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Dynamic Coordination With Network Externalites: Procrastination Can Be Efficient

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I analyze a dynamic coordination model under quasi-hyperbolic discounting. The main result is that present bias can induce a society to coordinate efficiently. When considering a transition from network A to B, higher present bias induces individuals to ask for higher relative quality of B, which is also what the central planner dictates, but for different reasons. A present biased agent overvalues relative quality because, when considering whether to initiate a transition, her momentary loss of network externalities is overvalued by myopic discounting. The planner's motives are the negative externalities inflicted on agents "stuck" in A during a transition.

Keywords: dynamic coordination, hyperbolic discounting

1 Introduction

Problems of dynamic coordination are frequent in economics. These are situations where the actions of others influence one's choice of action. Social networks are direct examples of these: if most of one's acquaintances were in Facebook rather than in the late Google +, even if Google + had attractive qualities, Facebook could be preferred. An economic framework for this problem is presented by Frankel and Pauzner (2000) (FP), where agents choose between two actions. They maximize utility through time, while knowing that there are timing frictions: a rate at which decisions can be reevaluated. The coordination problem here can be formulated as the answer to the question of how much better Google + would need to be in order to people start migrating to it, given that there is some initial share of individuals in each social network, some timing friction, and a discount rate.

In this model, as decisions are taken individually and externalities are present, one must suspect that resulting equilibria–or lack thereof–can be inefficient. Indeed, when agents are fully rational, externalities are roughly the same for each action, and utility is linear, in order to start a transition to an alternative action, a central planner would suggest agents to ask for higher values of intrinsic quality of this alternative action. That is because, when considering a change in action, individuals do not account for the decrease in the externality received by the agents who will take longer to change action, so that a higher increase in quality, in relation to the increase a free-market transition would need to take place, is required to justify transitioning.

In this paper, I study how the dynamic coordination framework can behave under present bias, in the form of hyperbolic preferences. The main result consists of showing that present bias nudges individuals in the same direction a central planner would do, being able to restrict or eliminate socially undesirable equilibria. The intuition for this result is that if a society is generally adopting action A, present bias induces agents to be more demanding in terms of relative intrinsic quality of action B in order to begin a transition to B, since the gain from the externality coming from more people adopting B comes only in the future, which is disproportionately discounted. To illustrate this result, I first present the theoretical model from FP, while adding hyperbolic discount, then proceed to a simplified model, where the result is shown and some comparative statics are analyzed to assess the likelihood that present bias is socially beneficial. In order to motivate the modelling, we can start by considering choices with complementarities.

Many decisions are taken in agreement to what is expected of others, because of complementarities. For instance, the best programming language to learn does not depend only on the specific qualities of a language, which are rarely clear for a beginner, but rather on which languages we imagine programmers will be writing code after the language is learned. If a language quickly loses users, significant costs may be imposed on the ones who are not able to migrate to the new one right away, which is likely the case for a beginner. In fact, even if someone already knows all relevant languages, the choice to conduct a project in some language may depend on which language other people use to conduct similar projects, as one may need to seek help or hire people with experience to assist.

A special characteristic of these decisions that involve complementarities is that they are subject to path dependency, that is, history matters. Once most settle for a standard programming language in some area, it can be quite hard to transition to another language, even if it seems that such language is better than the current standard. Another interesting example of this situation is the prevalence of QWERTY keyboards. David (1985) describes with detail the succession of events that culminated in the prevalence of the QWERTY standard on keyboards, even tough there is evidence that the Dvorak Simplified Keyboard (DSK), which has been available for decades, is more efficient for typing. More than exploring the QWERTY case, David (1985) motivated economists to be more attentive to history, exactly because many free market outcomes are path dependent. Cases like the keyboard choice were not the product of regulation or planning, but rather of the unravelling of history.

More recently, beginning with FP, a theoretical framework has been developed in the economics literature to deal with this kind of problem - a coordination problem - where there are externalities in choice and history may play an important role. Models in this literature of dynamic coordination succeeded in shedding new light on the possible equilibria and equilibrium choice in a society. Importantly, there is the possibility of multiple equilibria regions or unique equilibrium, depending on how many people belong to each standard (keybord style or programming language, in the examples) and on the intrinsic quality of each standard.

An economic model for dynamic coordination is also helpful as it allows for welfare analysis. Based on this class of models, Guimaraes and Pereira (2016) (GP) analyzes the QWERTY matter and the notion of efficiency in this case. It is argued that the prevalence of a lower quality state–as QWERTY over DSK–is likely to be overall a non-issue, as the central planner of the dynamic coordination problem suggests agents to ask for higher values, in relation to the selfishly chosen ones, of relative DSK quality in order to migrate from QWERTY to DSK. This implies that if QWERTY prevails, it is surely socially efficient. The rationale behind this need of higher relative DSK quality lies in the externalities that will vanish for the individuals which are stuck in the lower quality state in the case of a transition. It takes time for everyone to transition, and a transition which seems profitable for an individual may not be for society in general. In short, the person who changes state selfishly generates a form of negative externality to the individuals who are yet to change state, which is socially inefficient if the relative quality of DSK is not sufficiently high to justify the negative externality caused.

The result from Guimaraes and Pereira (2016) could be interpreted as either a paradox or a relief to those worried that there are too many cases where path-dependency is keeping society in inefficient equilibria, like in the QWERTY tale. In this paper, present bias is the missing ingredient that accommodates both stories. Present bias is a common suspect for explaining why an action with intertemporal consequences that should be taken is, maybe indefinitely, delayed. Indeed, present bias will make agents more demanding in terms of what they can get today–intrinsic quality–, making it harder for new, better standards, to reach popularity, allowing for a wider range of inefficient equilibria. Moreover, in light of the result that, under exponential discount, agents change networks under sub optimal intrinsic quality levels, present bias can also be a force that avoids undesirable equilibria.

Present biased individuals show higher discount rates between today and tomorrow than between tomorrow and the day after tomorrow–or, rather than a day, any fixed time interval. That is, they disproportionately enjoy rewards now rather than later, and also disproportionately push unpleasant efforts into the future. Early surveys on the subject, as Loewenstein and Prelec (1992) and Frederick et al. (2002) summarize field and laboratory evidence on present bias, analyzing an assortment of experiments that corroborates hyperbolic discounting, while references to more recent studies can be found in Andreoni and Sprenger (2015). Even though there is not a complete consensus in the literature about present bias, mostly because a good part of the evidence in favor of it arises from monetary choices and there are uncertainties associated with the future, two specific pieces of evidence help motivating the application of quasi-hyperbolic discounting in this paper.

The first is the evidence of present bias regarding non-monetary choices. In this paper, the decision problem regards whether one wants to change her action, costlessly, in a given moment. Hence, even though most of the documentations of present bias concern monetary decisions, studies like Augenblick et al. (2015) and Sadoff et al. (2015) provide relevant evidence in favor of present bias being important in non-monetary choices. The first study shows that individuals present time-inconsistency when distributing real work over time, rather then when choosing monetary rewards. The second one regards grocery consumption, where observed individuals also show time inconsistencies in their choices.

The second piece of evidence are the studies that focus on unravelling how dynamic inconsistencies come about, as Andreoni and Sprenger (2012) and Andreoni and Sprenger (2010). These studies indicate that present bias, in the form of quasi-hyperbolic preferences, may arise from the fact that individuals show a disproportional preference for certainty, along with the fact that the future is inherently uncertain. In a way, these conditions are met in the dynamic coordination framework. As utility arises from both complementarities and from the relative quality of actions, and each of these components carry different degrees of uncertainty and distribution though time, it can be reasonable to assume quasi-hyperbolic preferences in this paper's framework.[1]

With these ideas in mind, in Chapter 2, this paper adapts the dynamic coordination problem as posed by FP to the the existence of present bias, in the form of hyperbolic discounting. In Chapter 3, I specialize the framework to a linear utility function, so that I can analyze the problem in a tractable setting. Chapter 4 explores this linear model in order to understand in which situations can present bias ensure efficiency, so that conclusions can be drawn.

2 Set Up and Equilibrium Existence

In this chapter, I present the discount rate that introduces present bias in the model and briefly revisit the FP results under hyperbolic discounting. I show that the general form of the solution to the model is not affected by the introduction of a different discount factor. Most results follow almost immediately from FP and Burdzy et al. (1998), so that some proofs are either deferred to the Appendix or omitted for brevity. For a more comprehensive introduction to the dynamic coordination framework, see Guimares et al. (2017).

2.1 The Discount Function

Present bias has been commonly used in economic modelling through hyperbolic discounting. This type of utility discount was first formalized in discrete time as in Laibson (1997) and Rabin and O'Donoghue (1999) with preferences of the type

$$U_t = u_t + \beta\eta u_{t+1} + \beta\eta^2 u_{t+2} + \beta\eta^3 u_{t+3} + \dots$$

In order to extend this kind of behavior to continuous time, a discount function in continuous time is needed. There is more than one way to generate such a discounting factor, but, in this paper, I use the discounting

[1] This applies mostly to the case when the relative intrinsic quality follows a Brownian Motion with small shocks. In this scenario, the probability of a network transition is always positive, but there is little uncertainty about the intrinsic quality, so that the externality part of the utility bears uncertainty of a different nature than the intrinsic quality part.

factor suggested in Harris and Laibson (2013), in a deterministic setting:[2]

$$D(t,\tau,\lambda) = \begin{cases} e^{-\rho(t-\tau)} & \text{if } t-\tau \leq \lambda \\ \beta e^{-\rho(t-\tau)} & \text{if } t-\tau > \lambda \end{cases} \tag{1}$$

with $\rho > 0$ and $\beta \in [0,1]$

2.2 The Dynamic Coordination Problem

In its basic format, the dynamic coordination setting models a society where homogeneous agents choose between two actions in continuous time. Decisions are taken according to the rate of a Poisson clock: for any individual, for any equally sized period of time, there is a constant and independent chance of being able to reevaluate the action choice. This clock can be interpreted as an exogenous attention friction: individuals only update their choices over this matter in a countable number of random moments. Utility from choosing some action depends positively on the measure of individuals simultaneously taking that action and on the relative quality of action choices. The model yields different equilibria that depend on parameters and their evolution in time.

Let there be a continuum of individuals $i \in [0,1]$, which live forever. Agents are called upon to choose $a_{it} \in \{0,1\}$ according to a Poisson clock with rate δ. I will refer to a_{it} as an action, a choice of a network, or standard, interchangeably. Agents have flow utility of changing to action 1 equal to $\Delta u(\theta_t, n_t)$, continuously differentiable and increasing in both arguments. That is, the benefit of choosing action 1 over 0 increases with the intrinsic relative quality of action 1, θ_t, and with the number of individuals taking action 1, n_t. A present self believes she will be in command of choices forever, so agents are naive with respect to their time inconsistency. An individual drawn by the Poisson process at time τ chooses action 1 if

$$\mathbb{E}\left[\int_{\tau}^{+\infty} e^{-\delta(t-\tau)} D(t,\tau,\lambda) \Delta u(\theta_t, n_t)\, dt\right] > 0$$

where the LHS is the accumulated expected difference in utility from choosing action 1 in relation to action 0 until the moment the individual expects to reevaluate his choice, so that the discount rate is increased by

[2] Even though Webb (2016) suggests that other deterministic discount functions present better properties regarding desirable choice axioms, equation (1) presents all properties needed for this application while being simpler than the alternatives. In fact, the results presented here are somewhat similar if we use $D(t,\tau,\lambda) = \begin{cases} e^{-(\hat{\beta}/\lambda+\rho)(t-\tau)} & \text{if } t-\tau \leq \lambda \\ e^{-\hat{\beta}-\rho(t-\tau)} & \text{if } t-\tau > \lambda \end{cases}$, suggested by Webb (2016), instead. The main difference is that derivations and expressions are far more involved, while no extra clear economic intuition is derived.

δ.

2.2.1 Model With No Shocks

Let $\theta_t = \theta \in \Theta \subseteq \mathbb{R}$, for any point in time. Assume that there is $\underline{\theta}$ such that if $\theta < \underline{\theta}$, action 0 is dominant for any n_τ, that is, even if an individual believes everyone else will pick 1, which is the most optimistic belief, she will choose 0, for any n_τ. Also, assume there is $\bar{\theta}$ such that action 1 is dominant if $\theta > \bar{\theta}$, with an analogous meaning, for the most pessimistic belief. Then we have the following result:

Proposition 1 *There are functions $\theta^{opt} : [0,1] \to \Theta$ and $\theta^{pes} : [0,1] \to \Theta$ such that $\theta^{opt}(n_\tau) < \theta^{pes}(n_\tau)$ for any $n_\tau \in [0,1]$ and:*

- *If $\theta < \theta^{opt}(n_\tau)$, the unique equilibrium is everyone choosing 0*
- *If $\theta > \theta^{pes}(n_\tau)$, the unique equilibrium is everyone choosing 1*
- *If $\theta^{opt}(n_\tau) < \theta < \theta^{pes}(n_\tau)$, then there are multiple equilibria*

Proof. See the Appendix. ■

These functions are given as solutions of:

$$\int_\tau^{+\infty} e^{-\delta(t-\tau)} D(t,\tau,\lambda) \Delta u(\theta^{opt}(n_\tau), n_\tau^{\uparrow})\, dt = 0 \tag{2}$$

$$\int_\tau^{+\infty} e^{-\delta(t-\tau)} D(t,\tau,\lambda) \Delta u(\theta^{pes}(n_\tau), n_\tau^{\downarrow})\, dt = 0 \tag{3}$$

where $n_\tau^{\uparrow} = 1 - (1 - n_\tau)e^{-\delta(t-\tau)}$ and $n_\tau^{\downarrow} = n_\tau e^{-\delta(t-\tau)}$, the paths of monotonic motion of n_t. Notice that these thresholds are defined using the most pessimistic ($n^{\downarrow}$) and optimistic beliefs ($n^{\uparrow}$) about the adoption of action 1 by a society. These threshold functions are decreasing because higher values of relative quality imply that a smaller initial share of agents in a given standard is required for a transition to be worthwhile. A linear version of these thresholds are the dashed lines in Figure 1.

Hence, we conclude that the introduction of present bias via the $D(.)$ discount function has no qualitative impact on equilibria characterization under static relative quality. As in FP, for some initial distribution of agents between actions, there are values of θ low enough so that action 0 is dominant, followed by an interval of higher values of θ where no action is dominant, so that equilibrium depends on beliefs over the action of others, followed by an interval of even higher values of θ so that action 1 is dominant.

2.2.2 Model With Shocks

Now allow for

$$\theta_t = \mu dt + \sigma dZ_t \tag{4}$$

where Z_t follows a standard Brownian motion. Then we have:

Proposition 2 *Let the trend from the Brownian motion, μ, be a constant. There is a unique decreasing function $\theta^*(n_\tau)$ such that if $\theta_\tau < \theta^*(n_\tau)$, the unique equilibrium is everyone choosing 0. Conversely, if $\theta_\tau > \theta^*(n_\tau)$, the unique equilibrium is everyone choosing 1.*

The proof for the previous proposition follows the same steps as in FP and is omitted for brevity. The intuition for such a result is that the randomness in the path of the relative quality parameter triggers a systematic contagion process in both the thresholds derived from Proposition 1. Consider the pessimistic threshold, θ^{pes}, as in Figure 1. Previously, immediately to the left of θ^{pes}, any equilibria was possible. Now, there will be a region close to that curve, but still to the left, such that 1 will be a dominant choice if everyone plays according to θ^{pes}. Indeed, suppose a society where people play according to θ^{pes} and consider θ slighly to the left of it. While before one's action would would depend on beliefs, now there is a positive probability that θ_t moves to the right of θ^{pes}, making people choose 1 and increasing its utility, so that there is a new threshold, say, θ_1^{pes} to the left of θ^{pes} such that under the old beliefs, it is dominant to play 1 to the right of θ_1^{pes}. If beliefs are updated to "everyone plays according to θ_1^{pes}", there will be a θ_2^{pes} defined analogously, and so on. This series of thresholds converges to a limit threshold. This iteration process of eliminating strictly dominated strategies can be constructed also from θ^{opt}, going to the right, and one can show that the final set of equilibria is defined by a single threshold, θ^*.

2.2.3 Small Shocks

One last general result which will be useful for analyzing the specific examples is:

Proposition 3 *Allow for $\theta_t = \mu_t dt + \sigma dZ_t$, where $\mu_t = \alpha\psi(t, \theta_t)$. If $(\alpha, \sigma^2) \to 0$, then, θ^*, the unique decision threshold function, is the solution to:*

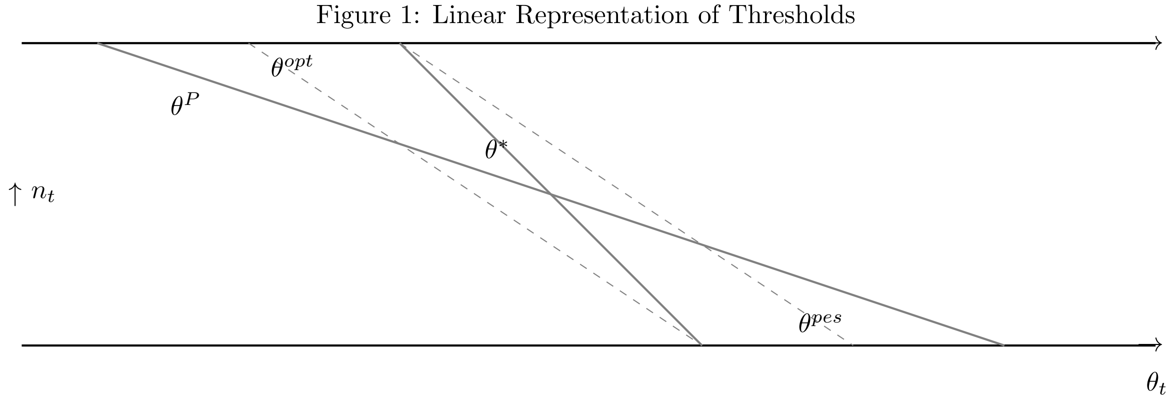
$$(1\text{-}n_\tau) \int_\tau^{+\infty} e^{-\delta(t-\tau)} D(t,\tau,\lambda)\Delta u(\theta^*(n_\tau), n_\tau^{\uparrow})\, dt + n_\tau \int_\tau^{+\infty} e^{-\delta(t-\tau)} D(t,\tau,\lambda)\Delta u(\theta^*(n_\tau), n_\tau^{\downarrow})\, dt = 0 \tag{5}$$

The proof for the previous proposition follows the same steps as in FP and is omitted for brevity. The result states that if shocks and trends of the Brownian motion of the relative quality parameter are small, then the uniqueness of the equilibrium is preserved, while it is defined as a convex combination of the thresholds in the static model. Furthermore, in this case, the trend can depend on parameters, and the relative speed with which trend and variance approach zero is not important. It can be the case that variance is very small relative to the trend, so that just a small a amount of persistence in shocks is enough to collapse the equilibria. At last, (5) is a convex combination of (2) and (3), with weights $(1 - n_\tau)$ and n_τ, which are the probabilities an economy bifurcates up, towards $n = 1$, or down, towards $n = 0$. That is because, under the previous assumptions, it can be shown that the time it takes an economy to bifurcate up or down approaches zero.

The threshold for this last case is illustrated with the θ^* curve in Figure 1. This threshold touches the one derived with fixed θ and optimistic beliefs, because the probability of bifurcating upwards is 1 at $n_\tau = 0$, what puts all weight in the optimistic threshold. The same argument applies for the threshold sharing a point with the pessimistic threshold.

At this point, then, we have checked that the general properties and results of the dynamic coordination problem as defined in Frankel and Pauzner (2000) extend to the case with hyperbolic discounting. Also, in doing that, we briefly reviewed that same framework.

Figure 1: Linear Representation of Thresholds



Thresholds derived in Chapter 2 and Chapter 3. In the plane, the relative quality of action 1 (DSK) increases to the right and the number of users of action 1 (DSK) increases upwards.

3 Linear Model

Now I specialize in a particular case, proposed by GP, in order to study the network choice problem, as the QWERTY case we previously described. Rename action 1 the use of the DSK keyboard standard, while action 0 is to use QWERTY. Let the flow utility of each action be:

$$u^Q(\theta_t^Q, n_t) = \theta_t^Q + \nu(1 - n_t)$$

$$u^D(\theta_t^D, n_t) = \theta_t^D + \nu$$

So that the difference in the utility flow is $\Delta u(\theta_t, n_t) = \theta_t + \gamma n_t$, with $\theta_t := \theta_t^D - \theta_t^Q - \nu \in \mathbb{R} \equiv \Theta$ and $\gamma := 2\nu > 0$. Notice that, in this specification, the externalities are symmetric for each action.[3]

3.1 No Shocks

Proposition 4 *For an economy where the intrinsic relative quality is fixed, the choice thresholds are given by:*

$$\theta^{opt}(\beta, n_t) = -\gamma\left(1 - \Omega(\beta)\right) - \gamma n_\tau \Omega(\beta) \tag{6}$$

$$\theta^{pes}(\beta, n_t) = -\gamma n_\tau \Omega(\beta) \tag{7}$$

where the notation now makes explicit the dependence of the threshold curves on β *and* $\Omega : [0,1] \to [\frac{\rho+\delta}{\rho+2\delta}, \frac{\rho+\delta}{\rho+2\delta}\frac{1-e^{-(\rho+2\delta)\lambda}}{1-e^{-(\rho+\delta)\lambda}}]$ *is a strictly decreasing function such that:*

$$\Omega(\beta) = \frac{\rho+\delta}{\rho+2\delta}\frac{1-(1-\beta)e^{-(\rho+2\delta)\lambda}}{1-(1-\beta)e^{-(\rho+\delta)\lambda}}$$

Proof. The thresholds are characterized by solving equations (2) and (3) for the specific linear utilities. ∎

Notice that both thresholds have the same slope in the (θ, n) plane. In this same plane, portrayed in Figures 1 and 2, a present biased individual presents a threshold less steep than the dynamically consistent one. That is because present bias makes the future disproportionately less valuable: for the optimistic threshold, it makes the future externalities from migrating to DSK less attractive, so that in every case, except at $(\underline{\theta}, 1)$, more intrinsic relative quality is asked to start a transition. For the pessimistic threshold, the intuition is reversed. Hence, we can conclude that more present bias induces a decrease in the area in $\mathbb{R} \times [0,1]$ where multiple equilibria are possible.[4]

[3] As showed in GP, if the externalities generated for each network are allowed to differ in magnitude, as long as the difference is not so big, the qualitative results of the model are similar

[4] Notice that this result is not exclusive to present bias. Any change in discount rates that makes future utility less important would do the same.

3.2 Small Shocks

Proposition 5 *When shocks and trends are vanishing, the unique equilibrium-defining threshold is:*

$$\theta^*(\beta, n_t) = -\gamma \left(1 - \Omega(\beta)\right) - \gamma n_\tau \left(2\Omega(\beta) - 1\right) \tag{8}$$

Proof. By Proposition 3, the unique threshold is a linear combination of (6) and (7). ■

We know that the threshold under vanishing trends and socks is contained between the fixed θ thresholds and that they share the points $(\theta^*(\beta, 0), 0)$ and $(\theta^*(\beta, 1), 1)$–see Figure 1. Hence, the "small shocks" and "fixed θ" thresholds are closely related and get closer together as the agents become more present biased, so that, for simplicity, from here on we focus on the "small shocks" case, where shocks and trends are vanishing.

When $\beta = 1$ or $\lambda = 0$ we have no present bias in preferences and the equation simplifies to:

$$\theta^*(1, n_\tau) = -\frac{\gamma\delta}{\rho + 2\delta} - n_\tau \frac{\gamma\rho}{\rho + 2\delta} \tag{9}$$

Hence, in the case of small shocks, in relation to the dynamically consistent case, letting $0 < \beta < 1$ induces a rotation of the threshold around the point $(\theta^*(1, 1/2), 1/2))$. Intuitively, that must happen because going from QWERTY to DSK provides an immediate gain in terms of relative quality of typing, while gains in terms of externalities take time to develop, as transition is not instantaneous. As hyperbolic discounters prefer rewards sooner than later, a greater relative quality is needed to induce a transition.

Consider a central planner problem where the planner weights every individual the same. GP shows that, under dynamically consistent exponential discounting, if a benevolent planner makes choices for each individual when she receives the chance to reevaluate her choice, the decision threshold is given by:

$$\theta^P(n_\tau) = \frac{\gamma\rho - 2\gamma\delta}{2(\rho + 2\delta)} - \frac{2\gamma\rho}{\rho + 2\delta} n_\tau \tag{10}$$

That is, the planner threshold is also a rotation of the individual's threshold, in the same direction of the rotation caused by hyperbolic preferences. Specifically, the planner's resulting threshold is half as steep as the dynamically consistent individual's. That happens because the planner is aware that, if a transition to DSK takes place, all individuals locked in QWERTY will be negatively affected by the decreasing QWERTY externality, since these agents will take some, possibly long, time to have their actions reevaluated. The central planner, then, asks for a higher value of DSK quality over QWERTY in order to compensate for the negative externality on QWERTY users and start a transition.

The central planner solution for the case analyzed here is the same, even though individuals are present-biased. That is justified as, ex-ante–at least λ units of time before a choice–agents would be dynamically consistent discounters. Hence, the main result:

Proposition 6 *Under vanishing shocks and trends, for any value of the parameters* $(\rho, \delta, \gamma, \lambda) \in \mathbb{R}_+^4$*, there is some* $\beta \in [0, 1)$ *such that the state space where socially inefficient coordination can occur is strictly smaller, i.e, there is a strictly smaller area in the* $\mathbb{R} \times [0, 1]$ *plane in which a transition that is not socially optimal will occur, in relation to the case where* $\beta = 1$*.*

Proof. $\Omega'(\beta) < 0$ for any $\beta \in [0, 1]$, so that any small decrease in β, starting from $\beta = 1$, continuously makes the threshold less steep. As the planner's threshold is always less steep than the dynamically consistent individual's threshold if $(\rho, \delta, \gamma, \lambda) \in \mathbb{R}_+^4$, then there is always $\tilde{\beta} \in [0, 1)$ such that $2(1-2\Omega(1)) \leq 1-2\Omega(\tilde{\beta}) < 1 - 2\Omega(1)$, that is, $\tilde{\beta}$ provides a value for $\partial\theta^*(\beta, n_\tau)/\partial n_\tau$, the slope of the present biased agent's threshold, such that some or all socially undesirable equilibria are avoided. ■

In fact, if

$$\frac{3\rho + 2\delta}{2(\rho + 2\delta)} \leq \frac{\rho + \delta}{\rho + 2\delta} \frac{1 - e^{-(\rho+2\delta)\lambda}}{1 - e^{-(\rho+\delta)\lambda}} \tag{11}$$

we can find:

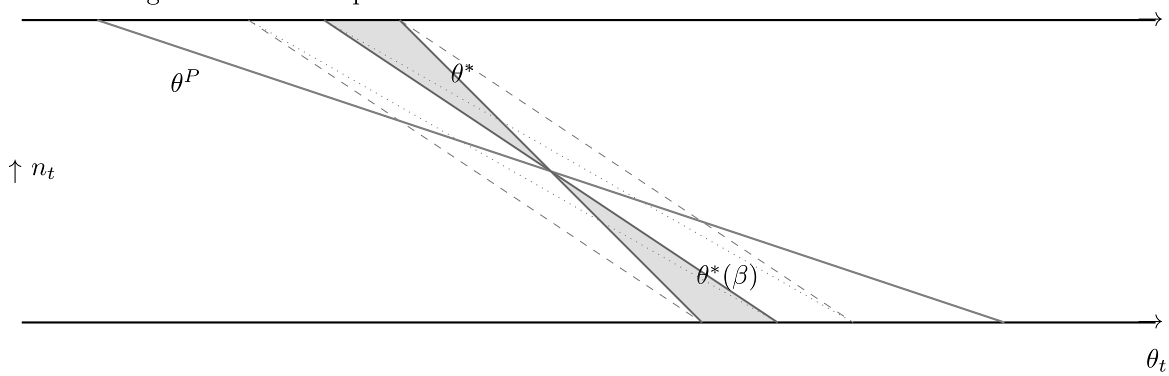
$$\beta^* = \Omega^{-1}\left(\frac{3\rho + 2\delta}{2(\rho + 2\delta)}\right) \tag{12}$$

which is the value for the present bias parameter that makes the planner's solution the same as the present biased individual's. If λ is close to zero, as one would imagine, the condition for the existence of β^* approaches $\rho \leq 2\delta$, that is, the long term discount rate cannot be too high relative to some level of timing friction, while less timing friction allows for a greater range of discount rates for which present bias can ensure optimal social choice. Intuition for that will be provided in the next chapter.

If the inequality in equation (11) is strict, then, there is the possibility that the individual's threshold is less steep than the planner's. This means that there can a be a level of present bias such that the coordination in QWERTY is actually inefficient. In the case without present bias, as in GP, an inefficient coordination in QWERTY could never happen.

Figure 2 depicts a situation where some present bias is present, but not enough for equating the the agents' and planner's thresholds. The $\theta^*(\beta)$ curve now stands for a threshold where there is some present bias. The shaded region then represents initial points in the $[0, 1] \times \mathbb{R}$ space where if individuals were dynamically consistent, they would implement a socially inefficient equilibrium. The area between θ^P and

Figure 2: Linear Representation of Thresholds With And Without Present Bias



Comparison of a situation where there is no present bias to a situation with some present bias. $\theta^*(\beta)$ is the threshold for a population with some present bias. The shaded area depicts points where a socially undesirable equilibrium is avoided.

$\theta^*(\beta)$ is constituted of points where socially inefficient equilibria may still happen.

4 Discussion and Conclusion

The presence of hyperbolic discounting makes individuals exaggeratedly sensitive to the advantages of intrinsic quality, whose benefits can start being enjoyed right away, in contrast to the benefits of the future externalities generated by transitioning. In this fashion, DSK needs to be substantially better than QWERTY to justify a transition under hyperbolic discount. A central planner is also more demanding in terms of θ, because of the negative externalities of a transition. Hence the society with an appropriate β is kinder to the individuals who would take long to adopt DSK in the event of a transition, even if it is due to individualistic reasons. That is true in the sense that this society is willing to transition to DSK, hurting those stuck with QWERTY, in fewer, more favorable, situations, that is, only when relative quality is higher.

An interpretation of the last result is that if a central planner could not directly make the individual's choices, he would always like individuals to be somewhat present biased, "procrastinating" the network change by asking for more relative intrinsic quality. This protects the individual that would take long to migrate from negative externalities, by preventing the ones who would like to change sooner, if there was no present bias, from doing so.

It is now useful to analyze what range of values for β are helpful, in the sense that it helps mimicking the central planner solution and how do these values depend on the parameters. Answering these questions can help us to understand the likelihood that procrastination is actually helpful and in which situations procrastination is effective. To narrow down this analysis, I will focus on the case where equation (12) is valid, i.e.: the present bias parameter equates the individual threshold to the socially efficient one. Once we understand these relationships, answering the first question will be easier.

For what follows, the baseline case is when $\rho = 0.02$, $\delta = 1$, $\lambda = 0.01$. That is, if the time unit is a year, then the discount rate is about 2% a year, individuals are expected to reevaluate keyboard choices about once a year, and the period of myopic discounting lasts about 4 days. This requires β^* to be nearly 0.5 in order to equalize the planner's and individual's threshold. Hence, any $1 > \beta \geq 0.5$ would be helpful in avoiding socially damaging equilibria, in relation to the case of no present bias.

4.1 The rate of discount

In Figure 3, the solid line depicts the relationship between β^* and ρ, for $\delta = 1$ and $\lambda = 0.01$, as in the baseline scenario. We can see that the need for present bias increases with ρ. That is the case because of two facts. First, the planner solution is very sensitive to the discount rate, making individuals ask for a higher quality of DSK in order to transition, because the discount rate determines the size of the externality loss suffered by all individuals locked in the network. Second, while the discount rate affects the individual's utility, its effect is smaller both because the individual only cares for his own utility and because $0 < \beta < 1$ implies a smaller base value where discount can happen, after λ units of time. Hence, for a given increase in ρ, β^* must decrease in order to make up for the smaller sensibility of the individual's threshold to the discount rate, so that the individual's demand for relative intrinsic quality stays the same as the planner's.

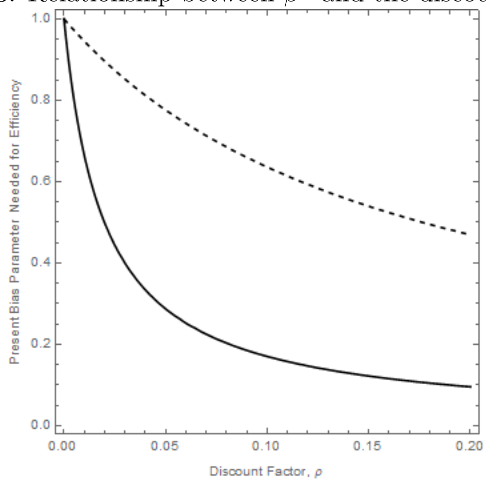
Because the planner's solution is more sensible to increases in ρ, for a high enough discount rate, there can be no degree of present bias that allows for the dynamically inconsistent threshold to equate the socially efficient one. This helps to explain why ρ cannot be too high, relative to δ–see equation (11)–in order to allow for the possibility of the present bias inducing a new socially inefficient region in the $\mathbb{R} \times [0, 1]$ space.

The dashed line in Figure 3 depicts the same relationship as in the solid line, but for $\delta = 3$. For any value of ρ, the level of present bias needed in order to make the economy efficient in every scenario is smaller. In what follows, I try to derive some intuition for that.

4.2 The Friction Parameter

In Figure 4, the solid line depicts the relationship between β^* and the friction parameter for $\rho = 0.02$ and $\lambda = 0.01$. We can see that the more often individuals rethink their decision, the smaller is the amount of present bias needed to induce an efficient threshold.

Figure 3: Relationship between β^* and the discount rate



Lines depict the relation between the present bias parameter needed to make the economy efficient, β^*, and the discount rate, ρ. The solid line assumes $\delta = 1$ and $\lambda = 0.01$, while the dashed one assumes $\delta = 3$ and $\lambda = 0.01$

In general, we expect the planner's threshold to be more sensitive to changes in δ, because of externalities. This is what is confirmed in Figure 4. As δ increases, transitions are faster and the thresholds become steeper, as the trade-off between intrinsic quality now and externalities in the future is changed by faster transitions. For $\delta \in (0, 20]$, this effect is stronger in the planner's threshold, so that the level of present bias needed to equate threshold is smaller for higher δ.

When frictions are small in relation to λ, the relationship depicted in Figure 4 is reversed.[5] That is because the moment the transition from present to future takes place is relevant in the sense that it can make utility from externalities more or less representative, making the present biased agents' threshold more or less sensitive to changes in δ. When δ increases, a relevant part of the transition happens in the present, before λ units of time, so that eventually the relation depicted in Figure 4 is reversed.

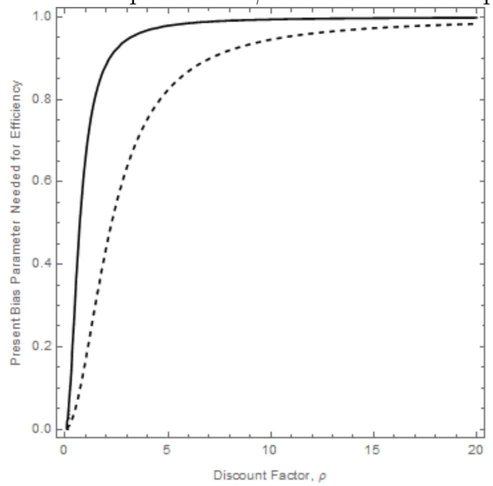
The dashed line in Figure 4 plots the same relationship as the solid line, but for $\rho = 0.1$.

4.3 Implications

Rethinking the choice of keyboard standard seems to be something rarely done by most people. For talking about the QWERTY coordination problem, it is reasonable to assume that timing frictions are quite significant. In this scenario, for some reasonable range of discount rates–see Figure 3–the level of present bias needed to make the economy efficient is significant, so there is a wide range of possible present bias levels that prevent undesirable equilibria.

[5]It becomes noticeably negatively sloped for $\delta > 500$

Figure 4: Relationship between β^* and the friction parameter



Lines depict the relation between the present bias parameter needed to make the economy efficient, β^*, and the timing friction, δ. The solid line assumes $\rho = 0.02$ and $\lambda = 0.01$, while the dashed one assumes $\rho = 0.10$ and $\lambda = 0.01$

For other subjects where the rate at which choices are revisited is higher, the amount of present bias needed to induce an efficient equilibrium is close to none, because in intermediately fast transitions the amount of negative externalities generated is small, while present bias is still effective in increasing the relative intrinsic quality needed to change standards. Hence, present bias can be detrimental, as one would expect, "delaying" a transition. This is likely to be the case in highly dynamic scenarios as the technology sector. For instance, the choice of the language a product will be written in by a firm who can easily access both competing languages is frequently revisited in light of expected future trends and productivity.

4.4 Conclusion And Final Remarks

Present bias can be incorporated in dynamic coordination problems without greater complications besides finding a suitable discounting factor through time. In particular, this paper shows that the main results, existence and uniqueness of equilibrium, in FP extend to a setting with present bias. In the specific case where utility is linear, externalities are the same for each network and there are vanishing shocks to the relative intrinsic quality, the threshold of a present biased society is a rotation of the dynamically consistent one and can prevent socially undesirable equilibria.

If decisions take long to be reevaluated and/or discount rates are not too low, loosely speaking, the existence of present bias keeps the economy in the socially adequate standard. This fits the QWERTY history. If the transitions are easier to implement as people reevaluate constantly and/or discount rates are low, then almost any level of present bias generates situations where the socially inadequate standard prevails. When timing frictions vanish, present bias, as well as the whole coordination problem, becomes irrelevant, as the only important variable is the relative intrinsic quality.

Results can be interpreted as if a central planner should suggest that everyone should procrastinate and "wait" for higher values of relative intrinsic quality to change to a competing network, so that the ones who take long to change action are spared from being left alone.

Important questions that remain open are: i) equilibrium characterization when network change is costly, which may be important in real examples such as when one wonders if she should learn a new programming language; ii) the existence of a relation between present bias and timing frictions, as intuition suggests these are linked; iii) other types of central planner objective functions, as these can be characterized in multiple ways.

Proofs of Propositions 1 to 3

Proposition 1

The proof for this results follows closely the proof for Proposition 1 from FP. We only need to change the discount rate and the utility function, so that it's possible to verify that all needed properties are still present. The proofs for Proposition 2 and 3 also follow the same parallel.

Indeed, let n_0 be the initial proportion of agents in action 1. All agents choosing 1 is an equilibrium only if when some agent chooses 1, all other follow, as Δu is strictly increasing in n. If everyone is to choose 1 from then on, to change from 0 to 1 provides the utility of:

$$\underline{U}(n_0, \theta) = \int_{\tau}^{+\infty} e^{-\delta(t-\tau)} D(t, \tau, \lambda) \Delta u(\theta, n_0^{\uparrow}) \, dt \tag{13}$$

There are values $\underline{\theta}$ and $\bar{\theta}$ such that 0 is always dominant if $\theta < \underline{\theta}$ and 1 is always dominant if $\theta > \bar{\theta}$. Also, $\underline{U}(n_0, \theta)$ is strictly increasing in both arguments, so there is some $\theta^{opt}(n_0)$ such that (13) is satisfied with equality. Observe that (13) describe the most optimistic belief about the adoption of 1, hence the threshold's name. For deriving $\theta^{pes}(n_0)$, the reasoning is the same. Also, for any point in time except the initial moment, $n_0^{\uparrow} > n_0^{\downarrow}$, so that $\theta^{opt}(n_0) < \theta^{pes}(n_0)$.

Proposition 2

The proof for this result is the same as the proof for Theorem 1 in FP. The main intuition is that the randomness in the path of the intrinsic quality triggers a process of contagion beginning with the boundaries from Proposition 1, as described in the main text.

Proposition 3

In the same way that Proposition 1 from this paper was derived by simply substituting the utility function and the discount rate in a proof from FP, the proof for this result olnly requires the same process, using the proof from Theorem 2 from FP.

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