TEXTO DE DISCUSSÃO Nº37

ESTIMATING THE OUTPUT GAP:

A SVAR Approach

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Estimating the Output Gap: a SVAR Approach[*]

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1   Introduction

The output gap measurement is one of the most difficult and important subjects in applied macroeconomics. It not only allows us to estimate the relationship between inflation and economic activity but also to evaluate the impact of interest rates movements and fiscal expansions on prices. There are three common strategies for estimating it: statistical filtering (e.g., the HP filter), production function approach, and demand shocks identification (such as Blanchard and Quah, in [4]). We use a combination of the second and third ones, applying the methology developed by [2] to identify demand shocks using the set of variables implied by the production function approach (and an external component).

2   Methodology

The variables were chosen to approximate a Cobb-Douglas production function (equation 1). Taking logs and derivatives of equation 1 and approximating the capital stock growth by the GDP's gross investment, we have a relationship between (HW) hours worked (IBRE), (CU) capacity utilization (IBRE), (GFCF) gross fixed capital formation (IBGE), (BGDP) Brazilian GDP (IBGE), and (UGDP) US GDP (BEA). Hence, these are the chosen variables for our SVAR. All data are quarterly (2004Q1 to 2013Q2) and seasonally adjusted.

$$GDP_t = Prod_t \left(K_t (1 - NUCI_t) \right)^{\alpha} \left(Hours_t \right)^{1-\alpha} \tag{1}$$

The VAR, introduced by [5], has the form of equation 2: a set of simultaneous equations explaining each of the variables from the vector Y_t as a linear combination of all variables from Y_{t-1} up to Y_{t-k} and a constant[1].

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[*]The views expressed here are from its author and do not represent the official opinions of FGV or IBRE

[1]Note that the lagged terms of Y_t are exogenous to Y_t (Y_t cannot have causal impact on Y_{t-k}, for every $k \geq 1$).

$$Y_t = \beta_0 + \sum_{j=1}^{k} Y_{t-j}\beta_j + Au_t \qquad (2)$$

In equation 2, u_t is a $5x1$ vector of shocks and A is a $5x5$ matrix that contemporaneously transmits the shocks in each of these 5 variables to all of them. However, the VAR estimation provides only residuals (its coefficients, β_j, are estimated under OLS and the residuals u_t are the difference between actual Y_t and predicted values for Y_t), not the structural shocks series. In order to identify them, assumptions on matrix A are needed.

A will be estimated by covariance-matching. The residual covariance of equation 2 is given by $E[Au_t u_t' A']$, and we know that, being Cov the estimated variance-covariance matrix, $Cov = E[Au_t u_t' A']$ (supposing A is deterministic and $E[u_t u_t'] = I$, $Cov = AA'$). Cov has 25 values and A has 25 variables; however, Cov and AA' are symmetric. Therefore, while $Cov = AA'$ has 25 equations, only 15 of them are linearly independent (the 10 on the superior triangle are identical to the 10 on the inferior one). As a consequence, we need to impose 10 identification restrictions in order to have 25 equations for 25 variables (15 come from $Cov = AA'$ and 10 come from the identification restrictions).

These restrictions are on the short run and long run properties of our shocks (following [2]). They are:

- 1) Brazil is a small open economy

- 2) An increase in GDP may be caused by an increase in the hours worked, but not the opposite

- 3) An increase in GDP may be caused by an increase in the NUCI, but not the opposite

All restrictions are imposed to A: short run restrictions have the form of equation 3 ($a_{i,j}$ is the element of A on the i-th row and j-th column), while long run restrictions have the form of equation 4 ($LR_{i,j}$ is the element of LR on the i-th row and j-th column).

$$a_{i,j} = 0 \qquad (3)$$

$$LR = (I - \sum_{j=1}^{k}\beta_j)^{-1}A^T$$
$$LR_{i,j} = 0 \qquad (4)$$

The first condition comprises 6 identification restrictions (since we allow for interdependence between Brazilian and American GDP, this restriction means that hours worked, NUCI, and Brazilian investment do not affect US GDP, neither on the short nor on the long run). Each one of the other two conditions

comprises two more restrictions (GDP and investment do not affect neither the number of hours worked nor the NUCI). All identification conditions can be summarized in the system of equations 5.

$$
\begin{aligned}
a_{BGDP,HW} &= 0\,(Condition2) \\
a_{BGDP,CU} &= 0\,(Condition3) \\
a_{HW,UGDP} &= 0\,(Condition1) \\
a_{CU,UGDP} &= 0\,(Condition1) \\
a_{GFCF,UGDP} &= 0\,(Condition1) \\
LR_{BGDP,HW} &= 0\,(Condition2) \\
LR_{BGDP,CU} &= 0\,(Condition3) \\
LR_{HW,UGDP} &= 0\,(Condition1) \\
LR_{CU,UGDP} &= 0\,(Condition1) \\
LR_{GFCF,UGDP} &= 0\,(Condition1)
\end{aligned}
\tag{5}
$$

After the identification of the elements of A we can compute the shocks that hit the economy for each period in our sample. This can be easily done using equation 6 (e_t is the residual of the VAR described in equation 2).

$$
u_t = A^{-1} e_t \tag{6}
$$

Therefore, we can calculate Brazilian GDP if only some of the five shocks occurred. Hence, using the decomposition method introduced by [1] we can calculate the contribution to economic growth for each of the 5 variables in our VAR. The first two variables (hours worked and NUCI) are interpreted as demand shocks, and so generate the output gap (the demand contribution to GDP growth is defined as the sum of the contributions of hours worked and NUCI). Similarly, the contribution of foreign shocks to the domestic growth are interpreted as the external gap. It is a natural consequence of this approach that the potential output be defined as the predicted value for GDP summed by the productivity and investment shocks.

3   The Output Gap

In figure 1 we show our estimates for the output gap. Since it captures demand shocks, it is indeed a very volatile series. In order to analyze our estimates, in figure 2 we show our four-quarter accumulated output gap and compare it with that obtained from the HP Filter. Both reflect percent deviations from the potential output. Interestingly, even our accumulated output gap is more volatile than the output gap derived from the HP Filter.

As shown in figure 2, most of the times our gap seems to be an antecedent indicator for the HP one (after 1Q2007 or 4Q2009, for example). After the moterary policy shock of 2Q2008, for instance, the HP Gap continues to rise,

Figure 1: Output Gap Measures
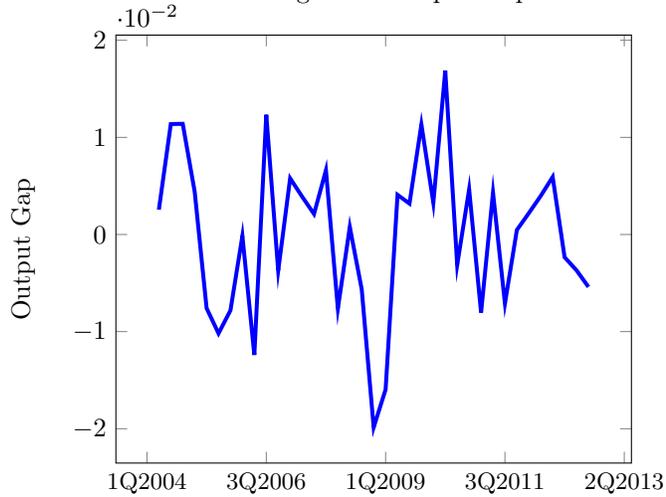
while ours immediately shows a rapid decline (there are negative demand shocks on 1Q2008, 3Q2008, and a strong negative shock in 4Q2008 and 1Q2009). Only in 4Q2008 the output gap derived from the HP is capable of indicating a deterioration in the aggregate demand. More importantly, after 2Q2012 the HP filter indicates a deflationary output gap, while our model shows there is inflationary pressure from aggregate demand. This is compatible with the rise in interest rates initiated in 2Q2013.

Figure 2: Output Gap Measures

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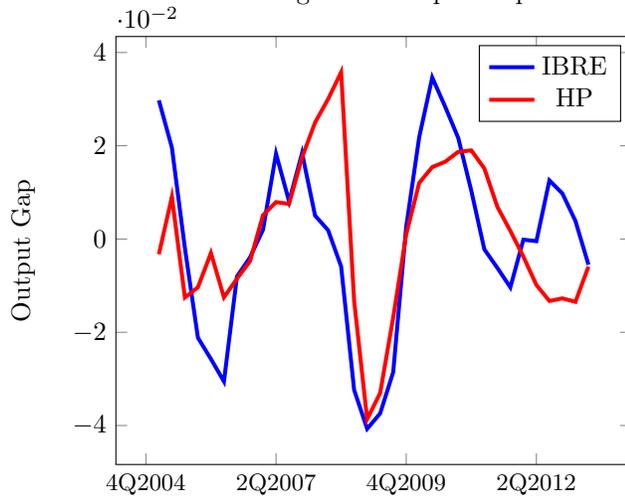
4 Small Economy Model (SEM)

Our SEM has 4 equations: the IS curve, the Phillips curve, the Taylor Rule, and the Expectations Rule. The Expectations Rule ensures that, in the long run, expected inflation equals actual inflation. The IS curve establishes a link between the neutral real interest rate *ex-ante* and the long run nominal public deficit. It also determines the impact of the real interest rate and external output gap on the domestic one. The equations were estimated using GMM with lagged variables as regressors.

4.1 Phillips Curve

The Phillips Curve is vertical in the long run and depends on expectations, output gap, and the ERPT (exchange rate pass-through). Its reduced-form is expressed in equation 7, where q_t is the real exchange rate in t, H_t is the output gap in t, H_t^* is the foreign output gap in t, Π_t^{t+3} is inflation accumulated from t to $t+3$, $E_t(x_{t+1})$ is the expectation operator for the random variable x_{t+1} based on an information set that comprises data only up to date t.

$$\Pi_t = \alpha_0 \Pi_{t-1} + \alpha_1 \Pi_{t-2} + (1 - \alpha_0 - \alpha_1) E_t(\Pi_{t+1}^{t+4}) +$$
$$\sum_{j_1=0}^{k_1} \beta_{j_1} H_{t-j_1} + \sum_{j_2=0}^{k_2} \gamma_{j_2} \Delta q_{t-j_2} + u_t^P \tag{7}$$

Note that, in the long run, H_t, H_t^*, and Δq_t are zero. It was estiamted using GMM, with lagged variables as instruments. Π_t is a core CPI measure. Monitored and food prices are exogenous.

Table 1: Phillips Curve estimates

Variable	Value
α_0	1.070
	(0.141)
α_1	-0.266
	(0.132)
β_3	0.295
	(0.102)
γ_4	0.026
	(0.009)

As table 1 suggests[2], these estimates indicate a short run exchange-rate pass-through (ERPT) of about 2.6% and very strong inertial effects. This is in

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[2]Standard erros in parentheses. All ommitted parameters were statistically insignificant and omitted on the reduced form estimation.

part due to the fact that inflation was measured as four-quarter accumulated inflation.

4.2 IS

The IS equation follows the process described in 8.

$$H_t = c + \alpha_R r_t + \sum_{w_1=1}^{f_1} \xi_{w_1} H_{t-w_1} + \sum_{w_2=0}^{f_2} \zeta_{w_2} H^*_{t-w_2} + \sum_{w_3=0}^{f_3} \eta_{w_3} g_{t-w_3} + u_t^H \quad (8)$$

Although H_t and H^*_t are zero in the long run, g_t is not. Therefore, there is a long run relationship between the real interest rate and the nominal deficit, as proportion of GDP.

Table 2: IS estimates

Variable	Value
α_R	-0.110
	(0.033)
ξ_2	0.360
	(0.057)
ζ_0	0.502
	(0.079)
η_0	0.135
	(0.060)
η_2	0.113
	(0.054)

As noted previously, for each level of permanent nominal deficit there is a real interest rate of equilibrium. For instance, for a nominal deficit of 2.8% of GDP, the real equilibrium interest rate is 6.3%; for a nominal deficit of 1.8% of GDP the real interest rate goes down to 4.1%.

The most recent data indicates a nominal deficit around 2.8% of GDP, which indicates a real interest rate of equilibrium lying around 6.3%. Therefore, the output gap calculated by this methodology seems to respond well to movements in interest rates and nominal government deficits, as expected.

However, this approach has one disadvantage: since the nominal deficit is a state variable, temporary changes in the fiscal surplus lead to permanent shifts in the interest rate of equilibrium. For instance, if the government assumes a temporary severe fiscal contraction, debt as a percentage of GDP will fall, and so will the interest expense. As a consequence, after the shock ends, for the same operational deficits of before the nominal deficit would be lower, and so would be the real interest rate of equilibrium.

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Figure 3: Interest Rates and Nominal Deficits
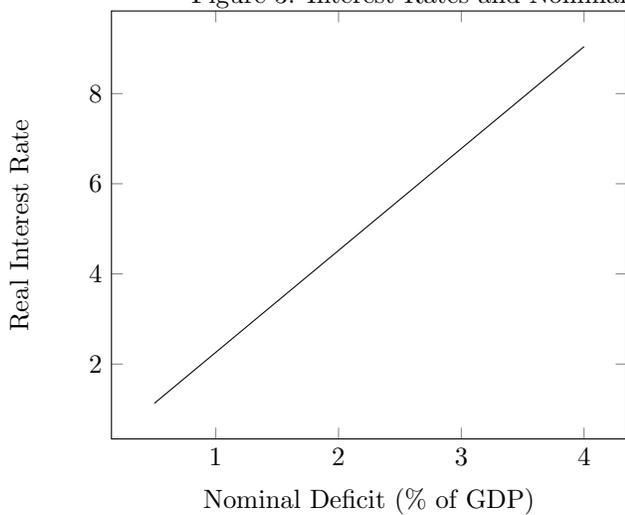
Also, the IRF (impulse response function) of four-quarter accumulated inflation after a shock of one standard deviation (0.8%) on the output gap is shown in figure 4.

Figure 4: 4Q accumulated inflation after a 1 standard deviation shock on the output gap
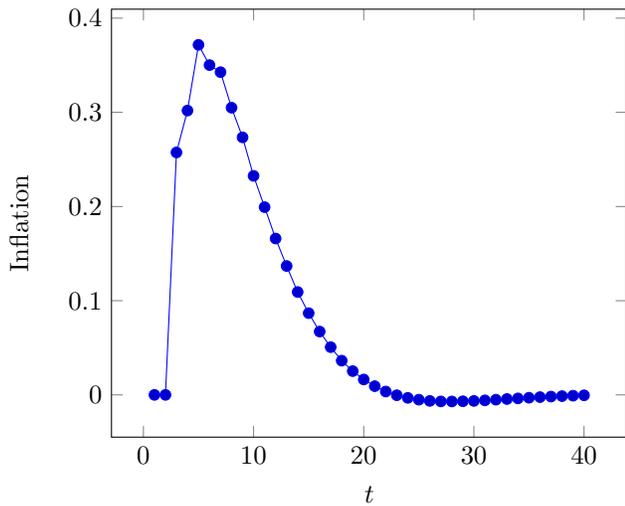
4.3 Taylor Rule

The monetary authority follows an inflation-targeting regime, defining the nominal interest rate. Therefore, as usual, it defines the level of inflation that will prevail in the long run (the inflation target). It assumes the relationship expressed in 9, where $\bar{\Pi}$ is the inflation target and u_t^i is a monetary policy shock.

$$i_t = \gamma_R i_{t-1} + (1 - \gamma_R)(i^* + \lambda_\Pi(\Pi_t - \bar{\Pi}) + \lambda_Y H_t) + u_t^i \qquad (9)$$

Note that i^* is the nominal interest rate of equilibrium (derived from the inflation target and the real interest rate of equilibrium), a clearly identified parameter in our model.

In the simulations, the real interest rate is computed using equation 10.

$$r_t = \frac{1 + i_t}{1 + E_t(\Pi_{t+1}^{t+4})} - 1 \qquad (10)$$

Table 3: Taylor Rule estimates

Variable	Value
constant	-1.800
	(0.527)
γ_R	0.951
	(0.030)
λ_Π	0.416
	(0.076)
λ_Y	0.533
	(0.137)

The Taylor Rule estimates an inflation target of 5.49% and greater sensitivity of the interest rate to output gap movements. This kind of sensitivity is not usually found in the literature, but it may be the result of a more accurate measurement of the output gap, since it is widely known that erratic measurement errors tend to underestimate their parameters.

4.4 Expectations

Expectations are assumed to be adaptative, as expressed in 11. This reduced form ensures that, in the long run, $E_t(\Pi_{t+1}^{t+4}) = \Pi_{t+1}^{t+4}$.

$$E_t(\Pi_{t+1}^{t+4}) - E_{t-4}(\Pi_{t-3}^t) = \lambda(\Pi_{t-3}^t - E_{t-4}(\Pi_{t-3}^t)) + u_t^E \qquad (11)$$
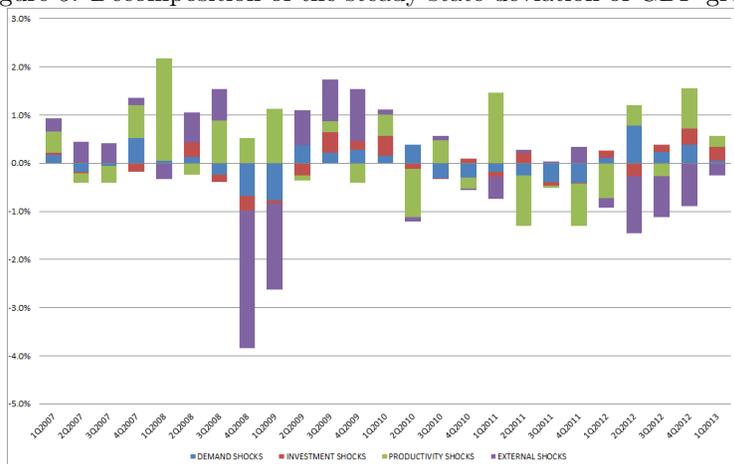
Table 4: Expectation Rule estimates

Variable	Value
λ	0.450
	(0.077)

5 Results

5.1 Growth Decomposition

The model allows us to decompose the steady-state deviations in four components: external, productivity, demand, and investment. The results for the period 2007Q1 up to 2013Q2 are in figure 5.

Figure 5: Decomposition of the steady-state deviation of GDP growth

Note that, as expected, in the global economic crisis of 2008, global growth is the main cause of the plunge in Brazilian GDP. Also, in 2012 the external component seems to be an important driver of our low growth (in contrast with 2011, when it seems to be caused by negative productivity shocks).

5.2 Impulse Response Functions (IRFs)

In figure 6 we have the impact on four-quarter accumulated inflation of a 100 bps increase on the Selic Rate for 4 quarters (after that, the economy follows the estimated Taylor Rule). These results are very similar to those presented on the Brazilian March Inflation Report ([3]). However, the impact of a temporary increase in the nominal deficit is much greater than the one estimated there. If

the nominal deficit is increased by 100 basis points for 4 quarters, going back to its original values after that, inflation will respond as in figure 7. The impacts estimated by this model are much greater than the ones showed in the Brazilian March Inflation Report ([3]).

Figure 6: Monetary Policy Impact on Inflation
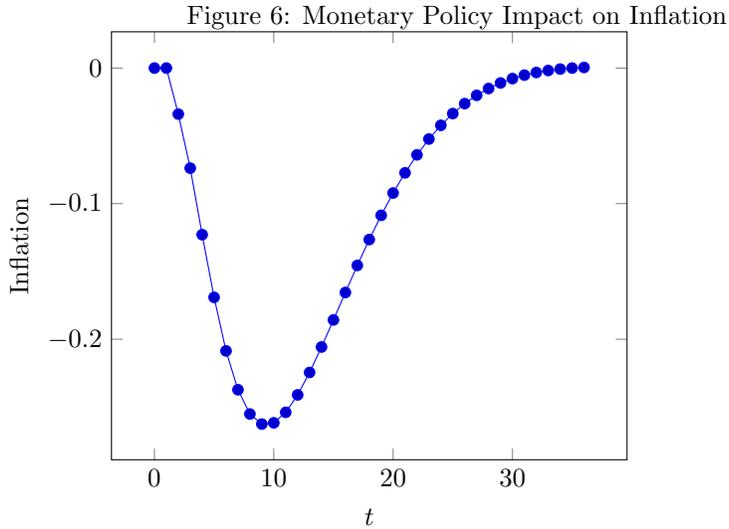
This is a very striking result, since the nominal deficit has the operational (which impacts GDP and inflation) and the non-operational (which does not impact GDP and inflation) components: inasmuch as it suffers from this measurement error, its impacts tend to be *underestimated* (bias toward zero). Other important observation is that a 100 basis points rise in the nominal deficit is much more unlikely than a 100 bps hike in the Selic. In fact, in our samples, a shock of one standard deviation in each of these variables would be a 350 bps hike in the Selic rate and a 140 bps hike in the nominal deficit. Therefore, although the Selic IRF shows an impact of approximately 0.25 bps after 9 quarters while the nominal deficit goes up to 0.40 bps, interest-rate movements are responsible for much more inflation fluctuation than the fiscal deficit (since it is 2.5 times more volatile).

It is also important to remark that our sample covers a period with several structural changes in the Brazilian economy. In particular, the average of the interest rate on the first half of the sample is 15.1%, while in the second is 9.9%. These structural changes are incorporated in the model as permanent shocks, rising the measured standard deviation. One important evolution from this work would be to test operational or structural measures of government deficit in this model, instead of using the nominal deficits.

Figure 7: Fiscal Policy Impact on Inflation
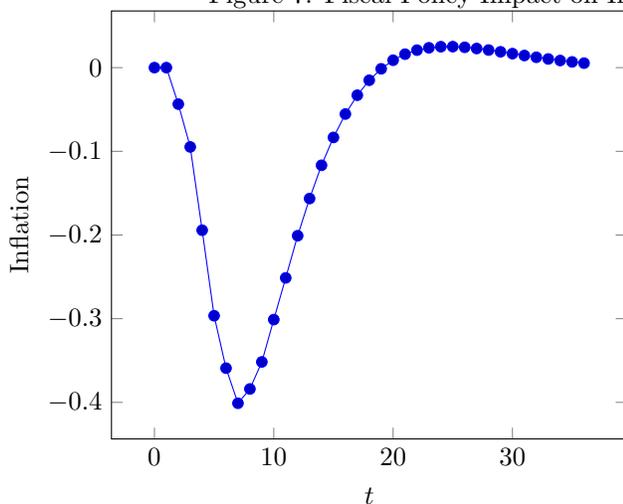
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