

# Informality in Labor Market and Welfare

Daniel R. Sanches

July 10, 2005

## **Abstract**

The neoclassical growth model with two sectors in production is employed in this paper in order to investigate how a change in the tax structure affects informality and welfare. We calibrate and simulate the model and find that welfare always increases when we reduce the tax rate on the demand for labor and adjust the tax rate on the value added so that the government revenue remains constant.

Escola de Pós-Graduação em Economia - EPGE  
Fundação Getúlio Vargas

## Informality in Labor Market and Welfare

Dissertação submetida à Escola de Pós-Graduação em Economia  
da Fundação Getúlio Vargas como requisito para obtenção do  
Título de Mestre em Economia

Aluno: Daniel Rocha Sanches

Orientador: Samuel Pessôa

Rio de Janeiro  
2005

I dedicate this work to my wife Ammanda.

# 1 Introduction

This paper analyzes the effect of taxation on informality and welfare. We investigate how a change in the tax structure affects informality in the labor market and total output. This investigation may yield new directions in the formulation of public policies, since there is evidence that informality has been increasing in both developed and developing economies (see Enste and Schneider, 2000). Therefore, an evaluation of the link between informality and welfare within a formal framework has become necessary to the design of public policies.

We define informality as a sector of the economy that contributes to the officially calculated gross domestic product but is currently unregistered. To be more precise, the informal sector of the economy includes unreported income from the production of legal products and services. This is the simplest and direct definition of informality employed in the literature<sup>1</sup>. Note that this definition does not differentiate the kind of goods and services produced in the informal sector of the economy, so that this fact must be taken into account when building a theoretical model. A key difference between the formal and informal sectors is that the informal sector is invisible to government authorities, which means that taxes are not levied on the production that is taken in this sector of the economy. In our analysis, technological aspects also distinguish the informal economy from the official.

The model employed in this paper is a variation of the neoclassical growth model which allows two sectors in production. As mentioned above, firms in the first sector have their plants officially registered and, therefore, taxes are levied on the production that is taken in this sector. For this reason, we call it the formal sector of the economy. Firms in the second sector are not taxed by the government because their plants are not official. This corresponds to the informal sector of the economy. There is a technological difference between firms in each sector. The firm in the formal sector operates with a technology that uses only reproducible factors of production and exhibits constant returns to scale. The firm in the informal sector employs a technology that requires a nonreproducible factor of production and, therefore, exhibits decreasing returns to scale in the reproducible factors of production.

We can alternatively think of this model in the following way. There are infinitely many identical firms in the economy. Each firm has two plants to

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<sup>1</sup>For other definitions and extensions, see Enste and Schneider (2000).

operate in the production of the unique good in the economy. The first plant uses a technology which exhibits constant returns to scale in capital and labor. The government levies taxes on the production that is taken in this plant. The second plant uses a technology which exhibits decreasing returns to scale in the reproducible factors of production. We assume that there is no taxation in the production taken in this plant. Therefore, a representative firm chooses optimally where to produce. This formulation represents an alternative interpretation of the model.

The variation of the neoclassical model adopted in this paper is the same as that employed by Hansen and Prescott (2002) and Restuccia (2004). In both papers, the authors assume the existence of a modern and a traditional sector in the economy. In the modern sector, production is taken in a plant that employs a technology which exhibits constant returns to scale. This corresponds to the formal sector in our model. In the traditional sector, the nonreproducible factor is land since this sector corresponds to agriculture in their model.

We apply the same model with a different interpretation of the sectors in production. In our model, the differences between the sectors are not exclusively due to technological factors. Taxation in our model is asymmetric and constitutes a characterization of the sectors in the economy. The reason for having decreasing returns to scale in the reproducible factors in the informal sector is that in some way businesses not officially registered cannot become larger simply by increasing all of its factors of production. We may think of the nonreproducible factor as land. However, land as a factor of production will not play the same role as it does in the model in which the second sector is viewed as agriculture (Restuccia (2004)). In our model, land simply represents a constraint to the growth of the informal business. Once a firm is in the informal sector of the economy, it will be constrained to remain invisible to government authorities.

Penãlosa and Turnovsky (2004) use this same interpretation of the neoclassical growth model with two sectors in production. Their main point is to find optimal taxation schemes in this class of models. The main difference of our paper is that we do not assume that aggregate capital stock provides an externality in production as they do in their paper. We do assume that the informal sector of the economy exhibits decreasing returns to scale in the reproducible factors of production. This is the main technological assumption of our paper.

Our objective is to analyze the impact of a change in the tax structure

on informality and welfare. In particular, we investigate the consequences of a reduction in the tax rate on the demand for labor followed by an adjust in the tax rate on the value added in order to keep the government revenue constant. The basic idea is to reduce the distortion in the labor market without changing the government revenue. The taxation on the value added is increased so that this can be done. We use the US economy as a benchmark and calibrate the model in order to evaluate the quantitative results of our experiment. Then, we extend the analysis to the Brazilian economy.

In Section 2, we present the model and define the competitive equilibrium. We calibrate the model in Section 3, and we conduct the simulations in Sections 4 and 5. In section 6, we present a welfare analysis. Conclusions are given in Section 7.

## 2 The Model

### 2.1 Households

The household's problem can be written as follows:

$$\begin{aligned} & \max_{\{(c_t, k_{t+1}, L_t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \\ s.t. & \begin{cases} c_t + k_{t+1} = w_t + r_t k_t + (1 - \delta) k_t + q_t L_t + \varkappa_t \\ c_t, k_{t+1} \geq 0 \text{ and } 0 \leq L_t \leq 1 \text{ for all } t \\ k_0 > 0 \text{ is given} \end{cases} \end{aligned}$$

where  $c_t$  represents the household's consumption in period  $t$ ;  $k_t$  is the stock of capital owned by the household in period  $t$ ;  $L_t$  is the stock of a nonreproducible factor, which is rented to firms at the price  $q_t$ ;  $\varkappa_t$  is the lump-sum transfer payment from the government to households; and  $w_t$  and  $r_t$  denote the wage and the rental rate received from the services provided by labor and capital, respectively.

The household takes the sequences of prices  $\{(q_t, w_t, r_t)\}_{t=0}^{\infty}$  and transfer payments  $\{\varkappa_t\}_{t=0}^{\infty}$  as given when choosing the optimal sequences for consumption and capital. We assume that each household is endowed with one unit of the nonreproducible factor.

Given the recursive nature of the household's problem, we can represent

it by writing the corresponding Bellman equation.

$$v(k) = \left\{ \begin{array}{c} \max_{h,c} [u(c) + \beta v(h)] \\ s.t \left\{ \begin{array}{l} c + h = w + rk + (1 - \delta)k + q + \varkappa \\ c, h \geq 0 \end{array} \right. \end{array} \right\} \quad (1)$$

The first order condition is given by

$$-u'(c(k)) + \beta v'(h(k)) = 0 \quad (2)$$

where  $h(k)$  and  $c(k)$  represent the optimal choices of end of period capital and consumption, respectively.

By the Envelope Theorem, we find an expression for the first derivative of the value function.

$$v'(k) = (1 + r - \delta) u'(c(k)) \quad (3)$$

Substituting (3) into (2), we obtain the equation that describes the household's optimal choice, that is, the Euler equation. This equation together with the household's budget constraint define the optimal choice of consumption and capital.

$$u'(c(k)) = \beta (1 + r' - \delta) u'(c(h(k))) \quad (4)$$

$$c(k) + h(k) = w + rk + (1 - \delta)k + q + \varkappa \quad (5)$$

We assume the following functional form for the utility function:

$$u(c) = \frac{1}{1 - \gamma} c^{1-\gamma} \quad (6)$$

Using this functional form, we can rewrite equations (4) and (5) as follows:

$$c(k)^{-\gamma} = \beta (1 + r' - \delta) c(h(k))^{-\gamma} \quad (7)$$

$$c(k) + h(k) = w + rk + (1 - \delta)k + q + \varkappa \quad (8)$$

## 2.2 Firms

There are two sectors in the economy: a formal sector in which taxes are charged and an informal sector which is not taxed by the government. The technology employed in the formal sector exhibits constant returns to scale in capital and labor (the reproducible factors of production in this model), while in the informal sector the technology used exhibits decreasing returns to scale in the reproducible factors. We have assumed that the proceeds from taxation in the formal sector are transferred in the form of lump-sum payments to households. Each sector is inhabited by identical competitive firms.

The technologies in the formal and informal sectors are assumed to have the following functional forms

$$F(K_f, N_f) = AK_f^{\alpha_f} N_f^{1-\alpha_f}$$

and

$$G(K_i, N_i, L) = AB \left( K_i^\psi L^{1-\psi} \right)^{\alpha_i} N_i^{1-\alpha_i}$$

where  $0 < \alpha_f < 1$ ,  $0 < \alpha_i < 1$ , and  $0 < \psi < 1$ .  $K_f$  and  $K_i$  denote the capital allocated to the formal and informal sectors, respectively;  $N_f$  and  $N_i$  denote labor allocated to the formal and informal sectors, respectively; and  $L$  is a nonreproducible factor. The term  $A$  represents total factor productivity in the formal sector of the economy and the term  $B$  represents the relative productivity of the informal sector. The capital share in the formal sector, given by  $\alpha_f$ , is assumed to be higher than that in the informal sector, given by  $\alpha_i\psi$ . We also assume that the labor share in the informal sector, given by  $1 - \alpha_i$ , is higher than that in the formal sector, given by  $1 - \alpha_f$ , which means that the informal sector is more labor intensive.

### 2.2.1 Formal Sector

Let  $w_f$  and  $r_f$  denote the wage rate and the rental rate of capital, respectively, that a firm in the formal sector takes as given. The profit of a representative firm in this sector is given by

$$\pi_f = (1 - \tau_{va}) AK_f^{\alpha_f} N_f^{1-\alpha_f} - (1 + \tau_l) w_f N_f - (1 + \tau_k) r_f K_f \quad (9)$$

where  $\tau_{va}$  is the tax rate on the value added (so that  $0 < \tau_{va} < 1$ ),  $\tau_k$  is the tax rate levied on the demand for capital services ( $\tau_k > 0$ ), and  $\tau_l$  is the tax rate levied on the demand for labor services ( $\tau_l > 0$ ).



We define  $T_{va}^{-1} \equiv (1 - \tau_{va})$ ,  $T_l \equiv (1 + \tau_l)$ , and  $T_k \equiv (1 + \tau_k)$ , so that equation (9) can be rewritten as

$$\pi_f = N_f [T_{va}^{-1} A k_f^{\alpha_f} - T_l w_f - T_k r_f k_f] \quad (10)$$

where  $k_f \equiv \frac{K_f}{N_f}$ .

The first order condition for the firm's profit maximization problem and the zero profit condition imply the following optimality conditions:

$$r_f = \frac{T_{va}^{-1}}{T_k} A \alpha_f k_f^{\alpha_f - 1} \quad (11)$$

$$w_f = \frac{T_{va}^{-1}}{T_l} A (1 - \alpha_f) k_f^{\alpha_f} \quad (12)$$

These conditions characterize the optimal choice of labor and capital for a firm in the formal sector.

### 2.2.2 Informal Sector

The profit of a representative firm in the informal sector is given by

$$\pi_i = AB \left( K_i^\psi L^{1-\psi} \right)^{\alpha_i} N_i^{1-\alpha_i} - w_i N_i - r_i K_i - qL \quad (13)$$

where  $q$  is the relative price of the nonreproducible factor. We assume that the firm in the informal sector takes as given the wage rate  $w_i$ , the rental rate  $r_i$ , and the relative price of land  $q$ .

The first order condition for the firm's profit maximization problem and the zero profit condition imply the following equations:

$$r_i = \frac{AB \alpha_i \psi \left( k_i^\psi l_i^{1-\psi} \right)^{\alpha_i}}{k_i} \quad (14)$$

$$q = \frac{AB \alpha_i (1 - \psi) \left( k_i^\psi l_i^{1-\psi} \right)^{\alpha_i}}{l_i} \quad (15)$$

$$w_i = AB (1 - \alpha_i) \left( k_i^\psi l_i^{1-\psi} \right)^{\alpha_i} \quad (16)$$

where  $k_i \equiv \frac{K_i}{N_i}$  and  $l_i \equiv \frac{L}{N_i}$ .

The above conditions define the optimal choice of labor, capital and the nonreproducible factor for a firm in the informal sector.

### 2.3 Perfect Factor Mobility

We assume that there is perfect mobility in the factors' markets. This assumption implies the following conditions:

$$w = w_f = w_i$$

and

$$r = r_f = r_i$$

Using equations (11), (12), (14), and (16), we find the following relation:

$$\begin{aligned} \frac{w}{r} &= \frac{T_k}{T_l} \frac{1 - \alpha_f}{\alpha_f} k_f \\ &= \frac{1 - \alpha_i}{\alpha_i \psi} k_i \end{aligned} \quad (17)$$

Therefore, the capital-to-labor ratios in each sector are proportional.

$$k_i = \frac{T_k}{T_l} \frac{\alpha_i}{\alpha_f} \frac{1 - \alpha_f}{1 - \alpha_i} \psi k_f \quad (18)$$

### 2.4 Government

We have assumed that the government taxed the firms in the formal sector and then transferred the proceeds to the households in the form of lump-sum payments. The government budget constraint is given by

$$\varkappa = \tau_{va} A K_f^{\alpha_f} N_f^{1-\alpha_f} + \tau_l w N_f + \tau_k r K_f \quad (19)$$

where  $\tau_{va}$ ,  $\tau_l$ , and  $\tau_k$  are the tax rates charged on the traditional sector.

### 2.5 Competitive Equilibrium

In equilibrium, the economy's resource constraint must be respected:

$$c(k) + h(k) - (1 - \delta)k = A K_f^{\alpha_f} N_f^{1-\alpha_f} + AB \left( K_i^\psi L^{1-\psi} \right)^{\alpha_i} N_i^{1-\alpha_i} \quad (20)$$

where  $K_f$  and  $N_f$  represent the optimal choice of capital and labor, respectively, by the firm in the formal sector and  $K_i$ ,  $N_i$ , and  $L$  represent the

optimal choice of capital, labor, and land, respectively, by the firm in the informal sector.

The market clearing conditions for the factors' markets must hold in equilibrium:

$$N_f + N_i = 1 \quad (21)$$

$$K_f + K_i = k \quad (22)$$

$$L = 1 \quad (23)$$

where total labor is normalized to unit and  $k$  represents the capital stock which is taken as given by households in the beginning of each period.

A competitive equilibrium in this economy is defined as an allocation  $\{(c_t, k_{t+1})\}_{t=0}^{\infty}$  for the representative household, an allocation  $\{(K_{ft}, N_{ft})\}_{t=0}^{\infty}$  for the firm in the formal sector, an allocation  $\{(K_{it}, N_{it}, L_t)\}_{t=0}^{\infty}$  for the firm in the informal sector, and a sequence of prices  $\{(r_t, w_t, q_t)\}_{t=0}^{\infty}$  such that:

- (i)  $\{(c_t, k_{t+1})\}_{t=0}^{\infty}$  solves (7) and (8) given the prices  $\{(r_t, w_t, q_t)\}_{t=0}^{\infty}$  and given  $k_0$ ;
- (ii)  $\{(K_{ft}, N_{ft})\}_{t=0}^{\infty}$  solves (11) and (12) given the prices  $\{(r_t, w_t, q_t)\}_{t=0}^{\infty}$ ;
- (iii)  $\{(K_{it}, N_{it}, L_t)\}_{t=0}^{\infty}$  solves (14), (15), and (16) given the prices  $\{(r_t, w_t, q_t)\}_{t=0}^{\infty}$ ;
- (iv) all markets clear: (20), (21), (22), and (23) hold for all  $t$ .

### 2.5.1 Long Run Equilibrium

In the long run, we find that

$$r = \frac{1}{\beta} - (1 - \delta) \quad (24)$$

Therefore, the capital-to-labor ratio in the formal and informal sectors are given, respectively, by

$$k_f = \left[ \frac{T_{va}^{-1}}{T_k} A \alpha_f \frac{\beta}{1 - (1 - \delta) \beta} \right]^{\frac{1}{1 - \alpha_f}} \quad (25)$$

and

$$k_i = \psi \frac{T_k}{T_l} \frac{\alpha_i}{\alpha_f} \frac{1 - \alpha_f}{1 - \alpha_i} \left[ A \alpha_f \frac{T_{va}^{-1}}{T_k} \frac{\beta}{1 - (1 - \delta) \beta} \right]^{\frac{1}{1 - \alpha_f}} \quad (26)$$

From the wage equalization across sectors, we find that the share of the labor force allocated to the informal sector is given by

$$N_i = \left( B \frac{T_l}{T_{va}^{-1}} \frac{1 - \alpha_i}{1 - \alpha_f} \frac{k_i^{\psi \alpha_i}}{k_f^{\alpha_f}} \right)^{\frac{1}{(1-\psi)\alpha_i}}. \quad (27)$$

Obviously, the share of the labor force allocated to the formal sector is given by  $N_f = 1 - N_i$ .

### 2.5.2 Short Run Equilibrium

We can also define the equilibrium for this economy when total capital is assumed to be fixed. We denote this situation as short-run equilibrium. In this case, the capital-to-labor ratio in the formal sector  $k_f$  is implicitly defined as a function of total capital  $k$  according to

$$N_i k_i + (1 - N_i) k_f = k \quad (28)$$

where  $k_i$  is given by (26) and  $N_i$  is given by (27).

## 3 Calibration

### 3.1 US Economy

Table 1 shows the values of the technology and preference parameters that we take from the existing literature. The depreciation rate  $\delta$  is set so that the investment-to-output ratio is 0.2, and the capital share in the formal sector is 0.35<sup>2</sup>. We assume that the labor share in the informal sector is 0.95, which is in line with our assumption that the informal sector is more labor intensive. We take the preference parameter  $\gamma$  from Penálosa and Turnovsky (2004).

Table 1 - Calibration

<i>parameter</i>	<i>value</i>
$\alpha_f$	0.35
$\alpha_i$	0.05
$\delta$	0.04
$\gamma$	2.5

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<sup>2</sup>See Restuccia (2004).

The parameter  $\psi$  defines the aggregate between capital and the nonreproducible factor. There is no observable counterpart that can be used in order to find a value for this parameter. Given this fact, we adopt the following strategy. We calibrate the model taking as given the parameter  $\psi$ . We then vary the value of  $\psi$  within its parametric space to investigate the robustness of the result. It can be seen from the tables below that the result of the calibration virtually does not change as we vary  $\psi$ .

There are three additional parameters ( $A$ ,  $B$ , and  $\beta$ ) that we must find the values so that the model reproduces the balance growth path of the US economy. In order to do this, we use equations (28), (25), (26), (27), and (20) together with the values  $\tau_k = 0.25$  (which corresponds to a tax rate of 0.2 on the supply of capital)<sup>3</sup>,  $\tau_l = 2/3$  (which corresponds to a tax rate of 0.4 on the supply of labor), and  $\tau_{va} = 0.03$ <sup>4</sup>. In this calibration, the targets are the following: a capital-to-output ratio of 2.5, a share of the labor force allocated to the informal sector of 0.1, and a unit output. Substituting equations (25) and (26) into (27), we find the following relation:

$$N_i = \left\{ \begin{array}{c} B \frac{T_l}{T_{va}^{-1}} \frac{1-\alpha_i}{1-\alpha_f} \times \\ \times \left[ \frac{T_k}{T_l} \frac{\alpha_i}{\alpha_f} \frac{1-\alpha_f}{1-\alpha_i} \psi \left( A \alpha_f \frac{T_{va}^{-1}}{T_k} \frac{1}{r} \right)^{\frac{1}{1-\alpha_f}} \right]^{\psi \alpha_i} \times \\ \times \left( A \alpha_f \frac{T_{va}^{-1}}{T_k} \frac{1}{r} \right)^{-\frac{\alpha_f}{1-\alpha_f}} \end{array} \right\}^{\frac{1}{(1-\psi)\alpha_i}} \quad (29)$$

We can now substitute this expression together with (25) and (26) into (28) in order to find the second equation that will be used to find the values of the parameters  $A$ ,  $B$ , and  $\beta$ . The third equation is obtained by substituting (25), (26), and (29) into (20). Table 2 shows the result of this calibration for each value of the parameter  $\psi$ .

**Table 2 - Calibration for each Value of the Parameter  $\psi$**

<i>parameter</i>	$\psi = 0.1$	$\psi = 0.3$	$\psi = 0.5$	$\psi = 0.7$	$\psi = 0.9$
$A$	0.74	0.74	0.75	0.75	0.75
$B$	0.53	0.55	0.57	0.59	0.61
$\beta$	0.94	0.94	0.94	0.94	0.94

We can see that the parameters  $A$  and  $\beta$  practically do not vary with the change in the value of  $\psi$ . Table 3 shows the share of the informal sector in

<sup>3</sup>Appendix B shows the equivalence in this model between taxation on the demand for factors and taxation on the supply.

<sup>4</sup>See Prescott (2004) and Enste and Schneider (2000).

total output and the revenue-to-output ratio for each value of the parameter  $\psi$ .

**Table 3 - Informal Sector Share and Revenue-to-Output Ratio**

<i>parameter</i>	$\psi = 0.1$	$\psi = 0.3$	$\psi = 0.5$	$\psi = 0.7$	$\psi = 0.9$
$y_i/y$	0.04	0.04	0.04	0.04	0.04
$\chi/y$	0.33	0.33	0.33	0.33	0.33

It follows that the values for  $y_i/y$  and  $\chi/y$  do not vary as  $\psi$  goes from 0.1 to 0.9. These values are in line with the empirical evidence; see Enste and Schneider (2000).

### 3.2 Brazilian Economy

The calibration for the Brazilian economy is accomplished in the following way. We assume that the preference parameter  $\beta$  and the technology parameters  $\alpha_i$  and  $\alpha_f$  are equal to those of the US economy. Therefore, there are two parameters left to calibrate:  $A$  and  $B$ . In order to do this, we use equations (29) and (20). The targets for this calibration are the following. We assume that the investment rate in Brazil is also 0.2, so that the depreciation rate is 0.04. The major differences are that the share of the labor force in the informal sector is 0.3 for the Brazilian economy, total output is 0.4 of the US economy<sup>5</sup>, and the tax rate on the demand for labor is 1 (which corresponds to a tax rate of 0.5 on the supply of labor). Table 4 shows the value of the productivity parameters  $A$  and  $B$  for the Brazilian economy for each value of the parameter  $\psi$ .

**Table 4 - Calibration for each Value of the Parameter  $\psi$**

<i>parameter</i>	$\psi = 0.1$	$\psi = 0.3$	$\psi = 0.5$	$\psi = 0.7$	$\psi = 0.9$
$A$	0.46	0.46	0.46	0.46	0.46
$B$	0.35	0.37	0.38	0.40	0.41

Table 5 shows the value of the quantities  $y_i/y$ ,  $\chi/y$ , and  $k/y$  for each value of the parameter  $\psi$ .

**Table 5 - Informal Sector Share, Revenue-to-Output Ratio, and Capital-to-Output Ratio**

<i>parameter</i>	$\psi = 0.1$	$\psi = 0.3$	$\psi = 0.5$	$\psi = 0.7$	$\psi = 0.9$
$y_i/y$	0.12	0.12	0.12	0.12	0.12
$\chi/y$	0.36	0.36	0.36	0.36	0.36
$k/y$	2.29	2.30	2.21	2.31	2.32

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<sup>5</sup>See appendix D.

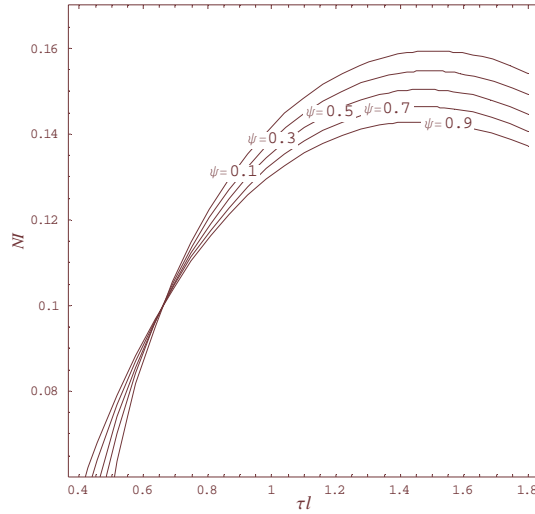
Note that the quantities  $y_i/y$  and  $\chi/y$  practically do not vary as  $\psi$  goes from 0.1 to 0.9. Only the capital-to-output ratio is sensitive to variations in the parameter  $\psi$ .

## 4 Simulation - Long Run

### 4.1 US Economy

Once the model is calibrated for the benchmark economy, we implement the following experiment. Consider a reduction in the tax rate on the demand for labor  $\tau_l$  and let the tax rate on the value added  $\tau_{va}$  vary so that the government revenue  $\varkappa$  remains constant. Graph 1 shows the value of the share of the labor force in the informal sector for each value of the parameter  $\psi$ . For relatively low levels of  $\tau_l$ , the labor share in the informal sector falls as  $\tau_l$  decreases in the simulation. When  $\tau_l$  is relatively large, the simulation shows that  $N_i$  may initially increase when we reduce  $\tau_l$ .

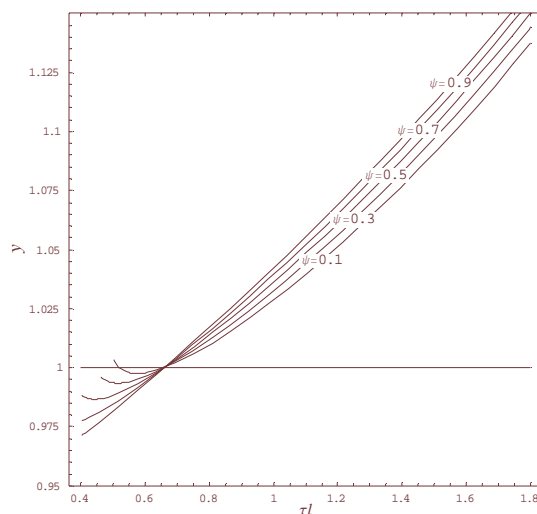
Graph 1 - Labor Share in the Informal Sector



Graph 2 shows total output for the same grid of values of  $\tau_l$ . We can see that total output always decreases as  $\tau_l$  falls and  $\tau_{va}$  adjusts so that the government revenue remains constant. This is true for all values of the parameter  $\psi$  adopted in the simulation. This occurs because the capital stock

allocated to the formal sector always decreases with the experiment. For relatively low initial levels of  $\tau_l$ , the capital stock allocated to the informal sector also decreases when the simulation is conducted, so that total capital diminishes. When the initial value of  $\tau_l$  is relatively high, the capital allocated to the informal sector may initially increase in the simulation. However, total capital decreases because the fall in the production of the formal sector more than compensates the rise in the production of the informal sector.<sup>6</sup>

**Graph 2 - Total Output**



The simulation taken above indicates that a policy that rises the tax rate on the value added and reduces the tax rate on the demand for labor, keeping the government revenue constant, reduces the long-run output of the US economy. Informality in the labor market may initially increase if the initial value of  $\tau_l$  is relatively high.

## 4.2 Brazilian Economy

We implement the same experiment in the Brazilian economy. The results observed are quite similar to those of the US economy. The simulation virtually does not change as the parameter  $\psi$  goes from 0.1 to 0.9, so that we will only show the results for  $\psi = 0.1$ . The variable  $N_i$  shows the same pattern as that observed for the US economy. For relatively low levels of  $\tau_l$ ,

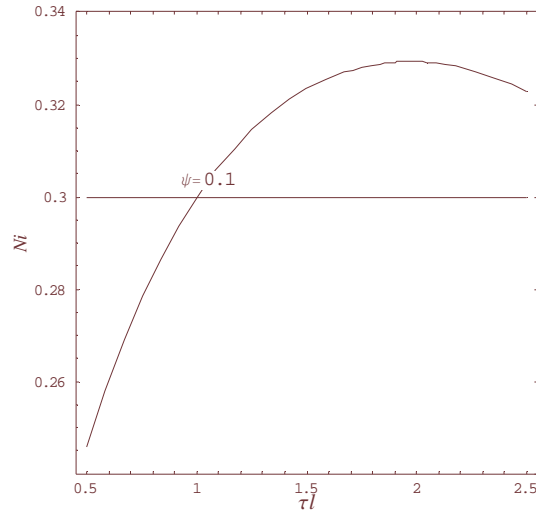
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<sup>6</sup>In appendix C, we conduct the analysis of the long-run equilibrium.



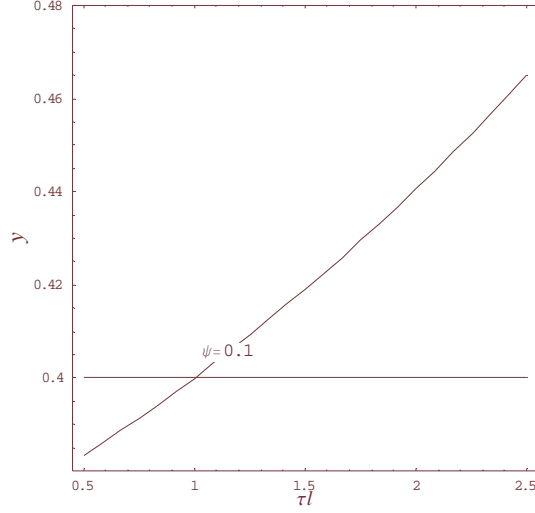
the labor share in the informal sector decreases as  $\tau_l$  falls and  $\tau_{va}$  adjusts so that the government revenue remains constant. When  $\tau_l$  is relatively high, the opposite occurs. Graph 3 shows the result of the simulation.

**Graph 3 - Labor Share in the Informal Sector**



Total output resulting from the simulation is shown in Graph 4. We use the same grid of values of  $\tau_l$ . Contrary to the result obtained for the US economy, both the capital allocated to the formal and informal sectors always fall.

Graph 4 - Total Output



The same conclusion that we had for the US economy holds for the Brazilian economy in the long run. The output of the economy in the long run always decreases in the simulation, although the share of the labor force allocated to the informal sector may initially increase.

## 5 Simulation - Short Run

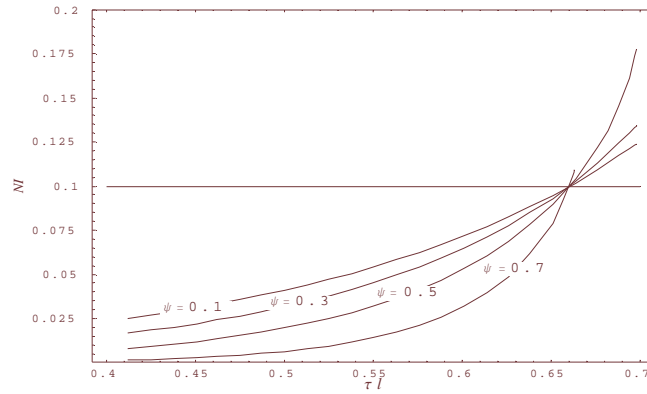
### 5.1 US Economy

In the previous section, we evaluated the effect of an experiment in which the tax rate on the demand for labor  $\tau_l$  was reduced and the tax rate on the value added  $\tau_{va}$  was adjusted so that the government revenue  $\varkappa$  remained constant. It was seen that this policy experiment led to a reduction in total output. We saw that this reduction was due to the fall in total capital with the fall in the capital allocated to the formal sector being larger. In this section, we analyze the consequences of a similar experiment on informality and total output. The only difference is that we do not allow total capital to change in the simulation. By doing this, we can see more clearly the allocative effects of our policy experiment.

Consider a reduction in  $\tau_l$  and let  $\tau_{va}$  vary so that both the government revenue  $\varkappa$  and total capital  $k$  remain constant. Graph 5 shows how the labor

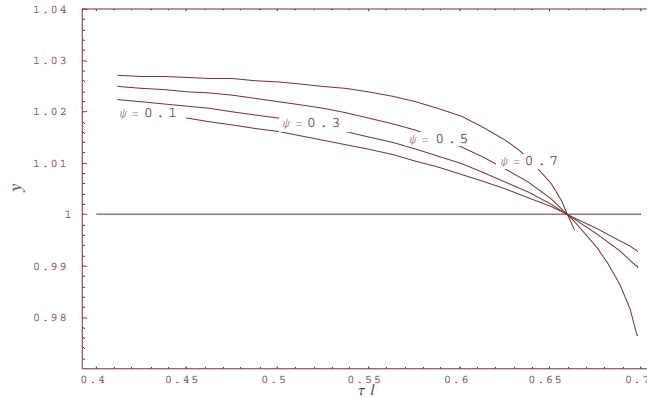
share in the informal sector varies in this simulation. It is not possible to simulate the model for the parametric value  $\psi = 0.9$ . In this case, we observe explosive behavior around the point  $\tau_l = 2/3$ . We can see that informality in the labor market always decreases as  $\tau_l$  falls and  $\tau_{va}$  adjusts so that both the government revenue  $\varkappa$  and total capital  $k$  remain constant.

**Graph 5 - Labor Share in the Informal Sector**



Graph 6 shows what happens to total output when the simulation is conducted. When total capital is fixed, we observe that total output always increases.

**Graph 6 - Total Output**

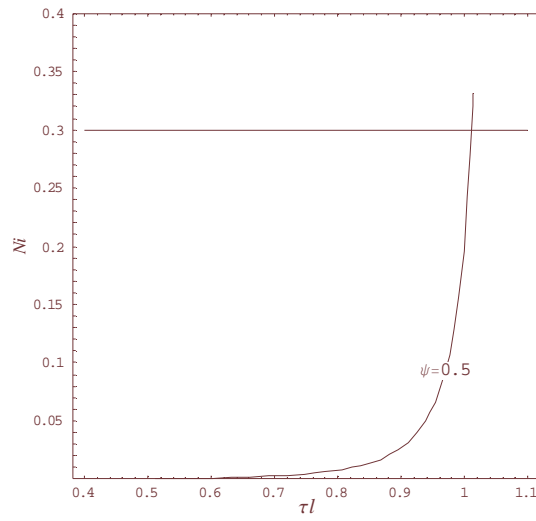


In the short run, we find a different result for the US economy. Total output always increases in the simulation, which is the opposite result to that obtained in the long run.

## 5.2 Brazilian Economy

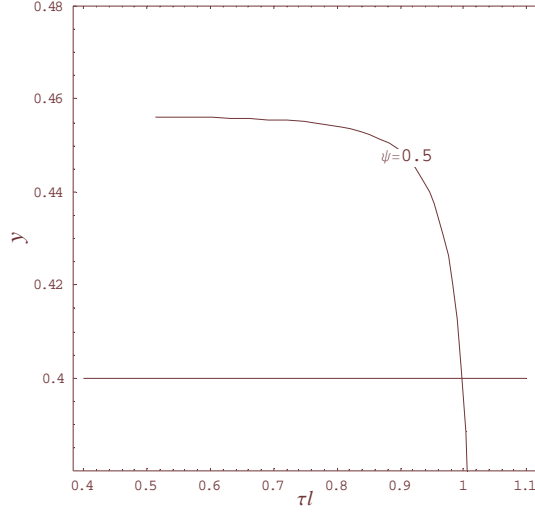
In this section, we conduct the same simulation for the Brazilian economy. It is possible to simulate the model only when  $\psi = 0.5$ . For the other parametric values, the model becomes explosive. Graph 7 shows the evolution of the labor share in the informal sector as  $\tau_l$  falls and  $\tau_{va}$  adjusts so that both the government revenue  $\varkappa$  and total capital  $k$  remain constant. We find the same pattern as that observed for the US economy.

Graph 7 - Labor Share in the Informal Sector



Graph 8 shows what happens to total output in the simulation. We observe that total output always increases, which is the same behavior as that observed for the US economy in the short run.

Graph 8 - Total Output



In the short run, we find the opposite result to that observed in the long run. This is true for both the US and Brazilian economies.

## 6 Welfare

### 6.1 Theoretical Analysis

In the last two sections, we evaluated the impact of our policy experiment on informality and total output. We found different results for the long-run and short-run simulations. In the short run, total output increases as  $\tau_l$  falls and  $\tau_{va}$  adjusts so that both the government revenue  $\varkappa$  and total capital  $k$  remain constant. In the long run, we observed just the opposite result when total capital is free to change. Therefore, a welfare analysis that takes into account the transition dynamics is needed. This is the aim of this section<sup>7</sup>.

The welfare in the model economy is given by

$$W = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

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<sup>7</sup>In this section, we follow Judd (1987).

The impact of  $\tau_l$  on  $W$  around the steady state is given by

$$\left. \frac{dW}{d\tau_l} \right|_* = \sum_{t=0}^{\infty} \beta^t u'(c^*) \left. \frac{dc_t}{d\tau_l} \right|_* = u'(c^*) \sum_{t=0}^{\infty} \beta^t \left. \frac{dc_t}{d\tau_l} \right|_* .$$

The equilibrium equations of the model are given by

$$\begin{cases} c_t + k_{t+1} - (1 - \delta) k_t = y_t \\ u'(c_t) = \beta (1 + r_{t+1} - \delta) u'(c_{t+1}) \end{cases}$$

where

$$\begin{aligned} y_t &= A k_{f,t}^{\alpha_f} (1 - N_{i,t}) + A B k_{i,t}^{\alpha_i \psi} N_{i,t}^{1 - \alpha_i (1 - \psi)} \\ k_{i,t} &= \frac{1 + \tau_k}{1 + \tau_l} \frac{\alpha_i}{\alpha_f} \frac{1 - \alpha_f}{1 - \alpha_i} \psi k_{f,t} \\ N_{i,t} &= \left( B \frac{1 + \tau_l}{1 - \tau_{va}} \frac{1 - \alpha_i}{1 - \alpha_f} k_{i,t}^{\psi \alpha_i} k_{f,t}^{-\alpha_f} \right)^{\frac{1}{(1 - \psi) \alpha_i}} \\ r_{t+1} &= \frac{1 - \tau_{va}}{1 + \tau_k} A \alpha_f k_{f,t+1}^{\alpha_f - 1} \end{aligned}$$

and

$$N_{i,t} k_{i,t} + (1 - N_{i,t}) k_{f,t} = k .$$

We can view both  $k_i$  and  $N_i$  as a function of  $\tau_{va}$ ,  $\tau_l$ , and  $k_f$  around the steady state. The equilibrium equation  $N_{i,t} k_{i,t} + (1 - N_{i,t}) k_{f,t} = k$  defines implicitly  $k_f$  as a function of  $k$ . Therefore, we can also view both  $y$  and  $r$  as a function of  $k$ , so that the system above can be rewritten as

$$\begin{cases} c_t + k_{t+1} - (1 - \delta) k_t = y_t(k_t, \tau_l) \\ u'(c_t) = \beta [1 + r_{t+1}(k_{t+1}, \tau_l) - \delta] u'(c_{t+1}) \end{cases} . \quad (30)$$

Let us derive this system with respect to  $\tau_l$  around the steady state.

$$\begin{cases} \left. \frac{dc_t}{d\tau_l} \right|_* + \left. \frac{dk_{t+1}}{d\tau_l} \right|_* = \left( \left. \frac{\partial y_t}{\partial k_t} \right|_* + 1 - \delta \right) \left. \frac{dk_t}{d\tau_l} \right|_* + \left. \frac{\partial y_t}{\partial \tau_l} \right|_* \\ \left. \frac{dc_t}{d\tau_l} \right|_* - \left. \frac{dc_{t+1}}{d\tau_l} \right|_* = \beta \frac{u'(c^*)}{u'(c^*)} \left. \frac{\partial r_{t+1}}{\partial k_{t+1}} \right|_* \left. \frac{dk_{t+1}}{d\tau_l} \right|_* + \beta \frac{u'(c^*)}{u'(c^*)} \left. \frac{\partial r_{t+1}}{\partial \tau_l} \right|_* \end{cases} \quad (31)$$

We define the following transforms

$$C_{\tau_l}(s) \equiv \sum_{t=0}^{\infty} s^t \left. \frac{dc_t}{d\tau_l} \right|_* \quad (32)$$

and

$$K_{\tau_l}(s) \equiv \sum_{t=0}^{\infty} s^t \left. \frac{dk_t}{d\tau_l} \right|_* \quad (33)$$

where  $0 < s < 1$ . Note that

$$\sum_{t=0}^{\infty} s^t \left. \frac{dc_{t+1}}{d\tau_l} \right|_* = \frac{1}{s} \left[ C_{\tau_l}(s) - \left. \frac{dc_0}{d\tau_l} \right|_* \right] \quad (34)$$

and

$$\sum_{t=0}^{\infty} s^t \left. \frac{dk_{t+1}}{d\tau_l} \right|_* = \frac{1}{s} K_{\tau_l}(s) \quad (35)$$

since  $\left. \frac{dk_0}{d\tau_l} \right|_* = 0$ .

If we multiply both sides of each equation in (31) by  $s^t$ , with  $0 < s < 1$ , and then sum from zero to infinity, we obtain the following system of equations:

$$\begin{pmatrix} \left. \frac{\partial y_t}{\partial k_t} \right|_* + 1 - \delta - \frac{1}{s} & -1 \\ -\beta \frac{u'(c^*)}{u(c^*)} \left. \frac{\partial r_{t+1}}{\partial k_{t+1}} \right|_* \frac{1}{s} & 1 - \frac{1}{s} \end{pmatrix} \begin{pmatrix} K_{\tau_l}(s) \\ C_{\tau_l}(s) \end{pmatrix} = \begin{pmatrix} -\frac{1}{1-s} \left. \frac{\partial y_t}{\partial \tau_l} \right|_* \\ lhs \end{pmatrix} \quad (36)$$

where

$$lhs = \frac{1}{1-s} \beta \frac{u'(c^*)}{u'(c^*)} \left. \frac{\partial r_{t+1}}{\partial \tau_l} \right|_* - \frac{1}{s} \left. \frac{dc_0}{d\tau_l} \right|_*.$$

We can solve this system for  $K_{\tau_l}(s)$  and  $C_{\tau_l}(s)$ , so that

$$K_{\tau_l}(s) = \frac{1}{\Delta} \left[ \frac{1}{1-s} \beta \frac{u'(c^*)}{u'(c^*)} \left. \frac{\partial r_{t+1}}{\partial \tau_l} \right|_* - \frac{1}{s} \left( \left. \frac{dc_0}{d\tau_l} \right|_* - \left. \frac{\partial y_t}{\partial \tau_l} \right|_* \right) \right] \quad (37)$$

and

$$C_{\tau_l}(s) = \frac{1}{\Delta} \left\{ \begin{aligned} & \left( \left. \frac{\partial y_t}{\partial k_t} \right|_* + 1 - \delta - \frac{1}{s} \right) \times \\ & \times \left[ \frac{1}{1-s} \beta \frac{u'(c^*)}{u'(c^*)} \left. \frac{\partial r_{t+1}}{\partial \tau_l} \right|_* - \frac{1}{s} \left. \frac{dc_0}{d\tau_l} \right|_* \right] + \\ & - \frac{1}{1-s} \left. \frac{\partial y_t}{\partial \tau_l} \right|_* \beta \frac{u'(c^*)}{u'(c^*)} \left. \frac{\partial r_{t+1}}{\partial k_{t+1}} \right|_* \frac{1}{s} \end{aligned} \right\} \quad (38)$$

where

$$\Delta \equiv \left( \left. \frac{\partial y_t}{\partial k_t} \right|_* + 1 - \delta - \frac{1}{s} \right) \left( 1 - \frac{1}{s} \right) - \beta \frac{u'(c^*)}{u'(c^*)} \left. \frac{\partial r_{t+1}}{\partial k_{t+1}} \right|_* \frac{1}{s}.$$

Let  $\mu$  be the unstable eigenvalue of the above matrix, that is,  $\mu > 1$ . From the second equation of the system, we have that

$$\mu \frac{dc_0}{d\tau_l} \Big|_* = \beta \frac{u'(c^*)}{u'(c^*)} \frac{\partial r_{t+1}}{\partial k_{t+1}} \Big|_* \mu K_{\tau_l} (\mu^{-1}) - (1 - \mu) C_{\tau_l} (\mu^{-1}) + \frac{\mu}{\mu - 1} \beta \frac{u'(c^*)}{u'(c^*)} \frac{\partial r_{t+1}}{\partial \tau_l} \Big|_*$$

since  $0 < \mu^{-1} < 1$ . The value  $\mu$  must also satisfy the following equation

$$\left( \frac{\partial y_t}{\partial k_t} \Big|_* + 1 - \delta - \mu \right) (1 - \mu) = \beta \frac{u'(c^*)}{u'(c^*)} \frac{\partial r_{t+1}}{\partial k_{t+1}} \Big|_* \mu .$$

Multiplying both sides of the above equation by  $K_{\tau_l} (\mu^{-1})$  and using the first equation of system, we find that

$$(1 - \mu) \left[ C_{\tau_l} (\mu^{-1}) - \frac{\mu}{\mu - 1} \frac{\partial y_t}{\partial \tau_l} \Big|_* \right] = \beta \frac{u'(c^*)}{u'(c^*)} \frac{\partial r_{t+1}}{\partial k_{t+1}} \Big|_* \mu K_{\tau_l} (\mu^{-1})$$

which means that the expression for the initial jump in the consumption can be simplified to

$$\frac{dc_0}{d\tau_l} \Big|_* = \frac{1}{\mu - 1} \beta \frac{u'(c^*)}{u'(c^*)} \frac{\partial r_{t+1}}{\partial \tau_l} \Big|_* + \frac{\partial y_t}{\partial \tau_l} \Big|_* .$$

We can rewrite the impact of  $\tau_l$  on welfare around the steady state as

$$\frac{dW}{d\tau_l} \Big|_* = u'(c^*) C_{\tau_l} (\beta) . \quad (39)$$

In order to find  $\frac{dW}{d\tau_l} \Big|_*$ , we need to compute the following derivatives:  $\frac{\partial r_{t+1}}{\partial \tau_l} \Big|_*$ ,  $\frac{\partial r_{t+1}}{\partial k_{t+1}} \Big|_*$ ,  $\frac{\partial y_t}{\partial \tau_l} \Big|_*$ , and  $\frac{\partial y_t}{\partial k_t} \Big|_*$ . Let us drop the time subscript in order to simplify the notation. The equilibrium equations can be rewritten as

$$k_i (\tau_l, \tau_{va}, k_f) = \frac{1 + \tau_k}{1 + \tau_l} \frac{\alpha_i}{\alpha_f} \frac{1 - \alpha_f}{1 - \alpha_i} \psi k_f ,$$

$$N_i (\tau_l, \tau_{va}, k_f) = \left\{ B \frac{1 + \tau_l}{1 - \tau_{va}} \frac{1 - \alpha_i}{1 - \alpha_f} [k_i (\tau_l, \tau_{va}, k_f)]^{\psi \alpha_i} k_f^{-\alpha_f} \right\}^{\frac{1}{(1 - \psi) \alpha_i}}$$

and

$$N_i (\tau_l, \tau_{va}, k_f) k_i (\tau_l, \tau_{va}, k_f) + [1 - N_i (\tau_l, \tau_{va}, k_f)] k_f = k .$$



The last equation defines  $k_f$  as an implicit function of  $\tau_l, \tau_{va}$ , and  $k$  around the steady state, that is,  $k_f = k_f(\tau_l, \tau_{va}, k)$  in a neighborhood of the steady state. By the Implicit Function Theorem, we find that

$$\left. \frac{\partial k_f}{\partial \tau_l} \right|_* = - \frac{\frac{\partial N_i}{\partial \tau_l} (k_i - k_f) + N_i \frac{\partial k_i}{\partial \tau_l}}{\frac{\partial N_i}{\partial k_f} (k_i - k_f) + 1 - N_i \left(1 - \frac{\partial k_i}{\partial k_f}\right)} \Big|_*, \quad (40)$$

$$\left. \frac{\partial k_f}{\partial \tau_{va}} \right|_* = - \frac{\frac{\partial N_i}{\partial \tau_{va}} (k_i - k_f) + N_i \frac{\partial k_i}{\partial \tau_{va}}}{\frac{\partial N_i}{\partial k_f} (k_i - k_f) + 1 - N_i \left(1 - \frac{\partial k_i}{\partial k_f}\right)} \Big|_* \quad (41)$$

and

$$\left. \frac{\partial k_f}{\partial k} \right|_* = \frac{1}{\frac{\partial N_i}{\partial k_f} (k_i - k_f) + 1 - N_i \left(1 - \frac{\partial k_i}{\partial k_f}\right)} \Big|_* \quad (42)$$

where  $k_f$  in steady state is given by (25).

The government budget constraint can be written as

$$\varkappa = [1 - N_i(\tau_l, \tau_{va}, k_f)] A k_f^{\alpha_f} \left[ \tau_{va} + \tau_l \frac{1 - \tau_{va}}{1 + \tau_l} (1 - \alpha_f) + \tau_k \frac{1 - \tau_{va}}{1 + \tau_k} \alpha_f \right]$$

where  $k_f = k_f(\tau_l, \tau_{va}, k)$ . In our policy analysis, we consider a small reduction in  $\tau_l$  and let  $\tau_{va}$  adjust so that the government revenue  $\varkappa$  remains constant. This means that the above equation defines  $\tau_{va}$  as an implicit function of  $\tau_l$  and  $k$  around the steady state, that is,  $\tau_{va} = \tau_{va}(\tau_l, k)$  in a neighborhood of the steady state. We define

$$\begin{aligned} H(\tau_l, \tau_{va}, k) &\equiv [1 - N_i(\tau_l, \tau_{va}, k_f)] k_f^{\alpha_f} A \times \\ &\times \left[ \tau_{va} + \tau_l \frac{1 - \tau_{va}}{1 + \tau_l} (1 - \alpha_f) + \tau_k \frac{1 - \tau_{va}}{1 + \tau_k} \alpha_f \right] - \varkappa \end{aligned}$$

where  $k_f = k_f(\tau_l, \tau_{va}, k)$  and compute its partial derivatives:

$$\begin{aligned} \frac{\partial H}{\partial \tau_l} &= -A k_f^{\alpha_f} \left[ \tau_{va} + \tau_l \frac{1 - \tau_{va}}{1 + \tau_l} (1 - \alpha_f) + \tau_k \frac{1 - \tau_{va}}{1 + \tau_k} \alpha_f \right] \times \\ &\times \left( \frac{\partial N_i}{\partial \tau_l} + \frac{\partial N_i}{\partial k_f} \frac{\partial k_f}{\partial \tau_l} \right) + \\ &+ (1 - N_i) A \alpha_f k_f^{\alpha_f - 1} \left[ \tau_{va} + \tau_l \frac{1 - \tau_{va}}{1 + \tau_l} (1 - \alpha_f) + \tau_k \frac{1 - \tau_{va}}{1 + \tau_k} \alpha_f \right] \times \\ &\times \frac{\partial k_f}{\partial \tau_l} + (1 - N_i) A k_f^{\alpha_f} \frac{(1 - \tau_{va})}{(1 + \tau_l)^2} (1 - \alpha_f) \end{aligned}$$

$$\begin{aligned}
\frac{\partial H}{\partial \tau_{va}} &= -Ak_f^{\alpha_f} \left[ \tau_{va} + \tau_l \frac{1 - \tau_{va}}{1 + \tau_l} (1 - \alpha_f) + \tau_k \frac{1 - \tau_{va}}{1 + \tau_k} \alpha_f \right] \times \\
&\quad \times \left( \frac{\partial N_i}{\partial \tau_{va}} + \frac{\partial N_i}{\partial k_f} \frac{\partial k_f}{\partial \tau_{va}} \right) + \\
&\quad + (1 - N_i) A \alpha_f k_f^{\alpha_f - 1} \left[ \tau_{va} + \tau_l \frac{1 - \tau_{va}}{1 + \tau_l} (1 - \alpha_f) + \tau_k \frac{1 - \tau_{va}}{1 + \tau_k} \alpha_f \right] \times \\
&\quad \times \frac{\partial k_f}{\partial \tau_{va}} + Ak_f^{\alpha_f} (1 - N_i) \times \\
&\quad \times \left( 1 - \tau_l \frac{1 - \alpha_f}{1 + \tau_l} - \tau_k \frac{\alpha_f}{1 + \tau_k} \right) \\
\frac{\partial H}{\partial k} &= \left[ \tau_{va} + \tau_l \frac{1 - \tau_{va}}{1 + \tau_l} (1 - \alpha_f) + \tau_k \frac{1 - \tau_{va}}{1 + \tau_k} \alpha_f \right] Ak_f^{\alpha_f} \frac{\partial k_f}{\partial k} \left[ (1 - N_i) \alpha_f k_f^{-1} - \frac{\partial N_i}{\partial k_f} \right]
\end{aligned}$$

By the Implicit Function Theorem, we find that

$$\left. \frac{\partial \tau_{va}}{\partial \tau_l} \right|_* = - \frac{\left. \frac{\partial H}{\partial \tau_l} \right|_*}{\left. \frac{\partial H}{\partial \tau_{va}} \right|_*} \quad (43)$$

and

$$\left. \frac{\partial \tau_{va}}{\partial k} \right|_* = - \frac{\left. \frac{\partial H}{\partial k} \right|_*}{\left. \frac{\partial H}{\partial \tau_{va}} \right|_*} \quad (44)$$

where  $k_f$  in steady state is given by (25).

We are now able to compute the derivatives  $\left. \frac{\partial r}{\partial k} \right|_*$ ,  $\left. \frac{\partial r}{\partial \tau_l} \right|_*$ ,  $\left. \frac{\partial y_t}{\partial \tau_l} \right|_*$ , and  $\left. \frac{\partial y}{\partial k} \right|_*$ . From the equilibrium equation (11), we find that

$$\begin{aligned}
\left. \frac{\partial r}{\partial k} \right|_* &= - \frac{1}{1 + \tau_k} \frac{\partial \tau_{va}}{\partial k} A \alpha_f k_f^{\alpha_f - 1} \Big|_* + \\
&\quad - \frac{1 - \tau_{va}}{1 + \tau_k} A \alpha_f (1 - \alpha_f) k_f^{\alpha_f - 2} \left( \frac{\partial k_f}{\partial k} + \frac{\partial k_f}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial k} \right) \Big|_* \quad (45)
\end{aligned}$$

and

$$\begin{aligned}
\left. \frac{\partial r}{\partial \tau_l} \right|_* &= - \frac{1}{1 + \tau_k} \frac{\partial \tau_{va}}{\partial \tau_l} A \alpha_f k_f^{\alpha_f - 1} \Big|_* + \\
&\quad - \frac{1 - \tau_{va}}{1 + \tau_k} A \alpha_f (1 - \alpha_f) k_f^{\alpha_f - 2} \left( \frac{\partial k_f}{\partial \tau_l} + \frac{\partial k_f}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial \tau_l} \right) \Big|_* \quad (46)
\end{aligned}$$

where  $k_f$  in steady state is given by (25).

To compute  $\left. \frac{dy}{dk} \right|_*$  and  $\left. \frac{\partial y}{\partial \tau_l} \right|_*$ , recall that total output is given by

$$\begin{aligned} y(\tau_l, \tau_{va}, k) &= Ak_f^{\alpha_f} [1 - N_i(\tau_l, \tau_{va}, k_f)] + \\ &\quad + AB [k_i(\tau_l, \tau_{va}, k_f)]^{\alpha_i \psi} [N_i(\tau_l, \tau_{va}, k_f)]^{1-\alpha_i(1-\psi)} \end{aligned}$$

where  $k_f = k_f(\tau_l, \tau_{va}, k)$  is the implicit function defined above. Then, the derivatives of  $y$  with respect to  $k$  and  $\tau_l$  evaluated in steady state are given by

$$\begin{aligned} \left. \frac{\partial y}{\partial k} \right|_* &= A\alpha_f k_f^{\alpha_f-1} (1 - N_i) \left( \frac{\partial k_f}{\partial k} + \frac{\partial k_f}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial k} \right) \Big|_* + \\ &\quad - Ak_f^{\alpha_f} \left[ \frac{\partial N_i}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial k} + \frac{\partial N_i}{\partial k_f} \left( \frac{\partial k_f}{\partial k} + \frac{\partial k_f}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial k} \right) \right] \Big|_* + \\ &\quad + AB\alpha_i \psi k_i^{\alpha_i \psi-1} N_i^{1-\alpha_i(1-\psi)} \times \\ &\quad \times \left[ \frac{\partial k_i}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial k} + \frac{\partial k_i}{\partial k_f} \left( \frac{\partial k_f}{\partial k} + \frac{\partial k_f}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial k} \right) \right] \Big|_* + \\ &\quad + AB [1 - \alpha_i (1 - \psi)] k_i^{\alpha_i \psi} N_i^{-\alpha_i(1-\psi)} \times \\ &\quad \times \left[ \frac{\partial N_i}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial k} + \frac{\partial N_i}{\partial k_f} \left( \frac{\partial k_f}{\partial k} + \frac{\partial k_f}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial k} \right) \right] \Big|_* \end{aligned} \quad (47)$$

and

$$\begin{aligned} \left. \frac{\partial y}{\partial \tau_l} \right|_* &= A\alpha_f k_f^{\alpha_f-1} (1 - N_i) \left( \frac{\partial k_f}{\partial \tau_l} + \frac{\partial k_f}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial \tau_l} \right) \Big|_* + \\ &\quad - Ak_f^{\alpha_f} \left[ \frac{\partial N_i}{\partial \tau_l} + \frac{\partial N_i}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial \tau_l} + \frac{\partial N_i}{\partial k_f} \left( \frac{\partial k_f}{\partial \tau_l} + \frac{\partial k_f}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial \tau_l} \right) \right] \Big|_* + \\ &\quad + AB\alpha_i \psi k_i^{\alpha_i \psi-1} N_i^{1-\alpha_i(1-\psi)} \times \\ &\quad \times \left[ \frac{\partial k_i}{\partial \tau_l} + \frac{\partial k_i}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial \tau_l} + \frac{\partial k_i}{\partial k_f} \left( \frac{\partial k_f}{\partial \tau_l} + \frac{\partial k_f}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial \tau_l} \right) \right] \Big|_* + \\ &\quad + AB [1 - \alpha_i (1 - \psi)] k_i^{\alpha_i \psi} N_i^{-\alpha_i(1-\psi)} \times \\ &\quad \times \left[ \frac{\partial N_i}{\partial \tau_l} + \frac{\partial N_i}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial \tau_l} + \frac{\partial N_i}{\partial k_f} \left( \frac{\partial k_f}{\partial \tau_l} + \frac{\partial k_f}{\partial \tau_{va}} \frac{\partial \tau_{va}}{\partial \tau_l} \right) \right] \Big|_* \end{aligned} \quad (48)$$

where  $k_f$  in steady state is given by (25).

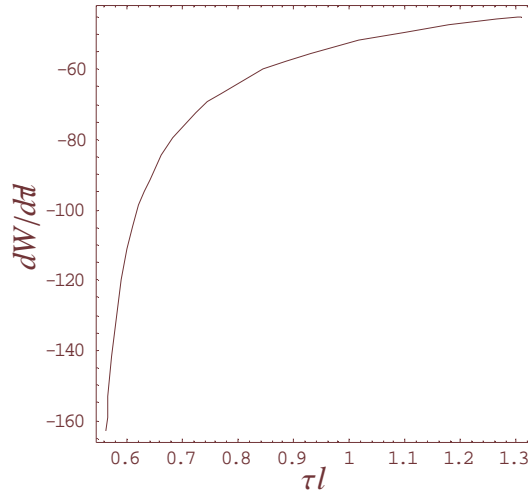
## 6.2 Quantitative Results

In this subsection we compute the value of  $\left. \frac{dW}{d\tau_l} \right|_*$  for different pairs of  $(\tau_l, \tau_{va})$ . These pairs are obtained in the same way as before: we let  $\tau_l$  vary and adjust  $\tau_{va}$  so that the government revenue remains constant. Note that each pair  $(\tau_l, \tau_{va})$  defines a different steady state. The only common feature among these steady states is that the government revenue is the same for all of them.

### 6.2.1 US Economy

Graph 9 shows the value of  $\left. \frac{dW}{d\tau_l} \right|_*$  for different pairs of  $(\tau_l, \tau_{va})$  when  $\psi = 0.1$ <sup>8</sup>. The welfare always decreases as  $\tau_l$  rises. This is true for each steady state defined by the pair  $(\tau_l, \tau_{va})$ .

Graph 9 - Welfare Analysis: US Economy ( $\psi=0.1$ )



In Appendix E, we see that when  $\psi = 0.9$  the derivative  $\left. \frac{dW}{d\tau_l} \right|_*$  becomes positive, indicating that a rise in  $\tau_l$  increases welfare. The value  $\psi = 0.9$  indicates that capital has a relatively higher importance in the technology of the informal sector than the nonreproducible factor. For the other values of  $\psi$ , we find a negative value for  $\left. \frac{dW}{d\tau_l} \right|_*$  for each pair  $(\tau_l, \tau_{va})$ . We conclude that a policy that reduces the tax rate on the demand for labor and rises the tax

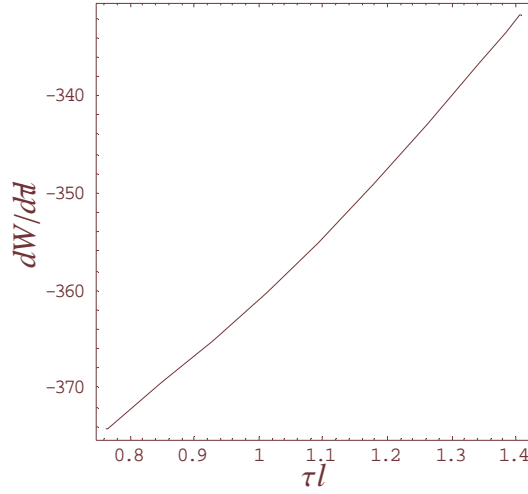
<sup>8</sup>In Appendix E, we show the results of the simulations for the other values of  $\psi$ .

rate on the value added keeping the government revenue constant increases the welfare.

### 6.2.2 Brazilian Economy

Graph 10 shows the value of  $\left. \frac{dW}{d\tau_l} \right|_*$  for different pairs of  $(\tau_l, \tau_{va})$  when  $\psi = 0.1$ . The welfare always decreases as  $\tau_l$  rises. This is true for each steady state defined by the pair  $(\tau_l, \tau_{va})$ .

Graph 10 - Welfare Analysis: Brazilian Economy ( $\psi=0.1$ )



The same conclusion that we have obtained for the US economy holds for the Brazilian economy: it is desirable to reduce the tax rate on the demand for labor even if it is necessary to increase the tax rate on the value added in order to keep the government revenue constant.

## 7 Conclusion

In the long run, we have seen that our policy experiment of reducing  $\tau_l$  and adjusting  $\tau_{va}$  so that the government revenue remains constant reduces total output regardless of the value of the parameter  $\psi$ . Informality in the labor market may initially increase for relatively high levels of  $\tau_l$ . But for lower initial levels of  $\tau_l$ , we observe a reduction in informality. In the short

run, when total capital is fixed, we observe that informality always decreases and total output always increases. This result is true both for the US and Brazilian economies.

Although we have had opposite results for the long and short run equilibrium, an analysis that takes into account transition dynamics indicates that welfare always increases when we reduce  $\tau_l$ . Therefore, a public policy that reduces the tax rate on the demand for labor and rises the tax rate on the value added so that the government revenue remains constant increases welfare.

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## Appendix A - The Informal Sector is Always Operated

**Proposition 1** *Even when taxes are zero, that is,  $\tau_{va} = \tau_l = \tau_k = 0$ , the informal sector is operated ( $Y_i > 0$ ).*

**Proof.** The cost function for a firm in the informal sector is defined as the value function for the following minimization problem:

$$\begin{aligned} \min_{K_i, N_i} \{ & rK_i + wN_i + q \} \\ \text{s.t. } & Y_i = ABK_i^{\psi\alpha_i} N_i^{1-\alpha_i} \end{aligned}$$

The Lagrangian for this problem is given by

$$L(K_i, N_i, \lambda) = rK_i + wN_i + q - \lambda \left( ABK_i^{\psi\alpha_i} N_i^{1-\alpha_i} - Y_i \right)$$

and the first order conditions for an interior solution are given by

$$\begin{aligned} \frac{\partial L}{\partial K_i} &= r - \lambda \psi \alpha_i ABK_i^{\psi\alpha_i-1} N_i^{1-\alpha_i} = 0 \\ \frac{\partial L}{\partial N_i} &= w - \lambda (1 - \alpha_i) ABK_i^{\psi\alpha_i} N_i^{-\alpha_i} = 0 \\ \frac{\partial L}{\partial \lambda} &= - \left( ABK_i^{\psi\alpha_i} N_i^{1-\alpha_i} - Y_i \right) = 0 \end{aligned}$$

It follows that

$$\frac{K_i}{N_i} = \frac{w}{r} \frac{\psi \alpha_i}{(1 - \alpha_i)}$$

and the conditional factor demand functions are given by

$$N_i(w, r, Y_i) = \left[ \frac{w}{r} \frac{\psi \alpha_i}{(1 - \alpha_i)} \right]^{-\frac{\psi \alpha_i}{1 - \alpha_i(1 - \psi)}} (AB)^{-\frac{1}{1 - \alpha_i(1 - \psi)}} Y_i^{\frac{1}{1 - \alpha_i(1 - \psi)}}$$

and

$$K_i(w, r, Y_i) = \left[ \frac{w}{r} \frac{\psi \alpha_i}{1 - \alpha_i} \right]^{\frac{1 - \alpha_i}{1 - \alpha_i(1 - \psi)}} (AB)^{-\frac{1}{1 - \alpha_i(1 - \psi)}} Y_i^{\frac{1}{1 - \alpha_i(1 - \psi)}}$$

This leads to the following cost function:

$$\begin{aligned} c_i(w, r, Y_i) &= w^{\frac{1 - \alpha_i}{1 - \alpha_i(1 - \psi)}} r^{\frac{\psi \alpha_i}{1 - \alpha_i(1 - \psi)}} \times \\ &\times \left[ \left( \frac{\psi \alpha_i}{1 - \alpha_i} \right)^{-\frac{\psi \alpha_i}{1 - \alpha_i(1 - \psi)}} + \left( \frac{\psi \alpha_i}{1 - \alpha_i} \right)^{\frac{1 - \alpha_i}{1 - \alpha_i(1 - \psi)}} \right] \times \\ &\times (AB)^{-\frac{1}{1 - \alpha_i(1 - \psi)}} Y_i^{\frac{1}{1 - \alpha_i(1 - \psi)}} + q \end{aligned}$$

Define

$$a_i \equiv w^{\frac{1 - \alpha_i}{1 - \alpha_i(1 - \psi)}} r^{\frac{\psi \alpha_i}{1 - \alpha_i(1 - \psi)}} \left[ \left( \frac{\psi \alpha_i}{1 - \alpha_i} \right)^{-\frac{\psi \alpha_i}{1 - \alpha_i(1 - \psi)}} + \left( \frac{\psi \alpha_i}{1 - \alpha_i} \right)^{\frac{1 - \alpha_i}{1 - \alpha_i(1 - \psi)}} \right] (AB)^{-\frac{1}{1 - \alpha_i(1 - \psi)}}.$$

Then, the cost function can be rewritten as

$$c_i(w, r, Y_i) = a_i Y_i^{\frac{1}{1 - \alpha_i(1 - \psi)}} + q \quad (49)$$

It follows that the marginal cost function is given by

$$cmg_i(Y_i) \equiv \frac{\partial c_i(w, r, Y_i)}{\partial Y_i} = \left[ \frac{1}{1 - \alpha_i(1 - \psi)} \right] a_i Y_i^{\frac{\alpha_i(1 - \psi)}{1 - \alpha_i(1 - \psi)}} \quad (50)$$

Let us find the cost function for a firm in the formal sector.

$$\begin{aligned} &\min_{K_i, N_i} \{rK_f + wN_f\} \\ &s.t. \ Y_f = AK_f^{\alpha_f} N_f^{1 - \alpha_f} \end{aligned}$$

The Lagrangian for this minimization problem is given by

$$L(K_i, N_i, \lambda) = rK_f + wN_f - \lambda \left( AK_f^{\alpha_f} N_f^{1 - \alpha_f} - Y_f \right)$$



The first order conditions for an interior solution are given by

$$\begin{aligned}\frac{\partial L}{\partial K_f} &= r - \lambda \alpha_f A K_f^{\alpha_f - 1} N_f^{1 - \alpha_f} = 0 \\ \frac{\partial L}{\partial N_f} &= w - \lambda (1 - \alpha_f) A K_f^{\alpha_f} N_f^{-\alpha_f} = 0 \\ \frac{\partial L}{\partial \lambda} &= - \left( A K_f^{\alpha_f} N_f^{1 - \alpha_f} - Y_f \right) = 0\end{aligned}$$

It follows that

$$\frac{K_f}{N_f} = \frac{w}{r} \frac{\alpha_f}{(1 - \alpha_f)}$$

and the conditional factor demand functions are given by

$$N_f(w, r, Y_f) = A^{-1} \left( \frac{w}{r} \right)^{-\alpha_f} \left( \frac{\alpha_f}{1 - \alpha_f} \right)^{-\alpha_f} Y_f$$

and

$$K_f(w, r, Y_f) = A^{-1} \left( \frac{w}{r} \right)^{1 - \alpha_f} \left( \frac{\alpha_f}{1 - \alpha_f} \right)^{1 - \alpha_f} Y_f$$

This leads to the following cost function:

$$c_f(w, r, Y_f) = A^{-1} w^{1 - \alpha_f} r^{\alpha_f} \left[ \left( \frac{\alpha_f}{1 - \alpha_f} \right)^{-\alpha_f} + \left( \frac{\alpha_f}{1 - \alpha_f} \right)^{1 - \alpha_f} \right] Y_f$$

Define  $a_f \equiv A^{-1} w^{1 - \alpha_f} r^{\alpha_f} \left[ \left( \frac{\alpha_f}{1 - \alpha_f} \right)^{-\alpha_f} + \left( \frac{\alpha_f}{1 - \alpha_f} \right)^{1 - \alpha_f} \right]$ , so that the cost function for a firm in the formal sector can be rewritten as

$$c_f(w, r, Y_f) = a_f Y_f \tag{51}$$

Hence, a firm in this sector faces a (positive) constant marginal cost:  $cmg_f(Y_f) \equiv \frac{\partial c_f(w, r, Y_f)}{\partial Y_f} = a_f > 0$ . It follows that there is a value  $\bar{Y}_i$  such that  $cmg_i(Y_i) < a_f$  for all  $Y_i \in (0, \bar{Y}_i)$  since  $cmg_i(Y_i)$  is strictly increasing and continuous at  $(0, \infty)$ , and  $\lim_{Y_i \rightarrow 0} cmg_i(Y_i) = 0$ . Therefore, it is profitable to produce some initial units in the informal sector. ■

## Appendix B - Equivalence Between Taxation on the Demand for Factors and Taxation on the Supply of Factors

Let us show the equivalence between taxation on the demand for capital and labor and taxation on the supply of capital and labor (that is, on the income from the capital and labor services supplied). Consider the same model with two sectors in production, but in which taxation occurs on the income from capital and labor services supplied to the formal sector of the economy. The household's problem can be written in the following way:

$$\begin{aligned} & \max_{\{c_t, k_{i,t+1}, k_{f,t+1}, n_{i,t}, n_{f,t}, L_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \\ & s.t \left\{ \begin{array}{l} c_t + k_{i,t+1} + k_{f,t+1} = (1 - \tau_l^s) w_{f,t} n_{f,t} + w_{i,t} n_{i,t} + \\ (1 - \tau_k^s) r_{f,t} k_{f,t} + r_{i,t} k_{i,t} + (1 - \delta) (k_{i,t} + k_{f,t}) + q_t L_t + \varkappa_t \\ k_{i,t} + k_{f,t} = k_t \\ n_{i,t} + n_{f,t} = 1 \\ c_t, k_{i,t+1}, k_{f,t+1}, n_{i,t}, n_{f,t} \geq 0 \text{ and } 0 \leq L_t \leq 1 \text{ for all } t \\ k_0 > 0 \text{ is given} \end{array} \right. \end{aligned}$$

The Bellman equation for this problem is

$$v(k) = \left\{ \begin{array}{l} \max_{c, h, n_f, k_f} \log c + \beta v(h) \\ s.t \left\{ \begin{array}{l} c + h = (1 - \tau_l^s) w_f n_f + w_i (1 - n_f) + \\ (1 - \tau_k^s) r_f k_f + r_i (k - k_f) + (1 - \delta) k + q + \chi \\ c \geq 0, h \geq 0, 0 \leq k_f \leq k, \text{ and } 0 \leq n_f \leq 1 \end{array} \right. \end{array} \right\}.$$

The first order conditions and the Envelope condition are given by

$$\begin{aligned} \frac{1}{c(k)} &= \beta v'(g(k)) \\ \frac{1}{c(k)} [(1 - \tau_k^s) r_f - r_i] &= 0 \\ \frac{1}{c(k)} [(1 - \tau_l^s) w_f - w_i] &= 0 \\ v'(k) &= \frac{1}{c(k)} [(1 - \delta) + r_i] \end{aligned}$$

The Euler equation

$$\frac{1}{c(k)} = \beta \frac{1}{c(g(k))} [(1 - \delta) + r'_i]$$

and the conditions  $(1 - \tau_k^s) r_f = r_i$  and  $(1 - \tau_l^s) w_f = w_i$  must be satisfied along the optimal path.

From the profit maximization problem of the representative firm in the formal sector, it follows that

$$w_f = A(1 - \alpha_f) K_f^{\alpha_f} N_f^{-\alpha_f}$$

and

$$r_f = A\alpha_f K_f^{\alpha_f-1} N_f^{1-\alpha_f} .$$

From the profit maximization problem of the representative firm in the informal sector, it follows that

$$w_i = AB(1 - \alpha_i) \left( K_i^\psi L^{1-\psi} \right)^{\alpha_i} N_i^{-\alpha_i}$$

$$r_i = AB\alpha_i \psi \left( K_i^\psi L^{1-\psi} \right)^{\alpha_i} N_i^{1-\alpha_i} K_i^{-1}$$

and

$$q = AB\alpha_i(1 - \psi) \left( K_i^\psi L^{1-\psi} \right)^{\alpha_i} N_i^{1-\alpha_i} L^{-1} .$$

In equilibrium, we have the following conditions:

$$AB(1 - \alpha_i) \left( K_i^\psi L^{1-\psi} \right)^{\alpha_i} N_i^{-\alpha_i} = (1 - \tau_l^s) A(1 - \alpha_f) K_f^{\alpha_f} N_f^{-\alpha_f}$$

and

$$AB\alpha_i \psi \left( K_i^\psi L^{1-\psi} \right)^{\alpha_i} N_i^{1-\alpha_i} K_i^{-1} = (1 - \tau_k^s) A\alpha_f K_f^{\alpha_f-1} N_f^{1-\alpha_f}$$

Therefore, it is possible to derive the following relation between the capital-to-labor ratio in each sector:

$$\frac{1 - \alpha_i}{\alpha_i \psi} \frac{K_i}{N_i} = \frac{1 - \tau_l^s}{1 - \tau_k^s} \frac{1 - \alpha_f}{\alpha_f} \frac{K_f}{N_f} \quad (52)$$

If we set  $1 - \tau_k^s = \frac{1}{1 + \tau_k}$  and  $1 - \tau_l^s = \frac{1}{1 + \tau_l}$ , then we obtain equation (18). Therefore, there is an equivalence between taxation on the demand for factors and taxation on the supply of factors.

## Appendix C - Long Run Analysis

We can view the capital-to-labor ratio in the long run in each sector as a function of  $\tau_{va}$  and  $\tau_l$ . Hence, we can define  $k_f : (0, 1) \times (0, \infty) \rightarrow R$  as

$$k_f(\tau_{va}, \tau_l) = \left[ A \frac{1 - \tau_{va}}{1 + \tau_k} \alpha_f \frac{\beta}{1 - (1 - \delta) \beta} \right]^{\frac{1}{1 - \alpha_f}}$$

and  $k_i : (0, 1) \times (0, \infty) \rightarrow R$  as

$$k_i(\tau_{va}, \tau_l) = \frac{1 + \tau_k}{1 + \tau_l} \frac{\alpha_i}{\alpha_f} \frac{1 - \alpha_f}{1 - \alpha_i} \psi \left[ \frac{1 - \tau_{va}}{1 + \tau_k} A \alpha_f \frac{\beta}{1 - (1 - \delta) \beta} \right]^{\frac{1}{1 - \alpha_f}}.$$

Let  $r = \frac{1}{\beta} - (1 - \delta)$ . Then, we have the following results

$$\frac{\partial k_f}{\partial \tau_l} = 0$$

$$\frac{\partial k_f}{\partial \tau_{va}} = -\frac{\alpha_f}{1 - \alpha_f} \frac{1}{1 + \tau_k} \frac{1}{r} A k_f^{\alpha_f} < 0$$

$$\frac{\partial k_i}{\partial \tau_l} = -\frac{1}{1 + \tau_l} k_i < 0$$

$$\frac{\partial k_i}{\partial \tau_{va}} = -\frac{k_i}{k_f} \frac{\alpha_f}{1 - \alpha_f} \frac{1}{1 + \tau_k} \frac{1}{r} A k_f^{\alpha_f} < 0$$

for any  $(\tau_{va}, \tau_l) \in (0, 1) \times (0, \infty)$ . We can also define  $N_i$  as a function of  $\tau_{va}$  and  $\tau_l$ , so that the function  $N_i : (0, 1) \times (0, \infty) \rightarrow R$  is given by

$$N_i(\tau_{va}, \tau_l) = \left( \frac{1 + \tau_l}{1 - \tau_{va}} B \frac{1 - \alpha_i}{1 - \alpha_f} k_i^{\psi \alpha_i} k_f^{-\alpha_f} \right)^{\frac{1}{(1 - \psi) \alpha_i}}$$

where  $k_f = k_f(\tau_{va}, \tau_l)$  and  $k_i = k_i(\tau_{va}, \tau_l)$ . We can also find the partial derivatives of  $N_i$ .

$$\begin{aligned} \frac{\partial N_i}{\partial \tau_{va}} &= \frac{1}{(1 - \psi) \alpha_i} \left( B \frac{1 + \tau_l}{1 - \tau_{va}} \frac{1 - \alpha_i}{1 - \alpha_f} k_i^{\psi \alpha_i} k_f^{-\alpha_f} \right)^{\frac{1}{(1 - \psi) \alpha_i}} \times \\ &\times \left( \frac{1}{1 - \tau_{va}} + \psi \alpha_i k_i^{-1} \frac{\partial k_i}{\partial \tau_{va}} + \right. \\ &\quad \left. - \alpha_f k_f^{-1} \frac{\partial k_f}{\partial \tau_{va}} \right) \end{aligned}$$

Since the term  $\frac{1}{(1-\psi)\alpha_i} \left( \frac{1+\tau_l}{1-\tau_{va}} B \frac{1-\alpha_i}{1-\alpha_f} k_i^{\psi\alpha_i} k_f^{-\alpha_f} \right)^{\frac{1}{(1-\psi)\alpha_i}}$  is strictly positive, it follows that the term in brackets on the left-hand side determines the sign of the derivative  $\frac{\partial N_i}{\partial \tau_{va}}$ . Note that we can rewrite this term in the following way:

$$\frac{1}{1-\tau_{va}} + \psi\alpha_i k_i^{-1} \frac{\partial k_i}{\partial \tau_{va}} - \alpha_f k_f^{-1} \frac{\partial k_f}{\partial \tau_{va}} = \frac{1}{1-\tau_{va}} + \frac{\alpha_f}{1-\alpha_f} \frac{1}{1+\tau_k} \frac{A}{r} k_f^{\alpha_f-1} (\alpha_f - \psi\alpha_i)$$

If  $\alpha_f - \psi\alpha_i > 0$ , then the derivative  $\frac{\partial N_i}{\partial \tau_{va}}$  is strictly positive. Note that this condition corresponds to the assumption that the formal sector is more capital intensive than the informal sector.

$$\begin{aligned} \frac{\partial N_i}{\partial \tau_l} &= \frac{1}{(1-\psi)\alpha_i} \left( B \frac{1+\tau_l}{1-\tau_{va}} \frac{1-\alpha_i}{1-\alpha_f} k_i^{\psi\alpha_i} k_f^{-\alpha_f} \right)^{\frac{1-\alpha_i(1-\psi)}{(1-\psi)\alpha_i}} \times \\ &\times B \frac{1}{1-\tau_{va}} \left( \frac{1-\alpha_i}{1-\alpha_f} \right) k_i^{\psi\alpha_i} k_f^{-\alpha_f} \left[ 1 + \psi\alpha_i (1 + \tau_l) k_i^{-1} \frac{\partial k_i}{\partial \tau_l} \right] \end{aligned}$$

It follows that

$$1 + \psi\alpha_i (1 + \tau_l) k_i^{-1} \frac{\partial k_i}{\partial \tau_l} = 1 - \psi\alpha_i .$$

Since  $0 < \psi\alpha_i < 1$ , we conclude that  $\frac{\partial N_i}{\partial \tau_l} > 0$ . Therefore,  $N_i$  is strictly increasing in  $\tau_{va}$  and  $\tau_l$ . Obviously, the function  $N_f : (0, 1) \times (0, \infty) \rightarrow R$  defined as

$$N_f(\tau_{va}, \tau_l) = 1 - N_i(\tau_{va}, \tau_l)$$

is strictly decreasing in  $\tau_{va}$  and  $\tau_l$ .

At last, we can also view the capital allocated to each sector as a function of  $\tau_{va}$  and  $\tau_l$ . We define the functions  $K_f : (0, 1) \times (0, \infty) \rightarrow R$  and  $K_i : (0, 1) \times (0, \infty) \rightarrow R$  as

$$K_f(\tau_{va}, \tau_l) = N_f(\tau_{va}, \tau_l) k_f(\tau_{va}, \tau_l)$$

and

$$K_i(\tau_{va}, \tau_l) = N_i(\tau_{va}, \tau_l) k_i(\tau_{va}, \tau_l)$$

respectively. Their partial derivatives are given by

$$\frac{\partial K_f}{\partial \tau_l} = k_f \frac{\partial N_f}{\partial \tau_l} < 0$$

$$\frac{\partial K_f}{\partial \tau_{va}} = N_f \frac{\partial k_f}{\partial \tau_{va}} + k_f \frac{\partial N_f}{\partial \tau_{va}} < 0$$

$$\frac{\partial K_i}{\partial \tau_l} = N_i \frac{\partial k_i}{\partial \tau_l} + k_i \frac{\partial N_i}{\partial \tau_l}$$

and

$$\frac{\partial K_i}{\partial \tau_{va}} = N_i \frac{\partial k_i}{\partial \tau_{va}} + k_i \frac{\partial N_i}{\partial \tau_{va}}$$

Note that the signals of the derivatives  $\frac{\partial K_i}{\partial \tau_l}$  and  $\frac{\partial K_i}{\partial \tau_{va}}$  are ambiguous.

## Appendix D - Total Output of the Brazilian Economy is 0.4 of that of the US Economy

Let  $K_{us}$  denote the aggregate capital stock of the US economy. Consider a technology that takes into account both human and physical capital. Let  $h_{us}$  denote the average years of schooling in the US. Total output is given by

$$Y_{us} = [A \exp(\phi h_{us}) L_{us}]^{1-\alpha} K_{us}^\alpha$$

where  $A$  is the effectiveness of labor and  $L_{us}$  is raw labor. Define

$$y_{us} \equiv \frac{Y_{us}}{L_{us}} .$$

We have that

$$\frac{y_{br} \exp(\phi h_{br})}{y_{us} \exp(\phi h_{us})} = 0.2 .$$

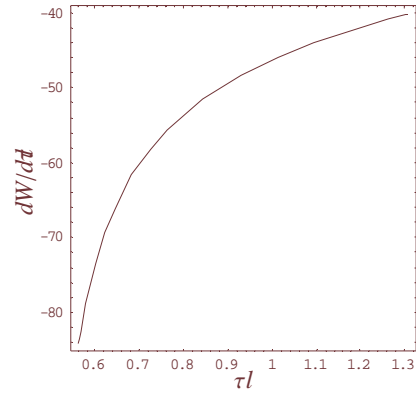
Since  $\exp[\phi(h_{us} - h_{br})]$  is about 2, we find that the ratio  $y_{br}/y_{us}$  is about 0.4.

## Appendix E - Welfare Simulation for $\psi = 0.3$ , $\psi = 0.5$ , $\psi = 0.7$ , $\psi = 0.9$ , and $\psi = 0.95$

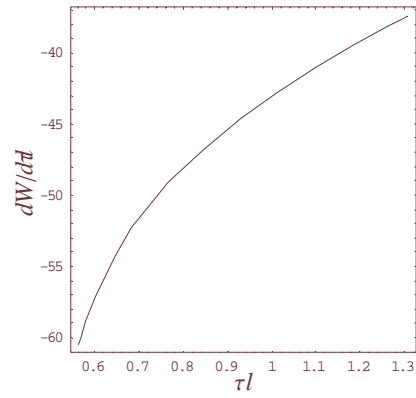
We show the value of  $\left. \frac{dW}{d\tau_l} \right|_*$  for the other values of the parameter  $\psi$ . Graphs 11, 12, 13, 14, and 15 refer to the US economy. Graphs 16, 17, 18,

19, and 20 refer to the Brazilian economy.

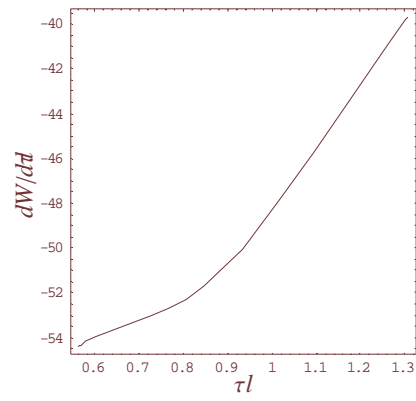
Graph 11 - Welfare Analysis: US Economy ( $\psi=0.3$ )



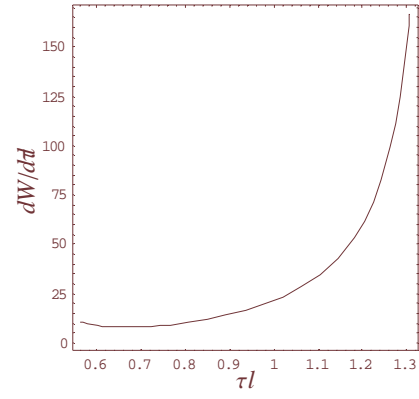
Graph 12 - Welfare Analysis: US Economy ( $\psi=0.5$ )



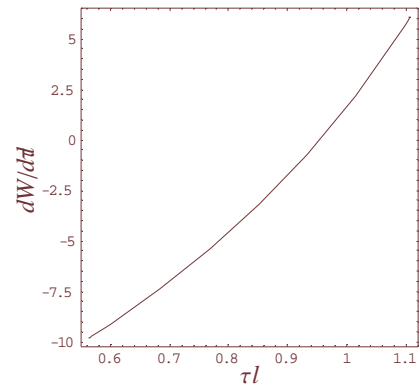
Graph 13 - Welfare Analysis: US Economy ( $\psi=0.7$ )



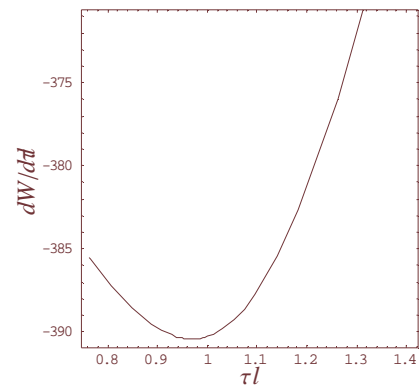
Graph 14 - Welfare Analysis: US Economy ( $\psi=0.9$ )



Graph 15 - Welfare Analysis: US Economy ( $\psi=0.95$ )

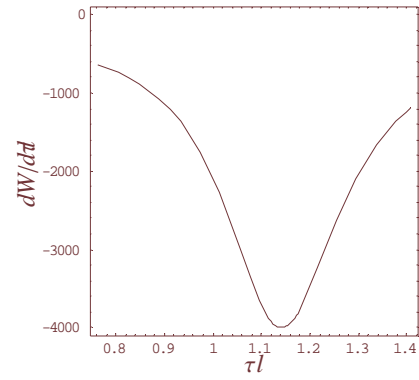


Graph 16 - Welfare Analysis: Brazilian Economy ( $\psi=0.3$ )

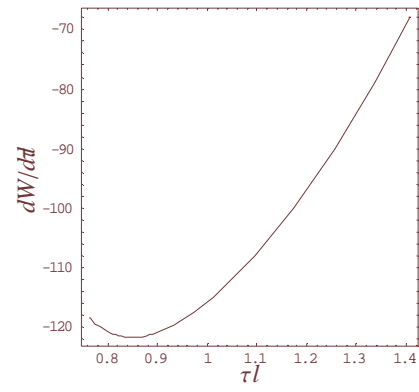




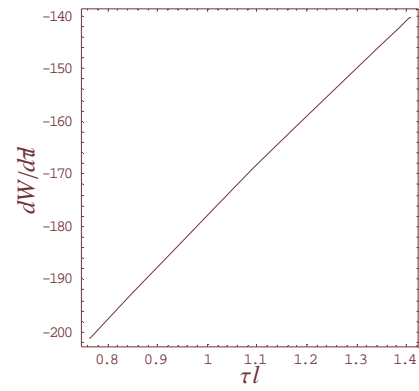
Graph 17 - Welfare Analysis: Brazilian Economy ( $\psi=0.5$ )



Graph 18 - Welfare Analysis: Brazilian Economy ( $\psi=0.7$ )



Graph 19 - Welfare Analysis: Brazilian Economy ( $\psi=0.9$ )



Graph 20 - Welfare Analysis: Brazilian Economy ( $\psi=0.95$ )

