

# **“Social Norms and Money”**

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# Social Norms and Money\*

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## Abstract

In an economy where there is no double coincidence of wants and without record-keeping of past transactions, money is usually seen as the only mechanism that can support exchange. In this paper, we show that, as long as the population is finite and agents are sufficiently patient, a social norm establishing gift-exchange can substitute for money. Notwithstanding, for a given discount factor, the growth of the population size eventually leads to the breakdown of the social norm, while money still works.

## 1 Introduction

The reasons for the existence of money and how it emerged in the economy are fundamental questions in monetary theory but only recently have they been addressed in a systematic way. In the work of Kiyotaki and Wright (KW) (1989, 1991, 1993) and the large literature that followed their initial idea, the answer they give for the existence question is: money exists to play the role of a medium of exchange. However, there are other mechanisms (or institutions) in society that can also play the role of a medium of exchange. Kocherlakota (1998a, 1998b) shows that every allocation that can be attained with money can be attained

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\*I am specially grateful to Randall Wright. I am also in debt with Jan Eeckhout, George Mailath, Andrew Postlewaite, Braz Camargo, Hamming Fang and Brandon Weber. As usual the remaining mistakes are mine.

with memory as long as there is perfect record-keeping of past transactions. Moreover, as it will be demonstrated latter, a norm establishing gift-exchange can also be a substitute of money.<sup>1</sup>

Memory is excluded, by construction, from the environment considered by KW. Despite departing from the Walrasian Economy in the sense that they make the exchange process non-trivial, they still preserve the anonymous market as the locus of trade. Hence since agents are indistinguishable from one another at all times, there is no scope for exploiting the history of past transactions.

Our objective is not to reject the idea that money actually plays the role of a medium of exchange. This is an obvious feature of modern societies and we believe that the KW environment captures in a natural and simple way the fact that money overcomes the difficulties of the exchange process in these societies. Our concern is the study of the conditions under which alternative ways of addressing the exchange problem survive (or not). By considering this problem, we can have better insight into the reasons money emerged as the dominant mechanism.

In the next section, it will be discussed first the conditions under which memory can be used in the KW environment as long as there is a perfect record-keeping of past transactions. But our main concern is the analysis of an equilibrium with gift-exchange in the standard KW environment (i.e., without record-keeping of past transactions). The main result establishes that money is not essential as a medium of exchange device in small populations but it is essential in a large population environment.<sup>2</sup>

## 2 The Model

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<sup>1</sup>The reference paper on norms, though not related to Monetary Theory, is Kandori (1992). See also Ellison (1994) and Okuno-Fujiwara & Postlewaite (1995).

<sup>2</sup>We say that money is essential if exists desirable allocations that can be achieved with money and not without money.

## 2.1 Memory

There is an economy with a continuum of infinitely-lived agents of  $k$  different types, and with  $k$  distinct indivisible goods.<sup>3</sup> Following KW (1989) assume that a type  $j$  agent only consumes good  $j$ , which yields utility  $u$ , and produces good  $j + 1$  (modulo  $k$ ) at a cost of  $c$  (with  $u > c$ ). Hence, as long as  $k > 2$ , there is no double coincidence of wants. At every date, each agent enters the exchange sector and interacts with others under a uniformly random matching technology; that is, there is a probability  $\frac{1}{k}$  that agent type  $j$  will meet, for example, an agent type  $m$ . Finally, the agents discount the future with a discount factor  $\beta$ .

Suppose that this economy has a record-keeping device that keeps track of all its transactions, with this device being common knowledge across the agents. Under these assumptions consider an exchange rule stating the following (denote it rule 1): every time an agent meets another who likes his good he produces the good for him, as long as everyone has done so in the past. If any agent deviates, the economy reverts to autarky.<sup>4</sup>

Note that autarky is always an equilibrium in this economy. But we will show that the above rule is an equilibrium that will make every agent better off than in autarky. Clearly, this rule is efficient.

**Lemma 1** *If  $\beta \geq \frac{ck}{ck+u-c}$ , the strategy profile associated with the exchange rule stated above (rule 1) constitutes a sub-game perfect nash equilibrium.*

**Proof.** Let  $V_c$  be the expected payoff of the strategy profile on the equilibrium path and  $V_a = 0$  the value function in autarky. The basic incentive to consider here is the incentive for agent  $j$  to give the good after meeting agent  $j + 1$ (modulo  $k$ ) for the first time. Since the environment is stationary, if the agent decides to cooperate for the first time, given that the others are following the described rule, he will always cooperate. If he gives the good he receives  $-c + \beta V_c$ . If the agent does not cooperate he will receive  $V_a = 0$ . We have that cooperation is optimal after a first meeting whenever  $-c + \beta V_c \geq 0$ . One can calculate:

<sup>3</sup>The assumption of indivisible goods is made only for simplicity, the results hold as well in the divisible good case.

<sup>4</sup>A more formal way of stating this rule is: If agent type  $j$  meets an agent type  $j + 1$ (modulo  $k$ ), he will produce one unit of the good and give it to  $j$ . If he meets an agent type  $j - 1$ (modulo  $k$ ), the latter will produce one unit of the good and give it to  $j$ . In any other situation, there is no exchange at all. Finally, if any agent deviates from this rule the economy reverts to autarky forever; that is, there is no more exchange.

$$V_c = \frac{1}{k}(u + \beta V_c) + \frac{1}{k}(-c + \beta V_c) + \frac{k-2}{k}(0 + \beta V_c) \implies V_c = \frac{1}{(1-\beta)} \frac{(u-c)}{k}$$

Hence, the agent will cooperate if and only if:

$$\beta \geq \frac{ck}{ck + u - c}$$

■

A natural question that arises from the previous lemma is whether it is reasonable to imagine such a stringent rule. After all, if society has full information regarding past transactions, why not retaliate against only the deviating agent, instead of the whole community? With this in mind the following result can be generated that implements the same exchange pattern with a more reasonable rule (rule 2), which is the following: every time an agent meets another who likes his good he produces the good to him, as long as everyone has done so in the past. If any agent deviates, he doesn't receive goods anymore.

**Lemma 2** *If  $\beta \geq \frac{ck}{ck+u-c}$ , the strategy profile associated with the exchange rule stated above (rule 2) constitutes a sub-game perfect nash equilibrium.*

**Proof.** The proof is a straightforward modification of lemma 1 and hence is omitted. ■

We can also modify the rule that implements this efficient outcome by allowing the possibility of forgiveness, i.e., that punishment occurs for only a finite time. The rule (rule 3) in this case is: every time an agent meets another who likes his good he produces the good to him, as long as everyone has done so in the past. If any agent deviates, he doesn't receive any good during  $T$  periods and then he is forgiven. A similar result arises:

**Lemma 3** *For every  $T \geq 1$ , if  $\beta(1 - \beta^T) \geq \frac{ck}{ck+u-c}$ , the strategy profile associated with rule 3 constitutes a sub-game perfect nash equilibrium.*

**Proof.** Now, if the agent deviates, he receives  $0 + \beta V_f$ , where  $V_f$  indicates the punishment and forgiving norm. We have that  $V_f = 0 + \beta^T V_c$ . Hence, he will cooperate whenever  $\beta(1 - \beta^T) > \frac{ck}{u+c(k-1)}$ . Clearly, there will be no deviations off the equilibrium path. ■

Note that the discount factor needed to guarantee cooperation in lemma 3 is bigger than in lemmas 1 and 2, since now the cost of a deviation is smaller. Of course, when  $T$  goes to infinity, the value of  $\beta$  in lemma 3 approaches that in the previous ones.

The objective of the analysis so far is to introduce the idea that equilibrium strategies can be seen as the result of rules in a society, and the three different lemmas try to capture the diversity of rules that can arise. The problem with this approach is that the result completely depends upon perfect record-keeping of past transactions and this is not a very reasonable assumption. In this sense, the results above are basically the same as those obtained by Kocherlakota (1998a) using gift-exchange strategies (actually, the gift-exchange procedure can always be thought of as coming from a rule). In the next model, the assumption of perfect record-keeping will be relaxed and it will be shown that exchange still can be implemented. With this result the contrast between an environment where cooperation is supported by the exploitation of the history of the transactions and one where cooperation is supported by the existence of a norm will be made clear.

## 2.2 Social Norms

In what follows we are going to use the notion of a contagious equilibrium developed by Kandori (1992).<sup>5</sup> Under a uniform random matching technology, Kandori shows that cooperation can be obtained in a one-shot prisoner's dilemma with a social norm establishing that: "...a single defection by a member means the end of the whole community trust, and a player who sees dishonest behavior starts cheating all of his opponents. As a result, defection spreads like an epidemic and cooperation in the whole community breaks down." (1992, page 69). The contagious defection differs from the rules of punishment considered in the previous section because it is not personalized and it cannot be personalized (since there is no record-keeping of transactions). Upon seeing a defection, an agent punishes the whole community, and this is why such a pattern is well represented by the idea of a social norm.

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<sup>5</sup>See also Ellison (1994) and Okuno-Fujiwara & Postlewaite (1995).

This structure of punishment will be used to prove the main result. For this to be possible, two additional assumptions in the standard KW environment are needed:

(i) There is a finite number of agents (but as many as we wish) rather than a continuum of agents.

(ii) After the matching, every agent is able to identify if the agent he matches has the good he likes or not.<sup>6</sup>

These assumptions are much weaker than the existence of a commonly known record-keeping device that keeps track of all past transactions, but still implements the same pattern of exchange.

To simplify the proof and to make the main result I am pursuing here more clear, I will assume that there are  $N$  agents in the economy, each one of them being of a different type. Now, consider the social norm:

“every time an agent meets another who likes his good he produces the good, as long as everyone has done so in the past for him. If at some point an agent does not receive a good he likes, from that period on he doesn’t give goods to anybody else”<sup>7</sup>.

**Proposition 1** *For every  $N$ , there exists a discount factor  $\beta'$  (depending on  $N$ ) such that, for every  $\beta \geq \beta'$ , the strategy profile associated with the above social norm constitutes a sequential equilibrium.*

**Proof.** Let  $V_n$  be the expected payoff associated with universal cooperation and let  $V_b$  be the expected payoff associated with a single agent triggering the contagious punishment. The only decision we need to consider is whether an agent will give a commodity when he is supposed to do so. If he follows the social norm and gives the good he receives  $-c + \beta V_n$ .

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<sup>6</sup>Actually, this assumption can be replaced by the following mechanism: after the matching and before the gift-exchange takes place or not, there is a cheap-talk game where each agent asks the other to show which good he produces. If he refuses to do that, you assume that he is a defector and start the contagious punishment. Clearly, it is a Nash Equilibrium of this cheap-talk game for every agent to show the good he produces.

<sup>7</sup>More formally, we have: If agent  $j$  meets agent  $j + 1$  (module  $N$ ), he will produce one unit of the good and give it to  $j$ . If he meets agent  $j - 1$  (module  $N$ ), the latter will produce one unit of the good and give it to  $j$ . In any other situation, there is no exchange at all. Finally, if any agent deviates from this rule, he triggers a contagious punishment where every agent who meets another which is supposed to cooperate but doesn’t, starts not to cooperate from that period on.

If he does not give the good he receives  $0 + \beta V_b$ . The agent will not deviate whenever  $\beta(V_n - V_b) \geq c$ . Let's calculate  $V_n$  and  $V_b$  (we will do the calculations for the case of  $N$  even but in the appendix A we have the results for  $N$  odd as well, and they are basically the same):

$$V_n = \frac{1}{N-1}(u - c + \beta V_n) + \frac{N-3}{N-1}(\beta V_n) \implies V_n = \frac{u - c}{(1 - \beta)(N - 1)}$$

$$V_b = \sum_{t=1}^{\infty} \beta^t [1 - P(T = t)] \frac{u}{(N - 1)}$$

$P(T = t)$  indicates the probability that the number of periods until the agent who deviates is reached by the contagious process is equal to  $t$ , and we have that when  $t$  goes to infinity  $P(T \leq t)$  goes to one.<sup>8</sup> Hence, an agent will not deviate whenever:

$$\beta(V_n - V_b) \geq c \implies \beta \left\{ \frac{u - c}{(1 - \beta)} - \sum_{t=0}^{\infty} \beta^t [1 - P(T = t)] u \right\} \geq c \quad (1)$$

When  $\beta$  converges to 1 the expression on the left goes to infinity (since as stated above and proved in the appendix,  $P(T \leq t)$  goes to one when  $t$  goes to infinity). So, there exists a discount factor  $\beta'$ , which is the value of  $\beta$  that solves the equation  $\beta \left\{ \frac{u - c}{(1 - \beta)} - \sum_{t=0}^{\infty} \beta^t [1 - P(T = t)] u \right\} = c$ , such that, for all  $\beta \geq \beta'$ , the above inequality is satisfied, implying that our strategy profile is a Nash equilibrium. It remains to prove that this equilibrium is sequential. Under the assumption that each agent specializes in the production of one good (and each agent only likes the good of another specific agent) this is a straightforward result. First, even if the agent who deviated first wanted to slow down the contagious process he can't since he already deviated with the only agent he can affect (which is the one that likes his good). Second, any other agent, after being contaminated by the contagious process has no incentive to slow down this process since there is no more good available to him in this economy. Hence, for any possible off the equilibrium path history, the agents have incentive to follow the prescription of the social norm and transmit the punishment. ■

As a remark, notice that this situation differs from Kandori (1992) and Ellison (1994) where there exists incentives to slow down the contagious process once it started. Kandori

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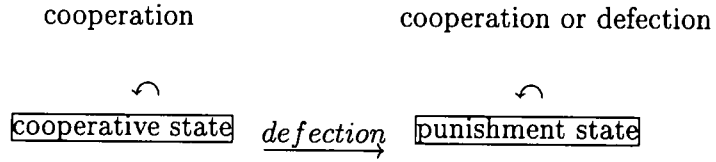
<sup>8</sup>In the appendix we calculate explicitly the value of  $V_p$ .



solves this problem by imposing a positive correlation between the size of the population and the cost of slowing down the contagion, and Ellison uses a public randomizing device. Here, these assumptions are not needed.

As stated before, the history of economy-wide transactions is not used, only the personal history of the agent, to implement the social norm. However, since it takes more time to punish a deviant agent in this environment as compared to one with memory, the discount factor needed to support the equilibrium here is higher than before.

One interesting feature of the above equilibrium is that it does not involve a complex strategy profile. Actually, we can represent behavior under this strategy profile by means of a simple automaton. We have the following:



Despite the proposition being a positive statement, since it establishes the conditions under which a social norm can support one-sided coincidence of wants exchange, its result can be used to determine under what conditions such a social norm cannot be implemented. In this respect there are two correlated factors: the discount factor  $\beta$  and the size of the population  $N$ . The idea is: when the size of the population increases, an initial defection requires a longer time in order to contaminate all the population. In other words, at a given point in time, the probability that one agent will meet a non-cooperative one is decreasing in population size. Formally this can be seen if we look to the probability of all the population be contaminated at a given point in time, that is (obtained from the appendix):

$$P(T = t) = \binom{t-1}{N-3, t-(N-2)} \left(\frac{1}{N-1}\right)^{N-2} \left(\frac{N-2}{N-1}\right)^{t-(N-2)}$$

Notice that for a fixed  $t$ ,  $P(T = t)$  is decreasing in  $N$ , and  $P(T = t)$  goes to zero when  $N$  goes to infinity. If the discount factor is stable over time and the size of the population increases, we will eventually reach a point where the social norm will fail to support exchange.

**Corollary 1** *For any  $\beta \in (0, 1)$ , there exists  $N'$  (depending on  $\beta$ ) such that the efficient outcome implemented by the social norm is attainable if and only if  $N < N'$ , where  $N'$  solves the equation:*

$$\beta \left\{ \frac{u - c}{(1 - \beta)(N - 1)} - \sum_{t=0}^{\infty} \beta^t [1 - P(T = t)] \frac{u}{(N - 1)} \right\} = c.$$

In the previous analysis it was assumed that the economy is totally specialized, with each agent being a different type. This assumption equates the population size and the number of types. The crucial variable that supports this equilibrium is population size, not the degree of specialization in the economy (number of types), as long as population is finite.

Fixing the discount factor, population growth will eventually lead to the breakdown of this norm. Notwithstanding this conclusion, the degree of specialization in the economy affects the incentives the agents have to follow the equilibrium strategy. The intuition is the following: suppose the economy is such that there are  $k$  types of agents, with  $\frac{N}{k}$  agents per type ( $N$  being even or odd depending on the value of  $k$ ). In this case, an increment in the value of  $k$  leads to a reduction in the future payoff, both on and off-the equilibrium path. This increment has a similar effect as a reduction in the value of  $\beta$ , which creates more incentives for an agent to deviate in order to avoid the cost of cooperating today. An increment in the value of  $k$  also affects the speed of the contagion process. Since interactions which involve gift-exchange become less frequent when  $k$  increases, the contagion process tends to be more slow as compared to an economy where  $k$  is small, and hence interactions involving gift-exchange are more frequent. So, our intuition is that specialization has an effect similar to population growth in our environment, i.e., it enhances the incentives for a deviation of the social norm. Our next objective is to verify this intuition. We will obtain a closed form expression for the contagion that can address the issue of the degree of specialization in the economy.

## 2.3 Specialization

In the economy analyzed so far specialization is measured by the relation between the number of types and the size of the population ( $\frac{k}{N}$ ), with a large value of  $\frac{k}{N}$  indicating a high degree of specialization. If there are many types, the chance of meeting an agent that has the good you like is small. Hence, to address the effects of specialization we just need to study how changes in the probability of a single-coincidence of wants meeting affects the performance of the social norm equilibrium. Under some modifications in our environment

this study turns out to be much simpler without losing explanatory power. Up to this point we have been using as baseline for our economy the KW model (1989), where the description of an agent type embodies both a specification of a preference over goods and a production technology. Now, we will consider an environment very similar to the KW model (1993), where a type is characterized only by a preference relation, with all the agents sharing the same production technology.

More specifically, consider the following economy: there are  $N$  agents and  $K$  distinct indivisible goods.<sup>9</sup> Each agent consumes  $k$  of these goods, with  $k$  varying from agent to agent. Before being pairwise matched in the market, every agent enters the production sector that yields one unit of the commodity, drawn randomly from the set of all commodities. An agent does not consume his own good. Agents are pairwise matched under a uniform random matching structure, and before trade takes place or not each agent knows the good the other agent is bringing to the market. Moreover, trade can take place in only one direction<sup>10</sup> and at a transaction cost of  $c$ , which is paid by the agent that gives the good.

Let  $x = \frac{k}{K}$ ;  $x$  captures the extent to which commodities and tastes are differentiated. If the economy is very specialized,  $x$  is low, i.e., there are too many goods but agents only like a small fraction of them.

Consider the following social norm, very similar to the previous one: “every time an agent meets another in a double-coincidence of wants meeting (which happens with probability  $x^2$ ), there is  $\frac{1}{2}$  chance that he will give the good and he will follow this rule as long as everyone has done so in the past for him. If he meets an agent in a single-coincidence meeting where the other agent likes his good (which happens with probability  $x(1 - x)$ ), he always gives the good as long as everyone has done so in the past for him. If at some point an agent does not receive a good he likes and he was supposed to receive, from that period on he doesn’t give goods to anybody else”.

We can use a reasoning very similar to the one considered in proposition 1 to prove that the strategy profile associated with this social norm constitutes an equilibrium. After

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<sup>9</sup>The assumption of indivisible goods is made only for simplicity, the results hold as well in the divisible good case.

<sup>10</sup>We introduced the assumption that trade can only take place in one direction because now there is a chance of a double-coincidence of wants meeting (which could not happen in the previous environment). This assumption makes it easier to calculate the payoffs after a deviation from the equilibrium but it can be relaxed without changing the main results.

a deviation from the equilibrium path, the finiteness of the population guarantees that eventually every agent will be a defector. Hence, for a sufficiently patient agent, there is no incentive to deviate from the equilibrium path.<sup>11</sup>

If an agent follows the equilibrium he obtains:

$$V_c = \frac{\theta(u - c)}{(1 - \beta)}$$

where  $\theta = \frac{x(2-x)}{2}$  indicates the probability of receiving or giving a good at each period. If an agent deviates from the equilibrium, he obtains (the calculation of  $V_d$  is done in the appendix B)<sup>12</sup>:

$$V_d = \sum_{t=1}^{\infty} \beta^t e_1 A^t \pi \theta u$$

where:

$e_1 = (1, 0, 0, \dots, 0)$ , N-dimensional.

$A = \{(a_{ij})\}_{N \times N}$ , where  $(a_{ij}) = \text{Prob}(\#defector\ s_{t+1} = j \mid \#defector\ s_t = i)$

$\pi = (\pi_i)$ , N-dimensional, where  $\pi_i = \text{Prob}(\text{defector meets a cooperator} \mid \text{there exists } i \text{ defectors})$

An agent will not deviate whenever:

$$-c + \beta V_c \geq V_d \implies (\beta V_c - V_d) \geq \frac{c}{\beta}$$

In order to see what are the effect of a change in the degree of specialization over the social norm equilibrium (if it reinforces the equilibrium or if it weakens it), we need to calculate

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<sup>11</sup>To prove that the equilibrium is sequential is not as straightforward as before, but we can use the same argument as Kandori (1992) to prove that for a sufficiently high cost  $c$ , there will be no incentive to slow down the contagion process once it started.

<sup>12</sup>To calculate  $V_d$ , we calculate first a closed form expression for the contagion process in this environment. It turns out that the process analyzed by Kandori (1992) can be seen as a particular case of the one analyzed here when  $\theta = 1$ .

$\frac{\partial(\beta V_c - V_d)}{\partial \theta}$ . If  $(\beta V_c - V_d)$  increases, increasing the degree of specialization in the economy weakens the equilibrium we have been considering so far. We did not obtain the analytical solution to  $\frac{\partial(\beta V_c - V_d)}{\partial \theta}$  but we analyzed the behavior of  $(\beta V_c - V_d)$  in terms of  $\theta$  under distinct population sizes. The tables below summarize the results for  $N = 6, N = 60$  for  $c = 1$ .\*\* In the appendix C we present additional results which analyzes in more detail the behavior of  $(\beta V_c - V_d)$ .

TABLE 1

Value of  $u$  such that  $(\beta V_c - V_d) = 1$

		$\theta$					
		0.9		0.5		0.1	
		N=6	N=60	N=6	N=60	N=6	N=60
$\beta$	0.99	1.02		1.04		1.2	
	0.9	1.32		1.6		4.6	
	0.5	4.8		8.5		47.3	
	0.1	47.4		87.2		447	

TABLE 2

$\frac{\partial(\beta V_c - V_d)}{\partial \theta}(\beta, \theta, u : (\beta V_c - V_d) = 1, 1)$

		$\theta$					
		0.9		0.5		0.1	
		N=6	N=60	N=6	N=60	N=6	N=60
$\beta$	0.99	1.68		3.67		21.15	
	0.9	2.34		4.22		17.95	
	0.5	1.90		3.10		11.53	
	0.1	1.27		2.17		10.17	

\*\* The calculations for  $N = 60$  are not finished yet, but the results obtained so far indicate that we will obtain the same conclusions as the ones for the case of  $N = 6$ .

In table 1, we calculate the minimum value of  $u$  that supports the social norm equilibrium (for various values of  $\beta$  and  $\theta$ ). Given this value of  $u$ , we obtained in table 2 the  $\frac{\partial(\beta V_c - V_d)}{\partial \theta}$  at the corresponding equilibrium points. We can see that  $\frac{\partial(\beta V_c - V_d)}{\partial \theta} > 0$ , does not matter the degree of specialization and the discount factor, which supports the idea that specialization of the economy reduces the incentives to follow the social norm equilibrium in a similar

way as the population growth does. In a more specialized economy the single-coincidence of wants meetings are more rare, which reduces the future benefits of giving a good today.

### 3 Conclusion

The objective of this paper is to show that social norms can support the exchange process in the Kiyotaki-Wright environment if the population is not too large and agents are sufficiently patient. The importance of this result is that it gives a possible explanation for the dominant role of money in modern economies as compared to social norms. As long as agents believe that money is valuable as a medium of exchange, we can support a monetary equilibrium.<sup>13</sup> Hence, even though social norms played a role in the past sustaining the exchange process, for example in village economies,<sup>14</sup> they cannot support exchange in modern economies, while money can.

In general, the basic result obtained in this paper is robust to some modifications of the environment. That is, as long as the population is finite and agents are sufficiently patient, gift-exchange can be supported in the KW environment as a Nash equilibrium. Notwithstanding, the fact that we could obtain a simple expression for the contagion process (and the fact that the incentives to follow the norm off the equilibrium path are trivially satisfied) depends on the assumption that each agent specializes in the production and consumption of only one good. Moreover, by considering an economy where heterogeneity among the agents is reflected only in the preferences and not in the production technology, we conclude that specialization can be a key factor leading to the breakdown of the social norm. Specialization, by reducing the value of future benefits and by reducing the speed of the contagion process, increases the incentives for the agents to deviate. Hence, even if the population size is fixed, the social norm equilibrium can collapse if the degree of specialization in the economy increases.

The results here are distinct from Kocherlakota (1998a,1998b). The conclusion from Kocherlakota's work is that if the cost of keeping track of the past is too high, memory cannot substitute money and money becomes essential. Our conclusion is that money is essential not due to the complexity of the past but to the complexity of the present, to the

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<sup>13</sup>This result is proved in KW(1989) and KW(1993).

<sup>14</sup>A good reference in this direction is Landa (1994), which gives a historical analysis of gift-exchange.

fact that is hard to punish a defector in a world that is either too large (wrt population size) or too specialized.

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## A Appendix

First we calculate the probability distribution of the random variable  $T_i$ ="number of periods until agent  $i$  is reached by the contagious process" since after this period, he will receive zero as a payoff. In the model we are analyzing here the distribution of  $T_i$  coincides with the distribution of  $T$ ="number of periods until all the population be contaminated by the contagious process". This happens for the following reason: the only way an agent can be affected by the contagious process is when he meets another that was supposed to give him a good but didn't. Moreover by assumption each agent is specialized in the production of one good. Hence, at each period, there is only one agent that can transmit the information of defection, namely, the last agent to be contaminated.<sup>15</sup> Finally, the agent that is supposed to give the good to agent  $i$  is the last one to be contaminated in the population, which leads the coincidence of these two distributions. So, from now on, we will restrict attention to the calculation of the distribution of  $T$ .

Consider a population of size  $N$ . After a deviation by agent  $i$  (WLOG, suppose that  $i=1$ ) we can summarize the relevant states of the world that can appear in the model by the following way (where the  $i^{th}$  position in the vector indicates the  $i^{th}$  agent, 1 indicates that he is a deviator and 0 indicates that he still cooperates):

$$S_2=(1,1,0,0,0,0,\dots,0,0)$$

$$S_3=(1,1,1,0,0,0,\dots,0,0)$$

$$S_4=(1,1,1,1,0,0,\dots,0,0)$$

....

$S_N=(1,1,1,1,1,\dots,1,1)$ , where the subscript indicates the number of deviators and each vector has dimension  $1 \times N$ .

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<sup>15</sup>We assume here that, since an agent has no incentive to inform about a defection by cheap-talk, we cannot count on this as a source of information transmission.



Now, let's calculate the probability of moving from state  $i$  to state  $j$ , where  $i, j=1,2,3,\dots,N-1$ . First, note that we cannot move backwards, that is, we cannot move from  $i$  to  $j$ , where  $j < i$ . Moreover, since only one agent can transmit the contamination at each period, we cannot move as well from  $i$  to  $j$ , where  $j > i+1$ . To obtain the probabilities  $P_{ij}$  (Probability of moving from  $i$  to  $j$ ), we need to know what are all the possible partitions of the population across matches. By induction, we can find that:

$N$  even

$$\#(N) = (N-1)\#(N-2)$$

$N$  odd

$$\#(N) = N\#(N-1),$$

where  $\#(N)$  indicates the number of possible partitions of the population across matches of size one (only for the odd number case) and two (for both cases). At each state  $i$ , the probability of moving to state  $(i+1)$  next period is equal to the number of partitions where  $i$  meets  $(i+1)$  divided by the total of partitions. But, for a population of size  $N$ , the number of partitions where  $i$  meets  $(i+1)$  is exactly equal to the number of partitions at  $(N-2)$  (for  $N$  even) and  $(N-1)$  (for  $N$  odd). Looking to the above formulas, we have that (for  $i \neq N$ ):

$N$  even

$$\begin{aligned} P_{ii} &= \frac{N-2}{N-1} \\ P_{ii+1} &= \frac{1}{N-1} \end{aligned}$$

$N$  odd

$$\begin{aligned} P_{ii} &= \frac{N-1}{N} \\ P_{ii+1} &= \frac{1}{N} \end{aligned}$$

Finally, since the state  $S_N$  is absorbing,  $P_{NN}=1$ .

We want to obtain the  $\Pr(T = t)$ , for every  $t$ . In order to see how this can be done, consider the following diagram:

[illegible]

Notice that to find  $\Pr(T = t)$  we need to consider all the possible paths that go from  $S_2$  at  $t = 0$  to  $S_N$  at  $t = t$  and then calculate the probability of each path. Let's consider a generic path  $P$ . The number of movements to the right that  $P$  needs to make in order to reach  $S_N$  at  $t = t$  is equal to  $N - 2$ . The number of down movements that  $P$  needs to make is equal to  $t - (M - 2)$ . This result holds for any size of the population and any number of periods. Moreover, since  $P_{ii}$  is the same for every  $i$  ( $i \neq N$ ),  $P_{ii+1}$  is the same for every  $i$  ( $i \neq N$ ), the probability of each path is always the same. Hence, we only need to calculate the number of paths.

For every path, the last movement is always to the right. So, we can find the total number of path by calculating the number of different ways of combining  $t - 1$  objects, the objects being of two different types: type 1 (movements to the right) and type 2 (down movements). This number is equal to:

$$\binom{t-1}{N-3, t-(N-2)} = \frac{(t-1)!}{(N-3)!t-(N-2)!}$$

Finally, we have the following result:

 $N$  even

$$P(T=t) = \binom{t-1}{N-3, t-(N-2)} \left(\frac{1}{N-1}\right)^{N-2} \left(\frac{N-2}{N-1}\right)^{t-(N-2)} \quad (2)$$

$N$  odd

$$P(T = t) = \binom{t-1}{N-3, t-(N-2)} \left(\frac{1}{N}\right)^{N-2} \left(\frac{N-1}{N}\right)^{t-(N-2)} \quad (3)$$

From the previous result, we can calculate the expected payoff of an agents after he deviates. It is equal to:

$N$  even

$$\sum_{t=1}^{\infty} \beta^t [1 - P(T = t)] \cdot \frac{u}{(N-1)}$$

$N$  odd

$$\sum_{t=1}^{\infty} \beta^t [1 - P(T=t)] \frac{u}{N}$$

Agent  $i$  will not deviate whenever (consider the case of an even number of agents):

$$\beta(V_c - V_p) \geq c \implies \beta \left\{ \frac{u - c}{(1 - \beta)(N - 1)} - \sum_{t=0}^{\infty} \beta^t [1 - P(T=t)] \cdot \frac{u}{(N - 1)} \right\} \geq c$$

From the above expressions we can adress questions like: What is the minimum cost of production that allows gift-exchange for a given discount factor, if we fix the utility  $u$  and the size of the population? By answering this question, we can have an exact notion of the degree of patience required to support gift-exchange, for some given population size.

## B Appendix

Let  $D_t$  be equal to the number of defectors at time  $t$ . We want to calculate the matrix  $A = (a_{ij})$ , where  $a_{ij} = \text{Prob}(D_{t+1} = j \mid D_t = i)$ . First, note that since the number of

defectors is non-decreasing over time,  $a_{ij} = 0$  whenever  $j < i$ . Moreover, if  $j \geq i$  and  $(j - i) > \min(D_t, N - D_t)$ ,  $a_{ij} = 0$  since in order to increase the number of defectors by  $(j - i)$  it is necessary to have at least  $(j - i)$  defectors and cooperators. Another situation where it is easy to identify the value of  $a_{ij}$  is when  $i=1$ . In this case, after a first deviation of a player, in the next period there will be exactly 2 defectors in the economy. That is,  $a_{1j} = 0$  if  $j \neq 2$  and  $a_{1j} = 1$  if  $j = 2$ . The non-trivial case is when  $i \neq 1$ ,  $j \geq i$  and  $(j - i) \leq \min(D_t, N - D_t)$ . In this case, we have the following lemma:

**Lemma 4** *If  $i \neq 1$ ,  $j \geq i$  and  $(j - i) \leq \min(D_t, N - D_t)$ , the probability of moving from  $j$  to  $i$  defectors ( $a_{ij}$ ) is equal to:*

$$a_{ij} = \frac{\sum_{\tau=0}^{\max(\tau | m-2\tau \geq j-i)} \binom{M}{m-2\tau} \binom{m}{m-2\tau} (m-2\tau)! \binom{m-2\tau}{j-i} \theta^{(j-i)} (1-\theta)^{[(m-2\tau)-(j-i)]} S(2\tau) S(M-(m-2\tau))}{S(N)}.$$

where:

$$m = \min(D_t, N - D_t)$$

$$M = \max(D_t, N - D_t)$$

$S(x)$  = number of different ways to make  $\frac{x}{2}$  pairs out of  $x$  pairs.

**Proof.** In order to generate  $(j - i)$  defectors, we need to have at least  $(j - i)$  meetings between defectors and cooperators. We have to consider situations where the number of meetings between cooperators and defectors is above  $(j - i)$  since not always a cooperator turns into a defector upon meeting a defector. More specifically we need to consider up to  $m$  meetings between a cooperator and a defector, since this is the maximum possible number of meetings of this kind.

Fix a number of meetings  $m - 2\tau$ , for some  $\tau \geq 0, \tau \leq \max(\tau | m - 2\tau \geq j - i)$ . The number of possible distinct partitions in groups of two (where one agent is a cooperator and another is a defector) with size  $m - 2\tau$  is equal to  $\binom{M}{m-2\tau} \binom{m}{m-2\tau} (m - 2\tau)!$ . Now, for each of these partitions, the chance of the number of defectors increase to  $(j - i)$  follows a binomial distribution with parameter  $\theta$ , and is equal to  $\binom{m-2\tau}{j-i} \theta^{(j-i)} (1 - \theta)^{[(m-2\tau)-(j-i)]}$ . Now, since we are fixing attention on only  $m - 2\tau$  meetings between a defector and a cooperator, the number of possible ways of choosing pairs with the rest of the population where cooperator meets cooperator and defector meets defector is equal to  $S(2\tau) S(M - (m - 2\tau))$ . By varying  $\tau$  for all its possible values, we obtain the number of possible partitions of the population in pairs such that the number of defectors increase to  $(j - i)$ , and dividing this number by  $S(N)$  gives the probability we are looking for. ■

Notice that the contagion process described by Kandori (1989, 1992) can be seen as a particular case of the one analyzed here, which happens when a cooperátor always starts to defect after meeting a defector, i.e., when  $\theta = 1$ . Now, we can calculate the payoff obtained by an agent who deviates from the proposed social norm equilibrium. First, an agent only has incentive to deviate when he meets another agent that he is supposed to give a good, according to the trade technology assumed here. In this case, by deviating he receives 0 today,  $\beta e_1 A \pi \theta u$  tomorrow,  $\beta e_1 A^2 \pi \theta u$  two days from now and so on. Hence, the overall payoff after a deviation is equal to:

$$V_d = \sum_{t=1}^{\infty} \beta^t e_1 A^t \pi \theta u$$

or

$$V_d = \left( \sum_{t=0}^{\infty} \beta^t e_1 A^t \pi \theta u \right) - \theta u$$

where:

$$e_1 = (1, 0, 0, \dots, 0)_{1 \times N}$$

$$A = (a_{ij})_{N \times N} = (\text{Pr ob}(\# \text{ defectors}_{t+1} = j \mid \# \text{ defectors}_t = i))_{N \times N}$$

$$\pi = (\pi_i) = (\text{Pr ob}(\text{defector meets a cooperátor} \mid \text{there exists } i \text{ defectors}))_{N \times 1}$$

Notice that  $A$  is not invertible but  $(I - \beta A)$  is invertible as long as  $\beta \neq 1$ . So we will restrict attention from now on for values of  $\beta$  such that  $\beta \neq 1$ . The invertibility of  $(I - \beta A)$  implies that we can substitute  $\left( \sum_{t=0}^{\infty} \beta^t e_1 A^t \pi \theta u \right)$  by  $e_1 (I - \beta A)^{-1} \pi \theta u$ . Hence, we can rewrite  $V_d$  as:

$$V_d = e_1 (I - \beta A)^{-1} \pi \theta u - \theta u$$

An agent will not deviate as long as:

$$\begin{aligned} -c + \beta V_c &\geq V_d \\ -c + \frac{\beta \theta (u - c)}{(1 - \beta)} &\geq e_1 (I - \beta A)^{-1} \pi \theta u - \theta u \end{aligned}$$

When  $\beta \rightarrow 1$ , the expression on the left-hand side goes to infinity while the expression on the right-hand side is finite. Hence, as long as agents are sufficiently patient, there will be no deviation from the social norm equilibrium<sup>16</sup>.

## C Appendix

Table 3 displays the value of  $V_d$  for distinct  $\beta$  and  $\theta$ . In table 4 we calculate the value of  $\frac{\partial V_d}{\partial \theta}(\beta = 0.9, \theta, u = 1)$  and  $\frac{\partial(\frac{V_d}{\theta})}{\partial \theta}(\beta = 0.9, \theta, u = 1)$ :

TABLE 3

		$V_d(\beta, \theta, 1)$					
		$\theta$					
		<b>0.9</b>		<b>0.5</b>		<b>0.1</b>	
		N=6	N=60	N=6	N=60	N=6	N=60
	<b>0.99</b>	1.44		1.33		1.14	
$\beta$	<b>0.9</b>	1.22		1.03		0.49	
	<b>0.5</b>	0.50		0.32		0.07	
	<b>0.1</b>	0.07		0.04		0.008	

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<sup>16</sup>Again, to prove that the equilibrium is sequential is not as straightforward as before, but we can use the same argument as Kandori (1992) to prove that for a sufficiently high cost of transaction  $c$ , there will be no incentive to slow down the contagion process once it started, and agents will always defect off the equilibrium path.

		TABLE 4		
		$\frac{\partial V_d}{\partial \theta}(\beta = 0.9, \theta, u = 1)$		$\frac{\partial(\frac{V_d}{\theta})}{\partial \theta}(\beta = 0.9, \theta, u = 1)$
		N=6	N=60	N=60
$\theta$	0.9	0.40		
	0.5	0.63		
	0.1	3.21		

\*\*The calculations for  $N = 60$  are not finished yet, but the results obtained so far indicate that we will obtain the same conclusions as the ones for the case of  $N = 6$ .

We can see that  $V_d(\beta, \theta, 1)$  reduces when  $\theta$  goes down (fixing  $\beta$ ), as shown in table 3. However, if we only consider the effect of  $\theta$  in reducing the contagion process (i.e., if we compute  $\frac{\partial(\frac{V_d}{\theta})}{\partial \theta}$ , as in table 4,  $V_d$  is decreasing in  $\theta$ . The intuition is the following: despite the fact that the speed of the contagion process declines when  $\theta$  gets smaller, the reduction on the frequency of single-coincidence meetings dominates the former effect, leading to an overall reduction on  $V_d$ .

Hence, the key feature that leads to a reduction on the incentives to follow the norm equilibrium after an increment in the degree of specialization is not the gain from deviating in an economy where the contagion process is slower. What drives our result is that the cost of cooperating today becomes too high in an environment where single-coincidence meetings are not as frequent as before.

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