literature

Parameter	**Symbol**	**Value**	**Source**
Income profile	hcj		Gourinchas and Parker (2002)
Elasticity of intertemporal substitution	ρ	0.75	Gourinchas and Parker (2002)
Capital / labor elasticity of substitution	σ	0.6	Antras (2004)
Depreciation rate	δ	12%	Jorgenson (1996)

The third category of parameters is set by the model through a minimizing loss function to match key data moments in 2015. The parameters from this category are: time preference rate (β), debt limit (D), bequest parameter (μ), capital share parameter (α), and retailer elasticity of substitution (θ). And, the specific moments in 2015 to be matched are: natural rate of interest, investment-to-output ratio (obtained from World Bank databse) [13], consumer-debt-to-output (available from BoE), labor share (from Berger and Wolff (2017)), and bequests-to-output ratio (only parameter we maintained the same for UK from Hendricks (2001)).

To calibrate the UK economy to 1970 figures, the type 1 parameters are adjusted to 1970, the type 2 parameters remained the same expect for relative price of investment goods, and the

[10] <https://www.bankofengland.co.uk/boeapps/database/>
[11] <https://www.ons.gov.uk/peoplepopulationandcommunity>
[12] <https://population.un.org/wpp/>
[13] <https://data.worldbank.org>

type 3 parameters are also kept the same but for consumer-debt-to-output and labor share, that are changed to 1970 values.

5.3 Results

5.3.1 United States

First, we re-estimated the OLG model for the United States only changing the natural rate of interest (table 6). We use a natural rate of interest of 0.75% as obtained by the BVAR Convenience Yield Model in section 4 for the United States economy in 2015, while Eggertsson, Mehrotra and Robbins (2019) considered a rate of -1.47%.

Table 6 – OLG - USA Targets for 2015

Targets	**Model/Data**	**Source**
Natural rate of interest	0.75%	BVAR Convenience Yield
Investment-to-output ratio	15.9%	Eggertsson, Mehrotra and Robbins (2019)
Consumer-debt-to-output ratio	6.3%	Eggertsson, Mehrotra and Robbins (2019)
Labor share	66%	Eggertsson, Mehrotra and Robbins (2019)
Bequests-to-output ratio	3.0%	Hendricks (2001)

Note – Parameters (β, D, μ, α, θ) chosen to match targets; data equal to Eggertsson, Mehrotra and Robbins (2019) except for natural rate of interest

Following Eggertsson, Mehrotra and Robbins (2019), we also compare change in parameters from 1970 to 2015. Before explaining main results, it is worth mentioning why the authors have chosen 1970 as a start point:

> Briefly, they are (i) children of the baby boom generation (born beginning in 1945) begin to enter the labor force; (ii) mortality rapidly falls with life expectancy increasing from an average of 70.8 in 1970 to 78.7 in 2010; (iii) according to Gordon (2016), the fall in the rate of productivity growth began in the 1970s; (iv) Greenwood, Hercowitz, and Krusell (1997) and Fernald (2012) have documented that the relative price of investment goods has fallen by 30 percent since 1970; and (v) Karabarbounis and Neiman (2014) have documented a significant drop in the labor share since 1970. Also, this period saw a significant rise in government and personal debt. (EGGERTSSON; MEHROTRA; ROBBINS, 2019)

Table 7 – OLG - USA Change in Parameters from 1970 to 2015

Data	**1970**	**2015**
Life expectancy	70.8	78.7
Total fertility rate	2.8	1.9
Productivity growth	2.02%	0.65%
Government debt (% of GDP)	42%	118%
Targets	**1970**	**2015**
Consumer-debt-to-output ratio	4.2%	6.3%
Labor share	72.4%	66.0%

Note – Equal to Eggertsson, Mehrotra and Robbins (2019)

As explained in the previous section (5.2), we targeted the consumer-debt-to-output ratio and the labor share to actual 1970 data to obtain the natural real interest rate and the investment-to-output ratio estimates in 1970 (table 7). Our simulation presents a rate of natural interest of 4.4% and an investment ratio of 18.8% versus, respectively, 2.5% and 19% reported by Eggertsson, Mehrotra and Robbins (2019). Since we considered a larger natural interest rate in 2015, we reached a higher simulate for the interest rate in 1970. Important to note that BVAR Convenience Yield Model indicates a natural rate of interest of 1.5% for 1970 (posterior coverage intervals of 95% between 0.74% and 2.21%), and HLW model estimates a rate of 3.8% in 1970 with 0.4% in 2015. Hence, OLG model seems to overestimate the natural interest rate compared to BVAR model but is reasonable aligned to HLW model.

Overall, we found similar results (table 8) in the decomposition of the natural interest rate decline from 1970 to 2015 for the United States to Eggertsson, Mehrotra and Robbins (2019) (figure 17). Our simulate indicated a drop in the natural interest rate of 3.74% compared to a decline of 4.02% reported by the aforementioned authors. The main factors pressuring interest rate down are the productivity growth deceleration (52% of total drop), the fell in fertility rates (44%), and the higher life expectancy (43%), with the increase in government debt (-59%) partially offsetting those forces.

Table 8 – OLG - USA Decomposition of the Natural Interest Rate Decline from 1970 to 2015

Forcing variable	**Δ in r**	**Percent of total Δ**
Total interest rate change	-3.7%	100
Productivity growth	-2.0	52
Total fertility rate	-1.7	44
Mortality Change	-1.6	43
Government debt (% of GDP)	+2.3	-59
Labor share	-0.5	12
Relative price of investment goods	-0.4	11
Change in debt limit	+0.1	-3

Figure 17 – OLG - USA Decomposition of the Natural Interest Rate Decline from 1970 to 2015: Extracted from Eggertsson, Mehrotra and Robbins (2019)

Forcing variable	Δ in r	Percent of total Δ
Total interest rate change	−4.02%	100
Mortality rate	−1.82	43
Total fertility rate	−1.84	43
Productivity growth	−1.90	44
Government debt (percent of GDP)	+2.11	−49
Labor share	−0.52	12
Relative price of investment goods	−0.44	10
Change in debt limit	+0.13	−3

Note – Natural interest rate for 2015 equal to -1.47%

5.3.2 United Kingdom

Second, we simulated the OLG model for the United Kingdom economy to decompose the decline in natural interest rate from 1970 to 2015. Table 9 shows main data considered for 2015, and table 10 summarizes the main changes in parameters between 1970 and 2015. We also used natural interest rate estimated through the BVAR Convenience Yield model as built in section 4.

Table 9 – OLG - UK Targets for 2015

Targets	**Model/Data**	**Source**
Natural rate of interest	-0.22%	BVAR Convenience Yield
Investment-to-output ratio	15.4%	World Bank
Consumer-debt-to-output ratio	9.5%	BoE
Labor share	60%	Berger and Wolff (2017)
Bequests-to-output ratio	3.0%	Hendricks (2001)

Note – Parameters (β, D, μ, α, θ) chosen to match targets

Table 10 – OLG - UK Change in Parameters from 1970 to 2015

Data	**1970**	**2015**
Life expectancy	72.0	81.0
Total fertility rate	2.4	1.8
Productivity growth	2.99%	0.53%
Government debt (% of GDP)	50.2%	107.1%
Targets	**1970**	**2015**
Consumer-debt-to-output ratio	3.0%	9.5%
Labor share	70%	60%

The simulated 1970 natural interest rate was 3.1%, higher than the BVAR Convenience Yield model estimate of 1.2% (posterior coverage intervals of 95% between -0.02% and 2.36%), but equal to HLW model of 3.1%. [14] Investment-to-output ratio simulated for 1970 was 18.5%, slightly below the observed data of 23.5%.

The OLG model simulated for the United Kingdom economy indicates a drop of 3.34% in natural interest rate from 1970 to 2015 (table 11). The demographic changes measured by life expectancy (55% of total drop) and fertility rate (32%) play the largest role in the interest rate drop, followed by the productivity slowdown (36%) and the labor share reduction (25%). The government and consumer debt increases counterbalanced the interest rate decline by, respectively, 50% and 11%.

Table 11 – OLG - UK Decomposition of the Natural Interest Rate Decline from 1970 to 2015

Forcing variable	**Δ in r**	**Percent of total Δ**
Total interest rate change	-3.3%	100
Mortality Change	-1.8	55
Productivity growth	-1.2	36
Total fertility rate	-1.1	32
Government debt (% of GDP)	+1.7	-50
Labor share	-0.8	25
Relative price of investment goods	-0.4	13
Change in debt limit	+0.4	-11

Worth noting the minor differences among the main forces forcing down the real interest rates in the United States and the United Kingdom. While the productivity deceleration is the main negative factor for the United States, the larger life expectancy is the main pressure for the United Kingdom. All in all, important to highlight that demographic changes coupled with lower productivity seem to be the main secular trends pressuring the interest rates down since the 1970s.

[14] HLW model estimates a real interest rate of 1.6% for 2015

6 CONCLUSION

A downward trend in the real interest rate over the last decades can be seen now almost as a stylized factor, this paper analyzed such fact for 5 countries in the last 20-50 years. We estimated the real interest rate through a BVAR model for the United States, the United Kingdom, Chile, Mexico, and Brazil. Main results of the BVAR Convenience Yield model show a significant decline in the real interest rates for the USA (-0.93 p.p.), Chile (-2.50 p.p.) and Mexico (-0.95 p.p) for the 1Q98-4Q19 period. There is also a material drop in the real interest rates for the UK (-0.89 p.p.), but it is not significant. For Brazil, results are even more unclear since consecutive economic and political crisis are not well captured by the model.

We concluded the convenience yield, the spread between government treasuries and corporate bonds that indicate the investor preference for the first, has risen since the 1990s for the advanced economies (USA and UK) and since 2008 for the emerging economies (Chile and Mexico), explaining a large part of this drop in the real interest rates for these countries. The increase in the convenience yield is roughly responsible for 77% of the interest rate decline in the USA, 67% in the UK, 49% in Chile, and 45% in Mexico, considering the 1Q98-4Q19 period. Such findings seem to be aligned to a global liquidity trap for investment grade rated countries, being them advanced economies or not.

On the other hand, the Secular Stagnation OLG model indicates a drop in the real interest rate for the advanced economies would have begun earlier by 1970s. Simulates for the USA and the UK from 1970 to 2015 resulted in the real interest rate fell of, respectively, 3.74 p.p. and 3.34 p.p. over the period. The main secular forces pressuring the interest rates down have been the demographic changes and the productivity slowdown, while the government debt increase has counterbalanced those factors. For the USA, the productivity growth deceleration explained 52% of total drop, the fell in fertility rates 44%, and the higher life expectancy 43%, with the rise in government debt offsetting 59%. For the UK, the life expectancy justified 55% of total decline, the fertility rate 32%, and the productivity slowdown 36%, with the government debt increase reducing drop by 50%.

Hence, it seems there are secular trends putting downward pressure on the real interest rates since the 1970s on the developed world. However, there is also an increase in the convenience yield, or the premium that investors are inclined to pay for safe assets, pressuring rates down since the 1990s on the advanced economies, with a spillover for the macroeconomic stable emerging economies since 2008.

For future research, it is interesting to develop a general framework combining the secular stagnation and convenience yield models, focusing on the emerging economies since the majority of literature is directed to the advanced economies. Indeed, a framework combining secular

stagnation and convenience yield for the developing world should likely require adjustments to the models presented here. One path would be adding fiscal and credit variables to the BVAR with common trends model to capture a broader decomposition of factors. Such model may overcome the challenge of estimating the natural interest rate for a longer time period for the emerging economies, considering all the economic and politic crises such countries faced in the 1960s-1990s.

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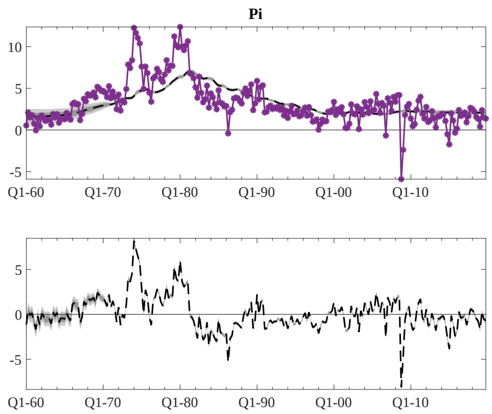
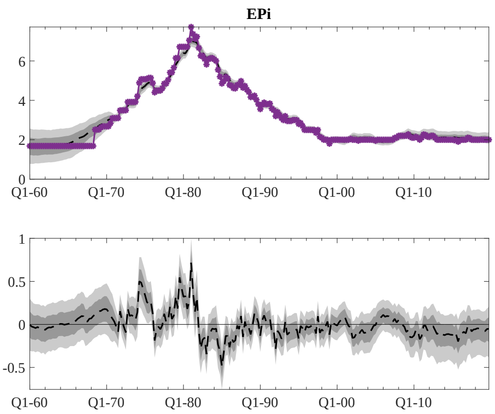
APPENDIX A – BVAR Baseline Model

For each variable, the top figure shows the original data and the trend component, and the bottom chart shows the stationary component. For trend and stationary components, the dashed black line shows the posterior median and the shaded areas show the 68% and 95% posterior intervals.

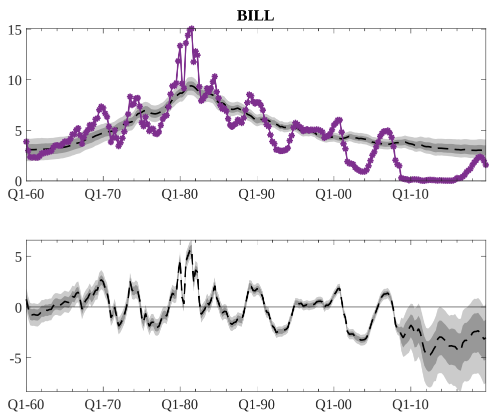
Figure 18 – BVAR Baseline Model: Trends and Observables for USA



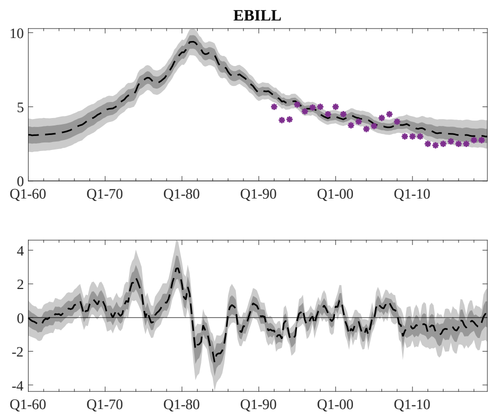
USA Inflation



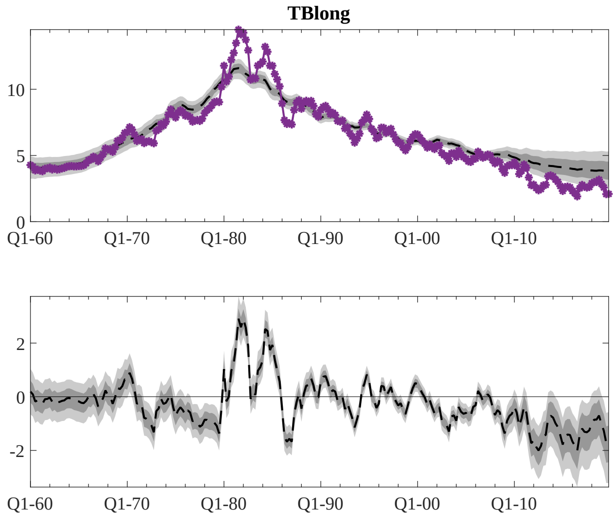
USA Inflation Expectation



USA 3-Month Interest Rate

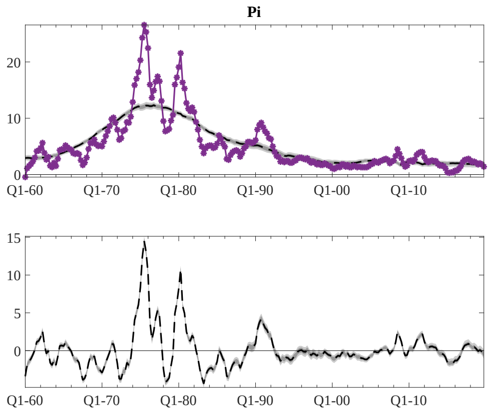


USA Interest Rate Expectation

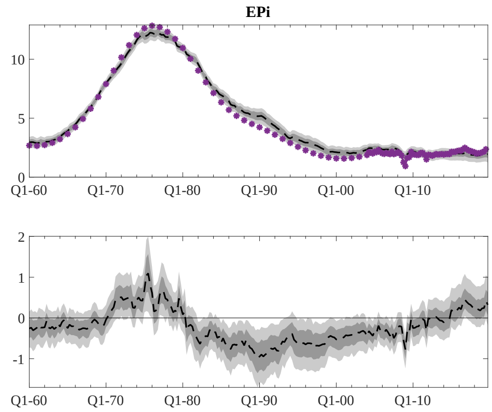


USA 20-Year Interest Rate

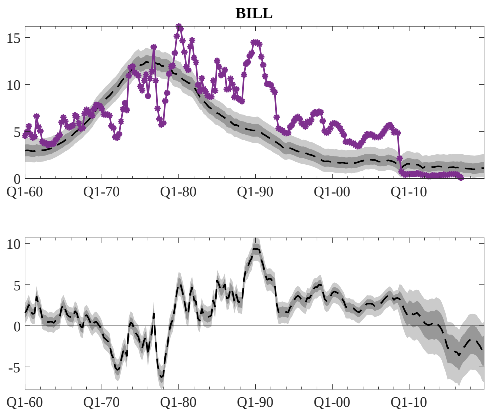
Figure 19 – BVAR Baseline Model: Trends and Observables for UK



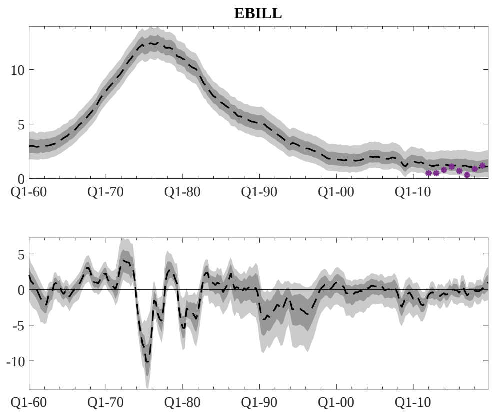
UK Inflation



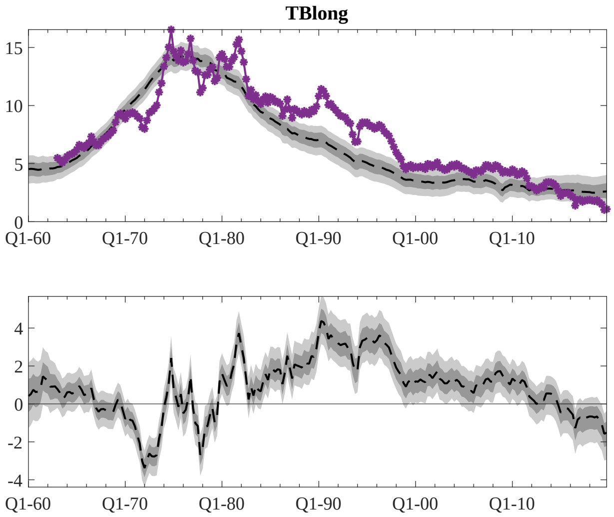
UK Inflation Expectation



UK 3-Month Interest Rate

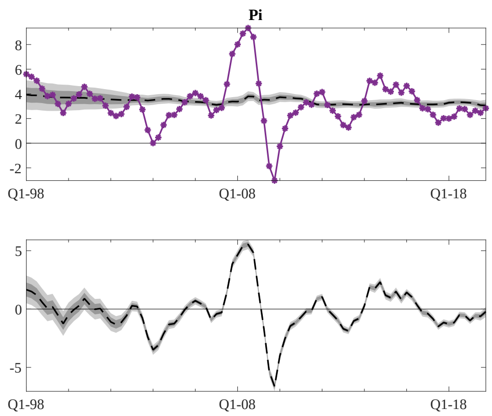


UK Interest Rate Expectation

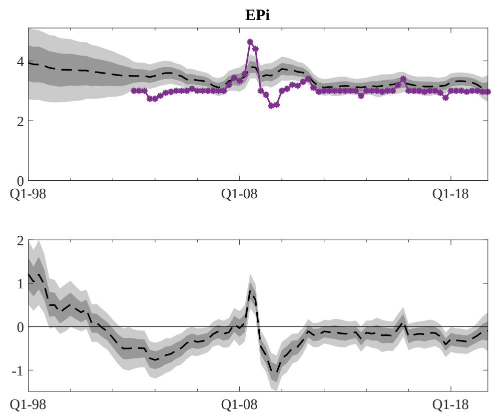


UK 20-Year Interest Rate

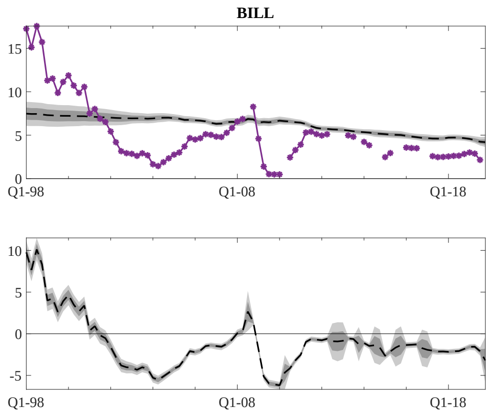
Figure 20 – BVAR Baseline Model: Trends and Observables for Chile



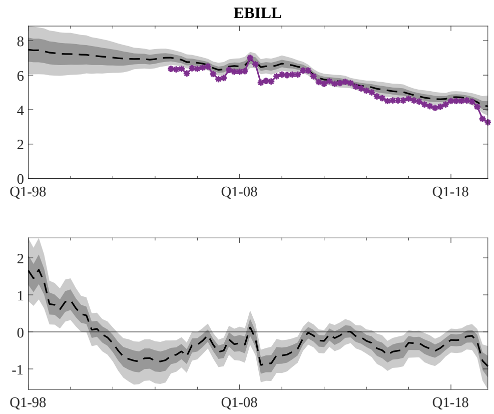
Chile Inflation



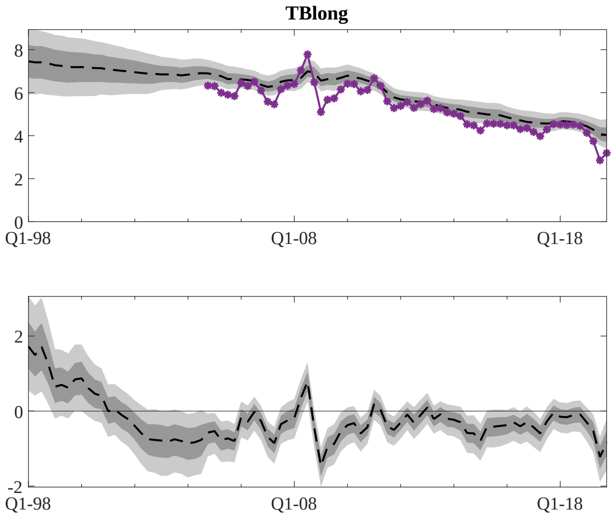
Chile Inflation Expectation



Chile 3-Month Interest Rate

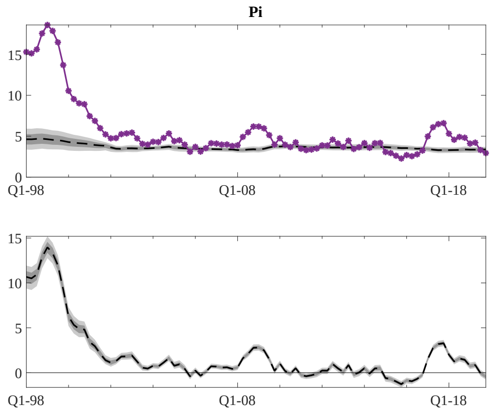


Chile Interest Rate Expectation

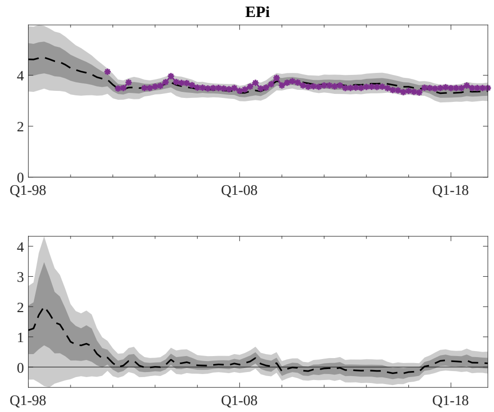


Chile 10-Year Interest Rate

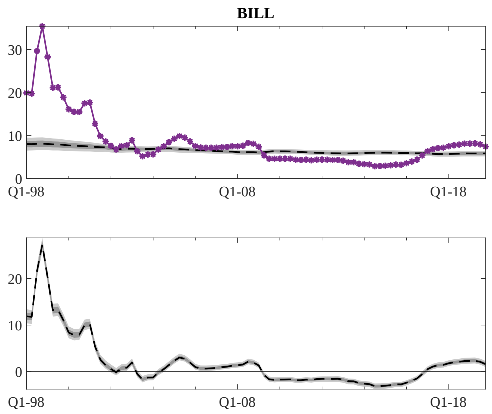
Figure 21 – BVAR Baseline Model: Trends and Observables for Mexico



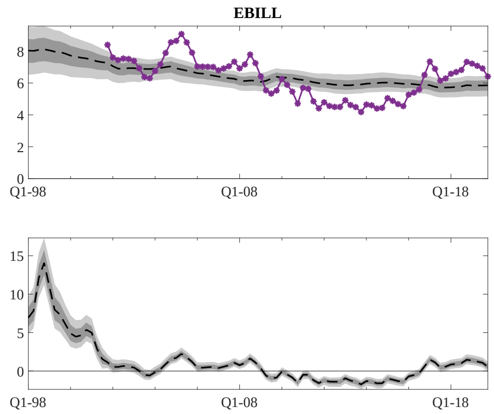
Mexico Inflation



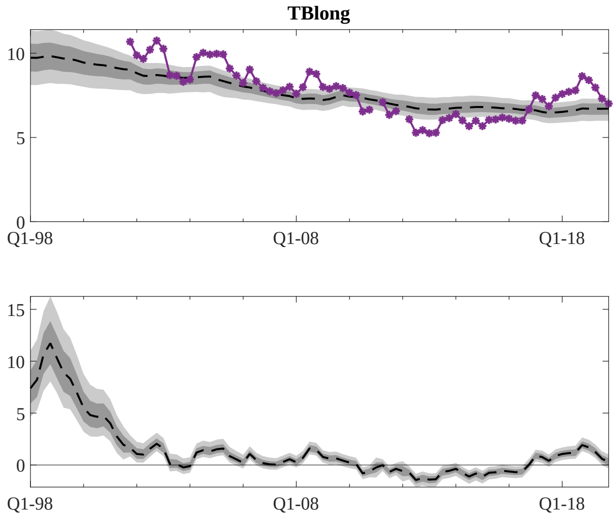
Mexico Inflation Expectation



Mexico 3-Month Interest Rate

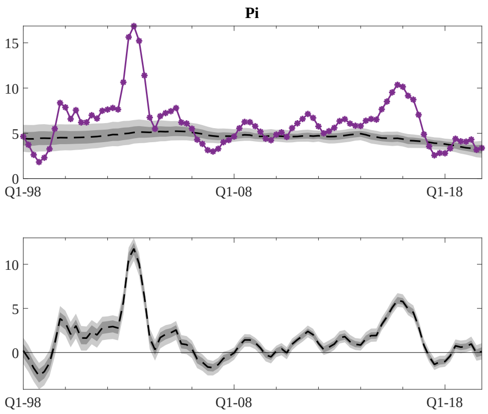


Mexico Interest Rate Expectation

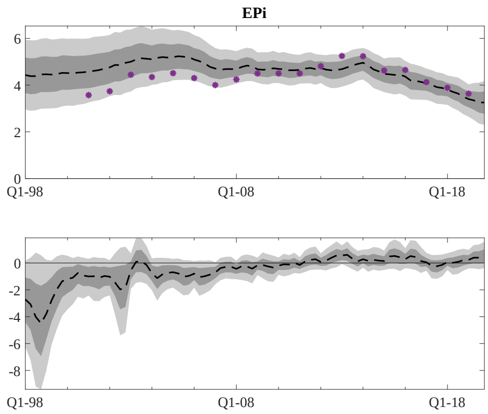


Mexico 10-Year Interest Rate

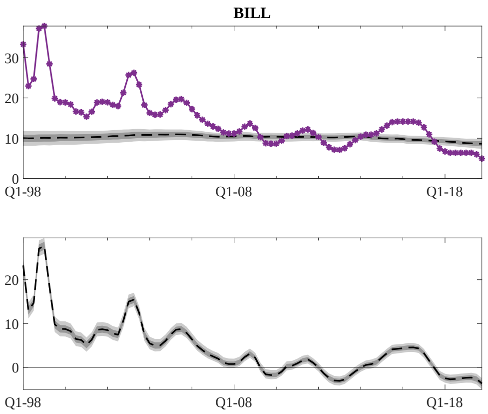
Figure 22 – BVAR Baseline Model: Trends and Observables for Brazil



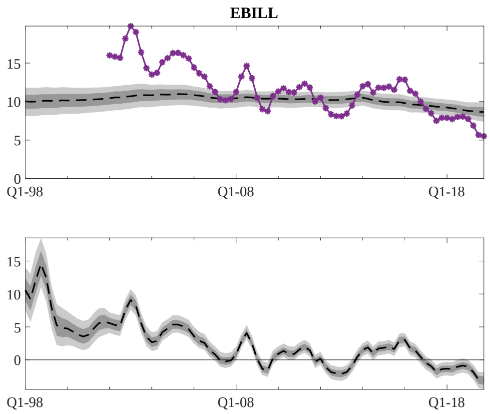
Brazil Inflation



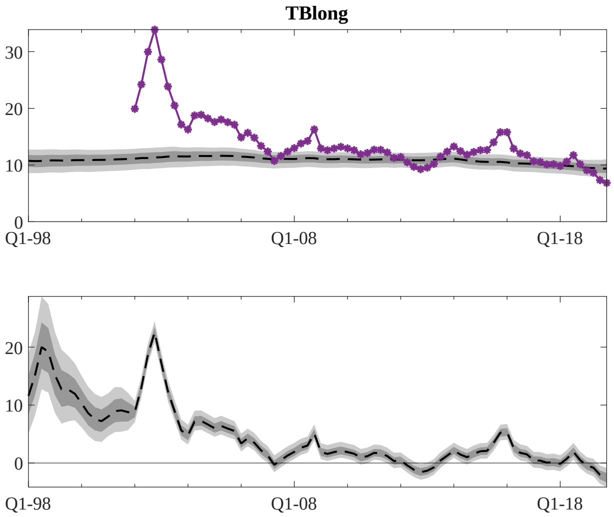
Brazil Inflation Expectation



Brazil Selic Interest Rate



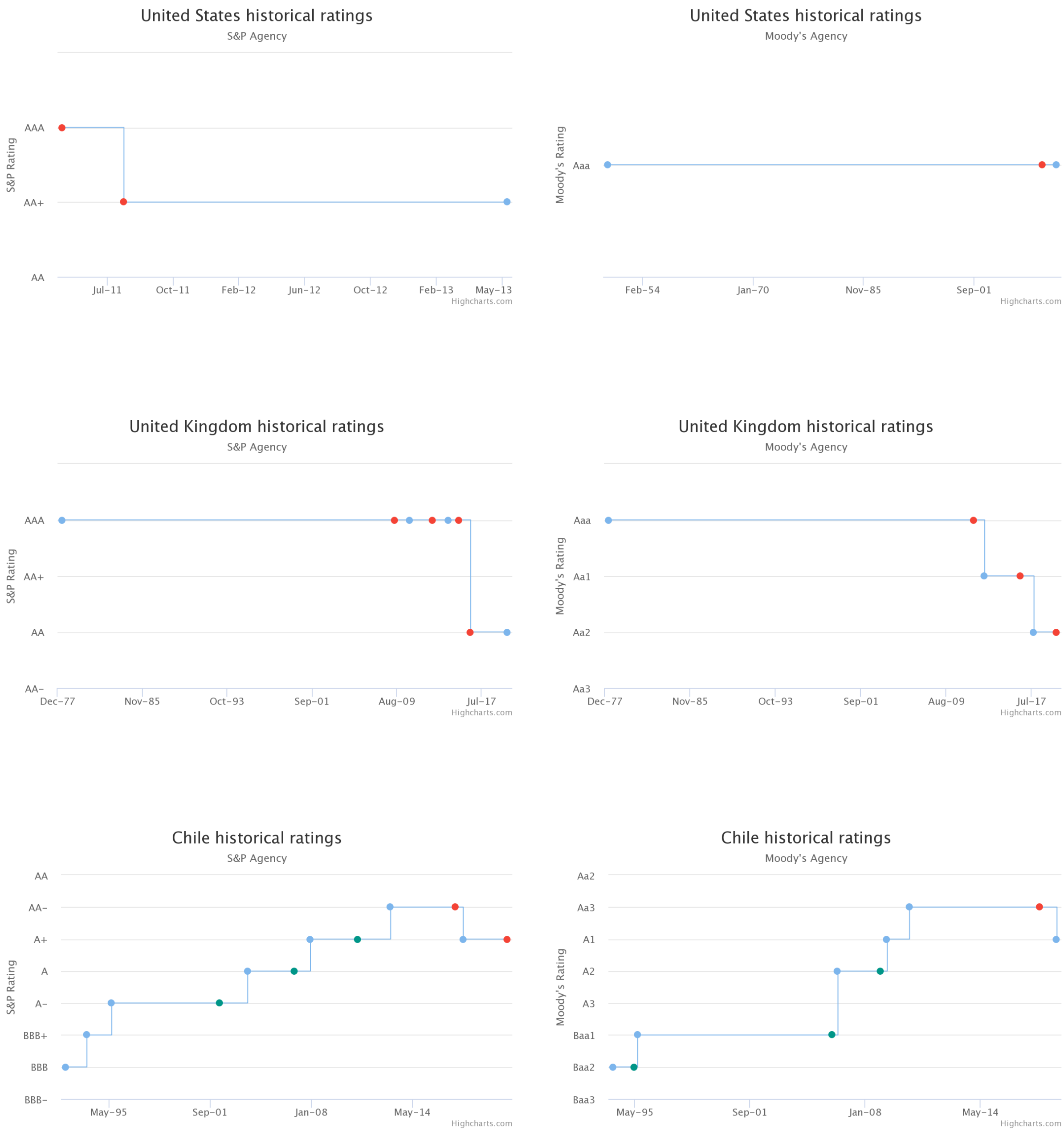
Brazil Interest Rate Expectation

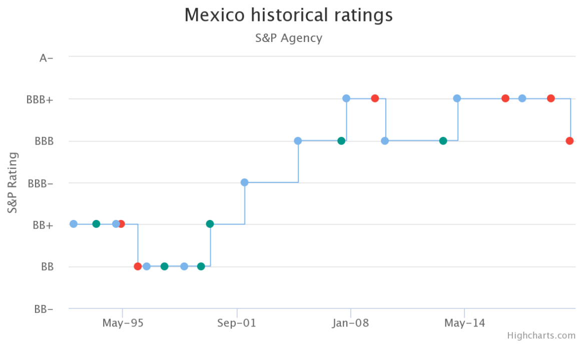


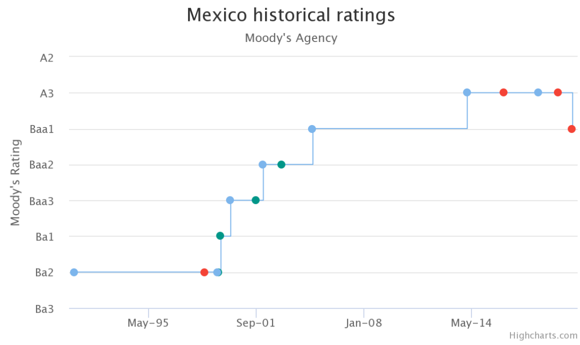
Brazil NTN-F Interest Rate

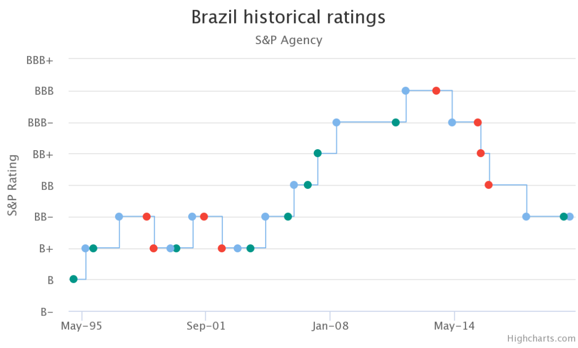
APPENDIX B – Credit Ratings

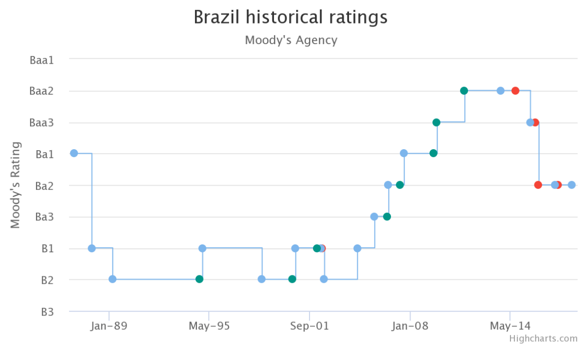
Figure 23 – Sovereign Credit Ratings











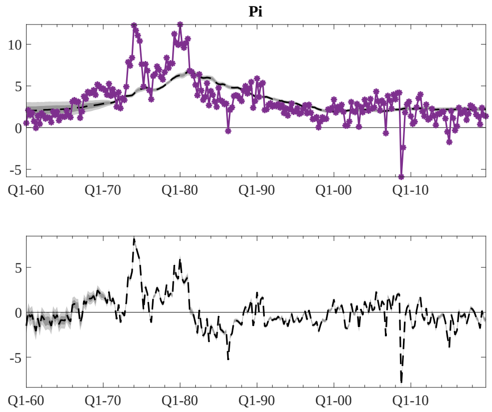
Symbol ● means a positive outlook assigned by the rating agency.
Symbol ● means a negative outlook.

Source – <http://www.worldgovernmentbonds.com/credit-rating/>

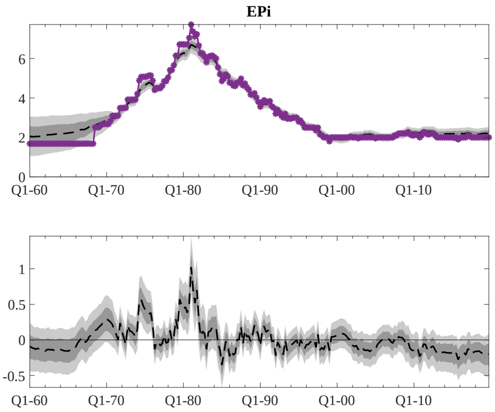
APPENDIX C – BVAR Convenience Yield Model

For each variable, the top figure shows the original data and the trend component, and the bottom chart shows the stationary component. For trend and stationary components, the dashed black line shows the posterior median and the shaded areas show the 68% and 95% posterior intervals.

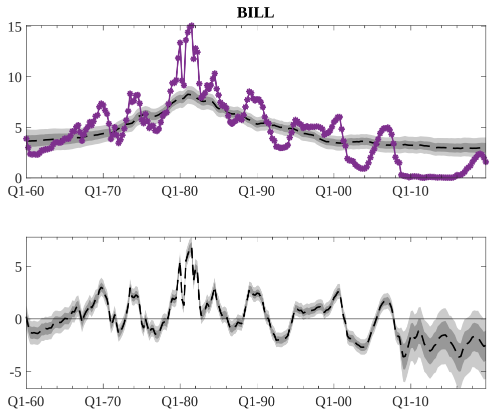
Figure 24 – BVAR Convenience Yield Model: Trends and Observables for USA



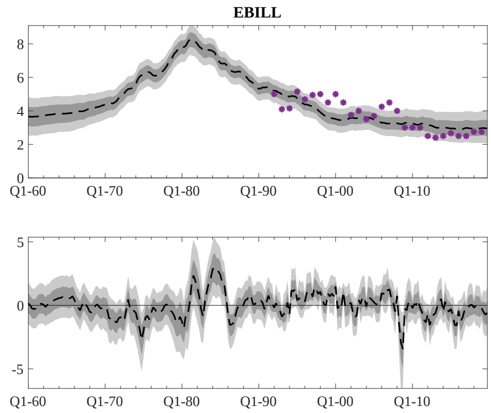
USA Inflation



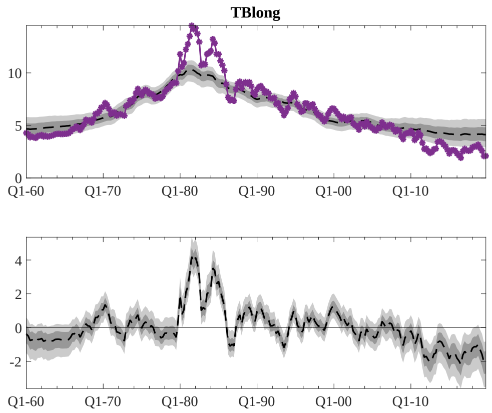
USA Inflation Expectation



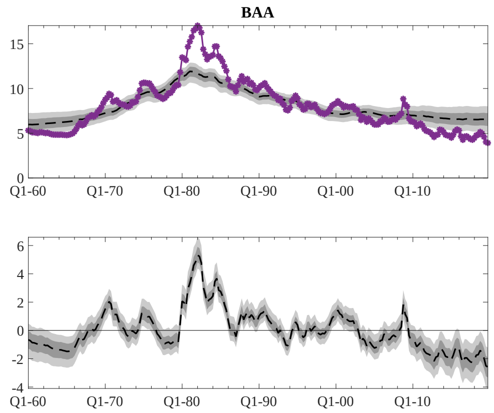
USA 3-Month Interest Rate



USA Interest Rate Expectation

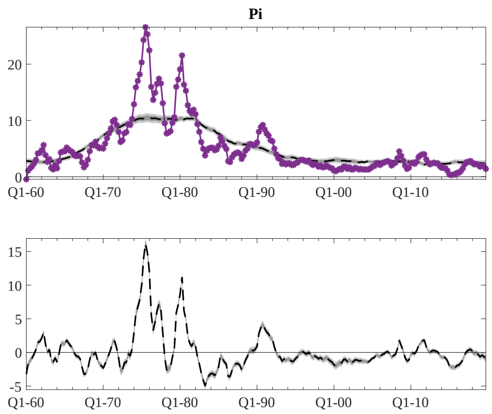


USA 20-Year Interest Rate

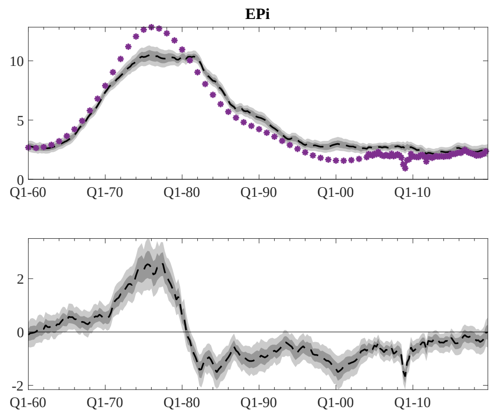


BAA Corporate Bond Yields

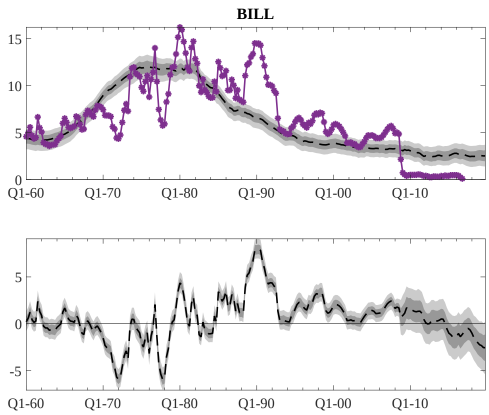
Figure 25 – BVAR Convenience Yield Model: Trends and Observables for UK



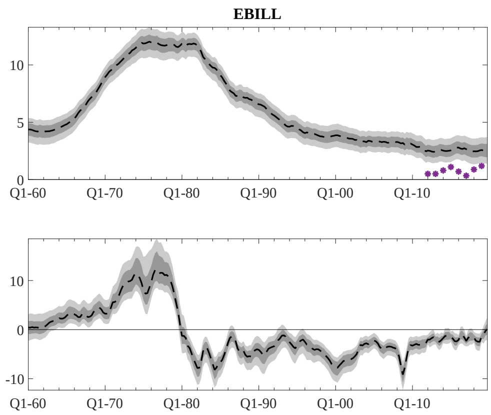
UK Inflation



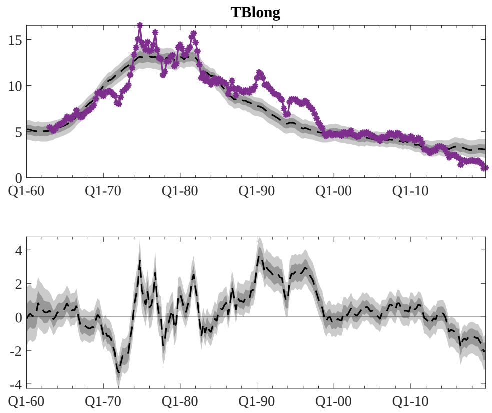
UK Inflation Expectation



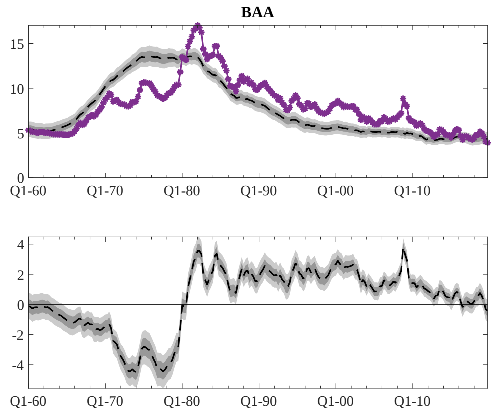
UK 3-Month Interest Rate



UK Interest Rate Expectation

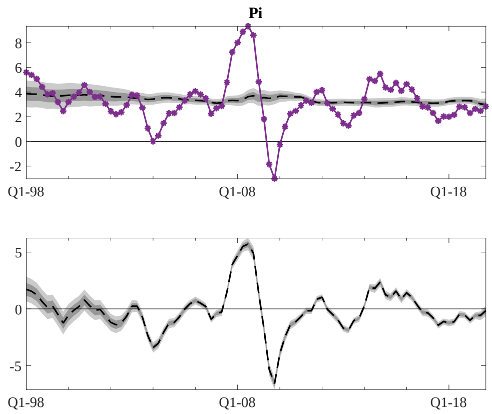


UK 20-Year Interest Rate

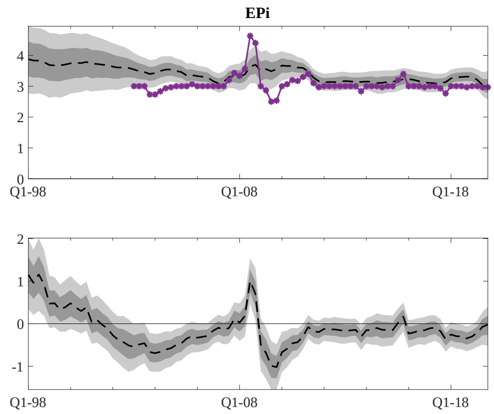


BAA Corporate Bond Yields

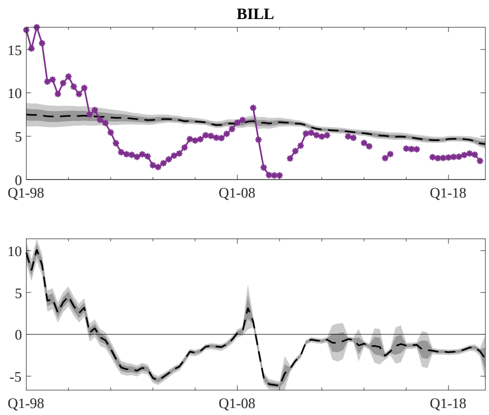
Figure 26 – BVAR Convenience Yield Model: Trends and Observables for Chile



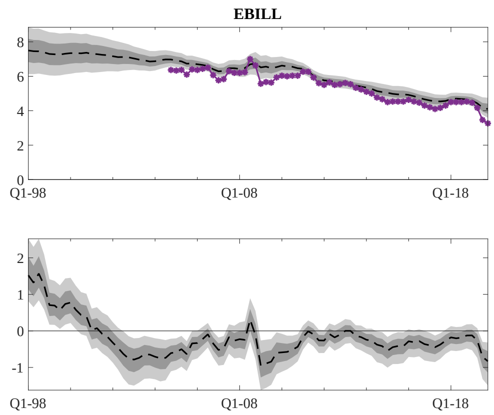
Chile Inflation



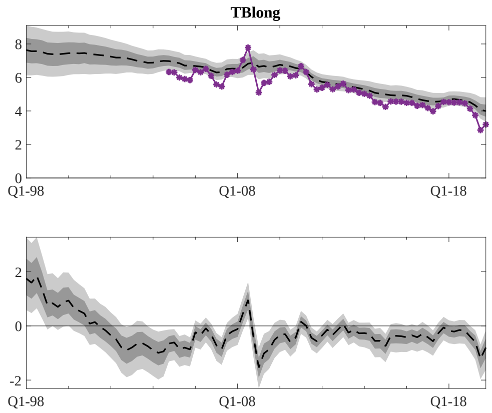
Chile Inflation Expectation



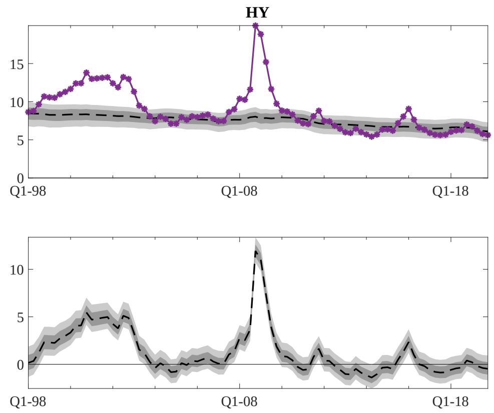
Chile 3-Month Interest Rate



Chile Interest Rate Expectation

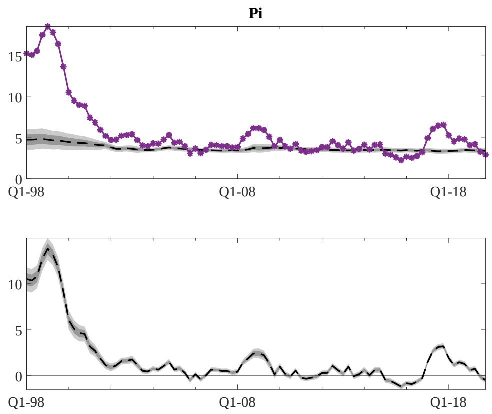


Chile 10-Year Interest Rate

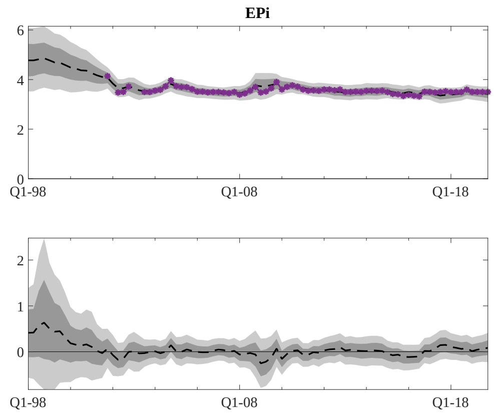


HY Corporate Bond Yields

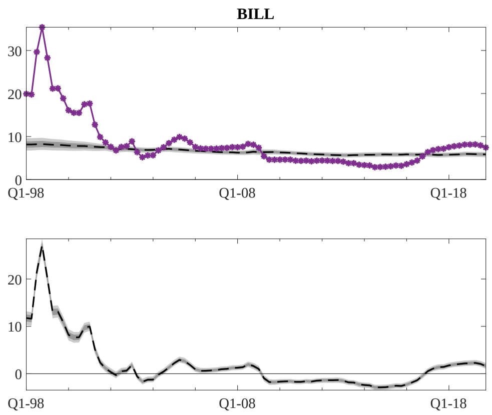
Figure 27 – BVAR Convenience Yield Model: Trends and Observables for Mexico



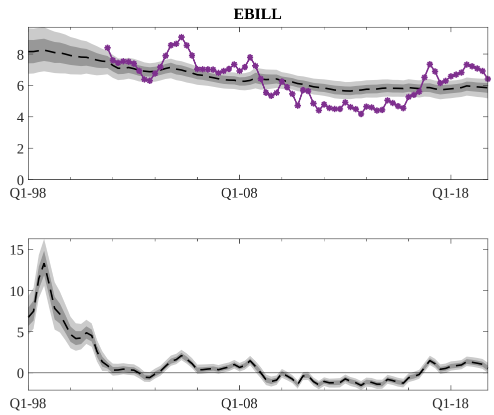
Mexico Inflation



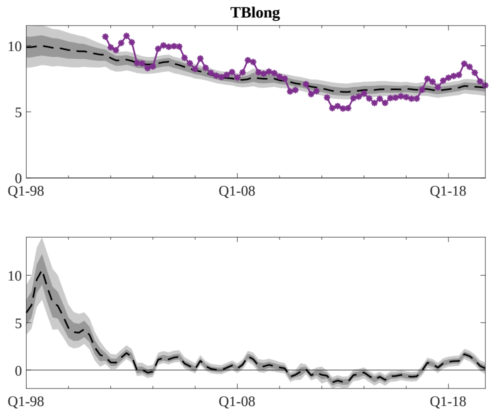
Mexico Inflation Expectation



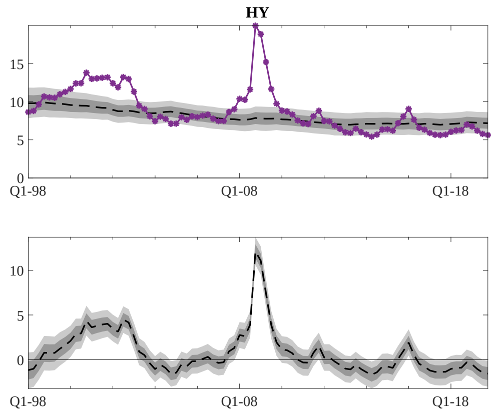
Mexico 3-Month Interest Rate



Mexico Interest Rate Expectation

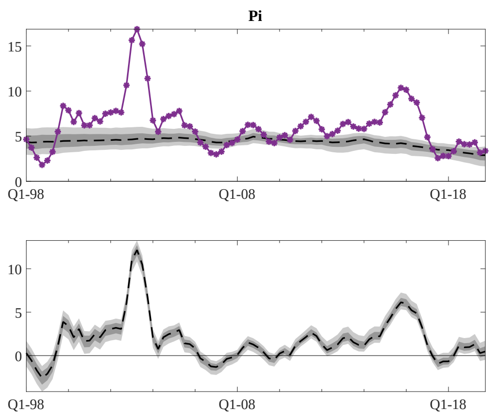


Mexico 10-Year Interest Rate

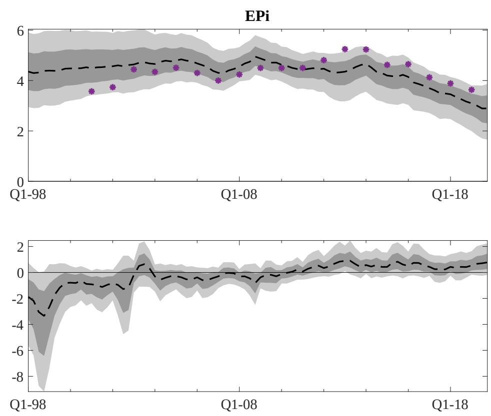


HY Corporate Bond Yields

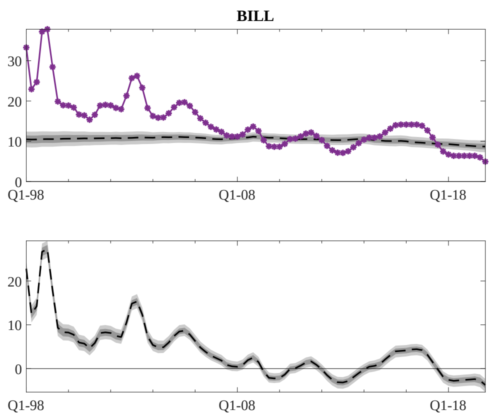
Figure 28 – BVAR Convenience Yield Model: Trends and Observables for Brazil



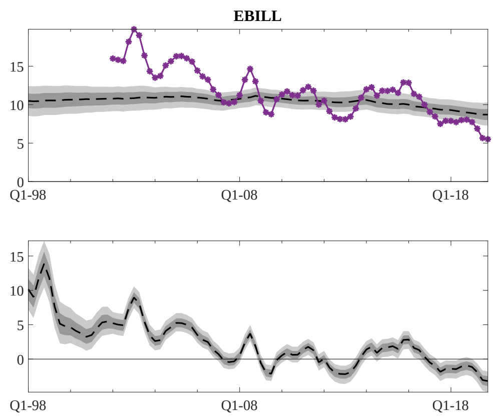
Brazil Inflation



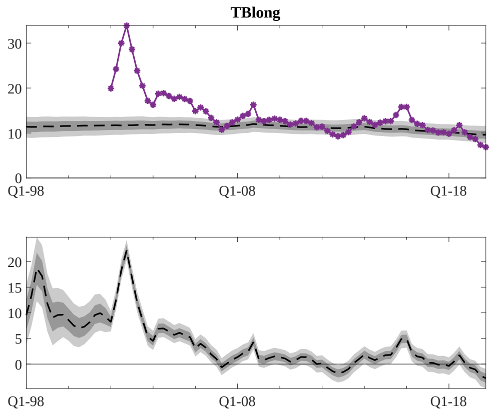
Brazil Inflation Expectation



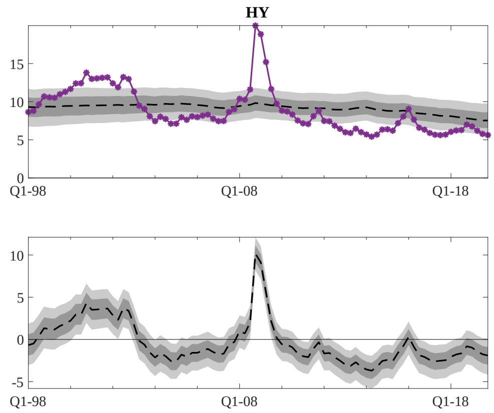
Brazil Selic Interest Rate



Brazil Interest Rate Expectation

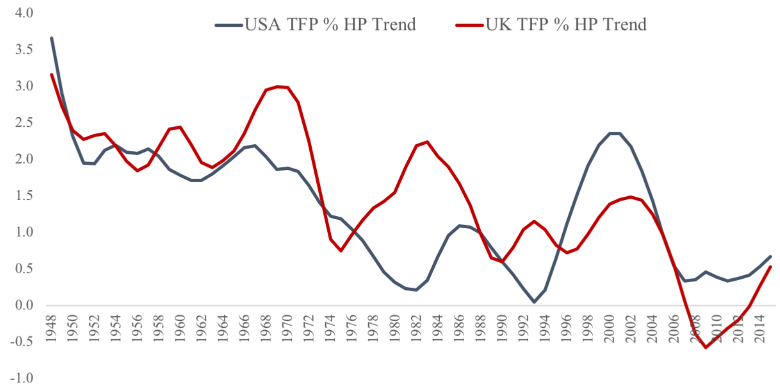


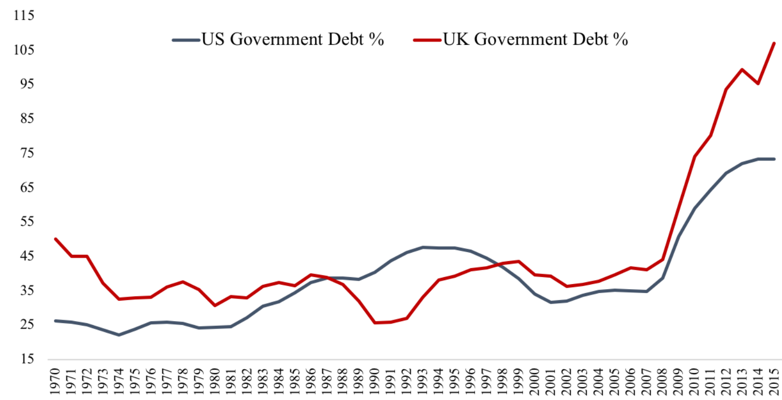
Brazil NTN-F Interest Rate

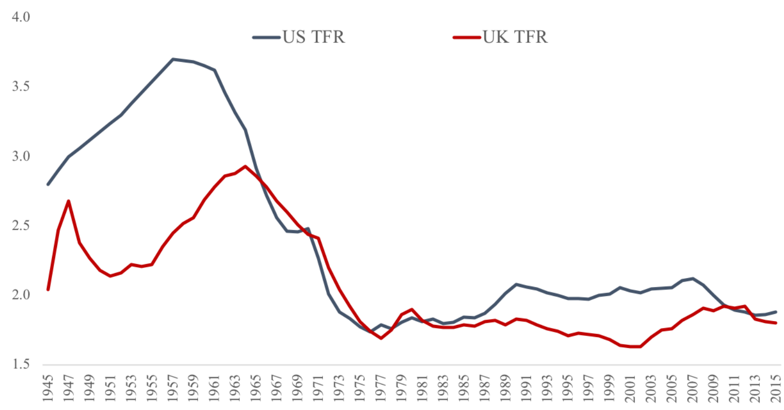


HY Corporate Bond Yields

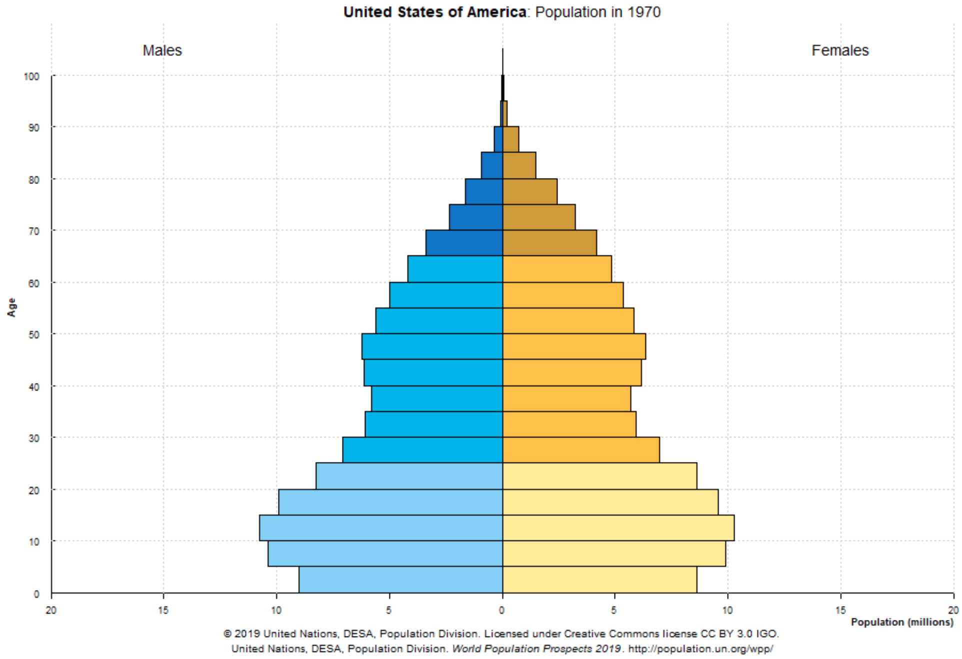
APPENDIX D – OLG Model Data

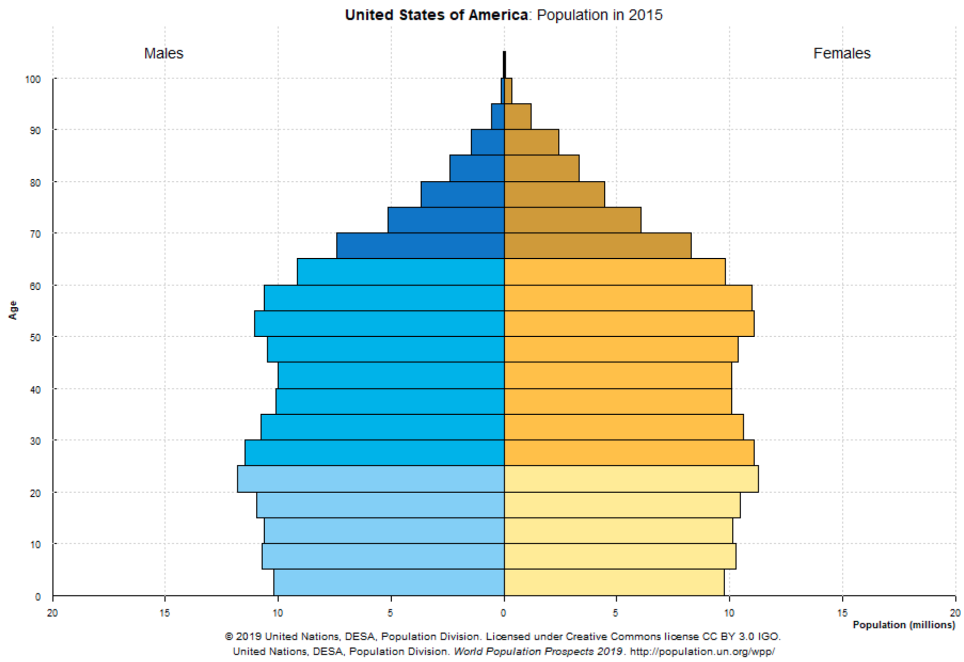


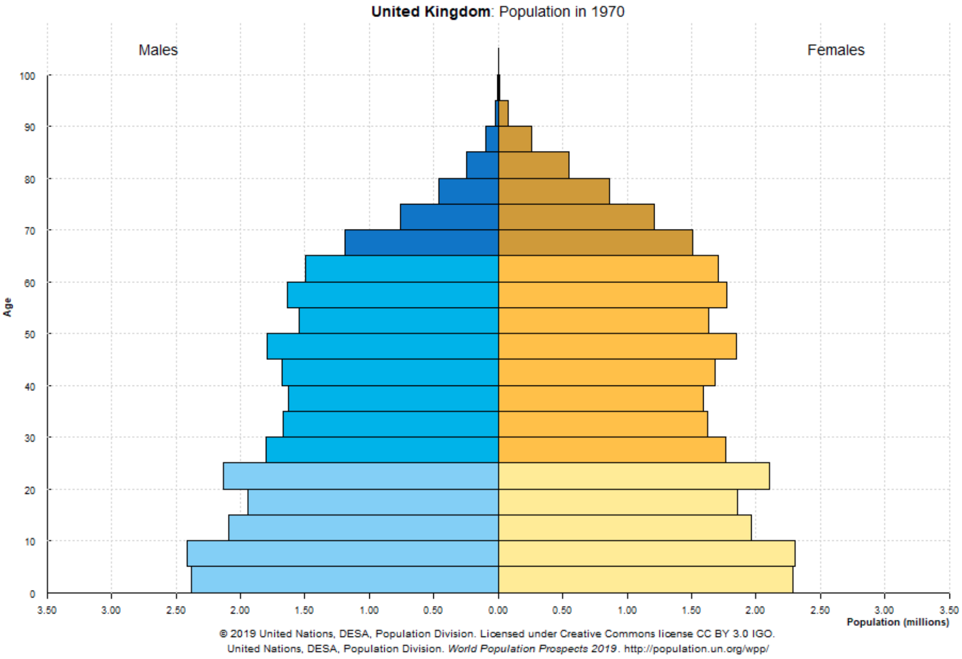




		Life table survivors, l(x), at exact age (x), both sexes												
Country	Period	25	30	35	40	45	50	55	60	65	70	75	80	85
United Kingdom	1970-1975	97041.4	96698.7	96258.6	95580.9	94392.8	92311.7	88971.7	83900.3	76367.5	65619.1	51564.9	35471.2	19727.0
United Kingdom	1975-1980	97480.0	97140.5	96726.2	96085.5	95005.9	93098.3	89862.3	84901.1	77543.9	67185.3	53477.6	37297.0	21185.5
United Kingdom	1980-1985	97965.3	97649.7	97264.8	96704.1	95752.2	94081.6	91223.0	86521.9	79445.9	69534.7	56254.1	40308.1	23718.8
United Kingdom	1985-1990	98189.6	97887.8	97499.0	96954.2	96085.6	94593.9	92110.3	87919.9	81207.8	71590.3	58740.7	43001.0	26207.9
United Kingdom	1990-1995	98498.1	98177.2	97784.7	97237.0	96405.6	95080.4	92857.8	89247.3	83309.2	74217.9	61760.8	46295.6	29121.8
United Kingdom	1995-2000	98647.9	98319.5	97924.4	97394.0	96583.9	95326.5	93305.0	90039.1	84872.8	76777.1	64899.8	49635.4	31962.4
United Kingdom	2000-2005	98804.4	98507.1	98126.2	97614.1	96851.8	95638.6	93776.3	90904.0	86345.3	79445.4	69018.5	54362.7	36348.9
United Kingdom	2005-2010	98921.4	98648.7	98286.3	97778.8	97039.0	95930.2	94195.7	91571.1	87566.8	81501.5	72476.8	59195.3	41534.0
United States of America	1970-1975	96294.4	95618.1	94826.9	93722.5	92061.0	89504.8	85793.1	80390.6	72996.4	63404.5	51101.5	36990.3	22438.1
United States of America	1975-1980	96874.9	96250.7	95575.6	94649.5	93230.2	91025.8	87674.2	82871.3	75983.9	67058.7	55328.7	41670.7	26714.5
United States of America	1980-1985	97373.9	96790.5	96144.7	95316.3	94080.0	92122.7	89075.6	84506.3	77991.5	69179.6	57753.6	44104.7	28758.8
United States of America	1985-1990	97547.9	96950.7	96225.2	95320.6	94119.7	92307.4	89473.1	85198.1	78935.4	70469.5	59240.6	45697.1	30191.6
United States of America	1990-1995	97778.9	97192.2	96435.7	95461.0	94202.4	92451.8	89841.3	85876.6	80040.6	71976.6	61323.7	48127.3	32446.1
United States of America	1995-2000	98086.7	97608.6	96998.8	96163.7	94981.1	93316.0	90903.9	87225.0	81706.1	74002.3	63511.0	50257.3	33939.3
United States of America	2000-2005	98212.6	97738.9	97191.4	96418.4	95267.4	93576.5	91202.9	87782.6	82721.9	75588.3	65760.1	52790.7	36416.2
United States of America	2005-2010	98315.7	97823.2	97276.1	96566.4	95504.2	93884.3	91520.8	88276.6	83685.2	77238.9	68260.6	56080.7	40466.9






United Kingdom: Population in 1970
Males
Females
Age
Population (millions)
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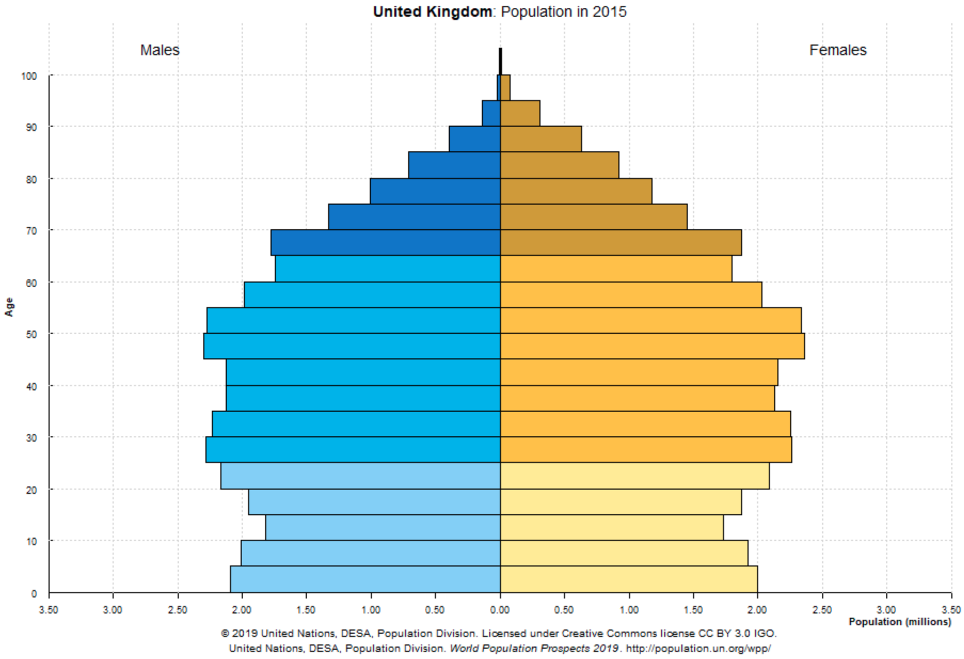
United Nations, DESA, Population Division. World Population Prospects 2019. http://population.un.org/wpp/


United Kingdom: Population in 2015
Males
Females
Age
Population (millions)
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United Nations, DESA, Population Division. World Population Prospects 2019. http://population.un.org/wpp/