On the origins of monetary exchange*

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Abstract

1 Introduction

2 Model

Time is discrete and indexed by $t \geq 0$. The economy is populated by a continuum of nonatomic agents and by one large agent, the government. Both the agents and the government are infinitely lived, have the same discount factor $\beta \in (0, 1)$, and maximize the present discounted sum of their per-period expected payoffs.

There are two types of non-storable goods: a special indivisible good and a general divisible good. Agents can produce and consume both types of goods, while the government does not produce any good and only derives utility from the consumption of the general good. An agent cannot consume the special good that he produces. The utility of consumption of the special good is u, and the cost of production is c, with $\beta u \geq c$. The utility of consumption of x units of the general good is x for both the agents and the government, and the agent incurs disutility x if he produces x units of the general good.

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Every period is divided in two sub-periods, morning and afternoon. In the morning, each agent decides between staying in the market and moving to autarky. If the agent stays in the market, he meets the government and pay taxes, i.e., make transfers to the government. Taxes are chosen by the government. If the agent moves to autarky, he does not pay taxes and he receives a constant payoff, which we normalize to zero. In the afternoon, all agents who paid taxes meet randomly and anonymously in pairs. In each meeting, at most one agent can produce, and the producer can produce only one unit of the special good.

Absent any technology of exchange, agents have no incentive to produce in the market. We consider two alternative technologies that may help exchange: credit and money. In what follows, we first consider the economy with credit and then we consider the economy with money.

2.1 Credit

There exists a monitoring technology that keeps track of the agents' behavior in the market and assigns to each agent either a good or a bad label. Participation in the monitoring technology is costly, i.e., in every period it requires $\phi(u-c)/2$ units of general goods from each agent that stays in the market, where $\phi \in [0,1]$. The agent incurs this cost when he meets the government and pays his taxes. All agents start with a good label. An agent with a good label can get a bad label only if he is a producer in his market meeting, his partner has a good label, and the agent fails to produce to his partner. In this case, the agent gets a bad label with probability $\rho \in [0,1]$.

Trade takes place as follows. After two agents meet, they observe each other's label and a coin toss determines the producer. We consider a "credit arrangement" where a producer produces to his partner only if both him and his partner have a good label. Hence, an agent with a bad label is permanently excluded from trade, and has no incentive to pay taxes and stay in the market. This arrangement provides the agents with the greatest incentive to produce.[1]

The government chooses a policy $\{x_t\}_{t=1}^{\infty}$ in order to maximize her payoff, where x_t is the quantity of the general good that an agent who chooses to stay in the market must produce to the government in period t. The government chooses her policy constrained by the participation constraint and by the incentive compatibility constraint of the agents.[2] We restrict attention to

[1] It will become clear from our analysis that the surplus the government can extract from the agents is increasing in the incentive they have to trade. Thus, considering equilibria in which agents with a bad label are permanently excluded from trade is without loss of generality; no other credit arrangement which respects agents' incentives can benefit the government more.

[2] The participation constraint corresponds to the agent's decision between staying in the market and moving to

stationary policies where $x_t = x$ for all t. We discuss this assumption at the end of this section.

If the policy is x, the payoff of an agent with a good label at the beginning of a period is

$$V^c(x) = \frac{1}{1-\beta}\left[-x + (1-\phi)\frac{u-c}{2}\right].$$

In the morning, the agent pays taxes x and incurs the cost $\phi(u-c)/2$ of participating in the monitoring technology. In the afternoon, he obtains u with probability 1/2 and c with probability 1/2. The agent is willing to produce to his partner if called upon to do so if and only if

$$-c + \beta V^c(x) \geq \beta(1-\rho)V^c(x),$$

which can be rewritten as

$$\frac{\beta\rho}{1-\beta}\left[-x + (1-\phi)\frac{u-c}{2}\right] \geq c.$$

The government chooses the largest x which satisfies the incentive compatibility constraint of the agent, which implies

$$x(\phi,\rho) = (1-\phi)\frac{u-c}{2} - \frac{(1-\beta)c}{\beta\rho}. \tag{1}$$

The government appropriates the surplus from market exchange but she needs to compensate the agent in the current period for her effort in the previous period and for the cost of participating in the monitoring technology.[3] A necessary condition for trade mediated by credit to occur is that $x(\phi,\rho) \geq 0$, which implies

$$\beta \geq \beta(\phi,\rho) \equiv \frac{c}{c + (1-\phi)\rho(u-c)/2}.$$

Note that $V^c(x(\phi,\rho)) > 0$, i.e., the participation constraint of the agent is satisfied. Proposition 1 summarizes our result.

Proposition 1 *Let $\beta \geq \beta(\phi,\rho)$. The credit arrangement is an equilibrium and the optimal policy in this equilibrium is given by (1).*

autarky, while the incentive compatibility constraint corresponds to the agent's decision on whether to produce to his partner if called upon to do so.

[3]The assumption that the government derives utility from the consumption of general goods can be replaced by the assumption that the government and the agents derive utility from public goods which the government produces using general goods. In fact, if $G(x)$ is the agent's utility from the consumption of x units of public goods, where $G' > 0$ and $G'' < 0$, the optimal policy $x^g(\phi,\rho)$ solves $x^g(\phi,\rho) = G[x^g(\phi,\rho)] + x(\phi,\rho)$, and it is strictly decreasing in ϕ and ρ. Note that, if the objective of the government is to maximize the agent's utility, taxes are set at $x^*(\phi,\rho)$, where $G'[x^*(\phi,\rho)] = 1$. Clearly, $x^g(\phi,\rho) > x^*(\phi,\rho)$.

Our notion of credit requires some justification. In general, in a credit transaction, the creditor delivers a good to the debtor in exchange for a promise of obtaining a good in the future. In our setting, we capture this promise with the idea that the creditor acquires a good label which entitles him the right to consume a good in the future. The difference from common credit transactions in modern economies is that the promise is not redeemed with the original debtor but with another agent in the economy. In this sense, it is like a gift-giving scheme or a multilateral credit arrangement. This is in line with much historical evidence, as it will be discussed later. However, there is also evidence that in various instances, credit was conducted bilaterally. In the Appendix, we consider a variant of our credit arrangement in which promises are redeemed with the original debtor.

2.2 Money

There exists a durable, costless and intrinsically useless indivisible object, labeled money. Agents are constrained to hold at most one unit of money at a time and money holdings are not observable. Money is randomly distributed by the government to a measure m of agents in the morning of period 0.

We consider a "monetary arrangement" where agents truthfully announce their money holdings in their meeting with the government and, in meetings between an agent with money and an agent without money, the latter produces to the former in exchange for money. There are no incentives for production to take place in meetings where both agents hold money and in meetings where both agents do not have money.

The government chooses a policy $\left(m_t, x_t^0, q_t^0, x_t^1, q_t^1\right)_{t=0}^{\infty}$ in order to maximize her payoff, where x_t^0 (x_t^1) is the quantity of the general good that an agent produces to the government if he announces zero (one) unit of money, and q_t^0 (q_t^1) is the probability that an agent without (with) money leaves the meeting with the government with money.[4] Thus, taxes can be set both in terms of goods and in terms of money. The government chooses her policy constrained by the participation constraint, the incentive compatibility constraint, and the truth-telling constraints of the agents. As in the economy with credit, we restrict attention to stationary policies where $\left(m_t, x_t^0, q_t^0, x_t^1, q_t^1\right) = \left(m, x^0, q^0, x^1, q^1\right)$ for all t. We discuss this assumption at the end of this section. Note that the stationarity of m

[4] Since $m_{t+1} = q^1 m_t + q^0(1 - m_t)$, the amount of money in the economy in all periods is completely determined by m and by the sequence of profiles $(x_t^0, q_t^0, x_t^1, q_t^1)$, for all $t > 0$.

implies

$$m = q^1 m + q^0(1-m). \tag{2}$$

Let W^i be the present discounted payoff of an agent with $i \in \{0,1\}$ units of money at the beginning of the morning, and V^i be the present discounted payoff of an agent with $i \in \{0,1\}$ units of money at the beginning of the afternoon. We have

$$W^i = -x^i + q^i V^1 + (1-q^i)V^0,$$

for $i \in \{0,1\}$, and

$$\begin{aligned} V^1 &= (1-m)\left(u+\beta W^0\right) + m\beta W^1 \\ V^0 &= m\left(-c+\beta W^1\right) + (1-m)\beta W^0. \end{aligned}$$

We first examine the constraints faced by the government when choosing the optimal policy. We need

$$W^0 \geq 0 \tag{3}$$

to make sure that an agent without money wants to participate in the market, We also need to consider the incentive compatibility constraint of the seller, i.e., an agent without money must be willing to produce in exchange for money. This requires

$$-c + \beta W^1 \geq \beta W^0. \tag{4}$$

An agent with money must also have an incentive not to hide the money when he meets the government, i.e.,

$$W^1 \geq -x^0 + V_1. \tag{5}$$

Lemma 1 *The constraints (3), (4), and (5) bind at the optimal policy.*

Proof. In the Apppendix. ■

Conditional on m, Lemma 1 implies that the optimal policy is given by the profile (x^0, q^0, x^1, q^1) which solves (2), (3), (4), and (5).[5] After some computation, we obtain (where $\Delta(m) \equiv (1-m)u + mc$)

$$\left(x_m^0, q_m^0, x_m^1, q_m^1\right) = \left(\Delta(m) - \frac{c}{\beta}, \frac{\beta\Delta(m)-c}{\beta\Delta(m)}, -\frac{(1-2m)\left[\beta\Delta(m)-c\right]}{\beta m}, 1 - \frac{(1-m)\left[\beta\Delta(m)-c\right]}{m\beta\Delta(m)}\right),$$

[5] The only constraints we need to worry about are (3), (4), and (5). The incentive compatibility constraint of an agent with money, given by $u + \beta W^0 \geq \beta W^1$, is implied by the fact that (4) binds. In turn, the participation constraint of agent with money, given by $W^1 \geq 0$, is implied by the fact that (3) and (4) bind.

and the expected tax in general goods $x(m) \equiv mx^1 + (1-m)x^0$ is given by

$$x(m) = m(1-m)(u-c) - \frac{(1-\beta)mc}{\beta}. \tag{6}$$

The optimal money supply is the choice of m which maximizes (6), given by

$$\widehat{m} = \frac{\beta u - c}{2\beta(u-c)}, \tag{7}$$

and the corresponding policy is

$$\left(x^0, q^0, x^1, q^1\right) = \left(\frac{\beta u - c}{2\beta}, \frac{\beta u - c}{\beta u + c}, \frac{-(1-\beta)c}{\beta}, \frac{2\beta c}{\beta u + c}\right). \tag{8}$$

Proposition 2 summarizes our result.

Proposition 2 *Let $\beta u \geq c$. The monetary arrangement is an equilibrium, and the optimal policy in this equilibrium is $\left\{\widehat{m}, \left(x^0, q^0, x^1, q^1\right)\right\}$, where $\widehat{m}$ is given by (7) and $\left(x^0, q^0, x^1, q^1\right)$ is given by (8).*

The monetary arrangement is an equilibrium whenever $\beta u \geq c$. In the absence of the government, a necessary condition for the existence of a monetary equilibrium is $\beta(u+c)/2 \geq c$, i.e., even though the government taxes agents, it enlarges the region of parameters where money is accepted as a medium of exchange. This is so because the government taxes less heavily agents who produce in exchange for money. In fact, the government gives a subsidy in general goods to these agents and taxes their money holdings. This way, she is able to transfer the money to agents without money and tax these agents in terms of general goods. If one interprets taxes on money holdings as inflation (e.g., Li, 1995, Li and Wright, 1998), the optimal policy from the government's perspective is inflationary.

The government maximizes her payoff subject to the participation constraint, the incentive compatibility constraint, and the truth-telling constraint of the agents. While it is immediate that the government must respect the first two constraints, the relevance of the truth-telling constraint is less clear. In order to understand its importance, consider the scenario where the government chooses taxes which violate truth-telling. In this case, each agent announces zero money holdings and all agents pay the same tax. Moreover, since agents hide their money holdings in their meeting with the government, taxes are only set in terms of general goods. In other words, the scenario where the truth-telling constraint is violated corresponds to no monetary intervention by the government.

In this case, a policy is simply given by $\widetilde{x}$, i.e., the transfer of general goods to the government. The optimal policy in this case can be found by using the fact that the participation constraint of an agent without money binds, and the fact that all agents face the same tax. Using V^1 and V^0, we obtain $\widetilde{x} = \beta m \left[(1-m)u + mc\right] - mc$, which, for all m, is strictly smaller than the expected tax under the optimal policy in the presence of monetary intervention.

The money supply which maximizes the government's payoff is smaller than the money supply which maximizes surplus, given by $m = 1/2$. To understand the reason for this difference, it is helpful to compare our result with the indivisible money and divisible goods environment of Trejos and Wright (1995). Our economy collapses into their economy if we eliminate the government and the morning period, and we assume that the special good is divisible. They obtain that the money supply which maximizes welfare is smaller than 1/2. Intuitively, there is a trade-off between the benefits of money in the extensive margin (which lean towards m equal to 1/2) and the benefits of money in the intensive margin (which lean towards m away from 1/2). In our case, there is no trade-off from an efficiency point of view, because the special goods is indivisible and the general good is linear and has no impact on welfare. However, like the planner in Trejos and Wright (1995), the government faces a trade-off: m closer to 1/2 improves the extensive margin and generate larger gains from trade, thus allowing to extract a larger surplus from the agents; m away from 1/2 improves the intensive margin and increases the incentive of an agent without money to pay a relatively large amount of taxes in his meeting with the government.

2.3 Comment

We restricted attention to stationary policies. This restriction prevents the government from choosing a policy consistent with the agent's incentive compatibility constraint in period $t-1$, and then reneging and implementing a policy consistent with the agent's participation constraint in period t. Precisely, in the credit economy, the optimal policy $x(\phi, \rho)$ ensures that an agent wants to produce in the market if called upon to do so in period $t-1$. However, once the agent's decision is sunk, the government has an incentive to deviate in period t, ask for $(1-\phi)(u-c)/2$ from all agents and announce that future taxes will revert to $x(\phi, \rho)$. This deviation violates the agent's incentive compatibility constraint in period $t-1$, i.e., if the agent anticipates that the government will ask for $(1-\phi)(u-c)/2$ in period t, he does not produce; but it satisfies the agent's participation constraints and incentive compatibility constraints from period t on. This non-stationary

policy delivers a strictly higher payoff to the government than the stationary policy that we consider. However, this policy is not time-consistent, as the government has an incentive to ask for $(1-\phi)(u-c)/2$ at the beginning of every period, and promise to revert to $x(\phi,\rho)$ in all future periods. By assuming that the government can only choose stationary policies, we ensure that her choice never violates the agents' incentive compatibility and participation constraints. A similar reasoning applies to the economy with money. In the Appendix, we provide a rationale for our restriction to stationary policies which aims at capturing this time-consistency problem.

We have also assumed that the government chooses her policy constrained only by each agent's incentive compatibility and participation constraints. In particular, there is no exogenous commitment technology, and there is no endogenous punishment scheme which prevents the government from reneging on her policy and choosing a new policy consistent with agent's current and future incentive compatibility and participation constraints This is without loss of generality under the assumption that only stationary policies are allowed, in which case it delivers a unique optimal policy. However, as said above, it creates time-consistency problems when we allow for non-stationary policies. Thus, an alternative to the assumption of stationary policies is the introduction of commitment or the possibility of punishment after policies which are consistent with agent's incentive compatibility and participation constraints.

For instance, in the economy with credit, it is easy to show that limited commitment suffices to ensure that $x(\phi,\rho)$ is the unique optimal policy. We simply need to assume that the government chooses taxes one period in advance. In this case, the agent is willing to produce in the market in period t if called upon to do so if $\beta\rho V^c_{t+1}=c$, where $V^c_{t+1}=-x_{t+1}+(1-\phi)(u-c)/2+\beta V^c_{t+2}$.Since $\beta\rho V^c_{t+1}=c$ for every t, we can use $\beta\rho V^c_{t+2}=c$ to obtain $x_{t+1}=x(\phi,\rho)$. Note that limited commitment is necessary to ensure that the government sets taxes in period t based on the agent's incentive compatibility constraint and does not renege in period $t+1$ and increases taxes up to the agent's participation constraint.[6]

Alternatively, we can allow agents to collectively punish the government with a global reversion to autarky even after policies which satisfy the agents' incentive compatibility and participation constraints. We see two problems with this approach. First, it critically depends on the market

[6]The fact that commitment is limited matters for the stationarity of the policy. If the government can commit at the beginning of the economy to a policy $\{x_t\}_{t=0}^{\infty}$, then the optimal policy is non stationary and consists in setting taxes equal to $(1-\phi)(u-c)/2$ in period 0, and setting all future taxes to $x(\phi,\rho)$. Clearly, this non-stationary policy is no longer credible if the government can only commit in advance to a finite number of periods.

being completely empty. That is, if an arbitrarily small measure of agents always participate in the market if participation is consistent with their incentive compatibility and participation constraints, all other agents would also want to participate in the market. Second, it gives rise to multiple equilibria due to folk-theorem type of results.

3 Credit and money

To understand the forces at play which impact the government's payoff under credit and money, we rewrite below the optimal tax under the credit arrangement, given by

$$x(\phi, \rho) = (1-\phi)\frac{u-c}{2} - \frac{(1-\beta)c}{\beta\rho},$$

and the optimal expected tax when the money supply is $\widehat{m}$, given by

$$x(\widehat{m}) = \widehat{m}(1-\widehat{m})(u-c) - \frac{(1-\beta)\widehat{m}c}{\beta}.$$

If the monitoring technology is too costly or too inefficient, taxes under money are higher than taxes under credit. For instance, if ρ becomes too small, say because it becomes harder to keep track of behavior when the society is large or complex, taxes under credit will fall below zero. In this case, the government is better off under the monetary arrangement, but so is society, i.e., the credit arrangement cannot be sustained as an equilibrium even if there is no taxation. This echoes Neil Wallace's idea that "we use money with strangers, and we don't with people we know" (2013, page 06), which underpins the standard view on the essentiality of money, i.e., the idea that money is essential because it allows to achieve desirable allocations that could not be achieved otherwise.

More interestingly, there are instances where the government is better off under money but the ex-ante welfare is higher under credit.[7] Indeed, if $\phi < \frac{1}{2}$, the ex-ante welfare is always larger under credit. However, irrespective of the value of $\rho \in [0,1]$, the government is better of under money if

$$\frac{1}{2} - \phi < \frac{1}{2}\left[\left(1 + \frac{1-\beta}{\beta}\frac{c}{u-c}\right)^2 - 1\right].$$

For every $\beta < 1$, there exists $\underline{\phi}(\beta) < \frac{1}{2}$ such that the above condition holds for all $\phi \in \left[\underline{\phi}(\beta), \frac{1}{2}\right]$. The intuition for this result runs as follows. Both under credit and under money, the government

[7] Note that general goods do not matter for the ex-ante welfare since each unit provides the same utility and disutility. They only matter for the distribution of surplus between the agents and the government.

appropriates the surplus from market exchange but she needs to compensate the agent in the current period for her effort in the previous period. Interestingly, even when $\rho = 1$ and monitoring is perfect, the amount of surplus the government must give up in order to preserve market exchange is always higher under credit than under money. This is so because, while agents are uniformly taxed under credit, under money, the government can actively use her policy instruments to condition taxes on money holdings. In particular, she only needs to give up surplus to agents who hold money in the current period.

Summarizing, the only reason credit may dominate money from the government's perspective is because it creates more surplus. If the difference between the surplus produced under each technology reduces, say because the monitoring technology worsens or because money becomes a more efficient medium of exchange, the government may prefer to tax under the monetary arrangement even though the credit arrangement is better from the ex-ante welfare point of view. This suggests a different view on the essentiality of money, one that is not driven by welfare considerations but by money's efficiency in transferring surplus from the agents to the government.

A weakness of our analysis is that we compare different economies, while the historical evidence suggests a transition from credit to money as well as the coexistence between these technologies of exchange. In what follows, we extend our model to deal with these shortcomings.

3.1 The coexistence between credit and money

In the environment of section 2, we assumed that the monitoring technology keeps track of agent's behavior in all market meetings. In order to allow for the coexistence between credit and money, assume instead that the monitoring technology is limited and only keeps track of behavior in a measure $\alpha(\phi) > \phi$ of meetings, where $\phi(u - c)/2$ is the quantity of general goods each agent must contribute to the technology. We further assume that, at the beginning of the economy, the government randomly distributes money to a measure m of agents. Throughout, to ease exposition, we let $\rho = 1$.

We consider a "mixed arrangement" where agents truthfully announce their money holdings in their meeting with the government and, in non-monitored meetings between an agent with money and an agent without money, the latter produces to the former in exchange for money. In turn, in monitored meetings, they observe each other's label and a coin toss determines the producer. As in the previous section, we restrict attention to stationary policies $\left(m, x^0, q^0, x^1, q^1\right)$.

Consider first the scenario where the government wants to maximize the agent's ex-ante welfare subject to his incentive compatibility, participation, and truth-telling constraints. The expected payoff of an agent with a good label and with one unit of money at the beginning of a period is

$$V_1^\omega = -\phi \frac{u-c}{2} + \alpha(\phi) \left(\frac{u-c}{2} + \beta V_1^\omega \right) + [1-\alpha(\phi)] \left[m\beta V_1^\omega + (1-m)(u+\beta V_0^\omega) \right],$$

while the expected payoff of an agent with a good label and no money is

$$V_0^\omega = -\phi \frac{u-c}{2} + \alpha(\phi) \left(\frac{u-c}{2} + \beta V_0^\omega \right) + [1-\alpha(\phi)] \left[m(-c+\beta V_1^\omega) + (1-m)\beta V_0^\omega \right].$$

If an agent refuses to produce in a monitored meeting, he does not need to contribute to the monitoring technology but he is permanently excluded from consumption in all future monitored meetings. Thus, an agent produces in monitored meeting meetings if and only if $-c+\beta V_i^\omega \geq \beta V_i^{\overline{\omega}}$, where $V_i^{\overline{\omega}}$ is the expected payoff of holding $i \in \{0,1\}$ units of money and only participating in exchange in non-monitored meetings. We can rewrite this condition as

$$\beta \left[\alpha(\phi) - \phi \right] (u-c) \geq 2(1-\beta)c. \tag{9}$$

We assume that agents are patient enough and (9) hold. In turn, an agent without money produces in a non-monitored meeting if and only if $-c+\beta V_1^\omega \geq \beta V_0^\omega$, which can be rewritten as

$$\alpha(\phi) \equiv \alpha^\omega(\phi) < \frac{\beta \left[(1-m)u + mc \right] - c}{\beta(1-m)\left(u-c\right)}, \tag{10}$$

and monitoring must be sufficiently limited. Indeed, if monitoring is abundant, agents have no incentive to produce in exchange for money, since they will rarely participate in a non-monitored meeting. This reasoning implicitly assumes that an agent does not use money in monitored meetings as a way to induce a partner without money to undertake the production opportunity. Precisely, it requires $(u-c)/2 + \beta V_0^\omega \geq -c + \beta V_1^\omega$, i.e., an agent without money does not accept money in exchange for his production in a monitored meeting. This condition can be rewritten as

$$\beta \left\{ c + [1-\alpha(\phi)] \left[(1-m)(u-c) \right] \right\} \leq (u+c)/2. \tag{11}$$

The ex-ante welfare is

$$(1-\beta)U^\omega = [\alpha(\phi) - \phi] \frac{u-c}{2} + [1-\alpha(\phi)]\, m(1-m)(u-c),$$

and the optimal choice of money supply is equal to $1/2$, as in the environment under the pure monetary arrangement. Note that (11) holds when $m = 1/2$.

Consider now the scenario where the government wants to maximize her own payoff. Let W_i^g be the present discounted payoff of an agent with $i \in \{0,1\}$ units of money at the beginning of the morning, and V_i^g be the present discounted payoff of an agent with $i \in \{0,1\}$ units of money at the beginning of the afternoon. We have

$$W_i^g = -x^i - \phi \frac{u-c}{2} + q^i V_1^g + (1-q^i) V_0^g,$$

and

$$\begin{aligned} V_1^g &= \alpha(\phi)\left(\frac{u-c}{2} + \beta W_1^g\right) + [1-\alpha(\phi)]\left[m\beta W_1^g + (1-m)(u+\beta W_0^g)\right] \\ V_0^g &= \alpha(\phi)\left(\frac{u-c}{2} + \beta W_0^g\right) + [1-\alpha(\phi)]\left[m(-c+\beta W_1^g) + (1-m)\beta W_0^g\right]. \end{aligned}$$

The same arguments used in the proof of Lemma 1 imply that, in the determination of the optimal policy, it suffices to find $\left(x^0, q^0, x^1, q^1\right)$ which solve for

$$W_0^g = 0, \tag{12}$$

$$-c + \beta W_1^g = \beta W_0^g, \tag{13}$$

and

$$W_1^g = -x^0 - \phi \frac{u-c}{2} + V_1^g, \tag{14}$$

together with the stationarity condition (2). The first constraint corresponds to the incentive of an agent without money to participate in the market, the second constraint corresponds to the incentive of an agent without money to produce in a non-monitored meeting, and the latter constraint captures the incentive of an agent with money to truthfully announce his money holdings in the meeting with the government.[8] After some computation, we obtain that, in the optimal policy, we must have

$$x^1 = [\alpha(\phi) - \phi]\frac{u-c}{2} + q^1\left\{c + [1-\alpha(\phi)](1-m)(u-c)\right\} - \frac{c}{\beta},$$

[8] As in the pure monetary arrangement, the participation constraint and the incentive compatibility constraint of an agent with money are implied by (12) and (13). However, in the presence of credit, we also need to make sure that an agent wants to produce in a monitored meeting. This condition is satisfied whenever (9) holds. Finally, we need to make sure that money is not used in monitored meetings. This requires $(u-c)/2 + \beta V_0^g \geq -c + \beta V_1^g$. Since $V_1^g = \phi(u+c)/2 + (1-\phi)[(1-m)u + mc]$, and $V_0^g = \phi(u-c)/2$, this condition is always satisfied, irrespective of the value of m.

and

$$x^0 = [\alpha(\phi) - \phi] \frac{u-c}{2} + q^0 \{c + [1-\alpha(\phi)] (1-m) (u-c)\}.$$

Intuitively, the government faces a trade-off between taxation of general goods and taxation of money holdings. Expected taxes are given by $x(\phi, m) = mx^1 + (1-m)x^0$, which can be rewritten as

$$x(\phi, m) = [\alpha(\phi) - \phi] \frac{u-c}{2} + [1-\alpha(\phi)] m(1-m)(u-c) - \frac{(1-\beta)mc}{\beta}.$$

As expected, $x(\phi, m)$ combines elements of the pure credit and the pure monetary arrangement. The government chooses m in order to maximize $x(\phi, m)$. The optimal money supply is

$$m(\phi) = \frac{\beta [1-\alpha(\phi)] u - [1-\beta\alpha(\phi)] c}{2\beta [1-\alpha(\phi)] (u-c)}.$$

A positive money supply requires

$$\alpha(\phi) \equiv \alpha^g(\phi) < \frac{\beta u - c}{\beta (u-c)}, \tag{15}$$

i.e., there must be a relatively large number of non-monitored meetings. Indeed, if most meetings are monitored, the government has no incentive to introduce money in the economy and simply taxes as in a pure credit arrangement. Note that $m(\phi) < \widehat{m}$ is strictly decreasing in ϕ and it converges to $\widehat{m}$ when ϕ goes to zero. Thus, the presence of credit further increases the inefficiency of monetary trades as compared to the environment under the pure monetary arrangement. Intuitively, m cannot be too far away from $1/2$ in the economy without credit because trades can only be conducted with money. As a result, the impact of the money supply on the extensive margin of trade is critical. This is not the case anymore when trade can also be conducted with credit.

More interestingly, by comparing (10) and (15), we obtain that, for all $m > 0$, there exists a region of parameters $\alpha(\phi) \in (\alpha^\omega(\phi), \alpha^g(\phi))$ where money cannot be part of the equilibrium if the objective of the government is to maximize the welfare, but money is part of the equilibrium if the objective of the government is to maximize her own payoff. Intuitively, even if monitoring is relatively abundant, the agent has an incentive to produce in exchange for money in a non-monitored meeting so he can reduce the transfer of general goods he needs to make to the government in the following period.

3.2 The transition from credit to money

In the environment of section 2, we assumed that the monitoring technology does not change over time. In order to examine the transition from credit to money, assume instead that the cost of the monitoring technology increases over time. Precisely, $\phi \equiv \phi(t) \in [0, 1/2]$ is a continuous, strictly increasing and strictly concave function, with $\phi(0) = 0$ and $\phi(\infty) = 1/2$. We further assume that the government differs from agents in three dimensions: (1) she can commit to her actions one period in advance; (2) she is relatively patient, and discounts the future with a factor $\delta \in (0, 1)$ with $\delta > \beta$; (3) she can store general goods one-to-one across periods. Throughout, we let $\rho = 1$, and $\beta > 2c/(u + c)$. We discuss these assumptions at the end of this subsection.

Consider first the scenario where the exchange arrangement is chosen by a government whose objective is to maximize the agent's ex-ante payoff subject to his incentive compatibility and participation constraints. In this case, the planner always choose the credit arrangement, i.e., there is no transition from credit to money. This is so because the credit arrangement always generates more surplus than the monetary arrangement and, since $\beta(\phi, \rho) = \beta(1/2, 1) < 2c/(u + c)$, it is an equilibrium.

Consider now the scenario where the exchange arrangement is chosen by a government whose objective is to maximize her own payoff. We proceed by backward induction, assuming that the economy transits from credit to money, and starting with the economy after the transition takes place. When the transition is complete, the analysis is the same as in the previous section, i.e., the monetary arrangement is an equilibrium and the optimal policy is given by $\left(x_m^0, q_m^0, x_m^1, q_m^1\right)$, where m is the amount of money distributed by the government in the first period of the monetary arrangement. Moreover, if one assumes that each agent follows the monetary arrangement whenever he is indifferent, then this arrangement is the unique equilibrium. This allows us to proceed backwards from a unique path.

The uniqueness of the monetary arrangement is not trivial because in monetary models without the government, autarky is always a strict equilibrium for purely coordination reasons. This is not the case in the presence of the government. The argument runs as follows. If an agent does not believe that money will be accepted as a medium of exchange by other agents, he may still produce in exchange for money if $-c - \beta\overline{x}^1 = 0$, i.e., if the government offers a subsidy $\overline{x}^1 = -c/\beta$ to agents with money which compensates for their effort. That is, in order to ensure that participation in the market and production in exchange for money is a weakly dominant action, the government

simply needs to offer a subsidy $\overline{x}^1$ for agents with money. This subsidy is larger than the subsidy x^1 set by the optimal policy. However, note that x^1 satisfies $-x^1 + q^1 V^1 + (1 - q^1)V^0 = c/\beta + V^0$. In words, if the agent believes that money is accepted as a medium of exchange, his payoff is the same under the optimal policy and under the alternative policy which sets a subsidy $\overline{x}^1$ and taxes the money of the agent with probability one. This implies that the taxation scheme which ensures that production in exchange for money is a weakly dominant action does not need to be applied on the equilibrium path. It is simply a device that prompt agents to coordinate in the use of money (see Goldberg (2012) for a similar result and a discussion on the tax-foundation of fiat money).

Let t^c be the last period of credit and let $\{x_t\}_{t=1}^{t^c+1}$ be a sequence of policies up to period t^c. From period $t^c + 2$ on, the expected tax $x(m)$ under the optimal money supply is given by (6). Consider the problem of an agent in the afternoon of period t^c. He anticipates that, if he deviates from the credit arrangement, he acquires a bad label with probability one, in which case he is punished with permanent exclusion from the market. He is willing to produce if and only if

$$\beta \left[-x_{t^c+1} + mV^1 + (1-m)\, V^0\right] \geq c.$$

At the beginning of period $t^c + 1$, there is a probability m that the agent receives money and a probability $1 - m$ that he does not receive money. Irrespective of whether the agent receives money or not, he is taxed in the amount x_{t^c+1}.[9] The value functions V^1 and V^0 are determined by m and the optimal policy $\left(x_m^0, q_m^0, x_m^1, q_m^1\right)$. We can rewrite the above condition as

$$\beta \left[-x_{t^c+1} + x(m) + \frac{mc}{\beta}\right] \geq c.$$

This implies that the highest tax the government can impose at $t^c + 1$ is

$$x_{t^c+1} = x(m) - \frac{(1-m)c}{\beta}.$$

Since x_{t^c+1} may be negative, we need to make sure the government has enough funds to cover this transfer (see below). The utility of the government if he distributes money to a measure m of agents in period $t^c + 1$ is

$$U(m) = \frac{x(m)}{1-\delta} - \frac{(1-m)c}{\beta}, \tag{16}$$

[9]The agent makes the decision between staying in the market and moving to autarky before knowing whether he will receive money or not. In this case, there is no loss in generality in assuming that the government does not condition taxes on whether the agent received money or not.

and the optimal money supply is given by

$$m^* = \frac{\beta(u-c) - (\delta - \beta)\, c}{2\beta(u-c)}. \tag{17}$$

Since $m^* > \widehat{m}$, the surplus under the monetary arrangement increases compared to the stationary case. Note that $m^* = 1/2$ when $\delta = \beta$ and $m^* = \widehat{m}$ when $\delta = 1$. Intuitively, if the government is relatively impatient, he chooses m close to $1/2$ to increase the initial tax (reduce the initial subsidy) x_{t^c+1}. In turn, if he is relatively patient, he considers the longer term payoff and chooses m closer to $x(m)$.

Consider now the problem of an agent in the afternoon of period $t^c - 1$. He is willing to produce if and only if

$$\beta \left\{ -x_{t^c} + [1 - \phi(t^c)] \frac{u-c}{2} + \beta \left[-x_{t^c+1} + m^* V^1 + (1 - m^*) V^0 \right] \right\} \geq c. \tag{18}$$

If the agent does not move to autarky in period t^c, he pays taxes x_{t^c} and obtains $(u-c)/2$ during the period. He then participates in the monetary arrangement from period $t^c + 1$ on. Substituting for x_{t^c+1} and rearranging terms, we obtain that the highest tax the government can impose at t^c is

$$x_{t^c} = [1 - \phi(t^c)] \frac{u-c}{2} - \frac{(1-\beta)c}{\beta}.$$

Proceeding backwards, consider the problem of an agent in the afternoon of period $t^c - 2$. He anticipates that, if enters the market in period $t^c - 1$, he pays x_{t^c-1} and obtains $(u-c)/2$ during the period. He then obtains V_{t^c} from the next period on, where

$$V_{t^c} = -x_{t^c} + [1 - \phi(t^c)] \frac{u-c}{2} + \beta \left[-x_{t^c+1} + m^* V^1 + (1 - m^*) V^0 \right]$$

corresponds to the expression inside brackets in (18). The agent is willing to produce if and only if

$$\beta \left\{ -x_{t^c-1} + [1 - \phi(t^c - 1)] \frac{u-c}{2} + \beta V_{t^c} \right\} \geq c,$$

which implies that the highest tax the government can impose at $t^c - 1$ is

$$x_{t^c-1} = [1 - \phi(t^c - 1)] \frac{u-c}{2} - \frac{(1-\beta)c}{\beta}.$$

A similar reasoning applies to all previous periods up to period 1, and we obtain that, for all $t \in \{1, ..., t^c\}$,

$$x_t = [1 - \phi(t)] \frac{u-c}{2} - \frac{(1-\beta)c}{\beta}.$$

Note that x_t is strictly decreasing and it converges to $(u-c)/4-(1-\beta)c/\beta$, which is strictly smaller than the expected payoff under the monetary arrangement $x(m)$. It remains to determine taxes in period 0. At the beginning of the economy, the government only needs to meet the agent's participation constraint and she can set x_0 such that

$$-x_0+[1-\phi(0)]\frac{u-c}{2}+\sum_{t=1}^{t^c}\beta^t\left[-x_t+[1-\phi(t)]\frac{u-c}{2}\right]+\beta^{t^c+1}\left[-x_{t^c+1}+m^*V^1+(1-m^*)V^0\right]=0,$$

which can be rewritten as

$$x_0=\frac{u+c}{2}.$$

Intuitively, since the agent anticipates that he will obtain a continuation payoff $\frac{c}{\beta}$, he is willing to pay a tax equal to $(u-c)/2+\beta(c/\beta)$ in the first period. This completes our characterization of taxes in all periods.[10]

We can now determine whether a transition takes place, and if it does, the period t^c in which it happens. The government's payoff under t^c is

$$U(t^c)=x_0+\sum_{t=1}^{t^c}\delta^t x_t+\delta^{t^c+1}x_{t^c+1}+\frac{\delta^{t^c+2}}{1-\delta}x(m^*),$$

and it can be rewritten as

$$U(t^c)=\frac{u+c}{2}-\frac{\delta(1-\beta)c}{(1-\delta)\beta}+\sum_{t=1}^{\infty}\delta^t\left[S_t+\delta^{t^c}\frac{(\delta-\beta)\,c}{2\beta}\frac{\beta(u-c)+(\delta-\beta)\,c}{\beta(u-c)}\right], \tag{19}$$

where

$$S_t=\begin{cases}[1-\phi\,(t)]\,\frac{u-c}{2}\text{ for all }t\leq t^c\\ \left[1-\frac{(\delta-\beta)c}{\beta(u-c)}\right]\left[1+\frac{(\delta-\beta)c}{\beta(u-c)}\right]\frac{u-c}{4}\text{ for all }t>t^c\end{cases}$$

corresponds to the surplus in every period. The first two terms inside the summation in (19) depend on t^c. The first term captures the total surplus in every period, while the second term captures the difference between the amount of surplus the government has to give up under credit and under money in every period. The first term is strictly increasing in t^c, reflecting the fact that the surplus under credit is always larger than the surplus under money, while the second term is strictly decreasing in t^c, reflecting the fact that the relative importance of the efficiency of the government in appropriating surplus increases when the monitoring technology becomes more costly.

[10] Note that x^0 ensures the government has enough resources to fund x_{t^c+1}, in case x_{t^c+1} is negative.

The function $U(t^c)$ gives the highest possible payoff to the government for any given transition period t^c. Thus, the government chooses the transition period t^c which maximizes $U(t^c)$. It is not always the case that the government wants to transit from credit to money. For instance, if the government is as patient as the agents, the surplus effect is the only effect and the economy does not transit to the monetary arrangement. Intuitively, there is a one-time yet substantial cost to the government if there is a transition, given by the tax/subsidy x_{t^c+1}. If $\delta = \beta$, the government's payoff after the transition, given by $U(m)$ in (16) is strictly lower than $(u-c)/4(1-\beta) - c/\beta$, which is a lower bound on the government's payoff if the transition never takes place. In order for a transition to take place, the government needs to be relatively patient and put more value on the long-run benefits of moving to the monetary arrangement, as given $x(m)$. Indeed, Proposition 3 shows that there is a transition from credit to money if the government is patient enough.

Proposition 3 *If the government is patient enough, there exists a transition from credit to money which maximizes the government's payoff.*

Proof. In the Appendix ■

We restricted the environment of Section 2 in various ways to accommodate a transition from credit to money. First, the assumption that ϕ increases over time captures the idea that, as time evolves and trade expands, monitoring becomes more difficult, requiring a larger fraction of the surplus. This assumption makes the environment non-stationary and constitutes the key exogenous force that drives the transition. We can partially endogenize this restriction by letting the population grow over time and let the monitoring technology exhibit decreasing marginal returns. This way, although it is true that more resources are poured into the technology as population grows, the technology needs to appropriate a larger fraction of the surplus in order to continue to operate properly.

The assumption that the government can commit to her choice of taxes one period in advance is important. It echoes the restriction to stationary policies in the previous section as it prevents the government from choosing a policy consistent with the agent's incentive compatibility constraint in period $t-1$, and then reneging and implementing a policy consistent with the agent's participation constraint in period t. Indeed, we conjecture that, in stationary environments, the ability to commit one period in advance implies stationary policies. One way in which one can rationalize this limited commitment is by assuming that the ruler, i.e., the agent in charge of the government, can always

choose policies consistent with agents' incentives, but she may be ousted from power and replaced by a new ruler if she reneges on such policies afterwards.

The assumption that the government is relatively patient is also important to make sure that she is willing to incur the one one-time cost of the transition, as captured by x_{t^c+1}. This cost is substantial because the government has to tax all agents equally, i.e., she cannot separate producers from consumers under the credit arrangement, and agents anticipate that the surplus will shrink after the transition to money comes into effect. A relatively patient government is willing to undertake this cost because she anticipates that she will be able to extract more surplus once the economy transits to the monetary arrangement. We conjecture that a similar result would go through if we assumed that the government is as patient as the agents but the tax base, i.e., the measure of agents paying taxes, increases over time. Intuitively, the government would be willing to incur the one-time cost of transition because she anticipates that the difference between the total surplus extracted under money and the total surplus extracted under credit would increase over time.

It remains to consider the storability assumption and the restrictions imposed on ϕ, ρ and β. The former allows the government to run a deficit by giving subsidies and not taxing during the transition period. Storage is an inferior savings technology but is needed because the government cannot produce general goods. Naturally, this assumption can be relaxed if the government can produce general goods. Moreover, it is not necessary if u is sufficiently larger than c and agents are patient enough, as in this case $x_{t^c+1} > 0$ and the government taxes even during the transition. Finally, by setting $\phi(t) < 1/2$ for all t, $\beta > 2c/(u+c)$, and $\rho = 1$, we highlight that the transition is not driven by welfare considerations or by the feasibility of the credit arrangement, but instead by the government's incentive to appropriate surplus. In the Appendix, we consider the case where $\phi(t) = \phi$ but $\rho(t)$ is strictly decreasing and convex, with $\rho(0) = 1$ and $\rho(\infty) = 0$. We obtain that there is always a transition from credit to money, but it takes place too early if one takes as reference the expected welfare of a representative agent.

4 Historical evidence

5 Final Remarks

6 Appendix

Proof of Lemma 01 We can use the expressions for W^i and V^i to rewrite V^0 as

$$(1-\beta)V^0 = \beta m\left[(1-m)u + mc\right] - mc - \beta\left[mx^1 + (1-m)x^0\right],$$

which allows us to rewrite (3), (4), and (5), respectively, as

$$-mc + \left[(1-\beta)\,q^0 + \beta m\right]\left[(1-m)u + mc\right] \geq (1-\beta)\,x^0 + \beta\left[mx^1 + (1-m)x^0\right], \tag{20}$$

$$\beta\left(x^0 - x^1\right) + \beta\left(q^1 - q^0\right)\left[(1-m)u + mc\right] \geq c, \tag{21}$$

and

$$x^0 - x^1 \geq (1-q^1)\left[(1-m)u + mc\right]. \tag{22}$$

The participation constraint of an agent without money must be binding at the optimal policy. If this constraint is not binding, we can make an arbitrarily small increase in x^0 and x^1 by the same amount. This change would not affect the remaining constraints, as they only depend on the difference $x^0 - x^1$, and it would strictly increase the payoff of the government. Moreover, at least one of the two other constraints must be binding. If this is not the case, we can make an arbitrarily small increase in x^1 and q^0 making sure that (20) holds with equality (note that, given (2),an increase in q^0 leads to a decrease in q^1). This change would increase the payoff of the government and would still satisfy (21) and (22) if these constraints are not binding.

We now show that both (21) and (22) are binding at the optimal policy. The proof is by contradiction. Consider, first, the case where the incentive compatibility constraint of the seller is binding but the truth-telling constraint is not binding. In this case, we can use the fact (20) and (21) are binding to obtain

$$x^0 = q^0\left[(1-m)u + mc\right]. \tag{23}$$

We can also use the fact that (21) is binding to rewrite (22) as

$$\beta(1-q^0)\left[(1-m)u + mc\right] < c.$$

This implies that we can make an arbitrarily small increase in x^0 and q^0 so that (23) is satisfied and (22) also holds. This change increases the payoff of the government, which contradicts the claim that the truth-telling constraint is not binding at the optimal policy.

Consider now the case where the truth-telling constraint is binding but the incentive compatibility constraint of the seller is not binding. Using the fact that (20) and (22) are binding, we obtain

$$x^0 = \left[q^0 (1-\beta m) + \beta m\right] \left[(1-m)u + mc\right] - mc. \tag{24}$$

We can then use the fact that (22) is binding to rewrite (21) as

$$\beta \left(1 - q^0\right) \left[(1-m)u + mc\right] > c.$$

We can make an arbitrarily small increase in x^0 and q^0 so that (24) is satisfied and (21) also holds. This change increases the payoff of the government, contradicting the claim that the incentive compatibility constraint of the seller is not binding at the optimal policy. Summarizing, at the optimal policy, it must be the case that (20), (21), and (22) are binding.

Proof of Proposition 03 The proof is by contradiction. Assume that, for all $\delta > \beta$, there is no transition. In this case, the term inside (19) becomes

$$\left[1 - \phi(t)\right] \frac{u-c}{2} - \frac{(1-\beta)c}{\beta}.$$

Since $\phi(t) \to 1/2$ when $t \to \infty$, there exists T large enough and δ sufficiently close to one such that, for all $t > T$

$$\left[1 - \phi(t)\right] \frac{u-c}{2} < \left[1 + \frac{\delta^{T-1}(\delta - \beta)c}{\beta(u-c)}\right] \frac{u-c}{4}.$$

Thus,

$$\widehat{U}(\infty) = \frac{u+c}{2} - \frac{\delta(1-\beta)c}{(1-\delta)\beta} + \sum_{t=1}^{T-1} \delta^t \left[1 - \phi(t)\right] \frac{u-c}{2} + \sum_{t=T}^{\infty} \delta^t \left[1 + \frac{\delta^{T-1}(\delta-\beta)c}{\beta(u-c)}\right] \frac{u-c}{4}$$

is an upper bound on $U(\infty)$. Now, if we assume that $m = 1/2$ and let the transition happen at $t = T - 1$, (19) becomes

$$\widehat{U}(T-1) = \frac{u+c}{2} - \frac{\delta(1-\beta)c}{(1-\delta)\beta} + \sum_{t=1}^{T-1} \delta^t \left[1 - \phi(t)\right] \frac{u-c}{2} + \sum_{t=T}^{\infty} \delta^t \left[\frac{u-c}{4} + \frac{\delta^{T-1}(\delta-\beta)c}{2\beta}\right],$$

which is a lower bound on $U(T-1)$ since m is not optimal. We obtain that $\widehat{U}(T-1) > \widehat{U}(\infty)$ if and only if

$$\frac{u-c}{4} + \frac{\delta^{T-1}(\delta-\beta)\,c}{2\beta} > \left[1 + \frac{\delta^{T-1}(\delta-\beta)\,c}{\beta(u-c)}\right]\frac{u-c}{4},$$

which is always true. Hence, there exists a transition from credit to money which gives a strictly higher payoff to the government than no transition.