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Sequential cost-reimbursement rules

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SEQUENTIAL COST-REIMBURSEMENT RULES.

Dissertação apresentada ao Curso de Mestrado em Economia da Escola de Pós-Graduação em Economia para obtenção do grau de Mestre em Economia.

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Abstract

This paper studies cost-sharing rules under dynamic adverse selection. We present a typical principal-agent model with two periods, set up in Laffont and Tirole's (1986) canonical regulation environment. At first, when the contract is signed, the firm has prior uncertainty about its efficiency parameter. In the second period, the firm learns its efficiency and chooses the level of cost-reducing effort. The optimal mechanism sequentially screens the firm's types and achieves a higher level of welfare than its static counterpart. The contract is indirectly implemented by a sequence of transfers, consisting of a fixed advance payment based on the reported cost estimate, and an ex-post compensation linear in cost performance.

Keywords: Cost-sharing contracts, dynamic regulation, sequential screening

Resumo

Este trabalho estuda regras de compartilhamento de custos sob seleção adversa dinâmica. Apresentamos um modelo típico de agente-principal com dois períodos, fundamentado no ambiente canônico de regulação de Laffont e Tirole (1986). De início, quando da assinatura do contrato, a firma possui incerteza prévia sobre seu parâmetro de eficiência. No segundo período, a firma aprende a sua eficiência e escolhe o nível de esforço para reduzir custos. O mecanismo ótimo efetua *screening* sequencial entre os tipos da firma e atinge um nível de bem-estar superior ao alcançado pelo mecanismo estático. O contrato é implementado indiretamente por uma sequência de transferências, que consiste em um pagamento fixo antecipado, baseado na estimativa de custos reportada pela firma, e uma compensação posterior linear no custo realizado.

Palavras-chave: Compartilhamento de custos, regulação dinâmica, screening sequencial

Contents

Chapter 1

Introduction

Cost-reimbursement rules figure at the center of modern procurement and regulation. Stemming from major contributions during the incentive revolution in regulation theory in the 1980s, cost-sharing arrangements are also relevant in practice, as seen from a myriad of contracts celebrated between private firms and the public sector. Simple cost-sharing contracts provide incentives for cost performance, addressing the well-known tradeoff between efficiency and informational rent in adverse selection environments.

This paper studies cost-reimbursement rules in a dynamic setting. Our main interest resides in the contractual relationship between an agency and a regulated firm under asymmetric information and risk neutrality.[1] The dynamics of the environment is reflected in an initial uncertainty about the firm's efficiency parameter, and subsequent learning of its true type before realization of the regulatory outcome.

We propose a model benchmarked on the canonical regulation framework of Laffont and Tirole (1986), with observable cost realizations and endogenous effort. However, we extend the model to a sequential setting with two periods. In the first period, the firm observes a private signal, from which she forms an unbiased cost estimate. In the second period, the firm learns its true efficiency parameter and chooses the level of cost-reducing effort. Although risk neutrality means both the regulator and the firm care only about expected values in period one, the contract must also provide incentives for effort in period two. We assume perfect commitment and complete contracting, which allows us to focus on the dynamic incentive issues at hand.

The contract is designed to screen the firm's unfolding private information in a sequential manner. Therefore, it stipulates two stages of interaction: the firm submits a cost estimate at the contracting date, and then provides cost data at the production stage. The firm is rewarded accordingly with a sequence of transfers, which can be paid at the end of each period, or else at the end of the relationship, since there are no commitment issues.

The optimal sequential mechanism achieves a higher level of welfare than its static counterpart. In the latter, the regulator leaves to contract with the firm in period two, when the firm already knows her efficiency with certainty, and the model collapses to the standard Laffont-Tirole benchmark. In our setting, the regulator extracts more surplus from being able to screen the firm's private information twice; therefore welfare cannot be lower. Moreover, the regulator is able to implement a higher level of effort and lower realized costs in the sequential mechanism.

[1] Likewise, our analysis applies to procurement settings between the government and private contractors.

Our main innovation, however, concerns implementation. The sequential mechanism is implemented with a sequence of transfers with the following property: the first-period transfer is a fixed payment reimbursing expected cost, while in the second-period it is an adjustment compensation which depends on cost performance, but is zero in expected value. Therefore, in equilibrium the firm expects no second-period reimbursement, which provides incentives for unbiased forecasting. We also show that, in the indirect mechanism, the second-period transfer can be replaced with a linear rule for sharing cost overruns, which decentralizes the effort decision to the firm.

We argue that our model constitutes a step towards rationalizing dynamic incentive schemes that are common in practice. We often see contractual arrangements featuring sequential devices, both formal and informal, for compensating the firm after the realization of uncertainty. Examples include refund policies and buy-back programs in private buyer-supplier relationships, where demand is uncertain at the time of contracting. In public contracts, there are often clauses offering guarantees and compensation rules to restore the contract's initial financial equilibrium.

Related literature

This paper contributes to the theory of optimal regulation. We build on Laffont and Tirole (1986), extending their results and providing additional support to the optimality of schemes that are linear on cost performance (for more in this subject, see Rogerson, 2003). Armstrong and Sappington (2007), in their comprehensive survey, stress the importance of modeling environments that resemble more accurately real-world circumstances and specific regulatory settings. Our work also takes one step in that direction.

We also contribute to the growing literature on dynamic mechanism design. Most importantly, our paper is an application of screening in the presence of sequential information, first proposed by Baron and Besanko (1984) and further analyzed in Courty and Li (2000), Eso and Szentes (2007) and Dai, Lewis and Lopomo (2006). Krahmer and Strausz (2011b) present a closely related model of sequential screening applied to procurement, adapting it to investigate incentives for information acquisition. All these studies have in common, however, that the mechanism's allocation is a binary decision rule (such as a probability of delivery), implemented by means of a cutoff in the type support. In our work, the allocation is represented by a continuous variable. Pavan, Segal and Toikka (2009) propose a method for characterizing optimal mechanisms in environments with dynamic information and full commitment, providing general results that are easily applied at our setting.

Related work includes seminal contributions to dynamic contract theory, as in Laffont and Tirole (1988; 1990). See Bergemann and Valimaki (2010), Athey and Segal (2013), and Maestri (2013) for recent developments. However, all these models feature persistent types and some form of dynamic distortion, such as noncommitment or renegotiation. For contributions with changing types, see Battaglini (2005) and Boleslavski and Said (2013).

Finally, there is a strand of the incomplete contracts literature that also attempts to explain dynamic schemes, such as guarantees. References include Hart and Moore (1988), Tirole (1986), and Bajari and Tadelis (2001). Our model innovates by proposing a rationale for ex post compensations ("renegotiations") without resorting to an incomplete contracting framework.

This paper is organized as follows. Section 2 presents the model with two initial types, to introduce the environment and present the main insights in a simple manner. Section 3 extends the model to continuous types, so the first-period signal can be interpreted as an unbiased estimate of the second-period type. Lastly, Section 4 provides further remarks on theoretical aspects of the model, and discusses extensions and implications for regulation and procurement. The proofs of all lemmas are left for the Appendix.

Chapter 2

The model: two types

Suppose the government wants to delegate the provision of a public service to a private firm. The service has a fixed social value S, and the cost of providing it is given by $C = \theta - e$, where θ represents an exogenous efficiency parameter and e denotes the regulated firm's choice of cost-reducing effort.

The model has two periods with no discounting:

1. **Contracting:** The regulator offers a take-it-or-leave-it contract to the firm. The firm observes a private signal i, and decides whether to accept the contract.
2. **Production:** The firm privately learns the true value of θ and chooses the level of cost-reducing effort $e \geq 0$.

At the contracting stage, the firm is uncertain about its own efficiency parameter, with a prior distribution given by $G_i(\theta)$, where $i \in \{L{,}H\}$ is a private cost signal. The exogenous efficiency parameter θ is drawn from an interval $\left[\underline{\theta}{,}\overline{\theta}\right]$, with positive densities g_L and g_H, $\forall\theta$. We assume below that $G_H(.)$ and $G_L(.)$ are ranked by first-order stochastic dominance (FOSD), which implies that a firm with a higher signal is more inefficient on average, i.e., $E_H\left[\theta\right] \geq E_L\left[\theta\right]$. Moreover, we make a technical assumption on the discrete version of Baron and Besanko's (1984) informativeness measure, which guarantees sufficient conditions for optimality of the second-best allocation.

Assumption 1. *The high-signal firm's distribution over its efficiency parameter $G_H(.)$ first-order stochastically dominates the low-signal firm's distribution $G_L(.)$. Formally, for every increasing function $\varphi : \Theta \to R$, we have $\int \varphi(\theta)dG_H(\theta) \geq \int \varphi(\theta)dG_L(\theta)$.*

Assumption 2. *The discrete-type informativeness of the distribution $G(\theta|\gamma)$ is given by $\frac{G_H(\theta)-G_L(\theta)}{g_H(\theta)}$ and satisfies:*

$$\frac{d}{d\theta}\left[\frac{G_H(\theta) - G_L(\theta)}{g_H(\theta)}\right] \leq 0 \tag{2.1}$$

We assume the regulated firm is risk neutral and has a type-independent outside option normalized to zero. The firm's only source of revenue is a direct transfer from the regulator T, which is agreed upon at contracting and embeds all rules regarding contingencies and timing. The firm's preferences are quasi-linear in monetary quantities (such as T and C) and there is a disutility of exerting cost-reducing effort, given by the continuously differentiable $\psi(e)$, satisfying $\psi',\psi'' > 0$ and $\psi''' \geq 0$.[1] It is useful to express the firm's payoff as a function of its types at each stage in the game. Thus, type-i's payoff evaluated from ex post and ex ante standpoints are, respectively:

$$u_i(\theta) = T_i(\theta) - C_i(\theta) - \psi(e_i(\theta)) \tag{2.2}$$

$$U_i = \int [T_i(\theta) - C_i(\theta) - \psi(e_i(\theta))] \, dG_i(\theta) \tag{2.3}$$

The regulator is able to observe the firm's ex post cost performance C, though not its composition between efficiency and effort. We assume the regulator is a risk neutral, utilitarian welfare maximizer. Let λ represent the shadow cost of public funds, which summarizes the distortion associated with a unit increase in tax revenues. Hence, the regulator's payoff when the firm's signal is i is given by the sum of the net consumer surplus and type-i firm's utility with equal weights:

$$\begin{aligned} W_i &= \int \{S - (1+\lambda)T_i(\theta) + T_i(\theta) - C_i(\theta) - \psi(e_i(\theta))\} \, dG_i(\theta) \\ &= S - (1+\lambda) \int [\theta - e_i(\theta) + \psi(e_i(\theta))] \, dG_i(\theta) - \lambda U_i \end{aligned}$$

2.1 The principal's problem

The regulator fully commits to a contract which specifies a monetary transfer $T = t + C$ to the regulated firm, consisting of the reimbursement the firm's cost plus a net payment t based on cost performance. By the revelation principle for multistage games (Myerson, 1986), the contract can be represented in terms of a Direct Incentive Compatible Mechanism (DICM) $\{(t_L(\theta), C_L(\theta)), (t_H(\theta), C_H(\theta))\}$, which assigns transfer $t_i(\theta)$ and recommends a cost realization $C_i(\theta)$ if the firm sequentially reports being of type (i,θ).[2] Note that the contract can be alternatively written in terms of the implemented cost-reducing effort $e_i(\theta) = \theta - C_i(\theta)$, $i = L,H$.

Define $u_i(\theta)$ as defined in (2.2) represent type-i firm's utility from truthfully revealing its efficiency parameter θ at the production stage. Second-period incentive compatibility requires that the firm optimally reveals its true ex post type after she knows it with certainty:

$$u_i(\theta) \geq t_i(\hat{\theta}) - \psi\left(\theta - C_i(\hat{\theta})\right), \quad \forall \theta, \hat{\theta}, \quad i \in \{L,H\} \tag{IC$_{2,i}$}$$

Now, first-period incentives assume that truthtelling is an optimal continuation strategy for the firm. Denote the firm's expected rent from reporting its true type i as U_i, defined in expression (2.3). Thus, incentive constraints at the contracting stage are:

[1] These conditions are standard in the canonical regulation model, and ensure the optimal regulatory is monotonic, in addition to ruling out stochastic mechanisms at optimum.

[2] Here, the transfer is interpreted as a payment agreement at the contracting stage, contingent on all possible realizations of θ. We will decompose it in sequential components later on.

$$U_L \geq \int [t_H(\theta) - \psi(\theta - C_H(\theta))]\, dG_L(\theta) \tag{IC$_{1,L}$}$$

$$U_H \geq \int [t_L(\theta) - \psi(\theta - C_L(\theta))]\, dG_H(\theta) \tag{IC$_{1,H}$}$$

The relevant participation constraints in this model require that each type of firm to attain a positive expected utility at the contracting stage, as the firm is risk neutral and decides to accept the contract prior to learning its actual efficiency parameter.[3] Thus, the individual rationality constraints are:

$$U_L \geq 0 \tag{IR$_L$}$$

$$U_H \geq 0 \tag{IR$_H$}$$

Denote W_i as the expected social welfare given the firm's type $i \in \{L,H\}$, that is, expectations about θ are taken according to the distribution G_i. Hence, social welfare in each state is given by:

$$\begin{aligned} W_L &= S - (1+\lambda) \int [\theta - e_L(\theta) + \psi(e_L(\theta))]\, dG_L(\theta) - \lambda U_L \\ W_H &= S - (1+\lambda) \int [\theta - e_H(\theta) + \psi(e_H(\theta))]\, dG_H(\theta) - \lambda U_H \end{aligned} \tag{2.4}$$

Given the regulator's prior $Prob(i = L) = \nu$ over the firm's signal, and thus over welfare states in (2.4), he chooses the optimal contract (DICM) solving the following program:

$$\begin{aligned} \underset{\{t_i(\theta), e_i(\theta)\}}{\text{Max}} \quad & \nu W_L + (1-\nu) W_H \\ \text{subject to } & (\text{IC-1})_i, (\text{IC-2})_i, (\text{IR})_i, i \in \{L,H\} \end{aligned}$$

2.2 Characterization

In order to characterize the optimal contract, we develop the steps to solve the principal's program in a series of lemmata. First, we replace the second period constraint (IC$_{2,i}$) by its first-order condition and a monotonicity condition.

Lemma 2.1. *The second period incentive constraint* (IC$_{2,i}$) *is equivalent to:*

$$\frac{d}{d\theta} u_i(\theta) = -\psi'(\theta - C_i(\theta)) \tag{2.5}$$

$$C_i'(\theta) \geq 0 \tag{MON-2$_i$}$$

[3] In fact, it is critical to the optimality of sequential discrimination that the firm is allowed to make a loss in some states, i.e., there must be no ex-post participation constraint. For further comments on this matter, see in Section 4.

The proof of Lemma 2.1 is standard, therefore omitted. Lemma 2.1 states that, in period-two, the firm has no incentive to pass for a slightly less efficient firm. Thus, in the second period there is equivalence between local and global incentives. We will not be so fortunate when it comes to first-period incentive compatibility; in fact, this is a well-known difficulty regarding dynamic mechanism applications.

The next lemma states that we can ignore type-L's participation constraint. The firm with a low cost signal can always mimic a less efficient one in expected terms, thus its participation is guaranteed.

Lemma 2.2. *The participation constraint for the low-signal firm* (IR_L) *is automatically satisfied in the optimal contract.*

Following the standard procedure, we first obtain the solution to an unconstrained version of the principal's program, and then verify *a fortiori* if the solution satisfies the omitted constraints. For now, it is useful to eliminate transfers from the incentive constraints, expressing them in terms of expected rents instead.[4]

Lemma 2.3. *The first period incentive constraint for a type-i firm (IC-1)$_i$ can be expressed as:*

$$U_i \geq U_j + \int \psi'(e_j(\theta))\,[G_i(\theta) - G_j(\theta)]\,d\theta, \quad j \neq i$$

Hence, the relaxed version of the principal's program is given by:

$$\begin{aligned}
\underset{\{U_i, e_i(\theta)\}}{\text{Max}} \quad & \nu \left\{ S - (1+\lambda) \int \theta - e_L(\theta) + \psi(e_L(\theta)) dG_L(\theta) - \lambda U_L \right\} \\
& +(1-\nu) \left\{ S - (1+\lambda) \int \theta - e_H(\theta) + \psi(e_H(\theta)) dG_H(\theta) - \lambda U_H \right\} \\
\text{subject to} \quad & U_H \geq 0 \\
& U_L \geq \int \psi'(e_H(\theta))\,[G_L(\theta) - G_H(\theta)]\,d\theta
\end{aligned}$$

As it is socially costly to leave rents to the firm, both constraints setting lower bounds for each type's rent, namely ($\text{IC}_{1,L}$) and (IR_H), are binding at optimum. After substituting the equality constraints in the objective function, pointwise optimization yields the first-order conditions to the unconstrained solution. Thus, we obtain the following lemma:

Lemma 2.4. *The solution to the relaxed problem is characterized by the following equations:*

$$\psi'(e_L(\theta)) = 1 \tag{2.6}$$

$$\psi'(e_H(\theta)) = 1 - \frac{\lambda}{1+\lambda} \frac{\nu}{1-\nu} \frac{G_L(\theta) - G_H(\theta)}{g_H(\theta)} \psi''(e_H(\theta)) \tag{2.7}$$

[4] A fundamental problem in sequential screening is that, unlike the static problem, single-crossing is not sufficient to guarantee equivalence between (IC-1) and local conditions (Courty and Li, 2000). Although this is not a problem in the discrete-type model, we still have to verify both first-period constraints to prove the optimality of the unconstrained solution.

Now we verify that the solution to the unconstrained problem characterized in Lemma 2.4 satisfies the property of monotonicity, a desirable property to ensure the DICM is fully separating (no bunching).

Lemma 2.5. *The cost schedule* $(C_L(\theta), C_H(\theta))$ *implemented in the solution to the unconstrained problem (equations* (2.6) *and* (2.7)*) satisfies the property of monotonicity, namely:*

(i) $C_H(\theta) \geq C_L(\theta)$*, for all* θ

(ii) $C_i(\theta)$ *is nondecreasing in* θ *for* $i = L,H$

2.3 Transfers

A central piece of the model, the transfer schedule $t_i(\theta)$ is the main instrument providing intertemporal incentives for truthful revelation and cost-reducing effort. It implements the DICM's effort at date zero, contingent on the many possible θ realizations. However, the sequential nature of the model allows us to decompose the transfer between an ex ante expected component and an ex post adjustment compensation:

$$t_i(\theta) = t_{1,i} + t_{2,i}(\theta) \tag{2.8}$$
$$\text{where } t_{1,i} \equiv \int t_i(\theta) dG_i(\theta)$$

Thus, the first-period component t_1 depends only on the reported cost signal, whereas $t_{2,i}$ has the role of inducing optimal behavior in the event of forecasting errors. Note the second-period transfer has a zero mean from an ex ante viewpoint; hence, in equilibrium, the firm expects no compensation at the contracting date.[5]

The next lemma uses the monotonicity results of Lemma 2.5 establish implementability (and thus second-best optimality) of the DICM, defined by transfers satisfying (2.8) and the optimal allocation in (2.6) and (2.7). See the proof of Lemma 2.6 for an explicit expression for the first- and second-period transfers, which are obtained from the discrete version of the envelope formula in Lemma 2.3, and pinned down by the binding constraint $U_H = 0$ in the relaxed problem.

Lemma 2.6. *There are transfers* $t_{1,i}$ *and* $t_{1,i}$*,* $i \in \{L,H\}$*, satisfying properties* (2.8)*, such that the optimal effort allocation satisfying FOCs* (2.6) *and* (2.7) *maximizes the original version of the principal's problem. Therefore, the DICM thus defined is second best optimal.*

The optimal mechanism distorts the high-signal firm's effort allocation, in comparison to the first best $\psi'(e(\theta)) = 1$, so as to limit the efficient firm's rent from reporting a low cost signal. On the other hand, the inefficient firm's informational rent is fully extracted, as seen from the binding participation constraint $U_H = 0$.

[5] The definition in text implies $\int t_{2,i}(\theta) dG_i(\theta) = 0$.

More interestingly, however, the optimal DICM implements first-best effort to the low-signal firm, even though its ex post efficiency is not observable. This is a consequence of the sequential nature of incentives. As the contract is signed ex ante and under risk neutrality, the firm may incur ex post losses. Nonetheless, incentives are such that, at the time of contracting, the firm anticipates that truthtelling is her best action in period two.

2.4 Indirect implementation

This section aims to show that the optimal DICM can be indirectly implemented by a particular incentive scheme: a sequence of transfers consisting of an advance payment $T_1(\bar{C})$ and an ex post adjustment $T_2(\bar{C},C)$, where $\bar{C}$ and C respectively stand for expected and realized cost. Hence, we are interested in representing the optimal transfers found in the proof of Lemma 2.6 as functions of observable information available at each period.

First, fix i as the true report of the firm's signal. In the second period, we can recover the firm's type by observing its cost performance, i.e. $\theta = \theta(i,C)$.[6] Thus we can write:

$$T_{2,i}(C) = t_{2,i}(\theta(i,C))$$

The indirect second-period transfer is nonincreasing and convex in the realized cost. We see that by taking simple derivatives:

$$\begin{aligned}
\frac{d}{dC}T_{2,i} &= \frac{d}{d\theta}t_{2,i}(\theta(i,C))\frac{1}{dC_i/d\theta} = -\psi'(e_i(\theta))C_i'\frac{1}{1-\dot{e}_i(\theta)} \\
&= -\psi'(e_i(\theta))
\end{aligned}$$

$$\begin{aligned}
\frac{d^2}{dC^2}T_{2,i} &= -\psi''(\theta(i,C)-C)\left[\frac{1}{C_i'}-1\right] \\
&= -\psi''(e_i(\theta))\frac{\dot{e}_i}{C_i'} \geq 0
\end{aligned}$$

where the last inequality follows from monotonicity and $\psi'' > 0$. Therefore, we can substitute the nonlinear schedule $T_{2,i}(C)$ by its family of tangents, by means of a simple Taylor expansion:

$$T_{2,i}(\hat{\theta},C) = t_{2,i}(\hat{\theta}) - \psi'(e_i(\hat{\theta}))\left[C - C_i(\hat{\theta})\right] \tag{2.9}$$

The above scheme is a menu of linear contracts which decentralizes effort choice to the firm. This is a central result discovered by Laffont and Tirole (1986). To see that, simply rewrite:

[6] As the cost schedule in the DICM is monotonic ($C_i(\theta) \geq 0$), it maps cost performance one-on-one to the firm's efficiency.

$$T_{2,i} = a_i(\hat{\theta}) - b_i(\hat{\theta})C$$

$$\text{where } a_i(\hat{\theta}) = t_{2,i}(\hat{\theta}) + \psi'(e_i(\hat{\theta}))C_i(\hat{\theta}) \tag{2.10}$$

$$b_i(\hat{\theta}) = \psi'(e_i(\hat{\theta})) \tag{2.11}$$

Given truthful report of i, the firm self-selects into the appropriate linear scheme and chooses second-best optimal effort, as stated in the next lemma.

Lemma 2.7. *Given truthful first-period report, the second-period incentive scheme $T_2 = a - bC$ with $a(.)$ and $b(.)$ given by equations (2.10) and (2.11) induces the type-i firm to choose:*

$$\hat{\theta} = \theta$$
$$e = e_i(\hat{\theta})$$

As to the first-period transfer, it should be a fixed payment just enough to cover the firm's expected cost and provide incentives for truthful reporting of its signal. As monotonicity implies $\bar{C}_H \geq \bar{C}_L$, we can recover the firm's private signal i from its reported cost forecast $\bar{C}$.[7] Therefore, we can write the first-period transfer as a function of the expected cost:

$$T_1(\bar{C}) = \begin{cases} t_{1,i} + \bar{C}_i, \text{ if } \bar{C} = \bar{C}_i, i \in \{L,H\} \\ 0, \text{ otherwise} \end{cases} \tag{2.12}$$

The next proposition summarizes the results of indirect implementation.

Proposition 1. *The indirect mechanism defined by the sequence of transfers in* (2.12) *and* (2.9) *implements the optimal DICM allocation characterized in Lemma 2.4.*

Proof. By Lemma 2.7, we know T_2 induces self-selection and optimal effort choice in the second period. It remains to prove that the combined transfer in period one, given by $T_1(\bar{C}) + E_i\left[T_2(\bar{C},C)\right]$, induces truthful reporting of the firm's signal i.

In period one, the firm anticipates that $T_2(\bar{C},C)$ is second-period incentive compatible, so she knows truthtelling is an optimal continuation strategy given her signal report. Therefore, in period one the firm anticipates $C = C_i(\theta)$ (or, equivalently, $e = e_i(\theta)$) when she announces i, which, in turn, implies she expects zero second-period compensation:

[7] Lemma 2.5 established monotonicity of the DICM cost allocation in two ways: $C_i(\theta)$ nondecreasing and $C_H(\theta) \geq C_L(\theta)$, for all θ. Alongside stochastic dominance (Assumption 1), this implies:

$$\bar{C}_H = \int C_H(\theta) dG_H$$
$$\geq \int C_L(\theta) dG_L = \bar{C}_L$$

$$\begin{aligned} E_i\left[T_2(\bar{C},C)\right] &= E_i\left[t_{2,i}(\theta) - b_i(\theta)(C - C_i(\theta))\right] \\ &= E_i\left[t_{2,i}(\theta)\right] \\ &= 0 \end{aligned}$$

Thus, in the first period the transfer must satisfy ($\text{IC}_{1,H}$):

$$\begin{aligned} U_H &\geq t_{1,L} + \bar{C}_L + \int \left[T_2(C,\bar{C}_L) - C_L(\theta) - \psi(e_L(\theta))\right] dG_H(\theta) \\ &= t_{1,L} + \int \left[t_2(\theta) - \psi(e_L(\theta))\right] dG_H(\theta) \end{aligned}$$

From that point, incentive compatibility follows from the same steps in the proof of Lemma 2.6. A similar argument applies to constraint ($\text{IC}_{1,L}$).

□

Our implementation by a sequence of cost reimbursements resembles Courty and Li's (2000) menu of refund contracts, where each consumer self-selects into a combination of an advance payment plus a refund policy. However, our result diverge from theirs in that the second period transfer is a nonlinear schedule depending on a continuous variable (though we can linearize it due to convexity). Courty and Li's mechanism instead defines a cutoff value on the second-period type support, which governs a dichotomic decision between consuming or obtaining the refund.

Chapter 3

Continuous cost signals

This section extends the model to the more realistic setting of continuous cost signals. Suppose the private signal observed by the firm in the first period is a continuous variable $\gamma \in \Gamma = [\underline{\gamma},\overline{\gamma}]$. This signal is interpreted as an initial estimate of the cost efficiency parameter θ, whose conditional distribution is given by $G(\theta|\gamma)$ with positive density $g(\theta|\gamma)$ in all points $\theta \in [\underline{\theta},\overline{\theta}]$.

The following assumptions over the dynamics of the firm's type mirror Assumptions 1 and 2 in the discrete-type version of the model. These assumptions impose the structure necessary to solve dynamic mechanism design problems with a standard Myersonian approach (see Pavan, Segal and Toikka, 2009):

Assumption 3. *The family $\{G(\theta|\gamma)\}_{\gamma\in\Gamma}$ is ranked by first-order stochastic dominance, i.e., for every increasing function $\varphi(.)$, $\int \varphi(\theta)dG(\theta|\gamma) \geq \int \varphi(\theta)dG(\theta|\hat{\gamma})$.*

Assumption 4. *For every θ and γ, the informativeness measure associated with $G(\theta|\gamma)$ is given by $\frac{\partial G(\theta|\gamma)/\partial\gamma}{g(\theta|\gamma)}$ and satisfies the following properties:*

$$\frac{\partial}{\partial\gamma}\left(\frac{\partial G(\theta|\gamma)/\partial\gamma}{g(\theta|\gamma)}\right) \leq 0 \tag{3.1}$$

$$\frac{\partial}{\partial\theta}\left(\frac{\partial G(\theta|\gamma)/\partial\gamma}{g(\theta|\gamma)}\right) \leq 0 \tag{3.2}$$

We can model the firm's signal as an unbiased estimate of its true efficiency parameter θ with an additive structure: $\theta = \gamma + s$, where s is an independent shock satisfying $E[s|\gamma] = 0$. This specification is without loss, since we can always redefine the shock as the difference between expected and realized efficiency parameters.[1] In the special case where γ and θ are independent, we have $G(\theta|\gamma) = \hat{G}(\theta - \gamma)$.

The contract is a revelation menu assigning to each sequence of reports (γ,θ) a pair $\{t(\gamma,\theta),C(\gamma,\theta)\}$, which specifies an overall transfer and a cost realization. The optimal contract maximizes social welfare subject to individual rationality and truthtelling at each report. Note that we can also

[1] The equivalence between state representations in environments with dynamic information is a result attributed to Eso and Szentes (2007).

express the contract in terms of the implemented cost-reducing effort, due to the one-to-one relationship between effort and realized cost:

$$C(\gamma,\theta) = \theta - e(\gamma,\theta)$$

In order to obtain incentive constraints, define $u(\gamma,\theta)$ the utility of reporting the true efficiency parameter θ in period two, given the type was truthfully revealed in period one. Moreover, define $U(\gamma)$ as the firm's rent from reporting its true cost signal γ at the contracting stage. Thus:

$$\begin{aligned} u(\gamma,\theta) &= t(\gamma,\theta) - \psi\left(\theta - C(\gamma,\theta)\right) \\ U(\gamma) &= \int_{\underline{\theta}}^{\overline{\theta}} \left[t(\gamma,\theta) - \psi\left(\theta - C(\gamma,\theta)\right)\right] dG(\theta|\gamma) \end{aligned}$$

Denote $F(\gamma)$ the regulator's prior over possible cost signals. We assume next that $F(.)$ satisfies the monotone hazard rate condition, which is standard in continuous mechanism design problems:

Assumption 5. *The regulator's prior distribution $F(.)$ has a monotonic hazard rate, i.e., the ratio $\frac{F(\gamma)}{f(\gamma)}$ is nondecreasing in γ.*

The regulator's problem is given by the following optimization:

$$\begin{aligned} \underset{\{t(\gamma,\theta),e(\gamma,\theta)\}}{\text{Max}} \quad & \int_{\underline{\gamma}}^{\overline{\gamma}} \left\{ S - (1+\lambda) \int \theta - e(\gamma,\theta) + \psi\left(e(\gamma,\theta)\right) dG(\theta|\gamma) - \lambda U(\gamma) \right\} dF(\gamma) \\ \text{subject to} \quad & u(\gamma,\theta) \geq t(\gamma,\theta') - \psi\left(\theta - C(\gamma,\theta')\right), \quad \forall \theta,\theta' \in \Theta, \quad \forall \gamma & \text{(IC-2)} \\ & U(\gamma) \geq \int_{\underline{\theta}}^{\overline{\theta}} t(\gamma',\theta) - \psi\left(\theta - C(\gamma',\theta)\right) dG(\theta|\gamma), \quad \forall \gamma,\gamma' \in \Gamma & \text{(IC-1)} \\ & U(\gamma) \geq 0, \quad \forall \gamma & \text{(IR)} \end{aligned}$$

In order to solve the principal's problem, we will follow a standard procedure consisting on a number of steps, which are developed in the following lemmata. First, like in the two-type model, we characterize the second period incentive constraint (IC-2) in terms of its first-order condition and a monotonicity condition.

Lemma 3.1. *The second period incentive constraint* (IC-2) *is equivalent to its associated first-order condition and a monotonicity condition:*

$$u_\theta(\gamma,\theta) = -\psi'\left(\theta - C(\gamma,\theta)\right) \tag{3.3}$$

$$C_\theta(\gamma,\theta) \geq 0 \tag{MON-2}$$

The proof of Lemma 3.1 is standard, therefore omitted. Unfortunately, the same procedure cannot be applied to the first period incentive constraint. It turns out that global incentive compatibility cannot be fully characterized by first and second order conditions in dynamic mechanism design problems. However, it is still useful obtain the derivative of the rent function (IC-1), in order to eliminate the transfer in the principal's objective function.

Lemma 3.2. *The first period incentive constraint* (IC-1) *implies:*

$$\dot{U}(\gamma) = \int_{\underline{\theta}}^{\overline{\theta}} \psi'\left(e(\gamma,\theta)\right) \frac{\partial G(\theta|\gamma)}{\partial \gamma} d\theta \tag{3.4}$$

Note from the expression in (3.4) that, since the FOSD assumption implies $\frac{\partial G(\theta|\gamma)}{\partial \gamma} \leq 0$, the firm's ex ante rent is decreasing in γ. Therefore, it allows us to drop all but the highest cost signal's participation constraints.

Lemma 3.3. *Under* (IC-1) *and equation* (3.3)*, the* (IR) *constraint is equivalent to* $U(\overline{\gamma}) \geq 0$*.*

Integrating the expression in equation (3.4):

$$U(\gamma) = U(\overline{\gamma}) - \int_{\gamma}^{\overline{\gamma}} \int_{\underline{\theta}}^{\overline{\theta}} \psi'\left(e(\tilde{\gamma},\theta)\right) \frac{\partial G(\theta|\tilde{\gamma})}{\partial \gamma} d\theta d\tilde{\gamma} \tag{3.5}$$

We obtain the expression for the expected rent by taking expectations with respect to the prior $F(\gamma)$ on both sides of equation (3.5), and subsequently integrating the RHS by parts:

$$\int_{\underline{\gamma}}^{\overline{\gamma}} U(\gamma) dF(\gamma) = U(\overline{\gamma}) - \int_{\underline{\gamma}}^{\overline{\gamma}} \int_{\underline{\theta}}^{\overline{\theta}} \psi'\left(e(\gamma,\theta)\right) \frac{\partial G(\theta|\gamma)}{\partial \gamma} d\theta \frac{F(\gamma)}{f(\gamma)} dF(\gamma)$$

Inserting this expression in the regulator's objective function, we can eliminate transfers from the principal's objective function (though not from the constraints), obtaining:

$$\begin{aligned} W(\gamma) = S - (1+\lambda) \int \left[\theta - e(\gamma,\theta) + \psi\left(e(\gamma,\theta)\right)\right] dG(\theta|\gamma) \\ + \lambda \frac{F(\gamma)}{f(\gamma)} \int \psi'\left(e(\gamma,\theta)\right) \frac{\partial G(\theta|\gamma)}{\partial \gamma} d\theta \end{aligned} \tag{3.6}$$

The above expression is the dynamic virtual welfare which the regulator seeks to maximize, in the sense of Myerson (1978), which consists of the net consumer surplus from the service plus a term correcting for the informational asymmetry. Note that virtual welfare's informational distortion (compared to first-best) due to the term representing the informativeness measure. In other words, expression (3.6) illustrates how the principal uses the sequential opportunities to screen the agent's private information in order to extract more surplus.

Moreover, expression (3.6) illustrates the importance of technical assumptions about both the principal's and the agent's priors. First, the log-concavity (MHRC) is important to guarantee the monotonicity of the optimal mechanism. Second, the assumption $\frac{\partial G(\theta|\gamma)}{\partial \gamma} \leq 0$ is important for the monotonicity of the sequential mechanism (see the proof of Lemma 3.5). This assumption means that the better the firm's initial estimate (in terms of a lower signal), the more informative is his distribution about the true cost realization. This assumption follows naturally from the FOSD assumption.

In order to characterize the relaxed solution to the principal's problem, we perform an optimization with the virtual welfare (3.6) as the objective function, while ignoring constraints (MON-2) and (IC-1):

$$\begin{aligned} \underset{\{e(\gamma,\theta)\}}{\text{Max}} \quad & \int_{\underline{\gamma}}^{\overline{\gamma}} W(\gamma) dF(\gamma) - \lambda U(\overline{\gamma}) \\ \text{subject to} \quad & U(\overline{\gamma}) \geq 0 \end{aligned}$$

Clearly, the constraint $U(\overline{\gamma}) \geq 0$ binds at optimum, and straightforward pointwise optimization allows us to compute the first-order condition which characterizes the unconstrained solution.

Lemma 3.4. *The unconstrained solution to the the principal's problem in the continuous-type model is characterized by the following equation:*

$$\psi'(e(\gamma,\theta)) = 1 + \frac{\lambda}{1+\lambda} \frac{F(\gamma)}{f(\gamma)} \frac{\partial G(\theta|\gamma)/\partial \gamma}{g(\theta|\gamma)} \psi''(e(\gamma,\theta)) \tag{3.7}$$

A desirable feature of our revelation mechanism is that it be monotonic, since it prevents bunching of types in the optimal contract. The following lemma establishes that the cost schedule implemented in the relaxed problem is monotonic in a strong sense (see Pavan, Segal and Toikka, 2009), which is necessary for proving second-best optimality.

Lemma 3.5. *The unconstrained effort allocation characterized in equation* (3.7) *implements a realized cost that is strongly monotonic, i.e.:*

(i) $C(\gamma,\theta)$ *is nondecreasing in* γ *for all* θ

(ii) $C(\gamma,\theta)$ *is nondecreasing in* θ *for all* γ

The main innovation about our model is the attainment of a transfer schedule $t(\gamma,\theta)$ that implements the cost schedule in equation (3.7). The direct computation of multidimensional transfer function from the envelope conditions in equations (3.3) and (3.4) would represent a difficult problem, involving a system of partial differential equations. However, we can find a suitable function $t(\gamma,\theta)$ by imposing the following properties:

$$t(\gamma,\theta) = t_1(\gamma) + t_2(\gamma,\theta) \tag{3.8}$$

$$t_1(\gamma) \equiv \int t(\gamma,\theta) dG(\theta|\gamma) \tag{3.9}$$

As in the discrete-type model, we separate the transfer function in two sequential components: an expected value transfer $t_1(\gamma)$ at the first stage of the game, which only depends on the cost signal report, followed by a zero-mean compensation transfer $t_2(\gamma,\theta)$ for forecasting errors after the second report.[2] The fact that the second period transfer is zero in expectation is crucial for the proving optimality of the sequential mechanism, as it will not affect the first period report.

The following lemma establishes the optimality of the mechanism's effort allocation characterized in (3.7).

Lemma 3.6. *Suppose* $e(\gamma,\theta)$ *satisfying* (3.7) *implements a monotonic cost schedule* $C(\gamma,\theta)$*, in the sense of Lemma 3.5. Then, there exists a sequential transfer schedule* $t(\gamma,\theta) = t_1(\gamma)+t_2(\gamma,\theta)$ *such that the contract* $(t(\gamma,\theta),C(\gamma,\theta))$ *satisfies* (MON-2) *and* (IC-1).

There only remains to compute expressions for the optimal sequence of transfers and the firm's rent, which are obtained residually from the problem's constraints.

Proposition 2. *The optimal regulatory contract is characterized by the following equations:*

$$\begin{aligned}
&C(\gamma,\theta) = \theta - e(\gamma,\theta)\\
&\psi'(e(\gamma,\theta)) = 1 + \frac{\lambda}{1+\lambda}\frac{F(\gamma)}{f(\gamma)}\frac{\partial G(\theta|\gamma)/\partial\gamma}{g(\theta|\gamma)}\psi''(e(\gamma,\theta))\\
&U(\gamma) = -\int_{\gamma}^{\overline{\gamma}}\int_{\underline{\theta}}^{\overline{\theta}} \psi'\left(e(\tilde{\gamma},\theta)\right)\frac{\partial G(\theta|\tilde{\gamma})}{\partial\gamma}d\theta d\tilde{\gamma}\\
&t(\gamma,\theta) = t_1(\gamma) + t_2(\gamma,\theta)\\
&t_1(\gamma) = \int t(\gamma,\theta)dG(\theta|\gamma)
\end{aligned}$$

3.1 An example with additive structure

The optimal effort implicit in equation (3.7) depends on a special term: $\frac{\partial G(\theta|\gamma)/\partial\gamma}{g(\theta|\gamma)}$. This is Baron and Besanko's (1984) informativeness measure, which can also be interpreted as an impulse-response function of the agent's current information on its future type (Pavan, Segal and Toikka,

[2] Note that $\int t_2(\gamma,\theta)dG(\theta|\gamma) = 0$. This is an immediate consequence of equation (3.9).

2009). Note the FOSD ordering of $G(.)$ implies that the optimal level of effort is higher the more informative is the firm's private signal.[3]

Additionally, the DICM implements higher effort (and thus lower costs) in comparison to the static regulatory mechanism. In order to illustrate this point, we present an example where the relationship between each period's type has an additive structure. Suppose we can write the efficiency parameter as:

$$\theta = \kappa\gamma + (1-\kappa)\varepsilon$$

where $\kappa \in [0,1]$, γ is the firm's private signal and ε is an i.i.d. shock over finite support, with density $h(.)$ and distribution $H(.)$. Then it follows that:

$$G(\theta|\gamma) = H\left(\frac{\theta - \kappa\gamma}{1-\kappa}\right)$$
$$\text{and} \quad \left|\frac{\partial G(\theta|\gamma)/\partial\gamma}{g(\theta|\gamma)}\right| = \kappa$$

Inserting this expression in the FOC which characterizes the optimal allocation (equation (3.7)), we obtain the following extreme cases:

1. **Independent case:** suppose $\kappa = 0$, then the optimal allocation is first-best:

$$\psi'(e(\gamma,\theta)) = 1$$

2. **Perfect correlation:** suppose $\kappa = 1$, then the optimal allocation is equivalent to the static mechanism:

$$\psi'(e(\gamma,\theta)) = 1 - \frac{\lambda}{1+\lambda}\frac{F(\gamma)}{f(\gamma)}\psi''(e(\gamma,\theta))$$

Note that, in the case of independent types, the principal is able to implement first-best effort regardless of the firm's ex post efficiency. On the other hand, perfect correlation collapses the model to the static Laffont-Tirole solution, as the firm knows its type with certainty from the beginning. Therefore, in the middle ground cases between independence and perfect correlation, the sequential mechanism achieves a higher level of welfare: the opportunity to screen the firm's private information twice allows the regulator to extract more surplus from the firm.

[3] By FOSD, we have $G(\theta|\hat{\gamma}) \geq G(\theta|\gamma)$, $\forall\hat{\gamma} > \gamma$, and thus $\frac{\partial G(\theta|\gamma)}{\partial\gamma} \leq 0$.

Chapter 4

Concluding remarks

Before we conclude, some technical comments are in order. First, we should note sequential screening models are akin to environments of multidimensional private information.[1] In such settings, the single-crossing property is harder to guarantee; as a consequence, optimal mechanisms might exhibit bunching or not be monotonic (see, for example, McAffee and McMillan, 1988, and Rochet and Chone, 1998). In our setting, however, the (IC-2) constraints impose additional restrictions on the instruments available to the regulator, which are absent from the multidimensional problem.

Similarly, we cannot in general guarantee equivalence between global and local incentive compatibility in dynamic mechanism design problems. In our model, this means the first-order and monotonicity conditions are not sufficient for incentive compatibility in the first period. Nonetheless, the FOSD assumption imposes additional structure on the type-space, which allows to pin down the firm's rent (and thus the transfer) from constraint (IC-1). This discussion is pursued further in Krahmer and Strausz (2011b).

Second, an important feature of the sequential screening design is that there must be no ex post participation constraint. Specific to our model, that means that there should be no device preventing the firm from making a loss in the second period, such as minimum income guarantees. Ex post participation constraints bring further complications to the model, and may even deem the sequential mechanism suboptimal in comparison to the static one. For more on this subject, see Krahmer and Strausz (2011a).

Finally, there is the issue of stochastic mechanisms. Courty and Li (2000) point that first-order dominance is not by itself sufficient to reduce the dimensionality of the sequential problem, so random allocation rules might help the principal fine tune the outcome when monotonicity fails. In contrast, Krahmer and Strausz (2011b) show that optimal monotonic schemes are globally incentive compatible precisely because they are deterministic. While such a concern is absent in our model since the principal enjoys a high degree of flexibility from effort being a continuous variable, failures of monotonicity are beyond the scope of this paper. See, for example, Strausz (2006).

Many instances of regulation and procurement indeed resemble sequential rules. In public works particularly, cost estimates tend to evolve as more information is revealed in later stages of

[1] We can imagine the firm chooses from a menu of reimbursement rules contingent on both her cost signal i and the ex post efficiency θ.

planning and execution. In fact, cost forecasts often have a bias towards underestimation at the time public contracts are awarded (Flyvbjerg et al., 2002). The incidence of cost escalation might suggest, for example:

(i) Incentives misalignment: dynamic incentives for truthful revelation of private information are not intuitive to the point of emerging as a long-term equilibrium of the institutional arrangement. Therefore, there would be room for reform in the manner such incentives are provided, as present contracts might be suboptimal.

(ii) There might be missing elements in our model: real-world situations are further complicated by commitment issues, regulatory capture and political economy considerations.

The second item points some interesting directions in which our model could be extended. For example, it would be instructive to explore how the sequential reimbursement mechanism behaves under standard dynamic distortions such as noncommitment and renegotiation. Our model assumed the regulator is not only committed to the second period transfer schedule, but not to renegotiate the terms even if it is mutually beneficial (which is possible for some realizations of θ).

Furthermore, our model could be extended to the complete Laffont-Tirole framework, with a variable size project, a market demand schedule, and a pricing rule as a part of the optimal mechanism. Such an effort might lead to a deeper insight on dynamic devices seen often in public contracts, like financial equilibrium clauses. For instance, if a mechanism resembling an extraordinary tariff review is found to be optimal, that might shed new light on requests to restore of the contract's financial equilibrium. Instead of being interpreted as rent-seeking, opportunistic renegotiations, such requests might be seen as the exercise of the sequential mechanism's second-period transfer (the "refund").

Appendix A

Proofs

Proof of Lemma 2.2.

If the optimal mechanism is $\{t_L(\theta), e_L(\theta), t_H(\theta), e_H(\theta)\}$, then it satisfies ($IC_{2,i}$), ($IC_{1,L}$) and ($IR_H$). As G_H stochastically dominates G_L, we have $\int \varphi(x)dG_H \geq \int \varphi(x)dG_L$, for any increasing function $\varphi(.)$. However, as $u_H(\theta)$ is decreasing in by ($IC_{2,i}$), it follows that:

$$\int u_H(\theta)dG_L \geq \int u_H(\theta)dG_L$$

By ($IC_{1,L}$), it follows that:

$$\int u_L(\theta)dG_L \geq \int u_H(\theta)dG_L \geq 0$$

The second inequality follows from FOSD, and the last one from (IR_H). Hence, the optimal mechanism satisfies (IR_L).

□

Proof of Lemma 2.3.

We prove for $i = L$. For $i = H$, simply reverse the roles of i and j. Under ($IC_{1,L}$):

$$\begin{aligned} U_L &\geq \int u_H(\theta)g_L(\theta)d\theta + \int u_H(\theta)g_H(\theta)d\theta - \int u_H(\theta)g_H(\theta)d\theta \\ &= U_H + \int u_H(\theta)\left[g_L(\theta) - g_H(\theta)\right]d\theta \end{aligned} \tag{A.1}$$

where $g_i(\theta)d\theta = dG_i(\theta)$. A simple integration by parts yields:

$$\int u_H(\theta)g_i(\theta)d\theta = u_H(\theta)G_i(\theta)\Big|_{\underline{\theta}}^{\overline{\theta}} - \int \dot{u}_H(\theta)G_i(\theta)d\theta$$
$$= u_H(\overline{\theta}) + \int \psi'(e_H(\theta))G_i(\theta)d\theta$$

where the last equality follows from equation (2.5) in Lemma 2.1.

Canceling out terms $u_H(\overline{\theta})$, it obtains that:

$$\int u_H(\theta)\left[g_L(\theta) - g_H(\theta)\right]d\theta = \int \psi'(e_H(\theta))G_L(\theta)d\theta - \int \psi'(e_H(\theta))G_H(\theta)d\theta \tag{A.2}$$

Together, (A.1) and (A.2) yield the result.

□

Proof of Lemma 2.5.

The first-order conditions in Lemma 2.4 imply that:

$$\psi'(e_H(\theta)) \leq 1 = \psi'(e_L(\theta)), \quad \forall\theta$$

which implies $e_H(\theta) \geq e_L(\theta)$.

Hence, from the definition $C_i(\theta) = \theta - e_i(\theta)$, it follows that $C_H(\theta) \geq C_L(\theta)$ for any θ.

To verify the monotonicity in θ of the allocations $\{C_L(\theta), C_H(\theta)\}$, note that:

$$C_i'(\theta) = 1 - \dot{e}_i(\theta), \quad i = L,H$$

implying $C_i'(\theta) \geq 0$ if, and only if $\dot{e}_i(\theta) \leq 1$. Thus, we need to compute the signs of the derivatives $\dot{e}_L$ and $\dot{e}_H$ implicitly from the first-order conditions (2.6) and (2.7).

(i) Differentiating equation (2.6):

$$\psi''(e_L)\dot{e}_L = 0$$

As $\psi'' > 0$ and the optimal allocation satisfies $e_L \geq 0$, it follows that:

$$\dot{e}_L(\theta) = 0$$

(ii) Differentiating equation (2.7):

$$\psi''(e_H)\dot{e}_H = -\frac{\lambda}{1+\lambda}\frac{\nu}{1-\nu}\left\{\frac{d}{d\theta}\left[\frac{G_L - G_H}{g_H}\right]\psi''(e_H) - \frac{G_L - G_H}{g_H}\psi'''(e_H)\dot{e}_H\right\}$$

$$\dot{e}_H = -\frac{\lambda}{1+\lambda}\frac{\nu}{1-\nu}\frac{\frac{d}{d\theta}\left[\frac{G_L-G_H}{g_H}\right]\psi''(e_H)}{\psi''(e_H) + \frac{\lambda}{1+\lambda}\frac{\nu}{1-\nu}\frac{G_L-G_H}{g_H}\psi'''(e_H)}$$

By assumption, we have $\psi'' > 0$ and $\psi''' \geq 0$. From the FOSD assumption, it follows that $G_L \geq G_H$. Finally, we need the technical assumption that $\frac{d}{d\theta}\left[\frac{G_H(\theta)-G_L(\theta)}{g_H(\theta)}\right] \leq 0$ in order to obtain:

$$\dot{e}_H(\theta) \leq 0$$

□

Proof of Lemma 2.6.

The unconstrained solution $\{e_L(\theta), e_H(\theta)\}$ is second-best optimal if it satisfies the omitted constraints (MON-2_i) and ($IC_{1,H}$), and ($IC_{1,L}$).

Constraint (MON-2_i) follows immediately from the monotonicity of the cost schedule $C_i(\theta)$, established in Lemma 2.5. In turn, verifying the first-period constraints requires computing expressions for the transfers $t_{1,i}$ and $t_{2,i}(\theta)$. We complete the the proof in three steps: first we define the transfers and then we verify the two incentive constraints.

Step 1. The first-period transfer $t_{1,i}$ is obtained residually from the binding constraints (IR_H) and ($IC_{1,L}$) and from the fact that $t_{1,i} = \int t_i(\theta)dG_i$. Hence:

$$t_{1,H} = \int \psi(e_H(\theta))dG_H \tag{A.3}$$

$$t_{1,L} = \int \psi(e_L(\theta))dG_L + \int \psi'(e_H(\theta))\left[G_L(\theta) - G_H(\theta)\right]d\theta \tag{A.4}$$

The second period transfer must satisfy the differential equation necessary to local optimality in ($IC_{2,i}$):

$$t'_{2,i}(\theta) + \psi'(\theta - C_i(\theta))C'_i(\theta) = 0 \tag{A.5}$$

Note that $t_{1,i}$ does not appear in equation (A.5), as it is invariant to θ. An initial condition is obtained from the zero mean requirement for the second period transfer, as follows:

$$
\begin{aligned}
0 &= \int t_{2,i}(\theta)g_i(\theta)d\theta \\
&= t_{2,i}(\theta)G_i(\theta)\Big|_{\underline{\theta}}^{\overline{\theta}} - \int t'_{2,i}(\theta)G_i(\theta)d\theta \\
\Rightarrow \quad t_{2,i}(\overline{\theta}) &= -\int \psi'(e_i(\theta))C'_i(\theta)G_i(\theta)d\theta
\end{aligned} \tag{A.6}
$$

where the second equality is an integration by parts and the third uses equation (A.5). Integrating equation (A.5) over the interval $[\theta,\overline{\theta}]$, subject to initial condition (A.6) yields:

$$
t_{2,i}(\theta) = \int_{\theta}^{\overline{\theta}} \psi'(e_i(\tilde{\theta}))C'_i(\tilde{\theta})d\tilde{\theta} - \int \psi'(e_i(\theta))C'_i(\theta)G_i(\theta)d\theta
$$

We can readily check that indeed $\int t_{2,i}(\theta)dG_i = 0$. More interestingly, let's compute the expected value of transfer $t_{2,i}$ from the point of view of type-j firm:

$$
\begin{aligned}
\int t_{2,i}(\theta)dG_j &= \int \left[\int_{\theta}^{\overline{\theta}} \psi'(e_i(\tilde{\theta}))C'_i(\tilde{\theta})d\tilde{\theta} - \int \psi'(e_i(\theta))C'_i(\theta)G_i(\theta)d\theta\right] dG_j \\
&= \underbrace{\int_{\theta}^{\overline{\theta}} \psi'(e_i(\tilde{\theta}))C'_i(\tilde{\theta})d\tilde{\theta}G_j(\theta)\Big|_{\underline{\theta}}^{\overline{\theta}}}_{=0} + \int \psi'(e_i(\theta))C'_i(\theta)G_j(\theta)d\theta \\
&\quad - \int \psi'(e_i(\theta))C'_i(\theta)G_i(\theta)d\theta \\
&= \int \psi'(e_i(\theta))C'_i(\theta)\left[G_j(\theta) - G_i(\theta)\right] d\theta \\
&\leq 0
\end{aligned}
$$

We will use this result in the verification of the first-period incentive constraints.

Step 2. In order to verify ($\mathrm{IC}_{1,H}$), we have to show that:

$$
U_H \geq \int \left[t_{1,L} + t_{2,L}(\theta) - \psi(e_L(\theta))\right] dG_H(\theta)
$$

Using the expressions for type-H's rent and transfer $t_{1,L}$, we obtain, equivalently:

$$\begin{aligned}
0 \geq & \int \psi(e_L(\theta))dG_L(\theta) + \int \psi'(e_H(\theta)) \left[G_L(\theta) - G_H(\theta)\right] d\theta \\
& + \int t_{2,L}(\theta)dG_H(\theta) - \int \psi(e_L(\theta)))dG_H(\theta) \\
\int \psi(e_L(\theta))dG_H(\theta) - \int \psi(e_L(\theta))dG_L(\theta) \geq & \int \psi'(e_H(\theta)) \left[G_L(\theta) - G_H(\theta)\right] d\theta + \int t_{2,L}(\theta)dG_H(\theta) \\
\int \psi'(e_L(\theta)) \left(1 - C_L'(\theta)\right) \left[G_L(\theta) - G_H(\theta)\right] d\theta \geq & \int \psi'(e_H(\theta)) \left[G_L(\theta) - G_H(\theta)\right] d\theta + \int t_{2,L}(\theta)dG_H(\theta) \\
\int \left[\psi'(e_L(\theta)) - \psi'(e_H(\theta))\right] \left[G_L(\theta) - G_H(\theta)\right] d\theta \geq & \int \psi'(e_L(\theta))C_L'(\theta) \left[G_L(\theta) - G_H(\theta)\right] d\theta + \int t_{2,L}(\theta)dG_H(\theta)
\end{aligned}$$

Note that the LHS is positive from FOSD and from the fact that $e_L(\theta) \geq e_H(\theta)$, $\forall\theta$, whereas the RHS is zero from the expression for $E_H\left[t_{2,L}\right]$ found in Step 1.

Step 3. In order to verify ($\text{IC}_{1,L}$), we have to show that:

$$U_L \geq \int \left[t_{1,H} + t_{2,H}(\theta) - \psi(e_H(\theta))\right] dG_L(\theta)$$

Adopting a similar strategy as in (i), we substitute the expressions for type-L's rent in binding constraints (IR_H) and the transfer in equations ($\text{IC}_{1,L}$), obtaining:

$$\begin{aligned}
\int \psi'(e_H(\theta)) \left[G_L(\theta) - G_H(\theta)\right] d\theta \geq & \int \psi(e_H(\theta))dG_H(\theta) + \int \left[t_{2,H}(\theta) - \psi(e_H)\right] dG_L(\theta) \\
\int \psi'(e_H(\theta)) \left[G_L(\theta) - G_H(\theta)\right] d\theta \geq & \int \psi(e_H(\theta)) \left[g_H(\theta) - g_L(\theta)\right] d\theta + \int t_{2,H}(\theta)dG_L(\theta) \\
\int \psi'(e_H(\theta)) \left[G_L(\theta) - G_H(\theta)\right] d\theta \geq & \int \psi'(e_H(\theta)) \left(1 - C_H'(\theta)\right) \left[G_L(\theta) - G_H(\theta)\right] d\theta + \int t_{2,H}(\theta)dG_L(\theta) \\
0 \geq & - \int \psi'(e_H(\theta))C_H^(\theta)\prime \left[G_L(\theta) - G_H(\theta)\right] d\theta + \int t_{2,H}(\theta)dG_L(\theta)
\end{aligned}$$

But the result for $E_L\left[t_{2,H}\right]$ ensures that the RHS of the last inequality is zero, so the constraint ($\text{IC}_{1,L}$) is satisfied with equality.

□

Proof of Lemma 2.7.

This proof follows Laffont and Tirole (1993), Chapter 1, Proposition 1.4 (p.59). The firm's maximization problem in the second period, given the transfer schedule $T_{2,i} = a_i(\hat{\theta}) - b_i(\hat{\theta})C$, is given by:

$$\underset{\hat{\theta},e}{\text{Max}} \quad t_{2,i}(\hat{\theta}) + \psi'(e_i(\hat{\theta})) \left[\hat{\theta} - e_i(\hat{\theta}) - e + \theta\right] - \psi(e)$$

The above problem is concave by monotonicity, thus the solution is given by the following first order conditions:

$$t'_{2,i}(\theta) + \psi''(e_i(\hat{\theta})) \left[\hat{\theta} - e_i(\hat{\theta}) - e + \theta\right] \dot{e}_i(\hat{\theta}) + \psi'(e_i(\hat{\theta})) \left[1 - \dot{e}_i(\hat{\theta})\right] = 0 \tag{A.7}$$

$$\psi'(e) = \psi'(e_i(\hat{\theta})) \tag{A.8}$$

Equation (A.8) implies $e = e_i(\hat{\theta})$, and equation (A.7), in turn, implies $\hat{\theta} = \theta$.

□

Proof of Lemma 3.3.

Take $\gamma,\hat{\gamma} \in \Gamma$ and suppose without loss $\hat{\gamma} > \gamma$. By (IC-1):

$$U(\hat{\gamma}) \geq U(\gamma) + \int [t(\gamma,\theta) - \psi(\theta - C(\gamma,\theta))] \left[g(\theta|\hat{\gamma}) - g(\theta|\gamma)\right] d\theta$$

Switching the roles of $\hat{\gamma}$ and $\hat{\gamma}$:

$$U(\gamma) \geq U(\hat{\gamma}) + \int [t(\hat{\gamma},\theta) - \psi(\theta - C(\hat{\gamma},\theta))] \left[g(\theta|\gamma) - g(\theta|\hat{\gamma})\right] d\theta$$

Combining both inequalities yields:

$$\int u(\gamma,\theta) \left[g(\theta|\hat{\gamma}) - g(\theta|\gamma)\right] d\theta \leq U(\hat{\gamma}) - U(\gamma) \leq \int u(\hat{\gamma},\theta) \left[g(\theta|\hat{\gamma}) - g(\theta|\gamma)\right] d\theta$$

Dividing all terms by $(\hat{\gamma} - \gamma)$ and taking limits when $\hat{\gamma} \to \gamma$, we obtain:

$$\begin{aligned}
\dot{U}(\gamma) &= \int_{\underline{\theta}}^{\overline{\theta}} u(\gamma,\theta) \frac{\partial g(\theta|\gamma)}{\partial \gamma} d\theta \\
&= u(\gamma,\theta) \frac{\partial G(\theta|\gamma)}{\partial \gamma} \Big|_{\underline{\theta}}^{\overline{\theta}} - \int_{\underline{\theta}}^{\overline{\theta}} u_\theta(\gamma,\theta) \frac{\partial G(\theta|\gamma)}{\partial \gamma} d\theta
\end{aligned}$$

where the last equality is a simple integration by parts.

Finally, using equation (3.4) in Lemma 3.1 obtains:[1]

$$\dot{U}(\gamma) = \int_{\underline{\theta}}^{\overline{\theta}} \psi'\left(\theta - C(\gamma,\theta)\right) \frac{\partial G(\theta|\gamma)}{\partial \gamma} d\theta$$

[1] We need boundary conditions such as zero values of $\frac{\partial G(\theta|\gamma)}{\partial \gamma}$ at corners $\underline{\theta}$ and $\overline{\theta}$.

□

Proof of Lemma 3.5.

To see that the unconstrained allocation $C(\gamma,\theta)$ is monotonic in the sense of the definition proposed in Lemma 3.5, note that the definition $C(\gamma,\theta) = \theta - e(\gamma,\theta)$ implies:

$$\nabla C(\gamma,\theta) = \begin{bmatrix} -e_\gamma(\gamma,\theta) \\ 1 - e_\theta(\gamma,\theta) \end{bmatrix}$$

Therefore, in order to establish monotonicity of $C(\gamma,\theta)$, we will find the sign of the derivatives of the effort schedule characterized in the first-order condition (3.7) with respect to each variable in turn.

(i) Differentiating (3.7) with respect to γ:

$$\psi''(e)e_\gamma = \frac{\lambda}{1+\lambda}\left[\frac{\partial}{\partial\gamma}\left(\frac{F}{f}\right)\frac{\partial G/\partial\gamma}{g}\psi''(e) + \frac{F}{f}\frac{\partial}{\partial\gamma}\left(\frac{\partial G/\partial\gamma}{g}\right)\psi''(e) + \frac{F}{f}\frac{\partial G/\partial\gamma}{g}\psi'''(e)e_\gamma\right]$$

Then some algebra leads us to the expression:

$$e_\gamma = \frac{\frac{\lambda}{1+\lambda}\psi''(e)\left[\frac{\partial}{\partial\gamma}\left(\frac{F}{f}\right)\frac{\partial G/\partial\gamma}{g} + \frac{F}{f}\frac{\partial}{\partial\gamma}\left(\frac{\partial G/\partial\gamma}{g}\right)\right]}{\psi''(e) - \frac{\lambda}{1+\lambda}\frac{F}{f}\frac{\partial G/\partial\gamma}{g}\psi'''(e)}$$

From the assumptions about the disutility of effort we have $\psi'' > 0$, $\psi''' \geq 0$. The monotone hazard rate condition guarantees that $\frac{\partial}{\partial\gamma}\left(\frac{F}{f}\right) \geq 0$, whereas the FOSD ordering of the $\{G(\theta|\gamma)\}_\gamma$ family implies $\frac{\partial G}{\partial\gamma} \leq 0$. Finally, we need property (3.1) the technical Assumption 4 made at the beginning of the model stating that:

$$\frac{\partial}{\partial\gamma}\left(\frac{\partial G/\partial\gamma}{g}\right) \leq 0$$

Therefore, we are able to compute the sign of the derivative of the unconstrained effort allocation with respect to the cost signal:

$$e_\gamma(\gamma,\theta) \leq 0, \quad \forall\theta$$

(ii) Differentiating (3.7) with respect to θ:

$$\psi''(e)e_\theta = \frac{\lambda}{1+\lambda}\frac{F}{f}\left[\frac{\partial}{\partial\theta}\left(\frac{\partial G/\partial\gamma}{g}\right)\psi''(e) + \frac{\partial G/\partial\gamma}{g}\psi'''(e)e_\theta\right]$$

After some manipulation we obtain:

$$e_\theta = \frac{\frac{\lambda}{1+\lambda}\frac{F}{f}\psi''(e)\frac{\partial}{\partial\theta}\left(\frac{\partial G/\partial\gamma}{g}\right)}{\psi''(e) - \frac{\lambda}{1+\lambda}\frac{F}{f}\frac{\partial G/\partial\gamma}{g}\psi'''(e)}$$

Analyzing all previous assumptions used in part (i), besides property (3.2) Assumption 4:

$$\frac{\partial}{\partial\theta}\left(\frac{\partial G/\partial\gamma}{g}\right) \leq 0$$

To complete the proof, we obtain the sign of the relevant derivative:

$$e_\theta(\gamma,\theta) \leq 0, \quad \forall\gamma$$

□

Proof of Lemma 3.6.

We will prove this Lemma in two steps. First we will propose transfer functions that satisfy the properties in the text. Then we will show that these transfers and the unconstrained allocation obtained in equation (3.7) satisfy the omitted constraints (MON-2) and (IC-1).

Step 1. The firm's ex ante rent is defined as:

$$U(\gamma) = \int \left[t(\gamma,\theta) - \psi(e(\gamma,\theta))\right] dG(\theta|\gamma)$$

Therefore, since $t_1(\gamma)$ is defined as the ex ante expected transfer, we have:

$$\begin{aligned} t_1(\gamma) &= E_\theta\left[\psi(e(\gamma,\theta))|\gamma\right] \\ &= \int \psi(e(\gamma,\theta))dG(\theta|\gamma) + U(\gamma) \end{aligned}$$

i.e., the first period transfer compensates the firm for the expected disutility of effort in addition to the ex ante informational rent. From equation (3.5), we can write the above transfer as:

$$t_1(\gamma) = \int \psi(e(\gamma,\theta))dG(\theta|\gamma) - \int_{\gamma}^{\overline{\gamma}} \int \psi'(e(\tilde{\gamma},\theta))\frac{\partial G(\theta|\tilde{\gamma})}{\partial \gamma}d\theta d\gamma \tag{A.9}$$

In order to obtain $t_2(\gamma,\theta)$, we depart from the second-period incentive constraint:

$$\theta \in \underset{\hat{\theta}\in\Theta}{\text{argmax}} \quad [t_1(\gamma) + t_2(\gamma,\hat{\theta})] - \psi(\theta - C(\gamma,\hat{\theta}))$$

with its associated first-order condition:

$$\frac{\partial}{\partial\theta}t_2(\gamma,\theta) = -\psi'(\theta - C(\gamma,\theta))C_\theta(\gamma,\theta) \tag{A.10}$$

The above differential equation characterizes the shape of the t_2 schedule. In order to obtain an initial condition, we use property (3.9) and integration by parts:

$$\begin{aligned} 0 &= \int t_2(\gamma,\theta)g(\theta|\gamma)d\theta \\ &= t_2(\gamma,\theta)G(\theta|\gamma)\Big|_{\underline{\theta}}^{\overline{\theta}} - \int \frac{\partial}{\partial\theta}t_2(\gamma,\theta)G(\theta|\gamma)d\theta \end{aligned}$$

to obtain:

$$t_2(\gamma,\overline{\theta}) = -\int \psi'(e(\gamma,\theta))C_\theta(\gamma,\theta)G(\theta|\gamma)d\theta \tag{A.11}$$

Integrating (A.10) subject to (A.11), we obtain:

$$\begin{aligned} t_2(\theta,\gamma) &= t_2(\gamma,\overline{\theta}) + \int_{\theta}^{\overline{\theta}} \psi'(e(\gamma,\tilde{\theta}))C_\theta(\gamma,\tilde{\theta}d\tilde{\theta} \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \psi'(e(\gamma,\theta))C_\theta(\gamma,\theta)\frac{1-G(\theta|\gamma)}{g(\theta|\gamma)}dG(\theta|\gamma) - \int_{\theta}^{\overline{\theta}} \psi'(e(\gamma,\tilde{\theta}))C_\theta(\gamma,\tilde{\theta})d\tilde{\theta} \end{aligned} \tag{A.12}$$

Step 2. Let $t(\gamma,\theta)$ be defined as the sum of components denoted in equations (A.9) and (A.12). Moreover, consider the unconstrained allocation $C(\gamma,\theta)$ characterized by equation (3.7) and the definition $C(\gamma,\theta) = \theta - e(\gamma,\theta)$. We will verify that such a pair of functions satisfies (MON-2) and (IC-1).

Constraint (MON-2) is trivially satisfied. It is a consequence of the allocation $C(\gamma,\theta)$ being monotonic in θ, as established in Lemma 3.5.

To verify (IC-1), we must show:

$$\Delta \equiv U(\gamma) - U(\hat{\gamma}|\gamma) \geq 0, \quad \forall \gamma, \hat{\gamma} \in \Gamma$$

First, note that:

$$\begin{aligned} \Delta &= U(\gamma) - U(\hat{\gamma}) + U(\hat{\gamma}|\hat{\gamma}) - U(\hat{\gamma}|\gamma) \\ &= \int_{\hat{\gamma}}^{\gamma} \dot{U}(z) - \frac{\partial}{\partial \gamma} U(\hat{\gamma}|z) dz \end{aligned}$$

Let's compute $\dot{U}(.)$ and $\partial U(\hat{\gamma}|.)/\partial \gamma$ in turn:

$$\begin{aligned} U(\gamma) &= \int \left[t_1(\gamma) + t_2(\gamma,\theta) - \psi(e(\gamma,\theta)) \right] dG(\theta|\gamma) \\ &= t_1(\gamma) - \int \psi(e(\gamma,\theta)) dG(\theta|\gamma) \\ \Rightarrow \quad \dot{U}(\gamma) &= \dot{t}_1(\gamma) - \int \psi'(e(\gamma,\theta)) e_\gamma(\gamma,\theta) dG(\theta|\gamma) - \int \psi(e(\gamma,\theta)) \frac{\partial g(\theta|\gamma)}{\partial \gamma} d\theta \end{aligned}$$

However, differentiating the expression for $t_1(\gamma)$ in (A.9):

$$\begin{aligned} \dot{t}_1(\gamma) = &\int \psi'(e(\gamma,\theta)) e_\gamma(\gamma,\theta) dG(\theta|\gamma) + \int \psi(e(\gamma,\theta)) \frac{\partial g(\theta|\gamma)}{\partial \gamma} d\theta \\ &+ \int \psi'(e(\gamma,\theta)) \frac{\partial G(\theta|\gamma)}{\partial \gamma} d\theta \end{aligned}$$

After canceling out opposite terms, we are left with:

$$\dot{U}(\gamma) = \int \psi'(e(\gamma,\theta)) \frac{\partial G(\theta|\gamma)}{\partial \gamma} d\theta$$

Moreover:

$$\begin{aligned} U(\hat{\gamma}|\gamma) &= \int \left[t(\hat{\gamma},\theta) - \psi(e(\hat{\gamma},\theta)) \right] dG(\theta|\gamma) \\ \Rightarrow \quad \frac{\partial}{\partial \gamma} U(\hat{\gamma}|\gamma) &= \int \underbrace{\left[t(\hat{\gamma},\theta) - \psi(e(\hat{\gamma},\theta)) \right]}_{u(\hat{\gamma},\theta)} \frac{\partial g(\theta|\gamma)}{\partial \gamma} d\theta \\ &= \int \psi'(e(\hat{\gamma},\theta)) \frac{\partial G(\theta|\gamma)}{\partial \gamma} d\theta \end{aligned}$$

where the last equality uses an integration by parts similar to the proof of Lemma 3.2.

Therefore, we have:

$$\Delta = \int_{\hat{\gamma}}^{\gamma} \int_{\underline{\theta}}^{\overline{\theta}} \left[\psi'(e(z,\theta)) - \psi'(e(\hat{\gamma},\theta))\right] \frac{\partial G(\theta|\gamma)}{\partial \gamma} d\theta dz$$

Now, suppose $\gamma > \hat{\gamma}$. Note that $z \geq \hat{\gamma}$, for all $z \in [\hat{\gamma},\gamma]$. Since the FOSD assumption implies $\partial G(\theta|\gamma)/\partial\gamma \leq 0$, we need the term in brackets to be nonpositive so to obtain $\Delta \geq 0$. But from the monotonicity of $e(\gamma,\theta)$, in addition to $\psi' > 0$, we have $\psi'(e(z,\theta)) \leq \psi'(e(\hat{\gamma},\theta))$. If $\gamma < \hat{\gamma}$, a similar argument applies.

□

Bibliography

Armstrong, M. and D. E. Sappington (2007, 00). *Recent Developments in the Theory of Regulation*, Volume 3 of *Handbook of Industrial Organization*, Chapter 27, pp. 1557–1700. Elsevier.

Athey, S. and I. Segal (2013). An efficient dynamic mechanism. *Econometrica 81*(6), 2463–2485.

Bajari, P. and S. Tadelis (2001, Autumn). Incentives versus transaction costs: A theory of procurement contracts. *RAND Journal of Economics 32*(3), 387–407.

Baron, D. P. and D. Besanko (1984). Regulation and information in a continuing relationship. *Information Economics and Policy 1*(3), 267–302.

Battaglini, M. (2005, June). Long-term contracting with markovian consumers. *American Economic Review 95*(3), 637–658.

Bergemann, D. and J. Valimaki (2010). The dynamic pivot mechanism. *Econometrica 78*(2), 771–789.

Boleslavsky, R. and M. Said (2013). Progressive screening: Long-term contracting with a privately known stochastic process. *Review of Economic Studies 80*(1), 1–34.

Courty, P. and H. Li (2000, October). Sequential screening. *Review of Economic Studies 67*(4), 697–717.

Dai, C., T. R. Lewis, and G. Lopomo (2006, 09). Delegating management to experts. *RAND Journal of Economics 37*(3), 503–520.

Eso, P. and B. Szentes (2007). Optimal information disclosure in auctions and the handicap auction. *Review of Economic Studies 74*(3), 705–731.

Flyvbjerg, B., M. S. Holm, and S. Buhl (2002). Underestimating costs in public works projects: Error or lie? *Journal of the American Planning Association 68*(3), 279–295.

Hart, O. D. and J. Moore (1988, July). Incomplete contracts and renegotiation. *Econometrica 56*(4), 755–85.

Krahmer, D. and R. Strausz (2011a, November). The benefits of sequential screening. CEPR Discussion Papers 8629, C.E.P.R. Discussion Papers.

Krahmer, D. and R. Strausz (2011b). Optimal procurement contracts with pre-project planning. *Review of Economic Studies 78*(3), 1015–1041.

Laffont, J.-J. and J. Tirole (1986, June). Using cost observation to regulate firms. *Journal of Political Economy 94*(3), 614–41.

Laffont, J.-J. and J. Tirole (1988, September). The dynamics of incentive contracts. *Econometrica 56*(5), 1153–75.

Laffont, J.-J. and J. Tirole (1990, October). Adverse selection and renegotiation in procurement. *Review of Economic Studies 57*(4), 597–625.

Laffont, J.-J. and J. Tirole (1993). *A Theory of Incentives in Procurement and Regulation*, Volume 1 of *MIT Press Books*. The MIT Press.

Maestri, L. (2013, September). Dynamic contracting under adverse selection and renegotiation. Working Papers Mimeo, Tolouse School of Economics.

McAfee, R. P. and J. McMillan (1988, December). Multidimensional incentive compatibility and mechanism design. *Journal of Economic Theory 46*(2), 335–354.

Myerson, R. B. (1978, December). Optimal auction design. Discussion Papers 362, Northwestern University, Center for Mathematical Studies in Economics and Management Science.

Myerson, R. B. (1986, March). Multistage games with communication. *Econometrica 54*(2), 323–58.

Pavan, A., I. Segal, and J. Toikka (2009, May). Dynamic mechanism design: Incentive compatibility, profit maximization and information disclosure. Discussion Papers 1501, Northwestern University, Center for Mathematical Studies in Economics and Management Science.

Rochet, J.-C. and P. Chone (1998, July). Ironing, sweeping, and multidimensional screening. *Econometrica 66*(4), 783–826.

Rogerson, W. P. (2003, June). Simple menus of contracts in cost-based procurement and regulation. *American Economic Review 93*(3), 919–926.

Strausz, R. (2006, May). Deterministic versus stochastic mechanisms in principal-agent models. *Journal of Economic Theory 128*(1), 306–314.

Tirole, J. (1986, April). Procurement and renegotiation. *Journal of Political Economy 94*(2), 235–59.