Central Bank Credibility and Inflation Expectations: A Microfounded Forecasting Approach

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Credibility is elusive and no generally agreed upon measure of it exists. Despite that, Blinder (2000) generated a consensus in the literature by arguing that "A central bank is credible if people believe it will do what it says". It is very hard to argue against such a definition of credibility, being the reason why it became so popular among central bankers and academics alike.

This paper proposes a measure of credibility that is based upon this definition. A main challenge to implement it is that one needs a measure of people’s beliefs, of what central banks say they do, and a way to compare whether the two are the same.

The core idea of our approach is as follows. First, we measure people’s beliefs by using survey data on inflation’s expectations, employing the panel-data setup of Gaglianone & Issler (2018), which allows filtering the conditional expectation of inflation from survey data. Second, we compare beliefs with explicit (or tacit) targets. Therefore, our measure of what central banks say they do are these targets, which serve as a binding contract between central banks and society. To compare people’s beliefs with what central banks say they do, we need to take into account the uncertainty in our estimate of people’s beliefs. To do so, we construct robust – heteroskedasticity and autocorrelation consistent (HAC) – 95% asymptotic confidence intervals for our estimates of the conditional expectation of inflation. Whenever the target falls into this interval we consider the central bank credible. We consider it not credible otherwise.

We apply our approach to study the credibility of the Brazilian Central Bank (BCB) by using the now well-known Focus Survey of forecasts, kept by the BCB on inflation
expectations. This choice is not merely geographic for us. The Focus Survey is a world-class database, fed by institutions that include commercial banks, asset management firms, consulting firms, non-financial institutions, etc. About 250 participants can provide daily forecasts for a large number of economic variables, e.g., inflation using different price indices, interest and exchange rates, GDP, industrial production, etc., and for different forecast horizons, e.g., current month, next month, current year, 5 years ahead, among others. At any point in time, about 120 participants are actively using the system that manages the database.

Based on this framework, we estimate on a monthly basis the conditional expectation of inflation 12-months ahead, coupled with a robust estimate of its asymptotic variance and the respective 95% robust confidence interval. Our estimates cover the period from January 2007 until April 2017. This is compared to the target in the Brazilian Inflation Target Regime. Results show that the BCB was credible 65% of the time, with the exception of a few months in the beginning of 2007 and during the interval between mid-2013 throughout mid-2016. We also constructed a credibility index for this period and compared it with alternative measures of credibility previously proposed in the literature.

Key words: Consensus Forecasts, Forecast Combination, Panel Data, Central Banking.
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1 Introduction

Over the last four decades, the question of central bank credibility has been extensively studied by the academic literature on monetary policy. It has also become a major concern for many central bankers around the world, which have taken a number of measures to enhance the credibility of monetary policy. Building central bank credibility was especially strong in countries under Inflation Targeting regimes, but this phenomenon was not restricted to these countries. This is not surprising, given the role that expectations have in most macroeconomic models, where credibility serves as a way of anchoring inflation expectations.

A main problem of measuring credibility is the fact that it is elusive and no generally agreed upon measure of it exists. Despite that, there is a consensus in the literature that Blinder (2000) offers an uncontroversial definition of credibility when he states that "A central bank is credible if people believe it will do what it says." It is very hard to argue against such a definition of credibility, being the reason why it became so popular among central bankers and academics alike.

For this reason, this paper proposes a measure of credibility that is based upon this definition. A main challenge to implement it is that one needs a measure of people’s beliefs, of what central banks say they do, and a way to compare whether the two are the same.

Compared to the literature on credibility, we approach this problem in a novel way. We focus on inflation expectations, since inflation is arguably the main variable central banks care about. Indeed, many countries that are users of Inflation Targeting regimes have explicit targets for inflation. Even in those which are not, there is usually a tacit agreement on what that target should be.

The core idea of our approach is as follows. First, we measure people’s beliefs by using survey data on inflation’s expectations, employing the panel-data setup of Gaglianone & Issler (2018), which allows filtering the conditional expectation of inflation from the survey. Second, we compare beliefs with explicit (or tacit) targets. Therefore, our measure of what central banks say they do are these targets, which serve as a binding contract between central banks and society. To compare people’s beliefs with what central banks say they do, we need to take into account the uncertainty in our estimate of people’s beliefs. To do so, we construct robust – heteroskedasticity and autocorrelation consistent (HAC) – 95% asymptotic confidence intervals for our estimates of the conditional expectation of inflation. Whenever the target falls into this interval we consider the central bank credible. We consider it not credible otherwise.

Of course, there has been active research in the area of credibility, especially in this millennium. Regarding measurement, a pioneering work is due to Svensson (1993), who proposed a simple test to check whether the inflation target is credible in the sense that market agents believe that future inflation will fall within the target range. A number of articles followed, trying to construct credibility measures and credibility indices in the last two decades; see Bomfim &

\[\text{We focus on 12-months ahead horizon since it is widely used in the literature.}\]

We advance with respect to this literature in three ways: (i) we measure beliefs properly: in most of this literature, people’s beliefs are not correctly estimated, since the survey consensus is used to measure beliefs. However, it has been shown that the consensus is a biased estimate of beliefs; (ii) We do not use ad hoc confidence intervals: most of these papers propose the use of ad hoc confidence bands in making comparisons between beliefs and targets. Even if these confidence bands are appropriate for a given country in a specific point in time, they may not be appropriate for a different country and/or for a different point in time in the same country; (iii) Our approach can be universally applied: it is straightforward to use it for countries under an inflation targeting program, but it extends naturally to other countries in which a tacit target is present, covering a wide range of countries.

We apply our approach to study the credibility of the Brazilian Central Bank (BCB) by using the now well-known Focus Survey of forecasts, kept by the BCB on inflation expectations. This choice is not merely geographic for us. The Focus Survey is a world-class database, fed by institutions that include commercial banks, asset management firms, consulting firms, non-financial institutions, etc. About 250 participants can provide daily forecasts for a large number of economic variables, e.g., inflation using different price indices, interest and exchange rates, GDP, industrial production, etc., and for different forecast horizons, e.g., current month, next month, current year, 5 years ahead, among others. At any point in time, about 120 participants are actively using the system that manages the database.

The survey has several key features: (i) participants can access the survey at any time and we can observe their decisions to update or not their forecasts; (ii) the confidentiality of information is guaranteed and the anonymity of forecasters is preserved, i.e., there are no reputational concerns; (iii) the Focus Survey has very strong incentives for participants to update their forecasts, especially on inflation expectations; see Marques (2013) for further details. To apply the techniques discussed above on the Focus Survey, we first extend the work of Gaglianone & Issler (2018) by deriving a robust HAC estimator for the asymptotic variance of the inflation expectation estimator proposed there.

Based on this framework, we estimate on a monthly basis the conditional expectation of inflation 12-months ahead, coupled with an estimate of its robust asymptotic variance and the respective 95% HAC confidence interval. Our estimates cover the period from January 2007 until April 2017. This is compared to the target in the Brazilian Inflation Target Regime. Results show that the BCB was credible 65% of the time, with the exception of a few months in the beginning of 2007 and during the interval between mid-2013 throughout mid-2016. We also constructed a credibility index for this period and compared it with alternative measures of credibility previously proposed in the literature.

The remainder of this paper is organized as follows. Section 2 discusses the existing literature...
on central bank credibility and brings a discussion about the existing indices. Section 3 gives an informal overview of the main results of Gaglianone & Issler (2018), a step-by-step description of the HAC covariance matrix estimation procedure and develops a new credibility index for the central bank based on this measure. Section 4 brings an empirical application of this framework to evaluate the credibility of the BCB in recent years, using the Focus Survey database. Section 5 concludes.

2 The literature on central bank credibility

The issue of whether it is better for the policymaker to operate with pure discretion and poor accountability or to commit to a policy has long been a central question for monetary economics. Kydland & Prescott (1977) showed that a regime where policymakers have to commit is preferable to a regime that allows policymakers discretion. The main idea behind the dynamic optimization argument is that expectations play a central role in macroeconomic dynamics, since economic agents choose their current actions based on the best possible forecasts of future outcomes given available information.

Several central banks have adopted a more systematic approach to maintain price stability since the early 1990s, particularly under an inflation targeting regime as a method of commitment, explicitly acknowledging that low and stable inflation is the goal of monetary policy, retaining constrained discretion, as argued by Bernanke & Mishkin (1997). At the same time, the theoretical debate in favor of monetary policy rules gained traction with Taylor (1993).

The evolution and improvement of the monetary policy institutional framework is related to better economic outcomes in empirical studies. Many articles in the literature (e.g., Rogoff (1985), Alesina (1988), Grilli et al. (1991), Alesina & Summers (1993), and Cukierman (2008)) show that independent, transparent, accountable, and credible central banks are able to deliver better outcomes. Cecchetti & Krause (2002) study a large sample of countries and find that credibility is the primary factor explaining the cross-country variation in macroeconomic outcomes. Carriere-Swallow et al. (2016) find evidence that price stability and greater monetary policy credibility are important determinants of exchange rate pass-through in a sample of 62 emerging and advanced economies.

Agents’ expectations regarding central bank policy is directly tied to the concept of credibility. In a clean and uncontroversial statement about how these two connect, Blinder (2000) argues that "A central bank is credible if people believe it will do what it says". The definition of central bank credibility in the literature is usually related to a reputation built based on

\[Cukierman & Meltzer (1986) define monetary policy credibility as the absolute value of the difference between the policymaker’s plans and the public’s beliefs about those plans.\] 

\[Rogoff (1985) suggested that monetary policy should be placed in the hands of an independent central bank run by a ‘conservative’ central banker who would have a greater aversion to inflation than that of the public at large. This would help to reduce the inflation bias inherent in discretion.\]
strong aversion to inflation\textsuperscript{5} or a framework that is characterized by an explicit contract with incentive compatibility\textsuperscript{6} or a commitment to a rule or a specific and clear objective.

Svensson (1993) was the first to propose a test to monetary policy credibility, in the sense that market agents believe that future inflation will be on target. He proposes to compare ex-post target-consistent real interest rates with market real yields on bonds. In the case where the central bank has an explicit inflation target, Svensson (2000) proposes to measure credibility as the distance between the expected inflation and the target.

Bomfim & Rudebusch (2000) suggest another approach, measuring overall credibility by the extent to which the announcement of a target is believed by the private sector when forming their long-run inflation expectations. Specifically, they assume that the expectation of inflation at time $t$, denoted by $\pi_e^t$, is a weighted average of the current target, denoted by $\pi_t$, and last period’s (12-months) inflation rate, denoted by $\pi_{t-1}$:

$$\pi_e^t = \lambda_t \pi_t + (1 - \lambda_t) \pi_{t-1}$$

(1)

The parameter $\lambda_t$, with $(0 < \lambda_t < 1)$ measures the credibility of the central bank. If $\lambda_t = 1$, there is perfect credibility, and private sector’s long-run inflation expectations will be equal to the announced long-run goal of the policymaker. If $\lambda_t = 0$, there is no credibility and intermediate values of $\lambda_t$ represent partial credibility.

Following Bomfim & Rudebusch (2000), Demertzis et al. (2008) model inflation and inflation expectations in a general VAR framework, based on the fact that the two variables are intrinsically related. When the level of credibility is low, inflation will not reach its target because expectations will drive it away, and expectations themselves will not be anchored at the level the central bank wishes. They apply their framework to a group of developed countries and compute an anchoring effect based on $\lambda_t$.

Cecchetti & Krause (2002) construct an index of policy credibility that is an inverse function of the gap between expected inflation and the central bank’s target level, taking values from 0 (no credibility) to 1 (full credibility). The index is defined as follows:

$$I_{CK} = \begin{cases} 
1 & \text{if } \pi_e < \bar{\pi}_t \\
1 - \frac{\pi_e - \pi_t}{20\% - \bar{\pi}_t} & \text{if } \bar{\pi}_t < \pi_e \leq 20\% \\
0 & \text{if } \pi_e \geq 20\% 
\end{cases}$$

where $\pi_e$ is the expected inflation and $\bar{\pi}_t$ is the central bank target. Between 0 and 1 the value

\textsuperscript{5}More recently, after the 2008 financial crisis, central bank credibility has also been very much tied to strong aversion to deflation in many advanced economies.

\textsuperscript{6}Canzoneri (1985), Persson & Tabellini (1993) and Walsh (1995) have analysed alternative ways of allowing discretionary use of monetary policy with an incentive to achieve low inflation on average. One suggestion is to create a penalty on either government or central bank if the average inflation rate over a period exceeds the level consistent with price stability. This allows the monetary authorities to respond to shocks without triggering expectations of an inflation bias to policy. One form which such a penalty could take is the announcement in advance of an explicit inflation target. There would be a penalty—political, reputational or, as in New Zealand, loss of tenure of the central bank governor—were the target not to be achieved.
of the index decreases linearly as expected inflation increases. The authors define 20% as an ad hoc upper bound \(^7\).

Levieuge et al. (2016) consider an asymmetric measure of credibility based on the linear exponential (LINEX) function, as follows:

\[
I_L = \frac{1}{\exp(\phi(\pi^e - \bar{\pi})) - \phi(\pi^e - \bar{\pi})}, \text{ for all } \pi^e
\]

where \(\pi^e\) is the inflation expectations of the private sector, \(\bar{\pi}\) is the inflation target and, for \(\phi = 1\), positive deviations \((\pi^e > \bar{\pi})\) will be considered more serious than negative deviations \((\pi^e < \bar{\pi})\) as the exponential part of the function dominates the linear part when the argument is positive. As the previous indices, when \(I_L = 1\) the central bank has full credibility and when \(I_L = 0\) there is no credibility at all.

Bordo & Siklos (2015) propose a comprehensive approach to find cross-country common determinants of credibility. They organize sources of changes in credibility into groups of variables that represent real, financial, and institutional determinants for the proposed central bank credibility proxy. Their preferred index to measure central bank credibility is:

\[
I_{BS} = \begin{cases} 
\pi_{t+1}^e - \pi_t & \text{if } \pi_t - 1 \leq \pi_{t+1}^e \leq \pi_t + 1 \\
(\pi_{t+1}^e - \pi_t)^2 & \text{if } \pi_t - 1 > \pi_{t+1}^e > \pi_t + 1 
\end{cases}
\]

where \(\pi_{t+1}^e\) is the one-year-ahead inflation expectations and \(\pi_t\) is the central bank target. Credibility is then defined such that the penalty for missing the target is greater when expectations are outside the 1% interval than when forecasts miss the target inside this 1% range.

There is also a strand of literature that applies state space models and the Kalman Filter to develop credibility measures, since it is a latent variable (See Hardouvelis & Barnhart (1989) and Demertzis et al. (2012)). In a recent contribution, Vereda et al. (2017) apply this framework to the term structure of inflation expectations in Brazil to estimate the long-term inflation trend, as it can be associated to the market perception about the target pursued by the central bank. They follow the methodology in Kozicki & Tinsley (2012) and treat the so called shifting inflation endpoint as a latent variable estimated using the Kalman filter.

\(^7\)Mendonça & Souza (2007) propose an extension to the index in Cecchetti & Krause (2002), considering that not only positive deviations but also negative deviations of inflation expectations from the target can generate a loss of credibility:

\[
I_{DMGS} = \begin{cases} 
1 & \text{if } \pi_t \leq \pi^e \leq \pi_t^{\max} \\
1 - \frac{\pi^e - \pi_t^{\max}}{20\% - \pi_t^{\min}} & \text{if } \pi_t^{\max} < \pi^e < 20\% \\
1 - \frac{\pi^e - \pi_t^{\min}}{\bar{\pi}} & \text{if } \pi^e < \bar{\pi} \\
0 & \text{if } \pi^e \geq 20\% \text{ or } \pi^e \leq 0
\end{cases}
\]

where \(\pi^e\) is the inflation expectation of the private sector and \(\pi_t^{\min}\) and \(\pi_t^{\max}\) represent the lower and upper bounds of the inflation target range, respectively. The central bank is viewed as non credible \((I_{DMGS} = 0)\) if expected inflation is equal or greater than 20% or lower than or equal to 0%.
The disagreement between forecasters is also related to a credibility measure, focusing not on the consensus forecast, but on the distribution of the cross-section of forecasts. Dovern et al. (2012) propose that disagreement among professional forecasters of inflation reflect credibility of monetary policy and find that it is related to measures of central bank independence among G7 economies, suggesting that more credible monetary policy can substantially contribute to anchoring of expectations about nominal variables. They argue that for expectations to be perfectly anchored it is necessary that their cross-sectional dispersion (disagreement) disappears. In that sense, Capistrán & Ramos-Francia (2010) finds that the dispersion of long-run inflation expectations is lower in targeting regimes.

Finally, Lowenkron et al. (2007) and Guillén & García (2014) also bring relevant contributions to the Brazilian literature. Lowenkron et al. (2007) study the relation between agents’ inflation expectations 12 months ahead and inflation surprises and also look at inflation-linked bonds to evaluate the relation with inflation risk premia. Guillén & García (2014) look at the distribution of inflation expectations using a Markov chain approach, based on the fact that if an agent is persistently optimistic or pessimistic about inflation prospects\(^8\), this implicitly reveals a bias, which is an evidence of lack of credibility.

Next, we discuss two important problems with the previous literature and explain how we intend to deal with them.

The first problem relates to how beliefs are measured. It is common to employ private-sector survey expectations of inflation, measured by the cross-sectional average (consensus) of 12-month ahead inflation expectations. But, from a theoretical point-of-view, survey-based forecasts can be a biased estimate of the conditional expectation of inflation, as long as agents have an asymmetric loss function; see, e.g., Bates (1969), Granger (1999) and Christoffersen & Diebold (1997).

At least since Ito (1990), research has shown that individual forecasts are biased. The consensus forecast is biased as well, and Elliott et al. (2008) establish that current rationality tests are not robust to small deviations from symmetric loss, having little ability to tell whether the forecaster is irrational or the loss function is asymmetric. Research done with Brazilian data by Guillén & García (2014) has also shown that the consensus of professional forecasters is biased, as they persistently overestimate or underestimate inflation. Gaglianone & Issler (2018) confirm these results using a different testing strategy. They fix the problem by extracting possible sources of bias from the consensus forecast. This is how we will deal with this problem here as well.

The second problem of the previous literature is that most of the credibility indices discussed above usually employ a threshold above (or below) which credibility is nil. But, in all cases, this threshold is ad hoc and cannot be suitable for all countries and/or all time periods, restricting the validity of these indices and their comparability.

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\(^8\)The concept of optimistic or pessimistic here is related to an inflation forecast that is below or above the inflation target, respectively.
To deal with ad hoc confidence intervals we will employ a statistical procedure in constructing them, based on robust (HAC) consistent estimates of the long-run variance of inflation expectations. This allows the construction of confidence intervals for any confidence level, which will be based on a sound statistical foundation.

3 Building a microfounded index

This section brings a discussion of the econometric methodology proposed by Gaglianone & Issler (2018)\(^9\) to combine survey expectations\(^10\) in order to obtain optimal forecasts in a panel-data context. Their main assumptions and propositions are listed on Appendix B in more detail. Here, we further discuss how to obtain the asymptotic distribution of their estimator of the conditional mean of inflation, using a robust (heteroskedasticity and autocorrelation consistent (HAC)) covariance matrix estimation methods.

3.1 Econometric Methodology

The techniques proposed by Gaglianone & Issler (2018) are appropriate for forecasting a weakly stationary and ergodic univariate process \(\{y_t\}\) using a large number of forecasts that will be combined to yield an optimal forecast in the mean-squared error (MSE) sense. The forecasts for \(y_t\) are taken from a survey of agents’ expectations regarding the variable in question and are computed using conditioning information sets lagged \(h\) periods. These \(h\)-step-ahead forecasts of \(y_t\) formed at period \((t - h)\) are labeled \(f_{i,t}^h\), for individual \(i = 1, ..., N\), in time period \(t = 1, ..., T\) and horizon \(h = 1, ..., H\).

The econometric setup includes two layers, where in the first layer the individual form his optimal point forecast of \(y_t\) \((f_{i,t}^h)\) and in the second layer the econometrician uses the information about individual forecasts to make inference about the conditional expectation \(E_{t-h}(y_t)\). They show that optimal forecasts, are related to the conditional expectation \(E_{t-h}(y_t)\) by an affine function:

\[
 f_{i,t}^h = k_{i}^h + \beta_{i}^h \cdot E_{t-h}(y_t) + \varepsilon_{i,t}^h
\]  

This is the optimal \(h\)-step-ahead feasible forecast of \(y_t\) for each forecaster \(i\) in the sample, where \(\theta_i^h = [\beta_i^h \, k_i^h]'\) is a \((2 \times 1)\) vector of parameters and \(\varepsilon_{i,t}^h\) accounts for finite sample parameter uncertainty. As we do not need identification of all \(\beta_{i}^h\) and \(k_{i}^h\), to be able to identify \(E_{t-h}(y_t)\), the only focus is on their means. Then, averaging across \(i\) and assuming that \(\frac{1}{N} \sum_{i=1}^{N} \varepsilon_{i,t}^h \overset{p}{\to} 0\), allows identifying \(E_{t-h}(y_t)\) as:

\(^9\)It should be mentioned that this methodology extends the previous literature of forecasting in a panel-data context (see Palm & Zellner (1992), Davies & Lahiri (1995), Issler & Lima (2009), Lahiri et al. (2015)).

\[
E_{t-h}(y_t) = \text{plim}_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} f_{i,t}^{h} - \frac{1}{N} \sum_{i=1}^{N} \beta_i^{h} \frac{1}{N} \sum_{i=1}^{N} k_i^{h} \quad (3)
\]

under the assumption that the terms of the limit above converge in probability.

Their basic idea is to estimate the random variable \(E_{t-h}(y_t)\) through GMM techniques relying mainly on \(T\) asymptotics. The loss function is known only by the individual forecaster and this is an important source of heterogeneity in the forecasts. Moreover, there is no reason why forecasters will use a symmetric loss function. Indeed, in many practical problems the use of an asymmetric loss function is called for. This, in turn, will make the optimal forecast, \(f_{i,t}^{h}\), a biased and error ridden measure of the conditional expectation.

To be able to proceed with the GMM framework, they need to eliminate the latent variable \(E_{t-h}(y_t)\) in the GMM restrictions. Their solution is to use an equation proposed by Issler & Lima (2009) in a similar context to decompose \(y_t\) as follows:

\[
y_t = E_{t-h}(y_t) + \eta_t^{h} \quad (4)
\]

where \(\eta_t^{h}\) is a martingale-difference sequence and, by construction, \(E_{t-h}(\eta_t^{h}) = 0\). Therefore, they can use this decomposition to express \(f_{i,t}^{h}\) in the following way:

\[
f_{i,t}^{h} = k_i^{h} + \beta_i^{h} \cdot y_t + \nu_{i,t}^{h} \quad (5)
\]

where \(\nu_{i,t}^{h} \equiv \beta_i^{h} \cdot \eta_t^{h} + \epsilon_{i,t}^{h}\) is a composite error term. Under the assumption that \(E(\nu_{i,t}^{h}|\mathcal{F}_{t-h}) = 0\)\(^{11}\), where \(\mathcal{F}_{t-h}\) is the information set available at \(t-h\), it follows that:

\[
E[\nu_{i,t}^{h} \otimes z_{t-s}] = E[(f_{i,t}^{h} - k_i^{h} - \beta_i^{h} \cdot y_t) \otimes z_{t-s}] = 0 \quad (6)
\]

for all \(i = 1, \ldots, N, t = 1, \ldots, T\) and all \(h = 1, \ldots, H\), and where \(z_{t-s}\) is a vector of instruments, \(z_{t-s} \in \mathcal{F}_{t-s}\) with \(s \geq h\) and \(\otimes\) is the Kronecker product. But, as these moment conditions have too many parameters and as the parameters to be estimated by GMM don’t depend on \(i\), they use cross-section averages to reduce parameter dimensionality, as long as there is convergence in probability, leading to:

\[
E[(\bar{f}_{t}^{h} - \bar{k}^{h} - \bar{\beta}^{h} \cdot y_t) \otimes z_{t-s}] = 0 \quad (7)
\]

for all \(t = 1, \ldots, N\) and \(h = 1, \ldots, H\) and where \(\bar{f}_{t}^{h} = \frac{1}{N} \sum_{i=1}^{N} f_{i,t}^{h}\), \(\bar{k}^{h} = \frac{1}{N} \sum_{i=1}^{N} k_i^{h}\), and \(\bar{\beta}^{h} = \frac{1}{N} \sum_{i=1}^{N} \beta_i^{h}\).

Note that averaging across \(i\) is a good strategy to be able to identify and estimate \(E_{t-h}(y_t)\) from a survey of forecasts, since \(E_{t-h}(y_t)\) does not vary across \(i\). If we stack all moment conditions implicit in equation 7 across \(h\), it is even more clear that the initial problem collapsed\(^{11}\) as we noted before, by construction, \(E(\nu_{i,t}^{h}|\mathcal{F}_{t-h}) = 0\), so the assumption needed to obtain \(E(\nu_{i,t}^{h}|\mathcal{F}_{t-h}) = 0\) is only that \(E(\epsilon_{i,t}^{h}|\mathcal{F}_{t-h}) = 0\).
to one where we have \( H \times \text{dim}(z_{t-s}) \) restrictions and \( 2H \) parameters to estimate:

\[
\mathbb{E} \begin{bmatrix}
(f_{1,t} - \bar{k}^1 - \beta^1 \cdot y_t) \\
(f_{2,t} - \bar{k}^2 - \beta^2 \cdot y_t) \\
\vdots \\
(f_{H,t} - \bar{k}^H - \beta^H \cdot y_t)
\end{bmatrix} \otimes z_{t-s} = 0
\]

There are two cases here, regarding the limit in equation (3). The first case is when we let first \( N \to \infty \) and then we let \( T \to \infty \). The second is when we first let \( T \to \infty \) and \( N \) is fixed or \( N \to \infty \) after \( T \).

In the first case, note that under suitable conditions, the cross-sectional averages in (7) would converge in probability to a unique limit as \( N \to \infty \), that is, \( \text{plim}_{N \to \infty} \sum_{i=1}^{N} f_{i,t} - \hat{k}^h = \beta^h, \beta^h \neq 0 \) and \( |\beta^h| < \infty; \text{plim}_{N \to \infty} \sum_{i=1}^{N} k_i = k^h, k^h \neq 0 \) and \( |k^h| < \infty \) and \( \text{plim}_{N \to \infty} \sum_{i=1}^{N} f_{i,t} = f^h, f^h \neq 0 \) and \( |f^h| < \infty \), for all \( t = 1, \ldots, T \). After taking moment conditions and noting that \( N \to \infty \):

\[
\mathbb{E}[\nu_{t,t}^h \otimes z_{t-s}] = \mathbb{E}[(f_{1,t}^h - \hat{k}^h - \beta^h \cdot y_t) \otimes z_{t-s}] = 0
\]

Gaglianone & Issler (2018) show that the feasible extended bias corrected forecast

\[
\hat{\theta}^h = [\hat{k}^h; \hat{\beta}^h]'
\]

based on \( T \)-consistent GMM estimates \( \hat{\theta}^h = [\hat{k}^h; \hat{\beta}^h]' \) obeys the following condition:

\[
\mathbb{E}_{t-h}(y_t) = \text{plim}_{(N,T \to \infty)} \left[ \frac{1}{N} \sum_{i=1}^{N} f_{i,t} - \hat{k}^h \right]
\]

where \((N,T \to \infty)_{seq}\) denotes the sequential asymptotic approach proposed by Phillips & Moon (1999), when first \((N \to \infty)\) and then \((T \to \infty)\).

In the second case, when first \((T \to \infty)\) and then \((N \to \infty)\) or \( N \) is fixed after \((T \to \infty)\), a stronger assumption is needed to validate the moment condition, that is, \( \mathbb{E}(\nu_{t,t}^h | F_{t-h}) = 0 \).

Based on this assumption, they conclude that the feasible extended BCAF based on consistent GMM estimates of the vector of parameters \( \theta^h = [\hat{k}^h; \hat{\beta}^h]' \) obeys:

\[
\mathbb{E}_{t-h}(y_t) = \text{plim}_{(T,N \to \infty)} \left[ \frac{1}{N} \sum_{i=1}^{N} f_{i,t} - \hat{k}^h \right]
\]

This result implies that \( \mathbb{E}_{t-h}(y_t) \) can be consistently estimated as:

\[
\hat{\mathbb{E}}_{t-h}(y_t) = \frac{1}{N} \sum_{i=1}^{N} \frac{f_{i,t} - \hat{k}^h}{\beta^h}, \text{ or,}
\]

\[
\hat{\mathbb{E}}_{t-h}(y_t) = \frac{1}{N} \sum_{i=1}^{N} \frac{f_{i,t} - \hat{k}^h}{\beta^h}
\]
for each $h = 1, ..., H$, depending on whether we let first $N \to \infty$ and then $T \to \infty$ or first $T \to \infty$ and then $N \to \infty$ or hold $N$ fixed after $T \to \infty$.

Therefore, we can state that:

$$
\mathbb{E}_{t-h}(y_t) = \text{plim}_{(N,T,\to \infty)} seq\left[ \frac{1}{N} \sum_{i=1}^{N} f_{i,t} - \hat{k}_{h} \beta_{h} \right] = \text{plim}_{(T,N,\to \infty)} seq\left[ \frac{1}{N} \sum_{i=1}^{N} f_{i,t} - \hat{k}_{h} \beta_{h} \right] 
$$

regardless of the order in which $N$ and $T$ diverge, using a microfounded structural model for the $h$-period-ahead expectations based on GMM estimates of the parameters under $T$ asymptotics.

The estimates for $\mathbb{E}_{t-h}(y_t)$ can be interpreted as bias-corrected versions of survey forecasts because if the mean $k^h$ or $\overline{k}^h$ is zero and the mean $\beta^h$ or $\overline{\beta}^h$ is one, $\mathbb{E}_{t-h}(y_t)$ will converge to the same probability limit of the consensus forecast ($\frac{1}{N} \sum_{i=1}^{N} f_{i,t}$).

As it is clear from equation (2), individual forecasts are in general not equal to the conditional expectation $\mathbb{E}_{t-h}(y_t)$. This happens essentially because of asymmetric loss. Despite that, all forecasts depend on a common factor $\mathbb{E}_{t-h}(y_t)$, which we can think of as a market forecast. Under a symmetric risk function for the market as a whole, it will be the optimal forecast for inflation, which justifies its label as the market expectation of inflation, or, as people’s beliefs, i.e., Blinder (2000). This is our main identification assumption here regarding people’s beliefs: we equate people’s beliefs with $\mathbb{E}_{t-h}(y_t)$. Its feasible versions are estimated in equations (11) and (12).

Next, we discuss how to obtain the asymptotic distribution of $\hat{\mathbb{E}}_{t-h}(y_t)$, applying non parametric methods. We will use the non parametric approach because the restrictions used in GMM estimation may have unknown autocorrelation and heteroskedasticity properties.

### 3.2 Heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimation

Heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimation refers to calculation of covariance matrices that account for conditional heteroskedasticity of regression disturbances and serial correlation of cross products of instruments and regression disturbances. The heteroskedasticity and serial correlation may be of known or unknown form, and this will determine if the estimation method will be parametric or non parametric.

The relevant technical issue in this paper is that $\hat{\mathbb{E}}_{t-h}(y_t)$ is estimated from an orthogonality condition that involves a composite error term, $\nu_{i,t}^h$, that has unknown distribution, hence unknown autocorrelation and heteroskedasticity properties, so that it is impossible to use a parametric model for consistent estimation of the covariance matrix of the orthogonality restrictions.

The most widely used class of non parametric estimators in a GMM context relies on smoothing of autocovariances. Newey & West (1987) and Andrews (1991) are the main references in
this field, building on the literature on estimation of spectral densities. We offer an informal treatment of the subject in this subsection, but Appendix A brings a more complete review of the theory.

Let $\Gamma_j$ denote the $j$-th autocovariance of a stationary mean zero random vector $h_t(\theta)$, for which a valid orthogonality condition of the form $E[h_t(\theta)] = 0$ applies, where $h_t(\theta)$ is a product between a vector of instruments $Z_t$ and a vector of regression errors $\epsilon_t$, i.e., $h_t(\theta) = Z_t \cdot \epsilon_t(\theta)$. Thus, $\Gamma_j = E[h_t(\theta)h_{t-j}(\theta)]$. The long-run variance of $h_t(\theta)$ is defined as the sum of all autocovariances. Since $\Gamma_j = \Gamma'_{-j}$, we can write the long-run variance as:

$$S = \Gamma_0 + \sum_{j=1}^{\infty} (\Gamma_j + \Gamma'_{-j}) \tag{14}$$

As White (1984a) argues, from the point of view of the estimation of asymptotic covariance matrices, there are three cases regarding this sum. The first is where $Z_t \cdot \epsilon_t$ is uncorrelated, so that $S = \Gamma_0$. The second case is where $Z_t \cdot \epsilon_t$ is finitely correlated, so that the sum can be truncated from $j = 1$ to $j = m$ because covariances of order greater than $m$ are zero, so that $S = \Gamma_0 + \sum_{j=1}^{m} (\Gamma_j + \Gamma'_{-j})$. The third and last case, and the most interesting one, is when $Z_t \epsilon_t$ is an asymptotically uncorrelated sequence. When we do not have information about its covariance structure, an essential restriction is that $\Gamma_j \rightarrow 0$ as $j \rightarrow \infty$. Therefore, we shall assume that $Z_t \epsilon_t$ is a mixing sequence, which suffices for asymptotic uncorrelatedness.

The idea of smoothing autocovariances in non-parametric covariance matrix estimation is to use a series of weights that obey certain properties to guarantee a positive semi-definite estimator $\hat{\Gamma}_j$ for $\Gamma_j$. Newey & West (1987) considered estimators defined as:

$$\hat{S} = \hat{\Gamma}_0 + \kappa(j,l) \sum_{j=1}^{l} (\hat{\Gamma}_j + \hat{\Gamma}'_{-j}) \tag{15}$$

where $\kappa(j,l)$ is a kernel weight that goes to zero as $j$ approaches $l$, the bandwidth parameter. The idea is that covariances of higher order have less weight, and as they are estimated with less accuracy. Therefore, the use of a HAC estimator involves the specification of a kernel function and bandwidth parameter. In our main application, we use the Bartlett (1950) kernel as proposed by Newey & West (1987)$^{12}$ and the data dependent method proposed by Newey & West (1994) to choose the bandwidth parameter.

As we have two asymptotic cases, we will begin by analyzing the case when we let first $N \rightarrow \infty$ and then $T \rightarrow \infty$. Later, we analyze the case when we first let $T \rightarrow \infty$ and $N$ is fixed or $N \rightarrow \infty$ after $T$.

$^{12}$The Bartlett (1950) kernel is defined as $\kappa(j,l) = 1 - \frac{j}{l+1}$.
3.2.1 First case: \( N \to \infty \) and then \( T \to \infty \)

Let us first recall that the population moment condition of Gaglianone & Issler (2018), after cross-sectional averaging, is given by:

\[
\mathbb{E}[h_t(\theta_0^h)] = \mathbb{E}[
u^h \otimes z_{t-s}] = \mathbb{E}[(f_{t,l}^h - \kappa_t^h \cdot y_t) \otimes z_{t-s}] = 0
\]

for each \( h = 1, ..., H \), where \( \theta_0^h = (\kappa_0^h, \beta_0^h) \) is the true parameter value and \( \otimes \) is the Kronecker product.

When we let \( N \to \infty \), under suitable conditions, the cross section averages in (16) converge in probability to certain limits, such that, \( \text{plim}_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} k^h_i = k^h_0 \), \( k^h_0 \neq 0 \) and \( |k^h_i| < \infty \) and \( \text{plim}_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} f^h_{i,t} = f^h_t \), \( f^h_t \neq 0 \) and \( |f^h_t| < \infty \), for all \( t = 1, ..., T \).

Assuming that standard regularity conditions for consistency\(^{13}\) hold, after \( N \to \infty \), we can replace cross-sectional averages by their probability limits:

\[
\mathbb{E}[h_t(\theta_0^h)] = \mathbb{E}[(f_{t,l}^h - k^h_0 \cdot \beta_0^h \cdot y_t) \otimes z_{t-s}] = 0
\]

where \( \theta_0^h = [k^h_0; \beta_0^h]' \) stack the true parameter values. In this context, the GMM estimator \( \hat{\theta}^h = [k^h; \beta^h]' \) is a consistent estimator, i.e., \( \hat{\theta}^h \overset{p}{\to} \theta_0^h \), for all \( h \). The heteroskedasticity and autocorrelation consistent (HAC) covariance estimator of the long-run variance of the moment conditions is given by:

\[
\hat{S}^{(h)} = \hat{\Gamma}_0(\hat{\theta}^h) + \sum_{j=1}^{l} \kappa(j, l)(\hat{\Gamma}_j(\hat{\theta}^h) + \hat{\Gamma}'_j(\hat{\theta}^h))
\]

where

\[
\hat{\Gamma}_j(\hat{\theta}^h) = \frac{1}{T} \sum_{t=j+1}^{T} h_t(\hat{\theta}^h) h_{t-j}(\hat{\theta}^h),
\]

and \( \kappa(j, l) \) is the kernel function weight used to smooth the sample autocovariance function, \( l \) is the bandwidth parameter. We will employ the Bartlett kernel \( \kappa(l, j) = 1 - \frac{j}{l} \), so that the HAC covariance estimator is the same as the one proposed by Newey & West (1987).

To find the asymptotic distribution of \( \hat{E}_{t-h}(y_t) \) entails the following steps. First, as proved by Hansen (1982), the efficient GMM estimator \( \hat{\theta}^h \) is asymptotically normal as follows:

\[
\sqrt{T}(\hat{\theta}^h - \theta_0^h) \overset{d}{\to} \mathcal{N}
\left(0, \left(G^h S^{(h)}(\hat{\theta}^h)^{-1} G^hight)^{-1}\right)
\]

\(^{13}\)For the general class of Extremum Estimators, where the GMM estimator is included, if: i) \( Q_0(\theta) \) is uniquely maximized (or minimized) at \( \theta_0 \); ii) \( \Theta \) is compact ; iii) \( Q_0(\theta) \) is continuous; iv) \( \hat{Q}_n(\theta) \) converges uniformly in probability to \( Q_0(\theta) \), then \( \hat{\theta} \overset{p}{\to} \theta_0 \). Here, uniform convergence and continuity are the hypotheses that are often referred to as "the standard regularity conditions for consistency". For more details, see Newey & McFadden (1994), p. 2120-2140.
where the optimal weighting matrix is $S^{-1}$, $S = \mathbb{E}[h(\theta^h)h'(\theta^h)]$ and $G^h = \text{plim}_{T \to \infty} \frac{\partial h'(\theta^h)}{\partial \theta^h} |_{\theta^h = \hat{\theta}^h}$.

Notice that $S^h$ can be consistently estimated as shown in equations (18) and (19).

The next step is to note that $\hat{E}_{t-h}(y_t)$ is a continuous function of $\hat{\theta}^h$, $f : \mathbb{R}^2 \to \mathbb{R}$, such that
\[ f(\hat{\theta}^h) = f(\tilde{k}^h, \tilde{\beta}^h) = \frac{1}{N} \sum_{i=1}^{N} f_{i,t}^{h} \cdot \frac{\tilde{k}^h}{\tilde{\beta}^h}. \]
Moreover, $f(\hat{\theta}^h)$ has continuous first derivatives, so that the delta method can be applied to find the asymptotic variance of $\hat{E}_{t-h}(y_t)$:
\[
\sqrt{T}(f(\hat{\theta}^h) - f(\hat{\theta}_0^h)) \xrightarrow{d} \mathcal{N}\left(0, D(f(\hat{\theta}^h))' \left(G^h S^{(h)-1} G^h'\right)^{-1} D(f(\hat{\theta}^h))\right) \tag{21}
\]
where $D(f(\hat{\theta}^h))$ is the jacobian of $f(\hat{\theta}^h)$: $D(f(\hat{\theta}^h)) = \left[-\frac{1}{\beta^h}, \frac{-\frac{1}{\beta^h} \sum_{i=1}^{N} f_{i,t}^{h} \cdot \tilde{k}^h}{(\beta^h)^2}\right]'$.

Therefore, a consistent estimator of the long-run variance of $\hat{E}_{t-h}(y_t)$ is given by:
\[
\hat{V}^{(h)} = \left[-\frac{1}{\beta^h}, \frac{\sum_{i=1}^{N} (-f_{i,t}^{h} \cdot \tilde{k}^h)}{(\beta^h)^2}\right] \left(\frac{\hat{G}^h S^{(h)-1} \hat{G}^h}{\beta^h}\right)^{-1} \left[-\frac{1}{\beta^h}, \frac{-\frac{1}{\beta^h} \sum_{i=1}^{N} f_{i,t}^{h} \cdot \tilde{k}^h}{(\beta^h)^2}\right]' \tag{22}
\]
where, the estimate of $S$ is given in equation (18), and the estimate of $G^h$ is given by $\frac{\partial h'(\theta^h)}{\partial \theta^h} |_{\theta^h = \hat{\theta}^h}$.

Then,
\[
\sqrt{T}(\hat{E}_{t-h}(y_t) - \mathbb{E}_{t-h}(y_t)) \xrightarrow{A_{sy}} \mathcal{N}(0, V^{(h)}) \tag{23}
\]
or:
\[
\hat{E}_{t-h}(y_t) \xrightarrow{A_{sy}} \mathcal{N}\left(\mathbb{E}_{t-h}(y_t), \frac{V^{(h)}}{T}\right) \tag{24}
\]
where $V^{(h)}$ can be consistently estimated by $\hat{V}^{(h)}$ as in equation (22).

3.2.2 Second case: $T \to \infty$ and $N$ is fixed or $N \to \infty$ after $T \to \infty$

The population moment condition is given by:
\[
\mathbb{E}[h_t(\theta^h_t)] = \mathbb{E}[\nu_t \otimes z_{t-s}] = \mathbb{E}[(f_{t,t}^{h} - \tilde{k}_0^{h} - \tilde{\beta}_0^{h} \cdot y_t) \otimes z_{t-s}] = 0 \tag{25}
\]
for each $h = 1, \ldots, H$, where $\theta^h = (k_0^h, \beta_0^h)$ is the true parameter value for each $h = 1, \ldots, H$.

As before, we assume there exists a consistent estimator of of the true parameter $\hat{\theta}_0^h$, such that $\hat{\theta}_0^h \xrightarrow{P} \tilde{\theta}^h$, for all $h$, as proven by Hansen (1982), and $z_{t-s}$ is a vector of instruments with $\text{dim}(z_{t-s}) \geq 2$.

Following the same steps as before, a consistent estimate of the long-run variance of $\hat{E}_{t-h}(y_t)$ is given by:
\[
\hat{V}^{(h)} = \left[ \frac{-1}{\hat{\beta}^2} + \frac{1}{\hat{\beta}^2} T \sum_{t=1}^{T} \left( -f_{i,t}^{(h)} + \hat{k}^2 \right) \right] \left( \hat{G}^{h} \hat{S}^{h} \right)^{-1} \left[ \frac{-1}{\hat{\beta}^2} + \frac{1}{\hat{\beta}^2} T \sum_{t=1}^{T} \left( -f_{i,t}^{(h)} + \hat{k}^2 \right) \right]^{-1}
\]

where the estimate of \( \hat{G}^{h} \) is given by \( \frac{\partial h^{(\hat{\theta}^{h})}}{\partial \theta^{h}} \bigg|_{\theta^{h} = \hat{\theta}^{h}} \) and the estimate of \( \hat{S}^{h} \) is given by:

\[
\hat{S}^{(h)} = \left[ \hat{\Gamma}^{0}(\hat{\theta}^{h}) + \sum_{j=1}^{l} \kappa(j, l)(\hat{\Gamma}^{j}(\hat{\theta}^{h}) + \hat{\Gamma}^{j}_{l}(\hat{\theta}^{h})) \right]
\]

where,

\[
\hat{\Gamma}^{j}(\hat{\theta}^{h}) = \frac{1}{T} \sum_{t=j+1}^{T} h_{t}(\hat{\theta}^{h}) \hat{h}_{t-j}^{'}(\hat{\theta}^{h}).
\]

Then, we obtain:

\[
\hat{E}_{t-h}(y_t) \overset{\text{Asy}}{\sim} N\left( \hat{E}_{t-h}(y_t), V^{(h)} / T \right)
\]

where \( V^{(h)} \) can be consistently estimated by \( \hat{V}^{(h)} \) from equation (26).

Based on this limiting distribution, we can construct asymptotic confidence intervals for \( \hat{E}_{t-h}(y_t) \) for each \( h \), which will be of great usefulness in the construction of our measure of credibility.

Once we have characterized the asymptotic distributions as in equations (24) and (29), we can construct 95% HAC robust confidence intervals to be compared with the explicit or tacit targets for inflation in each point in time. This will determine whether or not the central bank was credible in that period. As we all know, there is nothing magic about 95% confidence bands and perhaps a broader approach can be employed, considering different levels of credibility risk. The use of fan charts is an interesting approach, since it allows the assessment of uncertainties surrounding point forecasts. This technique gained momentum following the publication of fan charts by the Bank of England in 1996. Because we are using asymptotic results, we will not allow asymmetries as is common place when fan charts are employed. However, the technique of expressing risk in its different layers is an interesting extension of the current setup.

### 3.3 A credibility test and a new credibility index

Computing the asymptotic distribution of \( \hat{E}_{t-h}(y_t) \) for each \( h \), as discussed in the previous subsection, we can use this distribution to construct a new credibility index. The basic idea is that a centered version of \( \hat{E}_{t-h}(y_t) \) is asymptotically normally distributed with zero mean and covariance matrix \( V^{(h)} \), which can be consistently estimated by \( \hat{V}^{(h)} \) from equation (22) or (26). We will argue here that the area under the probability density function of the normal distribution between the inflation target and \( \hat{E}_{t-h}(y_t) \) can be used to construct an index of
central bank credibility. As before, this index will be based on a sound statistical basis.

In our application, we could use both cases discussed before, where we let \( N \to \infty \) first or when we let \( T \to \infty \) first. But, as we will argue in the next section for the brazilian case, most surveys approximate better the case when we let first \( T \to \infty \) and after we let \( N \to \infty \) or \( N \) is fixed, as there are usually thousands of time-series daily observations (more than 3,000) and a limited number of participants in the cross sectional dimension (about 120).

Based on \( \hat{E}_{t-h}(y_t) \sim \mathcal{N}\left(\mathbb{E}_{t-h}(y_t), \frac{V(h)}{T}\right) \), we can establish the cumulative distribution function (CDF) of \( \hat{E}_{t-h}(y_t) \), which we label as \( F(x) \). Here, we use \( F(x) \) to construct a credibility index (\( CI_{IS} \)) that has the following properties: (i) if \( \hat{E}_{t-h}(y_t) = \pi^* \), where \( \pi^* \) is the central bank inflation target midpoint, \( CI_{IS} \) attains its maximum value at 1 and this would be the perfect credibility case; (ii) \( CI_{IS} \) decreases as the distance between \( \pi^* \) and \( \mathbb{E}_{t-h}(y_t) = \pi^* \) increases, asymptotically going to zero.

Figure (1) describes the idea behind the index, that will be measured as the density of \( F(x) \) between \( \hat{E}_{t-h}(y_t) \) and \( \pi^* \), denoted in the graph by the blue area.

**Figure 1:** Normal Distribution

![Normal Distribution](image.png)

The new credibility index proposed here obeys:

\[
CI_{IS} = \left\{1 - \frac{|F(\hat{E}_{t-h}(y_t)) - F(\pi^*)|}{1/2}; \quad \text{if} \quad -\infty < \pi^* < \infty. \right.
\]

The proposed index has some advantages over the existing indices in the literature. First, it relies on a pure statistical criterion and it does not depend on ad hoc bounds, making its applicability much broader. Second, it is based on a structural model for survey expectations that yields a consistent estimate of the conditional expectation. Additionally, it also encompasses the case when the consensus forecast is free of bias.

Based on the distribution of \( \hat{E}_{t-h}(y_t) \), and on the inflation target \( \pi_t^* \), the definition of credibility is the following. A central bank is credible if:

\[
\pi_t^* \subset \left[\hat{E}_{t-h}(y_t) - 1.96\hat{V}^{(h)}1/2, \hat{E}_{t-h}(y_t) + 1.96\hat{V}^{(h)}1/2\right]
\]

(30)

In words, a central bank is credible if the confidence interval around \( \hat{E}_{t-h}(y_t) \) contains the official inflation target \( \pi_t^* \). It is not credible otherwise. Therefore, our credibility definition is
based on a statistical criterion, avoiding the use of *ad hoc* bounds.

## 4 Empirical application

### 4.1 Data

We use the data available in the *Focus Survey* of forecasts of the Brazilian Central Bank (BCB). This is a very rich database, which includes monthly and annual forecasts from roughly 250 institutions for every working day and for many important economic variables, such as different inflation indicators, exchange rates, GDP, industrial production, balance of payments series, etc. The survey collects data on professional forecasters, including banks, investment banks, other financial institutions, non-financial companies, consulting firms, academic institutions, etc.

The survey started in May 1999, initially collecting forecasts of around 50 institutions mainly on price indices and GDP growth. It quickly evolved to include around 250 institutions and survey data on a much broader basis. About 120 institutions are active in the system today. In November 2001, the online survey was created and in March 2010 it was improved, resulting in the present version of the Market Expectations System\(^{14}\).

Our focus is on the inflation rate measured by the Brazilian Broad Consumer Price Index (IPCA), because it is the official inflation target of the Brazilian Central Bank. The Focus Survey contains short-term monthly forecasts, 12-month-ahead forecasts, and also year-end forecasts from 2 to 5 years ahead. These year-end forecasts are fixed-point forecasts, where the forecast horizon changes monthly as time evolves. In our main application, we will focus on 12-month ahead inflation expectations, since those are fixed-horizon forecasts that can be collected at the monthly frequency. Also, this horizon is large enough for inflation shocks to dissipate, since we do not want to associate the lack of credibility with the presence of mean-reverting shocks to inflation.

Our sample covers monthly inflation forecasts collected from November 2001 until April 2017. The survey is available since 1999, but data regarding expectations 12 months ahead is available only since November 2001. In each month \(t, t = 1, \ldots, T\), survey respondent \(i, i = 1, \ldots, N\), may inform her/his forecast for IPCA inflation rates for five calendar years, including the current year. In our sample, \(T=208\) months and \(N\) is, on average, 60.

In recent years, the annual inflation rate in Brazil as measured by IPCA has increased considerably, reaching over 10% per year. Inflation expectations for all horizons have also increased, as Figure 2 shows. The data in Figure 2 are slightly different across horizons. There are fixed-horizon forecasts up to the 12-month horizon which can be extracted directly from the data base. For horizons from 24-months up to 48-months, there are no fixed-horizon forecasts.

\(^{14}\)For more information on the survey, see *Carvalho & Minella (2012)* and *Marques (2013)*
available. However, year-end forecasts are available and can be combined to generate synthetic fixed-horizon forecasts using the methodology proposed by Dovern et al. (2012)\textsuperscript{15}.

Figure 2: Focus Fixed Horizon Inflation Forecasts

![Figure 2: Focus Fixed Horizon Inflation Forecasts](image)

Figure 2 shows the target and the resulting series for monthly inflation forecasts, with fixed horizons of 12, and of synthetic horizons of 24, 36 and 48 months. Table 1 shows descriptive statistics for the consensus over different horizons. Data for the 12-month horizon comes directly from the survey. For the longer horizons we need to employ the synthetic interpolation procedure based on year-end expectations. Since the inflation target was kept stable for 4.5% for most of our sample, the average of 12-month ahead expectations above 5.7% is worrisome, and could be a sign of lack of credibility of the Brazilian Central Bank for the sample as a whole or for parts of it.

Figure 2 plots the same data from 2000 through 2017, together with the target at every

\begin{equation}
    f_t^h = 100 \left( \left( 1 + \frac{f_{t+j}^{y+1}}{100} \right)^{\frac{12(y+1)-1}{12}} \left( 1 + \frac{f_{t+j+1}^{y+1}}{100} \right)^{\frac{t+1}{12}} - 1 \right)
\end{equation}

where $h$ is the fixed horizon forecast period measured in years, $h = 1, 2, 3, 4$; $t$ is the month when the forecast is calculated, $t = 1, ..., T$ and $y$ is the calendar year of the forecast collected by the survey, $y = 2001, ..., 2022$ in our sample and $j = 0, 1, 2, ...$. 

\textsuperscript{15}This requires the interpolation of inflation expectations data, which converts fixed-point inflation expectations to fixed-horizon expectations according to the following formula:
point in time. It shows three distinct periods when we compare the consensus expectations with the inflation target. The first one is from 2002 until end-2003, being characterized by a sudden and strong deterioration of inflation expectations due to the uncertainty around the 2002 election. After the election, uncertainty was reduced and expectations fell. The second period is from the end of 2003 until the end of 2010, being characterized by the stability of long-term inflation expectation around the target midpoint. The third period starts in the beginning of 2011, where we observe a slow and sustained rise in all measures of inflation expectations. Their synchronized movement shows that something more than a price shock happened in that period. Towards the end of the sample, after President Rousseff’s impeachment, there was a change in economic policy and in the main team of policymakers, including a change in the Brazilian Central Bank. Soon after, expectations fell quickly towards the target midpoint.

In our main application, the inflation expectation measure is the 12-month-ahead consensus forecast which we denote by \( f_{h,t} \), for \( t = 1, \ldots, T \) and \( h = 12 \), where \( h \) is measured in months and the agents’ forecasts are cross-sectionally averaged in the survey to form the consensus forecast, \( \bar{f}_{h} = \frac{1}{N} \sum_{i=1}^{N} f_{i,h,t} \). The Focus survey approximates better the case where \( T \to \infty \) while \( N \) is small, and therefore we use the results discussed in section 3.2.

Figure 3 compares the monthly consensus forecasts for inflation rate with the actual inflation rate for the 12-month-ahead horizon. In this figure, the measure of consensus forecasts represented by the black line was forwarded 12 months to match inflation, which it is supposed to track. In each month, we can compare IPCA inflation over the last 12 months and its respective forecast made 12-months prior.

As is clear from figure 3, the consensus forecast underestimates actual inflation quite often, and this is true in 68% of the sample.
4.2 Results

Table 2 presents the results of the generalized method of moments (GMM)\textsuperscript{16} estimation for the parameters $k^h$ and $\beta^h$, based on information about $f^h,t$ and $y_t$. We use as instruments up to two lags of the consensuses forecasts ($f^h,t$), up to two lags of the output gap\textsuperscript{17} and up to four lags of the 12-month variation of the Commodity Price Index (IC-Br)\textsuperscript{18,19}.

Our sample starts at 2001 and out-of-sample forecasts are constructed for the period between 2007 and 2017\textsuperscript{20}, re-estimating the coefficients at the end of each year, and using them to construct true out-of-sample forecasts of inflation. In the first column of Table 2, the sample denotes the period used in estimation, which is used to perform out-of-sample forecasts up to 12-months ahead. For example, the sample from 2001 to 2006 generates out-of-sample forecasts for the year of 2007; the sample from 2001 to 2007 generates out-of-sample forecasts for 2008, and so on, in a growing-window setup.

\textsuperscript{16} The iterative procedure of Hansen et al. (1996) is employed in the GMM estimation, and the initial weight matrix is the identity. The HAC covariance matrix is obtained following the steps discussed previously and using the Bartlett kernel as Newey & West (1987) proposed and the bandwidth selection is according to the automatic procedure proposed by Newey & West (1994).

\textsuperscript{17} The output gap is calculated using HP filter.

\textsuperscript{18} IC-Br is published monthly by the Brazilian Central Bank and its weighting structure is designed to measure the impact of commodity prices on Brazilian consumer inflation. For more details, see "Transfer of Commodity Prices to the IPCA and the Commodities Index-Brazil", Brazilian Central Bank Inflation Report, December 2010, at "http://www.bcb.gov.br/htms/relinf/ing/2010/12/ri201012b5i.pdf".

\textsuperscript{19} We estimated the coefficients with alternative sets of instruments and obtained similar results.

\textsuperscript{20} Our sample ends in April, 2017.
Table 2: GMM estimation results for the 12-months ahead forecast horizon

<table>
<thead>
<tr>
<th>Sample*</th>
<th>( \hat{k}_{12} )</th>
<th>( \hat{\beta}_{12} )</th>
<th>( \hat{\kappa}_{12} )</th>
<th>( \hat{\beta}_{12} )</th>
<th>OIR test p-value</th>
<th>Joint significance test p-value</th>
</tr>
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<td>2001-2006</td>
<td>2.16</td>
<td>0.59</td>
<td>0.00</td>
<td>0.00</td>
<td>0.71</td>
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<td>(0.07)</td>
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<td>0.00</td>
<td>0.73</td>
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<td>(0.10)</td>
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<td>(0.72)</td>
<td>(0.13)</td>
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<td>2001-2009</td>
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<td>(0.12)</td>
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<td>2001-2010</td>
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<td>0.00</td>
<td>0.46</td>
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<tr>
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<td>(0.61)</td>
<td>(0.11)</td>
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<tr>
<td>2001-2011</td>
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<td>0.02</td>
<td>0.00</td>
<td>0.53</td>
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<td>(0.15)</td>
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<tr>
<td>2001-2012</td>
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<td>0.54</td>
<td>0.00</td>
<td>0.00</td>
<td>0.61</td>
<td>0.00</td>
</tr>
<tr>
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<td>(0.63)</td>
<td>(0.11)</td>
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<tr>
<td>2001-2013</td>
<td>2.03</td>
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<td>0.00</td>
<td>0.60</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.64)</td>
<td>(0.11)</td>
<td></td>
<td></td>
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<tr>
<td>2001-2014</td>
<td>2.52</td>
<td>0.47</td>
<td>0.00</td>
<td>0.00</td>
<td>0.74</td>
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<tr>
<td></td>
<td>(0.47)</td>
<td>(0.08)</td>
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<tr>
<td>2001-2015</td>
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<td>0.67</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.54)</td>
<td>(0.09)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>2001-2016</td>
<td>2.35</td>
<td>0.50</td>
<td>0.00</td>
<td>0.00</td>
<td>0.67</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.54)</td>
<td>(0.09)</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Notes: The set of instruments is up to two lags of the consensus forecasts, up to two lags of the output gap and up to four lags of the 12-month variation of the Commodity Price Index (IC-Br).

*The sample used in estimation and in forecasting up to 12-month ahead.

In Table 2, Hansen’s over-identifying restriction (OIR)\textsuperscript{21} test is employed in order to check the validity of instruments and GMM estimates. For all sets of estimates we are far from rejecting the null, which is evidence that the instrument set is appropriate and orthogonal to regression errors. In the last column, we test for joint significance of the coefficients \([H_0: \hat{k}_{12} = 0 \text{ and } \hat{\beta}_{12} = 1]\). If the null hypothesis is not rejected, the consensus forecast is equal to the Extended BCAF and there is no evidence of any bias in it. But, we reject the null hypothesis with great confidence in all cases, a strong evidence of biases for the consensus forecast, requiring their respective extraction to construct a consistent estimate for \(E_{t-h}(y_t)\).

Our estimates in Table 2 show that both mean intercept and mean slope parameters are statistically significant for all samples. The average intercept value is 2.26, ranging from 1.89 to 2.52 and the average slope value is 0.56, ranging from 0.47 to 0.63.

Figure 4 compares consensus forecasts with the Extended BCAF estimated, for the 12-months ahead horizon. The grey line represents the consensus forecasts for the 12-months

\textsuperscript{21}Hansen’s J statistic is used to determine the validity of the over-identifying restrictions in a GMM model. For more details, see Hansen (1982).
ahead horizon, while the black line represents the estimated Extended BCAF for the same horizon. Notice that the behavior of the two is quite distinct, with our estimate of $E_{t-h}(y_t)$ below the consensus on early years and above it on later years of the sample.

**Figure 4:** Consensuses Forecasts and Extended BCAF: 12-months-ahead horizon

The next step is to estimate robust (HAC) covariance matrices and standard deviations based on the asymptotic distribution of the estimated Extended BCAF, and then construct the respective 95% confidence interval. Figure 5 includes $\hat{E}_{t-h}(y_t)$, its respective 95% robust confidence interval, and the inflation target from 2007:1 through 2017:4.

In Figure 5, the 95% confidence interval contains the target slightly above 65% of the time and the Extended BCAF increases considerably between mid-2013 and mid-2016, such that the confidence interval around in this period does not contain the target. As noted before, these were times when the increase in inflation expectations were widespread, indicating a de-anchoring of inflation expectations not consistent with the target of 4.5% annual inflation rate. Based on a statistical criterion, there is a significant distance between inflation expectations and the inflation target, which indicates that central-bank policy was not credible.

From mid-2016 onward, there was a very steep fall in inflation expectations. This happened at the same time when there was a change in the economic team of the Brazilian government, including the central bank administration and its board of directors. Perhaps these changes helped to consolidate the fall in inflation expectations, which were re-anchored according to our
Figure 5: BCAF and Confidence Intervals: 12-months-ahead

Figure 6 shows the same result as Figure 5. However, comparisons between beliefs and targets are only made at the end of each year, since, technically, the Brazilian Inflation Targeting Program has only year-end targets. Results are virtually unchanged: there is no credibility in 2007 and from 2013 to 2016, with the BCB recovering credibility in 2017.

Figure 7 presents the credibility index proposed above. The shaded regions indicate when the target is outside the 95% confidence interval, therefore, when the central bank is not credible. From 2008 on, there is a decrease in the credibility index of the central bank, although with some volatility. This decrease is finally consolidated in 2013, with an almost complete loss of credibility according to our index. Agents’ forecasts were significantly above target, which shows a disbelief in the target and/or a disbelief in the capacity or the willingness of the central bank to bring current inflation towards the target. From August 2013 to July 2016, the credibility index fell close to zero and stayed around zero for three years, until August 2016, where there was a very steep rise in the index: it rose from 0.06 in July to 0.53 in September, reaching 1.0 in December. In the first months of 2017, the index stayed between 0.95 and 1.00, expectations falling below target. This is evidence of a strong recovery of central-bank credibility, and a strong re-anchoring of inflation expectations.

Figure 8 compares our measure of central bank credibility with other indices proposed in the literature. Our index is labelled as the "IS Index". The other indices are labelled as the "CK Index" (Cecchetti & Krause (2002)), the "DGMS Index" (De Mendonça et al. (2009)), the "DM Index" (Mendonça & Souza (2007)) and the "LL Index" (Levieuge et al. (2016)).
4.3 Comparing our credibility index with other indices in the literature

First, note that our index shows results in line with those of the DM and the LL Index. Index DGMS has its maximum value when inflation expectations are inside the inflation target band, therefore, it is equal to one for almost the entire sample. Second, the behavior of our index is very different from all other indices for 2007 and the beginning of 2008: ours points out to a loss in credibility, since inflation expectations fall too much during this period – the Extended BCAF was below 2% for some months – whereas for all other indices credibility is above 0.4 and in some cases close to 1.0. Third, our index shows a loss of credibility during the global crisis of 2008-9, whereas the other indices do not. Again, this is due to inflation expectations being very low vis-a-vis the target. After the crisis, our index shows a prolonged period where the central bank keeps its credibility, which is confirmed by all other indices. Finally, all three indices (IS, DM and LL) show credibility falling to close to zero between 2013 and 2016. However, the CK and the DGMS indices are kept above 0.8 during the same period. In our view, this is not consistent with a severe loss of credibility shown by a rise in expectations depicted in Figure 2. Despite that, all indices show strong evidence of a rise in central bank credibility by the end of the sample, being either equal or very close to one.
5 Conclusion

Although central bank credibility is elusive, Blinder (2000) generated a consensus when he wrote that "A central bank is credible if people believe it will do what it says". This paper proposes a measure of credibility based upon this definition. To implement it, one needs a measure of people’s beliefs, of what central banks say they do, and a way to compare whether these two are the same.

We approach this problem in a novel way. To keep it tractable, we focus on inflation expectations, since inflation is arguably the main variable central banks care about. We extract people’s beliefs about inflation by modelling inflation expectations data using the panel-data approach of Gaglianone & Issler (2018), which shows that optimal individual forecasts are an affine function of one factor alone – the conditional expectation of inflation. This allows the identification and estimation of people’s beliefs, as well as the construction of robust confidence intervals around it. The latter is as an additional contribution of this paper. Based on this approach, we say that a central bank is credible if its explicit (or tacit) inflation target lies within the 95% robust asymptotic confidence interval of people’s beliefs. We also propose a credibility index based upon these concepts.

Our methodology seems sensible from an economic and econometric (statistical) point of view. Given the consensus around Blinder’s definition of credibility, it is economically sensible to equate people’s beliefs to what central banks say they do. Based on Gaglianone and Issler’s
methodology, it is straightforward to extract market expectations from a panel of individual expectations, where market expectations are the common latent factor in the latter. Since a large number of important countries has adopted Inflation Targeting Programs, one can easily obtain data on what central banks say they do by looking at actual targets of inflation for these countries. Even if a given country has not adopted such programs, there usually exists tacit targets at any point in time that could serve to that end.

From an econometric point of view, the methodology is relatively simple: we have a consistent estimate of the conditional expectation of inflation, which is a random variable. Comparing it to explicit targets only requires constructing robust asymptotic confidence intervals for these estimates. If targets lie within confidence bands, we say that targets are credible, i.e., the central bank is credible. It is important to note that market expectations are extracted from a survey of expectations of professional forecasters, which are active financial stakeholders most of them. This further validates the methodology proposed here.

The approach discussed above is applied to the issue of credibility of the Brazilian Central Bank (BCB) by using the Focus Survey of professional forecasters. This is a world-class database, fed by institutions that include commercial banks, investment banks, asset management firms, consulting firms, large corporations, and academics, inter alia. About 250 participants can provide daily forecasts for a large number of economic variables, e.g., inflation using different price indices, interest and exchange rates, GDP, industrial production, etc., and for different forecast horizons, e.g., current month, next month, 12-month ahead, 2, 3, 4, and 27.
Based on the proposed methodology, we estimated monthly the conditional expectation of inflation (12-month-ahead) and its respective robust asymptotic variance, constructing 95% HAC robust confidence intervals for inflation expectations from January 2007 until April 2017. This was then compared to the target in the Brazilian Inflation Targeting Regime. Results show that the BCB was credible 65% of the time, with the exception of a few months in the beginning of 2007 and during the interval between mid-2013 throughout mid-2016. We also constructed a credibility index for the sample 2007-17 and compared it with alternative measures of credibility that are popular in the literature.

Our results agree with the conventional wisdom in Brazil. In describing our empirical findings, we related them to the existing literature on credibility.
References


A Technical Appendix

The central theoretical issue of this paper is consistent and efficient estimation of what is called in the econometric literature as ‘long-run variance’. This refers to the calculation of a covariance matrix that accounts for conditional heteroskedasticity of regression disturbances and serial correlation of cross products of instruments and disturbances, or in other words, heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimation. In our main application, heteroskedasticity and autocorrelation is of unknown form, but this poses no problem in this theory, as will be discussed here, due to the development of non parametric procedures, the focus of this appendix.

Let $\theta$ be a $(k \times 1)$ vector of unknown parameters and $h_t$ a $(q \times 1)$ vector of functions of the data and parameters, $q \geq k$. The generalized moments estimators (GMM) of the true value of the parameter $\theta$ uses an orthogonality condition, $E[h_t(\theta)] = 0$ and chooses $\hat{\theta}$ as the solution to:

$$\min_{\theta} h_T(\theta)'W_T h_T(\theta) \quad (32)$$

where $\theta \in \Theta \subset \mathbb{R}^k$, $h_T(\theta) = \frac{1}{T} \sum_{t=1}^{T} h_t(\theta)$ is the vector of sample moments of $h_t(\theta)$ and $W_T$ is a symmetric positive semi-definite matrix. Here, $h_t = Z_t \cdot u_t$ is an orthogonality condition used to identify a $k$-dimensional parameter $\theta$, for a $(q \times l)$ matrix of instruments $Z_t$ and a $(l \times 1)$ vector of regression disturbances $u_t$, where $u_t$ is unobservable. However, this poses no problem as we are able to write $u_t$ as a function of observable data and regression parameters, such that $u_t = u_t(\theta)$.

When we have an overidentified model, with more orthogonality restrictions than the number of parameters to be estimated, the GMM setup includes a weighting matrix that will attribute weights to the orthogonality conditions in the estimation process. This weighting matrix will have an important role in the covariance matrix estimation, as we will explain later. Hansen (1982) has shown that the asymptotic covariance matrix of $\hat{\theta}$ is given by:

$$V_T = (H_T'W_T H_T)^{-1}(H_T'W_T S_T W_T H_T)(H_T'W_T H_T)^{-1} \quad (33)$$

where $H_T = \frac{1}{T} E\left(\frac{\partial h_t(\theta)}{\partial \theta}\right)$, $W_T$ converges in probability to a deterministic matrix $W$ and the law of large numbers guarantees that $h_T(\theta) = \frac{1}{T} \sum_{t=1}^{T} h_t(\theta)$ converges in probability to the sample analog evaluated at $\theta_0$, $\frac{1}{T} \sum_{s=1}^{T} \sum_{t=1}^{T} E[h_t(\theta) h_s(\theta)']$. But we don’t know much about $S_T = \frac{1}{T} \sum_{s=1}^{T} \sum_{t=1}^{T} E[h_t(\theta) h_s(\theta)']$.

Thus, as it is possible to estimate all the terms in the equation (33) but $S_T$, the main concern in the literature regarding asymptotic estimation of GMM covariance matrix is related to the consistent estimation of $S_T$, accounting for possible heteroskedasticity and autocorrelation. This is relevant for the construction of confidence intervals, hypothesis tests and for the formation

\[\text{See Hansen (1982).}\]
of an optimal GMM estimator. Hansen (1982) has shown that the GMM estimator with the smallest possible variance is obtained when this weighting matrix is chosen optimally, and in this case the asymptotic variance takes the form \((H_T^T S_T^{-1} H_T)^{-1}\), where the optimal weighting matrix converges in probability to \(S^{-1}\).

Changing variables, we may rewrite \(S_T\) as:

\[
S_T = \Gamma_0 + \sum_{j=-T+1}^{T-1} (\Gamma_j + \Gamma_j')
\]  

(34)

where \(\Gamma_T(j) = \frac{1}{T} \sum_{t=j+1}^{T} \mathbb{E}[h_t(\theta)h_{t-j}(\theta)']\) if \(j \geq 0\) and \(\Gamma_T(j) = \frac{1}{T} \sum_{t=-j+1}^{T} \mathbb{E}[h_t(\theta)h_{t-j}(\theta)']\) if \(j \leq 0\). Therefore, \(S_T\) can be written as a sum of all sample autocovariances, with \(\Gamma_j\) denoting the \((q \times q)\) autocovariance.

If the autocovariances are zero after a lag \(n\), we could just replace population variables for sample variables in equation (34) to obtain the truncated estimator, which was the first estimator proposed in the literature:

\[
\hat{S}_T = \Gamma_0 + \sum_{j=1}^{m} (\hat{\Gamma}_j + \hat{\Gamma}_j')
\]  

(35)

The main obstacle to the truncated estimator is that the number of nonzero autocovariances of \(h_t(\theta)\) should be known a priori. But even in this case, truncated estimator need not yield necessarily a positive semi-definite matrix in many applications, as West (1997) shows through simulations, what turns this option into little use.

A wide strand of literature used spectral density estimation in the HAC estimation context. This was motivated by the fact that when the process \(h_t(\theta)\) is second order stationary, it has spectral density matrix equal to:

\[
f(\lambda) = \frac{1}{\pi} \sum_{-\infty}^{\infty} \Gamma(j)e^{-ij\lambda}
\]  

(36)

where \(\Gamma(j) = E h_t(\theta) h_{t-j}(\theta)'\) and \(i = \sqrt{-1}\). When \(T \to \infty\) in the equation 34, this limit equals \(2\pi\) times the spectral density matrix at frequency zero \((\lambda = 0)\). The estimators proposed by White (1984b), Gallant (1987) and Newey & West (1987) correspond to kernel density estimators evaluated at \(\lambda = 0\), and these were the main early contributions to the development of this literature, alongside the work of Andrews (1991).

Therefore, the most widely used class of estimators relies on smoothing of autocovariances. The idea is to use a series of weights that obey certain properties and will guarantee a positive semi-definite estimator. Andrews (1991) considers that the estimator can be represented as

\[
\hat{S} = \Gamma_0 + \sum_{j=1}^{l} \kappa(j,l)(\hat{\Gamma}_j + \hat{\Gamma}_j')
\]  

(37)
as $\Gamma_j = \Gamma_{-j}$, for a series of kernel weights $\kappa(j,l)$. To obtain consistent estimators, we need $\kappa(j,l)$ near zero for values of $j$ near $l$, the bandwidth parameter, as autocovariances at large lags are estimated imprecisely, and also $\kappa(j,l) \to 1$ for each $j$. Andrews considered the class of kernel estimators of the spectral density matrix proposed by Parzen (1957) and showed that the quadratic spectral kernel is asymptotically optimal within the class of kernels that generate positive semi-definite estimates.

Andrews & Monahan (1992) considered prewhitened kernel estimators with vector autoregression (VAR) employed in the prewhitening stage. They showed through Monte Carlo simulations that prewhitening is "effective in reducing bias, improving confidence interval coverage probabilities and reducing overrejection of $t$ statistics constructed using kernel HAC estimators". However, prewhitening inflate the variance and MSE of kernel estimators. They argue that when there is considerable temporal dependence in the data, standard kernel HAC covariance matrix estimators often yield confidence intervals whose test statistics reject too often. The idea behind prewhitening is to transform the data into an uncorrelated sequence before applying a kernel estimator when constructing a HAC covariance matrix estimator.

Therefore, the use of a HAC estimator involves the specification of a kernel function and bandwidth parameter. In our main application, we use the Bartlett kernel as proposed by Newey & West (1987)\(^{23}\) and, regarding the bandwidth parameter, we use the data dependent method proposed by Newey & West (1994). As we can see, many authors suggested different kernel functions as a rule for weighting the autocovariances. The main ones are summarized in table 3.

<table>
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<th>Kernel</th>
<th>Formula</th>
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<td>Truncated</td>
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</tr>
<tr>
<td>Bartlett</td>
<td>$k(x) = \begin{cases} 1-</td>
</tr>
<tr>
<td>Parzen</td>
<td>$k(x) = \begin{cases} 1-6x^2 + 6</td>
</tr>
<tr>
<td>Quadratic Spectral</td>
<td>$k(x) = \frac{25}{12\pi^2 x^2} \left( \frac{\sin(6\pi x/5)}{6\pi x/5} - \cos(6\pi x/5) \right)$</td>
</tr>
</tbody>
</table>

The use of a HAC estimator involves not only the specification of a kernel function, but also a choice of the bandwidth parameter, i.e., the number of autocovariances that will be

\(^{23}\)The Bartlett kernel is defined as $\kappa(j,l) = \frac{l+j-l}{l+j}$. 

35
included. This is relevant because Andrews (1991) showed that \( \hat{S}_T \) converges at different rates for different rules for choosing \( m \). Newey & West (1987) showed that for a given kernel it was necessary for consistency to let the bandwidth increase with the sample size, but left open the question of how many autocovariances to include, for a given sample.

Andrews (1991) and Andrews & Monahan (1992) proposed procedures for selecting the bandwidth optimally, but those require the knowledge of the ARMA model governing residual autocorrelation. Newey & West (1994) proposed a data dependent model to automatically choose the bandwidth parameter, even when the form of the autocorrelation is unknown. They show that this automatic selection procedure tends to lead to more accurately sized test statistics than traditional procedures, and one could obtain even more accurate statistics if prewhitening is combined with this procedure. They show, contrary to Andrews (1991), that if the bandwidth is selected accordingly to their optimal procedure, choice of kernel is of secondary importance.

More recently, there is a literature that argues that even if the estimator is not consistent, it is possible to find well defined test statistics to carry out hypothesis testing, setting bandwidth equal to the sample size (see Kiefer et al. (2000) and Kiefer & Vogelsang (2002)).

In this Appendix, our focus was to summarize the most important developments on nonparametric HAC estimation procedures, as we followed this methodology. However, the literature developed also classes of estimators when \( h_t(\theta) \) follows a parametric model, such that a moving average process\(^{24}\) or an autoregressive process\(^{25}\).

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**B Assumptions and Propositions of Gaglianone and Issler (2015)**

In this appendix, we replicate the main assumptions and propositions of Gaglianone & Issler (2015) that are relevant to obtain the main result of the paper, that survey forecasts are an affine function of the conditional expectation of the target variable.

The techniques reproduced here are appropriate for forecasting a weakly stationary and ergodic univariate process \( \{y_t\} \) using a large number of forecasts, in this case the result of an opinion poll on the variable in question. Individual forecasts of \( y_t \), computed using information sets lagged \( h \) periods, are \( f_{i,t}^h \), \( i = 1, 2, \ldots, N \), and \( t = 1, 2, \ldots, T \). Therefore, \( f_{i,t}^h \) are \( h \)-step-ahead forecasts of \( y_t \), formed at period \( t-h \), and \( N \) is the number of respondents of this opinion poll regarding \( y_t \).

They consider a setup which has two layers of decisions to be made. In the first layer, individuals (survey respondents) form their optimal point forecasts of a random variable \( y_t \) by using a specific loss function under different assumptions about knowledge of the DGP of \( y_t \).

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\(^{25}\)Den Haan & Levin (1997)
These optimal forecasts \( f_{t,t-h}^i \) will be available as survey results, where potentially \( N \to \infty \) and \( T \to \infty \). In the second layer of decisions, an econometrician will be the final user of this large number of forecasts and he operates under an MSE risk function\(^{26}\). Hence, his optimal forecast in this second layer of decision making is \( \mathbb{E}_{t-h}(y_t) \).

**Assumption A1 (Loss function)** [Gaglianone and Issler (2015), Patton and Timmermann (2007)] \( L^i \) depends solely\(^{27}\) on the forecast error \( e_{t,t-h}^i \equiv y_t - \tilde{f}_{t,t}^h \), that is, \( L^i = L(e_{t,t-h}^i) \).

**Assumption A2 (Shape of the Loss function)** [(Gaglianone and Issler (2015), Granger and Newbold (1986), Patton and Timmermann (2007))] The loss function exhibits the following properties: (i) \( L^i(0) = 0 \); (ii) \( L^i(e_i) \) is continuous, homogeneous and non-negative \( \forall e_i \in \mathbb{R} \); and (iii) \( L^i(e_i) \) is monotonic non-decreasing (for \( e_i > 0 \) or \( e_i < 0 \), and differentiable at least twice almost everywhere.

**Assumption A3 (Asymmetry of the Loss function)** [Gaglianone and Issler (2015), Granger and Newbold (1986), Patton and Timmermann (2007)] The loss function \( L^i(e_i) \) can be decomposed as \( L^i(e_i) = g^i(e_i)h^i(e_i) \), where \( g^i(e_i) \) is a non-negative and symmetric function about \( e_i = 0 \); \( g''(e_i) \) and \( g''(e_i) \) exist almost everywhere; \( h^i(e_i) = \begin{cases} \beta_1^i ; & e_i < 0 \\ \beta_2^i ; & e_i > 0 \end{cases} \) where \( \{\beta_1^i, \beta_2^i\} \) are positive constants.

**Assumption A4 (DGP - stationarity and regularity of the CDF)** [Gaglianone and Issler (2015)] The univariate time series \( y_t \) is a weakly stationary and ergodic process and the conditional cumulative distribution function (CDF) of \( y_t \), given \( F_{t-h} \), is absolutely continuous, with continuous densities \( f_{t,t-h} \) uniformly bounded away from 0 and \( \infty \) at the points \( F_{t,t-h}^{-1}(\tau) \), \( \forall \tau \in (0;1) \), where \( \tau \) denotes the quantile level with respect to the (conditional) CDF of \( y_t \).

**Proposition 1 (Asymmetric Loss)** [Gaglianone and Issler (2015)] Denote by \( Med_{t-h}(y_t) \) the conditional median of \( y_t \). If A1-A4 hold, then: (i) If \( \beta_1 \neq \beta_2 \) then \( F_{t,t-h}(\tilde{f}_{t,t}^h) \neq 0.5 \), where \( F_{t,t-h} \) is the conditional CDF of \( y_t \); (ii) If \( \beta_1 > \beta_2 \) then \( \tilde{f}_{t,t}^h < Med_{t-h}(y_t) \); (iii) If \( \beta_1 < \beta_2 \) then \( \tilde{f}_{t,t}^h > Med_{t-h}(y_t) \); and (iv) for two forecasters \( i \) and \( j \) such that \( \beta_1^i/\beta_2^i < \beta_1^j/\beta_2^j < 1 \), then, \( \tilde{f}_{t,t}^h > \tilde{f}_{j,t}^h > Med_{t-h}(y_t) \).

**Proposition 2 (DGP - parametric PDFs)** [Gaglianone and Issler (2015)] If A1-A4 hold and the conditional PDF of \( y_t \) is: (i) Gaussian, Two-piece Normal, or Logistic, then, \( \tilde{f}_{t,t}^h = k_i^h + \mathbb{E}_{t-h}(y_t) \); (ii) Log-normal or Weibull, then, \( \tilde{f}_{t,t}^h = \beta_i^h \mathbb{E}_{t-h}(y_t) \); (iii) \( (a = 1, b > 0) \), then, \( \tilde{f}_{t,t}^h = k_i^h + \beta_i^h \mathbb{E}_{t-h}(y_t) \); (iv) Beta\( (a > 0, b = 1) \), then, \( \tilde{f}_{t,t}^h = \beta_i^h \varphi(\mathbb{E}_{t-h}(y_t); \tau_i) \), where \( \varphi(\mathbb{E}_{t-h}(y_t); \tau_i) = \exp\left(\frac{\ln(\tau_i)}{\mathbb{E}_{t-h}(y_t)}\right) \) and \( \tau_i \equiv F_{t,t-h}(\tilde{f}_{t,t}^h) \).

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\(^{26}\)They argue that this assumption can be modified, but it potentially covers reasonably well some interesting cases, which are of practical importance, e.g., the government, a central bank, a large risk-neutral firm, etc.

\(^{27}\)According to Patton and Timmermann (2007), although it rules out certain loss functions (e.g., those which also depend on the level of the predicted variable), many common loss functions are of this form.
Assumption A5 (DGP - location-scale) [Gaglianone and Issler (2015)] The DGP of \( y_t \) follows a location-scale model, with conditional mean and variance dynamics defined as 
\[
y_t = X_{t-h}^\prime \delta + \left( X_{t-h}^\prime \gamma \right) \eta_t, \quad \text{in which } (\eta_t | F_{t-h}) \sim i.i.d. \ F_{\eta,h} (0, 1), \text{where } F_{\eta,h} (0, 1) \text{ is some distribution with zero mean and unit variance, which depends on } h \text{ but does not depend on } F_{t-h}; \ X_{t-h} \in F_{t-h} \text{ is a } m \times 1 \text{ vector of covariates (which includes the intercept, and that can be predicted using information available at time } t - h) \text{ and } \delta = [\delta_0; \delta_1; \ldots; \delta_{m-1}] \text{ and } \gamma = [\gamma_0; \gamma_1; \ldots; \gamma_{m-1}] \text{ are } m \times 1 \text{ vectors of parameters.}
\]

Proposition 3 (Location-scale model) [Gaglianone and Issler (2015)] If A1-A5 hold, then: (i) the optimal forecast is a linear function of the conditional mean of \( y_t \), so that 
\[
\tilde{f}^h_{i,t} = k_i^h + \beta_i^h \hat{E}_{t-h}(y_t);
\]
(ii) in the absence of scale effects on the DGP (\( \gamma_1 = \gamma_2 = \ldots = \gamma_{m-1} = 0 \)) it follows that \( \beta_i^h = 1 \), for all \( i \), i.e., 
\[
\tilde{f}^h_{i,t} = k_i^h + \hat{E}_{t-h}(y_t).
\]

Assumption A6 [Gaglianone and Issler (2015)] Define 
\[
[k_i^h; \beta_i^h] = [\hat{\alpha}_0(\tau_i) - \frac{\hat{\mu}}{\hat{\sigma}} \hat{\alpha}_1(\tau_i); \frac{\hat{\mu}}{\hat{\sigma}} \hat{\alpha}_2(\tau_i)],
\]
where \( [\hat{\alpha}_0(\tau_i); \hat{\alpha}_1(\tau_i)] \) are the resulting estimates (intercept and slope) of a standard linear quantile regression of \( y_t \) onto \( [1; x_{t-h}] \) at quantile level \( \tau_i \). In addition, let the average co-

Proposition 4 [Gaglianone and Issler (2015)] If A1-A6 hold, then, the optimal (feasible) forecast of \( y_t \) conditioned on \( F_{t-h} \) is of the form: 
\[
f^h_{i,t} = k_i^h + \beta_i^h \hat{E}_{t-h}(y_t) + \varepsilon_i^h \text{ for finite sample parameter uncertainty, and } [k_i^h; \beta_i^h] \text{ are consistent estimates of } [k_i^h; \beta_i^h].
\]

Assumption A7 [Gaglianone and Issler (2015)] Let \( \varepsilon_i^h = (\varepsilon_{1,i}^h, \varepsilon_{2,i}^h, \ldots, \varepsilon_{N,i}^h)' \) be a \( N \times 1 \) vector stacking the errors \( \varepsilon_i^h \) associated with all possible forecasts. Assume that the vector process \( \{ \varepsilon_i^h \} \) is covariance-stationary and ergodic for the first and second moments, uniformly on \( N \), and that \( \text{E} (\varepsilon_i^h) = 0 \) for all \( i \) and \( t \), given \( h \). Furthermore, assume that 
\[
\lim_{N \to \infty} \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \text{E} (\varepsilon_{i,t}^h \varepsilon_{j,s}^h) &= 0,
\]
for all \( t \) and \( s \), given \( h \).

Assumption A8 [Gaglianone and Issler (2015)] Assume that \( \text{plim}_{N \to \infty} \frac{1}{N} \beta_i^h = \beta^h \neq 0 \), \( |\beta^h| < \)

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28 Also assume that regularity conditions A1-A2 of Koenker (2005) on \( x_{t-h} \) are met, and that \( \alpha(\tau) \) is continuous and Riemann-integrable on \( [0, 1] \).

29 Assumption A7 guarantees that the errors \( \varepsilon_i^h \) can be diversified away, and that cross-sectional dependence is not a problem. It is required in a GMM context in order to ensure that 
\[
\text{E}_{t-h} \left( \text{plim}_{N \to \infty} \frac{1}{N} \varepsilon_i^h \right) = 0.
\]

30 Assumption A8 just requires finite convergence of different cross-sectional averages, which bounds the degree of cross-sectional and time-series dependence due to spatial dependence. They are expected to hold on a stationary-ergodic context.
\[ \lim_{N \to \infty} \left( \frac{1}{N} \sum_{i=1}^{N} h_i \right) = h, \quad \left| k^h \right| < \infty, \quad \text{and} \quad \lim_{N \to \infty} \left( \frac{1}{N} \sum_{i=1}^{N} f_{i,t}^h = f_{t}^h, \right) \mid f_{t}^h \right| < \infty, \text{for all } t = 1, 2, \ldots, T. \]

**Assumption A9** [Gaglianone and Issler (2015)] Assume that the identification conditions for GMM estimation are met and that there is a unique set of values \( \theta_0^h = [k_0^h; \beta_0^h]' \), \( h = 1, 2, \ldots, H \), that solve the orthogonality condition \( \mathbb{E} \left[ (f_{t}^h - k_0^h - \beta_0^h y_t) \otimes z_{t-s} \right] = 0 \), for each \( h \) separately\(^{32}\).

The following proposition is an important result, as it allows for consistent estimation of \( \mathbb{E}_{t-h}(y_t) \).

**Proposition 5** [Gaglianone and Issler (2015)] If A1-A9 hold, then, the feasible Extended BCAF (Bias Corrected Average Forecast) \( \frac{1}{N} \sum_{i=1}^{N} \frac{f_{i,t} - k_i^h}{\beta_i^h} \), based on \( T \)-consistent GMM estimates \( \hat{\theta} = \left[ \hat{k}^h; \hat{\beta}^h \right]' \), obeys the condition \( \lim_{N \to \infty} \left( \frac{1}{N} \sum_{i=1}^{N} \frac{f_{i,t} - k_i^h}{\beta_i^h} \right) = \mathbb{E}_{t-h}(y_t) \), where \( (N, T \to \infty)_{\text{seq}} \) denotes the sequential asymptotic approach proposed by Phillips & Moon (1999), when we let first \( N \to \infty \), and then let \( T \to \infty \).

We now turn into the more complicated case where in the sequential asymptotics we let first \( T \to \infty \), and then \( N \to \infty \), or, that we let \( T \to \infty \) with \( N \) fixed.

**Assumption A10** [Gaglianone and Issler (2015)] Let \( \varepsilon_t^h = (\varepsilon_{1,t}^h, \varepsilon_{2,t}^h, \ldots, \varepsilon_{N,t}^h)' \) be a \( N \times 1 \) vector stacking the errors \( \varepsilon_{t}^h \) associated with all possible forecasts. Assume that the vector process \( \{ \varepsilon_t^h \} \) is covariance-stationary and ergodic for the first and second moments, uniformly on \( N \), and that \( \mathbb{E}(\varepsilon_t^h |_{1-h}) = 0 \) for all \( t \), given \( h \).

**Assumption A11** [Gaglianone and Issler (2015)] Define \( \frac{1}{N} \sum_{i=1}^{N} \beta_i^h = \beta^h \) and \( \frac{1}{N} \sum_{i=1}^{N} k_i^h = k^h \). Assume that, for all \( N \), the identification conditions for GMM estimation are met and that there is a unique set of values \( \theta_0^h = [k_0^h; \beta_0^h]' \), \( h = 1, 2, \ldots, H \), that solves the orthogonality condition aforementioned for each \( h \) separately. We further assume that the additional regularity conditions used by Hansen (1982) in proving consistency of GMM are met as well.

**Proposition 6** [Gaglianone and Issler (2015)] If A1-A6 and A10-A11 hold, then, the feasible Extended BCAF (Bias Corrected Average Forecast) \( \frac{1}{N} \sum_{i=1}^{N} \frac{f_{i,t} - k_i^h}{\beta_i^h} \), based on \( T \)-consistent GMM estimates \( \hat{\theta} = \left[ \hat{k}^h; \hat{\beta}^h \right]' \), obeys \( \lim_{N \to \infty} \left( \frac{1}{N} \sum_{i=1}^{N} \frac{f_{i,t} - k_i^h}{\beta_i^h} \right) = \mathbb{E}_{t-h}(y_t) \), where \( (T, N \to \infty)_{\text{seq}} \) denotes the sequential asymptotic approach proposed by Phillips & Moon (1999).

**Assumption A12** [Gaglianone and Issler (2015)] Let first hold \( N, T, \) and \( h \) fixed, and define \( Y_1 = \left( \frac{1}{N} \sum_{i=1}^{N} \frac{f_{i,t} - k_i^h}{\beta_i^h} \right) \). Letting now \( T \to \infty \), this defines \( \lim_{T \to \infty} Y_1 \equiv Y_2 = \left( \frac{1}{N} \sum_{i=1}^{N} \frac{f_{i,t} - k_i^h}{\beta_i^h} \right). \)

\(^{31}\)Assumption A9 deals with GMM identification and is standard in the literature.

\(^{32}\)Also further assume that the additional regularity conditions used by Hansen (1982) in proving \( T \)-consistency of GMM estimates \( \hat{\theta} = [\hat{k}^h; \hat{\beta}^h]' \) are met as well.
both $N, T \to \infty$, this defines $\operatorname{plim}_{N,T \to \infty} Y_1 = Y_3 = y_t + \eta_i^h = \mathbb{E}_{t-h}(y_t)$.

**Assumption A13** [Gaglianone and Issler (2015)] $\limsup_{N,T} P \{ \| Y_1 - Y_2 \| > \varepsilon \} = 0, \forall \varepsilon > 0$, where $\| A \|$ is the Euclidean norm $(\operatorname{tr} (A' A))^{1/2}$.

**Proposition 7** [Gaglianone and Issler (2015)] If A1-A13 hold, then, both feasible extended BCAF s \( \left( \frac{1}{N} \sum_{i=1}^{N} f_{i,t} - \hat{k}^h \right) \) and \( \left( \frac{1}{N} \sum_{i=1}^{N} \frac{f_{i,t} - \hat{k}^h}{\hat{\beta}^h} \right) \), based respectively on $T$-consistent GMM estimates \( \hat{\theta}^h = \left[ \hat{k}^h; \hat{\beta}^h \right] \) and \( \hat{\theta}^h = \left[ \hat{k}^h; \hat{\beta}^h \right] \), obey $\operatorname{plim}_{(T,N \to \infty)} \left( \frac{1}{N} \sum_{i=1}^{N} \frac{f_{i,t} - \hat{k}^h}{\hat{\beta}^h} \right) = \mathbb{E}_{t-h}(y_t)$, regardless of the order in which $N$ and $T$ diverge.

Under different assumptions, their results above imply that we can estimate consistently $\mathbb{E}_{t-h}(y_t)$, respectively, as follows:

\[ \hat{\mathbb{E}}_{t-h}(y_t) = \frac{1}{N} \sum_{i=1}^{N} \frac{f_{i,t} - \hat{k}^h}{\hat{\beta}^h}, \text{ or,} \]

\[ \hat{\mathbb{E}}_{t-h}(y_t) = \frac{1}{N} \sum_{i=1}^{N} \frac{f_{i,t} - \hat{k}^h}{\hat{\beta}^h}, \]

depending on whether we let first $N \to \infty$, and then let $T \to \infty$, or, we either let first $T \to \infty$, and then let $N \to \infty$, or hold $N$ fixed after $T \to \infty$. In any case, estimation of $[k^h, \beta^h]$ or of $[\hat{k}^h, \hat{\beta}^h]$ is performed by GMM under $T$-asymptotics. These estimates of $\mathbb{E}_{t-h}(y_t)$ can be viewed as bias-corrected versions of survey forecasts.