Estimating common sectoral cycles

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Abstract

We investigate in this paper the degree of short-run and long-run comovement in U.S. sectoral output data by estimating sectoral trends and cycles. A theoretical model based on Long and Plosser (1983) is used to derive a reduced form for sectoral outputs from first principles. Cointegration and common-cycle tests are performed; sectoral output data seem to share a relatively high number of common trends and a relatively low number of common cycles. A special trend-cycle decomposition of the data set is performed, and the results indicate a very similar cyclical behavior across sectors and very different behavior for trends. In a variance decomposition analysis, prominent sectors such as Manufacturing and Wholesale Retail Trade exhibit relatively important transitory shocks.

Key words: Real business cycles; Common cycles; Sectoral outputs; Cointegration

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I. Introduction

Contemporary business cycle research focuses upon movements in aggregate output. Comparisons of simple statistical characteristics labeled 'stylized facts' with elaborate rigorous theoretical predictions are called 'calibration' and form the central paradigm in this research programme initiated by Kydland and Prescott (1982). In particular, the persistence of aggregate shocks is a central feature of such comparisons and one of the important developments was the discovery of theoretical models which could mimic this observed persistence.

This focus is, however, quite different from either the data-intensive analysis of the early NBER researchers, as documented in Burns and Mitchell (1946), or the macroeconometric analyses of the sixties perhaps best exemplified by the Brookings Model, as described in Duesenberry et al. (1965). In these studies, statistical analysis of comovements between sectors and products receives careful attention, but theoretical modelling plays a secondary role. Cycles are characterized by coordinate movements across sectors as well as persistence of shocks. The ability to identify cycles is greatly strengthened by the use of the additional information found in sectoral disaggregation.

Although the theoretical literature on business cycles has advanced considerably in the last fifteen years, e.g., Long and Plosser (1983), the empirical literature has not kept pace with it. One of the shortcomings is the lack of studies using sectorally disaggregated data. Notable exceptions are Long and Plosser (1987) and Pesaran et al. (1993). However, even these studies did not analyze the degree of short- and long-run comovement present in the data. There is a good reason for such incompleteness though, since only recently have the appropriate statistical tools been available to allow comovement comparisons.

The present paper examines jointly the degree of short- and long-run comovement present in U.S. output series by using a newly developed multivariate method called common trends and common cycles. The methodology is described in Vahid and Engle (1992), following Engle and Kozicki (1993). It works under the assumption that the data contain unit-roots, therefore stochastic trends, e.g., Stock and Watson (1988). Searching for common trends amounts to performing cointegration tests, which can be interpreted as long-run comovement tests. Thus, we build on the cointegration literature motivated by the work of Granger (1983, 1986) and Engle and Granger (1987). Cycles are modelled as transitory but persistent processes which may be common to several sectors. Such cycles reveal all synchronized persistence in output series, describing their short-run comovement in a way similar to the early NBER literature.

We focus our attention on sectoral per-capita GNP series at the single-digit SIC level in order to have broad coverage of private economic activity. We start with an extension of the theoretical model of Long and Plosser (1983) which accounts for the stylized facts of these data: they have unit roots and are cointegrated, e.g., Durlauf (1989) and Pesaran et al. (1993), and their fluctuations
display persistence and comovement, e.g., Lucas (1977, Sec. 2). After a special trend-cycle decomposition is performed, we measure the relative importance of transitory and permanent shocks to the variation of sectoral outputs. This is a central issue in much of the empirical work, e.g., Nelson and Plosser (1982), Watson (1986), Campbell and Mankiw (1987), Long and Plosser (1987), Blanchard and Quah (1988), King et al. (1991), Durlauf (1993), and Pesaran et al. (1993).

Section 2 reviews the concept of cointegration and common features applied to business cycles. Section 3 presents a macroeconomic model able to generate a multivariate system for sectoral outputs displaying the stylized facts outlined above. Section 4 presents system estimates, cointegration and common-cycle test results, trend-cycle decomposition results, and the results of variance decomposition of permanent and transitory shocks to the data. Section 5 concludes.

2. Common trends and common cycles

To motivate the following discussion on common trends and common cycles, a brief summary of the main ideas behind cointegration and common features is useful. Suppose sectoral output data to be well described as integrated [I(1)] processes which follow a Vector Autoregression (VAR) of order $k$:

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_k y_{t-k} + \nu_t,$$

where $y_t$ is an $N \times 1$ vector containing the variables of interest, the $A_i$’s are $N \times N$ matrices, and $\nu_t$ is a $N \times 1$ vector of white noise disturbances. Eq. (2.1) can be written compactly as $A(L)y_t = \nu_t$, where $A(L) = \sum_{i=0}^{k} A_i L^i$ is a matrix polynomial in the lag operator $L$ with $A_0 = I$. Its Error Correction (EC) form is

$$A_1 y_t = -(A(L)) y_{t-1} + A_1^* A_2 y_{t-2} + \cdots + A_k^* A_1 y_{t-k} + \nu_t,$$

where $A_i^* = -(A_i + \cdots + A_k)$, $\forall i = 1, 2, \ldots, k - 1$. The rank of $A(1)$ is called the cointegrating rank and labeled $r$. Rewriting $A(1) = \gamma x'$, where $\gamma$ and $x$ are $N \times r$ matrices, Engle and Granger (1987) show that the columns of $x$ are the cointegrating vectors, when stability conditions are satisfied. The matrix $\gamma$ is usually labeled the adjustment factor for $A_1 y_t$ given previous disequilibria $x'y_{t-1}$.

Vahid and Engle (1993) define a series to have a cycle if its first differences (growth rates for data in logs) displays persistence. This is an example of what Engle and Kozicki (1993) call a ‘feature’. A feature is called ‘common’ if there is a linear combination of the growth rates which has no cycle. Thus, a business cycle which is a component of the output in each sector will be common if its amplitude is different but its phase is the same across sectors.
A series is defined as persistent if it can be forecast based upon the past of all the series in the analysis. Thus, a random walk has no cycle, but a second-order or higher autoregression with a unit root does. Even stationary processes will have a cycle. More importantly, a business cycle which is generated by a nonlinear or asymmetric process will generally be classified as a cycle, given Wold’s decomposition theorem.

It is clear from (2.2) that all serial correlation of \( A \tau_i \) is captured by \( \left( A \tau_{t-1}, \ldots, A \tau_{t-k+1}, \tau_i \right) \), since \( \tau_i \) is white noise. Let us denote \( \tau_i \) as a cofeature vector, i.e., the linear combination that eliminates the serial correlation of \( A \tau_i \). This then implies that

\[
\tau_i A(1) = 0 \quad \text{and} \quad \tau_i A_i^* = 0, \quad \forall i, j. \tag{2.3}
\]

Note that cointegration neither prevents nor implies common serial correlation. If there is no cointegration, \( A(1) = 0 \). There may still be common serial correlation if \( \tau_i A_i^* = 0, \forall i \). If there is cointegration, \( A(1) \) is reduced rank, which of course does not guarantee that (2.3) holds.

If the data have common serial correlation, then \( \tau_i A \tau_i = \tau_i \epsilon_i, \forall j \). There are two important implications from this result: first, if we integrate \( \tau_i A \tau_i \), we find that \( \tau_i \tau_i \) is a random walk, thus with serially uncorrelated innovations. Therefore, the vector that removes the serial correlation of the \( \tau_i \) also removes the cyclical component of the \( \gamma_i \)'s if the trend is defined as a random walk. Second, \( \tau_i \) must be linearly independent from the cointegrating space, i.e., the space spanned by all linearly independent cointegrating vectors. This is a consequence of the fact that \( \tau_i \tau_i \) is I(1), while the linear combinations in the cointegrating space must generate only I(0) variates. This last result is a very important theoretical link between the cointegrating space and the cofeature space. It follows from it that if there are \( r \) linearly independent cointegrating vectors, there can be at most \( N - r \) linearly independent cofeature vectors. Notice that there is no guarantee that this upper bound will be achieved. When it is, however, a special trend-cycle decomposition of the \( \gamma_i \)'s is possible as discussed below.

To link our previous discussion with common trends and common cycles we next examine the Wold representation of \( A \gamma_i \). Since \( \gamma_i \) is I(1), its first difference has a Wold representation as follows:

\[
A \gamma_i = C(L)\epsilon_i, \quad \text{where} \quad C(L) = I + C_1 L + C_2 L^2 + \cdots. \tag{2.4}
\]

Now decompose \( C(L) \) as \( C(1)(1 - L)C^*(L) \), where \( C_i^* = \sum_j c_{ij} - c_{ii}, \forall i, \) and in particular \( C_i^* = 1 - C(1) \). Thus:

\[
A \gamma_i = C(1)\epsilon_i + (1 - L)C^*(L)\epsilon_i. \tag{2.5}
\]

Notice that all serial correlation of \( A \gamma_i \) is captured by \( (1 - L)C^*(L)\epsilon_i \), since \( C(1)\epsilon_i \) is a multivariate white noise. Integrating (2.5) yields the multivariate
Beveridge–Nelson representation of $y_t$ discussed in Stock and Watson (1988):\(^1\)

$$y_t = C'(1) \sum_{i=0}^r e_{t-i} + C^*(L)\psi_t.$$  
(2.6)

If there are $r$ linearly independent cointegrating relationships, $0 \leq r < N$, the long-run behavior of the $N$ variables is governed by $N - r$ common trends. Analogously, if the cofeature rank is $s$, $0 \leq s < N$, the $N$ variables will have their cyclical behavior governed by $N - s$ common cycles.

From our previous discussion on the VAR representation, cointegrating and cofeature vectors must satisfy

$$x'_i C'(1) = 0, \quad \forall i, \text{ and } \tilde{x}'_i C^*(L) = 0, \quad \forall j.$$  
(2.7)

If $r, s \neq 0$, we can decompose $C'(1)$ and $C^*(L)$ respectively as $\delta \beta'$ and $\psi \tilde{z}(L)$, where $\beta$ and $\delta$ are full column rank $N \times (N - r)$ matrices, $\psi$ is a full column rank $N \times (N - s)$ factor loading matrix, and $\tilde{z}(L)$ is a full rank $(N - s) \times N$ matrix polynomial. Thus, we can rewrite (2.6) as

$$y_t = \delta \tau_t + \psi \tilde{z}_t,$$  
(2.8)

where $\tau_t = \beta' \sum_{i=0}^r e_{t-i}$ is a $(N - r) \times 1$ vector of common trends and $\tilde{z}_t = \tilde{z}(L)\psi_t$ is a $(N - s) \times 1$ vector of common cycles. Notice that (2.8) allows for the presence of both common trends and common cycles. It is also a natural extension of the common trend representation discussed by Stock and Watson (1988), where only the reduced rank matrix $C'(1)$ is decomposed as $\delta \beta'$.

Although (2.8) allows both the presence of common trends and common cycles in $y_t$, it does not rule out the existence of idiosyncratic components in either trends or cycles. A simple example is given here for the case of common cycles, but an analogous example applies for the case of common trends as well. Suppose that $\tilde{z}(L)$ and $\psi$ are:

$$\tilde{z}(L) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \quad \psi = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$  
(2.9)

Then:

$$C^*(L) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$  
(2.10)

\(^1\)Beveridge and Nelson (1981) discuss in some detail the appropriateness of this representation for the univariate case. Since shocks to cycles dissipate in the infinite horizon and the conditional expected value of a random walk in the infinite future is its value today, one can think of the trend as being the value that the conditional long-run forecast of the series converges to.
Clearly, the first variable in $v_t$ has an idiosyncratic cycle, although the three variables in $v_t$ share only two cycles represented by $\hat{x}(L)v_t$. If idiosyncratic is taken to mean additive individual behavior for specific series, then there is still the possibility of reconciling idiosyncratic behavior and common cycles. If $c_t$ is white noise, we can still get idiosyncratic behavior with enough restrictions in the elements of $\psi$. In this case, cycles are still present, since the first difference of $c_t$ is serially correlated. Moreover, if $\psi$ is reduced rank we get both idiosyncratic behavior and common cycles.

Vahid and Engle (1993) discuss a special case on the dimensions of the cointegrating and cofeature ranks, which allows for a computationally simple trend-cycle decom position of the data. Suppose the cointegrating rank $r$ and the cofeature rank $s$ add up to the number of variables in the data set, i.e., $N = r + s$. Recall Eq. (2.5):

$$y_t = C(1) \sum_{i=0}^{r} \delta_{i} + C^\ast(L)v_t$$

(2.11)

When $N = r + s$, we can exploit the reduced rank condition of $C(1)$ and of $C^\ast(L)$ as follows: first collect all linearly independent cointegrating vectors in a $N \times r$ matrix $\hat{x}$ and all linearly independent cofeature vectors in a $N \times s$ matrix $\hat{\delta}$. We know that every element of the cointegrating space eliminates the stochastic trends and every element of the cofeature space eliminates the cycles. Thus:

$$\hat{x}' y_t = \hat{x}' C^\ast(L)v_t$$

(2.12)

and

$$\hat{\delta}' y_t = \hat{\delta}' C(1) \sum_{i=0}^{s} \delta_{i}.$$ 

(2.13)

Notice that (2.12) contains no stochastic trends, only stationary cycles, and that (2.13) contains no cycles, only stochastic trends. Suppose we now stack $\hat{x}'$ and $\hat{\delta}'$ as follows:

$$
\begin{bmatrix}
\hat{x}' \\
\hat{\delta}'
\end{bmatrix}
\begin{bmatrix}
y_t
\end{bmatrix}
= 
\begin{bmatrix}
\hat{x}' C(1) \sum_{i=0}^{r} \delta_{i} \\
\hat{\delta}' C^\ast(L)v_t
\end{bmatrix}.
$$

(2.14)

Define now:

$$A = 
\begin{bmatrix}
\hat{x}' \\
\hat{\delta}'
\end{bmatrix}$$
Since the cointegrating and cofeature spaces are linearly independent and \( r + s = N \), the matrix \( A \) is full rank. Consider the following partition of \( A^{-1} \), conformable to \( A \):

\[
A^{-1} = \begin{bmatrix}
\tilde{x} & \tilde{2} \\
\tilde{2}^T & \tilde{A}^T
\end{bmatrix}
\]

Premultiply (2.14) by \( A^{-1} \) to obtain Vahid and Engle's trend-cycle decomposition:

\[
y_t = \tilde{x}'y_t + \tilde{2}'x_t \sum_{i=0}^{n} r_{it} + x_t'C^*(L)e_t + \gamma_t + \eta_t, \tag{2.15}
\]

where \( \gamma_t^p \) is the random walk trend component and \( \eta_t \) is the serially correlated zero-mean \( I(0) \) cyclical component. Notice that we can carry out this decomposition without resorting to any knowledge of \( C^*(L) \). From (2.14) we have: \( y_t = \tilde{x}'y_t \) and \( \gamma_t^p = x't\gamma_t \), which shows that trend and cyclical components can be calculated as simple linear combinations of the data.

It is useful to contrast the trend-cycle decomposition discussed above with others available for \( I(1) \) data. King et al. (1991), following Blanchard and Quah (1989), use a similar decomposition, but do not allow for the possibility of common cycles, i.e., \( C^*(L) \) is full rank. Here, the rank of \( C^*(L) \) is to be determined by the data. The potential gains in proceeding this way are related to efficiency in the estimation of trends and cycles. Issler and Vahid (1993) discuss this issue, finding that a nontrivial efficiency loss may result from ignoring the existence of common cycles. Thus, the trend-cycle decomposition discussed here may be regarded as a natural extension of that in King et al., where the assumption of no common cycles is relaxed. Moreover, since trends and cycles in King et al.'s decomposition are just-identified, testing for common cycles in the VECM can also be viewed as an overidentification restriction test for their decomposition, with possible efficiency gains whenever these restrictions are rejected.

There are two important facts about the trend-cycle decomposition in (2.15): first, it is unique, i.e., performing linear transformations in the cointegrating space or cofeature space in isolation will not change trends and cycles. Second, the elements of \( y_t^c \) are just linear combinations of the Error Correction terms. Here, Error Correction terms can be viewed as cycle generators, which emphasizes their importance in macroeconometrics. Notice that an analogous role of trend generators can be ascribed to the cofeature vector linear combinations as well.

We now turn to estimation of the cointegrating and cofeature spaces. The testing procedure has two steps: first we estimate the cointegrating rank and conditional on the results we estimate the cofeature rank. For both steps, we use reduced rank regression methods; for cointegration, we use Johansen's (1988) technique, with critical values taken from Osterwald-Lenum (1992). We treat the
linear trend and intercept terms in the VAR as in Johansen (1992). These are important since the asymptotic distribution of cointegrating tests depend on possible restrictions on their coefficients. For common features, we use canonical correlation analysis.

Given the restrictions on the VAR found from the cointegrating tests, one can form a Vector EC Model (VECM) with the number of EC terms equal to the cointegrating rank. This VECM will itself have cross-equations restrictions if the variables have common serial correlation. We look for linear combinations of the $A_i y_i$'s which are uncorrelated with any linear combination of the variables in the RHS of (2.2). Such orthogonality tests are computed as canonical correlations between $A_i y_i' = (A_1 y_1, A_2 y_2, \ldots, A_N y_N)'$ and $w_i' = (A_1 y_1, \ldots, A_k y_{k+1}, (x_i y_i, y_i')')$.

Each statistically zero canonical correlation represents a linear combination of the $A_i y_i$'s uncorrelated with all linear combinations of the $w_i$'s, since it is uncorrelated with the one which provides maximal correlation between $A_i y_i$ and $w_i$. The cofeature rank, $s$, is the number of statistically zero canonical correlations, where $s \leq N - r$ and the number of common cycles is $N - s$. Clearly, the number of common cycles is the number of nonzero canonical correlations. The $N \times s$ full rank matrix $\tilde{z}_i$ which stacks all the $\tilde{z}_i$'s associated with the zero canonical correlations, is a basis for the cofeature rank; see Anderson (1984) for an introduction to canonical correlations.

To test the number of canonical correlations, standard distribution theory can be applied as in Ahn and Reinsel (1988) and Tiao and Tsay (1985). The distribution theory is predicated upon stationarity of the data. If the cointegration model is correctly specified, then the data will all be stationary; see Vahid and Engle (1993) for a discussior.

3. A real business cycles model for sectoral output

Long and Plosser (1983) is one of the few models attempting to derive persistence and comovement of sectoral output from optimizing behavior. This RBC model explains these features based solely on idiosyncratic technology shocks. As noted by Mankiw (1989), RBC models are an extreme version of dynamic Walrasian Equilibrium models in which money plays no active role. Trying to explain persistence and comovement using such extreme models can lead to incomplete or misleading explanations of how macroeconomic fluctuations come about. Nevertheless, RBC models are still useful, in that they are internally consistent theoretical models, with rational optimizing agents, which deliver some intuition of how macroeconomic variables interact.

Long and Plosser set up a dynamic programming problem solved in a Robinson Crusoe type of economy, where an infinitely lived agent maximizes discounted expected utility subject to sectoral technological constraints.
The production function for sector $i$, $i = 1, 2, \ldots, N$, is given by
\[ Y_{it} = \lambda_i + \sum_{j=1}^{N} X_{ij} \lambda_j, \]
where $Y_{it}$ is the produced quantity of commodity $i$ at time $t$ ($Y_t$ is a vector containing all sectoral production), $X_{ij}$ is the quantity of good $j$ used in producing good $i$, $L_i$ are labor units used, and the $\lambda_i$'s are sectoral productivity shocks. The vector $\lambda_i$, which stacks those productivity shocks, is assumed to be a time homogeneous Markov process. Constant Returns to Scale in all sectors is also assumed, i.e., $h_i + \sum_{j=1}^{N} a_{ij} = 1$, $\forall i$.

Using the optimal input decision rules for the Cobb-Douglas production function, Long and Plosser manage to derive the dynamic behavior of (log) sectoral output. The reduced form of the system is summarized by the following expression:
\[ \log Y_t = K + A \log Y_{t-1} + \log \lambda_i, \tag{3.1} \]
where $K$ is a function of the preference parameters of the problem and $A = (a_{ij})$.

The matrix $A$ plays an important role in the dynamics of sectoral outputs. Recall that the $a_{ij}$'s are input elasticities in production. Thus, $a_{ij} \geq 0$, $\forall i,j$. If labor is used in positive amounts from the constant returns to scale assumption on the production functions, $\sum_{j=1}^{N} a_{ij} < 1$ holds $\forall i$. In this case, from (3.1), unit roots for the $\log(Y)$'s cannot be achieved unless some of the $\log \lambda_i$'s have unit roots themselves. Consider now the following process for $\log \lambda_i$:
\[ \log \lambda_i = \Phi \log \lambda_{i-1} + \eta_i, \tag{3.2} \]
where $\eta_i$ is white noise. Assume further that all elements of $\log \lambda_i$ are $I(1)$. Notice that Long and Plosser set $\Phi = I_N$, i.e., they assume each individual productivity process to be a random walk. To discuss common trends and common cycles in this theoretical model it is useful to derive the VECM representation of (3.1) obtained combining it with (3.2):
\[ A \log Y_t = (I - \Phi)K - (I - \Phi)(I - A) \log Y_{t-1} + \Phi A \log Y_{t-1} + \eta_t. \tag{3.3} \]

We now state a very important result of the model (3.3):

**Proposition 1.** Given the model in Long and Plosser (1983) and assuming that \{log $\lambda_i$\} follows (3.2), with all elements of \{log $\lambda_i$\} being $I(1)$, if labor is used in all production processes, then there is cointegration among sectoral outputs if and only if there is cointegration among productivity shocks, i.e., if and only if $(I - \Phi)$ is reduced rank. Moreover, the two cointegrating ranks coincide.

If one element of log($\lambda$)'s has a unit root, the corresponding sectoral output will also have a unit root. However, this is not the only way to achieve $I(1)$ sectoral outputs. Some elements of log($Y$) may have a unit root not as a consequence of its corresponding element of log($\lambda$) being $I(1)$ but because it is a linear combination of lagged $I(1)$ sectoral outputs through the matrix $A$. 
Proof. See Appendix.

Proposition 1 delivers an intuitive result, since the log(Y)'s are integrated as a consequence of the elements of log(λ) being integrated. The interesting feature of this result is that it rules out production processes as a source of cointegration for the log(Y)'s, because (I - A) is necessarily full rank. Here, cointegration for sectoral outputs must be a consequence of the structure of the productivity process embodied in (I - Φ).

For the Long and Plosser case, where Φ = I₁, we get (I - Φ)(I - A) = 0. Thus, their model implies no cointegration among productivity shocks or sectoral outputs. In light of the evidence in Durlauf (1989) that sectoral outputs are cointegrated, this may be too restrictive an assumption. Thus, we continue under the assumption that Φ ≠ I₁, and that (I - Φ) is reduced rank. In this case, all the log(Y)'s are I(1) and there is cointegration among sectoral outputs, which must share common trends.

The conditions for the existence of common cycles are the existence of linear combinations ẑ_j ≠ 0, j = 1, ..., s, such that
\[ ẑ_j'(I - Φ)(I - A) = 0 \]  
\[ (3.4) \]
and
\[ ẑ_j Φ A = 0. \]  
\[ (3.5) \]

For the Long and Plosser case, Φ = I₁, (3.4) holds for any linear combination ẑ_j. Moreover, (3.5) collapses to ẑ_j A = 0, which will hold for some ẑ_j as long as A is reduced rank. This condition can be interpreted as a requirement for colinearity of input mixes across sectors. More generally, we have:

**Proposition 2.** A necessary condition for sectoral outputs to have common cycles is that A be reduced rank.

Proof. To seek a contradiction, assume common cycles do exist and that A is full rank. Since from Proposition 3.1 (I - A) is also full rank, the conditions for sectoral outputs having common cycles are simply
\[ ẑ_j'(I - Φ) = 0 \]  
\[ (3.4') \]
and
\[ ẑ_j Φ = 0. \]  
\[ (3.5') \]

Clearly (3.4') and (3.5') are contradictory, since the first implies ẑ_j Φ = ẑ_j ≠ 0 if common cycles do exist. Thus the result follows.

While Proposition 1 stresses the importance of the productivity process for sectoral outputs to share common trends (cointegration), Proposition 2 stresses the importance of the production function for sectoral outputs to have common
cycles. Here, as in the case analyzed by Long and Plosser, having $A$ reduced rank is a necessary condition for common cycles. To the contrary of that case, however, this is not sufficient to guarantee it.

To get some economic intuition from the conditions for common cycles, transform (3.4') to get $\tilde{x}_j^\prime \Phi = \tilde{x}_j^\prime$. Using this result in (3.5'), together with (3.4'), yields a different version of the common cycles conditions:

$$\tilde{x}_j(I - \Phi) = 0$$

(3.6)

and

$$\tilde{x}_j^\prime A = 0.$$  

(3.7)

**Proposition 3.** Conditions (3.6) and (3.7) have a nontrivial solution if and only if

$$\text{rank}(I - \Phi | A) < N,$$

where $(I - \Phi | A)$ represents the $N \times 2N$ stacked matrix.

**Proof.** If conditions (3.6) and (3.7) are achieved, we can then write:

$$\tilde{x}_j^\prime (I - \Phi | A) = (0|0).$$

This can be true only if the left null space of $(I - \Phi | A)$ is nonempty, thus, $\text{rank}(I - \Phi | A) < N$. Now, if $\text{rank}(I - \Phi | A) = N$, $(I - \Phi | A)$ has a right inverse and necessarily $\tilde{x}_j = 0$, establishing the only if.

We provide here one simple example that delivers common cycles, but several others can be imagined as well. Assume that we have only three produced goods but that two of these are final goods which are nowhere used as inputs. Then, $A$ has two columns of zeroes and is rank 1. Suppose further that the first two productivity shocks are random walks and that the third is a linear combination of the past of the first two. Then, $I - \Phi$ is rank 1. Since $\text{rank}(I - \Phi | A) \leq \text{rank}(I - \Phi) + \text{rank}(A) = 2$, the rank condition in Proposition 3 is achieved, and the three sectors will share common cycles. Notice that sectoral cycles are generated from the serial correlation present in the 'input good' process, which is transmitted to other sectors via the production function.

The goal of this section was to examine under what conditions the theoretical model could deliver common trends and common cycles. It seems that cointegrated sectoral outputs can be obtained from cointegrated productivity shocks, i.e., seem to be related to the impulse mechanism. Common serial correlation, on the other hand, seems to be possible as long as $(I - \Phi)$ and $A$ have a common left null space, i.e., it depends on the propagation mechanism through the restrictions on $A$. The next step is to examine whether or not the data show signs of common trends and common cycles.
4. Empirical evidence

The multivariate procedures described in Section 2 were applied to sectoral per-capita real GNP. Per-capita data are used since the theoretical model discussed in the previous section is that of a representative agent. Data consist of yearly (log) sectoral real GNP divided by total population, and is available from 1947 to 1989. Sectors are a subdivision of private GNP as follows: Agriculture, Forestry & Fisheries A, Construction Con, Mining Min, Wholesale and Retail Trade W, Manufacturing M, Transportation and Public Utilities T, Finance, Insurance & Real Estate F, and Services S. A plot of the data set appears in Fig. 1. Most series show the familiar upward trend of macroeconomic variables, it seems however that they are trending at different rates, e.g., Agriculture and Transportation. Fig. 2 shows sectoral GNP growth rates. In an amazing contrast with Fig. 1, it seems that growth rates across sectors have

\[ \begin{align*}
\text{A} & \ast \ast \ast \\
\text{M} & \ast \ast \ast \\
\text{F} & \ast \ast \ast \\
\text{W} & \ast \ast \ast \\
\text{Con} & \ast \ast \ast \\
\text{Min} & \ast \ast \ast \\
\text{T} & \ast \ast \ast \\
\text{S} & \ast \ast \ast \\
\end{align*} \]

Fig. 1 Sectoral private per-capita GNP, NBER recessions shown, no overlaps.

4. Empirical evidence

The multivariate procedures described in Section 2 were applied to sectoral per-capita real GNP. Per-capita data are used since the theoretical model discussed in the previous section is that of a representative agent. Data consist of yearly (log) sectoral real GNP divided by total population, and is available from 1947 to 1989. Sectors are a subdivision of private GNP as follows: Agriculture, Forestry & Fisheries A, Construction Con, Mining Min, Wholesale and Retail Trade W, Manufacturing M, Transportation and Public Utilities T, Finance, Insurance & Real Estate F, and Services S. A plot of the data set appears in Fig. 1. Most series show the familiar upward trend of macroeconomic variables, it seems however that they are trending at different rates, e.g., Agriculture and Transportation. Fig. 2 shows sectoral GNP growth rates. In an amazing contrast with Fig. 1, it seems that growth rates across sectors have

Almost all of the criticism relates to the Manufacturing and Services figure. Indeed, there have been criticisms of these particular data set as well as a response to them by the Bureau of Economic Analysis (BEA), see Survey of Current Business (1988, 1991) and the references therein. Almost all of the criticism relates to the Manufacturing and Services figures, and caused a methodological change by the BEA in 1977. Using a dummy for the post-1977 period we found no evidence of structural change for these two sectors. Nevertheless, there were significant changes in the growth rates for Construction, Wholesale and Retail Trade, and Finance. We believe these statistical findings are not related to the methodological change of 1977 and may just reflect some unknown form of model misspecification captured in our diagnostics test. Notice that this exact same data set was used in Durlauf (1989) and Pesaran et al. (1993) and no unusual behavior of the series is reported there.

1 All data were extracted from Citibase and are expressed in constant 1982 prices.

4 There have been criticisms of these particular data set as well as a response to them by the Bureau of Economic Analysis (BEA), see Survey of Current Business (1988, 1991) and the references therein. Almost all of the criticism relates to the Manufacturing and Services figures, and caused a methodological change by the BEA in 1977. Using a dummy for the post-1977 period we found no evidence of structural change for these two sectors. Nevertheless, there were significant changes in the growth rates for Construction, Wholesale and Retail Trade, and Finance. We believe these statistical findings are not related to the methodological change of 1977 and may just reflect some unknown form of model misspecification captured in our diagnostics test. Notice that this exact same data set was used in Durlauf (1989) and Pesaran et al. (1993) and no unusual behavior of the series is reported there.
a very similar shape and are well synchronized. They mostly drop during NBER
depressions and increase in periods of prosperity.

Durlauf (1989) tested the order of integration of sectoral per-capita GNP and
concluded that these data are well approximated by an $I(1)$ process. If sectoral
outputs contain stochastic trends, the next interesting question is to examine
whether or not some of these are common across sectors.

Before applying Johansen's methodology we must examine what type of
deterministic components are present in the VAR. This is a crucial step, since the
asymptotic distribution of the test statistic depends on possible restrictions on
these components. Thus, we first tested whether the VAR contains a deterministic
linear trend, allowing it to have a constant term. We used the Likelihood
Ratio (LR) test obtained from the concentrated likelihood function. The LR
statistic for this test is 48.35, which rejects the null that the VAR (and the EC)
does not contain a linear trend at usual significance levels. The next step was to
test if the linear trend is present only inside the EC term (null) against the
hypothesis that it is present also outside in the EC model (alternative). This test

Fig. 2. Sectoral per-capita GNP growth. NBER recessions shown. no overlaps.

4We use here the annual NBER chronology for recessions. Recessions peaks-troughs are as follows:

5Since Durlauf tested several series identical to the ones used here, we shall not be conducting
integration tests.

6We chose a VAR of order 2 in testing for cointegration. To be consistent, we used the estimated
cointegrating rank when searching for the 'best' deterministic structure.
Table 1
Cointegrating results Johansen's (1992) method

<table>
<thead>
<tr>
<th>( \lambda_{\text{max}} ) Stat.</th>
<th>Trace Stat.</th>
<th>Crit. Value at ( 5% )</th>
<th>Null Hypotheses</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T \ln(1 - \rho_j) )</td>
<td>( T \sum \ln(1 - \rho_j) )</td>
<td>( 1.9 )</td>
<td>( 3.7 ) at most 7 cointegrating vectors</td>
</tr>
<tr>
<td>10.5</td>
<td>12.4</td>
<td>18.2</td>
<td>( 1.69 ) at most 6 cointegrating vectors</td>
</tr>
<tr>
<td>15.2</td>
<td>27.6</td>
<td>34.5</td>
<td>( 23.81 ) at most 5 cointegrating vectors</td>
</tr>
<tr>
<td>20.3</td>
<td>47.9</td>
<td>54.6</td>
<td>( 30.31 ) at most 4 cointegrating vectors</td>
</tr>
<tr>
<td>25.3</td>
<td>73.2</td>
<td>77.7</td>
<td>( 36.4 ) at most 3 cointegrating vectors</td>
</tr>
<tr>
<td>32.9</td>
<td>106.1*</td>
<td>104.9</td>
<td>( 42.5 ) at most 2 cointegrating vectors</td>
</tr>
<tr>
<td>68.4**</td>
<td>174.5**</td>
<td>136.6</td>
<td>( 48.4 ) at most 1 cointegrating vectors</td>
</tr>
<tr>
<td>103.4**</td>
<td>282.8**</td>
<td>170.8</td>
<td>( 54.2 ) cointegrating vectors</td>
</tr>
</tbody>
</table>

Asterisks denote significance at the \( 5\% \) (*) and \( 1\% \) (**) levels

corresponds to testing \( H^*(r) \) versus \( H(r) \) in Johansen's (1992) notation and \( H^2(r) \) versus \( H^2(r) \) in Osterwald-Lenum's (1992) notation. The test statistic is 20.32, which rejects the restricted model at the \( 5\% \) significance level. As a consequence, when testing for cointegration, we consider the data as being approximately well described by a VAR with an unrestricted constant and a linear time trend. Therefore, the critical values of the asymptotic distributions for the trace and \( \lambda_{\text{max}} \) statistics correspond to Table 2 in Osterwald-Lenum.

Table 1 presents Johansen's trace statistic test for the system containing sectoral outputs. Given the results, we may conclude that there are either two or three cointegrating vectors. To avoid the risk of finding too much cointegration, typical of large systems, and since the third cointegration vector is only marginally significant, we opted for using only two cointegrating vectors in our

---

8 In this case, the Vector MA representation of the system has a linear time trend and also a quadratic time trend. As a consequence, the EC terms are trend-stationary. See Johansen (1992).
The estimates of these two cointegration vectors, using the modified VAR representation, are trend-stationary, as expected. In order to extract their deterministic components we run them on a constant and a linear trend. The detrended cointegrating vectors \((Z_{it} = z_{it} + \delta t, i = 1, 2)\) are plotted in Fig. 3. They both appear to be well-behaved long-run relationships.

Finding a small number of cointegrating vectors rules out the possibility that sectoral output data have one common stochastic trend. Indeed, since the rank of the cointegrating space is 2, the eight sectors will share six idiosyncratic common trends. This finding is consistent with the evidence found in Durlauf (1989), which notes that one should not expect to find very different sectors sharing common stochastic trends if these arise from technology shocks. As he notes, a technological improvement in Agriculture does not imply improvement in Manufacturing, due to limited spill-over effects across these sectors.

Table 2 presents the estimates of the VECM conditioning on two lags of the endogenous variables. This corresponds to a VAR of order 3, which, with yearly data, should be enough to capture the dynamics of the system. The VECM estimates are satisfactory, and all residuals passed autocorrelation and normality tests, a desirable feature since Johansen's test assumes independent Gaussian errors. Table 3 displays system significance levels for each regressor. Inclusion of up to two lags of the endogenous variables seems justified. Notice also the high

9 Using three cointegrating vectors did not imply any qualitative change in results. We were still able to perform the trend-cycle decomposition discussed in the previous section.
Table 2
System estimates of the EC model, t-statistics in parentheses

<table>
<thead>
<tr>
<th>Regressors</th>
<th>1 Log A,</th>
<th>1 Log Con,</th>
<th>1 Log Min,</th>
<th>1 Log M,</th>
<th>1 Log L,</th>
<th>1 Log W,</th>
<th>1 Log L,</th>
<th>1 Log N,</th>
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<tbody>
<tr>
<td>Z1t-1</td>
<td>1.47</td>
<td>0.09</td>
<td>1.57</td>
<td>2.47</td>
<td>0.73</td>
<td>0.96</td>
<td>0.67</td>
<td>0.67</td>
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<tr>
<td></td>
<td>(1.48)</td>
<td>(0.13)</td>
<td>(1.79)</td>
<td>(2.13)</td>
<td>(1.39)</td>
<td>(1.88)</td>
<td>(2.95)</td>
<td>(2.59)</td>
</tr>
<tr>
<td>Z2t-1</td>
<td>0.08</td>
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<td>0.02</td>
<td>0.06</td>
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<td>(1.67)</td>
<td>(1.20)</td>
<td>(0.39)</td>
<td>(0.95)</td>
<td>(0.77)</td>
<td>(2.56)</td>
<td>(0.54)</td>
<td>(2.52)</td>
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<td>0.00</td>
<td>0.01</td>
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<td>(1.39)</td>
<td>(2.70)</td>
<td>(3.04)</td>
<td>(0.88)</td>
<td>(4.89)</td>
<td>(0.42)</td>
</tr>
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<td>0.26</td>
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<td>0.16</td>
<td>0.07</td>
<td>0.03</td>
<td>0.04</td>
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<td></td>
<td>(3.20)</td>
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<td>(1.55)</td>
<td>(0.69)</td>
<td>(0.62)</td>
<td>(0.59)</td>
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<td>1 Log Con, 1</td>
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<td>0.25</td>
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<td>0.22</td>
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<td>(1.63)</td>
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Table 3
System significance tests (EC model, L-statistic... retained regressors F(8, 20))

<table>
<thead>
<tr>
<th>Regressor</th>
<th>$Z_{1,t}$</th>
<th>$Z_{2,t}$</th>
<th>Trend</th>
<th>$\log A_{t-1}$</th>
<th>$\log C_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>17.26</td>
<td>2.55</td>
<td>7.02</td>
<td>1.71</td>
<td>2.67</td>
</tr>
<tr>
<td>Pr &gt; $F$</td>
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<td>0.0427</td>
<td>0.0002</td>
<td>0.1587</td>
<td>0.0360</td>
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</table>

<table>
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<tr>
<th>Regressor</th>
<th>$\log M_{t-1}$</th>
<th>$\log M_{t-2}$</th>
<th>$\log T_{t-1}$</th>
<th>$\log W_{t-1}$</th>
<th>$\log F_{t-1}$</th>
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<tbody>
<tr>
<td>$F$</td>
<td>4.14</td>
<td>6.26</td>
<td>8.68</td>
<td>6.31</td>
<td>3.38</td>
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<td>Pr &gt; $F$</td>
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<td>0.0004</td>
<td>0.0000</td>
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<th>$\log A_{t-2}$</th>
<th>$\log C_{t-2}$</th>
<th>$\log M_{t-2}$</th>
<th>$\log M_{t-2}$</th>
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<tbody>
<tr>
<td>$F$</td>
<td>1.62</td>
<td>1.62</td>
<td>2.36</td>
<td>2.68</td>
<td>4.07</td>
</tr>
<tr>
<td>Pr &gt; $F$</td>
<td>0.1810</td>
<td>0.1807</td>
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<td>0.0353</td>
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<table>
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<th>$\log T_{t-2}$</th>
<th>$\log W_{t-2}$</th>
<th>$\log F_{t-2}$</th>
<th>$\log S_{t-2}$</th>
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</thead>
<tbody>
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<td>$F$</td>
<td>1.81</td>
<td>5.51</td>
<td>2.40</td>
<td>0.84</td>
</tr>
<tr>
<td>Pr &gt; $F$</td>
<td>0.1348</td>
<td>0.0009</td>
<td>0.0541</td>
<td>0.5813</td>
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</table>

explanatory power of the time trend in the system, corroborating the evidence of the LR test previously conducted.

Results of the canonical correlation analysis are presented in Table 4. As noted before, the cofeature rank will be equal to the number of statistically zero canonical correlations. At the 5% level, we conclude that the rank of the cofeature space is 6. This implies that eight sectors will share only two idiosyncratic serially correlated cycles. Thus, we should observe a very similar cyclical behavior for different sectors. This result is not surprising, given the similar pattern of sectoral GNP growth presented in Fig. 2. This feature of the data set exemplifies the basic thrust behind Burns and Mitchell's (1946) research and is cited as a stylized fact in Lucas (1977) and emphasized in Long and Plosser (1983, 1987).

Table 5 shows the bases for the cointegrating and cofeature spaces. The basis for the cointegrating space is spanned by the two estimated cointegrating vectors and that for the cofeature space by the six estimated cofeature vectors. Since these bases form a nonsingular matrix, i.e., $N = r + s$, we can use these estimates to construct trends and cycles as discussed above.

Trend and cycle estimates are presented in Figs. 4 through 10 and Table 6 presents a summary statistic of the data and these estimates. Since we want to

---

10The $F$-test used in this table provides better small sample results than the usual $\chi^2$ approximation (see Rao, 1973).
focus our attention on sectoral cycles, we present only a few sectoral outputs and their respective trends (Figs. 4 through 6). Not surprisingly, since the eight sectors share six idiosyncratic trends, they display a very distinct behavior across sectors. Moreover, many of them seem to be more volatile than sectoral outputs themselves.\footnote{Whenever the covariance between trend and cycle is negative and big enough in absolute value, individual sectoral GNP's will be smoother than their respective trends. This is the case for all sectors but Wholesale/Retail Trade.}

The cyclical components of sectoral output series are plotted in Figs. 7 through 10, which also include NLER recessions. In these plots, sectors are grouped according to the similarity of their cycles. Five out of eight sectoral
<table>
<thead>
<tr>
<th>Sectors</th>
<th>(log) Levels $\log(y_t)$</th>
<th>Cyclical component $1%$</th>
<th>Trend component $1%$</th>
<th>Correl. trend and cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{j}$</td>
<td>$\hat{\sigma}$</td>
<td>$\hat{\mu}$</td>
<td>$\hat{j}$</td>
</tr>
<tr>
<td>A</td>
<td>5.905</td>
<td>0.098</td>
<td>0.017</td>
<td>0.000</td>
</tr>
<tr>
<td>Con</td>
<td>6.644</td>
<td>0.153</td>
<td>0.023</td>
<td>0.000</td>
</tr>
<tr>
<td>Min</td>
<td>6.331</td>
<td>0.100</td>
<td>0.016</td>
<td>0.000</td>
</tr>
<tr>
<td>M</td>
<td>7.782</td>
<td>0.250</td>
<td>0.032</td>
<td>0.000</td>
</tr>
<tr>
<td>T</td>
<td>6.848</td>
<td>0.300</td>
<td>0.045</td>
<td>0.000</td>
</tr>
<tr>
<td>W</td>
<td>7.441</td>
<td>0.289</td>
<td>0.039</td>
<td>0.000</td>
</tr>
<tr>
<td>F</td>
<td>7.253</td>
<td>0.373</td>
<td>0.051</td>
<td>0.000</td>
</tr>
<tr>
<td>S</td>
<td>7.230</td>
<td>0.355</td>
<td>0.049</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Asterisks denote significance at the 5% (**) and 1% (***), levels.
cycles conform to NBER recessions and are therefore labelled pro-cyclical. They are: Mining, Construction, Manufacturing, Wholesale Retail Trade, and Finance. Three sectors do not conform with NBER recessions, with upward movements during these, and are labelled counter-cyclical. They are: Agriculture, Transportation, and Services.
Examining the cycles of pro-cyclical sectors reveals that these have similar shapes and durations but very different amplitudes. The sector with the cycle of highest amplitude is Construction. This is not surprising: since Construction includes housing construction, our evidence is in line with the stylized facts in Lucas (1977), who pos...s out that consumer durable output has relative high
amplitude. Construction is followed by Mining and Manufacturing, with amplitudes roughly half its size. Finally, the lowest amplitudes are found for Wholesale/Retail Trade and Finance, with amplitudes roughly a fifth of that of Construction.
Table 7
Factor loadings on EC terms, normalized to unit variance

<table>
<thead>
<tr>
<th>Sectors</th>
<th>$Z_{1t}$</th>
<th>$Z_{2t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>-0.019735</td>
<td>-0.059922</td>
</tr>
<tr>
<td>$Min$</td>
<td>0.0238157</td>
<td>0.184967</td>
</tr>
<tr>
<td>$Con$</td>
<td>0.0348906</td>
<td>0.305255</td>
</tr>
<tr>
<td>$M$</td>
<td>0.0103827</td>
<td>0.124335</td>
</tr>
<tr>
<td>$T$</td>
<td>-0.023833</td>
<td>-0.051567</td>
</tr>
<tr>
<td>$W$</td>
<td>0.0074991</td>
<td>0.0425257</td>
</tr>
<tr>
<td>$F$</td>
<td>0.0145785</td>
<td>0.0724686</td>
</tr>
<tr>
<td>$S$</td>
<td>-0.016855</td>
<td>-0.084293</td>
</tr>
</tbody>
</table>

All the counter-cyclical sectors have very similar cycles in shape and duration. They also share a very small cycle amplitude. Moreover, it seems that the shape of these counter-cyclical cycles is merely an upside down version of the procyclical ones. These findings reinforce the idea that per-capita sectoral outputs have a common cycle, if not in the statistical sense at least in the economic sense.

Table 7 shows sectoral cycles' factor loadings of the two EC terms. To make factor loadings comparable, we normalize the variance of the two EC terms to unity. For all sectors, the factor loadings of $Z_{2t}$ are much higher than that of $Z_{1t}$, suggesting that the variation in sectoral cycles is explained mainly by the variation of the first. Given the similarity in shape between $Z_{2t}$ and sectoral
cycles it would be surprising to find otherwise. Thus, it seems that although statistically sectoral cycles are generated by two idiosyncratic components, economically they are only a result of the variation of $Z_{2t}$. Thus, we found a particular rotation of the cointegrating space basis in which only one cointegrating vector explains most cyclical pattern of the data set.

To investigate further the findings of counter-cyclical sectors, a plot of the (log) level series for these sectors is presented in Fig. 11. During recessions, the behavior of Agriculture is definitely odd: while its series is almost flat, it increased in four out of seven recessions. Likewise, Services display little downward sensitivity in recession periods, which is most striking until the 1969–70 recession. Until then per-capita Services output not only increased during recessions but even showed no decrease in its growth rate vis-a-vis neighboring periods. After 1970, this feature is reversed. Transportation is the only counter-cyclical series which does not display any unusual behavior for recession periods. In that sense, observing it to display counter-cyclical behavior is surprising.

There is some empirical support for our findings of counter-cyclical sectors: using PSID data. Lougani and Rogerson (1989) found that the influx of workers into Services increases during recessions and that the outflow of workers from Services increases during booms. These findings are consistent with counter-cyclical behavior for Services, though they do not necessarily imply it. For Agriculture, Romer (1991), using factor analysis, found evidence that several agricultural goods have short run counter-cyclical behavior (see p. 27). Even

Fig. 11. Selected sectoral per-capita GNP. NBER recessions shown.
Table 8

Variance decomposition of sectoral output innovations, two orthogonalization procedures used

<table>
<thead>
<tr>
<th>Sectors</th>
<th>h = 1 yr.</th>
<th>h = 2 yrs.</th>
<th>h = 3 yrs.</th>
<th>h = 4 yrs.</th>
<th>h = x</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>27.4</td>
<td>24.6</td>
<td>25.0</td>
<td>21.0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.7)</td>
<td>(1.3)</td>
<td>(0.5)</td>
<td>(1.3)</td>
<td></td>
</tr>
<tr>
<td>Min</td>
<td>91.9</td>
<td>89.0</td>
<td>80.5</td>
<td>65.0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(10.5)</td>
<td>(11.2)</td>
<td>(8.9)</td>
<td>(3.7)</td>
<td></td>
</tr>
<tr>
<td>Con</td>
<td>93.6</td>
<td>86.3</td>
<td>72.6</td>
<td>55.6</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(4.9)</td>
<td>(1.9)</td>
<td>(0.0)</td>
<td>(0.9)</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>99.8</td>
<td>99.5</td>
<td>95.9</td>
<td>88.6</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(55.3)</td>
<td>(46.9)</td>
<td>(40.0)</td>
<td>(32.4)</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>24.9</td>
<td>18.5</td>
<td>15.3</td>
<td>12.4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(19.5)</td>
<td>(20.8)</td>
<td>(22.5)</td>
<td>(23.0)</td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>82.5</td>
<td>89.7</td>
<td>87.3</td>
<td>71.5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(65.8)</td>
<td>(59.2)</td>
<td>(52.6)</td>
<td>(46.4)</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>91.8</td>
<td>83.1</td>
<td>73.1</td>
<td>60.0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(8.8)</td>
<td>(7.1)</td>
<td>(4.9)</td>
<td>(1.0)</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>50.5</td>
<td>41.5</td>
<td>37.1</td>
<td>32.3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(7.0)</td>
<td>(6.4)</td>
<td>(4.3)</td>
<td>(3.9)</td>
<td></td>
</tr>
</tbody>
</table>

(a) Obtaining trend and cycle innovations: one-step-ahead innovations for trends were obtained by first-differencing them. For cycles, they are residuals of cycle projections on a lagged conditioning set containing four lags of the EC terms. H-step-ahead trend innovations were obtained by cumulating one-step-aheads. For cycles, they were obtained by shifting backwards the conditioning set.

(b) Trend and cycle innovations were found to display (significant) negative correlation for all sectors but Wholesale Retail Trade, for which this correlation was statistically zero. Transitory and permanent shocks were obtained by orthogonalizing trend and cycle innovations. These orthogonal shocks were then labelled permanent and transitory shocks to sectoral outputs.

(c) The orthogonalization method used above was: Denote \( \eta^h = (\eta^h_{tr}, \eta^h_{cy}) \) as a stack of period \( t \) innovations in sector \( i \) and horizon \( h \), where \( \eta^h_{tr} \) is the innovation in the trend and \( \eta^h_{cy} \) is the innovation in the cycle. The top number in Table 7 presents the results of decomposing the variance of \( \eta^h_{tr} = (1, 1) \eta^h_{tr} \) - the total period \( t \) innovation in sector \( i \), horizon \( h \), by using a lower triangular matrix \( D^h \), such that \( D^h \text{VAR}(\eta^h_{tr})D^h = \text{diagonal for all } i \). The matrix \( D^h \) used was

\[
D^h = \begin{bmatrix}
1 & 0 \\
-\sigma^h_{tr} & 1
\end{bmatrix}
\]

where \( \text{VAR}(\eta^h_{tr}) = \begin{bmatrix}
\sigma^h_{tr} & \sigma^h_{cy} \\
\sigma^h_{cy} & \sigma^h_{cy}
\end{bmatrix} \)

The number in parentheses performs the same exercise with \( \eta^h_{cy} = (1, 1) \eta^h_{cy} \) and

\[
D^h = \begin{bmatrix}
1 & 0 \\
-\sigma^h_{cy} & 1
\end{bmatrix}
\]

where \( \text{VAR}(\eta^h_{cy}) = \begin{bmatrix}
\sigma^h_{cy} & \sigma^h_{cy} \\
\sigma^h_{cy} & \sigma^h_{cy}
\end{bmatrix} \).
though her evidence is more compelling for the inter-war era, it holds for the post-war era as well. Evidence of low coherence for agricultural output is also mentioned in Lucas (1977) as a stylized fact of business cycles (see Section 2).

We next investigate the relative importance of transitory and permanent shocks for the variation of sectoral output data. The results are presented in Table 8. To construct those numbers, trend and cycle innovations were orthogonalized since they are negatively correlated in most cases. In Table 8, each cell contains two numbers: the top number represents the relative importance of transitory shocks when trend shocks come first in the orthogonalization procedure, and the number between parentheses represents the same measure when cycle innovations are put first. There is of course no consensus on how to orthogonalize shocks in performing variance decompositions. Nevertheless, in this specific issue, most authors prefer to put trend innovations first, e.g., King et al. (1991), since in theoretical RBC models, productivity (trend) shocks cause simultaneously trend and cyclical activity.

For given sectors, the results in Table 8 shows that the relative importance of permanent and transitory shocks may vary depending on the orthogonalization procedure. Despite this, some sectors displayed remarkable robustness in relation to the orthogonalization procedure employed. For example, the results for Wholesale/Retail Trade show unequivocally the importance of transitory shocks, explaining at least 45% of output variation up to the four-year horizon. Manufacturing is another example where, at least at short horizons, the bulk of total variance is explained by transitory shocks. Sectors in which permanent shocks are unequivocally important are: Agriculture, Transportation, and Services. Notice that these are the counter-cyclical sectors. The remaining sectors, Construction, Mining, and Finance, have results that seem to depend heavily on the ordering of innovations used, and thus constitute open issues. The picture that emerges from this analysis is that there is no clear evidence that either permanent or transitory shocks play a prominent role across all sectors. However, for essential sectors like Manufacturing and Wholesale/Retail Trade, transitory innovations are unequivocally important.

Our evidence from the variance decomposition analysis is very different from that presented in King et al. (1991). In their RBC model, permanent shocks explained the bulk of the variance of total output innovation. There are two possible explanations for the difference in results: first, we are using disaggregated data, and second, we are using common-cycle restrictions in performing the trend-cycle decomposition.

As a final investigation, we 'add up' sectoral outputs to get some idea of 'aggregate' cycles. Because we used data in logs, aggregating log GNP across sectors will not give us (log) Private Per-Capita GNP, but in some sense it is still a measure of aggregate GNP behavior. The sum of sectoral cycles is presented in Fig. 12. As expected, it conforms to NBER recession periods. Hence, although some sectors had counter-cyclical behavior, our aggregate cyclical measure does
The basic goal of this paper was to re-examine business cycles using a standard theoretical model and a new econometric technique which allows simultaneous discussion of short- and long-run comovement of multivariate data sets. The results indicate that Sectoral Per-Capita GNPs shares a relatively large number of idiosyncratic common trends but a relatively small number of idiosyncratic common cycles. Thus, trends have a very distinct behavior, whereas cycles seem almost identical in shape, duration, and timing. Furthermore, at least in the economic sense, sectoral cycles seem to be generated by a common component, in this case, the second EC term. The fact that cycles are so similar for different sectors is remarkable evidence, confirming the basic thrust of Burns and Mitchell (1946).

The evidence on the importance of permanent versus transitory shocks is ambiguous. For sectors like Agriculture, Transportation, and Services it seems that permanent shocks are the most important. However, for Manufacturing and Wholesale/Retail Trade, it seems that the opposite holds. Under the theoretical model discussed, sectoral cyclical fluctuations are closely related to the input/output interdependence among sectors, i.e., they are a consequence of
the propagation mechanism at work. On the other hand, trend fluctuations are closely related to productivity shocks, i.e., they are a consequence of the impulse mechanism at work. Although impulses may be very different, causing trends to differ across sectors, cycles may still be very similar, since the input/output relationship may work to that end.

Even though the theoretical RBC literature has gone far in modelling together economic growth and fluctuations under optimizing models, little empirical evidence has accumulated supporting these models. This paper is a step in this direction, showing that using production data alone one can describe economic fluctuations fairly well. Nevertheless, several issues remain open in the business cycle literature. Of these, the most critical for economic policy is the role of money: since we showed here that transitory shocks are an important source of noise for prominent sectors (Manufacturing and Wholesale/Retail Trade), one possible extension of this work is to investigate the relationship between money and transitory shocks. Although the objectives seem well defined, implementing this analysis may be difficult due to methodological issues. For this reason, we decline to pursue it here. Another important issue is the possible link between long-run shocks and technology. With that regard, our extended model offers an empirical test for Real Business Cycles, since it suggests that the cointegrating rank of productivity shocks and sectoral outputs should coincide. Although the objectives are clear once more, how to correctly estimate productivity shocks is still a controversial matter.

Appendix: Proof of Proposition 1

There is cointegration among the $\log(Y)$'s if and only if $(I - \Phi)(I - A)$ is reduced rank, e.g., Engle and Granger (1987). Recall the constant returns to scale assumption on production functions:

$$\sum_{j=1}^{N} a_{ij} + b_{i} = 1, \quad \forall i, \quad \text{and} \quad a_{ij} \geq 0.$$  

If labor is used in all production processes, $b_{i} > 0, \forall i$, thus:

$$\sum_{j=1}^{N} a_{ij} < 1, \quad \forall i, \quad \text{and} \quad a_{ij} \geq 0.$$  

Therefore, $A = (a_{ij})$ is a nonnegative matrix. Consider now $(I - A)$. Its eigenvalues are $1 - \lambda_{1}^{A}, 1 - \lambda_{2}^{A}, \ldots, 1 - \lambda_{N}^{A}$, where $\lambda_{1}^{A}, \lambda_{2}^{A}, \ldots, \lambda_{N}^{A}$ are the eigenvalues of $A$. Clearly, if $(I - A)$ is full rank, $(I - \Phi)(I - A)$ is full rank if and only if $(I - \Phi)$ is full rank. To prove the former, it suffices to show that

$$\max |\lambda_{i}^{A}| < 1.$$  

(A.1)
Since it implies that all eigenvalues of \((I - A)\) are nonzero. Eq. (A.1) follows from a theorem relating matrix norms and spectral radius, see Lancaster (1969, Theorem 6.13, p. 201). Define

\[
\| A \|_p \equiv \max_i \sum_{j=1}^{N} |a_{ij}|
\]

to be the row norm of the matrix \(A\). Then,

\[
\| A \|_p \geq \max_i |z_i^A|
\]

Since \(0 \leq \sum_{j=1}^{N} |a_{ij}| < 1, \forall i, \| A \|_p < 1\). Thus, \(\max_i |z_i^A| < 1\), and \((I - A)\) is full rank. Therefore, \((I - A)(I - \Phi)\) is reduced rank if and only if \((I - \Phi)\) is reduced rank. Moreover, \(\text{rank}(I - \Phi)(I - A) = \text{rank}(I - \Phi)\), and the result follows.

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