A New Class of Nonlinear Time Series Models for the Evaluation of DSGE Models

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Motivation

- Linear VARs have been useful benchmark for the evaluation of linearized DSGE models: Cogley and Nason (1994); Christiano, Eichenbaum and Evans (2005); Del Negro, Schorfheide, Smets, and Wouters (2007); Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007).
- Recent advances in development and estimation of nonlinear DSGE models, (e.g. Fernandez-Villaverde and Rubio-Ramirez, 2007), but no obvious benchmark for model evaluation: generalized autoregressive models? bilinear models? ARCH-M? regime-switching models? time-varying coefficient models? threshold autoregressions? smooth transition autoregressions?
- We propose a new class of time series models that can be used as a benchmark for the evaluation of DSGE models solved with nonlinear methods.
- In this talk we use a univariate version of the proposed class in a posterior predictive check to assess nonlinearities in a simple New Keynesian DSGE model.

Plan for the Talk

1. Introduce the class of Quadratic Autoregressive (QAR) Models:
    - Specification
    - Basic properties
    - Posterior inference

2. Use QAR(1,1) to construct posterior predictive checks for a small-scale New Keynesian DSGE model with asymmetric price and wage adjustment costs:
    - Do QAR estimates based on model-generated data look like estimates based on U.S. output growth, nom wage growth, inflation, and interest rate data? Some do, some don't!
    - What features of U.S. data is the QAR(1,1) capturing?

3. Related ongoing and future research.

Linear and Nonlinear DSGE Models

- Consider a DSGE model solved by second-order perturbation. State ($s_{i,t}$) and control ($c_{i,t}$) variables evolve according to

$$
\begin{aligned}
c_{i,t} &= \psi_{1i}(\theta) + \psi_{2ij}(\theta) s_{j,t} + \psi_{3ijk}(\theta) s_{j,t} s_{k,t} \\
s^{\text{end}}_{i,t+1} &= \zeta^{\text{end}}_{1i}(\theta) + \zeta^{\text{end}}_{2ij}(\theta) s_{j,t} + \zeta^{\text{end}}_{3ijk}(\theta) s_{j,t} s_{k,t} \\
s^{\text{exo}}_{i,t+1} &= \zeta^{\text{exo}}_{2i}(\theta) s^{\text{exo}}_{i,t} + \zeta^{\text{exo}}_{3i}(\theta) \epsilon_{i,t+1}.
\end{aligned}
$$

- Measurement:

$$y_{i,t} = A_{1i}(\theta) + A_{2ij}(\theta) c_{j,t} + A_{3ij}(\theta) s_{j,t} + e_{i,t},$$

- VARs do not capture nonlinear dynamics of the above state-space model.
- Goal of this research: generalize (V)ARs by introducing higher-order (nonlinear) terms.

QAR(1,1): Basics

- Consider the scalar process:

$$y_t = f(y_{t-1}, \sigma u_t)$$

- Use the idea of perturbation approximations (Holmes, 1995) to approximate solution to difference equation. Approximation error tends to zero as $\sigma \longrightarrow 0$.

  1. Postulate an approximate solution for y_t

  $$\tilde{y}_t = y_t^{(0)} + \sigma y_t^{(1)} + \sigma^2 y_t^{(2)} + o(\sigma^2)$$

  2. Use Taylor expansion of $f(\cdot)$ around $y_* = f(y_*, 0)$ and $\sigma = 0$, plug-in the guess $\tilde{y}_t$, determine the components $y_t^{(0)}, y_t^{(1)}, y_t^{(2)}$.

QAR(1,1): Basics

- Plugging our guess into a second-order approximation of $f(\cdot)$ yields

$$
\begin{aligned}
& y_t^{(0)} + \sigma y_t^{(1)} + \sigma^2 y_t^{(2)} \\
& \quad = \quad y_0 + f_y \sigma y_{t-1}^{(1)} + f_y \sigma^2 y_{t-1}^{(2)} + f_u \sigma u_t \\
& \qquad + \frac{1}{2} f_{yy} (\sigma y_{t-1}^{(1)} + \sigma^2 y_{t-1}^{(2)})^2 \\
& \qquad + \frac{1}{2} f_{yu} \sigma (\sigma y_{t-1}^{(1)} + \sigma^2 y_{t-1}^{(2)}) u_t + \frac{1}{2} f_{uu} \sigma^2 u_t^2 + o(\sigma^2)
\end{aligned}
$$

- Collecting "σ equivalent" terms, we obtain:

$$
\begin{aligned}
y_t^{(0)} &= y_0 \\
y_t^{(1)} &= f_y y_{t-1}^{(1)} + f_u \sigma u_t \\
y_t^{(2)} &= f_y y_{t-1}^{(2)} + \frac{1}{2} f_{yy} \left(y_{t-1}^{(1)} \right)^2 + \frac{1}{2} f_{yu} y_{t-1}^{(1)} u_t + \frac{1}{2} f_{uu} u_t^2
\end{aligned}
$$

- Substituting the above expressions in our guess for y_t and collecting terms, we obtain:

$$
\begin{aligned}
y_t &= y_0 + f_y (y_{t-1} - \phi_0) + \frac{1}{2} f_{yy} \left(y_{t-1}^{(1)} \right)^2 + \left(f_u + \frac{1}{2} f_{yu} y_{t-1}^{(1)} \right) u_t + \frac{1}{2} f_{uu} u_t^2 \\
y_t^{(1)} &= f_y y_{t-1}^{(1)} + f_u u_t
\end{aligned}
$$

QAR(1,1): Specification

- We set $f_{uu} = 0$ to maintain a conditional Gaussian distribution and consider the system as a nonlinear state-space model:

$$\begin{aligned} y_t &= \phi_0 + \phi_1(y_{t-1} - \phi_0) + \phi_2 s_{t-1}^2 + (1 + \gamma s_{t-1})\sigma u_t \\ s_t &= \phi_1 s_{t-1} + \sigma u_t \qquad\qquad u_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,1) \end{aligned}$$

- This model is closely related to other time series models proposed in the literature:
    - For $\gamma = 0$ and $\phi_2 = 0$, it is a standard AR(1) model.
    - For $\phi_2 = 0$, it is similar to a bilinear model (Granger and Andersen, 1978; Rao, 1980) and a LARCH model (Giraitis, Robinson, and Surgailis, 2000).
    - For $\gamma = 0$, it is a pruned version of Mittnik's (1990) generalized autoregressive (GAR) model:

$$y_t = \phi_0 + \phi_1(y_{t-1} - \phi_0) + \phi_2(y_{t-1} - \phi_0)^2 + \sigma u_t.$$

    - Observation-driven (as opposed to parameter driven) time series model.

QAR(1,1): Basic Properties

$$\begin{aligned} y_t &= \phi_0 + \phi_1(y_{t-1} - \phi_0) + \phi_2 s_{t-1}^2 + (1 + \gamma s_{t-1})\sigma u_t \\ s_t &= \phi_1 s_{t-1} + \sigma u_t \qquad\qquad u_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,1) \end{aligned}$$

- Recursively linear structure. Let $z_t = [s_{t-1}, s_{t-1}^2, u_t]'$. Then

$$y_t = \phi_0 + \sum_{j=0}^{\infty} \phi_1^j g(z_{t-j}).$$

For $|\phi_1| < 1$ z_t is strictly stationary and so is y_t.

- The model has a restricted 2nd order Volterra representation:

$$\begin{aligned} y_t &= \phi_0 + \sigma \sum_{j=0}^{\infty} \phi_1^j u_{t-j} \\ &\quad + \sigma^2 \sum_{j=0}^{\infty} \sum_{l=0}^{\infty} \left(\gamma \mathcal{I}\{l > j\} \phi_1^{l-j} + \phi_2 \sum_{k=0}^{\min\{j,l\}} \phi_1^{j+l-k} \right) u_{t-j} u_{t-l}. \end{aligned}$$

QAR(1,1): Basic Properties

$$\begin{aligned} y_t &= \phi_0 + \phi_1(y_{t-1} - \phi_0) + \phi_2 s_{t-1}^2 + (1 + \gamma s_{t-1})\sigma u_t \\ s_t &= \phi_1 s_{t-1} + \sigma u_t \qquad\qquad u_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,1) \end{aligned}$$

- It is fairly straightforward to derive the implied first, second, and higher-order moments for the QAR. E.g.

$$\mathbb{E}[y_t] = \phi_0 + \frac{\phi_2 \sigma^2}{(1-\phi_1)(1-\phi_1^2)}.$$

- Richer Dynamics than AR:
  - For $\gamma \neq 0$, the model produces conditional heteroskedasticity:

$$Var_t[y_t] = (1 + \gamma s_{t-1})^2 \sigma^2$$

  - For $\phi_2 \neq 0$, IRFs depend on the sign of shocks, on the size of shocks, and on the state s_{t-1}.

QAR(1,1): Basic Properties

$$\begin{aligned} y_t &= \phi_0 + \phi_1(y_{t-1} - \phi_0) + \phi_2 s_{t-1}^2 + (1 + \gamma s_{t-1})\sigma u_t \\ s_t &= \phi_1 s_{t-1} + \sigma u_t \qquad\qquad u_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,1) \end{aligned}$$

- Likelihood function can be computed recursively given (y_0, s_0).
- Bayesian inference via Gibbs sampler

The DSGE Model

- Small-scale DSGE model with nominal price and wage rigidities.
- Price and wage adjustment costs are potentially asymmetric to capture downward rigidity, see Kim and Ruge-Murcia (JME, 2009):

$$\Phi(x) = \phi\left(\frac{\exp(-\psi(x-x_*)) + \psi(x-x_*) - 1}{\psi^2}\right).$$

- Model consists of
  - households
  - intermediate goods producers
  - final goods producers
  - central bank / fiscal authority

DSGE Model - Key Features

- Final good producer: perfectly competitive, she has the following technology: $Y_t = \left(\int_0^1 Y_t(j)^{1-\lambda_{p,t}} dj \right)^{\frac{1}{1-\lambda_{p,t}}}$.
- Intermediate good producers: monopolists, they have the following technology:
  - Production: $Y_t(j) = A_t N_t(j)$
  - Price adjustment costs in terms of output: $AC_t(j) = \Phi\big(P_t(j)/P_{t-1}(j)\big)$.

DSGE Model - Key Features

- Households consists of continuum of family members k; perfect insurance.
- Utility function of family member:

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left(\frac{(C_{t+s}(k)/A_{t+s})^{1-\tau} - 1}{1-\tau} - \chi_{H,t} \frac{H_{t+s}^{1+1/\nu}(k)}{1+1/\nu} \right) \right].$$

- Monopolist in the supply of labor services of type k; posts wage $W_t(k)$.
- Nominal wages are costly to adjust. Labor income is given by $W_t(k)H_t(k)\big(1 - \Phi_w\big(\frac{W_t(k)}{W_{t-1}(k)}\big)\big)$.
- Aggregation of labor services:

$$H_t = \left(\int_0^1 H_t(k)^{1-\lambda_w} dk \right)^{\frac{1}{1-\lambda_w}}.$$

DSGE Model - Key Features

- Fiscal Authority: taxes Households, consumes a fraction ζ_t of final output.
- Monetary Authority: sets nominal bond rate using the feedback rule

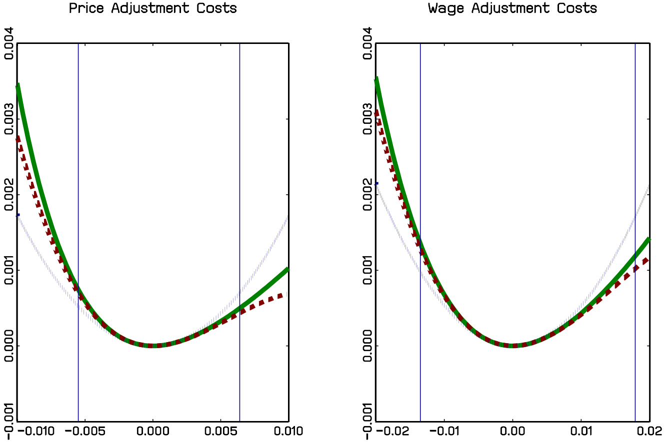
$$R_t = \left[r\pi^* \left(\frac{\pi_t}{\pi^*}\right)^{\psi_1} \left(\frac{\mathcal{Y}_t}{\gamma \mathcal{Y}_{t-1}}\right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R} e^{\epsilon_{R,t}}.$$

- Exogenous shocks:
  - technology A_t [RW with Drift and AR(1) errors],
  - demand / government spending $g_t = \frac{1}{1-\zeta_t}$ [AR(1)],
  - price mark-up $\ln \lambda_{p,t}$ [AR(1)],
  - monetary policy $\epsilon_{R,t}$ [WN].

DSGE Model Estimation

- U.S. Data: GDP growth per capita (using a smoothed population growth series), nominal wage growth (hourly compensation for business sector), the GDP deflator inflation Rate and the fed funds rate.
- Estimation samples: 1984:Q1-2007:Q4.
- Bayesian inference
- 2^{nd} order perturbation to solve the model; particle filter to evaluate the likelihood function.
- Details on prior.
- Details on posterior.

Estimated Adjustment Cost Functions

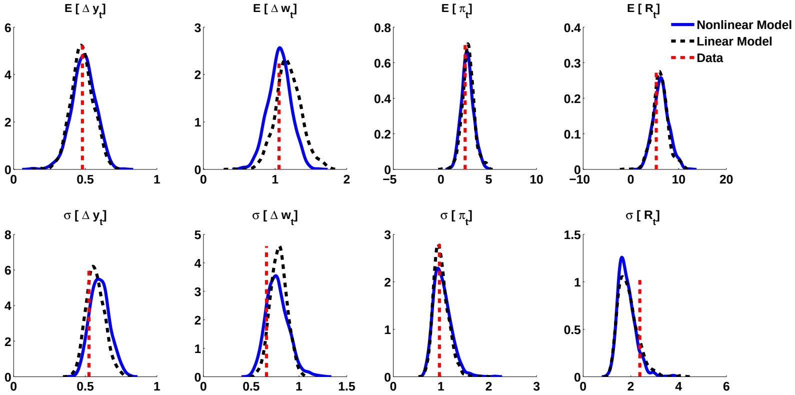


linex = green; 3rd-order approx = red; 2nd-order approx = blue

Posterior Predictive Checks to Assess Estimated DSGE Model

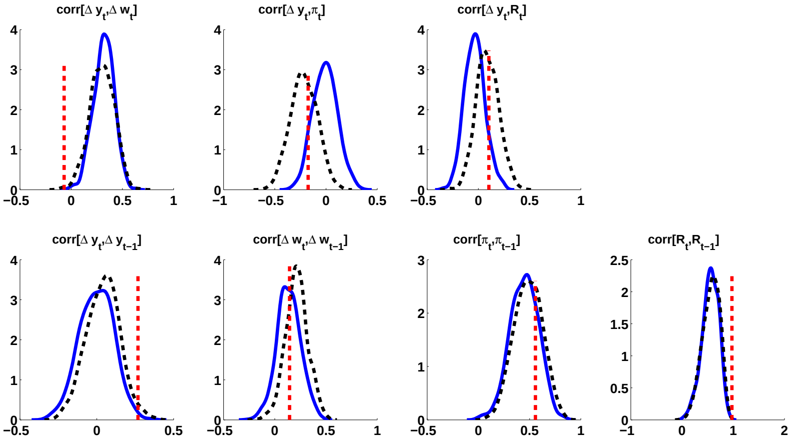
- For each posterior draw θ^m simulate trajectories of Y, π, and R (same length as in the data) from the DSGE model.
- Based on each simulated trajectory calculate sample moments and posterior mean estimates of QAR(1,1) parameters.
- Density estimate of posterior predictive density for sample moments and posterior mean of QAR(1,1) parameters.
- Compare posterior predictive density to statistics calculated from U.S. data.

Posterior Predictive Checks: “Linear” Statistics

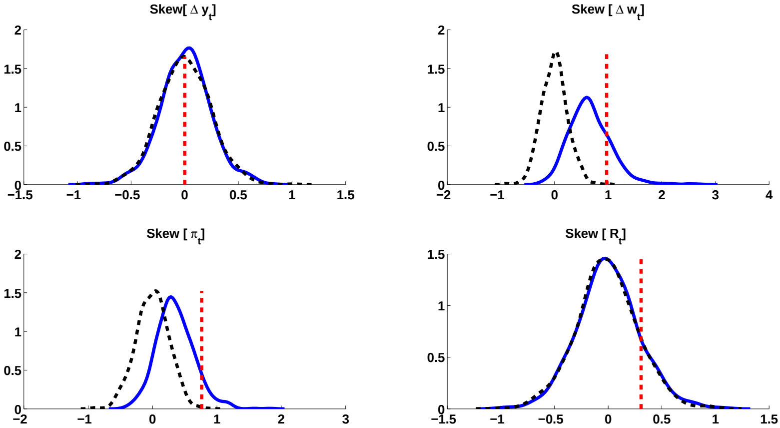


- Linearized and nonlinear DSGE model capture the means and standard deviations equally well.

Posterior Predictive Checks: Autocorrelations



Posterior Predictive Checks: Skewness



- The nonlinear model is able to reproduce some of the skewness in the data.

Posterior Predictive Checks Using the QAR(1,1) Model

- Step 1: We will first take a look at the QAR(1,1) estimates obtained from U.S. data.
- Step 2: We then compare them to the distribution of estimates generated from simulated data.

- Prior for QAR(1,1) parameters is "centered" around the AR(1) case.
- Priors for ϕ_0, ϕ_1, and σ are centered at pre-sample (1957:Q1 to 1983:Q4) mean, autocorrelation, and AR(1) innovation standard deviation.

	GDP Growth	Wage Growth	Inflation	Fed Funds Rate
ϕ_0	$N(0.52, 2)$	$N(1.66, 2)$	$N(4.47, 2)$	$N(6.03, 2)$
ϕ_1	$N^\dagger(0.28, 0.5)$	$N^\dagger(0.36, 0.5)$	$N^\dagger(0.85, 0.5)$	$N^\dagger(0.94, 0.5)$
σ	$IG(1.34, 4)$	$IG(0.93, 4)$	$IG(1.83, 4)$	$IG(1.48, 4)$
ϕ_2	$N(0, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$
γ	$N(0, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$

$\dagger$: Prior for ϕ_1 is truncated to ensure stationarity.

QAR Predictive Check: Step 1 – Use U.S. Data

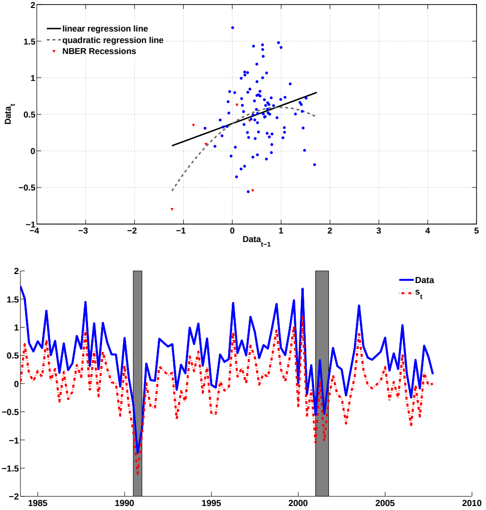
Table: Posterior Estimates for QAR(1,1) Model, 1984:Q1-2007:Q4

Data	ϕ_0	ϕ_1	ϕ_2	γ	σ
GDP	0.52	0.23	-0.13	-0.05	0.55
	[0.38,0.65]	[0.05,0.43]	[-0.27,0.01]	[-0.20,0.10]	[0.49,0.64]
WAGE					
INFL					
FFR					

Table: Log Marginal Data Density: QAR(1,1) and AR(1)

	GDP	Wage	Inflation	FFR
QAR(1,1)	-78.83			
AR(1)	-80.40			

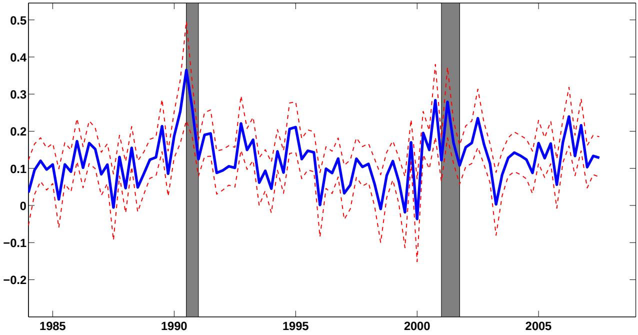
GDP Growth: What Are We Picking Up?



$\hat{\phi}_2 = -0.13;\ \hat{\gamma} = -0.05$

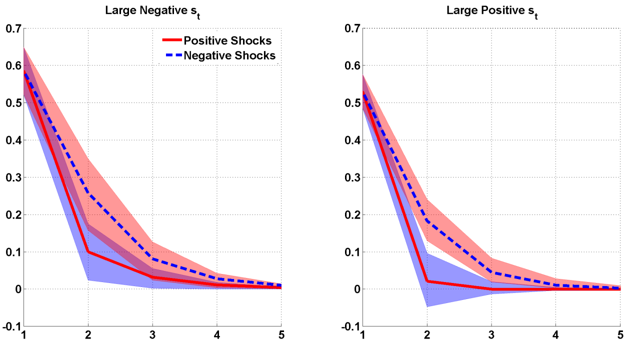
GDP Growth: The role of ϕ_2

Consider the expression $\mathbb{E}_t\left[\frac{\partial y_{t+1}}{\partial u_t}\right] = \phi_1(1+\gamma s_{t-1})\sigma + 2\phi_2 s_t \sigma$.



- Peaks of this elasticity during NBER recession dates: sudden and deep recessions

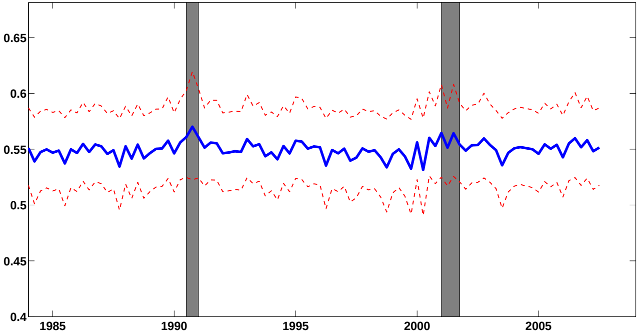
GDP Growth: The role of ϕ_2: QAR IRFs



- Recall: $\hat{\phi}_2 = -0.13$; $\hat{\gamma} = -0.05$.
- With $\phi_2 < 0$, negative shocks are more persistent than positive shock.
- More on QAR IRFs.

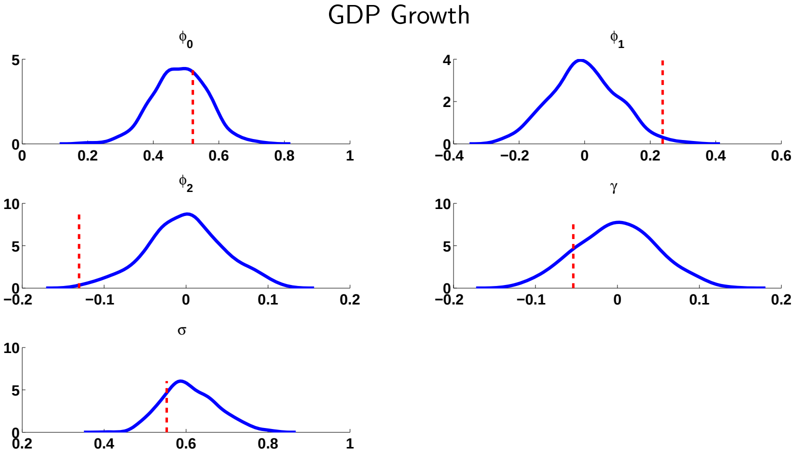
GDP Growth: The role of γ

Consider the expression $\text{Var}_t[y_t] = (1 + \gamma s_{t-1})^2 \sigma^2$.



- Output growth volatility increases in recessions.

QAR Predictive Check: Step 2 – Compare to DSGE Model



- DSGE model has difficulties to generate data that lead to large negative $\hat{\phi}_2$.

QAR Predictive Check: Step 1 – Use U.S. Data

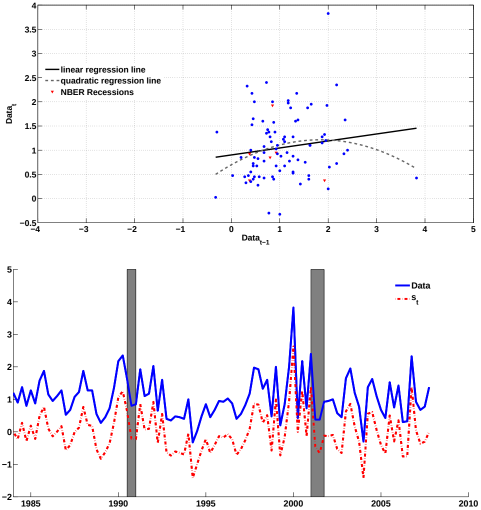
Table: Posterior Estimates for QAR(1,1) Model, 1984:Q1-2007:Q4

Data	ϕ_0	ϕ_1	ϕ_2	γ	σ
GDP	0.52	0.23	-0.13	-0.05	0.55
	[0.38,0.65]	[0.05,0.43]	[-0.27,0.01]	[-0.20,0.10]	[0.49,0.64]
WAGE	1.10	0.23	-0.08	0.08	0.68
	[0.94,1.28]	[0.04,0.42]	[-0.19,0.03]	[-0.03,0.19]	[0.59,0.76]
INFL					
FFR					

Table: Log Marginal Data Density: QAR(1,1) and AR(1)

	GDP	Wage	Inflation	FFR
QAR(1,1)	-78.83	-99.61		
AR(1)	-80.40	-101.28		

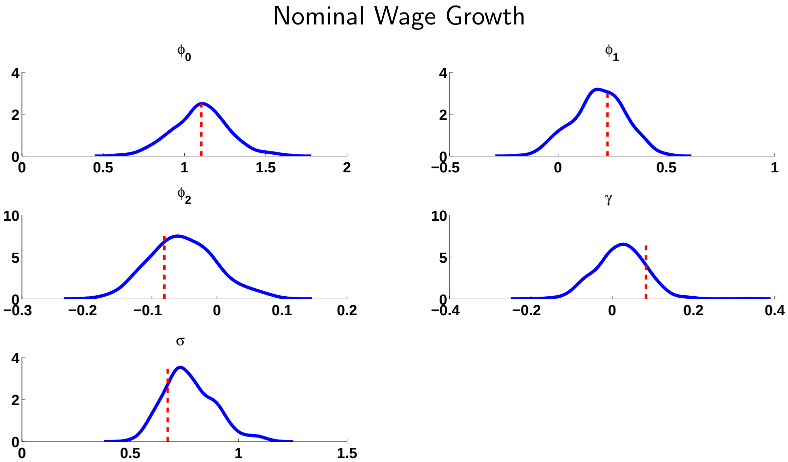
Wage Growth: What Are We Picking Up?



$$\hat{\phi}_2 = -0.08; \; \hat{\gamma} = 0.08$$

More on Wages

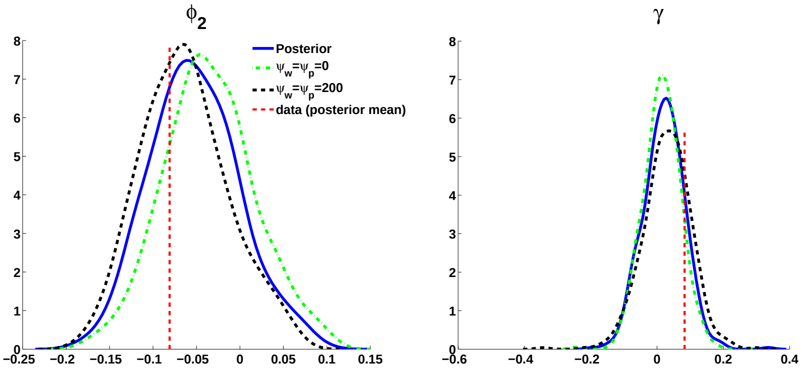
QAR Predictive Check: Step 2 – Compare to DSGE Model



- The DSGE model is able to generate data that imply a negative ϕ_2 estimate, similar to what is observed in the data.

QAR Predictive Check: Step 2 – What Happens If We Shut Down Adjustment Cost Asymmetry?

Nominal Wage Growth



- Without the asymmetric adjustment costs the distribution of the ϕ_2 estimates shifts toward zero.

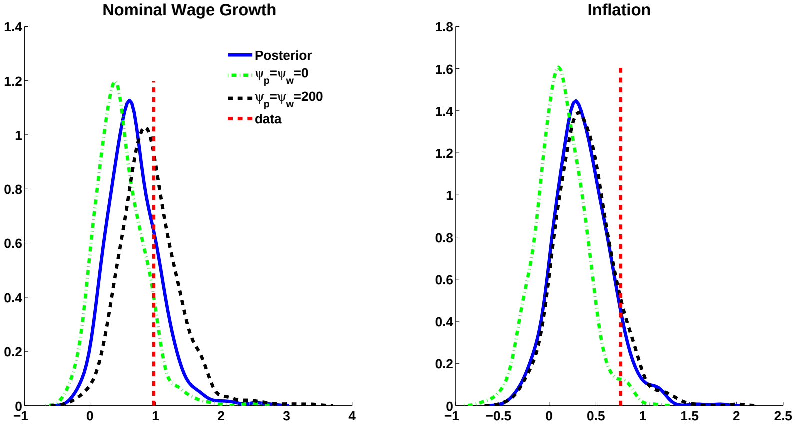
QAR Predictive Check: Step 2 – What Happens If We Shut Down Adjustment Cost Asymmetry?

	Posterior	$\psi_w = \psi_p = 0$	$\psi_w = \psi_p = 200$
Wage Growth (ϕ_2)	0.30	0.20	0.39
Wage Growth (γ)	0.15	0.11	0.25
Inflation (ϕ_2)	0.31	0.21	0.31
Inflation (γ)	0.18	0.04	0.19

- $p - values$;
- measure how far the observed sample moments lies in the tails of the predictive distribution generated by the DSGE model.
- Quantitative representation of the visual information in the graphs; values close to zero imply inconsistency between model and data.

Skewness Predictive Check: What Happens If We Shut Down Adjustment Cost Asymmetry?

Nominal Wage Growth



- Without the asymmetric adjustment costs the distribution of the sample skewness shifts toward zero.

Inflation and Federal Funds Rate

- Predictive check for Inflation
- Predictive check for Federal Funds Rate

Summary of Results

- Trajectories simulated from the estimated DSGE model have sample means, standard deviations, and autocorrelations that are similar to those in US data.
- The same trajectories do not always generate empirically plausible values for the QAR(1,1) model's parameters.
- The DSGE model does not generate nonlinear conditional mean and volatility dynamics of GDP growth that we find in US data.
- However, the DSGE model is able to pick up some of the nonlinearities in nominal wage growth dynamics.
- Posterior predictive checks based on the QAR(1,1) model detect forms of misspecification that are not detected by comparison with linear time series model.

Conclusion

- We have proposed a class of time series models for the evaluation of nonlinear DSGE models
- The QAR(p,q) model is less restrictive than the Nonlinear DSGE model, but capture similar dynamics
- We have shown how to use the QAR(p,q) model for the evaluation of the DSGE models
- Ongoing Research: multivariate generalization of the QAR model; other DSGE models with more interesting nonlinearities.

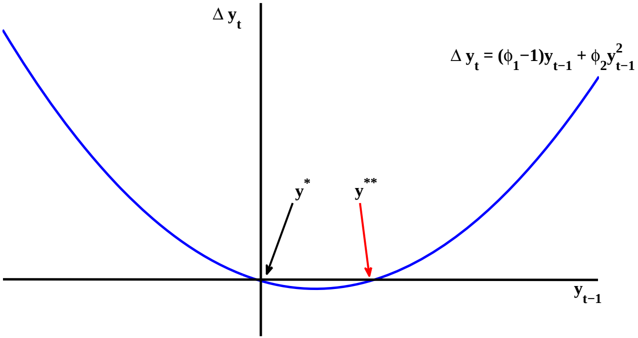
Supplementary Material

A "Naive" Quadratic Autoregressive Model

- Generalized auotregressive (GAR) models (e.g. Mittnik, 1990): add quadratic terms to a standard autoregressive:

$$y_t = \phi_0 + \phi_1(y_{t-1} - \phi_0) + \phi_2(y_{t-1} - \phi_0)^2 + \sigma u_t$$

- Unattractive features: (i) multiple steady states; (ii) explosive dynamics.



Return

QAR(1,1): Impulse Response Functions (IRFs)

IRFs in the QAR(1,1) model are calculated as follows:

$$
\begin{aligned}
\mathbb{E}_t\left[\frac{\partial y_t}{\partial u_t}\right] &= (1+\gamma s_{t-1})\sigma \\
\mathbb{E}_t\left[\frac{\partial y_{t+1}}{\partial u_t}\right] &= \phi_1 \mathbb{E}_t\left[\frac{\partial y_t}{\partial u_t}\right] + 2\phi_2\sigma(\phi_1 s_{t-1}+\sigma) \\
&\vdots \\
\mathbb{E}_t\left[\frac{\partial y_{t+k}}{\partial u_t}\right] &= \phi_1 \mathbb{E}_t\left[\frac{\partial y_{t+k-1}}{\partial u_t}\right] + 2\phi_2\phi_1^{2(k-1)}\sigma(\phi_1 s_{t-1}+\sigma)
\end{aligned}
$$

As long as $\phi_2 \neq 0$, one has that:

- IRFs depend on the initial state s_{t-1}
- IRFs depend on the **sign** of shocks
- IRFs depend on the **size** of shocks

[Return QAR] [Return GDP]

Priors for DSGE Model Estimation

Parameter	Distribution	Para (1)	Para (2)
τ	Gamma	2.00	1
ν	Gamma	0.5	1
$400\left(\frac{1}{\beta}-1\right)$	Gamma	2.00	1.00
π^A	Gamma	3.00	1
γ^A	Gamma	2.00	1.50
κ	Gamma	0.30	0.20
ψ_1	Gamma	1.50	0.5
ψ_2	Gamma	0.2	0.1
ρ_r	Beta	0.50	0.20
ρ_g	Beta	0.80	0.10
ρ_z	Beta	0.20	0.1
ρ_p	Beta	0.6	0.20
$100\sigma_r$	InvGamma	0.20	2.00
$100\sigma_g$	InvGamma	0.75	2.00
$100\sigma_z$	Beta	0.75	2.00
$100\sigma_p$	Beta	0.75	2.00
ϕ_w	Gamma	15	7.5
ψ_w	Uniform	-200	200
ψ_p	Uniform	-300	300
$1/g$	fixed	0.85	

Return

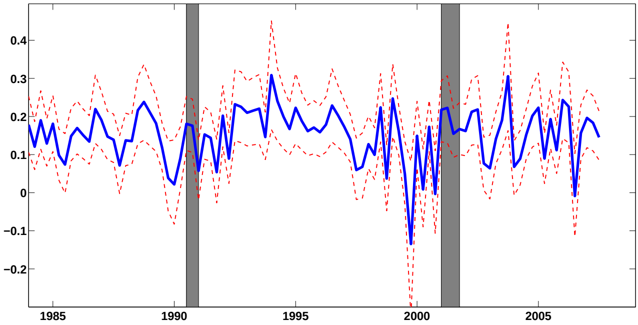
Posteriors from DSGE Model Estimation

	Linear Model		Non Linear Model	
Parameter	Mean	90% Interval	Mean	90% Interval
τ	4.78	[2.57, 8.70]	4.06	[2.37, 6.23]
ν	0.08	[0.03, 0.15]	4.78	[2.57, 8.70]
$400\left(\frac{1}{\beta}-1\right)$	1.31	[0.60, 2.17]	1.31	[0.28, 2.43]
π^A	2.80	[2.33, 3.29]	2.98	[2.32, 3.73]
γ^A	1.88	[1.53, 2.24]	2.03	[1.65, 2.39]
κ	0.18	[0.09, 0.30]	0.19	[0.10, 0.31]
ψ_1	2.67	[2.10, 3.30]	2.62	[2.08, 3.40]
ψ_2	0.76	[0.41, 1.11]	0.75	[0.41, 1.12]
ρ_r	0.71	[0.61, 0.79]	0.73	[0.64, 0.80]
ρ_g	0.96	[0.93, 0.98]	0.95	[0.93, 0.97]
ρ_z	0.07	[0.01, 0.19]	0.06	[0.01, 0.16]
ρ_p	0.93	[0.87, 0.98]	0.91	[0.83, 0.97]
$100\sigma_r$	0.18	[0.13, 0.25]	0.16	[0.12, 0.23]
$100\sigma_g$	0.76	[0.39, 1.34]	0.71	[0.46, 1.23]
$100\sigma_z$	0.47	[0.37, 0.56]	0.48	[0.40, 0.57]
$100\sigma_p$	7.63	[5.96, 9.48]	6.40	[4.60, 8.50]
ϕ_w	14.89	[6.15, 25.88]	10.68	[5.27, 19.53]
ψ_w			67.61	[52.29, 89.78]
ψ_p			179.97	[161.42, 195.72]

Return

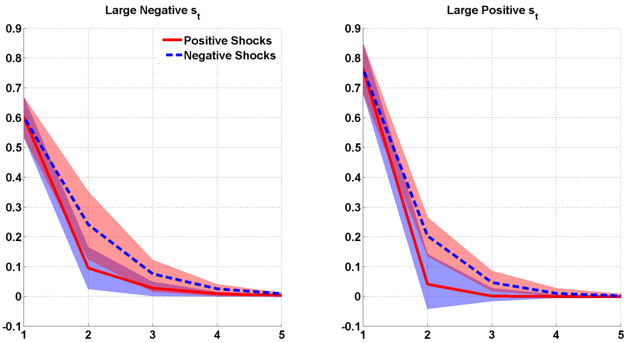
Wage Growth: The role of ϕ_2

Consider the expression $\mathbb{E}_t\left[\frac{\partial y_{t+1}}{\partial u_t}\right] = \phi_1(1+\gamma s_{t-1})\sigma + 2\phi_2 s_t \sigma$.



Return

Wage Growth: The role of ϕ_2

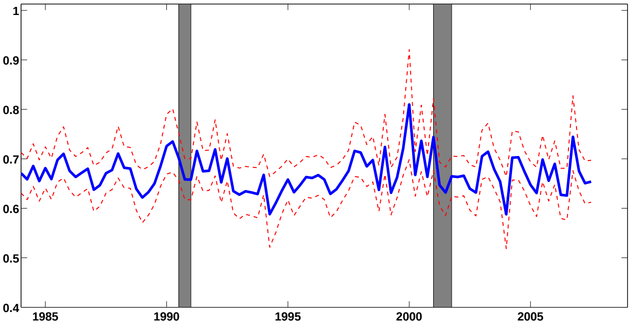


- Recall: $\hat{\phi}_2 = -0.08$; $\hat{\gamma} = 0.08$

Return

Wage Growth: The role of γ

Consider the expression $\text{Var}_t[y_t] = (1 + \gamma s_{t-1})^2 \sigma^2$.



Return

QAR Predictive Check: Step 1 – Use U.S. Data

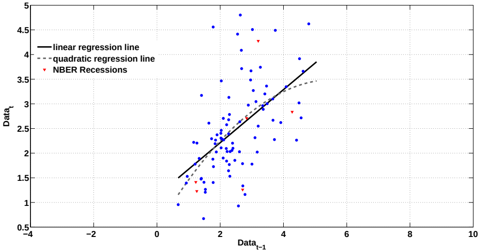
Table: Posterior Estimates for QAR(1,1) Model, 1984:Q1-2007:Q4

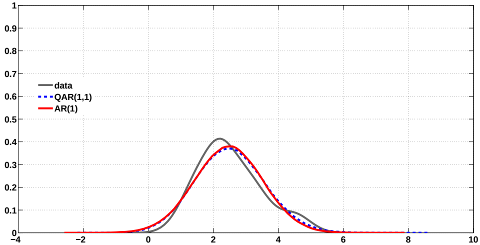
Data	ϕ_0	ϕ_1	ϕ_2	γ	σ
GDP	0.52	0.23	-0.13	-0.05	0.55
	[0.38,0.65]	[0.05,0.43]	[-0.27,0.01]	[-0.20,0.10]	[0.49,0.64]
WAGE	1.10	0.23	-0.08	0.08	0.68
	[0.94,1.28]	[0.04,0.42]	[-0.19,0.03]	[-0.03,0.19]	[0.59,0.76]
INFL	2.61	0.57	-0.04	0.08	0.87
	[2.20,3.07]	[0.41,0.72]	[-0.12,0.04]	[-0.05,0.20]	[0.76,0.99]
FFR					

Table: Log Marginal Data Density: QAR(1,1) and AR(1)

	GDP	Wage	Inflation	FFR
QAR(1,1)	-78.83	-99.61	-120.05	
AR(1)	-80.40	-101.28	-121.16	

Inflation: What Are We Picking Up?



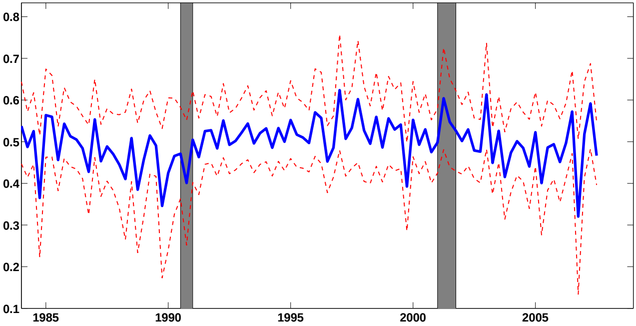


$\hat{\phi}_2 = -0.04; \; \hat{\gamma} = 0.08$

Return

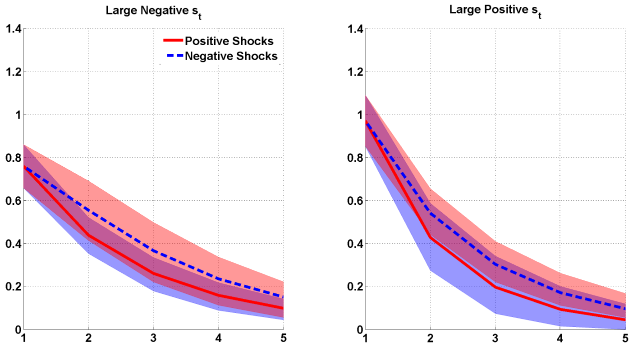
Inflation: The role of ϕ_2

Consider the expression $\mathbb{E}_t\left[\frac{\partial y_{t+1}}{\partial u_t}\right] = \phi_1(1+\gamma s_{t-1})\sigma + 2\phi_2 s_t \sigma$.



Return

Inflation: The role of ϕ_2

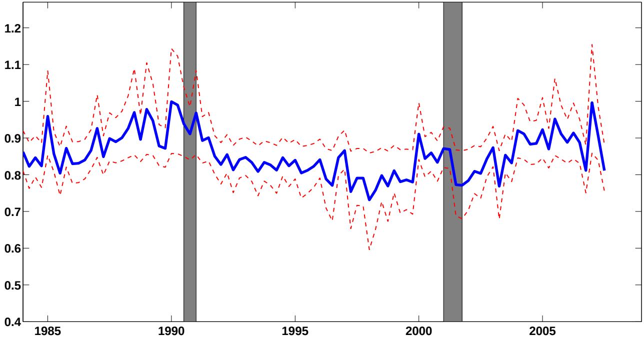


- Recall: $\hat{\phi}_2 = -0.04$; $\hat{\gamma} = 0.08$

Return

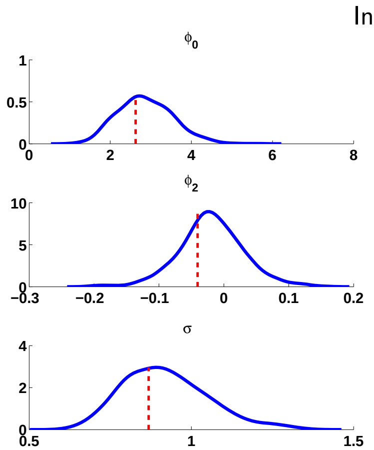
Inflation: The role of γ

Consider the expression $\text{Var}_t[y_t] = (1 + \gamma s_{t-1})^2 \sigma^2$.



Return

QAR Predictive Check: Step 2 – Compare to DSGE Model



Return

QAR Predictive Check: Step 1 – Use U.S. Data

Table: Posterior Estimates for QAR(1,1) Model, 1984:Q1-2007:Q4

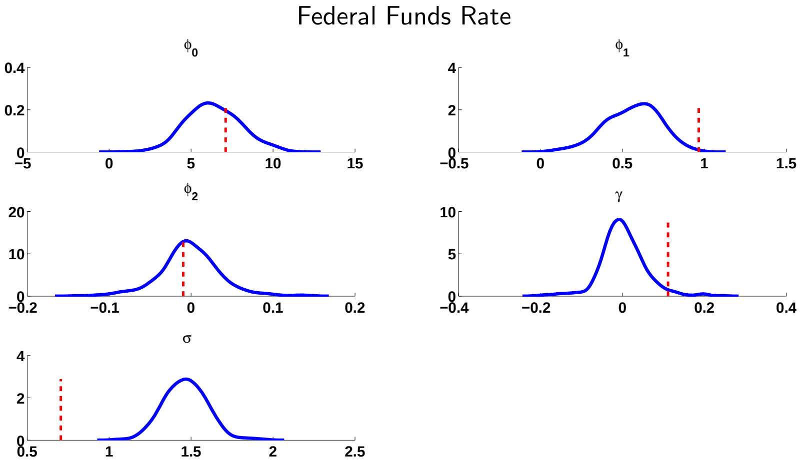
Data	ϕ_0	ϕ_1	ϕ_2	γ	σ
GDP	0.52	0.23	-0.13	-0.05	0.55
	[0.38,0.65]	[0.05,0.43]	[-0.27,0.01]	[-0.20,0.10]	[0.49,0.64]
WAGE	1.10	0.23	-0.08	0.08	0.68
	[0.94,1.28]	[0.04,0.42]	[-0.19,0.03]	[-0.03,0.19]	[0.59,0.76]
INFL	2.61	0.57	-0.04	0.08	0.87
	[2.20,3.07]	[0.41,0.72]	[-0.12,0.04]	[-0.05,0.20]	[0.76,0.99]
FFR	9.57	0.91	-0.08	0.09	0.67
	[7.64,11.61]	[0.86,0.96]	[-.16,-.01]	[0.05,0.12]	[0.57,0.79]

Table: Log Marginal Data Density: QAR(1,1) and AR(1)

	GDP	Wage	Inflation	FFR
QAR(1,1)	-78.83	-99.61	-120.05	-82.38
AR(1)	-80.40	-101.28	-121.16	-85.77

Return

QAR Predictive Check: Step 2 – Compare to DSGE Model



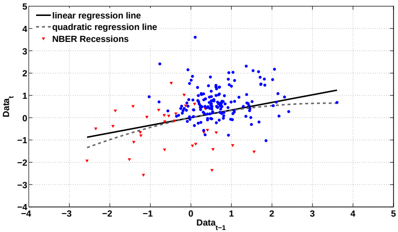
Return

Robustness Checks

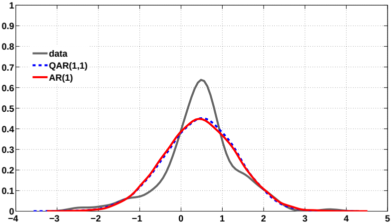
- We re-estimate the models over alternative sample periods

GDP Growth: 1966:Q1 to 2007:Q4 [TO BE UPDATED]

Scatter Plot



Density Plot



$$\hat{\phi}_2 = -0.25, \ \hat{\gamma} = -0.11$$