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RAONE BOTTEON COSTA

# ESSAYS IN ECONOMIC THEORY: PRICING AND INFORMATION DESIGN

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Tese apresentada à Escola de Economia de São Paulo da Fundação Getulio Vargas como requisito para a obtenção do título de Doutor em Economia. Campo de conhecimento: Teoria Econômica

Orientador: Daniel Monte

Coorientador: Bruno Ferman

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# Resumo

Essa tese é composta por dois ensaios em teoria econômica.

No primeiro capítulo estudo um problema de *screening* dinâmico onde o principal tenta influenciar um agente com informação privada através de Persuasão Bayesiana, visando tanto melhorar sua utilidade de curto prazo quanto aprender o tipo do agente para o longo prazo. Aplico o modelo para "períodos de experiência", como o enfrentado por trabalhadores ao ingressar em uma empresa na maior parte dos países antes da contratação final. Os resultados desse modelo têm paralelos próximos aos do modelo de *screening* tradicional sob um único preço, no sentido que o modelo terá um equilíbrio ótimo do tipo *pooling* e um ótimo do tipo separador, sem espaço para equilíbrios semi separadores. A intuição é que ainda que a estrutura de informação seja potencialmente complexa, ela atua apenas através de um instrumento simples: a crença esperada após o sinal. Dessa maneira, ela age como um contrato único. Mostro ainda que a informação privada do agente pode ser explorada pelo principal de forma que o mesmo potencialmente pode obter um payoff maior que no caso com informação perfeita. Isso é um *efeito Ratchet* reverso e é causado pelo fato que os interesses estratégicos do agente fazem ele se comportar de forma mais próxima ao que o principal deseja nesse modelo.

No segundo capítulo, desenvolvo um modelo teórico para atacar uma questão de política econômica típica das indústrias aéreas: se as bagagens devem ser precificadas de forma independentes do preço da passagem aérea. O modelo mostra que permitir dois preços distintos para esses bens nem sempre promove aumento de bem estar social, pois o mesmo depende do custo marginal de despacho da bagagem e do poder de mercado da empresa aérea. A intuição é simples: existe uma troca entre o excesso de consumo causado pela não existência de um mercado separado para as bagagens pela insuficiência de consumo causada pelos *markups* das empresas no caso de um preço separado para as bagagens. Assim, a ineficiência na discriminação de preços pode ser resultado de um excesso de consumo.

**Palavras-chaves**: Persuasão Bayesiana, Screening, Contratos Dinâmicos, Taxas de Bagagem, Tarifas em Duas Partes, Bem Estar

# Abstract

This thesis is composed by two essays on economic theory.

In the first chapter I study a dynamic screening problem where the principal tries to influence a privately informed agent through Bayesian Persuasion in the short run in order to both maximize its stage game payoff and learn about the receivers type for the long run. I apply the model to "testing periods" such as the experience time most countries allow for the firms to evaluate its workers before full hiring. The results of this model has close parallels to the traditional screening problem under a single price instrument, in the sense that there will be an optimal pooling and separating strategies with no room for semi-separating equilibria. The intuition is that although information is potentially complex, it affects the receiver only through a simple instrument: the expected posterior belief. Hence, it acts as a single contract. I also show that receiver's private information might be explored by the sender to allow for the latter to have a higher payoff than in the full information case. This is a reverse *Ratchet effect* and it is caused by the fact the receiver's strategic interests make them more aligned to sender's payoff in this model.

In the second chapter I develop a theoretical model to tackle a recurrent policy question in the airline industry: whether baggages should be priced independently from airline tickets. The model shows that allowing for two prices is not always welfare enhancing, as it depends on the marginal cost of luggage travel and market power of the firm. The intuition is simple: there is a trade-off between over-consumption caused by the non-existence of a baggage price against under-consumption caused by firm markups in the case of a separate price for baggages. Thus, inefficiency in price-discrimination might be the result of too much consumption.

**Key-words**: Bayesian Persuasion, Screening, Dynamic Contracting, Baggage Fees, Two Part Tariff, Welfare

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# 1 An Experience Model: Bayesian Persuasion Meets Screening

## 1.1 Introduction

It is well known that private information can be crucial to the understanding of equilibrium outcomes, especially in long-run relationships. In several principal-agent problems, it is common for the principal to have to distinguish between getting the best possible outcome in the short run while learning the agent's true type for a better payoff in the long-run.

For example, several countries allows for an experience time for a firm to "test" a worker before the final decision on the hiring, in which the worker has only limited employee benefits. During this time, the firm is trying to see if the worker is a good fit for the company, but may also be compelled to extract the most of the worker productivity to enhance its short-run goals. Those objectives are not always aligned, which can open space for strategic behavior on the worker. This blueprint can also be applied to other "testing" models, such as for when a buyer is enjoying a free-trial period of an unknown quality-seller.

Those issues can be understood as a persuasion problem within a screening model. In the context of the testing period model, the firm wants to persuade its workers to exert effort on the job, but the temporary worker have different incentives than the fully contracted worker, as its job security's status is also different. Hence, when dealing only with temporary workers, the firm must adapt its strategy in order to account for the screening issues.

Although both persuasion and screening models are well understood in economics, their combination has not received much attention. This paper aims to fill this gap, by developing an experience model that embodies both problems. In our model, a sender (firm) is dealing with a receiver (worker) in an experience period and must decide on whether to hire or fire the worker afterwards. During this period, the firm privately sees the draw of a random variable that is relevant for the workers payoff (for instance, demand for the firms product, which might influence wages) and must decide on a information structure to pass to its workers. Workers have two types, and while both types privately want to exert effort only when this variable high (which we interpret as the worker anticipating a higher return on effort), the high-type workers are easier to be persuaded to exert effort than the low-type ones (in the sense that they need lower posterior beliefs about the demand). To make things interesting, the firm derives short term payoff connected to the

effort of the worker, but wants to hire only the high-type workers, so it has to balance those needs during the tenure period.

The solution of this model has close parallels to the traditional screening problem under a single price instrument. There will be an optimal pooling and separating strategy leaving no room for semi-separating equilibria due to the absence of multi-dimensional menu of choices for the firm[1] (it can only influence the workers via the posterior beliefs on the state of the demand). In this sense, the persuasion setting is only significant in the design of the equilibrium, but the equilibrium choice is made through the screening part of the model.

Our second conclusion is that the strategic behavior of the receiver in this context might be beneficial to the senders. Depending on the parameters of the model, it is possible for the sender to be better off in a context where he does not know the worker's type than if he had full information. This happens because workers' desire to be hired might lead them to optimally decide to exert more effort in the tenure period to convince employers that they are of the high type, when they anticipate that deviations from this strategy would lead to being fired after the tenure period.

Our model structure is very similar to the seminal study of Bayesian persuasion on (KAMENICA; GENTZKOW, 2011), as we have an informed sender trying to persuade a receiver through the use of information design under commitment. There are a few important differences to note however. The first is that our model allow for dynamics, as we have two periods to be analysed. Moreover, as we have two types of workers, we add private information for the receivers. Finally, the decision on whether to hire or fire the worker after the tenure time turns our problem into a screening one.

Another study that is closely related to our work is (GERARDI; MAESTRI, 2016), which studies dynamic contracting under incomplete information. In it, a firm has a dynamic relationship with a worker that has private information about its efficiency. Like in this paper's model, the firm wants to balance the goal of maximizing short-term profits and learning the worker's true type. For that, it designs a menu of contracts spanning a continuum of quality and payments. The authors shows that, as the discount factor gets close to one, the firm optimal behavior is extreme, in the sense that it will offer either the best "firing menu" (i.e: a separating type menu in which only the non-efficient worker will accept the deal, earning no rents) or the best "pooling menu". This result is similar to the one we found under different circumstances. In our model the firm has full commitment, but have only one instrument (the information structure). Hence, our result suggests that using only persuasion the firm is as limited in its leaning capacities as a firm that is dealing with fully patient players in (GERARDI; MAESTRI, 2016)'s setting

[1] This result uses the classical hypothesis of breaking up ties in favor of the principal that is common in the Bayesian Persuassion literature

Apart from the already mentioned, many other studies are related to our work. (ELY, 2017) extended the Bayesian Persuasion model to the dynamic case, showing that in this case there are two restrictions on the optimal information structure: the familiar Bayesian-plausibility condition studied on (KAMENICA; GENTZKOW, 2011) and a new dynamic element: the effects that any deviations have on the future capacity to persuade the agent. Our study, however, simplifies the dynamic element of the analysis due to an $i.i.d$ assumption on the state of nature, focusing more on the effects this has on the screening problem. (TANEVA, 2018) and (BERGEMANN; MORRIS, 2016) study the general information design problem, characterizing Bayesian Persuasion under multiple senders and receivers. However, they focus on a various player setting, and not a privately-informed receiver like we do. This is important to our setting as the final period turns our receivers into strategic actors, potentially balancing short term gains in the tenure time against long term gains in the final period.

This paper is organized as follows. After this introduction we present our model. Section 3 then solves the model for the first-best case, that is, when the firm can condition its strategy on worker types. Section 4 solves the model in the second case when the sender is myopic and section 5 solves the full model. The final section presents brief conclusions and the proofs of the propositions are in the appendix.

## 1.2 The Model

There are two players: a sender and a receiver (which we interpret as an employer and a worker), who will play a game that lasts one period. In this game, the sender privately observes the state of random variable $\omega \in \{0, 1\}$ (demand for the firm's product, which might affect wages) for which both players have a common prior $\mu_1 \in \ ]0, 1[$. In each period, sender designs an information structure, which is composed by a finite signal realization space $S$ and a family of distributions $\{\pi(.|\omega)\}_{\omega\in\{0,1\}}$ over $S$ as is standard in the literature. Sender uses this structure to persuade the receiver's decision (tell the receiver about what he sees as the state of demand). After observing $s \in S$, receiver makes an action $a \in \{0, 1\}$ that affect both players stage game payoffs (decide whether to exert effort). We assume the sender is able to commit to this information structure[2]. During the experience time workers utility may be represented by one of the following functions: $u_1^H(a, \omega)$ or $u_1^L(a, \omega)$. We call these functions respectively worker type H (as in high-motivated) and L (low-motivated) and denote by $\Phi \in \{H, L\}$ the set of worker types. Those are private information. There is a prior $\phi_1 := \Pr(\Phi = H)$ about the probability that the worker is of type H, and in the beginning of the second period the employer

[2] Alternatively we can assume that workers union can check for aggregate demand and thus serve as a mediator for the employer message. As (BEST; QUIGLEY, 2017) shows, in this case the firm can achieve its commitment payoff using future credibility concerns

forms a posterior $\phi_2$. Employer's utility in the tenure time is given by $v_1(a, \omega)$.

After the experience time, the game has a final period, in which the sender will take a binary action $a_s$ that will affect both players payoffs (hire or fire the worker). We denote by $v_2$ and $u_2$ the stage game payoff in this final period and assume by simplicity that both worker types have the same payoff function in this period. Denote the common discount factor by $\delta$. The final payoff functions for the employer and both worker types are then given by:

$$
\begin{aligned}
V(a, a_s, \omega, \Phi) &:= v_1(a, \omega) + \delta v_2(a_s, \Phi) \\
U^H(a, a_s, \omega) &:= u_1^H(a, \omega) + \delta u_2(a_s) \\
U^L(a, a_s, \omega) &:= u_1^L(a, \omega) + \delta u_2(a_s)
\end{aligned}
$$

Let $x > 0$. To make things concrete we assume the following functional forms for the payoff function of the workers in the experience time:

$$
u_t^H := \begin{cases} 1 \text{ if } a_t = \omega_t \\ 0 \text{ if } a_t \neq \omega_t \end{cases} \qquad u_t^L := \begin{cases} 1 \text{ if } a_t = \omega_t \\ 0 \text{ if } (a_t, \omega_t) = (0, 1) \\ -x \text{ if } (a_t, \omega_t) = (1, 0) \end{cases}
$$

In words, we assume that both workers want to exert effort ($a = 1$) when demand is high ($\omega = 1$) and not exert effort otherwise[3]. What differentiates them is that L-type workers have a more pronounced aversion to exerting effort when demand is low that H-type workers. Hence the parameter $x$ measure the difference between worker types.

We assume that the employer always prefer that its workers exert effort no matter what is the value of $\omega$. This is represented in a simple way by setting

$$
v_1(a, \omega) := a
$$

By making those stylized assumptions we are effectively trading generality for simplicity, since this approach are enough to allow us to study the main features of the interaction between persuasion and screening. For the last period we will also follow this procedure: we will assume that both worker types prefer to get hired and the employer wants to hire only motivated workers. This is simply achieved using the following: let $a_s \in \{L, H\}$ and $y > 0$. Final period utilities are given by:

[3] One might interpret this reduced form model as the workers anticipating a higher return on effort when demand are higher, and thus are more prone to exert effort in this scenario

$$u_2 := \begin{cases} y \text{ if } a_s = H \\ 0 \text{ if } a_s = L \end{cases} \qquad v_2 := \begin{cases} y \text{ if } a_s = \phi \\ 0 \text{ if } a_s \neq \phi \end{cases}$$

In words, workers' final period is just $y > 0$ if he is hired and 0 otherwise and the employer's period is $y > 0$ if he matches is action to the worker type (hire the high motivated worker of fire the low motivated one) and 0 otherwise. Hence $y$ is the parameter that measures the importance of last period's payoffs and the only actor that directly cares about workers' type in the model is the employer.

We solve the model in three scenarios. In the first we assume the employer can condition its optimal information structure in the workers type. This is the full information scenario and its analogous to (KAMENICA; GENTZKOW, 2011). In the second scenario, we assume employer cannot make such condition, but cares only about the present ($\delta = 0$). This is the myopic scenario that deals only with Bayesian persuasion when the receiver has different types. Finally, in the third scenario, we solve the full problem.

## 1.3 Full Information

When the employer can condition its information structure in the workers type we have a standard Bayesian Persuasion problem. Let $\pi(s|\omega;\Phi)$ be the optimal employer signal structure for a given $\Phi$. Using the results of (KAMENICA; GENTZKOW, 2011) it is straightforward to see that we can set $S = \{0, 1\}$ without loss of generality. Also, whenever the employer wants to send an informative signal[4] the optimal structure is given by

$$\pi(s|\omega;H) := \begin{cases} 1 \text{ if } s = \omega = 1 \\ 0 \text{ if } s = 0 \text{ and } \omega = 1 \\ \frac{\mu_1}{1-\mu_1} \text{ if } s = 1 \text{ and } \omega = 0 \\ 1 - \frac{\mu_1}{1-\mu_1} \text{ if } s = \omega = 0 \end{cases} \qquad \pi(s|\omega;L) := \begin{cases} 1 \text{ if } s = \omega = 1 \\ 0 \text{ if } s = 0 \text{ and } \omega = 1 \\ \frac{\mu_1}{1-\mu_1}\frac{1}{1+x} \text{ if } s = 1 \text{ and } \omega = 0 \\ 1 - \frac{\mu_1}{1-\mu_1}\frac{1}{1+x} \text{ if } s = \omega = 0 \end{cases}$$

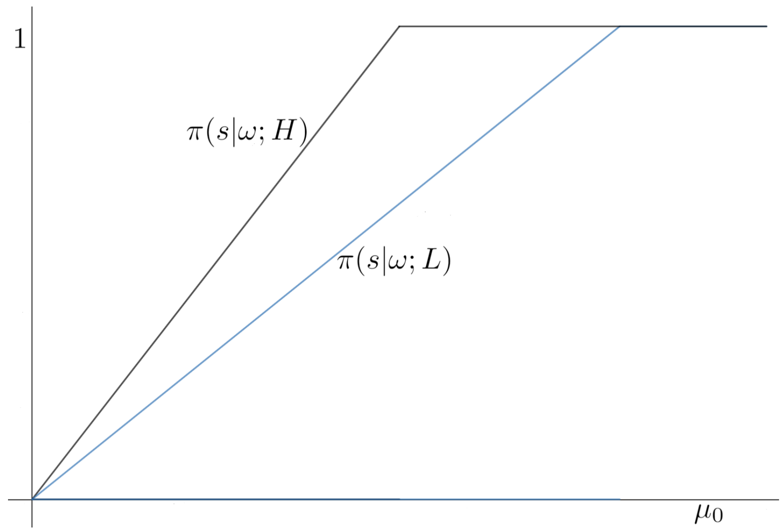
Note that in this structure the worker is always sure that demand is low if $s = 0$, so his action is to play $a = 0$ in this case. However, when $s = 1$, both worker types are

[4] This happens whenever the worker's default action is to play $a = 1$ as in this example workers belief's is clearly discrete at the prior in the sense developed by (KAMENICA; GENTZKOW, 2011). This would be the case if $\mu_1$ is sufficiently high, and it is not of interest to us, as then it would be trivially optimal for the receiver not to send any signal. In this example, simple algebra shows that the threshold is $\frac{1+x}{2+x}$ if $\Phi = L$ and $\frac{1}{2}$ if $\Phi = H$. In what follows in this and other sessions we will implicitly assume that $\mu_1$ is small enough to not be bothered by those issues

precisely indifferent between playing both actions. As is standard in the literature, we assume ties are broken in favor of the employer, so in this case both workers set $a = 1$. As (KAMENICA; GENTZKOW, 2011) shows, the value of the optimal signal is given by the concave closure of the sender's expected utility. Figure 1.1 illustrates this when $x = 3$. We can see that the sender obviously prefer worker type to be $H$. Indeed, the sender's expected payoff is $2\mu_1 + \delta y$ when he is dealing with type-H worker and $\frac{2+x}{1+x}\mu_1 + \delta y$ when dealing with type-L worker. Hence, ex-post expected payoff is always higher under type H-workers and ex-ante expected payoff is given by

$$\mu_1 \left[ 2\phi_1 + (1 - \phi_1) \left( \frac{2+x}{1+x} \right) \right] + \delta y$$

Figure 1.1 – First Best: Concave Closure of Sender's Expected Utility




## 1.4 Myopic Scenario

This case has many parallels with the classical screening problem when the principal is restricted to linear contracts. Here the employer would like to distinguish between the workers types and design a menu of contracts, but unlike the canonical screening problem, he does not have multiple instruments to design this menu such as in (GERARDI; MAESTRI, 2016). He only have a single instrument available: the information structure to be designed. Proposition 1.4.1 stated below shows that the solution to this case share several characteristics with the second-best optimal screening linear contract. For instance, the employer will decide if it is better to induce both worker types to play $a = 1$ or only the type that generated more utility to him (the most "efficient" one in the screening terminology). Moreover, the optimal information structure will leave one of the worker types indifferent between the two actions when $s = 1$, while the other will have positive gains from one of the actions. This is akin to the optimal linear contract in the

classical screening problem being the one that either completely extract consumer surplus from the least efficient group (case in which workers type $L$ are indifferent when $s = 1$) or the most efficient one (case in which workers type $H$ are indifferent when $s = 1$). In the former case the optimal information design gives a positive surplus for type $H$ workers when choosing $a = 1$, in the latter, the design gives surplus for type $L$ workers when choosing $a = 0$. Nonetheless, type $H$ workers will always play $a = 1$ when $s = 1$, while type $N$ workers may be incentivized to always play $a = 0$ (get out of the market in the screening terminology). Hence we get the familiar "no distortion at the top" property

**Proposition 1.4.1** ***Optimal Myopic Signal*** *Assume that employers cannot conditional their signals on worker's type and that $\delta = 0$. Then the optimal signal is given by*

$$\pi^*_M(s|\omega) := \begin{cases} \pi(s|\omega; H) \ \textit{if}\ 2\mu_1\phi_1 \geq \frac{2+x}{1+x}\mu_1 \\ \pi(s|\omega; L) \ \textit{if}\ 2\mu_1\phi_1 < \frac{2+x}{1+x}\mu_1 \end{cases}$$

*Moreover, if the optimal signal equals $\pi(s|\omega; H)$ then only type $H$ workers play $a = 1$ conditional on $s = 1$. If the optimal signal equals $\pi(s|\omega; L)$ then both workers play $a = 1$ conditional on $s = 1$. The expected employer payoff is given by $2\mu_1\phi_1$ if the optimal signal is $\pi(s|\omega; H)$ and $\frac{2+x}{1+x}\mu_1$ in the case of $\pi(s|\omega; L)$*

Proposition 1.4.1 shows that in the second best under a myopic sender, the optimal signal is one of the two possible optimal first best signals, but now the choice is dependent on beliefs $\mu_0$ and $\phi_1$, rather than on worker types[5]. Effectively, the sender is choosing between a separating and a pooling equilibrium, which is a choice between an easier persuasion (in the sense that lower beliefs are needed in equilibrium if sender plans to persuade only motivated workers) and higher share of persuaded workers. Clearly, higher values of $\phi_1$ (employer believes more workers are from type $M$) and $x$ (higher difference between worker types) leads to higher chance of the separating equilibrium to be the optimal choice.

The result for the nonexistence of semi-pooling equilibrium comes from the fact that information structure, although potentially complex in principle, affects payoffs only through a simple entity: the expected posterior it generates. As the state of nature is represented by a single random variable in our model, information design affects equilibrium through an uni-dimensional channel and so is similar to the screening case with only a single price instrument, which has the known property of being fully characterized by the best separating or pooling equilibrium.

[5] Hence, there is an abuse of notation when we write $\pi(s|\omega; H)$ as now we cannot condition on worker types

## 1.5 Full Model

In this scenario the problem is fundamentally different from the main Bayesian Persuasion model, as now the receiver is also strategic. Indeed, in this model both players are trying to influence each other as the sender is conflicted between trying to persuade receivers to improve its short run utility while also disentangling between receiver types for the long run. This adds another restriction to the information design problem, that is, the incentive constraints of both receiver types, just as in the main screening model.

To find the equilibrium in this model we can focus on information structures in which $S = \{0.1\}$. By the same reasoning as in the previous proposition, there is no possibility of semi-separating equilibria, so any signal will either induce the same action from both worker types or promote differentiation. Call the first case the pooling and the second the separating signal. Define $\lambda := \text{Max}\{\phi_1, 1-\phi_1\}$. Then the following lemmas give us the best signals for the first and the second case:

**Lemma 1.5.1** *Optimal Separating Signal*

*Assume that employers cannot conditional their signals on the worker's type and that $\delta > 0$. If the firm is restricted to separating signals then the following information structure is the optimal*

$$\pi_S^*(s|\omega) := \begin{cases} 1 \text{ if } s = \omega = 1 \\ 0 \text{ if } s = 0 \text{ and } \omega = 1 \\ \frac{\mu_1}{1-\mu_1}\frac{1+\delta y}{1-\delta y} \text{ if } s = 1 \text{ and } \omega = 0 \\ 1 - \frac{\mu_1}{1-\mu_1}\frac{1+\delta y}{1-\delta y} \text{ if } s = \omega = 0 \end{cases}$$

*This information structure requires $\delta y < 1$ to be chosen. If $\pi_S^*(s|\omega)$ is used then only type $H$ workers play $a = 1$ conditional on $s = 1$. In this case $a_s = H \iff a = 1$ and sender's expected utility is given by $\frac{2\mu_1\phi_1}{1-\delta y} + \delta y \left[\frac{2\mu_1}{1-\delta y} + \lambda\left(1 - \frac{2\mu_1}{1-\delta y}\right)\right]$.*

**Lemma 1.5.2** *Optimal Pooling Signal*

*Assume that employers cannot conditional their signals on the worker's type and that $\delta > 0$. If the firm is restricted to pooling signals then the following information structure is the optimal*

$$\pi_P^*(s|\omega) := \begin{cases} \pi(s|\omega; P) \text{ if } \phi_1 \geq \frac{1}{2} \\ \pi(s|\omega; L) \text{ if } \phi_1 < \frac{1}{2} \end{cases}$$

*Where $\pi(s|\omega; P)$ is defined as*

$$\pi(s|\omega;P) := \begin{cases} 1 \text{ } if \text{ } s=\omega=1 \\ 0 \text{ } if \text{ } s=0 \text{ } and \text{ } \omega=1 \\ \frac{\mu_1}{1-\mu_1}\frac{1+\delta y}{1+x-\delta y} \text{ } if \text{ } s=1 \text{ } and \text{ } \omega=0 \\ 1-\frac{\mu_1}{1-\mu_1}\frac{1+\delta y}{1+x-\delta y} \text{ } if \text{ } s=\omega=0 \end{cases}$$

*If $\pi^*_P(s|\omega)$ is used then both workers play $a=1$ conditional on $s=1$, but $a_s=H$ when $\phi_1 \geq \frac{1}{2}$ and $a_s=L$ otherwise. Sender's expected utility is given by $\frac{(2+x)\mu_1}{1+x-\delta y}+\delta y\lambda$ in the first case and $\frac{(2+x)\mu_1}{1+x}+\delta y\lambda$ in the latter.*

Taking together Lemma 1.5.1 and Lemma 1.5.2 results we have just shown the following proposition:

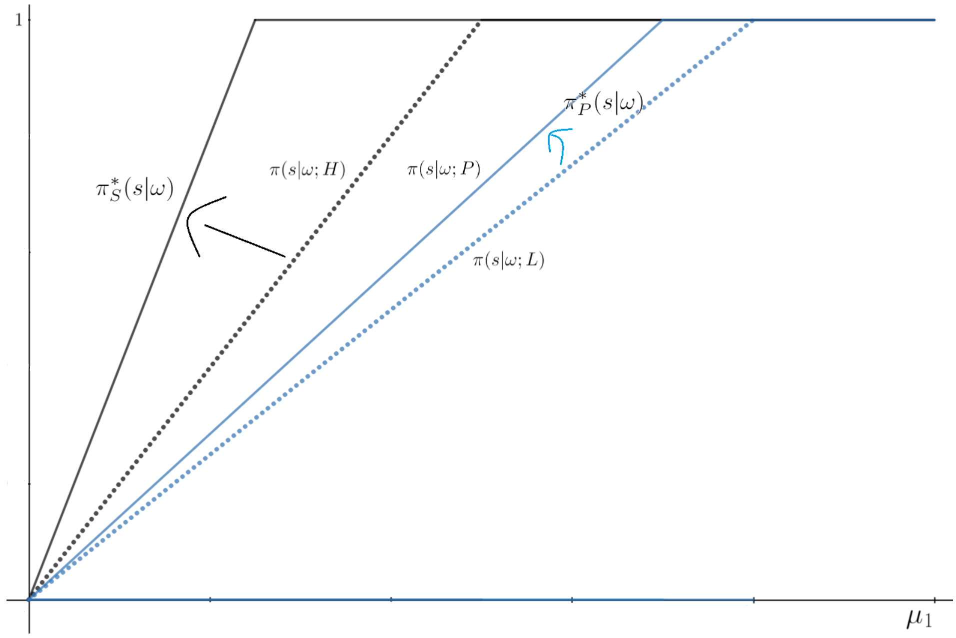
**Proposition 1.5.3** ***Optimal Signal in the Full Model*** *Assume that employers cannot conditional their signals on the worker's type and that $\delta>0$. Then the optimal signal is given by either $\pi^*_S(s|\omega)$ or $\pi^*_P(s|\omega)$. We call $\pi^*_S(s|\omega)$ the optimal separating signal and $\pi^*_P(s|\omega)$ the optimal pooling signal.*

Proposition 1.5.3 shows that, just like in the previous analysis, sender must choose between a separating and a pooling equilibrium, but the pooling equilibrium of choice is now dependent on $\phi_1$. More specifically, if $\phi_1$ is sufficiently high (so the worker know that if he does not deviate from equilibrium and play $a=1$ he will be hired), the optimal pooling strategy is to play $\pi(s|\omega;P)$. Note that $\pi(s|\omega;P)$ clearly induces lower beliefs than the previous $\pi(s|\omega;L)$ when $s=1$. This is translated into a higher expected utility for the sender in this equilibrium, as the expected payoff in the first period under $\pi(s|\omega;P)$ is higher, and in the second period they both equal $\lambda y$ due to the nature of the pooling equilibrium. Thus, perhaps not surprisingly, if the employer could commit not only to an information structure, but also to a final action, he could potentially increase its payoff whenever the currently equilibrium demands $\pi(s|\omega;L)$ to be played. Figure 2.2 portraits the value of optimal signals for this case for $(x,y,\delta)=(3,1,0.5)$ and illustrates this observation.

The main effect of this change is that for a subset of the parameter space long term concerns make the non-motivated worker to attempt to act like the motivated one, so the employer can persuade it in the short run with lower beliefs. As such, this is akin to a reverse *Ratchet effect*[6] in which the worker knows that its terms of trade will be

[6] The so-called *Ratchet Effect* is the desire of the worker to behave as a less efficient one in a world where its efficiency is negatively correlated with future terms of trade. This effect is taken to be a relevant factor to explain the poor productivity of the planning economy of the Soviet Union. A nice explanation of this effect and its consequences on dynamic relationships can be found on (GERARDI; MAESTRI, 2016)

Figure 1.2 – Second Best under Long-Term Sender: Concave Closure of Sender's Expected Utility




better if he behaves more closely aligned to the preferred principal action. This has the potential to make this scenario ex-ante preferable for the principal when compared to the full information scenario. To see this, note that as we take limits when $\phi_1 \to 1^-$, the optimal separating signal yields to the sender expected utility of

$$\frac{2\mu_1}{1-\delta y} + \delta y$$

As we require $\delta y < 1$ for the separating equilibrium to exist, it is easy to see that the expected utility generated by this equilibrium is higher than $2\mu_1 + \delta y$, which is the expected utility of the full information case when dealing with type $H$ workers. Similarly, the utility of the optimal pooling equilibrium when $\phi_1 \to 1^-$ converges to

$$\frac{2+x}{1+x-\delta y}\mu_1 + \delta y$$

which is higher than the expected utility of the full information case when dealing with type $L$ workers that is given by $\frac{2+x}{1+x}\mu_1 + \delta y$.

Another point that merits attention is that the choice for the separating versus the pooling equilibrium is proportional to the difference between the second terms of these equilibria expected utilities, which is given by

$$\alpha := \left[\frac{2\mu_1}{1-\delta y} + \lambda\left(1 - \frac{2\mu_1}{1-\delta y}\right)\right] - \lambda = 2\mu_1\frac{1-\lambda}{1-\delta y}$$

Those second terms measure the importance of second period payoffs in each case. For the separating equilibrium to be chosen we need $\delta y < 1$, hence whenever the separating equilibrium is the chosen one we have $\alpha > 0$. This is the value of learning worker's true type in this model.

Finally, as the lower the discount factor $\delta$ is, the lower is the value of $\alpha$ we can say that the more impatient the parties involved, the less relevant is the extra benefit of learning. This allows us to infer that, as we increase the tenure time - which can be seen as decreasing the discount factor - this extra benefit tends to get smaller, as both the low-motivated worker will have less incentive to behave as the high-motivated one (given that its benefit for doing so, namely, the possibility of being hired, is being postponed) and the firm will have less incentive to promote learning (once again, as its payoff for doing so is occurring further into the future).

## 1.6 Conclusion

This paper studied the introduction of a Bayesian persuasion problem into a dynamic screening model. The sender aims to influence the receiver though the choice of a information structure in a world where receivers have private information about their types and the sender have a final action with payoff connected to its learning capacities. The model shows that receiver's strategic behavior might improve the sender expected payoff. This happens because of a reversed *Ratchet Effect* as in our model receiver's strategic concerns make them behave as more aligned to the sender's stage game objectives. Those results suggest that trial periods, such as the worker's tenure model analyzed here, may increase the principal's payoff in the short-run, offering positive evidences for such practices in a world where the agents have private information about their productivity.

Another result we found is that by using only persuasion as a tool in a dynamic screening problem, the principal can behave as best as an "extreme" agent, in a sense that equilibria are restricted to the best pooling or fully separating strategies, leaving no room for semi-separating equilibria. This is due to the absence of multi-dimensional instruments such as a menu of contracts, as persuasion allows only for a single instrument (the information structure) when the state of the nature is also uni-dimensional, as is standard in most the literature.

The results on this paper was achieved using several simplifications like the restriction of the state of nature and types space to a binary case, the specific payoff functions and the assumption that discount rate were the same for senders and receivers. Although we believe none of these simplifications are crucial for the main results exposed above, extensions to the model seems to be a promising avenue of future research.

## 1.7 Appendix: Proof of the propositions and lemmas

### 1.7.1 Proof of Proposition 1.4.1

First note that the presence of different worker types do not alter the main results of (KAMENICA; GENTZKOW, 2011) analysis, in particular the revelation principle proved in the context of Bayesian Persuasion. Hence it is without loss of generality to focus on information structures in which $S = \{0, 1\}$. Let $\mu_2$ denotes worker posterior belief that $\omega = 1$ and $\mu_\pi(\omega|s)$ denote posterior beliefs under signal $\pi$. Standard math shows that type $H$ workers are indifferent between actions when $\mu_2 = \frac{1}{2} = \mu_{\pi(s|\omega;H)}(1|1)$ while type $L$ workers are indifferent when $\mu_2 = \frac{1+x}{2+x} = \mu_{\pi(s|\omega;L)}(1|1)$. Thus, if $\mu_2 \geq \frac{1+x}{2+x}$ both worker types chooses $a = 1$. Hence any signal $\pi$ that induces $\mu_\Pi(1|1) > \frac{1+x}{2+x}$ cannot be optimal as Bayesian plausibility is less restrictive for beliefs closer to the prior. Similarly, if $\frac{1}{2} \leq \mu_2 < \frac{1+x}{2+x}$ only type $H$ workers will choose $a = 1$, so, by the same argument, any signal $\pi$ with $\frac{1}{2} < \mu_\Pi(1|1) < \frac{1+x}{2+x}$ is also not optimal. Obviously any signal that induces beliefs lower that $\frac{1}{2}$ cannot be optimal as in this case the default action for both workers is 0. Hence the optimal signal is either $\pi(s|\omega; H)$ or $\pi(s|\omega; L)$. To select equilibrium we use the assumption of "breaking ties if favor of the principal" that is standard in the Bayesian Persuasion literature[7]. Under this assumption, it is easy to see that there is no equilibrium under mixed strategies as if any worker decide to use a mixed strategy and play $a = 1$ with probability $\gamma$ when indifferent, the best option for the principal would be to set $\gamma = 1$. Hence, the only equilibria that is left are the pure strategy ones. Standard math shows that the expected utility for the employer of $\pi(s|\omega; H)$ is $2\mu_1\phi_1$ while expected utility for $\pi(s|\omega; L)$ is $\frac{2+x}{1+x}\mu_1$

### 1.7.2 Proof of Lemma 1.5.1

As type H workers need lower posteriors to be induced to play $a = 1$ it is straightforward to check that the only separating signal that complies with the agents incentive constraints is the one that induces type $H$ to play $a = 1$ and type $L$ to play $a = 0$ for some $s \in S$. Label this signal realization as $s = 1$. In any separating equilibrium, the utility of $a = 1$ conditional on a posterior $\mu_2$ for type $H$ workers is given by $\mu_2 + \delta y$ and the utility of $a = 0$ for the same worker is given by $(1 - \mu_2)$. Hence, in order to induce type $H$ players to play $a = 1$ we need $\mu_2 + \delta y \geq (1 - \mu_2) \iff \mu_2 \geq \frac{1-\delta y}{2}$. Type $L$ players need to play $a = 0$ when type $H$ players play $a = 1$ for this to be a separating equilibrium. A similar analysis as the one just made shows that type $L$ players opts to play $a = 0$ whenever $\mu_2 \leq \frac{1+x-\delta y}{2+x}$. Hence, in order to get a separating equilibrium we need

$$\frac{1 + x - \delta y}{2 + x} < \frac{1 - \delta y}{2} \iff \delta y < 1$$

[7] Alternatively, we could add a small $\varepsilon > 0$ to the expected posterior to break workers indifference

As is never optimal to lie when $\omega = 1$ the optimal signal structure will be the one with the frequency of lies that promote $\mu_2 = \frac{1-\delta y}{2}$. By the Bayes Theorem then, if we let $z$ be such that $\frac{1-\delta y}{2} = \frac{\mu_1}{\mu_1 + z(1-\mu_1)}$ we get that the optimal separating signal is given by:

$$\pi_S^*(s|\omega) := \begin{cases} 1 \text{ if } s = \omega = 1 \\ 0 \text{ if } s = 0 \text{ and } \omega = 1 \\ z \text{ if } s = 1 \text{ and } \omega = 0 \\ 1 - z \text{ if } s = \omega = 0 \end{cases}$$

Using standard math we can see that $z = \frac{\mu_1}{1-\mu_1}\frac{1+\delta y}{1-\delta y}$ and that $z \in [0, 1]$ as long as $\mu_1 \leq \frac{1-\delta y}{2}$ (for otherwize the problem is trivial). Let $\lambda := \text{Max}\{\phi_1, 1 - \phi_1\}$. The optimal separating equilibrium yields to the employer expected utility of

$$\frac{2\mu_1\phi_1}{1-\delta y} + \delta y \left[\frac{2\mu_1}{1-\delta y} + \lambda\left(1 - \frac{2\mu_1}{1-\delta y}\right)\right]$$

### 1.7.3 Proof of Lemma 1.5.2

First, note that is never optimal to have a signal that always induces $a = 0$ from both types, as this is dominated by truth telling. Hence, we need to search for signals that induce both worker types to play $a = 1$ in at least some signal realization, which we once again will label as $s = 1$. As type $L$ workers require stronger beliefs to be persuaded, the optimal pooling signal will be the one that leaves them indifferent between actions. However, this indifference is contingent on worker's expectation of their last period payoffs. As we are dealing with a pooling equilibrium the employer is unable to learn anything about worker's types, so $\phi_1 = \phi_2$. Hence, employer's choice in the last period is trivial: $a_S = H \iff \phi_1 \geq \frac{1}{2}$. If $\phi_1 < \frac{1}{2}$ then we are effectively in the myopic sender case, so by the previous analysis the optimal signal is given by $\pi(s|\omega; L)$. If $\phi_1 \geq \frac{1}{2}$ a similar analysis to the one developed in case 1 shows that $L$ workers are indifferent between actions if and only if $\mu_T = \frac{1+x-\delta y}{2+x}$ and that the optimal signal to induce those beliefs is given by:

$$\pi(s|\omega; P) := \begin{cases} 1 \text{ if } s = \omega = 1 \\ 0 \text{ if } s = 0 \text{ and } \omega = 1 \\ \frac{\mu_1}{1-\mu_1}\frac{1+\delta y}{1+x-\delta y} \text{ if } s = 1 \text{ and } \omega = 0 \\ 1 - \frac{\mu_1}{1-\mu_1}\frac{1+\delta y}{1+x-\delta y} \text{ if } s = \omega = 0 \end{cases}$$

Whenever $\phi_1 \leq \frac{1}{2}$ the optimal pooling equilibrium yields to the employer expected utility of $\frac{(2+x)\mu_1}{1+x} + \delta y\lambda$ and when $\phi_1 > \frac{1}{2}$ it yields $\frac{(2+x)\mu_1}{1+x-\delta y} + \delta y\lambda$ if $\phi_1 \geq \frac{1}{2}$

# 2 Baggage fees in airlines: is this a good idea?

## 2.1 Introduction

In the last decade there was a growing movement of airline companies charging a separate amount for their ancillary services, such as baggage travel. This began in 2008, when U.S. airlines spearheaded the efforts, but quickly become an industrial trend worldwide. This phenomena is of clear interest to the social planner as not only it created a huge market - in 2008 baggage fees revenues amounted to U$ 0.5 bi in U.S. companies, while by the end of 2016 this number had quickly grown to more than U$ 4.0 bi[1] - but now governs more than 10% of the revenues of the majority of large airline carriers[2]. Understandably, this change generated political attention and cries for regulation: in 2011 U.S. Senator Mary Landreau attempted to banish baggage fees (HALSEY, 2011) and recently there's been a discussion about forcing airlines to disclose baggage fees to consumers at the point of sale political2 due to concerns about baggage fees salience. However, there is still limited evidence on the welfare effect of this change.

We construct a model to shed light on the issue. In it, consumers optimally choose whether to consume two goods - passenger travel and baggage travel - under the restriction that the second good is only available after the purchase of the first. The model is solved for two distinct scenarios: in the first, the monopolist firm is obliged to set the price of baggage travel to zero, and in the second it may optimally choose its prices an in a two-part tariff model. The results shows that allowing for two prices is not always welfare enhancing: it depends on the marginal cost of luggage travel. The intuition is simple: we have a trade-off between over-consumption and under-consumption of baggages in the different scenarios. When the airline can only charge a single price, some consumers with low valuation of baggages might opt to embark their luggage anyway (as the price for doing so is zero) leading to over-consumption. With two prices this do not happen, but due to the firm's monopolist power, it will set its prices in such a way that some consumers with baggage valuation higher than its marginal cost will still opt to withdraw from the market.

After this initial analysis, we extend the model to the duopoly case, and document the intuitive result that firms' market power is also important, as without it allowing for

[1] `https://www.bts.gov/content/baggage-fees-airline-2017`

[2] `http://www.amadeus.com/web/amadeus/en_US-US/Amadeus-Home/News-and-events/News/053111_Ancillary-Revenue-reported-by-airlines-grew-to-EUR15.11-b/1259071352352-Page-AMAD_DetailPpal?assetid=1319579863701&assettype=PressRelease_C`

two distinct prices is always welfare enhancing. To finish discussion, we study the issue of allocation. We show that firms are obviously at least neutral towards the change from a single to two distinct prices, but the effect on consumer utility are uncertain. In our main model consumers are only differentiated by their preferences over baggage travel, and that makes them at best indifferent towards this change as the monopolist firm is able to fully extract consumer surplus on the passenger market. However, once the model is extended to allow for heterogeneous preferences over both goods, we show that consumers might be also favorable to the change.

The literature on the impact of baggage fees on social welfare is still limited. (ALLON; BASSAMBOO; LARIVIERE, 2011) studies the problem using a different modelling approach. In their setting, consumers choose not whether to buy baggage transportation but whether to exert effort to avoid having to travel with a baggage. They conclude that moving from one to two prices for airlines services is unequivocally good for society, even in a monopoly setting. This result contrasts our main finding, that is, that the change from a single to two distinct price is not always social optimal. The difference is results is due to different modelling techniques: in their setting, we cannot have the possibility of under-consumption of baggages as those are undesired by both consumers and the firms. As such, the introduction of the explicit baggage fee in their setting leads naturally to the optimal effort level by the consumer, essentially solving the problem. In ours, this is not the case, as consumers derive utility directly from baggage travel. In this sense our model contributes to the discussion by analyzing the case in which consumers effectively want to embark their baggages.

Research generally agrees that explicit baggage fees are positive for firm metrics. We have evidence that baggage fees has a much lower elasticity than regular fares, which tends to increase firms market power (SCOTTI; DRESNER, 2015). They also tend to increase airlines stock price (BARONE; HENRICKSON; VOY, 2012) and the likelihood of on-time departure performance (NICOLAE et al., 2017). Moreover, contrafactual exercises on potential regulation of those fees indicates that banning baggage fees would have little to no effect in total travel prices (AGARWAL et al., 2014). The main criticism to allowing for dual prices comes from salience issues, as there is evidence that the opacity of baggage prices are relevant for revenues in this market (BRADLEY; FELDMAN, 2016) and that this feature might hurt consumers utility in behavioral ways (COY; CHIANG, 2012). Our model connects to this brand of research by showing that those improvements on firm's metrics might not be good for society even without bounded rationality and salience issues.

In methodological terms, as our model can be understood as a two-part tariff model in which consumers are restricted to only being allowed to purchase one of the goods after the consumption of the other, it connects to the larger literature on price discrimination

and bundling. Two good examples of such are (ARMSTRONG; VICKERS, 2010) and (ARMSTRONG, 2006). It also connects on the literature of multi-dimensional screening, as in our final extension we deal with two distinct sources of consumer private information. A good review on the subject can be found in (ARMSTRONG; ROCHET, 1999).

This rest of the paper is organized as follows. Section 2 describes our main model. Section 3 briefly discusses some interesting extensions and section 4 provides a conclusion, with an eye in potential applications of the model to different problems.

## 2.2 The Model

### 2.2.1 Setup

Consider an economy with 2 goods: airplane passenger travel and luggage travel. The goods are assumed to be offered in discrete quantities in which each consumer may opt to consume either one unit or no units of the good (i.e. they choose to travel or not). There is a monopolist firm that offers both goods and a continuum of measure one of consumers. Let $q_1$ denote consumption of airplane travel and $q_2$ denote consumption of luggage travel.

Each consumer $i$ have to choose between three travel options. He can either choose to travel heavy (i.e: travel with a dispatched baggage), travel light (i.e. to travel without any dispatched baggage) or not to travel at all. If he decides to travel, he gets utility $\overline{u} > 0$, which is assumed to be constant for all consumers by simplicity[3]. If he decides to embark his baggage, he gets utility $\delta_i$, which is a random variable draw from an uniform$[0,1]$. The utility of not travelling is normalized to 0. Note that this structures entails that consumers can purchase $q_2$ only if they also decided to purchase $q_1$

The firm have two distinct marginal costs: $c_1$ is the marginal cost of transporting passengers and $c_2$ the marginal cost of transporting baggage. We assume that both $c_1$ and $c_2$ are greater than 0 and that fixed costs are irrelevant.

There are two distinct prices: one for the consumption of $q_1$ (called $p_1$) and one for consumption of $q_2$ ($p_2$). Still, we analyze two distinct cases. In the first case, named "single price case", firms are restricted to set $p_2 = 0$. In the second case (the "dual price case") they may set both prices as they see fit.

In all cases, prices and consumer utilities are assumed to be measured in the same unit. Hence, the decision to consume one unit of passenger travel adds $\overline{u}$ utis to the consumer, but subtract $p_1$ utis.

[3] This will be relaxed in the extensions provided in the next session

### 2.2.2 Demand

Consumer demand comes from standard utility maximization. In the single price case (i.e. $p_2 = 0$) we have the following demand functions:

$$q_{1i}(p_1) := 1_{\{\overline{u} \geq p_1\} \vee \{\overline{u}+\delta_i \geq p_1\}} \quad \text{and} \quad q_{2i}(p_1) := 1_{\{\overline{u}+\delta_i \geq p_1\}} \tag{2.1}$$

Equation 2.1 tells us that consumers will opt to travel (and to embark their luggage) when the the utility of doing so is at least as high as the costs associated to it. For the case of the first good, this may happen by two reasons. Either travelling is good enough to be consumed on its own (that is, $\overline{u} \geq p_1$) or it might be a cost worth paying to travel heavy (which implies that, $\overline{u} + \delta_i \geq p_1$). Note that in this setting, due to the presence of a single price for both goods, the consumer will never prefer to travel light over travelling heavy[4]. Hence, we may simplify demand to $q_{1i}(p_1) = q_{2i}(p_1) := 1_{\{\overline{u}+\delta_i \geq p_1\}}$

In our second case, demand will be given by:

$$q_{1i}(p_1, p_2) := 1_{\{\overline{u} \geq p_1\} \vee \{\overline{u}+\delta_1 \geq p_1+p_2\}} \quad \text{and} \quad q_{2i}(p_1, p_2) := 1_{\{\overline{u}+\delta_i \geq p_1+p_2\} \wedge \{\delta_i > p_2\}} \tag{2.2}$$

Now, because of the two price structure, we cannot simplify demand for the first good like we did before, as the restrictions are not nested anymore. Moreover, demand for the second good now requires two conditions. Passengers will only opt to travel heavy only when this option is better than his other two options: not travelling (which requires $\overline{u} + \delta_i \geq p_1 + p_2$) and travelling light (which requires $\delta_i \geq p_2$)

### 2.2.3 First case: Single price

In this scenario, the firm is restricted to set $p_2 = 0$. Hence, it will chose $p_1$ to solve the following problem:

$$\text{Max } \Pi_1(p_1) := \mathbb{E}\{(p_1 - c_1)q_{1i}(p) - c_2 q_{2i}(p)\} \tag{2.3}$$

The subscript 1 on the above equation denotes that this is the profit function of our first case. Proposition 2.2.1 stated below characterize the solution to this case.

**Proposition 2.2.1** ***Single Price Characterization*** *The single price problem is characterized by the following function:*

$$p_1^* = \begin{cases} \overline{u} \ \textit{if} \ c_1 + c_2 \geq \overline{u} - 1 \\ \dfrac{1 + \overline{u} + c_1 + c_2}{2} \ \textit{if} \ \overline{u} - 1 \leq c_1 + c_2 \leq \overline{u} + 1 \\ \infty \ \textit{if} \ c_1 + c_2 \geq \overline{u} + 1 \end{cases}$$

[4] If $\overline{u} \geq p_1$ then obviously $\overline{u} + \delta_i \geq p_1$

Table 2.1 – Single Price Setting Resume

| | **X** | **Y** | **Z** |
|---|---|---|---|
| $p_1^*$ | $\overline{u}$ | $\left(\dfrac{1+\overline{u}+c_1+c_2}{2}\right)$ | $\overline{u}+1$ |
| $\Pi_1$ | $\overline{u}-c_1-c_2$ | $\left(\dfrac{1+\overline{u}-c_1-c_2}{2}\right)^2$ | 0 |
| $\Pr(\lambda_i=1)$ | 0 | 0 | 0 |
| $\Pr(\lambda_i=2)$ | 1 | $\left(\dfrac{1+\overline{u}-c_1-c_2}{2}\right)$ | 0 |

Proposition 2.2.1 tells us that the single price case divides the plane $(c_1, c_2)$ into three different regions. In the region defined by $c_1+c_2 \leq \overline{u}-1$, which we name **X**, costs are low enough so that the firm finds it optimal to bring all consumers to the market, as the potential cost of losing a fraction of consumers is higher than the benefit of an increased price for the fraction that remains. In order to do that, the firm sets $p_1^* = \overline{u}$, eliminating consumer surplus for the consumption of $q_1$. In this case, firms earn $\Pi_1(\overline{u}) = \overline{u} - c_1 - c_2$ and all consumers opt to travel heavy.

In the region $c_1 + c_2 \geq \overline{u} - 1$, which we define as **Z**, the opposite happens: costs are so high that the firms find it optimal to shut down the market entirely. In this case, profits are obviously null and consumers will always choose not to travel.

In between those two extremes, which we name **Y**, the firm find it optimal to only attend consumers that have a sufficiently high value of $\delta_i$. To see this point more clearly, let $\lambda_i$ be a random variable that assumes value 0 if consumer $i$ opts to not travel, 1 if he decides to travel light and 2 if he decides to travel heavy. We then have that in region **Y**:

$$\Pr(\lambda_i = 2) := \Pr(\delta_i \geq p_1^* - \overline{u}) = \left(\frac{1+\overline{u}-c_1-c_2}{2}\right) \tag{2.4}$$

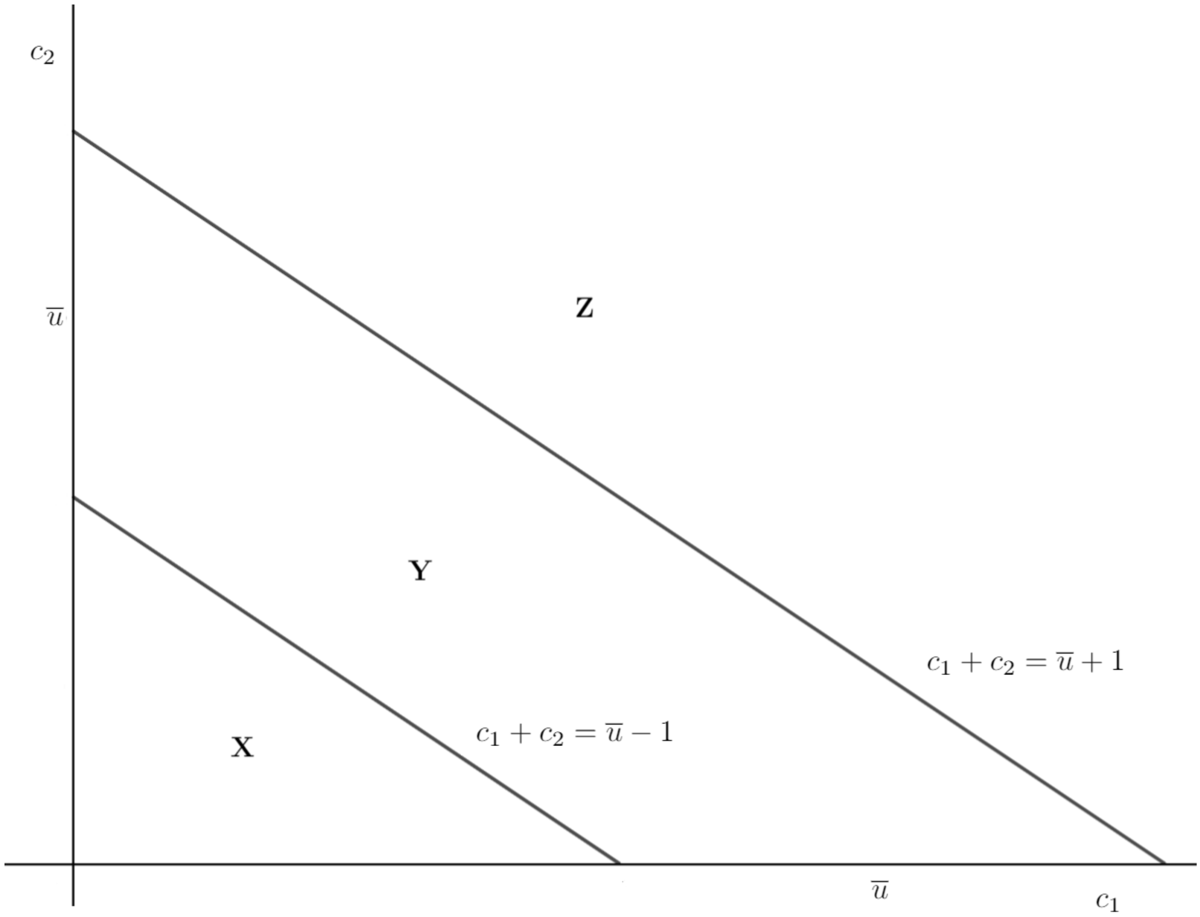
As with a single price the cost of travelling heavy and travelling light are the same, consumers will never opt for the latter option in this case, so $\Pr(\lambda_i = 1) = 0$ and $\Pr(\lambda_i = 0) = 1 - \Pr(\lambda_i = 2)$.

To finish our analysis, profits in this intermediate case will me given by

$$\Pi_1(p_1^*) = \left(\frac{1+\overline{u}-c_1-c_2}{2}\right)^2 \tag{2.5}$$

Table 2.1 summarizes our findings for the single price case and figure 1.1 exemplify the regions for the case $\overline{u} = 3$.

Figure 2.1 – Optimal Choice for the single price scenario with $\overline{u} = 3$




### 2.2.4 Second case: Two distinct prices

In this scenario, firms are free to set $p_2$ at its optimal value, so the firm problem turns into:

$$\text{Max } \Pi_2(p_1, p_2) := \mathbb{E}\{(p_1 - c_1)q_{1i}(p) + (p_2 - c_2)q_{2i}(p)\} \tag{2.6}$$

The subscript 2 on the above equation denotes that this is the profit function of our second case. Proposition 2.2.2 stated below characterize the solution to this case.

**Proposition 2.2.2** ***Two-Price Characterization*** *The two-price problem is characterized by the following function:*

$$(p_1^*, p_2^*) = \begin{cases} \left(\overline{u}, \dfrac{1+c_2}{2}\right) \; \textit{if } c_1 \leq \overline{u} \textit{ and } c_2 \leq 1 \\ (\overline{u}, \infty) \; \textit{if } c_1 \leq \overline{u} \textit{ and } c_2 > 1 \\ \left(\dfrac{1+\overline{u}+c_1+c_2}{2}, 0\right) \; \textit{if } c_1 > \overline{u} \textit{ and } c_1 + c_2 \leq \overline{u} + 1 \\ (\infty, \infty) \; \textit{if } c_1 > \overline{u} \textit{ and } c_1 + c_2 > \overline{u} + 1 \end{cases}$$

Proposition 2.2.2 tells us that the two price case divides the plane $(c_1, c_2)$ into four different regions. We will name those regions after the first letters of the alphabet, to differentiate from the letters used in the single price case.

If $c_1 < \overline{u}$ then firm behavior is effectively different from the single price case. In this case, the firm knows that passenger travel is a profitable endeavor by itself and can set $p_1$ to the value that maximizes profits in that activity, namely $\overline{u}$. Then, the firm evaluates whether of not it wants to price baggage travel in a way that attracts consumers. If $c_2 \leq 1$ it finds profitable to do so, and sets $p_2 = \frac{1+c_2}{2}$. In that case, it earns profits of $\Pi_2(\overline{u}, \frac{1+c_2}{2}) = (\overline{u} - c_1) + [\frac{(1-c_2)}{2}]^2$, a fraction $1 - \frac{(1-c_2)}{2}$ of passengers travel light and $\frac{1-c_2}{2}$ of passenger travel heavy. No passenger opts not to travel. We name this region as **A**. On the other hand, if $c_2 > 1$ then it is more profitable to shut down the baggage market. In that case, firm profit is given by $\Pi_2(\overline{u}, \infty) = (\overline{u} - c_1)$ and all consumers travel light. We name this region **B**. Note that in the single price case the firm could not act in this way because it lacked capacity to separate its consumers in the two markets (i.e. it was impossible for the firm to force a consumer to travel light in that environment).

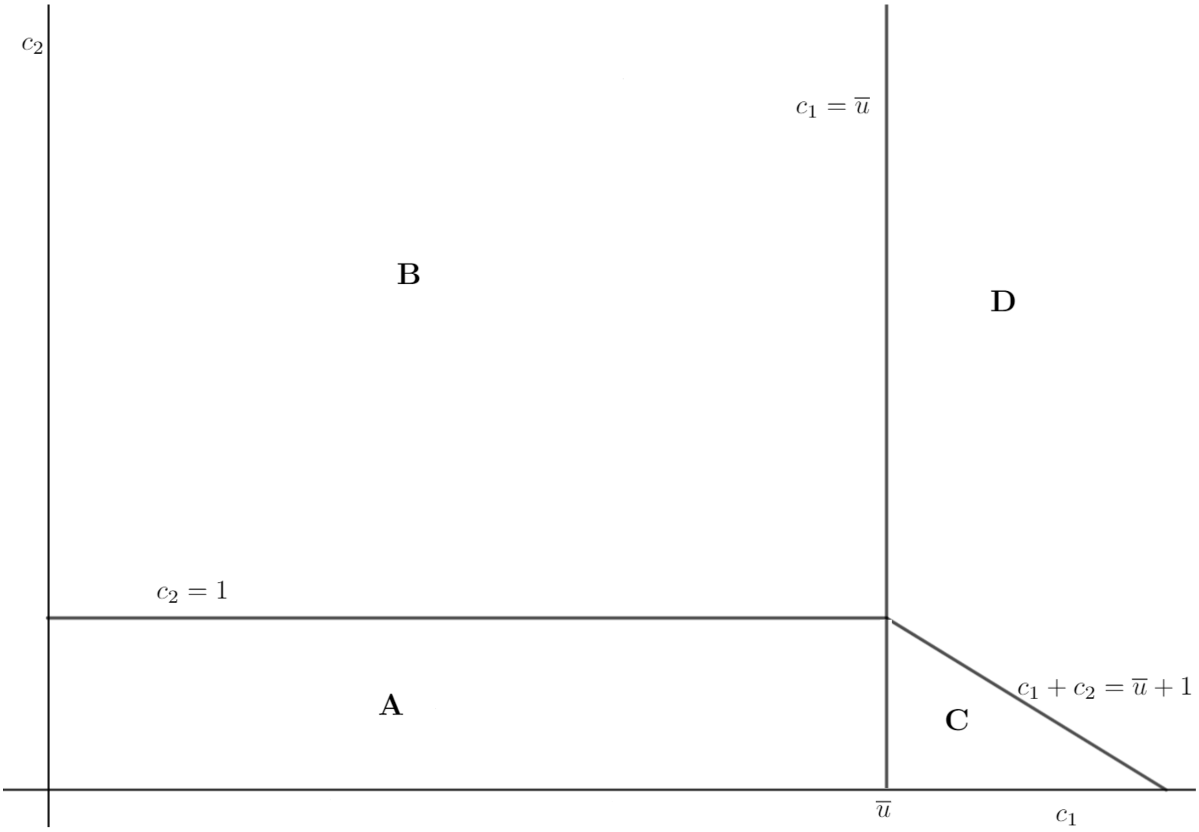
If $c_1 \geq \overline{u}$ the firm choices in the two price case is effectively the same as in the single price case. This is so because in this case passenger travel is never profitable enough to be sold on its own, so the firms is only interested in providing the consumption of baggage travel. As the model forces consumers to buy $q_1$ in order to buy $q_2$, the firm must offer $q_1$ at a loss to some consumers in order to earn profits from the sale of $q_2$. This strips away from the firm the possibility to differentiate light travellers from heavy travellers, making all its consumers heavy travellers, just as in the single price case. The solution is then equal to that case, that is, firms differentiate between consumers if $c_1 + c_2 \leq \overline{u} + 1$ and shot down the market if $c_1 + c_2 \geq \overline{u} + 1$. We name the first region as **C** and the second as **D**. Both profits and passenger behavior are obviously equal to the respective single price case in those scenarios.

Table 2.2 summarizes our findings for the two price case and figure 2.2 exemplify the regions for the case $\overline{u} = 3$.

Table 2.2 – Two Price Setting Resume

| | **A** | **B** | **C** | **D** |
|---|---|---|---|---|
| $p_1^*, p_2^*$ | $(\overline{u}, (1+c_2)/2)$ | $(\overline{u}, 1)$ | $((1+\overline{u}+c_1+c_2)/2, 0)$ | $(\overline{u}, 1)$ |
| $\Pi_2$ | $(\overline{u}-c_1)+[(1-c_2)/2]^2$ | $\overline{u}-c_1$ | $[(1+\overline{u}-c_1-c_2)/2]^2$ | 0 |
| $\Pr(\lambda_i=1)$ | $1-(1-c_2)/2$ | 1 | 0 | 0 |
| $\Pr(\lambda_i=2)$ | $(1-c_2)/2$ | 0 | $(1+\overline{u}-c_1-c_2)/2$ | 0 |

Figure 2.2 – Optimal Choice for the two distinct prices scenario with $\overline{u}=3$




## 2.2.5 Efficiency and Welfare Analysis

Both scenarios are inefficient in different contexts. The single price case suffers from over-consumption issues, that is, consumers with private benefit for luggage travel inferior to its marginal costs (i.e: $\delta_i < c_2$) might still opt to consume[5] as the firm cannot prevent the consumption of $q_2$ without also dampening consumption of $q_1$ with only a single price for both goods. This is obviously not an issue in the two-price analysis. However the two price scenario has a different source of inefficiency. With two prices, the firm can better separate its consumers between those that are profitable to serve in both goods and those that are not. By doing this, the effective markup of the firm increases, and in doing so we also increase its monopoly inefficiency. Thus, the two price case suffers from under-

[5] It is easy to see that this happens to at least some consumers in regions **X** and **Y**, as the choice to consume $q_2$ in this case does not depend solely on $p_2$ and $\delta_i$.

consumption issues, that is, consumers with private benefit for luggage travel superior to its marginal costs might be priced away from the market due to the monopoly power of the firm. This leads to a natural question: which scenario is the best for society in each region of the plane $(c_1, c_2)$? Naturally, firms should be at least indifferent when moving from a single price to dual price scenario as it can always set $p_2 = 0$ optimally if needed, but what about consumers and overall welfare? This is the question we tackle in this session.

To begin our analysis, let us define our aggregate welfare function and a bit of notation. Let $k \in \{1, 2\}$ denote the different price setting that we are dealing, with $k = 1$ the single price case and $k = 2$ for the two price case. Let $r \in \{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\} \cap \{\mathbf{X}, \mathbf{Y}, \mathbf{Z}\} := R_2 \cap R_1$ denote the region on the plane $(c_1, c_2)$ that we are analyzing. Finally, let $U_{kri}$ denote consumer $i$ utility in region $r$ of case $k$.

Using the previous section results and a bit of algebra, we have that aggregate utility in each case and region are given by:

$$\int_0^1 U_{1ri} \mathrm{d}i = \begin{cases} \dfrac{1}{2} & \text{if } r = \mathbf{X}, \\ \dfrac{(1 + \overline{u} - c_1 - c_2)^2}{8} & \text{if } r = \mathbf{Y}, \\ 0 & \text{if } r = \mathbf{Z}. \end{cases} \tag{2.7}$$

$$\int_0^1 U_{2ri} \mathrm{d}i = \begin{cases} \dfrac{(1 - c_2)^2}{8} & \text{if } r = \mathbf{A}, \\ 0 & \text{if } r = \mathbf{B}, \\ \dfrac{(1 + \overline{u} - c_1 - c_2)^2}{8} & \text{if } r = \mathbf{C}, \\ 0 & \text{if } r = \mathbf{D}, \end{cases} \tag{2.8}$$

Let $\Pi_{kr}$ denote aggregate profits for region $r$ and case $k$. As we have no reason to differentiate consumers, we define the aggregate welfare function is a natural way as

$$W_{kr} := \Pi_{kr} + \int_0^1 U_{kri} \mathrm{d}i \tag{2.9}$$

Define the correspondence $g : R_1 \to R_2$ as $g(\mathbf{X}) \mapsto \{\mathbf{A}, \mathbf{B}\}$, $g(\mathbf{Y}) \mapsto \{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$ and $g(\mathbf{Z}) \mapsto \{\mathbf{B}, \mathbf{D}\}$. We interpret $g$ as the correspondence that connects each region in the single price case to a possible counterpart in the two price case[6]. Now, define the map $f : x \to g(x)$. Using this notation, we are interested in the values of $\Delta W_{r,f(r)} := W_{2,f(r)} - W_{1,r}$ and $\Delta U_{r,f(r)} := \int_0^1 U_{2f(r)i} - U_{1ri} \mathrm{d}i$ that we interpret as variations in, respectively, aggregate

[6] For example, when we are in region $\mathbf{Z}$ of the single price case, we can be in two different regions of the two price case, region $\mathbf{B}$ if $c_1 \leq \overline{u}$ or region $\mathbf{D}$ otherwise, thus $g(\mathbf{Z}) \mapsto \{\mathbf{B}, \mathbf{D}\}$

welfare and aggregate consumer utility from moving to the dual price case from the single price case.

The following proposition, which is our main result, shows that the change from a single price to two prices is not always good for aggregate welfare as defined above. It also shows that the consumers are never better off with this change

**Proposition 2.2.3** ***Welfare Analysis is Uncertain*** *If* $r = \mathbf{X}$*, then* $\Delta W_{r,f(r)} < 0$ *if and only if* $c_2 < 1/3$*. If* $r = \mathbf{Y}$ *then* $\Delta W_{r,f(r)} < 0$ *if and only if* $c_2 < (\overline{u} - c_1)/2 - 1/3$*. Finally, if* $r = \mathbf{Z}$ *then* $\Delta W_{r,f(r)} \geq 0$*. Furthermore* $\Delta U_{r,f(r)} \leq 0$ *in all cases.*

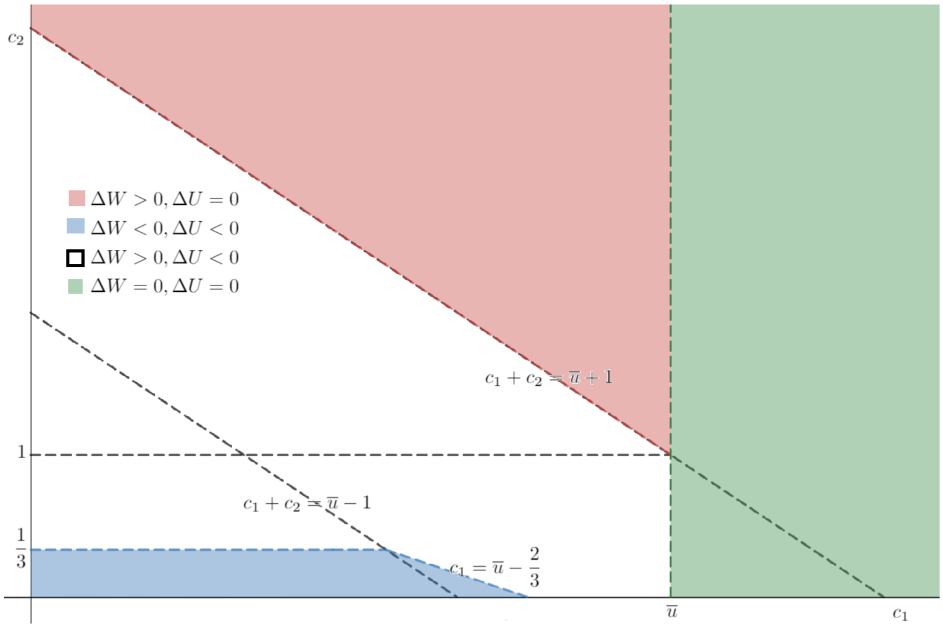
As regions $\mathbf{X}$ and $\mathbf{Y}$ are the ones in which we have an active market in the single price case, a corollary of proposition 2.2.3 is that whenever we have an active market in the single price case such that $c_1 < \overline{u} - 2/3$[7], then there is a positive value $c_2^*$ such that, if $c_2 \leq c_2^*$, then the change of a single price to a dual price case is negative to overall welfare. This is the case when the inefficiencies generated by the under-consumption of baggages in the dual price case are larger than the ones generated by the over-consumption of the single price case. Therefore, the value of $c_2$ is an important variable for a social planner to consider when promoting this change.

Proposition 2.2.3 also shows that consumers are never better off by the change from the single to the dual price case. There is however a region in which the move from a single to two prices in our model generates a Pareto-improvement. This is the region $\mathbf{Z} \cap \mathbf{B}$, as in this region we would have no market in the single price case and a full (and profitable) market for passengers in the dual price case. Consumers are not better off with this change because we simplified away the demand for passengers using $\overline{u}$ to determine the utility of passenger travel, so our monopolist firm is able to fully extract consumer surplus in this case. This suggests that if we relax either the assumption of the single value of $\overline{u}$ as the utility for all consumers of the monopoly of the firm then it would be possible for consumers to also be better off by the change in airline pricing depending on the parameters. This is indeed the case, as will be shown in the next session.

The chart below summarizes our conclusions. The blue area denotes the region in which aggregate welfare is reduced when we move from a single to the dual price scenario, that is, the region in which the under-consumption problem of the dual price case is more relevant than the over-consumption of the single price case. In the white region this change is positive for aggregate welfare, but negative to consumers. In red we have the Pareto-improvement region and finally in green the region where the change provokes no economic effect.

[7] To understand this inequality note that the equation $c_1 = \overline{u} - 2/3$ determines the point in which the value of $c_2$ that generates $\Delta W_{r,f(r)} < 0$ when $r = \mathbf{Y}$ is lower than 0

Figure 2.3 – Welfare and utility comparison in all regions




## 2.3 Extensions

In this section we attempt to briefly extent our main model in two ways. In the first we solve the model under a duopoly to understand the effect of market power in the analysis. In the second we differentiate consumer preference over passenger travel into two types, in order to clarify the allocative issues raised by the model in the previous analysis. The results shows that market power is crucial in the analysis, as in a perfect competition environment the two price case always generates at least the same aggregate welfare as the single price case. Moreover, consumers might benefit from this change when we allow for different valuations of the passenger travel, as in this case the firm will not be able to fully extract the surplus generate by the change.

### 2.3.1 Duopoly

In this section we assume that two firms exists in our previous model. The firms have equal costs and compete through prices to provide both goods. That simple change have profound effects in our model results. By allowing the presence of an extra firm, our model turns into a two part tariff competition in a homogeneous market duopoly, which is thoroughly analyzed by (KRINA; NIKOLAOS, 2015). In this context we can prove the following proposition

**Proposition 2.3.1** ***Bertrand Duopoly Characterization*** *Assume that two firms compete though prices into $q_1$ and $q_2$ with the same costs, given by $c_1$ and $c_2$. Then the single price case is characterized by $p_1^1 = p_1^2 = (c_1 + c_2)$ and the dual price case by*

$(p_1^1, p_2^1) = (p_1^2, p_2^2) = (c_1, c_2)$ *where* $p_i^j$ *denotes the optimal price for firm* $j$ *in good* $i$*. Moreover, both firms make 0 profits in both cases.*

*Proof:* In the single price case we have all the elements of the standard Bertrand model, which is well known to have its equilibrium characterized by the price being equal the marginal cost and profits begin null. As in this case all consumers that opt to purchase $q_1$ will also trivially want to purchase $q_2$ the result follows. In the dual price case the proof is a simple application of the first proposition of (KRINA; NIKOLAOS, 2015) .

Using proposition 2.3.1 it is straightforward to conclude that in the symmetric duopoly case both $\Delta W_{r,f(r)}$ and $\Delta U_{r,f(r)}$ are strictly positive whenever $\overline{u} > c_1$ and $c_2 > 0$ and neutral otherwise for all possible $r$. The intuition is that in this case we have no under-consumption issues, as the firms have no markup. As the problem of over-consumption in the single price case is still intact, the change from one to two distinct prices is at least neutral to overall welfare and consumer utility. Therefore, we conclude that a measure of market power is critical for the result of proposition 2.2.3.

### 2.3.2 Heterogeneous preferences over passenger travel

In this section we relax the assumption that every consumers gets the same utility value $\overline{u}$ for the purchase of $q_1$. Specifically, we assume that consumers are now of two distinct types: $L$ and $H$. While type $L$ consumers gets utility $u_L$ for the purchase of $q_1$, type $H$ consumers get $u_H$, with $0 < u_L < u_H$. Moreover, we assume that the probability of consumer $i$ being of type $L$ is given by $\gamma$ and that $u_i$ is independent of $\delta_i$. This effectively turns our model into a multidimensional screening one, as consumers have now two dimensions of private information. Those problems are notoriously hard to solve and have few conclusive results (see (ARMSTRONG; ROCHET, 1999) for a good review). As such, we do not attempt to fully present the solution, but rather to show its properties. The following proposition categorize the firm's optimal behavioral in the single price world

**Proposition 2.3.2** ***Single-Price Heterogeneous Consumers Characterization*** *Assume that the utility of passenger travel has two possible values* $u_L$ *and* $u_H$*, with* $0 < u_L < u_H$, $\Pr[u_i = u_L] = \gamma$ *and* $u_i\delta_i$*. Then, we can reduce the firms price choice* $p_1^*$ *of the single price case to one of the options below:*

*1)* $p_1^* = u_L$*, with* $\Pi_1(p_1^*) = u_L - c_1 - c_2$
*2)* $p_1^* = \dfrac{1+\gamma(u_L+c_1+c_2)}{2\gamma}$*, with* $\Pi_1(p_1^*) = \dfrac{1}{\gamma}\left(\dfrac{1+\gamma(u_L-c_1-c_2)}{2}\right)^2$*. This option is only available under the restrictions* $p_1^* \leq u_H$ *and* $u_L < p_1^* < 1 + u_L$
*3)* $p_1^* = \dfrac{1+\alpha_1+c_1+c_2}{2}$*, with* $\Pi_1(p_1^*) = \left(\dfrac{1+\alpha_1-c_1-c_2}{2}\right)^2$ *and* $\alpha_1 := \gamma u_L + (1-\gamma)u_H$*.*

*This option is only available under the restrictions* $u_H < p_1^* < u_H+1$ *and* $u_L < p_1^* < 1+u_L$
*4)* $p_1^* = u_H$*, with* $\Pi_1(p_1^*) = (1-\gamma)(u_H - c_1 - c_2)$*. This option is only available under the restriction* $p_1^* \geq 1 + u_L$
*5)* $p_1^* = \dfrac{1+u_H+c_1+c_2}{2}$*, with* $\Pi_1(p_1^*) = (1-\gamma)\left(\dfrac{1+u_H-c_1-c_2}{2}\right)^2$*. This option is only available under the restrictions* $u_H < p_1^* < 1+u_H$ *and* $p_1^* \geq 1+u_L$
*6)* $p_1^* = \infty$*, with* $\Pi_1(p_1^*) = 0$

Proposition 2.3.2 tells us that firms optimal policy in the single price case now depends on 4 parameters: $(c_1+c_2), \gamma, u_L$ and $u_H$. As this case requires $p_2 = 0$, the firm can still only differentiate consumers that will choose not to travel from those that will choose to travel heavy. The only difference is that as we have two types of consumers now, the firm has more effective options in order to do this split. If option 1 is found to be the best, then the firm will effectively allow for all consumers to travel heavy. Under option 2, the firm will segregate between type $L$ consumers, making that only those with high enough values of $\delta_i$ will decide travel heavy, while allowing for all type $H$ to be inside the market. Under option 3 the firm segregates among the two consumers types. Option 4 is the case when type $L$ are excluded from the market and all type $H$ will opt to travel heavy, while option 5 is the case when type $L$ are excluded and type $H$ are segregated. Finally, option 6 is the case when the firm wants to shut down the market. We now need to do the same analysis for the two price case. This is done in the proposition below.

**Proposition 2.3.3** ***Two-Price Heterogeneous Consumers Characterization*** *Assume that the utility of passenger travel has two possible values* $u_L$ *and* $u_H$*, with* $0 < u_L < u_H$*,* $\Pr[u_i = u_L] = \gamma$ *and* $u_i\delta_i$*. Then, we can reduce the firms price choice* $(p_1^*, p_2^*)$ *of the two price case to one of the options below:*

*1)* $(p_1^*, p_2^*) = \left(u_L, \dfrac{1+c_2}{2}\right)$*, with* $\Pi_1(p_1^*, p_2^*) = (u_L - c_1) + \left(\dfrac{1-c_2}{2}\right)$
*2)* $(p_1^*, p_2^*) = (u_L, \infty)$*, with* $\Pi_1(p_1^*, p_2^*) = (u_L - c_1)$
*3)* $(p_1^*, p_2^*) = \left(\dfrac{(1-\gamma)-\gamma^2(u_L+c_1)-\gamma(u_L-c_1)}{2\gamma(1-\gamma)}, \dfrac{c_2}{2} + \dfrac{\gamma}{1-\gamma}u_L\right)$*, with*

$$\Pi_1(p_1^*, p_2^*) = \frac{1}{4}\left[\left(\frac{1}{\gamma} - \frac{\gamma+1}{1-\gamma}u_L - c_1\right)(1+\gamma\alpha_2) + \left(\frac{2\gamma}{1-\gamma}u_L - c_2\right)(1-c_2+\gamma(u_L-c_1))\right]$$

*and* $\alpha_2 := (3u_L - c_1 - c_2)$*. This option is only available under the restrictions* $u_L < p_1^* < u_H$ *and* $u_L < p_1^* + p_2^* < 1 + u_L$*.*
*4)* $(p_1^*, p_2^*) = \left(u_H, \dfrac{1+c_2}{2}\right)$*, with* $\Pi_1(p_1^*, p_2^*) = (1-\gamma)\left[(u_H - c_1) + \left(\dfrac{1-{c_2}^2}{2}\right)\right]$*. This option is only available under the restrictions* $u_L < p_1^* \leq u_H$ *and* $p_1^* + p_2^* \geq 1 + u_L$*.*
*5)* $(p_1^*, p_2^*) = (u_H, \infty)$*, with* $\Pi_1(p_1^*, p_2^*) = (1-\gamma)(u_H - c_1)$
*6)* $(p_1^*, p_2^*) = (p_1^S, 0)$*, where* $p_1^S$ *is the optimal price of single price case as defined in propo-*

*sition 2.3.2*

Proposition 2.3.3 tells us that firms optimal policy in the dual price case now depends on 5 parameters: $c_1, c_2, \gamma, u_L$ and $u_H$. Now the firm can differentiate both types of consumers into the full three options. If options 1 or 2 is found to be the best, then the firm will effectively allow for all consumers to at least travel light, the difference among the options being if the market for baggage travel will be open (option 1) or not (option 2). This is analogous to what our firm did in regions **A** and **B** in the dual price case of our main model but now the firm is not fully extracting consumer surplus on the passenger travel market of the $H$ type consumers. Under option 3 the firm segregates among the two consumers types, but type $L$ consumers will decide whether to travel heavy or not to travel, and type $H$ will decide whether to travel light or travel heavy[8]. Note that, as $p_1^* < u_H$ we once again have the result that the firm is not able to perfectly extract consumer surplus of the passenger travel market in this case. Under option 4 and 5 the firm will opt to only serve type $H$ consumers. It will either allow for baggage travel (option 4) or not (option 5) depending on $c_2$. In both cases it will be able to fully extract consumer surplus as no type $L$ is present. Finally, under option 6 the firm behave as it did on the single price case, which will happen either if costs are too high (and the firm prefers to set $p_1^* > u_H$ effectively behaving just like our firm did in regions **C** and **D** in the dual price case of our main model) or if the firm wants to segregate type L consumers among those that will travel heavy and not to travel using only $p_1^*$ (that is, setting $u_L < p_1^* \leq u_H$) but allowing for all type $H$ passengers to travel heavy (that is, setting $p_2^* = 0$).

The analysis of propositions 2.3.2 and 2.3.3 suggests that the results of our main model about efficiency are still largely valid in this environment, as the issues of over and under consumption of baggage are unaltered. However, the allocative properties must be different now, as in some options of proposition 2.3.3 the firm is not able to fully extract consumer surplus on the passenger market, which it always did in our main model. The propositions below confirms this intuition:

**Proposition 2.3.4** ***For high values of*** $c_2$ ***welfare change is positive*** *Let* $U_i(c_2) := \int_0^1 U_{ij}(c_2)dj$ *denote aggregate utility understood as a function of* $c_2$*, with* $i = 1$ *denoting the single price case and* $i = 2$ *the dual price case. Let* $\Pi_i(c_2)$ *define optimal profits as a function of* $c_2$ *for each case, which is given as the maximal option in the list provided by propositions 2.3.2 and 2.3.3. Furthermore, define* $\Delta W(c_2) := \Delta\Pi(c_2) + \Delta U(c_2)$ *where* $\Delta\Pi(c_2) := \Pi_2(c_2) - \Pi_1(c_2)$ *and* $\Delta U(c_2) := U_2(c_2) - U_1(c_2)$*. Fix* $c_1, \gamma, u_L, u_H$*. Then, there exists* $\bar{c}_2(c_1, \gamma, u_L, u_H) \in R$ *such that* $\forall c_2 \geq \bar{c}_2 \Rightarrow \Delta W(c_2) \geq 0$.

[8] As $u_L < p_1^* \leq u_H$ the option to travel light is never better than the option not to travel for type $L$ consumers and always at least as good as this option for type $H$

**Proposition 2.3.5** ***It is possible for welfare to be negative*** *Let* $U_i$ *and* $\Pi_i$ *defined as in proposition 2.3.4. Then there exists values of* $(c_1, c_2, \gamma, u_L, u_H)$ *such that* $\Delta W > 0$ *and* $\Delta U > 0$*. There also exists a different set of parameters that generates* $\Delta W < 0$ *and* $\Delta U < 0$

Proposition 2.3.4 ensures us that for sufficiently high values of $c_2$ the change from the single to the dual price case is at least neutral for overall welfare. The last part of proposition 2.3.5 then guarantees that for small values of $c_2$ this change is not always beneficial to society, just like we had in our main model. The novelty is that the first part of proposition 2.3.4 ensures us that for some parameters, this change promotes gains not only for the firms, but also to aggregate consumer utility.

## 2.4 Conclusion

This paper models an airline company in two different scenarios. It it either restricted to a single price for the provision of two goods - passenger travel and baggage travel - or it might charge two different prices. We concluded that a change from the single price to dual price case leads to uncertain effects in overall welfare, which is a new result in the literature. Moreover, our model sheds light on the relevant factor for this analysis, that is, the magnitude of the marginal cost of baggage travel and the market power of the firm. The intuition for these results is that in markets with high market power (i.e: a monopoly) and low marginal cost of baggage travel, allowing for two distinct prices generates a problem of under-consumption of baggages due to the markup of the firms. Depending on the parameters, this problem might be higher than the more commonly thought problem of over-consumption of baggages generated by the single price scenario. In terms of allocative issues, we showed that the change from a single price to two distinct prices is always at least neutral for the firms and uncertain for the consumers, with the last result begin dependent on parameters and the capacity of the firm to extract consumer surplus.

Although specifically tailored to the airline sector, our model can be readily applied to other problems in which firms have two products and can restrict the consumption of one of them until the purchase of the other. A good example of such case is the mobile phone market. In this market, it is usual for the firm to have two distinct prices for its telephone and internet services, and consumers can usually only get internet access after purchasing a telephone plan. This paper' analysis suggests that this might not necessarily be optimal for overall welfare, so regulation might be desirable.

## 2.5 Appendix: Proofs of the propositions not proven on the text

### 2.5.1 Proof of Proposition 2.2.1

Using equation 2.1 and the uniform $[0,1]$ assumption on $\delta_i$ the above problem can be rewritten as

$$\text{Max } \Pi_1(p) = \begin{cases} (p_1 - c_1 - c_2)(1 - p + \overline{u}) & \text{if } p_1 \in [\overline{u}, \overline{u}+1] \\ (p_1 - c_1 - c_2) & \text{if } p_1 \leq \overline{u} \\ 0 & \text{if } p_1 \geq \overline{u}+1 \end{cases} \tag{2.10}$$

Let us first deal with the scenario where the firms choose $p_1 \in [\overline{u}, \overline{u}+1]$. We will first solve for the unrestricted problem and then look for boundary issues. As the objective function is concave, the first order condition for the unrestricted problem completely determines the interior optimal. This is given by:

$$p_1^* = \frac{1 + \overline{u} + c_1 + c_2}{2} \tag{2.11}$$

Let us now look for conditions for the optimal to this problem to be indeed interior. For that we need $p_1^* \in [\overline{u}, \overline{u}+1]$. This is equivalent to $\overline{u} - 1 \leq c_1 + c_2 \leq \overline{u} + 1$. If the lower bound on this inequality is not met, the firm would have to settle for a corner solution with the lowest possible price[9]. That is, in this case firms settle $p_1^* = \overline{u}$. Notice that this scenario is exactly the (trivial) solution for the case when the firms are restricted to choose price on the range $p_1 \leq \overline{u}$, so we do not need to bother with that case. If the upper bound is not met, the opposite happens and firms settle the maximum possible price at $p_1^* = \overline{u} + 1$. Once again, this (one of) the solution(s) to the case when firms are restricted to choose prices on the range $p_1 \geq \overline{u} + 1$, so we do not need to bother with that case as well. We pick $\infty$ to represent the optimal price in this case just to clarify that the firm will opt to shut down the market in that scenario.

### 2.5.2 Proof of Proposition 2.2.2

We will split the problem in two cases in order to facilitate solution. In the first, firms will solve the problem under the restriction $p_1 \leq \overline{u}$. In the second, they solve under $p_1 > \overline{u}$. We then compare profits to see which case is the relevant firm choice for each pair $(c_1, c_2)$. We start by proving the following two claims:

*Claim 1*: The second case is equivalent to the single price scenario.

[9] As the objective function is a negative square function of price

Proof: Assume first firms choose $p_1 \geq \overline{u}$. Then, equation 2.2 collapses to $q_{1i}(p_1, p_2) = q_{2i}(p_1, p_2) = 1_{\{\overline{u}+\delta_1 \geq p_1+p_2\}}$. Thus, we can rewrite equation 2.6 as:

$$\begin{cases} (p_1 - c_1 + p_2 - c_2)(1 - p_2 - p_1 + \overline{u}) & \text{if } \overline{u} \leq p_1 + p_2 \leq 1 + \overline{u} \\ 0 & \text{if } p_1 + p_2 \geq 1 + \overline{u} \\ (p_1 - c_1 + p_2 - c_2) & \text{if } p_1 + p_2 \leq \overline{u} \end{cases} \tag{2.12}$$

We first solve for the interior solution, then check borders. The first order conditions for the unrestricted problem is the same for both $p_1$ and $p_2$ and equals

$$1 - 2p_2^* - 2p_1^* + \overline{u} + c_1 + c_2 = 0 \tag{2.13}$$

Equation 2.13 defines $p_1^*$ implicitly as a function of $p_2^*$. Note that $p_1^*(0) = \frac{1+\overline{u}+c_1+c_2}{2}$. This is the same as in the interior solution of the single price case. Note that, for the optimal solution for this problem to be indeed interior, we need $\overline{u} \leq p_2^* + p_1^* \leq 1 + \overline{u}$. This is equivalent to $\overline{u} - 1 \leq c_1 + c_2 \leq \overline{u} + 1$, which is precisely the same restriction that defines region **Y** in the single price problem. We can check trivially that the border solutions are also the same. Thus we conclude that when firms set $p_1 \geq \overline{u}$ the solution is equivalent to the case with a single price price.

*Claim 2*: The first case solution is characterized by

$$(p_1^*, p_2^*) = \begin{cases} \left(\overline{u}, \dfrac{1+c_2}{2}\right) \text{ if } c_1 \leq \overline{u} \text{ and } c_2 \leq 1 \\ (\overline{u}, \infty) \text{ if } c_1 \leq \overline{u} \text{ and } c_2 > 1 \end{cases}$$

Its profits are given by

$$\Pi_2(p_1^*, p_2^*) = \begin{cases} \overline{u} - c_1 + \left(\dfrac{1-c_2}{2}\right)^2 \text{ if } c_1 \leq \overline{u} \text{ and } c_2 \leq 1 \\ \overline{u} - c_1 \text{ if } c_1 \leq \overline{u} \text{ and } c_2 > 1 \end{cases}$$

Proof: Using equation 2.2, this case problem can be simplified to:

$$\Pi_2(p_1, p_2) = \begin{cases} (p_1 - c_1) + [(p_2 - c_2)(1 - p_2)] & \text{if } 0 \leq p_2 \leq 1 \\ p_1 - c_1 & \text{if } p_2 > 1 \\ p_1 - c_1 + p_2 - c_2 & \text{if } p_2 < 0 \end{cases} \tag{2.14}$$

Once again, we solve for interior solution and then deal with boundary issues. Let $\mu_1$ and $\mu_2$ denote, respectively, the multiplier associated with the restriction $p_1 \leq \overline{u}$ and the restriction $0 \leq p_1$. The Kuhn-Tucker conditions for this problem are given by:

$$1 - \mu_1 + \mu_2 = 0 \tag{2.15a}$$
$$(1 - p_2) + (p_2 - c_2)(-1) = 0 \tag{2.15b}$$
$$\mu_1(p_1 - \overline{u}) = 0 \tag{2.15c}$$
$$\mu_2(-p_1) = 0 \tag{2.15d}$$
$$\tag{2.15e}$$

By 2.15a we get that $\mu > 0$. Hence we must have $p_1 = \overline{u}$ in this case. Condition 2.15b then imply that the optimal internal price is given by

$$p_2^* = \frac{1 + c_2}{2} \tag{2.16}$$

We need $0 \leq p_2^* \leq 1$ for an internal solution to be valid. This is equivalent to $-1 \leq c_2 \leq 1$. The lower bound on this inequality is true by assumption. The upper bound is equivalent to $c_2 \leq 1$. When the upper bound is not met, firms settle for a corner solution with the highest possible price. If that is the case, consumers would always travel light and profits equal

$$\Pi_2(\overline{u}, \infty) = (\overline{u} - c_1)$$

This is (one of) the solution(s) for the case when the firms are restricted to choose price on the range $p_2 > 1$ so we do not worry about this sub-case. When the firm is restricted to choose $p_2 \leq 0$ it will obviously set $(p_1^*, p_2^*) = (\overline{u}, 0)$. This provides profits of $\overline{u} - c_1 - c_2$, which is inferior to setting $p_2 > 1$, so this sub-case never happens.

To finish analysis, we need to compare profits in both cases. We claim that the firm prefer to set $p_1 \leq \overline{u}$ if $c_1 \leq \overline{u}$. To prove this statement we will split the plane $(c_1, c_2)$ in six regions and then compare the difference in profits between the first and the second case.

**Region 1:** $c_1 + c_2 \leq \overline{u} - 1$ *and* $c_2 \leq 1$.
First note that in this region, we always have $\overline{u} \geq c_1$. Then if the firm opts to set $p_1 \geq \overline{u}$, it will get profits equivalent to those earned in region **X** of the single price case. However, if it choose to set $p_1 = \overline{u}$ it will get $\Pi_2(\overline{u}, p_2^*) = \overline{u} - c_1 + \left(\frac{1-c_2}{2}\right)^2$. The difference in profits between those choices is then given by: $\overline{u} - c_1 + \left(\frac{1-c_2}{2}\right)^2 - (\overline{u} - c_1 - c_2) = \left(\frac{1-c_2}{2}\right)^2 + c_2$. This is greater than 0 as $c_2 > 0$. Hence in this region the firm will always prefer to put itself into the first case. We proceed in similar fashion for the other regions of the $(c_1, c_2)$ plane.

**Region 2:** $\overline{u} - 1 \leq c_1 + c_2 \leq \overline{u} + 1$ *and* $c_2 \leq 1$.
In this region, the difference between profits is given by:

$$\overline{u} - c_1 + \left(\frac{1 - c_2}{2}\right)^2 - \left(\frac{1 + \overline{u} - c_1 - c_2}{2}\right)^2 \tag{2.17}$$

Simple algebra shows that when $\overline{u} \geq c_1$, the condition for equation 2.17 to be positive is $2 + 2c_2 - (\overline{u} - c_1) \geq 0$. By assumption we know that $c_1 + c_2 \geq \overline{u} - 1$ and $c_2 \leq 1$. This implies $\overline{u} - c_1 < 1 + c_2 \leq 2 < 2 + c_2$. Thus this condition is always met implying that the firm will once again always prefer to put itself in the first case in this region when $\overline{u} \geq c_1$. On the other hand, when $\overline{u} \leq c_1$, the condition for equation 2.17 to be positive is $2 + 2c_2 - (\overline{u} - c_1) \leq 0$. This is obviously never met, so in this case the firm prefer to put itself in the second case.

**Region 3:** $c_1 + c_2 \geq \overline{u} + 1$ *and* $c_2 \leq 1$
First note that in this case we always have $\overline{u} \leq c_1$. The difference in profits is given by $\overline{u} - c_1 + \left(\frac{1-c_2}{2}\right)^2$. As in this region we assumed $c_1 + c_2 \geq \overline{u} + 1$ we get that $\overline{u} - c_1 \leq c_2 - 1$. As $c_2 \leq 1$ we also have $c_2^2 < 1$. Thus:

$$\begin{aligned}(\overline{u} - c_1) + \left(\frac{1 - c_2}{2}\right)^2 &\leq (c_2 - 1) + \left(\frac{1 - c_2}{2}\right)^2 \\ &= \frac{c_2^2 - 1}{2} < 0\end{aligned}$$

Thus in this region the firm will prefer to be in the second case.

**Region 4:** $c_1 + c_2 \leq \overline{u} - 1$ *and* $c_2 \geq 1$
In this case we always have $\overline{u} \geq c_1$. The difference in profits is given by $\overline{u} - c_1 - (\overline{u} - c_1 - c_2) = c_2 > 0$. Hence, in this region the firm prefer to be in the first case.

**Region 5:** $\overline{u} - 1 \leq c_1 + c_2 \leq \overline{u} + 1$ *and* $c_2 \geq 1$
In this case we always have $\overline{u} \geq c_1$. The difference in profits is given by

$$\overline{u} - c_1 - \left(\frac{1 + \overline{u} - c_1 - c_2}{2}\right)^2 \tag{2.18}$$

To see the signal of equation 2.18 assume first that $(\overline{u} - c_1) \geq 1/4$. As we know by assumption that $c_1 + c_2 \geq \overline{u} - 1$ we get $\overline{u} - c_1 \leq c_2 + 1$. Thus we have:

$$\begin{aligned}(\overline{u} - c_1) - \left(\frac{1 + \overline{u} - c_1 - c_2}{2}\right)^2 &\geq (\overline{u} - c_1) - \left(\frac{1 + 1 + c_2 - c_2}{2}\right)^2 \\ &= (\overline{u} - c_1) - \frac{1}{4} \geq 0\end{aligned}$$

Now assume $(\overline{u} - c_1) < 1/4$. As $c_2 > 1$ and $c_1 + c_2 \leq \overline{u} + 1$ we have $0 \leq \overline{u} - c_1$. Define $\alpha := \overline{u} - c_1 \in [0, 1/4[$. By $c_1 + c_2 \leq \overline{u} + 1$ we get that $c_2 \in ]1, 1+\alpha]$. We then have that:

$$\begin{aligned}(\overline{u} - c_1) - \left(\frac{1 + \overline{u} - c_1 - c_2}{2}\right)^2 &= \alpha - \left(\frac{1 + \alpha - c_2}{2}\right)^2 \\ &\geq \alpha - \frac{\alpha^2}{4} \geq 0\end{aligned}$$

Hence in this region firms will always prefer to put itself in the first case.

**Region 6:** $c_1 + c_2 \geq \overline{u} + 1$ *and* $c_2 \geq 1$
The difference in profits is given by $\overline{u} - c_1$. This is obviously positive whenever $\overline{u} \geq c_1$.

Collecting the results of the six regions proves our last claim and finishes the proof of the proposition

### 2.5.3 Proof of Proposition 2.2.3

We split the proof into three cases:

**Case 1:** $r = \mathbf{X}$: If $r = \mathbf{X}$, then $f(r) \in \{\mathbf{A}, \mathbf{B}\}$. If $f(r) = \mathbf{A}$, we can use the previous results for profits and utility to conclude that $\Delta U_{r,f(r)} = \frac{(1-c_2)^2}{8} - \frac{1}{2}$ and $\Delta W_{r,f(r)} = \frac{3c_2^2 + 2c_2 - 1}{8}$. As the definition of $\mathbf{A}$ ensures that $c_2 \leq 1$ we get that $\Delta U_{r,f(r)} < 0$ in this case. Moreover, it is straightforward to show that $\Delta W_{r,f(r)} \geq 0 \iff c_2 \geq 0$. If $f(r) = \mathbf{B}$ we have that $\Delta U_{r,f(r)} = -\frac{1}{2}$ and $\Delta W_{r,f(r)} = c_2 - \frac{1}{2}$. Trivially then $\Delta U_{r,f(r)} < 0$. As the restrictions on region $\mathbf{X} \cap \mathbf{B}$ force $c_2 > 1$ we get that $\Delta W_{r,f(r)}$ is always positive in this region. Taking these observations together prove the first part of the proposition.

**Case 2:** $r = \mathbf{Y}$: If $r = \mathbf{Y}$ then $f(r) \in \{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$. If $f(r) = \mathbf{A}$, we get that $\Delta U_{r,f(r)} = \frac{(1-c_2)^2}{8} - \frac{(1+\overline{u}-c_1-c_2)^2}{8}$. This is clearly non-positive, as both $(1 - c_2)$ and $(\overline{u} - c_1)$ are non negative due to the restrictions of the case. Moreover, it is straightforward to show that

$$\Delta W_{r,f(r)} = (\overline{u} - c_1)\frac{3(c_1 + c_2 - \overline{u} + 1) + 3c_2 - 1}{8} \tag{2.19}$$

To see the sign of equation 2.19, note that in region $\mathbf{A} \cap \mathbf{Y}$ we get that $\overline{u} - c_1 \geq 0$. Thus, the change in welfare in this region is proportional to $3(c_1 + c_2 - \overline{u} + 1) + 3c_2 - 1$. Rewriting this term, we conclude that $\Delta W_{r,f(r)} \geq 0 \iff c_2 \geq \frac{\overline{u} - c_1}{2} - \frac{1}{3}$.

If $f(r) = \mathbf{B}$, we get that $\Delta W_{r,f(r)} = -\frac{(1+\overline{u}-c_1-c_2)^2}{8}$. As the definition of region $\mathbf{Y}$ ensures that $c_1 + c_2 \leq \overline{u} + 1$ we get that $\Delta W_{r,f(r)} \leq 0$ in this case. Moreover, the previous results imply that

$$\Delta W_{r,f(r)} = (\overline{u} - c_1) - \frac{3}{8}(1+\overline{u}-c_1-c_2)^2 \tag{2.20}$$

The following claim guarantees that $\Delta W_{r,f(r)}$ is positive in this region:

*Claim*: For all $(c_1, c_2) \in \mathbf{Y} \cap \mathbf{B}$ we have $\Delta W_{r,f(r)} \geq 0$
Proof: Assume first that $(\overline{u} - c_1) \geq \frac{3}{8}$. Pick any $x \in \mathbf{B} \cap \mathbf{Y}$. As $x \in \mathbf{Y}$, we know that $c_1 + c_2 \geq \overline{u} - 1$. This implies $\overline{u} - c_1 \leq c_2 + 1$. Thus we have:

$$\begin{aligned}
(\overline{u} - c_1) - \frac{3}{8}(1+\overline{u}-c_1-c_2)^2 &\geq (\overline{u} - c_1) - \frac{3}{8}(1+1+c_2-c_2)^2 \\
&= (\overline{u} - c_1) - \frac{3}{8} \geq 0
\end{aligned}$$

Now assume that $(\overline{u} - c_1) < \frac{3}{8}$. As $x \in \mathbf{B}$ we know that $c_2 > 1$. As $x \in \mathbf{Y}$, we know that $c_1 + c_2 \leq \overline{u} + 1$. Together, these conditions imply $0 < c_2 - 1 \leq \overline{u} - c_1$. By $c_1 + c_2 \leq \overline{u} + 1$ we get that $c_2 \in ]1, 1 + \overline{u} - c_1]$. We then have that:

$$\begin{aligned}
(\overline{u} - c_1) - \frac{3}{8}(1+\overline{u}-c_1-c_2)^2 &= \overline{u} - c_1 - \frac{3}{8}((\overline{u} - c_1) - (c_2 - 1))^2 \\
&> \overline{u} - c_1 - \frac{3(\overline{u} - c_1)^2}{8} \geq 0
\end{aligned}$$

This finish the proof of the claim

Finally, if $f(r) = \mathbf{C}$ then trivially $\Delta U_{r,f(r)} = \Delta W_{r,f(r)} = 0$ as the single price and the dual price behavioral are the same in this case. Taking these observations together proves the second part of the proposition

**Case 3:** $r = \mathbf{Z}$: If $r = \mathbf{Z}$ then $f(r) \in \{\mathbf{B}, \mathbf{D}\}$. If $f(r) = \mathbf{B}$ then the previous results imply $\Delta U_{r,f(r)} = 0$. Moreover $\Delta W_{r,f(r)} = \overline{u} - c_1$ which is always positive by definition of region $\mathbf{B}$. If $f(r) = \mathbf{D}$ then trivially $\Delta U_{r,f(r)} = \Delta W_{r,f(r)} = 0$ as we have no market for both goods in all cases. This proves the last part of the proposition.

### 2.5.4 Proof of Proposition 2.3.2

Let $\Phi_\delta$ denotes the CDF of $\delta$. As we are in the single price case the firm's problem can by written as Max $(p_1 - c_1 - c_2)\Pr[\delta_i + u_i \geq p_1]$. By $u_i \delta_i$, this is equivalent to Max $(p_1 - c_1 - c_2)[1 - (\Phi_\delta(p_1 - u_L)\gamma + \Phi_\delta(p_1 - u_H)(1-\gamma))]$. As both $\Phi_\delta(p_1 - u_L)$ and $\Phi_\delta(p_1 - u_H)$ might assume value 0, 1 or an interior value we have 9 possible cases. We

deal with each one in what follows:

**Case 1**: $p_1$ is chosen such that $\Phi_\delta(p_1 - u_L) = \Phi_\delta(p_1 - u_H) = 0$.

This case's restrictions are $p_1 \leq u_H$ and $p_1 \leq u_L$ which collapses to $p_1 \leq u_H$. Thus, the firm's problem can be rewritten as

$$\text{Max } p_1 - c_1 - c_2 \text{ s.t. } p_1 \leq u_L$$

The trivial solution is $p_1^* = u_L$, which earns profits of $\Pi_1(p_1^*) = u_L - c_1 - c_2$. As the firm may always choose to put its price under $u_L$, we have no parameter restriction for this case to happen.

**Case 2**: $p_1$ is chosen such that $\Phi_\delta(p_1 - u_L) \in ]0, 1[$ and $\Phi_\delta(p_1 - u_H) = 0$.

This case's restrictions are $p_1 \leq u_H$ and $u_L < p_1 < 1+u_L$. Thus, the firm's problem collapses to

$$\text{Max } (p_1 - c_1 - c_2)[1 - (p_1 - u_L)\gamma] \text{ s.t. } \begin{cases} p_1 \leq u_H \\ u_L < p_1 < 1 + u_L \end{cases}$$

The objective function is strictly concave, so the problem's first order conditions are sufficient in the interior of the restrictions. Those are given by

$$(p_1 - c_1 - c_2)(-\gamma) + (1 - p_1\gamma + u_L\gamma) = 0$$

Which implies

$$p_1^* = \frac{1 + \gamma(u_L + c_1 + c_2)}{2\gamma}$$

Profits are then given by

$$\Pi_1(p_1^*) = \frac{1}{\gamma}\left(\frac{1 + \gamma(u_L - c_1 - c_2)}{2}\right)^2$$

For the solution to be indeed interior we need to verify the problems restrictions.

**Case 3**: $p_1$ is chosen such that $\Phi_\delta(p_1 - u_L) \in ]0, 1[$ and $\Phi_\delta(p_1 - u_H) \in ]0, 1[$.

This case's restrictions are $u_H < p_1 < 1 + u_H$ and $u_L < p_1 < 1 + u_L$. Thus, the firm's problem collapses to

$$\text{Max } (p_1 - c_1 - c_2)[1 - (p_1 - u_L)\gamma] \text{ s.t. } \begin{cases} u_H < p_1 < 1 + u_H \\ u_L < p_1 < 1 + u_L \end{cases}$$

The objective function is strictly concave, so the problem's first order conditions are sufficient in the interior of the restrictions. Those are given by

$$(p_1 - c_1 - c_2) + [1 - \gamma(p_1 - u_L) - (1 - \gamma)(p_1 - u_H)] = 0$$

Which implies

$$p_1^* = \frac{1 + \gamma u_L + (1-\gamma)u_H + c_1 + c_2}{2}$$

Profits are then given by

$$\Pi_1(p_1^*) = \left(\frac{1 + \gamma u_L + (1-\gamma)u_H - c_1 - c_2}{2}\right)^2$$

For the solution to be indeed interior we need to verify the problems restrictions.

**Case 4**: $p_1$ is chosen such that $\Phi_\delta(p_1 - u_L) = 1$ and $\Phi_\delta(p_1 - u_H) \in ]0, 1[$.

This case's restrictions are $u_H < p_1 < 1 + u_H$ and $p_1 \geq 1 + u_L$. Thus, the firm's problem collapses to

$$\text{Max } (p_1 - c_1 - c_2)[1 - (\gamma + (p_1 - u_H)(1-\gamma)] \text{ s.t. } \begin{cases} u_H < p_1 < 1 + u_H \\ p_1 \geq 1 \end{cases}$$

The objective function is strictly concave, so the problem's first order conditions are sufficient in the interior of the restrictions. Those are given by

$$-(p_1 - c_1 - c_2)(1-\gamma) + [1 - \gamma - (1-\gamma)(p_1 - u_H)] = 0$$

Which implies

$$p_1^* = \frac{1 + u_H + c_1 + c_2}{2}$$

Profits are then given by

$$\Pi_1(p_1^*) = (1-\gamma)\left(\frac{1 + u_H - c_1 - c_2}{2}\right)^2$$

For the solution to be indeed interior we need to verify the problems restrictions.

**Case 5**: $p_1$ is chosen such that $\Phi_\delta(p_1 - u_L) = 1$ and $\Phi_\delta(p_1 - u_H) = 0$.

This case's restrictions are $p_1 \leq u_H$ and $p_1 \geq 1 + u_L$. Thus, the firm's problem collapses to

$$\text{Max } (p_1 - c_1 - c_2)[1 - \gamma] \text{ s.t. } \begin{cases} p_1 \leq u_H \\ p_1 \geq 1 \end{cases}$$

The trivial solution is $p_1^* = u_H$, which earns profits of $\Pi_1(p_1^*) = (1-\gamma)(u_H - c_1 - c_2)$. For this solution to satisfy the restrictions we need $u_H \geq 1 + u_L$.

**Case 6**: $p_1$ is chosen such that $\Phi_\delta(p_1 - u_L) = \Phi_\delta(p_1 - u_H) = 1$.

This case's restrictions are $p_1 \geq u_H + 1$ and $p_1 \geq 1 + u_L$, which collapses to $p_1 \geq 1 + u_H$. Thus, the firm's problem can be rewritten as

$$\text{Max } 0 \text{ s.t. } p_1 \geq u_H + 1$$

Any price that satisfy the restrictions is a valid solution. Profits are obviously given by $\Pi_1(p_1^*) = 0$. As the firm may always choose to put its price above $u_H + 1$, we have no parameter restriction for this case to happen.

$$\textbf{Cases 7,8,9}: p_1 \text{ is chosen such that} \begin{cases} \Phi_\delta(p_1 - u_L) = 0 \text{ and } \Phi_\delta(p_1 - u_H) \in \ ]0,1[ \\ \Phi_\delta(p_1 - u_L) = 0 \text{ and } \Phi_\delta(p_1 - u_H) = 1 \\ \Phi_\delta(p_1 - u_L) \in \ ]0,1[ \text{ and } \Phi_\delta(p_1 - u_H) = 1 \end{cases}.$$

In the first two cases we have $\Phi_\delta(p_1 - u_L) = 0 \iff p_1 - u_L \leq 0 \rightarrow p_1 - u_H \leq 0 \rightarrow \Phi_\delta(p_1 - u_H) = 0$. In the third case we have $\Phi_\delta(p_1 - u_L) \in \ ]0,1[ \iff 0 \leq p_1 - u_L \leq 1 \rightarrow p_1 - u_H \leq 1 \rightarrow \Phi_\delta(p_1 - u_H) < 1$. Hence the restrictions gives rise to a maximization in an empty set for all cases

Taking together the 9 cases observation the desired result follows from continuity of $\Phi_\delta$

### 2.5.5 Proof of Proposition 2.3.3

During the proof of proposition 2.2.2 we stated that the dual price case turned effectively into the single price case whenever the optimal price $p_1^*$ of the dual price case is higher than $\overline{u}$. We start this proof by showing that this result is indeed general, and can be applied in the two-type consumer case as well with only minor changes. This is shown in the lemma below

*Lemma 1*: Assume that $u_i$ is a random variable with support in $[u_L, u_H]$. Let $\Phi_{u_i+\delta_i}$ denote the CDF of $u_i + \delta_i$. Then, if the optimal price for passenger travel $p_1^*$ of the dual price case is such that $p_1^* > u_H$ we get that the dual price case problem is analogous to the single price one

Proof: We start with the single price analysis in this environment. Using equation 2.1 the single price case can be written as:

$$\text{Max } (p_1 - c_1 - c_2)(1 - \Phi_{u_i+\delta_i}(p_1))$$

The first order condition is given by

$$(p_1 - c_1 - c_2)(-\Phi_{u_i+\delta_i}(p_1)) + (1 - \Phi_{u_i+\delta_i}(p_1)) = 0$$

. We now move to the analysis of the dual price problem. using equation 2.2 and the fact that $p_1^* > u_H$ we can rewrite our maximization problem as:

$$\text{Max } (p_1 + p_2 - c_1 - c_2)(1 - \Phi_{u_i+\delta_i}(p_1 + p_2))$$

The first order condition for both prices gives us the same result, which is:

$$(p_1 + p_2 - c_1 - c_2)(-\Phi_{u_i+\delta_i}(p_1 + p_2)) + (1 - \Phi_{u_i+\delta_i}(p_1 + p_2)) = 0 \tag{2.21}$$

Equation 2.21 defines $p_1^*$ implicitly as a function of $p_2^*$. Moreover, it collapses to the first order condition of the single price case when we set $p_2^* = 0$.

Lemma 1 allows us to focus on the cases where $p_1 \leq u_H$ to prove our desired result as long as we allow the firm the option to behave as it did in the single price case - which we did in option 6) of the proposition. To prove the remainder of the proposition, let us further divide our problem in two cases. In the first, the firms solve the problem under the additional restriction $p_1 \leq u_L$ and in the second they solve under $u_l < p_1 \leq u_H$. As in the previous proposition's proof, we assume $\Phi_\delta$ denotes the CDF of $\delta_i$

**Case 1**: Using equation 2.2 and $p_1 \leq u_L$ the firms problem collapses to

$$\text{Max } (p_1 - c_1) + (p_2 - c_2)(1 - \Phi_\delta(p_2))$$

This problem was already solved in claim 2 of the proof of proposition 2.2.2. The solution is given by

$$(p_1^*, p_2^*) = \begin{cases} \left(u_L, \dfrac{1+c_2}{2}\right) \text{ if } c_2 \leq 1 \\ (u_L, ) \text{ if } c_2 > 1 \end{cases}$$

The profits associated are given by:

$$\Pi_2(p_1^*, p_2^*) = \begin{cases} (u_L - c_1) + \left(\dfrac{1-c_2}{2}\right)^2 \text{ if } c_2 \leq 1 \\ u_L - c_1 \text{ if } c_2 > 1 \end{cases}$$

**Case 2**: We now assume $u_l < p_1 \leq u_H$. Hence our problem becomes

$$\begin{aligned} \text{Max } (p_1 - c_1)[1 - \gamma + \gamma(1 - \Phi_\delta(p_1 + p_2 - u_L))] + \\ + (p_2 - c_2)[(1-\gamma)(1 - \Phi_\delta(p_2)) + \gamma(1 - \Phi_\delta(p_1 + p_2 - u_L))] \end{aligned}$$

We will further simplify this problem into the following sub-cases to facilitate solution:

**Sub-Case 2.1**: $(p_1, p_2)$ are chosen such that $\Phi_\delta(p_2) \in ]0,1[$ and $\Phi_\delta(p_1 + p_2 - u_L) \in ]0,1[$.

This case's restrictions are $0 < p_2 < 1$, $u_L < p_1 + p_2 < 1 + u_L$ and $u_L < p_1 \leq u_H$. Thus, the firm's problem collapses to

$$\text{Max } (p_1 - c_1)[1 - \gamma(p_1 + p_2 - u_L)] + (p_2 - c_2)[1 - p_2 - \gamma(p_1 - u_L)] \text{ s.t. } \begin{cases} 0 < p_2 < 1 \\ u_L < p_1 + p_2 < u_L + 1 \\ u_L < p_1 \leq u_H \end{cases}$$

The objective function is strictly concave, so the problem's first order conditions are sufficient in the interior of the restrictions. Those are given by

$$\gamma(p_1 - c_1 + p_2 - c_2) = 1 - \gamma(p_1 + p_2 - u_L)$$
$$\gamma(p_1 - c_1) + p_2 - c_2 = 1 - p_2 - \gamma(p_1 - u_L)$$

Which implies

$$p_1^* = \frac{(1-\gamma) - \gamma^2(u_L + c_1) - \gamma(u_L - c_1)}{2\gamma(1-\gamma)} \qquad p_2^* = \frac{c_2}{2} + \frac{\gamma}{1-\gamma} u_L$$

Profits are then given by

$$\begin{aligned} \Pi_1(p_1^*, p_2^*) = & \frac{1}{4}\left[ \left( \frac{1}{\gamma} - \frac{\gamma + 1}{1-\gamma} u_L - c_1 \right) (1 + \gamma(3u_L - c_1 - c_2) \right. \\ & \left. + \left( \frac{2\gamma}{1-\gamma} u_L - c_2 \right) (1 - c_2 + \gamma(u_L - c_1)) \right] \end{aligned}$$

For the solution to be indeed interior we need to verify the problems restrictions However, the first restriction is already embedded in the last two, so we may as well ommit it.

**Sub-Case 2.2**: $(p_1, p_2)$ are chosen such that $\Phi_\delta(p_2) \in ]0,1[$ and $\Phi_\delta(p_1 + p_2 - u_L) = 1$.

This case's restrictions are $0 < p_2 < 1$, $p_1 + p_2 \geq 1 + u_L$ and $u_L < p_1 \leq u_H$. Thus, the firm's problem collapses to

$$\text{Max } (p_1 - c_1) + (p_2 - c_2)(1-\gamma)(1 - p_2) \text{ s.t. } \begin{cases} 0 < p_2 < 1 \\ p_1 + p_2 \geq u_L + 1 \\ u_L < p_1 \leq u_H \end{cases}$$

The problem is separable into two distinct problem, the first begin a maximization with $p_1$ as the choice variable (first term of the sum) and the second begin a maximization with $p_2$ as the choice variable (second term). In the first problem the obvious solution is $p_1^* = u_H$. In the second, we have a concave problem, which first order condition generates $p_2^* = \frac{1+c_2}{2}$. Taking these together, profits are then given by:

Profits are then given by

$$\Pi_1(p_1^*, p_2^*) = (1-\gamma)\left[(u_H - c_1) + \left(\frac{1-c_2}{2}\right)^2\right]$$

For the restriction $p_1 + p_2 \geq 1 + u_L$ to be respected we need $u_H - u_L \geq p_2^* = \frac{1-c_2}{2}$.

**Sub-Case 2.3**: $(p_1, p_2)$ are chosen such that $\Phi_\delta(p_2) = 1$ and $\Phi_\delta(p_1 + p_2 - u_L) = 1$.

This case's restrictions are $p_2 \geq 1$, $p_1 + p_2 \geq 1 + u_L$ and $u_L < p_1 \leq u_H$. Thus, the firm's problem collapses to Max $(p_1 - c_1)(1-\gamma)$ plus the restrictions. The obvious solution is to set $p_1^* = u_H$ and $p_2^*$ to any value that is higher than 1 (to accommodate the restrictions). Profits are then given by $\Pi_2(p_1^*, p_2^*) = (u_H - c_1)(1-\gamma)$

**Any other Sub-Case**:

In any other sub-case the restrictions either generates an empty set or the firms choice collapse to the single price problem. If $\Phi_\delta(p_2) = 0$, then the firm is restricted to set $p_2 \leq 0$. The obvious choice is then to set $p_2^* = 0$ as without the effect on demand, profit is a strictly positive function of the prices. As this is the forced choice of the single price case, any sub-case in which $\Phi_\delta(p_2) = 0$ makes the dual price problem to collapse to the single price one. If $\Phi_\delta(p_2) = 1$ and $\Phi_\delta(p_1 + p_2 - u_L) \in ]0,1[$ or if $\Phi_\delta(p_2) = 1$ and $\Phi_\delta(p_1 + p_2 - u_L) = 0$ we have empty sets as $p_2 \geq 1$ makes that $u_L < p_1 \rightarrow u_L + 1 < p_1 + 1 \leq p_1 + p_2 \rightarrow 1 < p_1 + p_2 - u_L \rightarrow \Phi_\delta(p_1 + p_2 - u_L) = 1$. Finally, if $\Phi_\delta(p_2) \in ]0,1[$ and $\Phi_\delta(p_1 + p_2 - u_L) = 0$ we have $u_L < p_1 \rightarrow 0 < p_1 + p_2 - u_L \rightarrow \Phi_\delta(p_1 + p_2 - u_L) > 0$, so this also generates an empty set.

Taking together all the cases analyzed the desired result follows from continuity of $\Phi_\delta$ and Lemma 1

### 2.5.6 Proof of Proposition 2.3.4

Take $c_2 \rightarrow \infty$. Then the set of options for the firm in the single price case collapse to options 1), 4) and 6), as the restrictions on the other options will not be met[10]. The obvious

[10] Taking limits under the optimal price of options 2), 3) and 5) when $c_2 \rightarrow \infty$ gives us $p_1^* \rightarrow \infty$, which is not possible given the restrictions of the options

choice is then option 6). For the dual price case, the options collapse to 2), 5) and 6), with option 6) being $\Pi_2(c_2) = 0$ for sure. These observations allows us to conclude that for a large enough value of $c_2$ we have $\Pi_1(c_2) = 0$ and $\Pi_2(c_2) = \text{Max}\ \{0, u_L - c_1, (u_H - c_1)(1-\gamma)\}$. It is then straightforward to check that, for large values of $c_2$ we have $U_1(c_2) = 0$ and

$$U_2(c_2) = \begin{cases} (1-\gamma)(u_H - u_L) \text{ if } \Pi_2(c_2) = u_L - c_1 \\ 0 \text{ in any other case} \end{cases}$$

As $u_H > u_L$ we get the desired result

### 2.5.7 Proof of Proposition 2.3.5

The easiest way to prove this proposition is through counterexamples. Let $(c_1, c_2, \gamma, u_L, u_H) = (1; 0.25; 0.3; 1; 1.3)$. The optimal options for the firm in this case is to choose option 3) of both propositions 2.3.2 and 2.3.3. This leads to $\Pi_1 \approx 0.23$ and $\Pi_2 \approx 0.30$. Utilities is then $U_1 \approx 0.12$ and $U_2 \approx 0.84$. This yields $\Delta U \approx 0.71$ and $\Delta W =\approx 0.78$, which proves the first part of the proposition

Let $(c_1, c_2, \gamma, u_L, u_H) = (0.9; 0.6; 0.5; 2.5; 3)$. The optimal options for the firm in this case is to choose option 2) of proposition 2.3.2 and option 1 of proposition 2.3.3. This leads to $\Pi_1 = 1.125$ and $\Pi_2 = 1.64$. Utilities is then $U_1 = 1.25$ and $U_2 = 0.27$. This yields $\Delta U = -0.98$ and $\Delta W = -0.465$, which proves the last part of the proposition

# Bibliography

AGARWAL, S. et al. A simple framework for estimating consumer benefits from regulating hidden fees. *The Journal of Legal Studies*, v. 43, p. S239–S252, 2014. 24

ALLON, G.; BASSAMBOO, A.; LARIVIERE, M. Would the social planner let bags fly free? 2011. 24

ARMSTRONG, M. Recent developments in the economics of price discrimination. *Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress*, v. 2, p. 97–141, 2006. 25

ARMSTRONG, M.; ROCHET, J.-C. Multi-dimensional screening: A user's guide. *European Economic Review*, v. 43, n. 959, 1999. 25, 34

ARMSTRONG, M.; VICKERS, J. Competitive non-linear pricing and bundling. *The Review of Economic Studies*, v. 77, p. 30–60, 2010. 25

BARONE, G.; HENRICKSON, K.; VOY, A. Baggage fees and airline performance: A case study of initial investor misperception. *J. Transp. Res. Forum*, v. 51, p. 5–18, 2012. 24

BERGEMANN, D.; MORRIS, S. Bayes correlated equilibrium and the comparison of information structures in games. *Theoretical Economics*, v. 11, n. 2, p. 487–522, 2016. 12

BEST, J.; QUIGLEY, D. Persuasion for the long run. 2017. 12

BRADLEY, S.; FELDMAN, N. E. Hidden baggage: Behavioral responses to changes in airline ticket tax disclosure. 2016. 24

COY, J.; CHIANG, E. Are explicit baggage fees the answer to rising airline operating costs? *Proceedings of the IABE- KeyWest, Florida – Winter Conference*, v. 11, p. 178–183, 2012. 24

ELY, J. C. Beeps. *American Economic Review*, v. 107, n. 1, p. 31–53, 2017. 12

GERARDI, D.; MAESTRI, L. *Dynamic Contracting with Limited Commitment and the Ratchet Effect*. [S.l.], 2016. 11, 15, 18

HALSEY, A. I. Bill targets airline fees for checked luggage. *Washington Post*, November 2011. 23

KAMENICA, E.; GENTZKOW, M. Bayesian persuasion. *American Economic Review*, v. 101, p. 2590–2615, 2011. 11, 12, 14, 15, 21

KRINA, G.; NIKOLAOS, V. On two-part tariff competition in a homogeneous product duopoly. *International Journal of Industrial Organization*, v. 41, p. 30–41, 2015. 33, 34

NICOLAE, M. et al. Do bags fly free? an empirical analysis of the operational implications of airline baggage fees. *Management Science*, v. 63, n. 10, 2017. 24

SCOTTI, D.; DRESNER, M. The impact of baggage fees on passenger demand on us air routes. *Transport Policy*, v. 43, p. 4–10, 2015. 24

TANEVA, I. Information design. 2018. 12