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# Uncertainty and Countervailing Incentives in Procurement

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Dissertação submetida a Escola de Pós-Graduação em Economia como requisito parcial para a obtenção do grau de Mestre em Economia.

Orientador: Humberto Luiz Ataíde Moreira

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## Abstract

This thesis develops a simple model to represent a procurement situation with two main features. The first is that the optimal level of production cannot be fully anticipated when suppliers build their plants due to demand shocks. The second is that producers competing for a supply contract typically have different technologies within an efficient frontier, characterized by a trade-off between the marginal cost of production and the fixed cost per unit of capacity. With this framework in mind, we investigate how the shape of the frontier and the distribution of shocks affect efficient technology choices when the planner knows firms' technologies (first-best) and when she doesn't (second-best). In addition, we characterize how and when a well established real-life mechanism such as a quasi-linear score auction may implement second-best social welfare. We find that, if there is a strict preference over technologies in first-best, a quasi-linear score auction may implement second-best allocations. However, there is a non-neglectable case in which countervailing incentives arise, i.e. firms' allocations may be distorted either upwards or downwards with respect to first-best depending on their technologies. In that case, the planner may optimally choose to hire more than one firm, and there is no quasi-linear score auction that provides the social welfare achieved in second-best.

KEYWORDS: *Procurement, Countervailing Incentives, Score auctions, Second-Best, First-Best.*

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# 1 Introduction

In some markets demand for a good may either spike or drop due to exogenous shocks, while it is infeasible or too costly to adjust productive capacity accordingly, at least in the short term. One can think of natural phenomena as possible causes, for instance, as heat waves are often followed by an excess demand for air-conditioners and earthquakes usually motivate an excess supply of hotel vacancies in affected areas. Besides, preferences may simply shift for some reason, e.g. if people are suddenly more concerned about animals and their well-being, one might expect leather and fur products to accumulate on stock during the following months. In this sense, the broad question we pose in this paper is how these frictions affect outcomes in procurement situations in which firms with different technologies compete.

This question is motivated by a situation in Brazilian electricity market. In Brazil, the predominant source of electric power is hydroelectric, which is an intermittent source. Due due to environmental concerns, reservoirs cannot be large enough to guarantee a sufficient provision for long dry periods. Hence, despite having a substantially higher marginal cost of production, thermal firms are necessary to fulfil this residual demand. As the cost of construction is regarded as a relevant entry barrier, the Brazilian government decided, in 2004, to promote a separate procurement auction for thermal plants in construction phase to stimulate new investments. In these New Energy Auctions, thermal firms of different technologies are auctioned together and a trade-off arises between marginal costs of production and the costs of construction. As the problem of comparing different technologies is not trivial, bids are multidimensional and evaluated by a scoring function. Nevertheless, as pointed out in Rego (2013), the function utilized is overly dependent on assumptions about the probability distribution of rain, which is being underestimated and, therefore, thermal plants with higher marginal cost of production are being privileged.

With that example in mind, we want to highlight to the trade-off between marginal costs of production and marginal cost of construction by assuming all firms are on an efficient technology frontier, i.e. the latter is a strictly decreasing function of the former. Our aim is to understand how the interaction between the shape of the frontier and the distribution of shocks affect: i) efficient allocations, including total de-

mand for the good and preferences over technologies; ii) the direction of distortions of second-best allocations with respect to the first-best benchmark; iii) implementation of second-best allocations using the most standard kind of score auctions, which have been discussed in literature and used in real life procurement problems.

In order to address these questions, we create a basic two-stage model in which the production capacity, which proxies the size of plants, is decided in the first stage, while production takes place in the second. Inverse demand functions are subject to demand shocks, which are not known until the beginning of the second stage. Our main finding is that both the shape of technological frontier and the distribution of shocks can radically affect technology choices in first-best and second-best environments. More importantly, interaction of these two inputs may affect implementability of second-best allocations by standard real-life mechanisms.

The paper is organized as follows: Section 2 discusses the related literature. Section 3 presents our framework and relates it to a model that proxies our motivation. In Section 4, we develop a first-best benchmark and derive efficient allocations. In Section 5 we derive second-best allocations, assuming firm types are unknown by the buyer. In Section 6 we approach implementation of second-best allocations with a quasi-linear score auction. Section 7 concludes. You may find our proofs in the Appendix.

# 2 Literature

This work is closely related with two branches of literature.

The first is concerned with countervailing incentives, defined as situations in which firm's outside option depends on their private information parameters leading to either understate of overstate of their types. This leads to the appearance of an interior pooling region, interpreted as a scenario in which simplicity of contracts is the optimal thing to do. The seminal paper by Lewis and Sappington (1989) developed a model in which countervailing incentives occurred as a consequence of a trade-off between marginal and fixed costs in a regulation problem. There are other possible causes to countervailing incentives. In Maggi and Rodríguez-Clare (1995), for instance, countervailing incentives arise as a consequence of agent's outside options. An innovation of our framework with relation to the core of this literature is that there are two agents in the model, which may be excluded, and, in addition to the private information of the firms, there is also a source of uncertainty related to the shock, which is common to all agents. In this case, there is an additional concern with government's preferences over technologies and protection against risk.

The second branch of literature is on score auctions, which are presented as mechanisms that are compatible with real-life implementation when there is more than one relevant variable for procurement. Che (1993) develops a model for the procurement auction of one license when both quality and price matter and private information is one-dimensional. In that case, second-best is in fact implemented by a score action with an optimal scoring rule. Important extensions have been made to this framework. Asker and Cantillon (2008), for instance, extend the analysis to the case where the participants' private information is multidimensional. Although results on second-best allocation do not carry out so easily, the paper shows that this mechanism is superior to other alternatives used in practice. Branco (1997) on the other hand extends Che (1993) analysis allowing for correlated types. However, this is the first paper to our knowledge to relate score auctions with countervailing incentives.

# 3 Framework

This procurement model has two periods $t = \{1, 2\}$. In the first period, the central planner may procure a subset of the two available producers of a good, each of them a potential investor. If a contract is signed, the entrant must build a new facility in $t = 1$, which only becomes operative in $t = 2$. Production only takes place in the second period.

However, the society's demand $t = 2$ cannot be anticipated in $t = 1$ due to the existence of an exogenous shock $\theta \in [0, 1]$, which is a random variable, not unravelled until the beginning of period 2. We define $M$ as the probability measure induced by $\theta$ on $([0, 1], B([0, 1]))$. Expectations $E_\theta$ are defined as Lebesgue integrals with respect to $M$.

The effect of shocks is captured by an inverse demand function $P(Q, \theta)$, where $Q$ stands for the amount of consumption good. About $P$ we assume the following:

**Assumption 1.** $P$ is a continuous function that is strictly decreasing on $Q$ and $\theta$ if $Q + \theta \leq 1$ and zero otherwise. Also, $P(0, 0)$ is arbitrarily high (yet finite) and consumers do not discount the future. [1]

Consequently, we define for each firm $i \in \{1, 2\}$ a capacity level $k_i$ to be built in $t = 1$ and a contingent plan of production $\boldsymbol{q_i} : \theta \to q_i(\theta)$ for the second period, after the value of the shock is revealed. To simplify notation, let $\boldsymbol{q} = (\boldsymbol{q}_1, \boldsymbol{q}_2)$ and $\boldsymbol{k} = (k_1, k_2)$. The important feasibility condition in this problem is that production can never exceed capacity. Therefore, the feasibility set is given by:

$$FA = \{(\boldsymbol{q}, \boldsymbol{k}) \geq 0 : q_i(\theta) \leq k_i \,\forall\, \theta \in [0, 1] \text{ and } i = 1, 2\}.$$

And the expected social benefit the good proportions is given by:

$$V(\mathbf{q}) = E_\theta \left( \int_0^{Q(\theta)} P(y, \theta) dy \right), \text{ where } Q(\theta) = \sum_{i=1}^{2} q_i(\theta).$$

For normalization purposes, consumers and firms do not discount the future.

[1] In this model, we consider the shocks to be non-negative because the lower bound makes some proofs more direct and has an important meaning to the applied situation that inspired us: it stands for the scenario in which there is a drought and the demand for the good (energy) the highest possible. However, this framework could be easily rewritten to have zero-measure shocks.

In terms of technology, we consider both firms to be efficient. A firm $i \in \{1,2\}$ of type $c_i$, produces $q_i$ units of the good on $t = 2$ at the cost of $c_i q_i$ and builds a plant of capacity $k_i$ in period 1 at the cost of $\psi(c_i)k_i$. The cost of each firm in both periods is therefore given by:

$$C_i(c_i, \boldsymbol{q}_i, k_i) = c_i q_i + \psi(c_i)k_i, \quad q_i = E_\theta(q_i(\theta)).$$

As $\psi$ characterizes the shape of the technology frontier, we must assume that no type in $c \in [\underline{c}, \bar{c}]$ is dominated by some convex combination of other types by having strictly higher costs for all feasible contingent plans $\{\boldsymbol{q}_i, k_i\}$. Therefore, we assume the following.

**Assumption 2.** $\psi$ is twice continuously differentiable, strictly positive, strictly decreasing and convex.

About firms' types we assume:

**Assumption 3.** $c_i \sim G$ i.i.d. with support on $[\underline{c}, \bar{c}]$, where $G \in C^2$ with density $g$.

The next assumption guarantees firms are committed:

**Assumption 4.** If a firm commits to providing a given plan $(\boldsymbol{q}_i, k_i)$ and fails to do so, the government inflicts a punishment of $-\infty$ on its payoff.

## a Example

Suppose the government holds one state-owned hydro firm in periods 1 and 2. The production of hydroelectric power has no financial costs. However, is limited by reservoir level $r^t$ for $t \in \{1,2\}$, measured in terms of the maximum number of energy units it may produce. Reservoir $r^t$ is known at the very beginning of each period, therefore $r^1$ is a parameter of the model, and $r^2$ is regarded as a non-degenerate random variable on period 1. The source of uncertainty is rain $\theta$.
The reservoir may be stored to period $t = 2$ up to an exogenous limit of 1 unit[2] by reducing production in $t = 1$: each unit of energy produced in $t = 1$ ($h^1$) spends one unit of the reservoir level available in the next period. Hence, $r^2(\theta) = \min\{r_1 - h_1 +$

[2]which stands for the size of the reservoir. If more water is poured on it it starts to overflow.

$\theta, 1\}$, $\theta \in [0,1]$. As in period 0, for each contingent state $\theta$ in $t = 2$ it must be that the production is some $h^2(\theta) \in [0, r^2(\theta)]$. To summarize information, call $\mathbf{h} = (h^1, \boldsymbol{h}^2)$.

The set of feasible allocations is therefore:

$FA_h = \{(\mathbf{h}, \mathbf{q}, \mathbf{k}) \geq 0 : h^1 \in [0, r^1], h^2(\theta) \in [0, \min\{1, r^1 - h^1 + \theta\}]$ and $q_i(\theta) \leq k_i$ for all $\theta = H, L$, $i = 1, 2\}$

Moreover, if we call $P_h(Q)$ the inverse demand for $Q$ units of electricity, we may define the social benefit of energy production as:

$$V_h(\mathbf{h}, \mathbf{q}) = \int_0^{h^1} P_h(Q)dQ + E_\theta \left( \int_0^{Q_h(\theta)} P_h(Q)dQ \right) \text{ for } Q_h(\theta) = \sum_{i=1}^{2} q_i(\theta) + h^2(\theta)$$

Note that, if $P_h$ is continuous and strictly decreasing, $V_h$ is strictly concave. Also, the hydro firm is the only producer available for period 1. Also, if $r_0 \leq E(\theta)$, the expected consumption of the good in period 2 is weakly greater than that of period 1 for all feasible allocations. Thus, it is always true that optimal $\boldsymbol{h}$ obeys $h^1 = r_0$ and $h^2(\theta) = \theta \, \forall \theta \in [0,1]$. As a consequence, the (residual) inverse demand for thermal energy in period 2 depends only on contingent production $Q(\theta)$ and $\theta$. This statement is formally made in the next Proposition.

**Proposition 0.** If $P_h$ is a continuous, strictly decreasing function and $r_0 \leq E(\theta)$, then $P(Q, \theta) = P_h(Q + \theta)$ is the inverse demand function for thermal energy and $V(\boldsymbol{q}) = \int_0^{Q(\theta)} P(Q, \theta)dQ$ as social benefit of thermal production.

With this result in mind, we turn a supply shock into a demand shock and treat this example as a particular case of our framework.

# 4 First-best benchmark

Consider at first the benchmark case in which the government knows the firms' types $\boldsymbol{c} = (c_1, c_2) \in [\underline{c}, \overline{c}]^2$. Since all she cares about is the consumers, all rent from firms will be extracted and the problem she is primarily interested in is a social welfare maximization:

$$(\boldsymbol{q}^{FB}, \boldsymbol{k}^{FB}) = \underset{(\boldsymbol{q},\boldsymbol{k})\in FA}{\operatorname{argmax}} V(\boldsymbol{q}) - \sum_{i=1}^{2} C(c_i, \boldsymbol{q}_i, k_i)$$

In order to be sure that the efficient total amount of capacity $\sum_{i=1}^{2} k_i^{FB}(\mathbf{c})$ is strictly positive for all $\mathbf{c}$ without any additional restrictions on parameters, it is necessary and sufficient to make the following assumption (as proved in Lemma A in the Appendix):

**Assumption 5.** $E_\theta \left[ \max \left\{ P(0,\theta) - c, 0 \right\} \right] > \psi(c)$ for all $c \in [\underline{c}, \overline{c}]$

The term on the left stands for the expected social benefit of building the first marginal unit of capacity from a firm of type $c$, while the term on the right is the marginal cost of doing so. This condition ensures that it is not optimal to exclude a given type $c \in [\underline{c}, \overline{c}]$ *a priori* for being too costly.

The next proposition shows sufficient conditions on $\psi$ and $M$ to guarantee that for a full-measure set of types there is one technology strictly preferable to the other and allocations are as if only the best type was available. Therefore, the following definition is useful:

**Definition 1.** Let:

i) $\mu_1(c) = M(\{\theta : P(0,\theta) > c\})$ be the probability of the event where demand for the good that costs $c$ is strictly positive in $t = 2$ when the firm of type $c$ is the lowest cost supplier available.

ii) $K^*(c) = \sum_{i=1}^{2} k_i^{FB}(c,c)$ and $\boldsymbol{Q}_i^*(c) = \sum_{i=1}^{2} \boldsymbol{q}_i^{FB}(c,c)$ be the optimal allocation when $c$ is the only type available.

ii) $\mu_2(c) = M(\{\theta : P(K^*(c),\theta) > c\})$ be the probability of the event in which it is

optimal to use all available capacity from type $c$ when this type is the highest one to be hired.

**Proposition 1.** Given Assumptions 1-5 $\mu_1 > \mu_2 > 0$ are well-defined decreasing functions and, if we pick a type vector $\boldsymbol{c}$ with $c_1 < c_2$:

i) For any type vector $\boldsymbol{c}$ with $c_1 < c_2$ the condition $|\psi'(c_1)|$, $|\psi'(c_2)| > \mu_1(c_1)$ is sufficient to ensure that it is optimal to hire only firm 2. In that case,$k_1^{FB}(\mathbf{c}) = 0$ and $k_2^{FB}(\mathbf{c}) = K^*(c_2)$.

   If $|\psi'| \geq \mu_1(\underline{c})$ there is a full-measure set of type vectors $\boldsymbol{c}$ such that the lowest cost firm is not hired.

ii) For any type vector $\boldsymbol{c}$ with $c_1 < c_2$ the condition $|\psi'(c_1)|, |\psi'(c_2)| < \mu_2(c_2)$ is sufficient to ensure that it is always optimal to hire only firm 1. In that case, $k_1^{FB}(\mathbf{c}) = K^*(c_1)$ and $k_2^{FB}(\mathbf{c}) = 0$.

   If $|\psi'| \leq \mu_2(\bar{c})$ there is a full-measure set of type vectors $\boldsymbol{c}$ such that the lowest cost firm is not hired.

The intuition for this result is that $\psi'$ represents the intensity of the trade-off between the cost of building new capacity, which leads to "sunk" cost on $t = 2$, and the cost of actually producing the good on $t = 2$ after the realization of the shock. If $\psi'$ is sufficiently close to zero, then the effect of a lower marginal cost of production is more important because a sharp decrease in marginal costs is accompanied by a very slight increase in fixed costs. This effect becomes more important as $M$ attributes higher probability to low realizations of shocks. In that case, production is higher, capacity constraints become more likely to bind and a lower marginal cost saves relatively more money. On the other hand, if $\psi'$ is very low, the social benefit of reducing fixed costs is always predominant, especially as $M$ concentrates more mass on high shocks. In that case, production happens less often and the difference in "sunk" costs become more relevant relatively to difference on marginal costs.

However, for intermediate values of $\psi'$ it is not clear which effect is more important and it may be the case that having two technologies is optimal. In order to address this matter we will start by showing that, for every $\mathbf{c}$, the set probability measures $M$ over $[0, 1]$ that admit $k_1^{FB}(\mathbf{c}), k_2^{FB}(\mathbf{c}) > 0$ for any given $\psi$ is open. In

the sequence, we show with an example that those sets are non-empty for a positive measure set of type vectors $\boldsymbol{c}$ when there is some $c$ such that $\mu_1(c) > |\psi'(c)| > \mu_2(c)$.

In order to define an open set in the space of probability measure, we use operator's norm, i.e, $d(M, M') = \sup\left\{\left|\int x(\theta)dM(\theta) - \int x(\theta)dM'(\theta)\right| : x \in C([0,1]) \sup_{\theta\in[0,1]} = |x(\theta)| = 1\right\}$.

**Lemma 1.** Consider any efficient frontier represented by $\psi$. Then:

i) For any given probability measure $M$ over $\theta$, if $k_1^{FB}(\mathbf{c}), k_2^{FB}(\mathbf{c}) > 0$ for some $\mathbf{c}$, then there is a neighbourhood $A$ of $\mathbf{c}$ such that both firms are hired for all type vector in $A$.

ii) Given a type vector $\boldsymbol{c}$, there is an open set of probability measures with respect to $d$ such that both firms are hired in first-best.

That being said, we will now show, by example, that the set of measures described by Lemma 1 is not empty. In order to do so, we carry out the following simplification:

**Assumption 6.** Given the parameters $\mu, H \in (0,1)$, $\theta$ is a binary variable assuming value $H$ with probability $1-\mu$ and $L = 0$ with probability $\mu$.

Then, if $M$ is consistent with Assumption 6, we know $\mu_2(c) \in \{\mu, 1\}$ for all $c \in [\underline{c}, \bar{c}]$, otherwise $K^*(c)$ would never be optimal because with probability 1 the optimal contingent production level would be strictly below $K^*(c)$. As a consequence, we guarantee that:

i) $|\psi'| > 1$ only the highest cost firm is hired.

ii) $|\psi'| < \mu$ only the lowest cost firm is hired.

However, if $\psi'$ can assume intermediate values, the government's preference for technologies may depend on the parameter $H$ and on firms' types. The reason is that high-cost firms' capacity is always built to produce only in high-shock scenarios, while low cost firms are built to work in both scenarios. If $H$ is intermediate, then the optimal thing to do is to use both firms to hedge risk because $H$ is not high enough to make low-cost firm useless when the shock is high and not low enough to justify letting low cost firm supply all demand for capacity as high-cost firms can build new

capacity at a lower cost. Consequently, it will be optimal to have a low marginal cost firm to provide the basic level of consumption, and a high marginal cost firm to build the extra units of capacity needed when the shock is low. This result is formalized in Proposition 2 below.

**Proposition 2.** Suppose there is $c$ such that $\mu < \psi'(c) < 1$ and Assumptions 1-6 hold. Then, there is a non-degenerate interval $(c', c'') \subseteq [\underline{c}, \bar{c}]$ such that, for all $\mathbf{c} \in (c', c'')^2$ with $c_1 < c_2$ there are thresholds $H^-(\mathbf{c}) < H^+(\mathbf{c}) \in (0, 1)$ such that:

i) If $H \in [0, H^-(\mathbf{c})]$, then it is optimal to hire only firm 1, who must build a plant of size $k_1^{FB}(\mathbf{c}) = K^*(c_1)$. All available capacity will be used with probability one ;

ii) If $H \in (H^-(\mathbf{c}), H^+(\mathbf{c}))$, then both firms are hired. Firm 1 is supposed to provide for the basic level of consumption and all its capacity is used with probability one. Firm 2 is hired only to fulfill a residual demand that will exist if the shock turns out to be low. Therefore, total capacity is determined by the high cost $c_2$, i.e. $\sum_{i=1}^{2} k_i^{FB}(\mathbf{c}) = K^*(c_2)$;

iii) If $H \in [H^+(\mathbf{c}), 1]$, then it is optimal to hire only firm 2, who must build a plant of size $k_2(\mathbf{c}) = K^*(c_2)$. The plant is supposed to be turned off in if the shock is high and produce all it can if the shock is low.

If $\psi'(c_1), \psi'(c_2) \in (-1, -\mu)$, the thresholds $H^-(\mathbf{c})$ and $H^+(\mathbf{c})$ are monotonically decreasing functions of $\mathbf{c}$.

An interesting feature of the case with two states and two firms is that the capacity of the low cost firm can be pinned down to be the optimal minimum level of consumption of the good, which will be demanded with probability 1 and, therefore, cheaper to be produced by the lowest type firm because in that case the marginal cost effect is more important. The capacity of the high type, however, is hand picked to provide for residual demand on the low shock scenario and, as it is less frequent, it will be more efficiently provided by the high type firm as it has a lower sunk cost. This causes production plans to be binary: for every realization of the shock a firm can either it produce all its capacity or do nothing.

In order to make the interaction between trade-offs on costs and the intensity of high shocks visually clear, we provide the following numeric example.

**Example 1.** Let $P(Q, \theta) = 40(1 - Q - \theta)$, $\psi(c) = 4 - \alpha(c - 1)$, $\mu = 0.4$, $\underline{c} = 1$, $\bar{c} = 3$, $\boldsymbol{c} = (2, 2.5)$.

*Figure 1 depicts for a given* $\mathbf{c}$ *the pairs of* $|\psi'| = \alpha$ *and* $H$ *where: only low type firms are hired (R1), both firms are hired (R2) and only the high cost firm is hired (R3).*

*This example shows us how the shape of the frontier and the distribution of shocks affect first-best allocations: we measure the importance of marginal costs of capacity relative to marginal cost of production by* $\alpha$*. The parameter* $H$*, on the other hand, shows us how different shocks can be.*

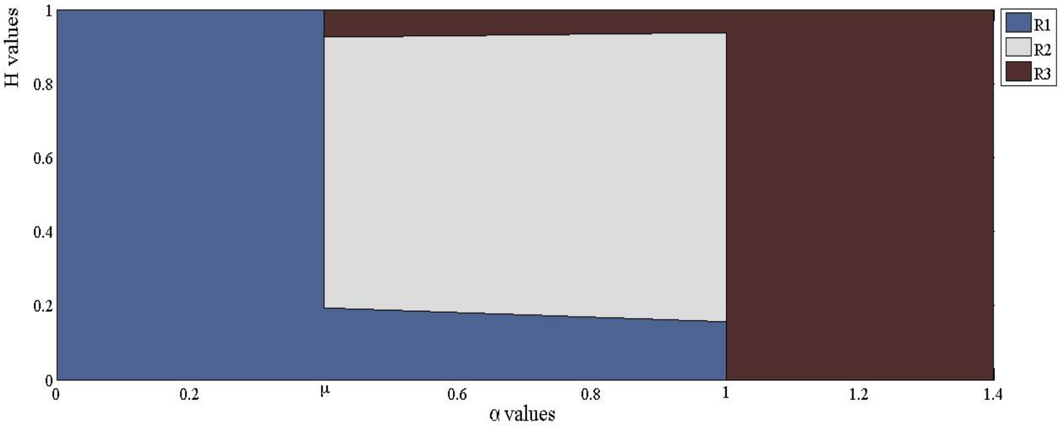


Figure 1: First-best solutions.

The following corollary concludes that the diversification result is not restricted to the particular case of Assumption 6. In fact, there is an open, non-empty set of probability measures that admit more than one firm being hired in the first-best solution.

**Corollary 1.** Consider a type vector $\mathbf{c}$ with $c_1 < c_2$, a frontier $\psi$ and a probability measure $M$ that satisfies Assumption 6. The following conditions on $\psi$ and $M$ are sufficient to guarantee that there is a neighbourhood of $M$ with respect to $d$ such that both firms are hired.

i) $\mu < |\psi'(c_1)|, |\psi'(c_2)| < 1$;

ii) $H \in (H^-(\mathbf{c}), H^+(\mathbf{c}))$.

However, if $M$ does not satisfy Assumption 6, it may be the case that both firms are hired and for a positive measure set of shocks one of the firms is producing only part

of its capacity. This means that, although our main finding on technology choices does not rely on binary distributions, the all-or-nothing property does not carry on so easily if distributions are richer. This result is stated in Proposition 3.

**Proposition 3.** If $M$ does not follow Assumption 6 and for a given type vector $\boldsymbol{c}$ both firms are hired, we might have one of the firms' capacity being used only partially.

# 5 Second-best benchmark

This section aims to find and characterize the optimal mechanism when types $\mathbf{c}$ cannot be observed. In order to facilitate the exposition, we use Assumptions 1-5 from this moment on. Also, we introduce profits $\boldsymbol{\pi}(\boldsymbol{c}) = (\pi_1(\boldsymbol{c}), \pi_2(\boldsymbol{c}))$ as firms will receive informational rents from now on given that information is asymmetric.

By finding the optimal direct mechanism we have, by Revelation Principle, the second-best solution.

**Definition 2** (Feasible direct mechanism)**.** A feasible direct mechanism is a lottery in $\Gamma$, where:

$$\Gamma = \{\boldsymbol{m} : \mathbf{c} \to (\boldsymbol{q}(\mathbf{c}), \boldsymbol{k}(\mathbf{c}), \boldsymbol{\pi}(\mathbf{c})) \mid (\boldsymbol{q}(\mathbf{c}), \boldsymbol{k}(\mathbf{c})) \in FA \text{ for all } \mathbf{c} \in [\underline{c}, \overline{c}]^2\}.$$

An optimal mechanism in $\Gamma$ solves:

$$\begin{aligned} &\max_{(\boldsymbol{q},\boldsymbol{k},\boldsymbol{\pi})\in\Gamma} E_{\mathbf{c}}\left[V(\boldsymbol{q}(\mathbf{c})) - \sum_{i=1}^{2} C(c_i, \boldsymbol{q}_i(\mathbf{c}), k_i(\mathbf{c})) - \sum_{i=1}^{2} \pi_i(\mathbf{c})\right] \\ &\text{s.t.} \quad (IR_i), (IC_i) \quad i = 1, 2 \end{aligned}$$

where participation constraints $(IR_i)$ are given by:

$$E_{-i}(\pi_i(z, .)) \geq 0 \qquad z \in [\underline{c}, \overline{c}]$$

and each incentive constraint $(IC_i)$ can be written as:

$$E_{-i}[\pi_i(z, .) - (c_i - z)q_i(z, .) - (\psi(c_i) - \psi(z))k_i(z, .)] \leq E_{-i}[\pi_i(z, .)] \quad \forall z \in [\underline{c}, \overline{c}]$$

where $q_i(\boldsymbol{c}) = E_\theta(q_i(\theta, \mathbf{c})|\mathbf{c})$.

**Lemma 2.** The optimal mechanism in $\Gamma$ is optimal in the class of feasible direct mechanisms. As a consequence, we cannot find any feasible direct (or indirect) mechanism that gives the planner a strictly higher welfare.

Lemma 2 comes as a direct consequence of risk aversion and the Revelation Principle. That been said, we can derive all important results by computing the optimal mechanism in $\Gamma$, which we call $(\boldsymbol{q}^{SB}, \boldsymbol{k}^{SB}, \boldsymbol{\pi}^{SB})$.

In order to properly relate second-best allocations to first-best solutions, we need the following assumption on virtual types:

**Assumption 7.** Virtual type functions $\rho_1(c_i) = c_i + \frac{G(c_i)}{g(c_i)}$ and $\rho_2(c_i) = c_i - \frac{1-G(c_i)}{g(c_i)}$ are strictly increasing on types $c_i$. Moreover, $\rho_2(\underline{c}) > 0$.

In order to avoid exclusion, the following modified version of Assumption 5 will be a necessary and sufficient condition:

**Assumption 8.** If $\rho_j$ is the distorted marginal cost of capacity for each $j \in \{1, 2\}$, no virtual type will be excluded *ex antes*, that is:

$$E_\theta \left[\max\left\{P(0,\theta) - \rho_j(c), 0\right\}\right] > \psi_j(\rho_j(c)) \text{ for all } c \in [\underline{c}, \overline{c}] \text{ and } j \in \{1, 2\},$$

Where, for each $j \in \{1, 2\}$, $\psi_j > 0$ is the distorted technological frontier, written as a function of virtual types $z \in [\rho_j(\underline{c}), \rho_j(\bar{c})]$ given by $\psi_j(z) = \psi(\rho_j^{-1}(z)) + \psi'(\rho_j^{-1}(z))(z - \rho_j^{-1}(z))$.

**Proposition 4.** If Assumptions 1-4 and 6-8 hold, we can say the following about the optimal mechanism in $\Gamma$ :

i. If $|\psi'| < \mu$, $(\boldsymbol{q}^{SB}(\mathbf{c}), \boldsymbol{k}^{SB}(\mathbf{c}))$ is almost surely equal to the solution to the first-best problem with virtual types $\boldsymbol{\rho}_1(\mathbf{c}) = (\rho_1(c_1), \rho_1(c_2))$ and $\psi_1$ as marginal cost of building new capacity. The firm with the lowest type is the only one hired with positive capacity and no informational rent is given to the highest type $\overline{c}$.

ii. If $|\psi'| > 1$, $(\boldsymbol{q}^{SB}(\mathbf{c}), \mathbf{k}^{SB}(\mathbf{c}))$ is almost surely equal to the solution to the first-best problem with virtual types $\boldsymbol{\rho}_2(\mathbf{c}) = (\rho_2(c_1), \rho_2(c_2))$ and $\psi_2$ as marginal cost of building new capacity. The firm with the highest type is the only one hired with positive capacity and no informational rent is given to the lowest type $\underline{c}$.

As in the first-best case, however, now we need to understand what happens in intermediate cases. Therefore, we make a final simplification to the model:

**Assumption 9.** $\psi(c) = \psi(\underline{c}) - \alpha(c - \underline{c})$, where $\alpha \in (\mu, 1)$ is a known parameter.

**Lemma 3.** Consider Assumptions 1, 3, 4 and 6-9. Then, if $\hat{\rho}(c_i) = \rho_1(c_i)1_{c_i<n_i} + \rho_2(c_i)1_{c_i>n_i}$ for some threshold vector $\boldsymbol{n} = (n_1, n_2)$, the optimal mechanism solves the following program:

$$\max_{(\boldsymbol{q},\boldsymbol{k},\boldsymbol{\pi})\in\Gamma,\, \boldsymbol{n}\in[\underline{c},\bar{c}]^2} E_{\mathbf{c}}\left(V(\mathbf{q}(\mathbf{c})) - \sum_{i=1}^{2}\hat{\rho}_i(c_i)q_i(\mathbf{c}) - \sum_{i=1}^{2}\psi(\hat{\rho}(c_i))k_i(\mathbf{c})\right)$$

Subject to:

$$\begin{aligned}
&\bar{\pi}'(c_i) \leq 0 \qquad \forall c_i \leq n_i \qquad && (R_i^-)\\
&\bar{\pi}'(c_i) \geq 0 \qquad \forall c_i \geq n_i \qquad && (R_i^+)\\
&\bar{\pi}'(c_i) \text{ non-decreasing } && (MON)
\end{aligned}$$

for every $i \in \{1, 2\}$.

This comes straight from envelope and monotonicity conditions when we allow for interior types to receive zero rents for each $i \in \{1, 2\}$. As in literature, countervailing incentives will happen if and only if non-negativity conditions $(R_i^+)$ and non-positivity conditions $(R_i^-)$ bind only in a subset of $(\underline{c}, \bar{c})$, which leads to an interior pooling interval on the worse types. The contribution of this paper, however, is to relate the presence of countervailing incentives to the distribution on shocks.

Call $\bar{H}^{FB} = sup\{H^+(\mathbf{c}) : \mathbf{c} \in [\underline{c}, \bar{c}]^2\}$ the lowest $H$ for which there is a full-measure set of types $\mathbf{c}$ such that type $\underline{c}$ does not build a new plant. As a consequence, if $H \geq \bar{H}^{FB}$ the lowest type must receive zero rents in second-best solution and types must be distorted (weakly) downwards. If $H \in [\bar{H}^{FB}, \bar{H}^{SB})$, where $\bar{H}^{SB} = sup\{H^+(\boldsymbol{\rho}_2(\mathbf{c})) : \mathbf{c} \in [\underline{c}, \bar{c}]^2\} > H^{FB}(\alpha)$ (by Proposition 2), then monotonicity conditions are binding in the lowest types as we distort them to $\rho_2(c_i) < c_i$ and, consequently, we have a pooling interval that includes $\underline{c}$. If $H \geq H^{SB}(\alpha)$, then there is no pooling and solutions are just the same as when $\alpha > 1$.

The same kind of observation can be made for high types. We can define: $\underline{H}^{FB} = \inf\{H^-(\mathbf{c}) : \mathbf{c} \in [\underline{c}, \bar{c}]^2\}$ as the highest $H$ such that, for a full-measure subset of $[\underline{c}, \bar{c}]^2$, only the lowest types will be contracted with positive capacity in first-best solutions

and $\underline{H}^{SB}(\alpha) = \inf\{H^{-}(\boldsymbol{\rho}_1(\mathbf{c})) : \mathbf{c} \in [\underline{c}, \overline{c}]^2\}$, which does the same for the virtual type vector $\boldsymbol{\rho}_1(\mathbf{c})$. Then, as types are distorted upwards, we have $\underline{H}^{FB} > \underline{H}^{SB}$ and, if $H \in [0, \underline{H}^{SB}]$ solutions are just as if $\alpha < \mu$, while if $\theta_H \in (\underline{H}^{SB}, \underline{H}^{FB}]$ only the (weakly) lowest types are hired, but there is a pooling in the highest types.

Finally, we have a non-empty interval [3] given by $H \in (\underline{H}^{FB}, \bar{H}^{FB})$ where constraints $(R^i_-)$ and $(R^i_+)$ do not bind for extreme types $\underline{c}$ and $\overline{c}$. This result is stated in Proposition 5.

**Proposition 5.** If Assumptions 1, 3, 4 and 6-9 hold, second-best solutions are unique and may be characterized in the following way:

i) If $H \in [0, \underline{H}^{FB}]$, only firms weakly lower virtual types can be hired with positive probability. If $H \in [0, \underline{H}^{SB}]$, any firm's type $c$ will be distorted upwards to $\rho_1(c)$ and there is no pooling. If $H \in (\underline{H}^{SB}, \underline{H}^{FB}]$, there is pooling on the highest types and the virtual type function for each firm is $\rho = \rho_1$ outside the pooling region.

ii) If $H \in [\bar{H}^{FB}, 1]$ only firms weakly lower virtual types can be hired with positive probability. If $H \in [\bar{H}^{SB}, 1]$, each firm's type $c$ will be distorted downwards to $\rho_2(c)$ and there is no pooling. If $H \in [\bar{H}^{FB}, \bar{H}^{SB})$, there is pooling on the lowest types and the virtual type function for each firm is $\rho = \rho_2$ outside the pooling region.

iii) If $H \in (\underline{H}^{FB}, \bar{H}^{FB})$, we have countervailing incentives, that is, types may be distorted either upwards or downwards depending on the realization of types. Virtual type function is given by:

$$\rho(c) = \begin{cases} \rho_1(c) \text{ if } c \in [\underline{c}, \rho_1^{-1}(c_\alpha)] \\ c_\alpha \text{ if } c \in [\rho_1^{-1}(c_\alpha), \rho_2^{-1}(c_\alpha)] \\ \rho_2(c) \text{ otherwise} \end{cases}$$

The emergence of countervailing incentives is a direct consequence of Proposition 2. If $H \in (\underline{H}^{SB}, \bar{H}^{SB})$, there are non-empty intervals $(\underline{c}', \underline{c}'')$ and $(\bar{c}', \bar{c}'')$ such that: 1) when a firm $i$ of type $\underline{c}$ faces an opponent $c_{-i} \in (\underline{c}', \underline{c}'')$, both firms must be hired and firm $i$ produces with probability $1 > \alpha$ for being the lowest type. 2) when a firm $i$ of virtual type $\bar{c}$ faces an opponent $c_{-i} \in (\bar{c}', \bar{c}'')$, both firms must be hired and $i$ must

[3] Proposition 2 guarantees that this interval is non-empty.

produce with probability $\mu < \alpha$ for being the highest type. In other words, the effect of having lower marginal costs of production is more relevant than the effect of having lower costs of construction if $c$ is low, while it is the opposite for high values of $c$. As a consequence, both extremely low and extremely high marginal cost firms must receive informational rent.

**Corollary 2.** The following events occur for positive measure subsets of $[\underline{c}, \overline{c}]^2$:

i) Second-best solutions for total capacity hired is distorted upwards/downwards relative to first-best solutions;

ii) The second-best capacity of the lowest type and high type firms are distorted downwards/upwards relatively to first-best solutions;

iii) The capacity of the lowest/highest type firm is distorted downwards while capacity of the highest/lowest type firm is distorted upwards relatively to first-best solutions;

iv) The government does not differentiate the two firms (virtual types are the same).

**Example 1 (continued).** *Using all functions and parameters from Example 1 and assuming $G \sim U[1,3]$, we illustrate the possible second-best allocations described in Proposition 5. Figure 2 depicts the following parametric regions of interest: R1 is the set of pairs $(\alpha, H)$, where only the lowest-cost firm is chosen and types are distorted to $\boldsymbol{\rho} = \boldsymbol{\rho}_1$, R2 is the set of $(\alpha, H)$ where only the (weakly) lowest pseudotype firm is chosen and there is pooling of high types; R3 is the set of $(\alpha, H)$ with countervailing incentives (and an interior pooling interval); R4 stands for the parametric values of $\alpha$ and $H$ in which only the (weakly) higher pseudotype firms are chosen and there is pooling at low types; R5, is the region in the $\alpha, H$ Cartesian where only the highest cost firms are hired and types are distorted to $\boldsymbol{\rho} = \boldsymbol{\rho}_2$.*

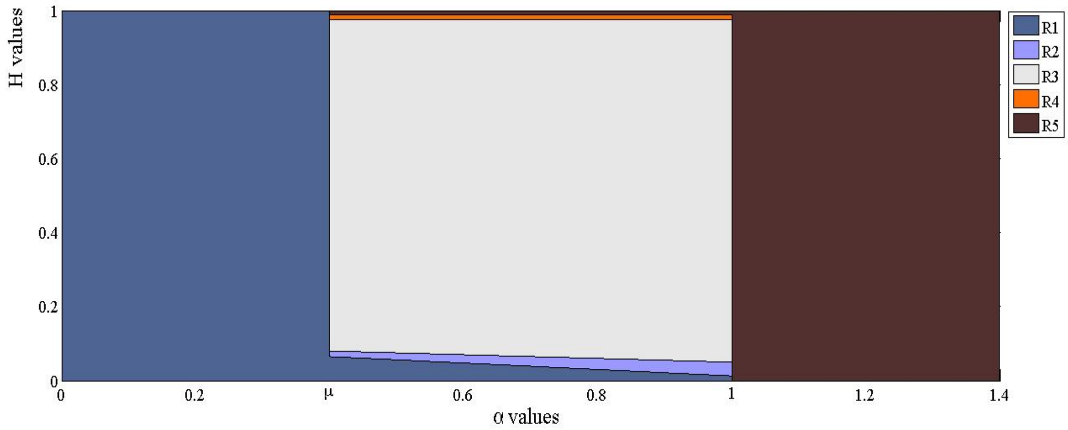


Figure 2: Second-best solutions

# 6 Implementation with a score auction

## a Motivation and definitions

An important question to make at that point is whether the second-best solutions we found can be implemented with indirect mechanisms similar to those that can be put in practice in the real world.

We set the strategy space to be as broad as possible in order to allow firms to make a detailed offer on every detail that affects its payoff. With that in mind, strategy spaces are defined as follows:

**Definition 3.** Given $i \in \{1, 2\}$, a strategy space as the set of functions $f : c_i \to S_i$, where $S_i$ is the set of vectors $(k_i, \boldsymbol{q}_i, p_i, F_i)$ such that:

i) A capacity $k_i \geq 0$ is built for next period;

ii) A vector of $\boldsymbol{q}_i = (q_i(L), q_i(H)) \in [0, k_i]^2$ of energy production levels for each state of nature in period 1;

iii) A unit price $p_i$ the government is supposed to pay for every unit of energy supplied;

iv) A fixed revenue $F_i$ that must be given to the firm in period 1 regardless of the amount of energy produced as a payment for making the capacity available to the government.

A well-established mechanism we can use when the firm's offer is not unidimensional is a score auction.

**Definition 4.** A score auction is a mechanism $\{\boldsymbol{b}, \boldsymbol{x}\}$ comprising a score rule $\boldsymbol{b}$ and an attribution rule $\boldsymbol{x}$. A score rule $b_i : S_i \to \mathbb{R}$ attributes a real number (score) to every offer $s_i \in S_i$ a firm $i$ can make. An attribution rule $x_i : b_1(S_1) \times b_2(S_2) \to [0, 1]$ accounts for the probability of a firm $i$'s offer $s_i$ being accepted (or, alternatively, the share of the firm's offer on $\{\boldsymbol{q}_i, k_i\}$ accepted by the principal), which must depend on the scores [4].

[4] Note that we do not restrain the principal from hiring both firms, which is important given that it might be optimal to do so.

In order to make the argument simple, we restrict our attention scoring rules that are quasi-linear on fixed payments $b_i(s_i) = \phi(k_i, \boldsymbol{q}_i, p_i) - F_i$, which are the standard case in the literature on score auctions. [5]

Quasi-linear score auctions have been used in many real life procurement problems where the cost of a service is not the only important variable to be considered. In Brazil, New Energy Auctions employ a score function to evaluate firms' bids. Hence, although this paper does not reproduce the mechanism employed in those tenders, having a basic insight on whether score auctions implement second-best allocations may be a helpful starting point. After all, the score rule used in Brazil has received several critics for favouring high marginal cost firms relatively to low marginal cost and sensitive to model inputs as the estimated probability distribution of rain (see, for instance, Rego (2013)). As a consequence, it is important to access how and if the parameters of the model affect the efficiency of quasi-linear score auctions relative to a second-best benchmark.

In this sense, the main message of the section that $\psi'$ and the distribution of shock do matter. We argue that a score auction with quasi-linear score rule may implement second-best allocations if and only if there are no countervailing incentives, that is, if $H \notin (\underline{H}^{FB}, \bar{H}^{FB})$ and the direction in which types are distorted by $\boldsymbol{\rho}$ is the same all over $[\underline{c}, \bar{c}]^2$.

## b Results

In quasi-linear score auctions, a firm $i's$ problem may divided in two parts: picking the right offer $(k_i, \boldsymbol{q}_i, p_i)$ in order to maximize profits subject to achieving a given score $s_i \in \mathbb{R}$ and choosing an optimal score $b_i$ by choosing the fixed payment $F_i$ appropriately. That means every offer compatible with second-best allocations must maximize profits conditional on attaining a given score $b_i$.

If we define the set of such offers for each firm $i$ and type $c \in [\underline{c}, \bar{c}]$ as:

$$B_i^{SB}(c) = \{(\boldsymbol{q}_i^{SB}(\boldsymbol{c}), k_i^{SB}(\boldsymbol{c}), \rho(c)) : k_i^{SB}(\mathbf{c}) > 0, c_i = c\}$$

We may formalize this condition in the following lemma:

[5]Note that this class encompasses all uni-dimensional first-price and second-price auctions, for instance.

**Lemma 4.** Given $i \in \{1,2\}$, the following condition is necessary to implement second-best allocations with a score auction $\{\boldsymbol{b}, \boldsymbol{x}\}$:

For every type $c \in [\underline{c}, \bar{c}]$:

$$B_i^{SB}(c) \subseteq \underset{\{k_i, \boldsymbol{q}_i, p_i\}}{\operatorname{argmax}} \{\phi(k_i, \boldsymbol{q}_i, p_i) - c_i q_i - \psi(c_i) k_i : (q_i(L), q_i(H)) \in [0, k_i]^2\}$$

This condition imposes an important restriction to the set of allocations we may implement with a quasi-linear score auction. This result is key to find how and when we may build a scoring rule that implements second-best solutions.

The following lemma is a direct consequence of Proposition 5 and gives us a hint that constructing a quasi-linear score auction $\{\boldsymbol{b}, \boldsymbol{x}\}$ that satisfies the conditions in the previous lemma is not difficult if there are no countervailing incentives.

**Lemma 5.** $B_i^{SB}(c)$ is a singleton if and only if there are no countervailing incentives.

That being said, we show that if we do not have countervailing incentives we may adapt the argument in Che (1993) to our problem in order to construct a quasi-linear score rule that implements second-best solutions. This is done by setting a rule such that induces each firm $i$ of type $c$ to choose as a best response the offer in $B_i^{SB}(c)$ and the fixed payment that is consistent with its second-best expected profit. We formalize this result in Proposition 5.

**Proposition 6.** Suppose at least one of the following conditions is satisfied:

i) If Assumptions 1-4 and 6-8 hold and either $|\psi'| > 1$ or $|\psi'| < 1$;

ii) If Assumptions 1, 3, 4 and 6-9 hold and there are no countervailing incentives.

Then, we may implement second-best allocations with a quasi-linear score auction $\{\boldsymbol{b}, \boldsymbol{x}\}$.

The score rule has the shape:

$$b_i(s_i) = \begin{cases} \phi(p_i) - F_i, \text{ if } k_i(c_i) = K_i^*(p_i), \boldsymbol{q}_i = \boldsymbol{Q}_i^*(p_i) \text{ and } p_i \in [\rho(\underline{c}), \rho(\bar{c})] \\ -\infty \text{ otherwise,} \end{cases}$$

where $\phi :\in [\rho(\underline{c}), \rho(\bar{c})] \to \mathbb{R}$ is a non-linear function.

The attribution rule is the following:

$$x_i(\boldsymbol{s}) = \begin{cases} 1, \text{ if } b(s_i) > b(s_{-i}) \\ \frac{1}{2}, \text{ if } b(s_i) = b(s_{-i}) \\ 0, \text{ if } b(s_i) < b(s_{-i}) \end{cases}$$

However, if there are countervailing incentives, not only does this argument fails, but the condition in Lemma 4 cannot be met by a quasi-linear score auction. The essence of the proof is that $B_i^{SB}(c)$ must not vary among pooled types $c \in [\rho_1^{-1}(c_\alpha), \rho_2^{-1}(c_\alpha)]$. However, not only is this set not a singleton, but also the expected cost of fulfilling the offers in $B_i^{SB}(c_\alpha)$ typically varies among types in the pooling interval. As a consequence, there are second-best allocations that cannot be implemented in a quasi-linear score auction as they are never best responses, as shown in the next Proposition.

**Proposition 7.** If we are under Assumptions 1, 3, 4 and 6-9 and there are countervailing incentives, i.e. $H \in (\underline{H}^{FB}, \bar{H}^{FB})$ and $\alpha \in (\mu, 1)$, there is no quasi-linear scoring rule that implements second-best allocations. Consequently, the welfare attained by the planner with any quasi-linear score auction will be strictly below second-best benchmark.

This result proves that the shape of technology and the distribution of shocks also matter for the optimality of quasi-linear score auctions. If we cannot guarantee that they implement second-best solutions, it may be useful to consider other mechanisms.

# 7 Conclusion

In this paper we developed a basic model in which demand for a good is uncertain and technological frontier is characterized by a trade-off between marginal costs and ”sunk” costs, as in Lewis and Sappington (1989). We derive three results.

The first is that, in the first-best benchmark, we show that both the shape of the frontier and the distribution of shocks matter when it comes to defining optimal choices on technologies. If ”sunk” costs decrease very slightly with marginal costs, then only lowest marginal costs are hired. On the other hand, if this decrease is very sharp, firm with the highest marginal cost is hired with exclusivity. There is, however, an intermediate case in which efficient technology choice depends heavily on the distribution of shocks: if there is considerable mass in high realizations, then only high-cost firms are chosen. If shocks are predominantly low, then only low-cost firms are chosen. Otherwise, it is optimal to build both plants. The reason is that the there is not enough mass on small shocks to make it optimal to use all available capacity with probability one and not enough mass on shocks that are high enough to justify zero consumption. Hence, it is optimal to use low marginal cost firms for the provision of the ”basic” consumption, which will be needed even on high shock scenarios, and high marginal cost firms to provide for the extra demand on low shock realizations.

The second result is about the sense of distortions that arise from asymmetric information. If we are in the case where only one firm is chosen for every type vector, deviations are standard: second-best solutions for capacities are distorted downwards as virtual types are always worse than the firms' actual types. However, it is possible for both types to be hired in second-best allocations. In that case, we have countervailing incentives as types may be distorted either upwards or downwards, depending on their realization.

Finally, our third result shows how the shape of the technological frontier and the distribution of shocks affect the efficiency of quasi-linear score auctions. We show that a quasi-linear score rule as the one used in Brazil may implement the second-best outcome if and only if there are no countervailing incentives.

Our results represent a word of caution when it comes to dealing with procure-

ment situations of this type given that standard mechanisms may not work as well as we expect. However, this raises (at least) two research questions that remain unanswered. The first is about optimizing procurement: which real-life procurement mechanism would be the best in terms of social welfare when types are not known and countervailing incentives arise? It may be useful to compare, as in Asker and Cantillon (2008), mechanisms such as score auctions, menu auctions and beauty contests in order to find out which one is the closest to second-best. The second is a deeper market design question: Would it be beneficial for society to allow for oligopolistic competition instead of promoting centralized procurement? For instance, we may assume a two-stage competition as in Kreps and Sheinkman (1983), in which firms engage a Cournot competition in the first period while choosing capacity and then face a Bertrand competition with capacity constraints in the second period. We leave these ideas for future research.

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# 9 Appendix

## a First-best allocation

First of all, note that we can write the Lagrangian of this problem as:

$$\mathcal{L}(\boldsymbol{q},\boldsymbol{k},\boldsymbol{\lambda}) = E_\theta\bigg(\int_0^{Q(\theta)} P(Q,\theta)dQ - \sum_{i=1}^{2}[c_i q_i(\theta) + \psi(c_i)k_i] + \sum_{i=1}^{2}\lambda_i(\theta)(k_i - q_i(\theta))\bigg)$$

The contingent production on $t = 2$, $\boldsymbol{q}^{FB}$ that solves this problem is a.s. equivalent to the pointwise solution to this problem. That been said, we may prove the following:

Lemmas A-B are useful to prove Propositions 1 and 2 and Lemma 1 in the main text.

**Lemma (A).** We can use the following technical properties can be verified for first-best solutions:

i) If $c_1 \neq c_2$, first-best solutions $\{\boldsymbol{q}^{FB}(\boldsymbol{c}), \boldsymbol{k}^{FB}(\boldsymbol{c})\}$ are a.s. unique and continuous on the point $\boldsymbol{c}$.

ii) If $c_1 = c_2 = c$, $\{\boldsymbol{Q}^*(c), K^*(c)\}$ are unique and continuous on $c$, where $\boldsymbol{Q}^*(c) = \sum_{i=1}^2 \boldsymbol{q}_i^{FB}(c,c)$ and $K^*(c) = \sum_{i=1}^2 k_i^{FB}(c,c)$.

Also, solutions may be uniquely pinned down by first-order conditions on $\boldsymbol{q}$ and $\boldsymbol{k}$.

***Proof of Lemma A***. Note that, if types are the same, we can see this problem as if there was only one potential entrant of type $c$ and derive the results.

As $\int_0^{Q(\theta)} P(Q,\theta)dQ$ is concave on $Q(\theta)$ by Assumption 1, first-order conditions are sufficient. The fact that $\int_0^{Q(\theta)} P(Q,\theta)dQ$ is strictly concave in $Q(\theta)$ if $P(Q(\theta),\theta) > 0$ and $c_i > 0$ for every $i$ guarantees the uniqueness of solutions. Note that, if types are different, this function will be strictly concave on each $q_i(\theta)$ if $P(Q(\theta),\theta) > 0$. The immediate consequence of concavity and differentiability of the objective function is that first-order conditions are necessary and sufficient to characterize the optimal.

For the results on continuity and non-emptiness of policy functions, just apply Berge's Maximum Theorem (Berge (1959)). □

First-order conditions on $q_i(\theta)$ determine that:

$$\lambda_i(\theta) = \max\{P(Q(\theta), \theta) - c_i, 0\}.$$

While first-order conditions on $\boldsymbol{k}$ impose that, for each $i \in \{1, 2\}$:

$$E_\theta[\lambda_i(\theta)] - \psi(c_i) \leq 0 \text{ (with equality if } k_i > 0).$$

As a consequence, we may express the first-order condition on $k_i$ by:

$$R_i(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) = E_\theta(\max\{P(Q(\theta), \theta) - c_i, 0\}) - \psi(c_i) \leq 0 \text{ with equality if } k_i > 0 \tag{1}$$

If $c_1 = c_2 = c$, we determine $K^*(c)$ as the solution to:

$$E_\theta((P(K^*(c), \theta) - c)^+) - \psi(c) = 0 \tag{2}$$

And

$$Q^*(c, \theta) = \begin{cases} 0 \text{ if } P(0, \theta) < c \\ K^*(c) \text{ if } P(K^*(c), \theta) > c \\ Q \text{ satisfying } P(Q, \theta) = c \text{ otherwise} \end{cases} \tag{3}$$

If $c_1 < c_2$, equation 1 must hold for every $i \in \{1, 2\}$. In addition, first-order conditions on $\boldsymbol{q}$ impose that: if $\lambda_i(\theta) > 0$, $q_i(\theta) = k_i$. Also $P(q_1(\theta), \theta) - c_2 < 0$ implies $q_2(\theta) = 0$ and if $P(0, \theta) - c_1 < 0$, $q_1(\theta) = 0$.

**Lemma (B).** Assumption 5 is necessary and sufficient to ensure that the total capacity hired is a.s. strictly positive. Moreover, every interior type $c \in (\underline{c}, \bar{c})$ gets hired with positive probability $\left(\text{i.e. } \int_{B(c)} dG(c_{-i}) > 0 \text{ for } B(c) = \{c_{-i} \neq c : k_i^{FB}(c, c_{-i}) > 0\}\right)$.

***Proof of Lemma B.*** The first sentence follows directly from equation 1 and the fact that $P$ is strictly decreasing on $Q$. To see the second part, note that, by Lemma A, solutions are continuous on $\mathbf{c}$ if $c_1 < c_2$. As a consequence, one of the following must be true: i) $k_1^{FB}(\mathbf{c}) > 0$ for all $c_2 \in [c', c'']$, where $c'' > c' > c_1$, ii) $k_1^{FB}(\mathbf{c}) = 0$ for all $c_2 > c_1$. If $i)$ is true, we have our result because $g > 0$. If $ii)$ is true, then there must be $c_2' < c_1$ such that $k_1(c_1, c_2') > 0$ because solutions are continuous and the problem is symmetrical. Also, there is a neighbourhood around $\boldsymbol{c} = (c_1, c_2')$ such that

$k_1(c_1, c_2') > 0$, which gives us the result. The same point can be easily made for firm 2. $\square$

***Proof of Proposition 1***. With probability 1 we have $c_1 \neq c_2$. In that case, suppose without loss of generality that $c_1 < c_2$.

In this proposition, our approach to show a firm $i$ will not be hired will consist on demonstrating that if $R_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) = 0$, then $R_i(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) < R_{-i}(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c})$.

To start with, we use equation 1 and first-order conditions on $\boldsymbol{q}$ to see that:

$$R_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) - R_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) =$$
$$E_\theta((P(k_1, \theta) - c_2)1_{P(k_1,\theta)\in(c_1,c_2)}) + (c_1 - c_2)E_\theta(1_{P(k_1,\theta)\leq c_1}) - (c_1 - c_2) - (\psi(c_1) - \psi(c_2)). \quad (4)$$

Where $1_A$ stands for an indicator function receiving the value $1$ if $\theta \in A$ and $0$ otherwise. Note that the term $(c_1 - c_2) + (\psi(c_1) - \psi(c_2))$ stands for the difference in unitary costs of capacity that is used with probability one regardless if the cost of production of this unit is $c_1$ or $c_2$. To get the differences on net marginal benefits of new capacity between the two firms, we must discount it by two terms: $E_\theta((P(k_1, \theta) - c_2)1_{P(k_1,\theta)\in(c_1,c_2)})$ accounts for the fact that if $\theta \in \{\theta' : P(k_1, \theta') \in (c_1, c_2)\}$ firm 1 is capacity constrained but the marginal benefit of one more unit of production does not reach $c_2$, the necessary level to get firm 2 turned on. On the other hand, $(c_1 - c_2)E_\theta(1_{P(k_1,\theta)\leq c_1})$ is a necessary discount for the $\theta$ scenarios where it is not efficient to produce the good at all because the shock is high enough to make consumers unwilling to pay for it.

If we use first-order conditions on $\boldsymbol{q}$ and equation 2 on $c = c_2$, we get:

$$R_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) = E_\theta\left[\left(P\left(\sum_{i=1}^{2} k_i, \theta\right) - c_2\right)^+\right] - \psi(c_2). \quad (5)$$

As a consequence, we know that, if $k_2 > 0$ at the optimal, $\sum_{i=1}^{2} k_i = K^*(c_2)$. That been said, we may proceed to the demonstration.

i) If $|\psi'(c_1)| \geq |\psi'(c_2)| > E_\theta(1_{P(0,\theta)>c_1})) = \mu_1(c_1)$, then it will be optimal to exclude firm 1 and hire firm 2 ($k_1 = 0$, $k_2 > 0$). To see that, note that, if $\boldsymbol{k} = (k_1, k_2)$ for

$k_1, k_2 \geq 0$, first-order conditions on $\boldsymbol{k}$ impose:

$$R_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) - R_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) \leq E_\theta((P(0,\theta) - c_2)1_{P(0,\theta)\in(c_1,c_2)}) + (c_1 - c_2)E_\theta(1_{P(0,\theta)\leq c_1})$$
$$-(c_1 - c_2) - (\psi(c_1) - \psi(c_2)) \leq [\mu_1(c_1) - |\psi'(c_2)|](c_2 - c_1) < 0. \quad (6)$$

Hence, any solution must satisfy $k_1 = 0$. Also, by Assumption 5 and the continuity of $P$ there is $k_2 > 0$ such that

$$R_2(\mathbf{q}, (0, k_2), \mathbf{c}) = E_\theta((P(k_2, \theta) - c_2)^+) - \psi(c_2) = 0.$$

Since $E_\theta(P(k,\theta) - c_2)^+)$ is a strictly decreasing function of $k$ in the interval where this function is strictly positive, $k_2$ is uniquely determined by equation 2 when $c = c_2$. Hence, as $(0, k_2)$ is the only point satisfying first order conditions on capacity and quantity, we have the result. Note that Assumption 5 also gives us that $\{\theta : P(0,\theta) > c_1\}$ is a strictly positive measure set for every $c \in [\underline{c}, \bar{c}]$. As $\mu_1(c_1)$ is clearly decreasing on $c_1 < c_2$, $|\psi'(c)| \geq \mu_1(\underline{c})$ for all $c \in [\underline{c}, \bar{c}]$ guarantees that with probability one (with respect to $G$) the highest cost firm is the only one hired.

ii) Suppose $|\psi'(c_2)| \leq |\psi'(c_1)| < E(1_{(P(K^*(c_2),\theta)>c_2}) = \mu_2(c_2)$ where $K^*(c_2)$ is the capacity level that solves equation 2 for $c = c_2$. As we know $K^*(c)$ may be uniquely pinned down for every $c \in [\underline{c}, \bar{c}]$, we know $\mu_2$ is a well-defined function.

We must then show that firm 2 will necessarily be excluded, while firm 1 is hired ($k_1 > 0$ and $k_2 = 0$).

Because $E_\theta((P(k_1,\theta) - c_2)1_{P(k_1,\theta)\in(c_1,c_2)}) \geq E_\theta[(c_1 - c_2)1_{P(k_1,\theta)\in(c_1,c_2)}]$ for all $k_1 \geq 0$, we have:

$$R_1(\mathbf{q}, \mathbf{k}, \mathbf{c}) - R_2(\mathbf{q}, \mathbf{k}, \mathbf{c}) \geq (c_1 - c_2)E_\theta(1_{P(k_1,\theta)\leq c_2}) - (c_1 - c_2) - [\psi(c_1) - \psi(c_2)] \geq$$
$$(E_\theta(1_{P(k_1,\theta)>c_2}) - |\psi'(c_1)|)(c_2 - c_1)$$

To obtain a contradiction, we suppose $k_2 > 0$ at the optimal. Then, $k_1 \leq$

$\sum_{i=1}^{2} k_2 = K^*(c_2)$ and, consequently:

$$R_1(\mathbf{q}, \mathbf{k}, \mathbf{c}) - R_2(\mathbf{q}, \mathbf{k}, \mathbf{c}) \geq (E_\theta(1_{P(k_1,\theta)>c_2}) - |\psi'(c_1)|)(c_2 - c_1) \geq$$
$$(E_\theta(1_{P(K^*(c_2),\theta)>c_2}) - |\psi'(c_1)|)(c_2 - c_1) = (\mu_2(c_2) - |\psi'(c_1)|)(c_2 - c_1) > 0 \qquad (7)$$

Which shows us that $k_2 > 0$ cannot be optimal because the optimal $k_1$ is strictly greater then $K^*(c_2)$. As a consequence, $\sum_{i=1}^{2} k_1 = k_1 = K^*(c_1) > K^*(c_2)$.

To see that $\mu_2(c_2)$ is decreasing on $c_2$ note that, if $K^*(c_1) < K^*(c_2)$, $E_\theta(1_{P(K^*(c_1))>c_1}) \geq E_\theta(1_{P(K^*(c_1))>c_2}) \geq E_\theta(1_{P(K^*(c_2))>c_2})$. As a consequence, if $|\psi'| \leq \mu_2(\bar{c})$, we know the highest cost firm is excluded with probability 1. As $K^*(\bar{c}) > 0$ by Assumption 5 and $\psi > 0$, we know $\mu_2(\bar{c}) > 0$.

Finally, Assumption 5 gives us $\mu_1(c_1) > \mu_2(c_1) \geq \mu_2(c_2)$ because $K^*(c) > 0$ for every $c \in [\underline{c}, \bar{c}]$. □

***Proof of Lemma 1***. The solution comes in two parts.

i) Straight from the continuity of First-Best solutions, proved in Lemma A.

ii) Given a measure $M$, we start by rewriting equations 4 and 5 in a convenient way and showing the necessary and sufficient first-order conditions for having both firms hired.

Call

$\xi_1(k_1|M) = \int 1_{P(k,\theta)\leq c_2}(\max\{c_1, P(k,\theta)\} - c_2)dM(\theta)$ for all $k_1 \geq 0$;

$\xi_2(k|M) = \int 1_{P(k,\theta)>c_2}(P(k,\theta) - c_2)dM(\theta)$ for $k \geq 0$;

$k = \sum_{i=1}^{2} k_i$ for $k_1, k_2 \geq 0$.

Then, we may rewrite equation 4 as:

$$R_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) - R_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) = \xi_1(k_1|M) - [(c_1 + \psi(c_1)) - (c_2 + \psi(c_2))]\,;$$

Likewise, we may rewrite equation 5 as:

$$R_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) = \xi_2(k|M) - \psi(c_2).$$

Consequently, we have both firms hired for a given $\boldsymbol{c}$ if and only if:

$$\xi_1(k_1|M) - [(c_1 + \psi(c_1)) - (c_2 + \psi(c_2))] = 0 \tag{8}$$

$$\xi_2(k|M) - \psi(c_2) = 0 \tag{9}$$

Now we prove some technical properties of $\xi_1$ and $\xi_2$ that will help establish our results on continuity:

**Claim 1.** Let $M$ be the probability measure induced by shocks, $k_1 > 0$ the scalar that solves equation 8 and $k$ the scalar solving equation 9. Then:

i) $\xi_1(.|M)$ is a continuous non-increasing function, which is injective in a neighbourhood $N_1$ of $k_1$.

ii) $\xi_2(.|M)$ is a continuous non-increasing function, which is injective in a neighbourhood $N_2$ of $k$.

*Proof.* Let's divide this proof in two parts:

i) $\xi_1$ is always non-positive and, if $k_1'' > k_1'$, $\{\theta : P(k_1', \theta) \leq c_2\} \subseteq \{\theta : P(k_1'', \theta) \leq c_2\}$ and:

$$0 \leq \int 1_{P(k_1',\theta)\leq c_2}(\max\{c_1, P(k_1', \theta)\} - \max\{c_1, P(k_1, \theta)\})dM(\theta) \leq \xi_1(k_1'|M) - \xi_1(k_1''|M) \leq \int 1_{P(k_1'',\theta)\leq c_2}(\max\{c_1, P(k_1', \theta)\} - \max\{c_1, P(k_1'', \theta)\})dM(\theta).$$

To see that, note that $\xi_1(k_1|M) \leq \int 1_{P(k_1',\theta)\leq c_2}\Bigg(\max\{c_1, P(k_1, \theta)\} - c_2\Bigg)dM(\theta)$ and $\xi_1(k_1'|M) \leq \int 1_{P(k_1,\theta)\leq c_2}\Bigg(\max\{c_1, P(k_1', \theta)\} - c_2\Bigg)dM(\theta)$.

Hence, we get immediately that $\xi_1$ is non-increasing and that its continuity is a direct consequence of the continuity of $\max\{c_1, P(k, \theta)\} - c_2$. To see why it has to be injective in a neighbourhood of $k_1$, just note that, by Lemma A, first-best solutions are unique and there can be but one $k_1$ such that $\xi_1(k_1|M) = (c_1 - c_2) + (\psi(c_1) - \psi(c_2))$. As a consequence, $\xi_1$ must be strictly decreasing on $k_1$. Because $\xi_1$ is continuous, $\xi_1(k_1''|M) - \xi_1(k_1'|M)$ is continuous on $(k_1'', k_1')$ and we have the result.

ii) note that $\xi_2$ is non-negative and, if $k' > k$, $\{\theta : P(k,\theta) > c_2\} \subseteq \{\theta : P(k',\theta) > c_2\}$:

$$0 \leq \int 1_{P(k',\theta)>c_2}(P(k',\theta) - P(k,\theta))dM(\theta) \leq \xi_2(k', M) - \xi_2(k, M) \leq \int 1_{P(k,\theta)>c_2}(P(k',\theta) - P(k,\theta))dM(\theta)$$

Hence, $\xi_2$ is continuous and non-increasing as a consequence of $P$ being continuous and decreasing.To see why it has to be injective in a neighbourhood of $k$, just note once again that first-best solutions are unique by Lemma A and therefore $\xi_2$ is strictly decreasing on $k$. By continuity, there must be a neighbourhood $N_2$ of $k_2$ such that this property holds.

□

That being said, we know that if $z_1 \in N_1$ and $z \in N_2$

$$\forall\, \delta > 0 \text{ there is } \varepsilon_\delta > 0 \text{ such that } |\xi_1(z_1|M) - \xi_1(z_1'|M)| < \varepsilon_\delta \iff |z_1 - z_1'| < \delta$$

$$\forall\, \delta > 0 \text{ there is } \sigma_\delta > 0 \text{ such that } |\xi_2(z|M) - \xi_2(z'|M)| < \sigma_\delta \iff |z - z'| < \delta$$

Now let $k_1'$ be the solution to equation 8 when we replace $M$ for $M'$ and $k'$ the solution to equation 9 when we replace $M$ for $M'$.

If $\upsilon = P(k',0)\min\{\xi_2(k'|M'), \xi_1(k_1'|M'))\}$, we can take $\epsilon > 0$ such that $\epsilon \leq \min\{\varepsilon_\delta, \sigma_\delta, \upsilon\}$ for a given $\delta > 0$ and we guarantee that, if $d(M, M') \leq \frac{\epsilon}{P(k_1',0)} = \frac{\epsilon}{\|P(k_1',.)\|}$ then:

$|\xi_1(k_1'|M) - \xi_1(k_1'|M')| = |\xi_1(k_1'|M) - \xi_1(k_1|M)| = |\xi_1(k_1'|M) - (c_1 - c_2) - (\psi(c_1) - \psi(c_2))| < \epsilon$;

$|\xi_2(k'|M) - \xi_2(k'|M')| = |\xi_2(k'|M) - \xi_2(k|M)| = |\xi_1(k_1'|M) - \psi(c_2))| < \epsilon$.

As a consequence, $k_1' \in B_\sigma(k_1)$ and $k' \in B_\sigma(k)$. If we make $\sigma$ small enough, we get $k' > k_1' > 0$.

□

***Proof of Proposition 2***. Consider the first order conditions of the first-best problem for binary distributions and $c_1 < c_2$ such that $|\psi'(c_1)|, |\psi'(c_2)| \in (\mu, 1)$. If we want both firms to be hired, we need the capacity constraint of firm 1 to be binding in both scenarios and capacity constraint of firm 2 to be binding only in the low state. Hence,

first-order conditions impose that there must be $k_1, k_2 > 0$ such that

$$R_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) - R_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) = 0 \iff P(k_1, H) = c_2 - \frac{(c_2 - c_1)(1 - \alpha(\mathbf{c}))}{1 - \mu}. \tag{10}$$

For $\alpha(\boldsymbol{c}) = -\frac{\psi(c_2) - \psi(c_1)}{c_2 - c_1} \in (\mu, 1)$ and:

$$R_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) = 0 \iff P(k_1 + k_2, L) = c_2 + \frac{\psi(c_2)}{\mu}. \tag{11}$$

Note that because $\alpha(\mathbf{c}) \in (\mu, 1)$ inverse demands are consistent with $\lambda_1(H), \lambda_1(H), \lambda_2(L) > 0$ and $\lambda_2(H) = 0$. They are also consistent with $q_2(H) = 0$, $q_2(L) = k_2$ and $q_1(\theta) = k_1$ for all $\theta \in \{L, H\}$.

That being said, we prove two important claims:

**Claim 2.** There are thresholds $H^+(\boldsymbol{c}) > H^-(\boldsymbol{c})$ such that:

a) $H^+(\boldsymbol{c})$ solves for $P(0, H^+(\boldsymbol{c})) = c_2 - \frac{(c_2 - c_1)(1 - \alpha(\mathbf{c}))}{1 - \mu}$ and $k_1^{FB}(\mathbf{c}) > 0 \iff H < H^+(\boldsymbol{c})$.

b) $H^-(\boldsymbol{c})$ solves for $P(K^*(c_2), H^-(\boldsymbol{c})) = c_2 - \frac{(c_2 - c_1)(1 - \alpha(\mathbf{c}))}{1 - \mu}$ and $k_2^{FB}(\mathbf{c}) > 0 \iff H > H^-(\boldsymbol{c})$.

*Proof.*

a) Let's begin by showing the existence and unicity of $H^+(\boldsymbol{c})$ . If $P(0; 1) = 0$, $P(0; 0)$ is sufficiently high and $P(0, H)$ is continuous and strictly decreasing on H for $H \in [0, 1]$, there is exactly one $H^+(\boldsymbol{c}) \in (0, 1)$ such that $P(0, H^+(\boldsymbol{c})) = c_2 - \frac{(c_2 - c_1)(1 - \alpha(\mathbf{c}))}{1 - \mu}$.

If $H \geq H^+(\mathbf{c})$, then $R_1(\mathbf{q}, \mathbf{k}, \mathbf{c}) - R_2(\mathbf{q}, \mathbf{k}, \mathbf{c}) < 0$ and we may conclude that any solution must involve $k_1(\mathbf{c}) = 0$. If $H < H^+(\mathbf{c})$, by equation 10, $R_1(\mathbf{q}, (0, k_2), \mathbf{c}) - R_2(\mathbf{q}, (0, k_2), \mathbf{c}) > 0$ for all $k_2 \geq 0$ and it must be that $k_1^{FB}(\mathbf{c}) > 0$.

To see that $H^+(\boldsymbol{c})$ decreases monotonically with $\mathbf{c}$, note that $P$ is strictly decreasing in $\theta$ and $c_2 - \frac{(c_2 - c_1)(1 - \alpha(\mathbf{c}))}{1 - \mu} = \frac{c_1 + \psi(c_1) - [\psi(c_2) + \mu c_2]}{1 - \mu}$ is strictly increasing in $c_1$ and $c_2$.

b) Note that $K^*(c_2) > 0$ is a well-defined function of $c_2$. Now, for similar reasons, there must be exactly one $H^-(\boldsymbol{c})$ such that $P(K^*(c_2), H^-(\boldsymbol{c})) = c_2 - \frac{(c_2-c_1)(1-\alpha(\mathbf{c}))}{1-\mu}$. Because $K^*(c_2) > 0$ by Assumption 5, we guarantee that $H^-(\boldsymbol{c}) < H^+(\boldsymbol{c})$.

   Now we show that, if $H > H^-(\boldsymbol{c})$ $k_2^{FB}(\mathbf{c}) > 0$. If $H^+(\boldsymbol{c}) > H > H^-(\boldsymbol{c})$ we know that there is some $k_2 > 0$ $k_1 > 0$ such that $k_1$ solves equation 10 and $\sum_{i=1}^{2} k_i$ solves equation 11 and, consequently, both firms are hired. If $H^+(\boldsymbol{c}) \geq H^+(\boldsymbol{c})$, we know $k_1^{FB}(\boldsymbol{c})$ and, by Lemma B it must be that $k_2^{FB}(\boldsymbol{c}) > 0$.

   If $H \leq H^-(\mathbf{c})$, then $R_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) - R_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) > 0$ for all $k_1 \leq K^*(c_2)$ and $k_2 \geq 0$. As $K^*(c_2)$ solves for 11 $k_2^{FB}(\mathbf{c}) = 0$ in that case.

□

**Claim 3.** Firm 1 produces all its capacity with probability 1 and firm 2 produces all its capacity with probability $\mu$ and nothing with probability $1 - \mu$.

*Proof.* The result has already been shown for the case where $H \in (H^-(\boldsymbol{c}), H^+(\boldsymbol{c}))$.

If $H \geq H^+(\boldsymbol{c})$, we hire only firm 2. Because $P(0, H) < c_2$ by definition of $H^-(\boldsymbol{c})$ and $\alpha(\boldsymbol{c}) \in (\mu, 1)$, there is no reason to have firm 2 produce in the high shock scenario.

Suppose now $H \leq H^-(\boldsymbol{c})$, then clearly $P(0, H) > P(K^*(c_2), H) > c_1$, so the low cost firm produces in the high shock scenario. To see that all its capacity will be used, note that, if $k_1$ is such that $P(k_1, H) \leq c_1$, then $R(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) = P(k_1, H) - \left[c_2 - \frac{(c_2-c_1)(1-\alpha(\boldsymbol{c}))}{1-\mu}\right] < 0$ for every $k_2 \geq 0$, which is a contradiction with $k_1$ being optimal. □

Itens $i)$, $ii)$ and $iii)$ are direct consequences of Claim 2 and Claim 3.

□

***Proof of Corollary 1***. If $\mu$ such that $\mu \leq |\psi'(c_i)| \leq 1$ for every $i \in \{1, 2\}$ and $H \in (H^-(\boldsymbol{c}), H^+(\boldsymbol{c}))$, then Proposition 2 gives us that both firms are hired with positive capacity when we consider measure $M$. As a consequence, there is an open set of measures including $M$ such that this property holds by Lemma 1. □

***Proof of Proposition 3***. We can show this by example. Let $\boldsymbol{c}$ be a type vector such that $\psi'(c_1)|, |\psi'(c_2)| < 1$. If $H \in (H^-(\boldsymbol{c}), H^+(\boldsymbol{c}))$ and $\mu < \psi'(c_1)|, |\psi'(c_2)|$, we may build a probability measure $M$ in the following way:

1. $M(\{0\}) = \mu$;

2. $M([0,1] \setminus \{0, H\}) = \gamma$ for $\gamma > 0$ sufficiently small;

3. $M$ has full support.

By corollary 1, both firms are hired in this case, i.e., $0 < k_1^{FB}(\boldsymbol{c}) < \sum_{i=1}^{2} k_i^{FB}(\boldsymbol{c})$. As a consequence, $A_1 = \left\{\theta : P(k_1^{FB}(\boldsymbol{c}), \theta) < c_1 < P(0, \theta)\right\}$ and $A_2 = \left\{\theta : P(k_1^{FB}(\boldsymbol{c}), \theta) > c_1, P(k_1^{FB}(\boldsymbol{c}) + k_1^{FB}(\boldsymbol{c}), \theta) < c_2\right\}$ are non-empty and non-degenerate intervals.

Using first-order conditions on $\boldsymbol{q}$, we see that, if $\theta \in A_1$, then $q_1^{FB}(\boldsymbol{c}, \theta) \in (0, k_1^{FB}(\boldsymbol{c}))$. Also, if $\theta \in A_2$, then $q_2^{FB}(\boldsymbol{c}, \theta) \in (0, k_2^{FB}(\boldsymbol{c}))$.

Because $M$ has full support, $M(A_1), M(A_2) > 0$ and we have our result. $\square$

## b Second-best allocation

In the demonstrations of this section, call:

i) $\bar{\pi}_i(z) = E_{-i}(\pi_i^{SB}(z, .))$

ii) $\bar{q}_i(z) = E_{-i}(q_i^{SB}(z, .))$

iii) $\bar{k}_i(z) = E_{-i}(k_i^{SB}(z, .))$

We also simplify notation by calling $\frac{d}{dz}\bar{\pi}(z) = \bar{\pi}'(z)$.

***Proof of Lemma 2.*** To see that, suppose the central planner can choose in the space of lotteries in $\Gamma$, i.e., choose any direct mechanism. First note that the expectation of any lottery $\sigma \in \Delta(\Gamma)$, $E(\sigma|\boldsymbol{c})$ must be an element of $\Gamma$ because feasibility conditions are linear.

Note also that, as firms are risk-neutral, incentive compatibility and participation constraints depend only on the expectations of that lottery. The principal's payoff is also linear in expenditure, so the way informational rents affect her payoff also depends only on expectations. As a consequence, the lottery $\sigma$ respects incentive compatibility and participation constraints if and only if $E(\sigma|\boldsymbol{c})$ also does. Because $V$ is strictly concave on the amount of good consumed, the principal always prefers $E(\sigma|\boldsymbol{c})$ to $\sigma$.

Hence, the optimal mechanism in $\Gamma$ has to be optimal among all direct mechanisms. To see that it will be optimal among all direct and indirect mechanisms we use the revelation principle. $\square$

Lemmas C-D are helpful to prove Propositions 4 and 5. Lemma E is important for propositions 4 and 7.

**Lemma (C).** Consider the following conditions and $i \in \{1, 2\}$:

i) $\partial\bar{\pi}(z) = -\bar{q}_i(z) - \psi'(z)\bar{k}_i(z)$ (Envelope)

ii) $\partial\bar{\pi}(z)$ non-decreasing (MON)

Condition $i)$ is a necessary condition for $(IC_i)$ be satisfied. If $i)$ and $ii)$ hold, we have sufficiency. If $\psi$ is linear, $ii)$ is also necessary and $i)$ and $ii)$ are equivalent to $(IC_i)$.

***Proof of Lemma C.*** Using the envelope theorem and the fact that $(IC)_i$ is equivalent to
$c_i \in \text{argmax}_z \left\{\bar{\pi}_i(z) - (c_i - z)\bar{q}_i(z) - (\psi(c_i) - \psi(z))\bar{k}_i(z)\right\}$, we have that $i)$ is a necessary condition.

Using condition $i)$, we must show sufficiency of $i)$ and $ii)$ and the necessity of $ii)$ if $\psi$ is linear.

i) If $z > c_i$

$$\begin{aligned}\bar{\pi}_i(c_i) - \bar{\pi}_i(z) - (z - c_i)\bar{q}_i(z) - (\psi(z) - \psi(c_i))\bar{k}_i(z) &= \\ \int_{c_i}^{z} \left[\bar{q}_i(\upsilon) + \psi'(\upsilon)\bar{k}_i(\upsilon)\right] d\upsilon - (z - c_i)\bar{q}_i(z) - (\psi(z) - \psi(c_i))\bar{k}_i(z) &\geq \\ \int_{c_i}^{z} \left\{ \left[\bar{q}_i(\upsilon) + \psi'(\upsilon)\bar{k}_i(\upsilon)\right] - \left[\bar{q}_i(z) + \psi'(z)\bar{k}_i(z)\right] \right\} d\upsilon\end{aligned}$$

The last inequality holds because $\psi$ is convex. If $\psi$ is linear, this inequality becomes an equality. Hence, for $\psi$ convex $i)$ and $ii)$ are sufficient to ensure a type $c_i$ does not deviate upwards. If $\psi$ is linear, $ii)$ condition is also necessary.

The same point can be made for $z < c_i$:

ii) If $z < c_i$,

$$\bar{\pi}_i(c_i) - \bar{\pi}_i(z) - (z - c_i)\bar{q}_i(z) - (\psi(z) - \psi(c_i))\bar{k}_i(z) \geq$$
$$\int_z^{c_i} \Big\{ \big[-\bar{q}_i(\upsilon) - \psi'(\upsilon)\bar{k}_i(\upsilon)\big] - \big[-\bar{q}_i(z) - \psi'(z)\bar{k}_i(z)\big] \Big\} d\upsilon$$

Where equality holds if $\psi$ is linear. Hence, we have sufficiency of $i)$ and $ii)$ and necessity of $ii)$ when $\psi$ is linear. □

**Lemma (D).** If (MON) and (envelope) hold, then there is $n_i \in [\underline{c}, \bar{c}]$ receiving zero rents and the unconditional expectation on $i$'s profit is given by:

$$E(\pi_i(\boldsymbol{c})) = E\Bigg(\left(\frac{1 - G(c_i)}{g(c_i)}\right)(-q_i^{SB}(\boldsymbol{c}) - \psi'(c_i)k_i^{SB}(\boldsymbol{c}))1_{c_i<n_i} + \frac{G(z)}{g(z)}(q_i^{SB}(\boldsymbol{c}) + \psi'(c_i)k_i^{SB}(\boldsymbol{c}))1_{z>n_i}\Bigg)$$

The same formula can be derived if there is $c \in [\underline{c}, \bar{c}]$ such that $\bar{\pi}(z) \geq 0$ for all $z \geq c$ and $\bar{\pi}(z) \leq 0$ for all $z \leq c$.

***Proof of Lemma D.*** If $\bar{\pi}_i'(z)$ is weakly increasing, one of the following must be true:

1. $\bar{\pi}_i'(a) \geq 0$ for all $a \in [\underline{c}, \bar{c}]$;
2. $\bar{\pi}_i'(a) \leq 0$ for all $a \in [\underline{c}, \bar{c}]$;
3. There is at least an intermediate $n_i$ such that $\bar{\pi}_i'(c_i) \leq 0$ for all $c_i < n_i$ and $\bar{\pi}_i'(c_i) \geq 0$ for all $c_i > n_i$.

In any case, the optimal way of respecting participation constraints and envelope condition is making $\bar{\pi}_i(n_i) = 0$ and:

$$E(\pi_i(\boldsymbol{c})) = E_{c_i}\left[\left(\int_{n_i}^{c_i}(-\bar{q}_i(a) - \psi'(a)\bar{k}^i(a))da\right)1_{c_i>n_i} + \left(\int_{c_i}^{n_i}(\bar{q}_i(a) + \psi'(a)\bar{k}_i(a))da\right)1_{c_i<n_i}\right].$$

As a consequence, we can write:

$$E(\pi_i(\boldsymbol{c})) = \int_{n_i}^{\bar{c}}\left(\int_{n_i}^{c_i}\left(\int_{\underline{c}}^{\bar{c}}(-q_i^{SB}(a,c_{-i}) - \psi'(a)k_i^{SB}(a,c_{-i}))g(c_{-i})dc_{-i}\right)da\right)g(c_i)dc_i +$$
$$\int_{\underline{c}}^{n_i}\left(\int_{c_i}^{n_i}\left(\int_{\underline{c}}^{\bar{c}}(q_i^{SB}(a,c_{-i}) + \psi'(a)k_i^{SB}(a,c_{-i}))g(c_{-i})dc_{-i}\right)da\right)g(c_i)dc_i =$$
$$\int_{n_i}^{\bar{c}}\left(\int_{n_i}^{c_i}\left(\int_{\underline{c}}^{\bar{c}}\frac{(-q_i^{SB}(a,c_{-i}) - \psi'(a)k_i^{SB}(a,c_{-i}))}{g(a)}g(c_{-i})dc_{-i}\right)g(a)da\right)g(c_i)dc_i +$$
$$\int_{\underline{c}}^{n_i}\left(\int_{c_i}^{n_i}\left(\int_{\underline{c}}^{\bar{c}}\frac{(q_i^{SB}(a,c_{-i}) + \psi'(a)k_i^{SB}(a,c_{-i}))}{g(a)}g(c_{-i})dc_{-i}\right)g(a)da\right)g(c_i)dc_i.$$

By changing the order of integrals, the previous expression can be written as:

$$E(\pi_i(\boldsymbol{c})) = \int_{n_i}^{\bar{c}}\left(\int_{\underline{c}}^{\bar{c}}\left(\int_{a}^{\bar{c}}g(c_i)dc_i\right)\frac{(-q_i^{SB}(a,c_{-i}) - \psi'(a)k_i^{SB}(a,c_{-i}))}{g(a)}g(c_{-i})dc_{-i}\right)g(a)da +$$
$$\int_{\underline{c}}^{n_i}\left(\int_{\underline{c}}^{\bar{c}}\left(\int_{\underline{c}}^{a}g(c_i)dc_i\right)\frac{(q_i^{SB}(a,c_{-i}) + \psi'(a)k_i^{SB}(a,c_{-i}))}{g(a)}g(c_{-i})dc_{-i}\right)g(a)da =$$
$$\int_{n_i}^{\bar{c}}\left(\int_{\underline{c}}^{\bar{c}}(1-G(a))\frac{(-q_i^{SB}(a,c_{-i}) - \psi'(a)k_i^{SB}(a,c_{-i}))}{g(a)}g(c_{-i})dc_{-i}\right)g(a)da +$$
$$\int_{\underline{c}}^{n_i}\left(\int_{\underline{c}}^{\bar{c}}G(a)\frac{(q_i^{SB}(a,c_{-i}) + \psi'(a)k_i^{SB}(a,c_{-i}))}{g(a)}g(c_{-i})dc_{-i}\right)g(a)da =$$
$$\int_{\underline{c}}^{\bar{c}}\int_{\underline{c}}^{\bar{c}}\left[\left((1-G(a))\frac{(-q_i^{SB}(a,c_{-i}) - \psi'(a)k_i^{SB}(a,c_{-i}))}{g(a)}\right)1_{a>n_i} +\right.$$
$$\left.\left(G(a)\frac{(q_i^{SB}(a,c_{-i}) + \psi'(a)k_i^{SB}(a,c_{-i}))}{g(a)}\right)1_{a<n_i}\right]dG(a)dG(c_{-i}) =$$
$$E_{c_i}\left(E_{c_{-i}}\left(\frac{-q_i^{SB}(\boldsymbol{c}) - \psi'(c_i)k_i^{SB}(\boldsymbol{c})}{g(c_i)}(1-G(c_i))1_{c_i>n_i} + \frac{q_i^{SB}(\boldsymbol{c}) - \psi'(c_i)k_i^{SB}(\boldsymbol{c})}{g(c_i)}G(c_i)1_{c_i<n_i}\right)\right)$$

And we have our result.

□

**Lemma (E).** Suppose we are under Assumptions 1-8:

i) If $|\psi'|> 1$, $-Q^*(c) - \psi'(c_i)K^*(c) \geq 0$ is continuous and strictly increasing on $c$.

ii) If $|\psi'|< \mu$, $-Q^*(c) - \psi'(c_i)K^*(c) \leq 0$ is continuous and strictly increasing on $c$.

For $Q^*(c) = E_\theta(Q_i^*(c,\theta))$.

*Proof.* i) To see that, note that it is a.s. true that there is $i$ such that $c_i > c_{-i}$. If $c_1 < c_2$, there is only one solution to this problem in which only firm 2 is hired. Hence, the probability of having strictly positive profits (by being the highest

type) increases as $c$ increases. Now note that, as $c + \psi(c)$ is a strictly decreasing function, the value $K^*(c)$ solving equation 2 strictly increases with $c$. Note that, by feasibility reasons, $Q^*(c) \leq K^*(c)$ and $(P(Q,\theta) - c)^+$ is a decreasing function of $c$ for all $\theta$. As a consequence, if $c' > c$, $Q^*(c') < Q^*(c)$ and we have the result.

ii) If $c_1' < c_1$, note that $K^*(c_1') > K^*(c_1)$ because, as full capacity is always used in the low shock scenario, $E_\theta((P(K^*(c_1),\theta) - c_1')^+) - \psi(c_1') > 0$ once $\mu c + \psi(c)$ is an increasing function of $c$. As a consequence, $\boldsymbol{Q}^*(c_1,\theta) \leq \boldsymbol{Q}^*(c_1',\theta)$ because $(P(Q,\theta) - c)^+$ is a decreasing function of $c$ and capacity levels decrease with types. Thus, $[-Q^*(c_1') - \psi'(c_1')K^*(c_1'))] - [-Q^*(c_1) - \psi'(c_1)K^*(c_1'))] \geq (-\mu - \psi'(c_1'))[K^*(c_1') - K^*(c_1)] < 0$.

For results on continuity just apply Lemma A. □

***Proof of Proposition 4.*** We have two cases:

i) If $|\psi'| > 1$:

Note that participation constraints will bind for $\underline{c}$ because, if

$|\psi'(c_i)| > 1$, $\bar{\pi}_i'(c_i) = -\bar{q}_i(c_i) - \psi'(c_i)\bar{k}_i(c_i) > 0$ for every $c_i \in [\underline{c}, \bar{c}]$. Then, using the formula in Lemma D, envelope condition gives us:

$$E\left(\pi_i(\boldsymbol{c})\right) = E\left[\left(\frac{1 - G(c_i)}{g(c_i)}\right)\left[-q_i(\boldsymbol{c}) - \psi'(z)k_i(\boldsymbol{c})\right]\right]. \tag{12}$$

By replacing 12 on the problem of finding the optimal mechanism in $\Gamma$ we get that the pointwise solution is exactly the first-best problem for types $(\rho_2(c_1), \rho_2(c_2))$ and marginal cost of capacity $\psi_2$. Note also that, if $|\psi'| > 1$, $|\psi_2'| > 1$. Hence, solutions are as in Proposition 1. As in first-best problem, Assumption 8 guarantees that all interior types are hired with positive probability and that total capacity is always positive. Note that this is a relaxed version of the problem in which we impose only a necessary condition for incentive compatibility.

To conclude the argument that this is actually the second-best solution, we use Lemma E and the fact that higher types have higher probability of being hired to show monotonicity holds, and, therefore, we have $(IC_i)$.

ii) If $|\psi'| < \mu$, then it is clear that $\bar{\pi}_i'(z) = -\bar{q}_i(c_i) - \psi'(c_i)\bar{k}_i(c_i) < 0$ for every $c_i \in [\underline{c}, \bar{c}]$

because $q_i(L) = k_i(\boldsymbol{c})$. Hence, using the formula in Lemma (D) one more time:

$$E\left(\pi_i(\boldsymbol{c})\right) = E\left[\left(\frac{G(c_i)}{g(c_i)}\right)\left[q_i(\boldsymbol{c}) + \psi'(z)k_i(\boldsymbol{c})\right]\right] \tag{13}$$

By plugging 13 on the problem of finding the optimal mechanism in $\Gamma$ we find that the pointwise solution is the same as a first-best problem in which $\psi_2$ is the marginal cost of building new capacity and $(\rho_2(c_1), \rho_2(c_2))$ are the types. Once again, we use Assumption 8 to avoid exclusion. Note that, if $|\psi'|< \mu$ $|\psi_2'|< \mu$ and, therefore, only the lowest types are hired. To verify monotonicity, just use Lemma E and the fact that lower types have a higher probability of being lower than their opponents. As a consequence, the solution to this problem is second-best solution.

□

***Proof of Lemma 3.*** The maximization problem can be written as we plug $E(\pi(\boldsymbol{c}))$ derived in Lemma D into the problem of finding the optimal mechanism in $\Gamma$ with a linear $\psi$ (assumption 9). Note that envelope and conditions are equivalent to incentive compatibility. □

Lemma F will be useful for Propositions 5 and 6.

**Lemma (F).** $-q_i^{FB}(\boldsymbol{c})+\alpha k_i^{FB}(\boldsymbol{c})$ is continuous on $\boldsymbol{c}$. If $k_i^{FB}(\boldsymbol{c}) > 0$, it is strictly increasing on $c_i$. Moreover, if Assumption 5 is true, $E_{-i}(-q_i^{FB}(c_i,.) + \alpha k_i^{FB}(c_i,.))$ is strictly increasing on $c_i$.

*Proof.* To see the continuity, just use Lemma A.

To see that $-q_i^{FB}(\boldsymbol{c})+\alpha k_i^{FB}(\boldsymbol{c})$ is strictly increasing on $c_i$ in the region where capacity of firm $i$ is positive, use Proposition 2 to state that, if $c_1 < c_2$, $-q_1^{FB}(\boldsymbol{c}) + \alpha k_1^{FB}(\boldsymbol{c}) = (-1 + \alpha)k_1^{FB}(\boldsymbol{c}) \leq 0 \leq -q_2^{FB}(\boldsymbol{c}) + \alpha k_2^{FB}(\boldsymbol{c}) = (-\mu + \alpha)k_2^{FB}(\boldsymbol{c})$. We can also verify using equations 10 and 11 with $\alpha(\boldsymbol{c}) = \alpha$ for all type vectors $\boldsymbol{c}$ to see that $k_2^{FB}(\boldsymbol{c})$ is strictly increasing on $c_2$ because $\mu c_2 + \psi(c_2)$ is a decreasing function, which makes $K^*(c_2)$ increasing on $c_2$, and also $k_1^{FB}(\boldsymbol{c})$ is strictly decreasing on $c_1$ and $c_2$ when $k_2^{FB}(\mathbf{c})$ and $K_1^*(c_1)$ strictly decreasing on $c_1$ when $c_1 < c_2$. The fact that $H^+, H^-$ are strictly decreasing on both types completes this part of the proof because the probability of a firm i being the only firm hired conditional on being the lowest type, that

is, $\int 1_{\{c_{-i}:H^-(\boldsymbol{c})>H\}} dG(c_{-i})$, decreases with $c_i$ and the probability of being hired at all conditional on being the lowest type, $\int 1_{\{c_{-i}:H^+(\boldsymbol{c})>H\}} dG(c_{-i})$, also decreases. On the other hand, both the probability of being hired with exclusivity and being hired conditional on being the highest type increase with types.

To see that the conditional expectation $E_{-i}(-q_i^{FB}(c_i,.) + \alpha k_i^{FB}(c_i,.))$ is strictly increasing, use the fact that higher types have a higher probability of being higher than opponents and that, by Lemma B, all types have positive probability of being hired. To see the continuity, just note that $G$ is continuous.

□

***Proof of Proposition 5.*** Suppose we drop the monotonicity constraint and impose only feasibility, non-negativity and non-positivity constraints.

We proceed by guess and verify, assuming there are Lagrange multipliers $\gamma(c_i)$ for $(R_i^-)$ and $\delta_i(c_i)$ for $(R_i^+)$ and then verifying that they actually exist. As we know, by Lemma D, that for every $i \in \{1,2\}$ we may find $n_i \in [\underline{c}, \bar{c}]$ receiving zero rents, if multipliers exist we may write:

$$v_i(c_i) = \rho_1(c_i)1_{c_1<n_i} + \rho_2(c_i)1_{c_1>n_i} - \gamma_i(c_i)1_{c_i<n_i} + \delta_i(c_i)1_{c_i>n_i} \tag{14}$$

as virtual types. Although the argument is omitted to simplify notation, note that the virtual types also depend on an optimal $n_i$. Note that, by Lemma D, we know that there is at least one pair of $\boldsymbol{n}$ that solve this problem.

Consequently, the problem in Lemma 3 may be written as:

$$\begin{aligned} \max E\Bigg( V(\mathbf{q}(\mathbf{c})) - \sum_{i=1}^{2} v_i(c_i,n_i)q_i(\mathbf{c}) - \sum_{i=1}^{2} \psi(v_i(c_i,n_i))k_i(\mathbf{c})\Bigg) \\ \text{subject to: } \{\boldsymbol{q}(\boldsymbol{c}),\boldsymbol{k}(\boldsymbol{c})\} \in FA \text{ for every } \boldsymbol{c} \\ \boldsymbol{n} \in [\underline{c},\bar{c}]^2 \end{aligned} \tag{15}$$

Once we choose $\boldsymbol{n}$, the profit maximizing $\boldsymbol{q}^{SB}, \boldsymbol{k}^{SB}$ are optimal first-best allocations with types distorted as equation 14. By Lemma A we know solutions exist. As a consequence, we know the optimal mechanism in $\Gamma$ will exist.

In addition $v_i(c_i)$ must be a continuous function of $c_i$ for every $i \in [\underline{c}, \bar{c}]$ because

the function $E_{-i}(-q_i^{FB}(\boldsymbol{\rho}(\boldsymbol{c})) + \alpha k_i^{FB}((\boldsymbol{\rho}(\boldsymbol{c}))))$ in the constraints is a continuous function of $\boldsymbol{\rho}(\boldsymbol{c}))$, as proved in Lemma A, as multipliers must be just enough to make restrictions work.

Given , we may write two types of possible distortions for firm $i$'s types:

$\boldsymbol{z}_i^1(\boldsymbol{c}) = (v_{-i}(c_{-i}), \rho_1(c_i))$;

$\boldsymbol{z}_i^2(\boldsymbol{c}) = (v_{-i}(c_{-i}), \rho_2(c_i))$.

Once we define these functions, the following claim can be made:

**Claim 4.** One of the following options must be true for firm $i$:

i) $E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^2(\underline{c},.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^2(\underline{c},.))) \geq 0$;

ii) $E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^1(\bar{c},.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^1(\bar{c},.))) \leq 0$;

iii) There is only one $c_{i,\alpha} \in (\underline{c}, \bar{c})$ such that:

$$E_{-i}(-q_i^{FB}(c_{i,\alpha}, v_{-i}(.)) + \alpha k_i^{FB}(c_{i,\alpha}, v_{-i}(.))) = 0 \tag{16}$$

In addition, there is a non-degenerate interval $[a_i, b_i]$, for $a_i = \max\{\rho_1^{-1}(c_{i,\alpha}), \underline{c}\}$ and $b_i = \min\{\rho_2^{-1}(c_{i,\alpha}), \bar{c}\}$ such that, for any $n_i \in [\underline{c}, \bar{c}]$, either non-negativity or non-positivity will bind. All types in $[a_i, b_i]$ must be pooled, i.e., $v(c_i) = c_{i,\alpha}$ for all $c_i \in [a_i, b_i]$.

*Proof.* We know by Lemma F that $E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^1(c_i,.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^1(c_i,.)))$ and $E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^2(c_i,.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^2(c_i,.)))$ are strictly increasing functions of type $c_i$ because $\rho_1$ and $\rho_2$ are strictly increasing functions. [6]

As a consequence, if there is any $c_{i,\alpha} \in (\underline{c}, \bar{c})$ as defined by equation 16, it must be unique.

If $E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^2(\underline{c},.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^2(\underline{c},.))) \geq 0$, then:

$E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^1(c_i,.)) - \alpha k_i^{FB}(\boldsymbol{z}_i^1(c_i,.))) \geq 0$ for all $c_i \in [\underline{c}, \bar{c}]$. In that case, we may choose $n_i = \underline{c}$ and all constraints are satisfied.

Similarly, if, $E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^1(\bar{c},.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^1(\bar{c},.))) \leq 0$ then:

[6] Note that the argument in Lemma B carries on because $v_i$ is continuous for every $i$ and, as a consequence, interior virtual types are never excluded.

$E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^1(c_i,.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^1(c_i,.))) \leq 0$ for all $c_i \in [\underline{c}, \bar{c}]$. If we choose $n_i = \bar{c}$, all constraints are satisfied.

If:

$$E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^2(\underline{c},.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^2(\underline{c},.))) < 0$$
$$E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^1(\bar{c},.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^1(\bar{c},.))) > 0$$

By continuity there must be either $\underline{n}_i = \rho_1^{-1}(c_{i,\alpha}) \in (\underline{c}, \bar{c})$ satisfying:

$$E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^1(\underline{n}_i,.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^1(\underline{n}_i,.))) = 0;$$
$$E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^1(c_i,.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^1(c_i,.))) > 0 \text{ for all } c_i > \underline{n}_i;$$
$$E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^1(c_i,.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^1(c_i,.))) < 0 \text{ for all } c_i \leq \underline{n}_i;$$

or $\bar{n}_i = \rho_1^{-1}(c_{i,\alpha}) \in (\underline{c}, \bar{c})$ satisfying:

$$E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^2(\bar{n}_i,.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^2(\bar{n}_i,.))) = 0;$$
$$E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^2(c_i,.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^2(c_i,.))) > 0 \text{ for all } c_i \geq \bar{n}_i;$$
$$E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^2(c_i,.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^2(c_i,.))) < 0 \text{ for all } c_i < \bar{n}_i.$$

Consider $c_i \in [a_i, b_i]$ for $a_i = \max\{\rho_1^{-1}(c_{i,\alpha}), \underline{c}\}$, $a_i = \min\{\rho_2^{-1}(c_{i,\alpha}), \bar{c}\}$. Then, for every $c_i \in [a_i, b_i]$ we have $E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^2(c_i,.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^2(c_i,.))) < 0$ and $E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^1(c_i,.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^1(c_i,.))) > 0$. As a consequence, either non-negativity or non-positivity constraints will bind in this interval for all possible choices of $n_i$. Because the functions in the restrictions $(R^+)$ and $(R^-)$ are injective, it must be that $a_i < b_i$ because $\rho_1(c) < \rho_2(c)$ for all $c \in [\underline{c}, \bar{c}]$.

Finally, regardless of the $n_i$ we choose all types in $[a_i, b_i]$ will have to be pooled to $c_\alpha$. After all, because the functions in $(R^+)$ and $(R^-)$ are strictly increasing, this is the lowest distortion on types causes optimal allocations to satisfy all restrictions.

If both $\underline{n}_i$ and $\bar{n}_i$ can be found, it must be that $\rho_1(\underline{n}_i) = \rho_2(\bar{n}_i) = c_{i,\alpha}$.

$\square$

**Claim 5.** Given $v_{-i}$, the policy correspondence for $n_i$ is given by:

$$n_i^*(\boldsymbol{c}) = \begin{cases} \{\underline{c}\} \text{ if } E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^2(\underline{c},.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^2(\underline{c},.))) \geq 0 \\ \{\bar{c}\} \text{ if } E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^1(\bar{c},.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^1(\bar{c},.))) \geq 0 \\ [a_i, b_i] \text{ otherwise} \end{cases}$$

*Proof.* Call $\{k_i(\boldsymbol{c}), \boldsymbol{q}_i(\boldsymbol{c})\}$ the optimal first-best allocation for virtual types $\boldsymbol{\rho}$ when we choose $n_i$ according to this rule, i.e. $v_i(c_i) = c_\alpha 1_{[a_i,b_i]} + \rho_1(c_i)1_{[\underline{c},a_i)} + \rho_2(c_i)1_{(b_i,\bar{c}]}$ and $v_{-i}$ as given.

If we choose $n_i'$ that does not follow this policy rule, $\{k_i(\boldsymbol{c}), \boldsymbol{q}_i(\boldsymbol{c})\}$ will either violate $(R^+)$ for a non-degenerate interval or we violate $(R^-)$ for a non-degenerate interval. To see that, note that, if we choose $n' > n$ for all $n \in n_i^*(\boldsymbol{c})$, then $\bar{n}_i$ exists and:

$$E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^1(\bar{n}_i,.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^1(\bar{n}_i,.))) > 0$$

Because $\rho_1 > \rho_2$. For continuity reasons, there will be a non-degenerate interval $(b_i, c')$ for $b_i < c' \leq n'$ such that $(R^-)$ does not hold. As a consequence, these types will also have to be pooled to $c_{i,\alpha}$.

Similarly, if we choose $n' < n$ for all $n \in n_i^*(\boldsymbol{c})$, then $\underline{n}_i = \rho_1^{-1}(c_{i,\alpha}) \in (\underline{c}, \bar{c})$ and:

$$E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^2(\underline{n}_i,.)) + \alpha k_i^{FB}(\boldsymbol{z}_i^1(\underline{n}_i,.))) > 0$$

Because $\rho_1 > \rho_2$. For continuity reasons, there will be a non-degenerate interval $(c', a_i)$ for $a_i > c' \geq n'$ such that $(R^-)$ does not hold. As a consequence, these types will also have to be pooled to $c_\alpha = \rho_1(\underline{n}_i)$.

As a consequence, we will have a pooling interval $A_i' = [a_i', b_i']$ such that $[a_i, b_i] \subseteq A_i'$.

Now note that, if $\{k_i'(\boldsymbol{c}), \boldsymbol{q}_i'(\boldsymbol{c})\}$ is the first-best solution when types are distorted to $\rho^i(c_i) = \rho_1(c_i)1_{[\underline{c},a_i']} + c_{i,\alpha}1_{A_i'} + \rho_2(c_i)1_{[b_i',\bar{c}]}$, then $\{k_i'(\boldsymbol{c}), \boldsymbol{q}_i'(\boldsymbol{c})\}$ satisfies $(R^+)$ and $(R^-)$ for $n \in n_1^*(\boldsymbol{c})$.

By the Weak Axiom of Revealed Preference, we know that the planner must have a strict preference for $\{k_i(\boldsymbol{c}), \boldsymbol{q}_i(\boldsymbol{c})\}$ over $\{k_i'(\boldsymbol{c}), \boldsymbol{q}_i'(\boldsymbol{c})\}$. □

Given an $n_i \in n^*(c_i)$, we may find Lagrange multipliers for the restrictions, which are:

For $c_i < n_i$:

$$\gamma(c_i) = \begin{cases} 0, \text{ if } E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^1(\bar{c}, .)) + \alpha k_i^{FB}(\boldsymbol{z}_i^1(\bar{c}, .))) \leq 0 \\ \max\{0, \rho_1(c_i) - c_\alpha\} \text{ otherwise} \end{cases}$$

For $c_i > n_i$:

$$\delta(c_i) = \begin{cases} 0, \text{ if } E_{-i}(-q_i^{FB}(\boldsymbol{z}_i^2(\underline{c}, .)) + \alpha k_i^{FB}(\underline{z}_i^1(\bar{c}, .))) \leq 0 \\ \max\{0, c_\alpha - \rho_2(c_i)\} \text{ otherwise} \end{cases}$$

**Claim 6.** The virtual type function is the same for both types.

*Proof.* We argue that, if there is a pooling, $c_{1,\alpha} = c_{2,\alpha}$. To see that, suppose $c_{1,\alpha} > c_{2,\alpha}$ for some i. Then, it must be that $a_1 \geq a_2$ and $b_1 \geq b_2$, with at least one strict inequality, because $a_i, b_i$ are increasing functions of $c_{i,\alpha}$. As $-q_i^{FB}(\boldsymbol{c}) + \alpha k_i^{FB}(\boldsymbol{c})$ is strictly increasing, we know $v_1(c) \geq v_2(c)$ (with strict inequality for a positive-measure set). As a consequence, the distribution of $v_1$ first-order stochastically dominates the distribution of $v_{-1}$ and, consequently, $\bar{\pi}_1'(c) \geq \bar{\pi}_{-1}'(c)$ for every $c$ as a consequence of Lemma F. Then, as non-positivity restrictions will bind and non-negativity restrictions will become slack for lower types, $\underline{n}_1 \leq \underline{n}_2$ and $\bar{n}_1 \leq \bar{n}_2$, a contradiction. □

As a consequence, we may call $v_1 = v^2 = \rho$, $c_{1,\alpha} = c_{2,\alpha} = c_\alpha$. Also, we can use $\bar{n}_i = \rho_2^{-1}(c_\alpha)$ and $\underline{n}_i = \rho_1^{-1}(c_\alpha)$.

As a direct consequence of Proposition 2, $\underline{H}^{SB} < \underline{H}^{FB} < \bar{H}^{FB} < \bar{H}^{SB}$ because thresholds are strictly decreasing on types. Five cases are then possible:

i) If $H \in (\underline{H}^{FB}, \bar{H}^{FB})$. Then, by Proposition 2 both $\underline{c}$ and $\bar{c}$ have positive probability of being hired with a positive capacity. Because $q_i^{FB}(\underline{c}, c_{-i}) = k_i^{FB}(\underline{c}, c_{-i})$ with probability one and $q_i^{FB}(\bar{c}, c_{-i}) = \mu k_i^{FB}(\bar{c}, c_{-i})$ with probability one, $\bar{\pi}'(\underline{c}) < 0$, $\bar{\pi}'(\bar{c}) > 0$. Consequently, the continuity of $\bar{\pi}'(z)$ proved in Lemma F gives us that there are thresholds $\underline{n}, \bar{n} \in [\underline{c}, \bar{c}]$. To see that, note that $\rho_1(\underline{c}) = \underline{c}$ and $\rho_2(\bar{c}) = \bar{c}$.

ii) If $H \in [\bar{H}^{FB}, \bar{H}^{SB})$ then it is true that $\bar{n} > \underline{c}$ and $\underline{n} \leq \underline{c}$ because type $\underline{c}$ is hired with probability zero in first-best, but if we distort types to $\boldsymbol{\rho}_2$ the lowest type is $\rho_2(\underline{c})$, it is hired with positive probability, so $E_{-i}(-q_i^{FB}(\boldsymbol{\rho}_2(\underline{c}, .)) + \alpha k_i^{FB}(\boldsymbol{\rho}_2(\underline{c}, .)) < 0$. As

a consequence, there will be a pooling interval on the lowest types and the types $c$ not pooled will be distorted to $\rho_2(c)$.

iii) If $H \in (\bar{H}^{SB}, \bar{H}^{FB}]$, then the same point can be made to show that there will be pooling on the highest types and the types not pooled will be distorted to $\rho_1(c)$.

iv) If $H \geq \bar{H}^{SB}$, then, when the type vector $\boldsymbol{c}$ is distorted to $\boldsymbol{\rho}_2(\boldsymbol{c})$, $\rho_2(\underline{c})$ is hired with probability zero ($\bar{n}_i \leq \underline{c}$). Hence, $\bar{\pi}_i'(z) = E(-q_i^{FB}(\boldsymbol{\rho}_2(z,.)) + \alpha k_i^{FB}(\boldsymbol{\rho}_2(z,.))) \geq 0$ for all $z$ and we have the optimal second-best solution by distorting all types $c$ to $\rho_2(c)$.

v) Using the symmetrical argument, if $H \leq \underline{H}^{SB}$, the solution involves distorting all types $c$ to $\rho_1(c)$.

**Claim 7.** The $\{\boldsymbol{k}^{SB}(\boldsymbol{c}), \boldsymbol{q}^{SB}(\boldsymbol{c})\}$ is almost everywhere unique.

*Proof.* If there is no pooling, this result is immediate from Lemma A. If there is pooling, then, because the problem is symmetric, expected informational rents $\bar{\pi}_1(c) = \bar{\pi}_2(c)$ almost surely. As $\Gamma$ does not admit non-degenerate lotteries and solutions are unique when virtual types are different, it must be that pooled types are treated exactly the same, i.e. $\boldsymbol{k}^{SB}(\boldsymbol{c}) = \left(\frac{K^*(c_\alpha)}{2}, \frac{K^*(c_\alpha)}{2}\right)$ and $\boldsymbol{q}^{SB}(\boldsymbol{c}) = \left(\frac{\boldsymbol{Q}^*(c_\alpha)}{2}, \frac{\boldsymbol{Q}^*(c_\alpha)}{2}\right)$ for all $\boldsymbol{c} \in [\rho_1^{-1}(c_\alpha), \rho_2^{-1}(c_\alpha)]^2$. □

□

***Proof of Corollary 2.*** Just note that, by Proposition 5, if $H \in (\underline{H}^{FB}, \bar{H}^{FB})$, then there is a positive-measure set of types such that:

a) $\boldsymbol{c} \in [\underline{c}, \rho_1^{-1}(c_\alpha))^2$ and, therefore, $\boldsymbol{\rho}(\boldsymbol{c}) > \boldsymbol{c}$,

b) $\boldsymbol{c} \in (\rho_2^{-1}(c_\alpha), \bar{c}]^2$ and, therefore, $\boldsymbol{\rho}(\boldsymbol{c}) < \boldsymbol{c}$

c) Types are pooled.

d) $c_1 < \rho_1^{-1}(c_\alpha)$, $c_2 > \rho_2^{-1}(c_\alpha)$ and, consequently, $\rho(c_1) > c_1$ and $\rho(c_2) < c_2$.

Now we just use equation 10 to show that $k_1^{FB}(\boldsymbol{c})$ decreases with both types because $R_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c}) - R_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c})$ is a decreasing function of both types. Also, equation 11 shows

that the total capacity $\sum_{i=1}^{2} k_i^{FB}(\boldsymbol{c})$ is increasing on both types because $R_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{c})$ increases with $c_2$ and does not depend on $c_1$. As a consequence, $k_2^{FB}(\boldsymbol{\rho}(\boldsymbol{c}))$ is increasing on both types.

By relating this information with $a)$, $b)$, $c)$ and $d)$ we get the results. $\square$

## c Implementation

***Proof of Lemma 4.*** The best response of a firm of type $c_i$ must solve:

$$\begin{aligned} \max_{\{k_i, \boldsymbol{q}_i, p_i, F_i\}} & E_{-i}(x_i(\bar{b}_i, .))(F_i - C(c_i, \boldsymbol{q}_i, k_i)) \\ b_i(k_i, \boldsymbol{q}_i, p_i, F_i) = & \phi_i(k_i, \boldsymbol{q}_i, p_i) - F_i = \bar{b}_i \\ & 0 \leq \boldsymbol{q}_i \leq (k_i, k_i) \end{aligned} \tag{17}$$

Because the score rule is quasi-linear, the problem of finding a solution $\{k_i, \boldsymbol{q}_i, p_i, F_i\}$ to the problem in equation 17 is equivalent to finding the solution to the following two-part problem:

1. Fixing a score value $\bar{b}_i$ such that:

   $\bar{b}_i \in \mathrm{argmax}_{b_i} \, E_{-i}(x_i(b_i, b_{-i})|b_{-i})\hat{\pi}_i(c_i, \bar{b}_i)$

   Where $\hat{\pi}_i(c_i, \bar{b}_i)$ is the highest possible winning profit a firm $i$ of type $c_i$ can get when it is forced to attain a score $\bar{b}_i$, that is:

   $\hat{\pi}_i(c_i, \bar{b}_i) = \max_{k_i, \boldsymbol{q}_i, p_i}\{F_i - C(c_i, \boldsymbol{q}_i, k_i) : \bar{b}_i = \phi_i(k_i, \boldsymbol{q}_i, p_i) - F_i, 0 \leq \boldsymbol{q}_i \leq (k_i, k_i)\}$

2. After fixing a score value $\bar{b}_i \in \mathbb{R}$, an offer $\{k_i, \boldsymbol{q}_i, p_i\}$ is made only if it maximizes winning profits conditional on attaining score $\bar{b}_i$, that is:

$$\begin{aligned} \{k_i, \boldsymbol{q}_i, p_i\} \in \underset{k_i, \boldsymbol{q}_i, p_i}{\mathrm{argmax}} & \{F_i - C(c_i, \boldsymbol{q}_i, k_i) : \bar{b}_i = \phi_i(k_i, \boldsymbol{q}_i, p_i) - F_i, 0 \leq \boldsymbol{q}_i \leq (k_i, k_i)\} = \\ & \underset{k_i, \boldsymbol{q}_i, p_i}{\mathrm{argmax}} \{\phi_i(k_i, \boldsymbol{q}_i, p_i) - C(c_i, \boldsymbol{q}_i, k_i) : 0 \leq \boldsymbol{q}_i \leq (k_i, k_i)\}. \end{aligned} \tag{18}$$

To see that, just note that the $\mathrm{argmax}$ set in item 2 does not depend on $\bar{b}_i$.

As a consequence, if we want to implement second-best, we must have:

$$B_i^{SB}(c) \subseteq \underset{k_i,\boldsymbol{q}_i,p_i}{\text{argmax}}\{\phi_i(k_i,\boldsymbol{q}_i,p_i) - C(c_i,\boldsymbol{q}_i,k_i) : 0 \leq \boldsymbol{q}_i \leq (k_i,k_i)\} \tag{19}$$

For a full-measure set of types. □

***Proof of Lemma 5****.* Straight from proposition 5. □

***Proof of Proposition 6****.*

i) Suppose we are under Assumptions $1-4$ and $6-8$ and either $|\psi'|> 1$ or $|\psi'|< \mu$. If $|\psi'|> 1$, we know virtual types are given by $\boldsymbol{\rho}_2(\boldsymbol{c})$ and, if $|\psi'|< \mu$, virtual types are given by $\boldsymbol{\rho}_1(\boldsymbol{c})$.

   By lemma 5, we can call $(\boldsymbol{q}_i^w(c), k_i^w(c), \rho(c))$ the only element of $B_i^{SB}(c)$ for each type $c$.

   We proceed, as in Lemma 4, by finding whether $(\boldsymbol{q}_i^w(c_i), k_i^w(c_i), \rho(c_i))$ solves:

$$\max_{p_i\in[\rho(\underline{c}),\rho(\bar{c})]} \phi(p_i) - (c_i - p_i)q_i^w(\rho^{-1}(p_i)) - \psi(c_i)k_i^w(\rho^{-1}(p_i)). \tag{20}$$

   As $\rho$ is a bijective function, the whole problem turns into a unidimensional problem of a firm truthfully revealing its type. By incentive compatibility of the problem in equation 17, for any score value $b$ a firm must achieve the following profit by the Envelope Theorem:

$$\hat{\pi}_i(c_i,b) = \begin{cases} \int_{\underline{c}}^{c_i}[-q_i^w(z) - \psi'(z)k_i^w(z)]dz - b, \text{ if } \boldsymbol{\rho} = \boldsymbol{\rho}_2; \\ \\ \int_{c_i}^{\bar{c}}[q_i^w(z) + \psi'(z)k_i^w(z)]dz - b, \text{ if } \boldsymbol{\rho} = \boldsymbol{\rho}_1. \end{cases}$$

   Note that know $\frac{\partial\hat{\pi}(c_i,b)}{\partial c_i}$ to be non-decreasing because of Lemma E, which gives us that this format for conditional profits $\hat{\pi}$ is also sufficient for Incentive Compatibility of the problem in equation 17.

   Now we prove that it is an equilibrium behaviour for best types to choose higher scores in the auction, that is, $b_i^*(c_i) = b_i(s_i(c_i))$ is decreasing on types when

$\boldsymbol{\rho} = \boldsymbol{\rho}_1$ and increasing when $\boldsymbol{\rho} = \boldsymbol{\rho}_2$ when $s_i(c_i)$ is the equilibrium strategy for $i$. To see that, we proceed by guess and verify, assuming there is a symmetric equilibrium where $b_i^* = b_{-i}^* = b^*$ which is strictly increasing on types if $\boldsymbol{\rho} = \boldsymbol{\rho}_2$ and strictly decreasing if $\boldsymbol{\rho} = \boldsymbol{\rho}_1$. This gives us:

$$E_{-i}(x_i(b^*(c_i), b^*(c_{-i})|c_i) = \begin{cases} G(c_i) \text{ if } \boldsymbol{\rho} = \boldsymbol{\rho}_1 \\ 1 - G(c_i), \text{ if } \boldsymbol{\rho} = \boldsymbol{\rho}_2 \end{cases}$$

To derive the optimal $b^*(c_i)$, we solve the following equation:

$$E_{-i}(x_i(b^*(c_i), b^*(c_{-i}))|c_{-i})\hat{\pi}_i(c_i, b^*(c_i)) = \bar{\pi}_i(c_i)$$

Where $\bar{\pi}_i(c_i)$ follows the envelope condition in Lemma C as a necessary condition for incentive compatibility of the problem as a whole. As a consequence:

$$b^*(c_i) = \begin{cases} \int_{\underline{c}}^{c_i} \frac{G(c_i)-G(z)}{G(c_i)}[-q_i^w(z) - \psi'(z)k_i^w(z)]dz, \text{ if } \boldsymbol{\rho} = \boldsymbol{\rho}_2 \\ \\ \int_{c_i}^{\bar{c}} \frac{G(z)-G(c_i)}{1-G(c_i)}[q_i^w(z) + \psi'(z)k_i^w(z)]dz \text{ if } \boldsymbol{\rho} = \boldsymbol{\rho}_1 \end{cases}$$

Which is strictly increasing function of $c_i$ if $\boldsymbol{\rho} = \boldsymbol{\rho}_2$ and strictly decreasing function of $c_i$ if $\boldsymbol{\rho} = \boldsymbol{\rho}_1$. As a consequence, we confirm that there is an equilibrium in which best types choose the highest scores and second-best is implemented.

We can find the optimal $\phi$ by doing:

$$\hat{\pi}_i(\rho^{-1}(p), b_i) = \phi(p) - (\rho^{-1}(p) - p)q_i^w(\rho^{-1}(p)) - \psi(\rho^{-1}(p))k_i^w(\rho^{-1}(p)) - b_i = \hat{\pi}_i(\rho^{-1}(p), b_i)$$

This gives us:

$$\phi(p) = \begin{cases} \int_{\rho_2(\underline{c})}^{p} [-q_i^w(\rho_2^{-1}(z)) - \psi'(\rho_2^{-1}(z))k_i^w(\rho_2^{-1}(z))] \frac{\partial \rho_2^{-1}(z)}{\partial z} dz + (\rho_2^{-1}(p) - p) q_i^w(\rho_2^{-1}(p)) + \\ \psi(\rho_2^{-1}(p)) k_i^w(\rho_2^{-1}(p)), \text{ if } \boldsymbol{\rho} = \boldsymbol{\rho}_2 \\ \\ \int_{p}^{\rho_1(\bar{c})} [q_i^w(\rho_1^{-1}(z)) + \psi'(\rho_1^{-1}(z))k_i^w(\rho_1^{-1}(z))] \frac{\partial \rho_1^{-1}(z)}{\partial z} dz + (\rho_1^{-1}(p) - p) q_i^w(\rho_1^{-1}(p)) + \\ \psi(\rho_1^{-1}(p)) k_i^w(\rho_1^{-1}(p)) \text{ ,if } \boldsymbol{\rho} = \boldsymbol{\rho}_1 \end{cases}$$

ii) Now suppose are under assumptions 1, 3, 4 and $6 - 9$ and we do not have countervailing incentives. We show that the rule just proposed can implement second-best in this problem as well. If $H \in [0, \underline{H}^{SB}] \cup (\bar{H}^{SB}]$, the exact same proof applies.

If $H \in [\underline{H}^{SB}, \underline{H}^{FB})$, then we know that there is a pooling $[d, \bar{c}]$ for some interior $d$ and, because strictly higher cost firms are never hired, the only point in $B_i^{SB}(c)$ for $c$ in the pooling interval is $\left(\frac{Q^*(d)}{2}, \frac{K^*(d)}{2}, \rho(d)\right)$[7] and, because second-best expected profits in the pooling region are always 0, we must have $Q^*(d) = \alpha K^*(d)$. Using Lemma E, $-q_i^{FB}(\boldsymbol{c}) + \alpha k_i^{FB}(\boldsymbol{c})$ is non-decreasing on types and $q_i^{FB}(\boldsymbol{c}) \leq \alpha k_i^{FB}(\boldsymbol{c})$ if $c < \rho_1^{-1}(c_\alpha)$ for every $c_{-i}$.

Then, every type in the pooling region produces $\left(\frac{Q^*(d)}{2}, \frac{K^*(d)}{2}\right)$ at the same cost, therefore having the same payoff in case of offering the pooling contract and getting hired. However, for every $z \in (d, \bar{c}]$ it is more costly to deviate outside the pooling region than it is for $d$. So, if $d$ does not have incentive to deviate, neither does a type $z > d$.

The same point can be made to show that, if $H \in [\bar{H}^{FB}(\alpha), \underline{H}^{SB})$, the pooling region will be $[\underline{c}, d]$ for some interior $d$ and if $d$ does not deviate, no other type in the pooling region will have incentive to do so.

□

***Proof of Proposition 7***.

First of all, note that, if there are countervailing incentives, there is an interior pool-

[7] Note that it is payoff equivalent for the firm to bid $\{\frac{Q^*(d)}{2}, \frac{K^*(d)}{2}, \rho(d), \frac{F_i}{2}\}$ and have its contract fulfilled with probability $a \in [0, 1]$ and bidding $\{Q^*(d), K^*(d), \rho(d), F_i\}$ and winning with probability $\frac{a}{2}$.

ing interval $[\rho_1^{-1}(c_\alpha), \rho_2^{-1}(c_\alpha)]$ such that $B_i^{SB}(c) = B_i^{SB}(\tilde{c}) = B$ for all $c, \tilde{c} \in [\rho_1^{-1}(c_\alpha), \rho_2^{-1}(c_\alpha)]$. By Lemma B, there is a positive-measure set $A = \{c \neq c_\alpha : k_i^{FB}(\boldsymbol{c}) > 0, \boldsymbol{c} = (c_\alpha, c_{-i}),\ c_{-i} = c\}$. By continuity, there is an open interval $(c', c'') \subseteq A$ such that either $c_\alpha < c' < c''$ or $c' < c'' < c_\alpha$.

Take any two offers

$$(\boldsymbol{q}^1, \boldsymbol{k}^1, c_\alpha) = (\boldsymbol{q}_i^{SB}(c, c_{-i}), k_i^{SB}(c, c_{-i}), c_\alpha) = (\boldsymbol{q}_i^{SB}(c', c_{-i}), k_i^{SB}(c', c_{-i}), c_\alpha);$$
$$(\boldsymbol{q}^2, \boldsymbol{k}^2, c_\alpha) = (\boldsymbol{q}_i^{SB}(c, c'_{-i}), k_i^{SB}(c, c'_{-i}), c_\alpha) = (\boldsymbol{q}_i^{SB}(c', c'_{-i}), k_i^{SB}(c', c'_{-i}), c_\alpha);$$

where $c, c' \in [\rho_1^{-1}(c_\alpha), \rho_2^{-1}(c_\alpha)]$ and $\rho(c'_{-i}), \rho(c_{-i}) \in (c', c'')$. We know $(\boldsymbol{q}^j, \boldsymbol{k}^j, c_\alpha) \in B$ for $j \in \{1, 2\}$. By Proposition 2, either $-q^j + \alpha k^j = (-\mu + \alpha)k^j > 0$ or $-q^j + \alpha k^j = (-1 + \alpha)k^j > 0$ for all $j \in \{1, 2\}$.

However, if both points are best response choices for types $c$ and $c'$, both types have to be indifferent between these two offers and, by Lemma 4, $\phi(\boldsymbol{q}^1, k^1, c_\alpha) - \phi(\boldsymbol{q}^2, k^2, c_\alpha) = c(q^1 - q^2) - \psi(c)(k^1 - k^2) = c'(q^1 - q^2) - \psi(c')(k^1 - k^2)$. This implies $(q^1 - q^2) = \alpha(k^1 - k^2)$, which is not true. As a consequence, for every quasi-linear score auction we have a positive-measure set of types $\boldsymbol{c}$ such that second-best allocation for these types is not implemented. As we know the policy correspondences are single-valued for if virtual types are different (Lemma A), we know that the welfare will be strictly lower than second-best. □