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JEFFERSON DONIZETI PEREIRA BERTOLAI

ESSAYS ON MONETARY AND BANKING THEORY

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Tese para obtenção do grau de Doutor em Economia apresentada à Escola de Pós-Graduação em Economia da Fundação Getulio Vargas

Área de concentração: Economia

Orientador: Ricardo de Oliveira Cavalcanti

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Resumo

Esta tese de Doutorado é dedicada ao estudo de instabilidade financeira e dinâmica em Teoria Monetária. É demonstrado que corridas bancárias são eliminadas sem custos no modelo padrão de teoria bancária quando a população não é pequena. É proposta uma extensão em que incerteza agregada é mais severa e o custo da estabilidade financeira é relevante. Finalmente, estabelece-se otimalidade de transições na distribuição de moeda em economias em que oportunidades de trocas são escassas e heterogêneas. Em particular, otimalidade da inflação depende dos incentivos dinâmicos proporcionados por tais transições.

O capítulo 1 estabelece o resultado de estabilidade sem custos para economias grandes ao estudar os efeitos do tamanho populacional na análise de corridas bancárias de Peck & Shell. No capítulo 2, otimalidade de dinâmica é estudada no modelo de monetário de Kiyotaki & Wright quando a sociedade é capaz de implementar uma política inflacionária. Apesar de adotar a abordagem de desenho de mecanismos, este capítulo faz um paralelo com a análise de Sargent and Wallace (1981) ao destacar efeitos de incentivos dinâmicos sobre a interação entre as políticas monetária e fiscal. O capítulo 3 retoma o tema de estabilidade financeira ao quantificar os custos envolvidos no desenho ótimo de um setor bancário à prova de corridas e ao propor uma estrutura informacional alternativa que possibilita bancos insolventes. A primeira análise mostra que o esquema de estabilidade ótima exibe altas taxas de juros de longo prazo e a segunda que monitoramento imperfeito pode levar a corridas bancárias com insolvência.

Palavras-chave: Teoria Monetária. Teoria Bancária. Fragilidade Financeira.

Abstract

This thesis is dedicated to the study of both financial instability and dynamics in monetary theory. It is shown that bank runs are costless prevented in the standard model of banking theory when population is not small. An extension is proposed where aggregate uncertainty is more severe and financial stability cost is relevant. Finally, transitions in the distribution of money are shown to be optimal in an economy where exchanges opportunities are scarce and heterogeneous. In particular, optimality of inflation depends on dynamic incentives provided by such transitions.

Chapter 1 establishes the costless result for large economies by studying the effects of population size in the Peck-Shell analysis of bank runs. In chapter 2, dynamics optimality is studied in Kiyotaki-Wright monetary model when society is able to implement a inflationary policy. Despite adopting the mechanism design approach, this chapter parallels Sargent and Wallace (1981) analysis in highlighting dynamic incentives to the interaction between fiscal and monetary policies. Chapter 3 returns to the issue of financial stability by quantifying the costs involved in optimally designing a run-proof banking sector and by proposing an alternative information structure which allows for insolvent banks. Former analysis shows that optimal stability scheme features high long term interest rates, and the latter that imperfect monitoring can lead to bank runs with insolvency.
***Keywords*:** Monetary Theory. Banking Theory. Financial fragility.

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Introduction

This thesis is comprised of three chapters about monetary and banking theory. Money and banks are institutions that has been extensively studied in Economics, perhaps for their relevance in shaping economic relations among individuals. Among main questions on this subject are bank instability, that has frequently been associated to economic crisis, and inflationary policies which decreases expected return of the medium of exchange. Current work seeks to contribute to this discussion by studying the welfare cost of financial stability in the standard model of bank runs and the implications of dynamic incentives implied by transitions in the distribution of money to the optimality of inflation.

Diamond and Dybvig (1983) model is considered the first to show that an illiquid bank system can be seen as a property of the optimal allocation. They also advocate the existence of a bank run equilibrium under such allocation. In this equilibrium, people who prefer late consumption demand early withdrawals to protect themselves from a run they believe is happening. As shown by Wallace (1988), such result requires payments to be done sequentially in the sense that a given withdrawal size cannot depend on the choices made by late withdrawals. Later, Green and Lin (2003) has shown that truth-telling equilibrium is uniquely implemented if depositors knows their positions in the queue to access the bank and individuals type are independent. In this context, Peck and Shell (2003) model emerges as the standard model to study bank runs as a possible outcome of the optimal allocation. They study an economy where depositors do not know their position in line and provide examples where run equilibrium exists. Chapter 1 study the effects increasing the population size in their setup and found that a contract featuring equal-treatment for almost all depositors of the same type approximates the optimum. Because the approximation also satisfies Green-Lin incentive constraints, when the planner discloses positions in the queue, welfare in these alternative specifications are sandwiched. Disclosure, however, is not needed since our approximating contract is not subject to runs. As a consequence, bank stability in large economies can be implemented without welfare costs.

An alternative function attributed to bank sector is creation of money. In order to study such phenomenon, Cavalcanti and Wallace (1999) extend Kiyotaki and Wright (1989) monetary model by making society able to perfectly monitor actions taken by a proportion of the population. Without monitoring technology, the society faces scarcity of

opportunities of exchange and individual anonymity which together give an essential role to an otherwise useless durable object called money. It is used as a proof of past production to society which is expected to compensated by future consumption, i.e., as a medium of exchange. Monitoring ability creates a sector in the economy that can credibly promise to redeem money they issue, called inside money. Recently, Deviatov and Wallace (2012) has studied optimality of inflation in this setup. Specifically, they study a policy of destructing money that resembles the effects of inflation in the sense that it decreases expected return of money and concentrates money distribution. Positive inflation is a property of the optimal steady-state allocation in that it improves the distribution of opportunities of exchange. Chapter 2 analyses how dynamics in the distribution of money can change such result. Main finding is that distribution of exchange opportunities can be improved using transitions with virtually no use of inflation. Scarcity of money is initially high and progressively deteriorates with inside money creation. On the other hand, monitored people are taxed by society which demands them gifts to people without money. In later result, monitoring technology is used to tax the group of people issuing money, a new dimension to enrich the interaction between monetary and fiscal policy studied by Sargent and Wallace (1981). In particular, a fiscal policy which restrict taxation would make transition less attractive and therefore higher inflation more desirable.

Chapter 3 returns to the issue of financial stability by quantifying the costs involved in optimally designing a run-proof banking sector and by proposing an alternative information structure which allows for insolvent banks. First analysis shows that optimal stability scheme features high long term interest rates. When applied to the setup studied in chapter 1, it is verified that the cost is zero in large economies. More important, such policy renders virtually no cost also in small economies. An alternative specification is proposed to generate the costly crisis that are common in history. Inspired on Kocherlakota and Wallace (1998) and Prescott and Weinberg (2003) formulation of information frictions, second section studies effects of imperfect monitoring of individuals actions on the existence of bank runs with insolvency.

Chapter 1

Convergence of Peck-Shell and Green-Lin mechanisms in the Diamond-Dybvig model

Chapter Abstract

We[1] study the effects of population size in the Peck-Shell analysis of bank runs. We find that a contract featuring equal-treatment for almost all depositors of the same type approximates the optimum. Because the approximation also satisfies Green-Lin incentive constraints, when the planner discloses positions in the queue, welfare in these alternative specifications are sandwiched. Disclosure, however, is not needed since our approximating contract is not subject to runs.
***Keywords*:** bank fragility, role of population size, role of aggregate uncertainty
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1.1 Introduction

In the seminal model by Diamond and Dybvig (1983), an atomless population faces private liquidity needs. They remark that aggregate uncertainty poses a major problem for financial stability since the timing of expenditures becomes unpredictable. In this case, typical suspensions of payments should be avoided because they cannot remove bank panics and, at the same time, support the optimal provision of liquidity.

These conclusions led to the Diamond-Dybvig follow-up analysis of deposit insurance, received with reservations for ignoring constraints implied by the sequential nature of information flows. Reinstating the theory in a sequential-service environment became a goal for which finite traders and independent liquidity shocks are convenient assumptions.

[1]This is a joint work with Ricardo de Oliveira Cavalcanti and Paulo Klinger Monteiro.

But then Green and Lin (2003) proved that bank runs cannot become equilibria using a specification with slacking incentive constraints.[2] The leading alternative in the field is now Peck and Shell (2003). They bring back the possibility of bank panics in examples with active constraints, and under the assumption that depositors are *not* informed about their relative position in the sequence of bank service.[3]

After the Peck-Shell model, it is natural to expect a renewed interest in arrangements giving rise to strong implementation (unique outcomes), possibly resembling the Green-Lin setting. Nosal and Wallace (2009) have already noticed that incentive constraints are relaxed when the planner withholds information from depositors. As a result, 'Green-Lin disclosure' (of positions) can only eliminate 'Peck-Shell runs' at an average-utility loss. In this note, we offer another perspective on this issue by showing that the optimal contracts of Peck-Shell and Green-Lin specifications essentially converge to the same mechanism as the population size increases. We also find a high speed of convergence in a numerical example. These results directly imply that the welfare cost of disclosure is zero. But there are other conclusions that remind us of intuitive ideas in Diamond and Dybvig (1983).

We find also that financial stability does not require disclosure of information. This important feature is revealed by approximations which we call *step* mechanisms. They are constructed for economies with finite population as follows. The first step is to target contracts that are desirable when the number of impatient depositors is about average. The planner proceeds making transfers sequentially, until indicators of the state of the world—the depositors' withdrawals—trigger an one-shot correction forcing feasibility. In this second step, the new regime has impatient depositors receiving a lower level of consumption. We find three facts: (i) step mechanisms are also implementable with disclosure; (ii) they approach the Peck-Shell optimum as the population increases; (iii) they feature no bank runs since patient depositors are fully insured.

In summary, although Peck and Shell (2003) do reintroduce runs as equilibrium phenomena, their model does not generate sufficient aggregate uncertainty in order to make the problem quantitatively relevant. Hence, the Diamond-Dybvig emphasis on aggregate uncertainty is still an important issue. Progress requires avoiding approximations like ours in order to restore a trade-off between welfare and multiplicity in banking arrangements.[4]

1.2 The environment

A typical economy in our analysis is hit by a shock ω with support $\Omega \equiv \{0,1\}^N$ according to the probability $P(\omega) = p^{N-|\omega|}(1-p)^{|\omega|}$, where $|\omega| = \sum_{i=1}^{N} \omega_i$. There are N ex ante

[2]See Andolfatto et al. (2007) for an extension which assumes disclosure of all announcements made by early traders.

[3]See Ennis and Keister (2009) for examples of runs in a Green-Lin setting with correlated shocks.

[4]It also seems necessary to rule out a larger message space, such as the one used by Cavalcanti and Monteiro (2011) to achieve strong implementation even with correlated shocks.

identical depositors that live for two dates and derive utility from pairs (c_1, c_2) of consumption provided by a bank—the benevolent social planner, who controls the aggregate endowment Y—according to positions and announcements about preferences that are private information. Each individual draws a unique position i in $\{1, ..., N\}$ with probability $\frac{1}{N}$ and, as a result, the realization ω_i, without knowing the other coordinates of ω. As a benchmark, we assume that the individual is not informed of his position i. We shall keep the parameter p and the per capita endowment $e = \frac{Y}{N}$ constant when we consider changes in the population size N. Person i is called *impatient* if $\omega_i = 0$ and called *patient* otherwise. The utility in the former case is $Au(c_1)$ and in the latter is $u(c_1 + c_2)$, where $A \geq 1$ and u is continuous, strictly increasing, concave, twice differentiable and satisfies the Inada condition $u'(0) = +\infty$. Thus only patient individuals can substitute consumption across dates. The resources not consumed in date 1 are reinvested at gross rate-of-return $R > 1$. These assumptions include the preferences in Green and Lin (2003) and Peck and Shell (2003) as particular cases.

Feasible transfers must be incentive-compatible and satisfy a sequential-service constraint. The sequential-service constraint prevents date-1 consumption transferred to a person in position i to depend on information provided by someone at position n for $n > i$.

A compact description of candidates for optimal allocations follows from additional notation. Let us denote by ω^i the vector $(\omega_1, \omega_2, ..., \omega_i)$, and by (ω_{-i}, z) the profile that results from substituting the i-th coordinate of ω by z. Given that $R > 1$ we can restrict attention to transfers that assigns $x_i(\omega^i)$ units of date-1 consumption to someone at position i if that person is impatient ($\omega_i = 0$), and $y_i(\omega)$ units of date-2 consumption if that person is patient ($\omega_i = 1$), where $1 \leq i \leq N$. The sequential-service requirement has thus shaped the domains of x_i and y_i. We notice next that $(x_i, y_i)_{i=1}^N$ is feasible if

$$\sum_{i=1}^{N} \big((1-\omega_i)\, x_i(\omega_i) + \omega_i R^{-1} y_i(\omega)\big) \leq Y, \tag{1.1}$$

and incentive-compatible if

$$E\left[\frac{1}{N}\sum_{i=1}^{N} u\left(y_i\left(\omega_{-i}, 1\right)\right)\right] \geq E\left[\frac{1}{N}\sum_{i=1}^{N} u\left(x_i\left(\omega^{i-1}, 0\right)\right)\right], \tag{1.2}$$

that is, when patient individuals that are not informed of their positions agree with revelation.

We also say that $(x_i, y_i)_{i=1}^N$ is robust to *disclosure* if, in addition,

$$E\left[u\left(y_i\left(\omega_{-i}, 1\right)\right)\right] \geq E\left[u\left(x_i\left(\omega^{i-1}, 0\right)\right)\right], 1 \leq i \leq N, \tag{1.3}$$

that is, when a patient individual agrees with revelation after being informed of his posi-

tion.

The planner's problem is that of maximizing the representative-agent utility, before types and positions are assigned,

$$E\left[\frac{1}{N}\sum_{i=1}^{N}\left(\left(1-\omega_i\right)Au\left(x_i\left(\omega_i\right)\right)+\omega_i u\left(y_i\left(\omega\right)\right)\right)\right], \tag{1.4}$$

subject to (3.1) and (3.2).

1.3 A 'continuum' with active constraints

In order to build some intuition on optimality, let us consider the following *continuum* maximization problem. The goal is to choose $(c_1, c_2) \in \mathbb{R}_+^2$ to solve

$$\max pAu\left(c_1\right)+\left(1-p\right)u\left(c_2\right)\text{s.t. } pc_1+\left(1-p\right)R^{-1}c_2\leq e\text{ and }c_2\geq c_1.$$

Lemma 1 *If $A \geq R$ the solution to this problem is*

$$c_1\left(p\right)=c_2\left(p\right)=\frac{Re}{p\left(R-1\right)+1}\in\left(e,Re\right).$$

Proof. Since u is strictly increasing at the optimum we have $pc_1+\left(1-p\right)R^{-1}c_2=e$. Thus $c_1=c_1\left(c_2\right)=\frac{e}{p}-\frac{(1-p)}{p}\frac{c_2}{R}$. If $f\left(c_2\right)=pu\left(c_1\left(c_2\right)\right)+\left(1-p\right)u\left(c_2\right)$ and $c_1\leq c_2\leq\frac{Re}{1-p}$ then

$$f'\left(\cdot\right)=-\frac{\left(1-p\right)}{Rp}pAu'\left(c_1\left(\cdot\right)\right)+\left(1-p\right)u'\left(\cdot\right)=\left(1-p\right)\left(u'\left(\cdot\right)-\frac{A}{R}u'\left(c_1(\cdot)\right)\right).$$

Since $f'\left(\frac{Re}{1-p}\right)=\left(1-p\right)\left(u'\left(\frac{Re}{1-p}\right)-\frac{A}{R}u'\left(0\right)\right)<0$ necessarily $c_2<\frac{Re}{1-p}$ at the optimum. However if $f'\left(c_2\right)=0$ then $Ru'\left(c_2\right)=Au'\left(c_1\right)\geq Ru'\left(c_1\right)$ implies that $c_1\geq c_2$ or that the optimum cannot be interior. Therefore $c_2=c_1$. ■

Remark 2 *The proof of the lemma also demonstrates that any solution satisfies $c_2\left(p\right)>c_1\left(p\right)$ if $A<R$.*

Given the previous remark, we restrict attention to the case $A \geq R$ (also assumed by Peck and Shell (2003)), in order to derive a simple comparison between the solution of the planner's problem for finite economies and that for the continuum problem. It is a straightforward extension to consider the case $A < R$.

Assumption $A \geq R$.

Proposition 3 *Suppose $Y = Ne$. Let $\alpha\left(N\right)$ be the optimal welfare (3.3) in a finite economy and $\beta\left(e\right)$ the maximum of the 'continuum' problem. Then $\alpha\left(N\right)\leq\beta\left(e\right)$.*

Proof. We shall start with a candidate solution $(x_i, y_i)_{i=1}^N$ for the planner's problem, use it to define a feasible candidate for the continuum problem, and then rank the corresponding objectives. Define the numbers

$$\bar{y}_i = E\left[y_i\left(\omega_{-i}, 1\right)\right];$$
$$\bar{\bar{y}} = \frac{1}{N}\sum_{i=1}^N \bar{y}_i;$$
$$\bar{x}_i = E\left[x_i\left(\omega^{i-1}, 0\right)\right], 1 \le i \le N.$$

And define implicitly

$$u\left(\tilde{x}_i\right) = E\left[u\left(x_i\left(\omega^{i-1}, 0\right)\right)\right],$$
$$u\left(\tilde{\tilde{x}}\right) = \frac{1}{N}\sum_{i=1}^N u\left(\tilde{x}_i\right).$$

From (3.2) and Jensen's inequality we get

$$u\left(\bar{\bar{y}}\right) \ge \frac{1}{N}\sum_{i=1}^N u\left(\bar{y}_i\right) \ge E\left[\frac{1}{N}\sum_{i=1}^N u\left(y_i\left(\omega_{-i}, 1\right)\right)\right] \ge$$
$$E\left[\frac{1}{N}\sum_{i=1}^N u\left(x_i\left(\omega^{i-1}, 0\right)\right)\right] \ge \frac{1}{N}\sum_{i=1}^N u\left(\tilde{x}_i\right) = u\left(\tilde{\tilde{x}}\right)$$

and therefore

$$\bar{\bar{y}} \ge \tilde{\tilde{x}}. \tag{1.5}$$

If we take expectations in the feasibility constraint (3.1) we obtain

$$\sum_{i=1}^N \left(p\tilde{x}_i + (1-p)\frac{\bar{y}_i}{R}\right) \le Ne. \tag{1.6}$$

From this we also obtain

$$p\tilde{\tilde{x}} + (1-p)\frac{\bar{\bar{y}}}{R} \le e$$

since $\tilde{\tilde{x}} \leq \frac{1}{N}\sum_{i=1}^{N}\tilde{x}_i$. We are now ready to evaluate the objective (3.3):

$$\begin{aligned}
&E\left[\frac{1}{N}\sum_{i=1}^{N}\left((1-\omega_i)\,Au\left(x_i\left(\omega^i\right)\right)+\omega_i u\left(y_i\left(\omega\right)\right)\right)\right]= \\
&\frac{1}{N}\sum_{i=1}^{N}\left(pAE\left[u\left(x_i\left(\omega^{i-1},0\right)\right)\right]+(1-p)\,E\left[u\left(y_i\left(\omega_{-i},1\right)\right)\right]\right) \\
&\leq \frac{1}{N}\sum_{i=1}^{N}\left(pAu\left(\tilde{x}_i\right)+(1-p)\,u\left(\bar{y}_i\right)\right) \leq pAu\left(\tilde{\tilde{x}}\right)+(1-p)\,u\left(\overline{\overline{y}}\right) \\
&= pAu\left(\tilde{\tilde{x}}\right)+(1-p)\,u\left(\overline{\overline{y}}\right) \leq \beta\left(e\right),
\end{aligned}$$

which completes the proof. ■

1.4 A 'lower bound' without runs

Any mechanism $(x_i, y_i)_{i=1}^{N}$ for a finite economy defines a game of announcements, and a bank *run* is a Bayesian-Nash equilibrium of this game featuring misrepresentation of types. In this section we construct mechanisms that approximate the optimum and that are *immune* to runs. Because only the patient individuals consider misrepresentation, in order to be immune to runs it suffices to have transfers to the patient that are invariant to ω. For $0 \leq q \leq 1$ we define

$$x\left(q\right)=y\left(q\right)=\frac{Re}{q\left(R-1\right)+1},$$

and construct the *step* transfer-function according to

$$\begin{aligned}
&y_i\left(\omega\right)=y\left(q\right), \text{ if } \omega_i=1; \\
&x_i\left(\omega\right)=\begin{cases} x\left(q\right), & \text{if } \ \omega_i=0 \text{ and } \sum_{j=1}^{i}\left(1-\omega_j\right)\leq Nq; \\ R^{-1}x\left(q\right), & \text{if } \ \omega_i=0 \text{ and } \sum_{j=1}^{i}\left(1-\omega_j\right)> Nq. \end{cases}
\end{aligned}$$

Thus an impatient agent receives $x(q)$ if less than Nq impatient agents are positioned before him. And he receives $R^{-1}x(q)$ if there are at least Nq impatient agents before him.

We first notice that the constructed transfer function is feasible. We shall use $a \wedge b$ to represent $\min\{a, b\}$ and a^+ to mean $\max\{a, 0\}$ in what follows. For each $\omega \in \Omega$ let $I_\omega = N - |\omega|$ be the number of impatient agents. Now,

$$I_\omega \wedge (Nq)\,x\left(q\right)+\left(I_\omega-Nq\right)^+\frac{x\left(q\right)}{R}+\left(N-I_\omega\right)\frac{y(q)}{R}$$

is equal, for $I_\omega \leq Nq$, to

$$I_\omega x(q) + (N - I_\omega)\frac{x(q)}{R} \leq Nqx(q) + (N - Nq)\frac{x(q)}{R} = Ne$$

and otherwise to

$$Nqx(q) + (N - Nq)\frac{x(q)}{R} = Ne.$$

Hence, by construction, the step function is a feasible and incentive-compatible (since $y_i \geq x_i$) transfer scheme which we call the *step mechanism*.

The following simple lemma is useful for measuring convergence.

Lemma 4 *There is a constant K such that if $q > p$ is sufficiently near p,*

$$|u(x(p)) - u(x(q))| \leq K|q - p|. \tag{1.7}$$

Proof. Since

$$|x(q) - x(p)| = \frac{Re(R-1)|q-p|}{(q(R-1)+1)(p(R-1)+1)},$$

a Lipschitz constant M for u in a neighborhood of $x(p)$ yields

$$|u(x(q)) - u(x(p))| \leq M|x(q) - x(p)| \leq M\frac{Re(R-1)}{(p(R-1)+1)^2}|q-p|$$

and the desired constant K. ■

We now assess the optimality of a step mechanism. Its average utility, W, can be written as $\frac{1}{N}$ times

$$E\left[I_\omega \wedge (Nq)\,Au(x(q)) + (I_\omega - Nq)^+ Au\left(\frac{x(q)}{R}\right) + (N - I_\omega)u(x(q))\right].$$

We notice that $E[(N - I_\omega)] = N(1-p)$ and

$$E[I_\omega \wedge (Nq)] = E\left[I_\omega - (I_\omega - Nq)^+\right] = Np - E\left[(I_\omega - Nq)^+\right],$$

since $E[I(\cdot)] = Np$. Given these observations, W can be easily compared with the maximum of the continuum problem according to the expression

$$W = \bar{A}u(x(q)) - \frac{1}{N}E\left[(I_\omega - Nq)^+\right]A\left[u(x(q)) - u\left(\frac{x(q)}{R}\right)\right],$$

where $\bar{A} \equiv pA + (1-p)$. We shall see that $\frac{1}{N}E\left[(I_\omega - Nq)^+\right]$ can be bounded using inequalities, in the spirit of Tchebycheff's, regarding sums of bounded, independent random

variables. A tight result is provided by Hoeffding (1963).[5] Because

$$\int (I_\omega - Nq)^+ dP \leq \int_{I_\omega > Nq} (N - Nq)dP = N(1-q)P(I_\omega > Nq),$$

it follows that

$$\frac{1}{N}E\left[(I_\omega - Nq)^+\right] \leq (1-q)\Pr\left(I_\omega > Nq\right)$$

and hence that

$$\frac{1}{N}E\left[(I_\omega - Nq)^+\right] \leq (1-q)\Pr\left(\frac{I_\omega}{N} - p \geq q - p\right) \leq (1-q)\,e^{-2(q-p)^2 N}.$$

Now, having bounded this component, it follows that

$$\beta(e) - W \leq \bar{A}k_1 + (1-q)\,e^{-2(q-p)^2 N} A k_2$$

where

$$k_1 = u(x(p)) - u(x(q))$$

and

$$k_2 = u(x(q)) - u\left(\frac{x\,(q)}{R}\right).$$

The next step is to use the Lipschitz constant (independent of N), derived in the lemma, to put

$$k_1 \leq K\,|q-p|$$

and

$$\beta(e) - W \leq \bar{A}K\delta + (1-q)\,e^{-2\delta^2 N} A k_2, \tag{1.8}$$

where $\delta = |q-p|$. Now, if $\varepsilon > 0$ is given, we claim that a choice of a suitable δ leads to an error-term in (1.8) less than ε. In order to have each of the two terms in the right-hand side of (1.8) less than $\frac{\varepsilon}{2}$, notice that setting $\delta = \frac{\varepsilon}{2\bar{A}K}$ takes care of the first, and then a choice of N sufficient large (see remark below) takes care of the second, implying $|\beta\,(e) - W| < \varepsilon$.

Remark 5 *Since $p < q$ and $A < \bar{A}$, the right-hand side of (1.8) is bounded by $AK\delta + (1-p)e^{-2\delta^2 N}Ak_2$. The cutoff value of N that makes this bound equal to ε for $\delta = \frac{\varepsilon}{2\bar{A}K}$ can be found analytically. An approach that produces a tighter N is to choose δ_N minimizing the bound in δ, which gives*

$$\frac{K}{4N(1-p)k_2} = \delta e^{-2\delta_N^2 N},$$

and then choose N such that the objective is made equal to ε.

[5] If $\bar{X} = (X_1 + ... + X_n)/n$ and $\mu = E[\bar{X}]$, where $0 \leq X_i \leq 1$ for $i = 1,..,n$ are independent random variables, then Theorem 1 of (Hoeffding, 1963, p. 15) establishes that $\Pr\{\bar{X} - \mu \geq t\} \leq e^{-2nt^2}$.

We have thus proved the following.

Theorem 6 *By choosing the population size, the step-mechanism welfare can be made arbitrarily close to the optimal one.*

Another obvious result is recorded in the following.

Remark 7 *Since patient consumption is invariant to positions, the step mechanism is robust to disclosure.*

Because we have shown that the optimal welfare is bounded above by the continuum maximum, it follows that W sandwiches the welfare of optimal mechanisms with and without disclosure as the population increases.

1.5 Numerical examples

In this section we further explore the example of direct mechanisms in the Appendix B of Peck and Shell (2003). It follows by assuming homothetic preferences represented by $u(c) = (1-\gamma)^{-1}c^{1-\gamma}$.[6] In order to find results for large N we use the algorithm discussed in chapter 3, inspired by penalty functions, that iterates on candidate Lagrangian multipliers. Since the Peck-Shell model has a single (truth-telling) constraint, the problem is well behaved for large range of values of N. When the planner is forced to disclose positions, as in Green and Lin (2003), there is one truth-telling constraint for each position and we can compare Lagrangian multipliers in both settings for $N \leq 15$.

Figure 1 summarizes the main results.

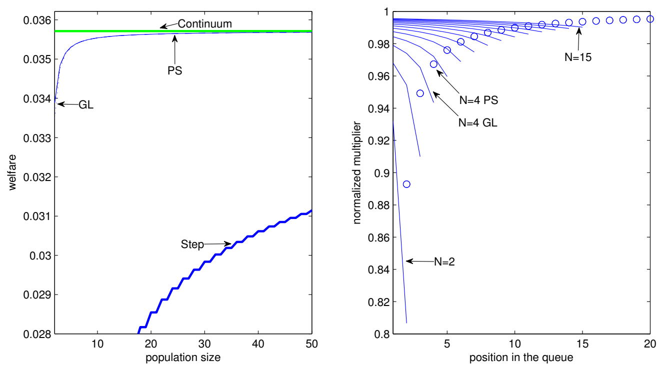

Figure 1.1: Convergence under the parameterization in Peck and Shell (2003)

The graph on the left of Figure 1 displays welfare, for each N, in terms of its difference to average utility under autarky. The lowest curve is that for step mechanisms. It is

[6] The parameters in the example are $A = 10$, $R = 1.05$, $\gamma = 2$, $p = 0.5$, and $e = 3$.

computed using $Q(j) \equiv \binom{N}{j} p^j (1-p)^{N-j}$ to represent the probability of $I_\omega = j$, which allow us to write average utility for the homothetic u as

$$W = u(x(p))[\bar{A} + A(1 - R^{\delta-1}) \sum_j Q(j)(\frac{j}{N} - p)^+].$$

The top line in the graph is the maximum for the continuum. In the scale established by these two bounds, the curves corresponding to the benchmark (PS) and the disclosure (GL) cases are almost identical.

The graph on the right of Figure 1 gives an explanation for this result. It plots the Lagrangian multiplier for the benchmark as a dot for each N. It also plots the set of multipliers for the disclosure case, linked by straight lines in a piecewise fashion, also for each N. The multipliers are also normalized by the value of the multiplier of the continuous problem.[7] It is evident that the set of GL multipliers converge rapidly to their PS counterparts, and that around $N = 15$ there is no significant difference to the multiplier in the limit (represented by unity).

We also compute how much per capita endowment should be reduced in the continuum economy in order to deliver the same welfare as in the benchmark economy. The endowment reduction that does the job for $N = 2$ is .102%. That for $N = 15$ is .0061%, and for $N = 100$ is .0008%. This documents the fact that the Peck-Shell specification with independent shocks does not produce significant aggregate uncertainty for values of N beyond 15.

By studying the best-response correspondence for each position in the game defined by disclosure (GL), we do not find banks runs for $N \leq 15$. Because the payoffs with and without disclosure change continuously and converge rapidly to the continuous case, we do not expect existence of runs for larger values of N under disclosure. Hence, provided that disclosure can be adopted freely, there is virtually no welfare cost of strong implementation. In terms of per capita endowment, the reduction that takes welfare from benchmark to disclosure for $N = 2$ is .022%, but for $N = 15$ is a tiny .00003%.

For completeness, we report the reduction in the benchmark endowment that is necessary for delivering the welfare of step mechanisms for some population levels in Table 1. In a region of population sizes, doubling the population halves the error, so that convergence is roughly linear in this metric.

[7] After straightforward algebra, the multiplier for the continuum problem can be expressed as $\frac{(1-p)p(A-R)}{1+p(R-1)} u'(x(p))$. Dividing this by $u'(x(p))$ yields the multiplier when the truth-telling constraint is written in utility levels.

Table 1.1: Benchmark-endowment reduction for step mechanisms

Population (N)	2	15	100	200	300	400
e reduction	1.022%	0.468%	0.279%	0.127%	0.104%	0.090%

1.6 Final Remarks

This note identifies two simple concepts for the Peck-Shell model with independent shocks. The first is a bound on welfare. Given convexity in preferences, we show that equal treatment is both desirable and feasible when the population is large. The shape of incentive constraints, with only patient individuals consuming in both dates, facilitates the argument, but it should hold more generally. The second construct is a simple mechanism that fully insures patient depositors, allocating ex-post distortions to impatient ones at the end of the queue. It is robust to disclosure and its welfare converges to the bound as the population increases. These ideas put together produce a sandwich result for both Peck-Shell and Green-Lin mechanisms.

Using the same functional forms and parameters of Peck-Shell examples, we find that the approximation error, expressed in terms of per capita endowment, is about .1% for a population near 300 individuals. Hence, the problem of eliminating runs is easily addressed by our simple mechanism at such population sizes.

For smaller populations, one can rely on a faster convergence between Peck-Shell and Green-Lin mechanisms. When the population size is 15 and the other parameters are maintained, allocations in both specifications are similar because the Lagrangian multipliers are essentially the same. No runs are however found for the setup with disclosure. The conclusion is that strong implementation does not have a bite when the population has size 15 either.

Future research thus seems necessary to find specifications in which aggregate uncertainty plays a more significant role in limiting the provision of insurance.

Chapter 2

Transitions when money distribution matters

Chapter Abstract

Specialization[1] of analysis to concepts of *steady-state* allocation and equilibria is pervasive in economics, especially in monetary models where dimensionality is a critical problem. Sometimes such assumption can be justified by establishing optimality, but usually it is just a strategy to avoid non-stationary complexities. In this chapter, we show that dynamics is a fundamental property of the optimal allocation for an economy where money distribution matters for the exchange pattern of the economy.

A natural question for this class of monetary models is the optimality of inflation, since it implies a distributive effect which concentrates heterogeneity. A well established result for economies without exchange frictions, the Friedman rule, states that return of money (medium of exchange) should maximized by minimizing the cost of holding such asset, i.e., the inflation. Because existence of exchange frictions gives money an additional role here, optimality of such policy is not clear anymore. We contribute to this discussion by studying the role played by transitions in an economy where steady-state optimum involves positive inflation.
***Keywords*:** dynamics, inside money, inflation, optima
***JEL codes*:** E52, E58

2.1 Introduction

Specialization of analysis to concepts of *steady-state* allocation and equilibria is pervasive in economics, especially in monetary models where dimensionality implied by heterogeneity is a critical problem. Sometimes such assumption is not problematic since it is possible to show that the optimum is stationary. However, it usually is just a strategy to avoid

[1]This is a joint work with Ricardo de Oliveira Cavalcanti and Paulo Klinger Monteiro.

non-stationary complexities on choosing allocation in the space of sequences. In this chapter, we show that dynamics is a fundamental property of the optimal allocation for an economy where money distribution matters for the exchange pattern of the economy.

In an economy where it is not easy to find someone to trade, distribution of money is relevant to determine expected utility of individuals. In a given period, a person can meet a rich producer who is not willing to produce for the payment she is able to make. Sometimes, she meets a consumer without money to buy her production. Therefore, how money is distributed among people is important to assess exchange opportunities a person have. On the other hand, when it is possible to find a lot of people or when you know and can access the location of each person, it is easier to select a good partner to trade. This usually leads to degenerate distribution of heterogeneity and, therefore, more predictability of trading opportunities. We study the class of economies pioneered by Kiyotaki and Wright (1989) as a model to the first situation. Exchange frictions are obtained by limiting the number of people each individual meets to only *one* person per period and by making such pairing *involuntary* and *random*. Money is shown to be essential (its use improves social welfare) in this economies if no record keeping technology is available.

A natural question for this class of monetary models is the optimality of inflation, since it implies a distributive effect which concentrates heterogeneity. People with more money suffer more than those without money, for example. A well established result for economies without exchange frictions, known as Friedman rule, states that return of money (medium of exchange) should maximized by minimizing the cost of holding such asset, i.e., the inflation. Because existence of both exchange frictions and anonymity gives money an additional role here, optimality of such policy is not so clear.

We contribute to this discussion by studying the role played by transitions in the economy studied by Deviatov and Wallace (2012), where steady-state optimum involves *positive* inflation and, therefore, Friedman rule is not optimal. The model is the extension of Kiyotaki and Wright (1989) economy proposed by Cavalcanti and Wallace (1999) where a fraction of the population is perfectly monitored and the remaining people remains perfectly anonymous. In addition, every person has a printing press that allows her to produce identical, indivisible, and durable objects. Trading among monitored people can be done using credit arrangements and does not require use of money, but exchanges between non-monitored people remain as before, consumer should give money to get production from her partner. Even more interesting relation takes place when a monitored person meets a non-monitored partner, where the former can print money and delivers it to the latter. People believe in the promise of future consumption embedded in such object since society can punish monitored people if they refuse to redeem it. As a result, this money is actually used in all types of meetings as a perfect substitute to ordinary money.

Such creation of money is *per se* a source of dynamics to the distribution of money among non-monitored, unless it is accompanied by an equivalent destruction through either monetary policy or redemption of money by monitored people. In the Deviatov and Wallace (2012)'s result, planner allows monitored people to create as many money as they want when they have consumption opportunities and asks them to redeem and destruct as many money as they can. Monetary policy probabilistically destructs money in order to sustain a stationary distribution of money where the number of exchanges involving creation of money exceeds the number of trades with redemption.

In taxing who has more money, inflation improves the distribution of meetings. In effect, rich non-monitored people are not willing to produce, only to consume, while poor non-monitored people always wants to produce and can consume when meets monitored people. Because inflation deteriorates individual wealth, it increases the number of productive meetings. It must be clear that the consumption without monetary payments when a poor person meets a monitored person is essential to this result. Such gift would not be feasible without the monitoring technology. Therefore, optimality of inflation comes from the same source of the optimality of inside money creation. Furthermore, because monitoring technology is used to tax the group of people issuing money, this represents a new dimension to the interaction between monetary and fiscal policy studied by Sargent and Wallace (1981). The question we pose in this chapter is how stationarity shapes these results.

2.2 Environment

The economy is a version of Cavalcanti and Wallace (1999) model of inside money, which extends monetary model of Trejos and Wright (1995) and Shi (1995). Time is discrete and the horizon is infinite. The economy is inhabited by a continuum of people of measure 1. There are $K > 2$ types of consumption goods, which are divisible and perishable. Individual's type is defined according to the goods she is able to consumes and to produce. Type-i individual produces good i and consumes good $i+1$. Each period individuals meet in pairs. As a result, the probability of meeting a type-$i+1$ individual is $\frac{1}{K}$. In this case, the type-i individual is a potential consumer to her partner. Similarly, the probability of meeting a type-$i-1$ individual is $\frac{1}{K}$, when she is a potential producer to her partner. In all other meetings, there is no coincidence of interest.

When consuming y units of good, an individual derives instantaneous utility $u(y)$ and when producing y units of good, she has instantaneous disutility y. Function u is assumed to be continuous, differentiable, $u(0) = 0$, $u'() > 0$, $u''() < 0$, $\lim_{y\to\infty} u'(y) > 1$. In addition, it is not possible to produce more than $\bar{y}$, which equals the strictly positive production such that such that $u(y) = y$. Because we only consider allocations which are symmetric with respect to the type of the individuals, we do not index quantities

consumed and produced. Preferences are separable in time, so that individuals seeks to maximize expected utility with discount $\beta \in (0, 1)$.

Society is able to perfect monitor a proportion $\alpha \in [0, 1]$ of the individuals, which we refer as monitored people. Each person is endowed with a printing press that allows her to produce identical, indivisible, and durable objects. Cavalcanti and Wallace (1999) has shown that if it is possible to determine who printed a given object, then those printed by monitored people can be used as a media of exchange called inside money and that it is a perfect substitute for outside money. We assume to be that our case.

Finally, we suppose that individual cannot carry more than one unit of money. This is not without loss of generality, but it keeps the model's dimensionality simple.

2.2.1 Feasible Policy

As in Deviatov and Wallace (2012), we study a probabilistic version of monetary policy intended to mimic the effect of inflation in decreasing the expected return of money. We assume that planner is able randomly tax a proportion ξ of the population after exchanges take place. As a consequence, a person loses her unity of money with probability ξ if she has money to be taxed. The following transition matrix summarizes the effect of such policy

$$\Psi = \begin{bmatrix} 1 & 0 \\ \xi & 1-\xi \end{bmatrix}_{2\times 2}$$

where Ψ_{ij} is the probability of a person who ended current period with $i-1$ units of money to start next period with $j-1$ units of money[2].

Observe that such policy is able to capture distributional effect of inflation even in a model where individuals cannot carry more than one unit of money. Inflation taxes more who have more money and therefore concentrates money distribution.

2.3 Optimal allocation

Let $S \equiv \{n0, n1, m0, m1\}$ be the set of types, where s_1 denotes person's monitoring status and s_2 her amount of money. The space of probability mass distribution over S is denoted by Θ and each meeting is indexed by $(s, s') \in S \times S$, where first coordinate refers to the producer type and second one to the consumer type.

Definition 8 *A* ***allocation*** *is a sequence* $\{(y, \lambda, \xi, \theta)_t\}_{t=0}^{\infty}$*, where for each* $t \geq 0$

- $y = (y^j)_{j \in S\times S}$ *defines the production/consumption for each* $j \in S \times S$*;*

[2] They also consider money transfer from planner to non-monitored people and find that it is not optimal to do so. We do not allow such possibility since we guess (and must verify) that it is not optimal here also.

- $\lambda = (\lambda_p^j, \lambda_c^j)_{j \in S \times S}$ *defines the monetary payments for each* $j \in S \times S$. $\lambda_p^{ss'} = [\lambda_p^{ss'}(0), \lambda_p^{ss'}(1)]$ *is a lottery over producer's monetary holdings after trading and* $\lambda_c^{s's} = [\lambda_c^{s's}(0), \lambda_c^{s's}(1)]$ *is a lottery over consumer's monetary holdings after trading*

- $\xi \in [0,1]$ *is the monetary policy; and*

- $\theta \in \Theta$ *is the distribution of people over* S

It is ***feasible*** *if for all* $t \geq 0$ *we have* $(y, \lambda, \xi, \theta)_t$ *such that for all* $(s,s') \in S \times S$ *producer is able to produce* $y^{ss'}$

$$y^{ss'} \in [0, \bar{y}],$$

consumer payments induces λ *such that* $\sum_k \lambda_p^{ss'}(k) = \sum_k \lambda_c^{ss'}(k) = 1$ *and*

$$\lambda_p^{ss'}(s_2 + k) = \lambda_c^{ss'}(s_2' - k) \quad \textit{for} \quad k = 0, \cdots, \min\{s_2, 1 - s_2'\}$$

and next money distribution is given by

$$\theta_{t+1}^{n1} = (1-\xi)\left(\theta^{n1} + \frac{1}{K}\left[\alpha\theta^{n0}\lambda_p^{(n0,m)}(1) - \alpha\theta^{n1}\lambda_c^{(m,n1)}(0)\right]\right)$$

The dynamics of money distribution deserves comments. As noted by Deviatov and Wallace (2012), a general result for the current class of inside money models is that monitored people always destruct money they receives and, therefore, they start every period without money ($\theta_t^{m0} = \alpha$ and $\theta_t^{m1} = 0$ for every $t \geq 0$). Because $\theta_t^{n1} = (1-\alpha) - \theta_t^{n0}$, money distribution dynamics can be summarized by the evolution of the mass of non-monitored people with money.

In order to define implementability of an allocation, it is useful define the expected utility of an individual in state s before meeting someone. It is

$$v_s = \frac{1}{K}\sum_{s' \in S} \theta^{s'}\left[\pi^p(s,s') + \pi^c(s',s) + (K-2)\pi^0(s)\right] \tag{2.1}$$

for each $s \in S$. Here,

$$\pi^p(s,s') = -y^{ss'} + \beta\lambda_p^{ss'}\Psi v_{s_1}'$$

is the expected utility a producer gets from participating in meeting (s,s') and $v_{s_1}' = [v_{s_1,0}', v_{s_1,1}']^t$. Similarly,

$$\pi^c(s',s) = u(y^{s's}) + \beta\lambda_c^{s's}\Psi v_{s_1}'$$

is the expected utility a consumer gets from participating in meeting (s',s). Finally, if individual is in a non-coincidence meeting her expected utility is

$$\pi^0(s) = \beta\delta_{s_2}\Psi v_{s_1}'$$

where $\delta_{s_2} = [1, 0]$ if $s_2 = 0$ and $\delta_{s_2} = [0, 1]$ otherwise. Equations (2.1) is called Markov equations by Cavalcanti and Monteiro (2006). It says is that a person in state s at the beginning of a given period (i) becomes a producer (meets a consumer) in meeting (s, s') with probability $\theta^{s'}/K$ and gets $\pi^p(s, s')$ from participating in exchanges; (ii) becomes a consumer (meets a producer) in meeting (s', s) with probability $\theta^{s'}/K$ and gets $\pi^c(s', s)$ from participating in exchanges; and (iii) has no coincidence of interest with her partner with probability $1 - 2/K$ and gets $\pi^0(s)$.

A deviation from exchange (y, λ) is punished by not trading today and, when deviator is a monitored person, her money becomes valueless and she becomes a non-monitored person. Therefore, the producer in meeting (s, s') agrees with the exchange only if

$$\pi^p((s_1, s_2), s') \geq \pi^0((n, s_2)) \tag{2.2}$$

and the consumer in meeting (s', s) agrees with the exchange only if

$$\pi^c(s', (s_1, s_2)) \geq \pi^0((n, s_2)). \tag{2.3}$$

Definition 9 *An allocation is* ***implementable*** *if it is feasible and there exist a sequence* $(v_t)_{t=0}^{\infty}$ *such that*

- *incentive constraints* (2.2) *and* (2.3)*; and*
- *Markov equations* (2.1)*.*

are satisfied for all $t \geq 0$*. It is* ***optimal*** *if it is implementable and it maximizes*

$$\sum_{t=0}^{\infty} \beta^t \frac{1}{K} \sum_{s,s'} \theta_t^s \theta_t^{s'} [u(y_t^{ss'}) - y_t^{ss'}].$$

2.4 Planner's recursive problem

We built on Cavalcanti and Monteiro (2006) to construct an algorithm to compute optimal non-stationary allocation. They study a version of the current economy without monitored people ($\alpha = 0$) and without limit in the individual money holding. Their proof of existence and their recursive formulation of the planner problem are fundamental to the computational strategy here.

While Cavalcanti and Monteiro (2006) use expected utility as state variable to ensure planner commitment, we follow Cavalcanti and Erosa (2008) in demanding planner to honor only differentials of expected utility. In order to see that this is enough to ensure recursivity here, observe that when considering to agree or disagree to the exchange, individuals care only about $r'_m = v'_{m0} - v'_{n0}$ in the case of monitored people and $r'_n = v'_{n1} - v'_{n0}$ in the case of non-monitored people. In effect, (2.2) and (2.3) can be written as

s, s'	$\pi^p(s,s') \geq \pi^0(s)$	$\pi^c(s',s) \geq \pi^0(s)$
$(n0, n1)$	$y^{ss'} \leq \beta(1-\xi)r'_n$	$u(y^{ss'}) \geq \beta(1-\xi)r'_n$
$(n0, m0)$	$y^{ss'} \leq \beta(1-\xi)r'_n$	$u(y^{ss'}) \geq -\beta r'_m$
$(m0, n0)$	$y^{ss'} \leq \beta r'_m$	$u(y^{ss'}) \geq 0$
$(m0, n1)$	$y^{ss'} \leq \beta r'_m$	$u(y^{ss'}) \geq \beta(1-\xi)r'_n$
$(m0, m0)$	$y^{ss'} \leq \beta r'_m$	$u(y^{ss'}) \geq -\beta r'_m$

where conditions for meetings in the set $\{(n0,n0),(n1,n0),(n1,n1),(n1,m0)\}$ were omitted because they are trivially satisfied since no trade is possible in these cases. When producer's type is $n1$, she is not willing to produce since an additional unit of money is useless given maximum individual money holding. When producer is type $n0$, she produces only in exchange to money. When consumer's type is also $n0$, no payment is possible. In writing previous table, we also have specialized in the case of no lotteries in monetary payments. This is motivated by Deviatov and Wallace (2012)'s result that they are not used in the stationary optimum. We guess that this will also be the case for the non-stationary optimum, a property to be verified.

In order to induce individuals to agree to current exchanges, planner must be able to honor promises r'_m and r'_n. Otherwise, such promises would not be credible. Let $\Gamma(\theta)$ be the set of sustainable (credible) promises under distribution θ. Then, by using (2.1) we have that $r = (r_m, r_n)$ is sustainable if, and only if, exist implementable exchanges (y, λ) and future promises $r' = (r'_m, r'_n) \in \Gamma(\theta')$ such that

$$r_m = v_{m0} - v_{n0} = \frac{1}{K}\sum_{s'} \theta^{s'}\left[\Pi^p_m(s') + \Pi^c_m(s')\right] + \beta r'_m \tag{2.4}$$

$$r_n = v_{n1} - v_{n0} = \frac{1}{K}\sum_{s'} \theta^{s'}\left[\Pi^p_n(s') + \Pi^c_n(s')\right] + \beta(1-\xi) r'_n \tag{2.5}$$

where

$$\Pi^p_{s_1}(s') = [\pi^p(s,s') - \pi^0(s_1)] - [\pi^p(n0,s') - \pi^0(n0)]$$

is the surplus type-s producer gets from trading relative to the surplus type-$n0$ producer gets from trading, and

$$\Pi^c_{s_1}(s') = [\pi^c(s',s) - \pi^0(s_1)] - [\pi^c(s',n0) - \pi^0(n0)].$$

is the surplus type-s consumer gets from trading relative to the surplus type-$n0$ consumer gets from trading. Equations (2.4) and (2.5) demand planner to pay r only by making new *sustainable* promises r' or by providing *implementable* exchange surplus to those engaged in trade.

We are now able to state planner's recursive problem,

$$W(\theta, r_m, r_n) = \max_{(y, r', \xi)} \left\{ \frac{1}{K} \sum_{s,s'} \theta^s \theta^{s'} \left[u(y^{ss'}) - y^{ss'} \right] + \beta W(\theta_+, r'_m, r'_n) \right\}$$

subject to (2.2-2.5), the distribution dynamics

$$\theta_+^{n1} = (1-\xi) \left(\theta^{n1} + \frac{1}{K} \left[\alpha(1-\alpha-\theta^{n1}) - \alpha\theta^{n1} \right] \right) \tag{2.6}$$

and $r' \in \Gamma(\theta')$. Optimal allocation is obtained using the policy function calculated in the recursive planner problem above evaluated from initial state $(\theta_0, r_0) = \arg\max_{(\theta,r)} W(\theta, r)$ to the infinite.

2.5 Results

First-best allocation in this economy is stationary and consists of production and consumption given by $y^* = \arg\max\{u(y) - y; 0 \le y \le \bar{y}\}$ without use of money since no incentive is necessary to induce this production. In general, such allocation is not implementable[3]. In particular, for the economies studied by Deviatov and Wallace (2012), stationary optimum involves production lower than y^* and use of money as shown in the last column of the following table for the case where half of population is monitored ($\alpha = .5$)[4]. The last five rows report the ratio between optimal production and y^*.

	Optimal allocation							DW
	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t>6$	SS
$\theta^{n1}/(1-\alpha)$	0.070	0.210	0.305	0.364	0.408	0.437	0.454	.371
W/W^*	0.682	0.654	0.607	0.578	0.567	0.564	0.562	.599
ξ	0.015	0.005	0.017	0.002	0.004	0.010	0.033	.104
r_m	0.240	0.294	0.371	0.416	0.427	0.427	0.427	.319
r_n	0.235	0.281	0.348	0.393	0.404	0.404	0.404	.288
$y^{n0,n1}/y^*$	0.709	0.888	0.990	1.034	1.033	1.026	1.002	.663
$y^{n0,m0}/y^*$	0.709	0.888	0.990	1.034	1.033	1.026	1.002	.663
$y^{m0,n0}/y^*$	0.748	0.604	0.242	0.079	0.026	0.013	0.001	.141
$y^{m0,n1}/y^*$	0.748	0.950	1.055	1.040	1.002	0.986	1.043	.818
$y^{m0,m0}/y^*$	0.748	0.950	1.000	1.000	1.000	1.000	1.000	.818

[3]For example, for economies with sufficiently low β, future payoffs are not high enough to induce producer to deliver such product. Think about the case $\beta = 0$. Producer will never be rewarded by the disutility she incur to produce today. In the neighborhood of such β this results remains valid.

[4]Remaining specifications are $\beta = .59$, $K = 3$, and $u(y) = 1 - \exp(-10y)$

In general, production is closer to y^* when producers are monitored people. This confirms that credit is superior to money in inducing better exchanges, as pointed out by Kocherlakota (1998). When participating in exchanges, $m0$ producers know that planner (society) will provide them return r'_m in the future. Because they are monitored, no money is necessary to prove current production to society in the future. There is a record keeping of all such contributions. In this sense, such exchanges involves credit operation, not a monetary one.

Credit is also explored in meeting $(m0, n0)$, where a gift is given to $n0$ people since no monetary payment is possible. Production is much lower in this case. Differently of meetings $(m0, n1)$ and $(m0, m0)$, where production is constrained by producer compatibility incentive, the gift here is limited by the promise r_m planner should honor. Higher gifts in meeting $(m0, n0)$ would reduce r_m, which implies a reduction in r'_m in steady-state situation. As a consequence, incentive to other monitored producers would be decreased.

Monetary payments in meeting $(n0, m0)$ consist of creation of money, in the sense that more money are held by non-monitored people. Similarly, destruction of money happens in meeting $(m0, n1)$. Because no lottery is used in the optimum steady-state, creation and destruction are fully explored. The former is the standard way to induce production towards y^*. Even more interesting is the latter, where producer do not demand money to produce. Planner recommends such payments in order to preserve monetary scarcity among non-monitored people and consequently the future return, r'.

Scarcity of money could also be preserved by increasing inflation, ξ. But while redemption does not involve incentive distortions (consumer incentive constraint slacks in meeting $(m0, n1)$), higher inflation would imply less incentive to non-monitored producers. In this sense, it is cheaper to preserve return of money using redemption.

The presence of positive inflation in the optimum would then be unexpected. However, studying distribution dynamics (2.6) we see that only distribution

$$\bar{\theta}^{n1} = \frac{(1-\alpha)\lambda_p^{n0m0}(1)}{\lambda_p^{n0m0}(1) + \lambda_c^{m0,n1}(0)} = \frac{1-\alpha}{2}$$

can be sustained in a steady-state without inflation and with full creation and destruction of money[5]. This distribution is not attractive because the quantity of productive meetings (extensive margin) are low. In effect, define

$$P_\alpha(\theta) = \sum_{ss' \in \bar{S}} \theta^s \theta^{s'} = \alpha + (\alpha + \theta^{n1})(1 - \alpha - \theta^{n1})$$

where $\bar{S} = \{(n0, n1), (n0, m0), (m0, n0), (m0, n1), (m0, m0)\}$ is the set of productive meetings. This is the quantity of productive meetings when distribution is θ and a proportion

[5]The capacity to preserve scarcity using monitored people is limited by θ^{n1} and the amount of injection of money is proportional to θ^{n0}. Under inflation zero, we must have $\theta^{n1} = \theta^{n0}$.

α of the population is monitored. It is easy to see that $P_\alpha(\theta)$ is strictly decreasing in θ^{n1} for

$$\theta^{n1} > \bar{\theta}^{n1} - \frac{\alpha}{2}.$$

Extensive margin could be improved by decreasing injection of money, but this would make non-monitored producers less willing to produce. Intensive margin would be sacrificed in the same way higher ξ would do. But now, inflation is superior (cheaper) because the cost destruction of money can be shared among all individuals with money. The cost of reducing injection of money, however, is borne only by producers in meeting $(n0, m0)$. As a conclusion, inflation in optimum steady-state allows society to sustain more scarcity of money and, consequently, to improve both the extensive and the intensive margin. In the particular case reported in the table, inflation implied by $\xi = .104$ is able to sustain a steady-state distribution given by $\theta^{n1} = .371(1-\alpha) < \bar{\theta}^{n1}$.

Intuitively, inflation improves the distribution of meetings by taxing more who has more money. In effect, rich non-monitored people are not willing to produce, only to consume, while poor non-monitored people always wants to produce and can consume when meets monitored people. Because inflation deteriorates individual wealth, it increases the number of productive meetings. It must be clear that the consumption without monetary payments when a poor person meets a monitored person is essential to this result. Such gift would not be feasible without the monitoring technology. Therefore, optimality of inflation comes from the same source of the optimality of inside money creation. Furthermore, because monitoring technology is used to tax the group of people issuing money, this represents a new dimension to the interaction between monetary and fiscal policy studied by Sargent and Wallace (1981) to be explored later.

When planner is able to choose non-stationary sequences of distribution of money, inflation plays a much smaller role in the optimum. For the grid of distributions we use, ξ is lower than .02 for all optimal transition reaching .033 in the limiting allocation. These results are not only much more smaller than that Deviatov and Wallace (2012) found in the optimal steady-state, but also they always are the smallest possible ξ given all possible transitions θ_+ from θ.[6]

Although inflation (or reduction in the injection of money) is not used in the optimal transition, planner is able to maximize the number of productive meetings (extensive margin) in the beginning of the optimal path (non-monitored people has essentially no money). Such distribution were not chosen as optimal steady-state distribution because this would require an extremely high ξ. The intensive margin would then be very poor since money disappears almost certainly. With transition possibility, however, reasonable production can be implemented under this distribution by decreasing ξ and allowing net creation of money. The cost involved in this strategy is a progressive deterioration of the

[6] Such property has shown to be robust to enriching the grid for Θ.

extensive margin, but it is slowed by the monetary destruction in meeting $(m0, n1)$.

Transition also allows improvements in the intensive margin. Because planner can offer r'_m different from the promise r_m made in the previous period, it is able to implement much higher gift in meeting $(m0, n0)$. It increases from $.141y^*$ in the optimal steady-state to $.748y^*$ in the first period of transition[7]. As time goes on, such taxation is decreased in order to sustain higher returns to monitored people. As shown in the table, essentially no gift is delivered in the limiting allocation.

Dynamics allows society to explore its taxation capability without the high incentive distortion it implies in steady-state situation, as discussed above. A decreasing taxation schedule implements increasing future returns r'_m and, therefore, induces high levels of production in the optimal path. Despite coming from a mechanism design approach, this result parallels Sargent and Wallace (1981) analysis in highlighting dynamic incentives to the interaction between fiscal and monetary policies. In particular, a low tax policy would make transitions less attractive and therefore higher inflation more desirable as a way to sustain the quantity of exchanges.

Finally, optimal dynamics exhibits increasing paths for both the return to money r_n and the return to monitored people r_m, which are sustained by a roughly increasing production to consumers with money and decreasing gift to $n0$-type consumers. In some cases,‘ high returns are paid through consumption levels exceeding y^*. This seems to be a suboptimal way to pay previous promises and could be improved upon by decreasing monetary payments and production in meetings $(n0, n1)$ and $(n0, m0)$. Because our algorithm does not allow for lotteries, we are unable to capture such mechanism. However, observe that our main result would be reinforced by allowing lotteries since they would decrease injection of money slowing deterioration of extensive margin and making inflation even more unnecessary in the limit distribution.

2.6 Final Remarks

We have provided an example where transition is a fundamental property of optimal allocation. It allows tax instrument to be widely explored, as evidenced by temporarily high gifts from monitored people to non-monitored people, and makes inflation unnecessary to sustain high quantity of exchange, as is the case in the steady-state optimum.

Previously, Cavalcanti and Erosa (2008) has shown that transition is also essential in a similar economy, where there are no monitored people ($\alpha = 0$) and preferences are subject to an aggregate shock. Roughly, planner distorts production in a state to increase expected return of money in order to relax producer incentive constraint when preference shock

[7]This improvement is certainly responsible for a big share of the welfare gain from allowing transitions. Specially if we remember that the quantity of such meetings has increased a lot when choosing extensive margin.

implies high marginal utility. As in our case, such distortion makes production higher than y^* since lotteries in monetary payments are unfeasible. Later, Huang and Igarashi (2012) showed that transition disappears if planner can recommend probabilistic monetary payments. Our results, on the other hand, are robust to lotteries since optimality of transition is mainly driven by distributional motivation, not for distortion of return of money. In effect, (i) dynamics optimality would still hold in the presence of lotteries. As shown by Deviatov and Wallace (2012), optimum steady-state does not use lotteries; and (ii) even if they are used to sustain high return at a lower cost, this would be done by reducing money creation and then making inflation even more unnecessary.

Optimality of transition here differs from that in Cavalcanti and Erosa (2008) mainly because money distribution can evolve in time. In their economy transitions is used only to sustain high returns to money. For the current economy, it also allows to increase the quantity of exchanges (number of productive meeting) without the deterioration of the quality of exchange (consumption) required by state-state allocations.

Finally, current results show that optimality of transition is not innocuous. Questions like desirability of inflation conjectured by Wallace (2012) should take transitions into account. At least for the current economy, alternative policy instruments must be considered. Also, it suggests current model is a rich environment to enrich the analysis of the iteration of monetary and fiscal policy studied by Sargent and Wallace (1981).

Chapter 3

High interest rates: the golden rule for bank stability in the Diamond-Dybvig model

Chapter Abstract

In[1] chapter 1 we build on Peck-Shell (2003) economies and obtain strong implementation in perturbations of optimal contracts. Since bank runs are eliminated with distortions that become very small when the population grows, a pressing issue is whether an alternative specification can generate the costly crisis that are common in history. We find, in this chapter, an affirmative answer in the context of the Diamond and Dybvig (1983) model, and uncover the role played by societal weights on future consumption and solvency risk. An extension of the Ennis and Keister (2009) algorithm shows the impact of run strategies and implicit rates of interest on the formation of expectations, in line with some classical views.
***Keywords*:** severe aggregate-uncertainty, mixed-strategy bank runs, insolvency, dynamic programming
***JEL codes*:** E4, E5

3.1 Introduction

Perhaps less forgiving than modern counterparts, financial arrangements based on paper credit and fractional reserves, in the era of gold standard, gave to classical economists Adams Smith and Henry Thornton a special perspective on what today are called financial regulation and central banking. For Thornton, an illiquid system can accomplish more — despite the inherent risk of suspension of convertibility — when the industry seeks long-term payoffs, in contrast to narrow objectives associated to bank panics, high demand for gold and extinction of debt. One possible interpretation of Thornton ideas is that

[1]This is a joint work with Ricardo de Oliveira Cavalcanti.

society should commit to policies making future growth attractive to a substantial set of agents, and that in doing so the potential for panics is minimized. There is, however, a considerable degree of ambiguity in the modern literature about the trade-offs involved in the provision of liquidity from a macro perspective. The purpose of this chapter is to make a simple and transparent connection between bank stability and the social discounting of future utilities using variations of the Diamond-Dybvig (1983) model. In particular, we discuss which implicit interest rates society should adopt when aggregate uncertainty is severe or when insolvency becomes a real threat.[2]

This chapter builds on a literature inspired by Diamond and Dybvig (1983) but focusing on frictions in the diffusion of information that are fundamental. Wallace (1988) was the first to notice that a social planner should not attempt to treat *impatient* consumers equally in a banking model with sequential service and aggregate uncertainty. For him [see also Wallace (1990)], this basic property has deep roots, and should be interpreted as a single-asset explanation of actual bank suspensions observed in history. Since then, new tools have been developed, and in this chapter we shall advance them to bring to bear constraints needed to insure truth-telling and uniqueness of equilibrium, two issues not pursued at the time. We find that variations in consumption at the first date of Wallace's sequential-service model is best interpreted as part of a movement in interest rates meant to give future utilities a proper weight and which should be modified by incentives and stability considerations. We also think that ideas for bringing insolvency issues into focus, which we also formalize, can increase our understanding of historic suspensions. We find, for instance, that in general the planner should not seek equal-treatment of *patient* consumers either.

Wallace's paper falls short of characterizing optimal allocations if depositors of different types — with different liquidity needs — arrive at random times to the bank. Green and Lin (2000, 2003) reach much further, including examples that can be computed in close form. In more general specifications with independent shocks, reserves need not be distorted in order to provide incentives for truth telling and, remarkably, there is no need to worry about bank stability either: they prove that the optimum is interior and uniquely implemented in their economies. Peck and Shell (2003) propose a new landscape, however, showing that runs can become pervasive with new truth-telling constraints resulting from a less informed depositor (see also Andolfatto et al. (2007), for more on the role of disclosure).

Peck and Shell (2003) and, more recently, Ennis and Keister (2009), manage to open up several directions for future work, with examples of runs, design of strong implementation

[2] In contrast to the proposals of matching maturity of assets and liabilities, by Adams Smith, or prohibition of fractional reserves, by Milton Friedman [see Wallace (1988) for references], Henry Thornton (1802) warns against reductions in the creation of liquid liabilities by the central bank for preventing financial crises. An alternative, a policy of extending liquidity in exchange for good collateral, is largely credited to Walter Bagehot (1999), whose proposal includes a recommendation for higher "penalty" rates.

in pure strategies (Peck-Shell) and an algorithm for optimal reserves (possibly susceptible to runs) for a particular case of correlated shocks and inactive truth-telling constraints (Ennis-Keister). In chapter 1, however, the issue of strong implementation becomes very subtle when the distribution of types is easily predicted. They show that if the population is not too small and types are independent, then optimal reserves are essentially invariant to typical changes in truth-telling constraints (differences are particularly small in numerical examples with homothetic preferences).[3] This suggests that reserve management is more meaningful — eliminating runs is more costly — if aggregate uncertainty remains substantial even after the intentions of an initial subset of depositors can be sampled. In this chapter, we shall provide an algorithm for strong implementation generalizing the weak-implementation method in Ennis and Keister (2009) in other respects: it allows for active truth-telling constraints in the spirit of Peck and Shell (2003), and for a different notion of aggregate uncertainty that avoids the limit result in chapter 1.

Our extension to insolvency risk in the Diamond and Dybvig (1983) model is also novel. The need to avoid a bank run becomes more appealing when financial distress can leave a number of depositors without consumption (or with very low utility). We borrow from Kocherlakota and Wallace (1998), and Peck and Shell (2010), monitoring assumptions that can lead to insolvency. In the former, a planner learns about actions in a monetary setting only after a time lag.[4] In the latter, a bank cannot store information provided by those willing to visit without withdrawing. In our formulation, patient individuals — with high desire for late consumption — are further divided into two groups: in one case (type 1), actions are monitored as in the conventional model, and in the other (type 2), a given individual is able to mimic the actions of a subset (of size two, in our numerical examples) of impatient depositors. Embezzlement behavior by a type-2 person at the first period, if it happens to occur in a run equilibrium, can be detected only in the second period, thus leaving the bank insolvent. We find that reserve management should direct extra second-period resources to these potential 'insiders' in order to promote bank stability.

Golden rule: less discounting, more savings

Our basic construction, employed throughout the chapter, can be explained in simple terms for the particular case of independent shocks and standard monitoring (no type-2 people). There are two dates and two types of depositors. The aggregate state is $\omega \in \{0,1\}^N$, where N is the population size. A null coordinate i, $\omega_i = 0$, means that an individual arriving to the bank in position i consumes only at the first date (c_1) and has

[3]A different but somewhat related subtlety is found by Cavalcanti and Monteiro (2011). They propose circumventing the revelation principle in order to achieve strong implementation in Ennis-Keister and Peck-Shell environments through large message-spaces, allowing for partial withdrawals and two-price mechanisms. Since, to some degree, such mechanisms are subject to the criticism of being unrealistic, we do not pursue them here.

[4]See also Calomiris and Kahn (1991), and Prescott and Weinberg (2003), as well as other references on the treatment of imperfect monitoring in monetary theory reviewed by Cavalcanti (2011).

utility $Au(c_1)$, where $A \geq 1$ and $u(c)$ stands for $\frac{1}{1-\delta}c^{1-\delta}$ with $\delta > 1$. Otherwise, when $\omega_i = 1$, 'person i' can enjoy consumption at both dates (c_1 and c_2) and her utility is instead $u(c_1 + c_2)$. Individuals are ex-ante identical and later experience idiosyncratic shocks to preferences and to positions. The probability of drawing type 0 is p and, for any given type realization, the probability of position i is $\frac{1}{N}$. As in Peck and Shell (2003), individuals must announce types without knowing their positions; the planner — the bank — must transfer a quantity of date-1 consumption to the person in position i based on history $(\omega_1, ..., \omega_i)$ and taking expectations about $\omega_{i+1}, ..., \omega_N$ (sequential service). For a given i, date-2 consumption is a function of the whole list ω, and the aggregate date-2 consumption is bounded above by reserves saved at date 1 and reinvested at (gross) rate of return $R > 1$. The bank starts with initial reserves Y. A reserve-management rule should be designed so as to maximize ex-ante utility.

Because $R > 1$, a necessary condition for welfare maximization is that type-1 individuals consume zero at the first date, enabling society to make the best use of its growth potential. Also, it can be shown that in this simple economy all type-1 individuals should share the same level of consumption at the second date (this also holds with binding constraints here but not in our general formulation of aggregate uncertainty below).

Ennis and Keister (2009) establish that optimal reserves for this homothetic economy follow from a dynamic-programming solution, exploring the fact that the marginal benefit of a unit saved to date 2 and shared with any j patient individuals can be found analytically. Also, the level of date-1 consumption that should be transferred to an impatient person at position i (defining reserves saved for position $i+1$) is an explicit fraction

$$\frac{1}{1 + f_i(\omega_1, ..., \omega_i)}$$

of previous savings, where the coefficient f_i vary with the number of zeros in history $(\omega_1, ..., \omega_i)$. And, more importantly, Ennis and Keister (2009) show that these coefficients can be computed recursively.

One problem with the Ennis-Keister calculation is that it may discount future utility too much if A is sufficiently high. This is the case when types are private information, a situation assumed throughout this chapter, and when A is sufficiently high. Increases in A lead to higher marginal utility of date-1 consumption and, if no corrections are made, a patient individual will choose to misrepresent her type in order to reach a better payoff. We show, in this chapter, that the problem can be corrected as follows. Instead of assuming that the type-1 utility is $u(c_1 + c_2)$, the planner can apply the Ennis-Keister algorithm to an alternative economy where the patient utility function is now $\beta u(c_1 + c_2)$, and where β is a correction factor to be determined. With this abstraction in mind, we construct an algorithm that computes the slackness of truth-telling constraints associated to the Ennis-Keister program for the 'β-economy'. If for $\beta = 1$ the constraint slacks then

the optimum has been found. Otherwise, a search for β is constructed, with increments that depend on the degree of individual gain that a type misrepresentation generates.

The previous exposition refers to weak implementation. Let us suppose now that a sought-after value β, organizing bank reserves according to the modified Ennis-Keister algorithm, has been found. By construction, a patient individual reveals her type under the assumption that others report truthfully. But nowhere in this design it has been taken into account what individuals prefer to do in the scenario of a bank run, when other patient depositors choose to withdraw. This leads to the question of how reserves should be designed within the class of mechanisms without multiple equilibria. Our approach is to incorporate into our method *no-run constraints* introduced by Peck and Shell (2003). Our version is broader, eliminating all runs in mixed, symmetric strategies. We do find another dynamic programming leading to the desired solution. It applies new shadow prices: date-1 consumption is socially valued according to $\alpha A u(c_1)$, while that for date 2 is valued according to $\beta u(c_2)$, for suitably chosen α and β that vary along the history but are still computable by a single-dimensional search protocol as before.

This intuitive approach is shown to be applicable to new questions alluded to above. The main conclusion is that sufficiently high (implicit) interest rates guarantee incentive compatibility and bank stability. But even within the basic environment of independent shocks, new findings are demonstrated. We show, for instance, that bank runs are pervasive. While Peck and Shell (2003) appeal to a strong demand for liquidity — setting the constant A equal to ten — in order to find bank fragility, we show that an increase in the supply of liquidity plays the same role, without additional assumptions. All that is needed is an increase in the population size, making insurance easier to provide. In particular, we find two equilibria besides the optimal one. Namely a run in pure strategies and a run in mixed strategies.

The distortions that make strong implementation possible, not surprisingly, are found to vary with insurance levels. We use simulations to document properties not explored in the literature to date. For instance, and again in the basic economy with independent shocks, the size of the economy produces two opposing effects. First, it facilitates the provision of insurance as a larger population can more easily pool risks. And, with higher insurance, run strategies become more attractive. Second, in order to discourage runs, an economy with many traders can spread distortions across many events. Hence, a distortion placed after a long sequence of withdrawals produces a small tax on the average depositor (a kind of 'backloading' property common in the public finance literature). The final effect confirms a seemingly paradoxical outcome: with independent shocks, runs can be removed in very large economies without substantial welfare losses for the average person. In particular, a run-proof management of reserves should tax heavily only the impatient arriving late to unusually long withdrawal events. This conclusion, however, is not valid when correlation is introduced, and we present a careful discussion for this reversal below.

The rest of the chapter is divided as follows. Section 2 presents the basic environment with the standard monitoring assumption. Section 3 introduces a shadow-price approach to take distortions into account in simulations. Section 4 illustrates how the method can be used to characterize financial fragility. Section 5 extends the analysis to the design of run-proof reserves. Section 6 presents the environment with imperfect monitoring and the corresponding findings. Section 7 concludes.

3.2 The environment without insolvency

The benchmark economy in our analysis is hit by a shock ω with support $\Omega \equiv \{0,1\}^N$. The parameter N stands for the number of ex ante identical depositors that live for two dates and derive utility from pairs (c_1, c_2) of consumption provided by a bank—the benevolent social planner, who controls the aggregate endowment Y—according to positions and announcements about preferences that are private information. While for now an individual can be of two types, 0 or 1, in the last section of the chapter we add a third type in order to discuss insolvency.

Person i is called *impatient* if $\omega_i = 0$ and called *patient* otherwise. The utility in the former case is $Au(c_1)$ and in the latter is $u(c_1 + c_2)$, where $A \geq 1$ and $u(c) = \frac{1}{1-\delta}c^{1-\delta}$ and $\delta > 1$. Thus only patient individuals can substitute consumption across dates. The resources not consumed in date 1 are reinvested at gross rate-of-return $R > 1$. These preferences have been used by Green and Lin (2000), with $A = 1$, and Peck and Shell (2003), with $A = 10$.

Feasible transfers must be incentive-compatible and satisfy a sequential-service constraint. The sequential-service constraint prevents date-1 consumption transferred to a person in position i to depend on information provided by someone at position n for $n > i$.

In the standard model, each individual draws a unique position i in $\{1, ..., N\}$ with probability $\frac{1}{N}$ and, as a result, the realization ω_i, without knowing the other coordinates of ω. As in Peck and Shell (2003), we shall assume that the individual is not informed of his position i. But we also want to consider a case in which shocks are correlated and let the degree of correlation vary in a simple manner, having independence as a particular case. The details about this more general stochastic process for types are given at the end of this section. In comparative-statics exercises with variations in population sizes, we shall keep the per capita endowment $e = \frac{Y}{N}$ constant as the population size, N, varies.

A compact description of candidates for optimal allocations follows from additional notation. Let us denote by ω^i the vector $(\omega_1, \omega_2, ..., \omega_i)$, and by (ω_{-i}, z) the profile that results from substituting the i-th coordinate of ω by z. Given that $R > 1$ we can restrict attention to transfers that assigns, to someone at position i, $x_i\left(\omega^{i-1}, 0\right)$ units of date-1 consumption, if that person is impatient, or $y_i\left(\omega\right)$ units of date-2 consumption, otherwise. The sequential-service requirement has thus shaped the domains of x_i and y_i. We shall

denote by (x, y) a typical pair of transfer functions. We notice that (x, y) is feasible if

$$\sum_{i=1}^{N} \left((1-\omega_i)\, x_i \left(\omega^{i-1}, 0\right) + \omega_i R^{-1} y_i(\omega) \right) \leq Y, \tag{3.1}$$

and incentive-compatible if

$$E\left[\frac{1}{N}\sum_{i=1}^{N} u\left(y_i\left(\omega_{-i}, 1\right)\right)\right] \geq E\left[\frac{1}{N}\sum_{i=1}^{N} u\left(x_i\left(\omega^{i-1}, 0\right)\right)\right], \tag{3.2}$$

that is, when patient individuals that are not informed of their positions agree with revelation. [5]

The planner's problem is that of maximizing the representative-agent utility, before types and positions are assigned,

$$E\left[\frac{1}{N}\sum_{i=1}^{N} \left((1-\omega_i)\, A u\left(x_i\left(\omega^{i-1}, 0\right)\right) + \omega_i u\left(y_i\left(\omega\right)\right)\right)\right], \tag{3.3}$$

subject to (3.1) and (3.2).

A realization ω is interpreted as the type-composition of a queue for being served by the bank. We let the probability distribution on types for someone in position i depend on the realized type for the person in position $i-1$. In particular, the first-position person is impatient with probability p. And the person in position i is assumed to have the same type of the previous individual in the queue with probability θ. Accordingly, the probability distribution for position i on $\{0,1\}$ is $(1-\omega_{i-1}, \omega_{i-1})$ times

$$T \equiv \begin{bmatrix} \theta & 1-\theta \\ 1-\theta & \theta \end{bmatrix}.$$

Therefore, a history ω has probability

$$P(\omega) = \bar{\omega}_1 \begin{bmatrix} p \\ 1-p \end{bmatrix} \prod_{i=2}^{N} \bar{\omega}_{i-1} T \bar{\omega}_i'^{1-\omega_1} (1-p)^{\omega_1} \left[\theta^{N-1-s(\omega)} (1-\theta)^{s(\omega)}\right]$$

if $s(\omega) = \sum_i^{N-1} |\omega_{i+1} - \omega_i|$ is the number of type switches and $\bar{\omega}_i = \begin{pmatrix} 1-\omega_i & \omega_i \end{pmatrix}$.

3.3 The Lagrangian approach

The planner's problem defined above aims to find the best incentive-compatible mechanism (x, y). As it will become clear further below, every (x, y) defines a game of an-

[5]The expectations in (3.2) are taken with respect to the distribution of ω_{-i} on $\{0,1\}^{N-1}$, given that $\omega_i = 1$.

nouncements and it can happen that this game have multiple equilibria, with the intended revelation-equilibrium being just one of them. But before we can address multiplicity, it is important to be able to compute the optimal (x, y) defined above. While Green and Lin (2000) finds a closed form solution for $N = 3$ and independent shocks, Ennis and Keister (2009), henceforth EK, derive a recursive method that handles correlated shocks (although with a different specification compared to the setup above). Our objective is to extend the Ennis-Keister (2009) approach. But one problem is that, like in the situation addressed by Green and Lin (2000), the incentive constraints could be ignored, while this is not the case for (3.2) above when A and θ are arbitrary. We shall see, however, that an intuitive extension is possible if we shift the attention away from the objective (3.3) and focus instead on a lagrangian version.

3.3.1 The objective for weak implementation

Proposition 10 *The optimum can be computed by [ignoring (3.2) and] replacing the objective (3.3) with*

$$\max_{(x,y)} \sum_{\omega} P(\omega) \sum_{i} \left[(1-\omega_i) \left(\alpha_i^{\omega_{i-1}}\right)^{\delta} u(x_i) + \omega_i \left(\beta_i\right)^{\delta} u(y_i) \right],$$

where $(\alpha_i, \beta_i)_{i=1}^{N}$ are functions of ω [determined in close form for each candidate Lagrange multiplier for (3.2)].

Proof. A patient agent when in position i believes that announcement profile ω_{-i} happens with probability

$$\Pr(\omega_{-i}|\omega_i = 1) = \frac{\Pr(\omega_{-i}, [\omega_i = 1])}{\Pr([\omega_i = 1])} = \frac{P(\omega_{-i}, 1)}{\sum_w P(w_{-i}, 1)} = \frac{P(\omega_{-i}, 1)}{\sum_{w^{i-1}} P(w^{i-1}, 1)}$$

where the denominator can be written as $q_i = (p, 1-p)T^{i-1}([0, 1]')$[6]. Therefore, the incentive constraint (3.2) can be rewritten as

$$\frac{1}{N} \sum_i \frac{1}{q_i} \sum_{[\omega:\omega_i=1]} P(\omega) \Big[u\left(y_i\left(\omega_{-i}, 1\right)\right) - u\left(x_i\left(\omega^{i-1}, 0\right)\right) \Big] =$$

$$\frac{1}{N} \sum_{\omega} P(\omega) \sum_i \frac{1}{q_i} \Big[\omega_i u\left(y_i\left(\omega_{-i}, 1\right)\right) - (1-\omega_i)\gamma_i^{\omega_{i-1}} u\left(x_i\left(\omega^{i-1}, 0\right)\right) \Big] \geq 0$$

where, for $t \in \{0, 1\}$, γ_i^t stands for $((1-\theta)/\theta)^{1-2t}$ when $i > 1$, and for $(1-p)/p$, otherwise. The term $\gamma_i^{\omega_{i-1}}$ is actually equal to the ratio between $P(\omega^{i-1}, 1)$ and $P(\omega^{i-1}, 0)$ which appears in the expression above when the summation is taking place over ω in Ω [as in (3.3)] instead of ω in $\omega \in \Omega : \omega_i = 1\}$ [as in (3.2)]. After this adjustment the slack in the

[6]Note that $p = 0.5$ implies that $q_i = 0.5$ for all i

truth-telling constraint, which appears in the lagrangian, becomes written as a summation with weights $P(\omega)$ as in the objective function.

If λ denotes the lagrangian multiplier for (3.2) then the planner's problem can be stated as

$$\max_{(x,y)} \left\{ \frac{1}{N} \sum_{\omega} P(\omega) \left(\sum_i \left[(1-\omega_i) \left(\alpha_i^{\omega_{i-1}}\right)^{\delta} u(x_i) + \omega_i \left(\beta_i\right)^{\delta} u(y_i) \right] \right) ; (3.1) \right\}$$

where $\alpha_i^t = \left(A - \frac{\lambda}{q_i}\gamma_i^t\right)^{1/\delta}$ and $\beta_i = \left(1 + \frac{\lambda}{q_i}\right)^{1/\delta}$. Notice that the q's and γ's can be computed using only properties of the distribution of partial histories, and nothing else (they are invariant to λ). Hence the coefficients α's and β's are computed in close form, and the proof is now complete. ■

3.3.2 Candidate policy functions

Now the EK dynamic-programming approach can be applied to solve the modified problem for each candidate multiplier. In effect, consider the date-2 partial problem faced by the planner after history ω:

$$\max_{y} \left\{ \sum_i \left(\beta_i\right)^{\delta} u(y_i); \sum_i \omega_i y_i(\omega) \leq Ra \right\},$$

where a denotes the sum of resources not consumed in date 1. Its solution must satisfy $y_i = \beta_i/\mu^{1/\delta}$ if μ is the multiplier for the resource constraint. Since it binds at the optimal solution, then $\mu^{1/\delta} = (\sum_i \omega_i \beta_i)/Ra$, which yields

$$y_i = \frac{\beta_i}{\sum_k \omega_k \beta_k} Ra. \tag{3.4}$$

Therefore, the corresponding optimal value is $(f_N(\omega))^{\delta} u(a)$, where $f_N(\omega) = R^{1/\delta - 1} \sum_k \omega_k \beta_k$. Keeping this contingent-solution fixed, the planner faces after position $N-1$ the partial problem

$$\max_{c \leq a} \left\{ T_{t+1,1} \Big(\left(\alpha_N^t\right)^{\delta} u(c) + f_N(\cdot, 0)^{\delta} u(a-c) \Big) + T_{t+1,2} \Big(f_N(\cdot, 1)^{\delta} u(a) \Big) \right\}$$

where a now denotes resources not consumed at the first $N-1$ positions, $t = \omega_{N-1}$, and $T_{i,j}$ denotes the element (i,j) in matrix T. The solution is now

$$c = \frac{\alpha_N^t}{\alpha_N^t + f_N(\cdot, 0)} a,$$

which produces the corresponding value

$$u(a)\Big(T_{t+1,1}[\alpha_N^t + f_N(\cdot,0)]^\delta + T_{t+1,2}[f_N(,\cdot,1)]^\delta\Big) = u(a)\Big(f_{N-1}(\cdot)\Big)^\delta.$$

Similar algebra can be done to show that the optimal solution for $i < N-1$ is always

$$c_i = \frac{\alpha_i^t}{\alpha_i^t + f_i(\cdot,0)}a$$

and, likewise, the optimal value is $u(a)\Big(f_{i-1}(\cdot)\Big)^\delta$. The f's are coefficients for 'splitting the pie' and are functions of partial histories. They are moreover fully determined by the system given by

$$f_i(\cdot) = \Big(T_{t+1,1}[\alpha_{i+1}^t + f_{i+1}(\cdot,0)]^\delta + T_{t+1,2}[f_{i+1}(\cdot,1)]^\delta\Big)^{1/\delta}, \tag{3.5}$$

if $i > 0$, and

$$f_0 = \Big(p[\alpha_1 + f_1(0)]^\delta + (1-p)[f_1(1)]^\delta\Big)^{1/\delta}, \tag{3.6}$$

otherwise. The value for objective in the lagrangian/planner problem is therefore $N^{-1}u(Y)[f_0]^\delta$.

3.3.3 Iterating on multipliers

Unlike EK, we must verify whether a given discount β is correct. This is done by computing (3.2) under the implied allocation. If the incentive constraint is violated (slacks), β must be increased (reduced). The following proposition establishes a recursive method for computing the slack in the constraint. In the same way that we have found policies $f_i(\omega^i)$, determining consumption as a fraction of existing reserves at partial history ω^i, we shall look for partial sums in the expectations that define the slack in truth-telling constraints, and organize these terms by partial histories, using the notation $g_i^{\omega^i}$ as follows. Let us inspect the expression for the slack for given ω, as in the proof of Proposition 1, isolate the term

$$\sum_i \frac{1}{q_i}\Big[\omega_i u\left(y_i\left(\omega_{-i},1\right)\right) - (1-\omega_i)\gamma_i^{\omega_{i-1}} u\left(x_i\left(\omega^{i-1},0\right)\right)\Big]$$

and, in particular, consider the part associated to date-2 consumption. We write

$$g_N^\omega = \sum_i \frac{1}{q_i}\omega_i u\left(y_i\right)$$

where y_i is given by (3.4) for ω.

Proposition 11 *For each history ω, define $g_N^{\omega} = \sum_i \frac{\omega_i}{q_i}\left(\frac{\langle \omega,\beta\rangle}{\beta_i R}\right)^{1/\delta}$. If $g_i^{\omega^i}$ is defined as*

$$T_{t+1,1}\left[g_{i+1}^{(\cdot,0)}\left(1+\frac{\alpha_{i+1}^t}{f_{i+1}(\cdot,0)}\right)^{\delta-1}\right]+T_{t+1,2}\left[g_{i+1}^{(\cdot,1)}-\frac{1}{q_{i+1}}\left(1+\frac{f_{i+1}(\cdot,0)}{\alpha_{i+1}^t}\right)^{\delta-1}\right]$$

for $i>0$, and

$$p\left[g_1^{(0)}\left(1+\frac{\alpha_1}{f_1(0)}\right)^{\delta-1}\right]+(1-p)\left[g_1^{(1)}-\frac{1}{q_1}\left(1+\frac{f_1(0)}{\alpha_1}\right)^{\delta-1}\right]$$

for $i=0$, then $N^{-1}u(Y)g_0$ equals the slack in the truth-telling constraint when evaluated at the optimal mechanism implied by β.

Proof. The patient's payoff, in terms of an average across positions, given ω, is $u(a)g_N^{\omega}$ divided by N, where a is the endowment saved for date 2. Using now (3.4), we have $g_N^{\omega} = \sum_i \frac{\omega_i}{q_i}\left(\frac{\omega\beta}{\beta_i R}\right)^{\delta-1}$. A patient person can now proceed to compute the slack in his or her constraint according to deviation payoffs which vary across positions. We can compute the slack by restating the payoff $u(a)g_N^{\omega}$, with adjustments for either the opportunity from lying or for the law of motion of reserves, and according to a series of contingencies. If this person has drawn position N ($\omega_N = 1$) the corresponding expression is

$$g_N^{(\cdot,1)}u(a)-\frac{1}{q_N}u(x_N)$$

where x_N is the date-1 transfers at $(\omega^{N-1},0)$. If this person has not drawn position N ($\omega_N = 0$) , there is still an impact of x_N on available reserves that, when taken into account yields

$$g_N^{(\cdot,0)}u(a-x_N).$$

Integrating over these two contingencies using the Markov chain gives us a function of ω_{N-1} compactly written as

$$T_{t+1,1}\left(g_N^{(\cdot,0)}u(a-x_N)\right)+T_{t+1,2}\left(g_N^{(\cdot,1)}u(a)-(1/q_N)u(x_N)\right)$$

where a is now the endowment saved after the first $N-1$ positions and $t=\omega_{N-1}$. Now, using optimal value for x_N this expression amounts to $u(a)g_{N-1}^{\omega^{N-1}}$ as in the statement of the proposition.

Similar and straightforward algebra can be used to show that this partial measure of incentives to tell the truth for position $i<N$ equals $u(a)g_{i-1}^{\omega^{i-1}}$ if a is the quantity saved for position i, as in the statement of the proposition. ■

The proof demonstrates that one can start with the measure g_N^{ω} , summarizing the expected payoff from telling the truth at ω, and then recursively compute adjustments

according to the what happens if the patient person draws each potential position. Integrating over all contingencies is facilitated by the Markov assumption. In summary, date-1 consumption affects the incentives of a patient person by reducing resources saved for date 2 and by its effect on misrepresentation payoffs. The former is accounted by the factor multiplying $g_{i+1}^{(\cdot,0)}$ in the first term of $g_i^{\omega^i}$ in the statement of the proposition, while the latter is the expression being subtracted in the second term.

The proposition gives us g_0 as a proxy for the slack on truth-telling constraints, which can then guide increases in implicit interest rates through changes in the multiplier λ (proposition 1). The EK method is, in this way, extended along the lines of penalty-function methods. The algorithm performs a simple search, by bracketing the positive orthant, until either $g_0 = 0$ is found or $g_0 > 0$ for $\lambda = 0$.

3.4 Financial stability

In this section we study the existence of run equilibria, and the cost in avoiding them. The optimal mechanism defines a game of announcements where patient agents can lie with probability π. *Run* equilibria are symmetric Bayesian-Nash equilibria of this game such that $\pi > 0$. In order to study the existence of runs for a fixed mechanism (x, y), we let for $\pi \in [0, 1]$

$$w(\pi) = \frac{1}{N} \sum_i E_\pi \Big[\omega_i u(y_i(\omega)) - (1 - \omega_i) u(x_i \left(\omega^{i-1}, 0\right)) \Big] \tag{3.7}$$

denote the relative payoff of truth-telling when other patient individuals are lying with probability π. In terms of the signal of the function w, a patient individual tells the truth (runs) if w is positive (negative), and the best reply to π is the interval $[0, 1]$ when $w(\pi) = 0$. Notice that w is a property of a weakly implementable (x, y) that describes its potential fragility, and that $w(0)$ can be computed according to the construction of g_0 derived above and the corresponding iterations of multipliers in our algorithm.

3.4.1 Pervasive runs

The computation of $w(\pi)$ for $\pi > 0$, as in Figure 1, is a by-product of the algorithm for strong implementation outlined below.

Figure 1 presents the function w for some economies with *iid* shocks, which is the case when $\theta = .5$ and $p = .5$ (we keep $p = .5$ throughout the chapter). The numbers in parenthesis correspond to the values of the pair of parameters (A, N). The three decreasing curves are representations of weak implementation discussed so far. The other curve, which bounces back after a tangency at the horizontal axis, has to be explained later because it corresponds to a w after judiciously selected distortions on (x, y) are

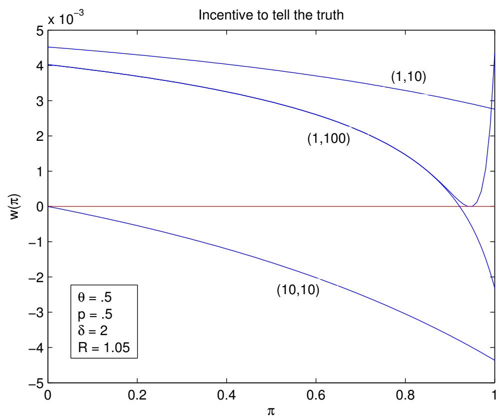

Figure 3.1: A mixed-strategy run equilibrium

introduced and which accomplishes strong implementation.

In these examples, weak implementation with independent shocks feature $w(\pi)$ decreasing in π because a patient person thinks that telling the truth becomes increasing unattractive as more people are running, leaving less and less resources left for date 2. The fact that reserves are planned to be constantly reduced with a sequence of zeros — say, along the contingency $\omega = (0, 0, ...)$ — is easily understood when we take into account the role of homothetic preferences. After a given sequence of zeros, the planned transfers are computed as if future shocks are drawn from P, and not from a transformation of P generated by run strategies. Since preferences are homothetic, the drop in reserves up to this point has no effect on how the pie is planned to be divided from this point onwards. As result, a plan of providing insurance based on P for the remaining traders is maintained, which means that some sharing of the growth potential R with impatient people will continue to drive reserves down during a run. A key question is thus whether the function w eventually crosses into negative territory, when panic is widespread and π approaches one, demonstrating that an equilibrium-run exists. Intuitively, one factor leading to w negative is the level of liquidity insurance 'demanded' by the impatient. This factor has been emphasized by Peck and Shell (2003), with A increasing from 1 [as in Green and Lin (2000, 2003)] to 10, and by EK, with increases in δ across examples. But the preceding discussion suggests that N also plays a big role. For, if N is large and shocks are independent, more insurance is planned and the consequent depletion of reserves lasts longer as the sequence of 0-announcements keep coming.

It can also happen that w is everywhere negative but at point $\pi = 0$. This case is illustrated by the lowest curve, which corresponds to a similar parametrization in Peck and Shell (2003, appendix B). The optimal allocation is constrained since $w(0) = 0$, and there is a run equilibrium in pure strategies since $w(1) < 0$. The highest curve shows that the the best-reply correspondence changes, and no runs are found, in the same economy, when $A = 1$. This exercise confirms that run equilibria can be obtained in small

population economies by increasing A. And, in the another direction, one can increase the population size N to 100 to show that runs do not require a high value for A (neither a high value for δ used in the run example constructed by EK with independent shocks). The conclusion is that runs are always motivated by too much risk sharing. In addition, the large population example illustrates that there are mixed-strategy equilibria whenever the optimum is unconstrained and there is a run equilibrium in pure strategies.

The role of N in these examples take us to the limit result in chapter 1. They show that welfare of a Peck-Shell economy with independent shocks approaches that of a of an economy with a continuum of people as N grows to infinity. Their proof does not rely on homothetic preferences nor bounds on risk aversion. And moreover, they show the truth-telling constraint of the 'continuum' problem is active if and only if $A \geq R$. Bearing in mind this characterization of large-population economies, we guess and verify, numerically, the following claim for our homothetic economy, given the intuition constructed for the law of motion of reserves during a run.

Claim Assume that shocks are independent. There exist mixed-strategy runs in all economies with $A < R$ and N sufficiently high. On the converse, there are no mixed-strategy runs in economies with $A \geq R$.

3.4.2 Persistence and weak implementation

Because persistence introduces a load on memory requirements for numerical work that grows significantly with N, we have not attempted to study limit properties with $\theta = .75$ in general. The characterization of economies featuring active constraints is more subtle in this case, and the theorem in chapter 1 cannot serve as guidance. But we do find that the propensity to run increases when N is fixed and θ shifts from .5 to .75, for N is a large range. This tells us that mixed-strategy runs are still common, but the way that (A, R, N) determine the slack in truth-telling constraints changes.

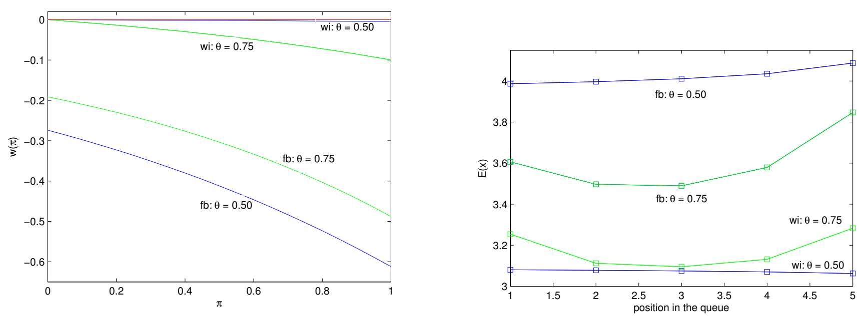

Figure 3.2: Aggregate-uncertainty effects.

Figure 2 illustrates the effects of introducing aggregate uncertainty with Markov transitions. The parameter values are the same as before, but $(A, N) = (10, 5)$. The two

lowest curves in the graph on the left refers to first-best (fb) allocations for the cases of independence ($\theta = .5$) and persistence ($\theta = .75$). These are mechanisms obtained by ignoring truth-telling constraints, as if types are no longer private information. The other curves refers to weak implementation (wi).

Let us fix attention to the first-best curves on both graphs momentarily. For both θ's the first best is not implementable since $w(0) < 0$. In addition, the fb-w is lower (misrepresentation is more attractive) when there is less persistence because there is more room for risk sharing. Intuitively, there is more mixing in P. The graph on the right confirms such intuition: it can be seen that the average date-1 consumption in the first-best allocation is higher when shocks are independent. It can also be noticed that transfers are more sensitive to positions on average when shocks are persistent because current shocks are very important for predicting the future, even close to the end of the queue. That is, the sequential-information friction proposed by Wallace (1988) becomes stronger under correlation. A novel result, that we shall document below, is that Lagrange multipliers fall under correlation and, in the case of active constraints, there is again in the capacity of providing insurance because weak-implementaion distortions can be reduced. The reason for the fall in multipliers is explained below. We should see, however, that the corresponding increase in the provision of insurance makes runs more likely.

When forced to provide incentives to patient agents to tell the truth — when forced to change (x, y) so that $w(0) \geq 0$) — the planner distorts the allocation in a way that misrepresentation becomes more attractive for higher θ. The reason for this inversion in w's can be found in the second graph of figure 2: planner is able to provide incentives in the persistence case with a smaller reduction in date-1 consumption, so that the deviation payoff does not decrease as much as in the independent case. Intuitively, it is easier to convince patient individuals to not run when shocks are persistent. In effect, due to persistence, a patient person knows that with high probability announcements will be mostly patient ones. Telling the truth in this scenario is good since date-2 payments are high. On the other hand, the deviation payoff for this individual is mainly concentrated (in a probability sense) on withdrawals just after a patient announcement. Knowing this, the planner can concentrate distortions on impatient announcements which happens after a type innovation. That way, the planner is able to preserve a high insurance level on average since distortions placed on withdrawals immediately after impatient announcements can be made temporary, easily preventing lies (again in a probability sense).[7]

Figure 3 illustrates the effects of population growth on economies with persistence. The graph on the right shows that the ability to maintaining risk sharing under weak implementable allocations plays an important role when the population increases. Welfare

[7]More formally, we notice, from proposition 1, that date-1 consumption has weight $\alpha_i^t = (A - (\lambda/q_i)\gamma_i^t)^{1/\delta}$ in the modified problem that planner solves under weak implementation. Because in general $\gamma_i^t = (1/\theta - 1)^{1-2t}$, such weight is invariant to t when shocks are independent ($\theta = .5$), while it is high for $t = 0$ and low for $t = 1$ when there is persistence ($\theta > .5$).

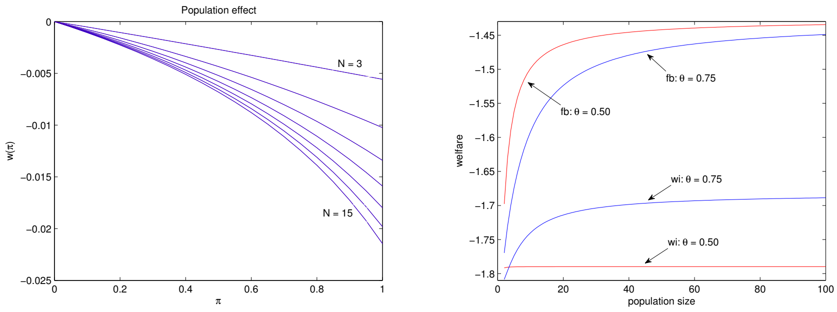

Figure 3.3: Population effects.

significantly improves even in a constrained optimum when $\theta = .75$, but not so much when $\theta = .5$. If there was no incentive problem, insurance would increase in both cases as can be concluded from first-best curves (fb) in the second graph. When incentives are taken into account (wi), the curve for $\theta = .5$ confirms that the limit result in chapter 1 occurs for very low populations with Peck-Shell preferences: in the scale presented by the graph, welfare with small N coincides with that of a very large economy. The corresponding speed of convergence is much lower when $\theta = .75$. We shall see that, although welfare eventually flattens up with persistence, the economy with $\theta = .75$ is qualitatively different because the incentives to run in a panic are magnified, and the remedy cost is maintained considerable as the population grows, in contrast to the perturbation result in chapter 1. There, with independent shocks, the cost of removing panics converges to zero exponentially as the population grows.

When $\theta = .75$, increases in population is reflected in smaller and smaller values for w.[8] The graph on the left of Figure 3 shows such shifts in w, and we have done the same exercise for $\theta = .5$ (not shown in the graph). In both cases, the shifts become smaller as N increases, but they are much larger when θ is high. In addition, w is always much smaller in the persistent case, with $w(1) \approx -10^{-2}$ for $N = 11$ in the independent case. As discussed above, the impact of increases in N on deviation payoffs is driven by the insurance effect. It is always higher in the persistence case because the planner can induce truth-telling at a lower cost, allowing for higher date-1 consumption on average. As the population grows, the distortions needed to provide incentives under weak implementation are reduced, and the consequent impact on the provision of insurance causes shifts on the w's curves that keep changing, even though in lower rates in both cases.

3.4.3 The cost of eliminating runs

Next we address the issue of what is the least costly way to promote distortions in (x, y) in order to eliminate runs. We propose to measure the 'welfare cost of financial fragility'

[8] We have seen in Figure 1 that a similar effect is occurring when shocks are independent, which is tatonnement to the pervasiveness of run equilibria documented in the claim that follows the figure.

by studying the effects of imposing 'no-run constraints' that have been defined in Peck and Shell (2003) for a context of pure strategies. We shall approach this matter by extending the concept to mixed strategies, and extending the numerical work to cover these constraints simultaneously with the standard truth-telling constraints. The new method is responsible for the construction of the non-monotone w curve in Figure 1 which remains always nonnegative.

We call *cost of financial stability* the reduction in the per capita endowment that has to applied in order to produce an optimal welfare, under weak implementation, of the same magnitude as that under strong implementation. In other words, we seek the least costly way to introduce distortions in the economy in order to eliminate runs. We have seen that deviation payoffs are higher when there is more persistence and when the population is larger. We recall that in chapter 1 we construct a simple mechanism featuring strong implementation and which converges to the weakly-implementable optimum when $N \to \infty$ and shocks are independent. Hence, we must confirm numerically, using the Peck-Shell concept of no-run constraints, that the cost of financial stability indeed converges to zero when $\theta = .5$.

The methods developed so far generates a w function such that $w(0)$ is nonnegative. The necessary changes are guided by the following definition.

Definition 12 *The optimal allocation associated to a revelation equilibrium ($\pi = 0$) for a given mechanism is strongly implementable if the w for this mechanism is nonnegative for all π.*

The idea is to force, if necessary, a better treatment of patient individuals so that, generically, runs are not best replies. In particular, we seek to solve the following.

SI problem Maximize (3.3) in (x, y), subject to (3.1) and to $w(\pi) \geq 0$ for all $\pi \in [0, 1]$.

The following results shows that a simple extension of the current algorithm is able to solve the SI problem.

Proposition 13 *Assume that the solution to the SI problem features $w(\pi) = 0$ at finitely many π's and that the respective multipliers λ_π's are known. Then the this solution can be computed by modifying the objective as*

$$\sum_{\omega} P(\omega) \sum_{i} \left[(1 - \omega_i)\alpha_i(\omega^i)^\delta u(x_i) + \omega_i \beta_i(\omega)^\delta u(y_i)\right]$$

where

$$\alpha_i(\cdot) = \left(A - \frac{1}{q_i}\int_0^1 \lambda_\pi h_\pi^i(\cdot, i)d\pi\right)^{1/\delta}$$
$$\beta_i(\cdot) = \left(1 + \frac{1}{q_i}\int_0^1 \lambda_\pi h_\pi(\cdot, i)d\pi\right)^{1/\delta}$$

and $(h_\pi, h_\pi^i)_{i=1}^N$ *are functions of* ω *determined in close form.*

Proof. Let η stand for the history of announcements. A patient agent believes that an impatient announcement happens in position i with probability

$$T_{t+1,1}^\pi = T_{t+1,1} + \pi T_{t+1,2} = \langle \bar{\omega}_{i-1} T, (1,\pi) \rangle,$$

if $i > 1$, and $\langle (p, 1-p), (1,\pi) \rangle$, otherwise. Accordingly,

$$P_\pi(\eta) = \sum_\omega P_\pi(\eta|\omega) P(\omega) = \sum_\omega P(\omega) \prod_{i=1}^N \bar{\omega}_i M_\pi \bar{\eta}_i'$$

If in position i, he or she believes that announcement η_{-i} happens with probability

$$\begin{aligned}
\Pr(\eta_{-i}|\omega_i = 1; \pi) &= \frac{P_\pi(\eta_{-i}, [\omega_i = 1])}{\Pr([\omega_i = 1])} = q_i^{-1} \sum_\omega P_\pi(\eta_{-i}, [\omega_i = 1]|\omega) P(\omega) \\
&= q_i^{-1} \sum_{[\omega:\omega_i=1]} P_\pi(\eta_{-i}|\omega) P(\omega) \\
&= q_i^{-1} \sum_{[\omega:\omega_i=1]} P(\omega) \left[\prod_{j\neq i} \left(\bar{\omega}_j M_\pi \bar{\eta}_j' \right) \right] = \frac{P(\eta_{-i}, t)}{q_i} h_\pi((\eta_{-i}, t), i)
\end{aligned}$$

where $M_\pi = \begin{bmatrix} 1 & 0 \\ \pi & 1-\pi \end{bmatrix}$ and

$$h_\pi((\eta_{-i}, t), i) = \sum_{[\omega:\omega_i=1]} \frac{P(\omega)}{P(\eta_{-i}, t)} \left[\prod_{j\neq i} \left(\bar{\omega}_j M_\pi \bar{\eta}_j' \right) \right]. \tag{3.8}$$

That way, we have

$$\begin{aligned}
w(\pi) &= \sum_i \frac{1}{N} \sum_{\eta_{-i}} \Pr(\eta_{-i}|\omega_i = 1; \pi) \left[u(y_i(\eta_{-i}, 1)) - u(x_i(\eta^{i-1}, 0)) \right] \\
&= \frac{1}{N} \sum_i q_i^{-1} \sum_\eta P(\eta) h_\pi(\eta, i) [\eta_i u(y_i(\eta_{-i}, 1)) - (1-\eta_i) u(x_i(\eta^{i-1}, 0))] \\
&= \frac{1}{N} E \sum_i q_i^{-1} \Big[\omega_i h_\pi(\omega, i) u\left(y_i\left(\omega\right)\right) - (1-\omega_i) h_\pi^i(\omega^i, i) u\left(x_i\left(\omega^i\right)\right) \Big]
\end{aligned}$$

where expectation is now taken with respect to ω, and the h's are given by

$$h_\pi^k(\cdot, i) = \left\langle \bar{\omega}_k T, \left(h_\pi^{k+1}((\cdot, 0), i), h_\pi^{k+1}((\cdot, 1), i) \right) \right\rangle$$

for each ω^k if $h_\pi^N(\omega, i) = h_\pi(\omega, i)$.

If λ_π denotes the lagrangian multiplier on (3.7), then SI problem can be stated as

solving

$$\max_{(x,y)}\left\{\frac{1}{N}\sum_{\omega}P(\omega)\left(\sum_{i}\left[(1-\omega_i)\alpha_i(\omega^i)^{\delta}u(x_i)+\omega_i\beta_i(\omega)^{\delta}u(y_i)\right]\right);(3.1)\right\}$$

and the proof is now complete. ■

Again the EK dynamic-programming approach can be applied to solve such a problem for each candidate multiplier set, $\boldsymbol{\lambda}=\{\lambda_\pi:\pi\in[0,1]\}$. The optimal solution is still given by (3.5-3.6), but now α_i depends on ω^i, not only on ω_{i-1} as in the weak-implementation case. It turns out that $w(\pi)$ can be computed recursively in the same fashion as done for the slack in the truth-telling constraint in Proposition 2.

Proposition 14 *For a fixed belief* π*, define* $g_i^{\omega^i}$ *as*

$$T_{t+1,1}\left[g_{i+1}^{(\cdot,0)}\left(1+\frac{\alpha_{i+1}(\cdot,0)}{f_{i+1}(\cdot,0)}\right)^{\delta-1}\right]+T_{t+1,2}\left[g_{i+1}^{(\cdot,1)}-\frac{h_\pi^i((\cdot,1),i)}{q_{i+1}}\left(1+\frac{f_{i+1}(\cdot,0)}{\alpha_{i+1}(\cdot,0)}\right)^{\delta-1}\right]$$

for $0<i<N$*, and*

$$p\left[g_1^{(0)}\left(1+\frac{\alpha_1}{f_1(0)}\right)^{\delta-1}\right]+(1-p)\left[g_1^{(1)}-\frac{h_\pi^1((1),1)}{q_1}\left(1+\frac{f_1(0)}{\alpha_1}\right)^{\delta-1}\right]$$

for $i=0$*, together with* $g_N^{\omega}=\sum_i\frac{\omega_i h_\pi(\cdot,i)}{q_i}\left(\frac{\langle\omega,\beta\rangle}{\beta_i R}\right)^{1/\delta}$*. Then* $N^{-1}u(Y)g_0$ *equals* $w(\pi)$ *when evaluated at the solution associated to* $\boldsymbol{\lambda}$*.*

Proof. The argument is identical to the one used in Proposition 2, after the adjustment required by the introduction of the terms h and h^i, and is thus omitted. ■

The non-monotone curve in Figure 1 illustrates the typical case found by the algorithm when mixed-strategy runs are rule out. In this case there is only on critical π to be found (and only one multiplier for the solution): it corresponds to the pivotal (in the language of linear programming) strategy for which the corresponding no-run constraint is weakly active. Once this critical π is found, all other constraints slack. And because the resulting non-monotone curve is positive at 0, the standard incentive constraint (implied by the revelation principle) also slacks. We conclude that the machinery used here to address mixed strategies are important for an accurate assessment of the costs of financial stability.

In order to quantify the distortion in the optimal allocation, we compute by how much the per capita endowment should be reduced under weak implementation to achieve the welfare under strong implementation. The following table summarizes the results for selected economies.

θ	Endowment reduction $N=3$	$N=5$	$N=7$	$N=15$	Interest rate: $E(\bar{y}/\bar{x})$ first-best	weak	strong
.50	.0022%	.0021%	.0017%	.0009%	.4027	.9258	.9262
.51	.0059%	.0071%	.0064%	.0038%	.4031	.9202	.9208
.52	.0112%	.0150%	.0142%	.0091%	.4034	.9136	.9147
.75	.1543%	.3173%	.3786%	.2421%	.3572	.5874	.6092

$\bar{y}(\omega) = y(\omega)/|\omega|$ and $\bar{x}(\omega) = \sum_i x_i(\omega)/(N - |\omega|)$

The left side of the table shows that for all considered population sizes, cost of financial stability is much larger when there is persistence. We also confirm that the cost is very low in the independent case. While in chapter 1 there was a noticeable convergence of Peck-Shell and Green-Lin mechanisms (when there is disclosure of positions and traders use elimination of dominated strategies, in computed cases, to choose truth-telling) for such low N's. Hence, because there were no runs for the Green-Lin setting in the numerical examples of chapter 1, the conclusion was that by letting traders be informed of their positions in the queue — if such disclosure can be considered feasible, what is debatable — the planner can achieve strong implementation at a low cost (since the transfer functions of Peck-Shell and Green-Lin mechanisms quickly converge to each other and there no runs with disclosure). Here, by contrast, we show that disclosure is not needed. By placing the appropriate distortions, according to our extended algorithm, we find that the cost falls to zero very quickly when $\theta = .5$. The effects of an increase in θ on stability costs are very strong because the distortions needed are more severe when there is a threat that a whole group of people is running (the case $\pi > 0$ that has to be addressed by strong implementation). With persistence, when forced to provide incentives to patient agents to not run, when they believe other patient individuals are misrepresenting (when forced to change (x, y) so that $w(\pi) \geq 0$ for all $\pi \in [0, 1]$), the planner must distort withdrawals in a larger range of 0's. Intuitively, a deviator thinks that he or she will be bunched with other zeros. Hence, a belief $\pi > 0$ shifts (in a probabilistic sense) the deviation payoffs compared to the weak-implementation case. Now there is a need to reassure patient people whose deviation payoff accrues after a type innovation, as well as those whose deviation payoff accrues after a type persistence. Thus the planner must now reduce the payment schedule for a larger group.

The right-side of the table documents the result for $N = 5$. It shows that the ratio of average consumption in date 2 to that in date 1 increases the whenever planner must provide incentives. And indeed the necessary increase in this proxy for interest rates, as one moves from weak to strong implementable allocations, is low for $\theta = .5$ and much higher for $\theta = .75$. Provided that correlations in types is a reason for being concerned about runs, the table illustrates a basic result across our experiments: a high level of interest rates is the golden rule for financial stability.

We found a bit surprising that the cost increases with N for these small economies. We have seen, while analyzing Figure 3, that w is much more negative, and shifts cause by population increases are larger when θ is high. This pattern points towards a higher cost and slower convergence under persistence, since patient agents demand more to not run, and becomes relatively more demanding when N increases. On one hand, as with independence, the cost of fulfilling such demands is expected to decrease as population grows and the aggregate state becomes more predictable. On the other hand, while the table shows that the reduction in aggregate uncertainty is sufficient to offset increasing demands under independence, it is not under persistence. But since the uncertainty about the fraction of impatient people in the population should vanish as $N \to \infty$ even when $\theta = .75$ (as N grows the number of Markov switches also grows, generating equal representation of impatient and patient people with increasingly high probability), we expect the cost to reach a maximum at larger N.

3.5 Insolvency

A fundamental feature of the economies studied until here is that depositor actions are very easy to monitor. We now consider an extension in which the bank is not able to recognize people's identities during the whole first date. We assume that it is possible for some patient people to make two successive accesses to the bank, and that this ability is available with probability $q > 0$. These are the potential 'insiders' that can cause large losses to the financial system in case of a bank run. We also assume that, at the second date, the actions taken at date 1 are matched to the people claiming transfers at the date 2. Hence the identities are recognized and matched to actions at the second date, but this may happen too late to make a difference in case of a run. In summary, the record of actions become updated with a lag, as in Kocherlakota and Wallace (1998). We shall see that the bank has to become more conservative. It has to keep a high level of reserves because some insiders may appear with a positive probability, and in order to keep these insiders away from embezzlement options the bank must increase interest rates with regards to the consumption of this subgroup.

The main purpose of this investigation is to show another side of weak implementation, meaning that stability programs should be taken seriously, using the basic ideas for calculation stability costs outlined above. For simplicity we focus below on some basic properties for this economy under weak implementation and independent shocks. We believe that, at least for small q, the main results about correlation and population increases apply. In what follows, we want to show how imperfect monitoring affects the level of insurance that can be provided, and the corresponding distortions. With this small change in the model, patient people will not be treated equally.[9]. And, we should keep

[9]We also find unequal treatment of patient people under weak implementation for the conventional

in mind, that if run are possible now (and their existence follows from similar parameter configurations as those seen above) then it is demonstrated that a reasonably small change in the Diamond-Dybvig, following the contributions of Peck and Shell (2003) and EK, can produce cases of insolvency in the sense that a group of people consume zero in equilibrium, thus facing in a very low utility in this scenario.

In the standard model, when actions are fully monitored, this double access assumed here would have no effect on optimal allocations: once date-1 consumption is first transferred to someone, and this person is identified at the second access, the optimum arrangement would trivially give no consumption at the second access. But the situation changes with the imperfect monitoring that we are assuming now. It is true that, under truth-telling, patient people will be lead to consume at date 2, even those with the special abilities. But it is now important to convince the ones with double access to reveal their status freely. Under the current monitoring assumption, the bank is not able to detect this second access and, therefore, must distort allocations if it wants this information revealed.

As mentioned above, we now restrict attention to specifications with independent shocks. The economy can be seen as populated by three types of agents. Type 0 individuals are impatient and have only one access to the bank. A person draws this type with probability $p_0 = p$. Patient agents with single access are designed by type 1. This is drawn with probability $p_1 = (1-p)(1-q)$. Finally, double-access patient people are called type 2. This type occurs with probability $p_2 = (1-q)q$. Since we are interested in computing optimal allocation under truth-telling beliefs, the second access can viewed as just a way for type 2 people to separate themselves from type 1. We shall see that we can leave the issue of double access confined to the introduction of a new truth-telling constraint, which allows us to keep the same structure with N access to the bank as before for much of the accounting that is needed. Accordingly, we set the aggregate state as $\omega \in \{0,1,2\}^N$, which occurs with probability $P(\omega) = p_0^{|\omega|_0} p_1^{|\omega|_1} p_2^{|\omega|_2}$, where $|\omega|_i \equiv \sum_k I_{[\omega_k=i]}$.

A mechanism is (x, y, z), where in addition to impatient date-1 consumption x and date-2 consumption y for type 2, it is introduced date-2 consumption z for type 2. The list (x, y, z) is feasible if

$$\sum_{i=1}^{N} \left(I_{[\omega_i=0]} x_i(\omega^i) + R^{-1} \left[I_{[\omega_i=1]} y_i(\omega) + I_{[\omega_i=2]} z_i(\omega) \right] \right) \leq Y. \tag{3.9}$$

It is incentive-compatible if

$$E\left[\frac{1}{N} \sum_{i=1}^{N} u\left(y_i\left(\omega_{-i}, 1\right)\right) \right] \geq E\left[\frac{1}{N} \sum_{i=1}^{N} u\left(x_i\left(\omega_i^{-}, 0\right)\right) \right] \tag{3.10}$$

Peck-Shell economy when there is persistence and $p \neq \frac{1}{2}$, for the Green-Lin economy with independence and active constraints, and under strong implementation for Peck-Shell economies with persistence.

and

$$E\left[\frac{1}{N}\sum_{i=1}^{N}u\left(z_i\left(\omega_{-i},1\right)\right)\right]\geq E\left[\frac{1}{N}\sum_{i=1}^{N}u\left(x_i\left(\omega_i^-,0\right)+x_{i+1}\left(\omega_i^-,0,0\right)\right)\right], \tag{3.11}$$

where $x_{N+1}(\omega^{N-1},0,0) = 0$. The planner's problem is that of maximizing $U(x,y,z)$, defined as

$$E\left[\frac{1}{N}\sum_{i=1}^{N}\left(I_{[\omega_i=0]}Au\left(x_i\left(\omega_i\right)\right)+I_{[\omega_i=1]}u\left(y_i(\omega)\right)+I_{[\omega_i=0]}u\left(z_i(\omega)\right)\right)\right], \tag{3.12}$$

subject to (3.9), (3.10) and (3.11).

A computational method similar to that described for environment without insolvency can be designed for this economy. The main difference between the two cases is that the deviation payoff for type-2 individuals are not separable among positions [there are two transfers x_i and x_{i+1} inside the utility function on the right-hand side of (3.11)]. In this situation the recursive formulation remains valid, but we are not able to get a close solution for the optimal transfers at each position anymore. We propose to guess a solution by approximating first-order conditions and then to iterate on them in order to achieve a numerical convergence to the true solution. The procedure is outlined in the appendix B.

3.5.1 Numerical findings

We study the existence of pure-strategy run equilibria under the imperfect-monitoring assumption. The following table summarizes the basic results. It presents patient's expected payoff in telling the truth (net of the expected payoff in lying) when he or she believes that all other patient agents (type 1 and type 2) are lying. Parameterization is $N = 5$, $\delta = 2$, $p_0 = 1/2$, $y = 3$, and $R = 1.05$. The table shows the effects of changes in q, the average fraction of insiders among patient individuals, and in A.

A	type	second access probability (q)			
		0.0%	0.5%	1.0%	2.0%
1	1	0.0087	0.0105	0.0123	0.0157
	2	$\cdots$	−0.0299	−0.0296	−0.0290
10	1	−0.0043	−0.0033	−0.0024	−0.0006
	2	$\cdots$	−0.0315	−0.0313	−0.0310

There are eight examples in the table, one for each pair (A, q). Accordingly, first rows (after labels row) refers to a economy in which $A = 1$, and first column (after column *type*) corresponds to iid economy with perfect monitoring (PM) studied in the previous sections. It can be seen that there exists a complete (all patient runs) pure-strategy run only when $A = 10$. Such result is consistent with the PM iid case: there is not insurance

enough to sustain a run equilibrium since A, δ, and N are low. This suggests that run equilibria is as easy to find in IM case as in the PM case. However, the fact that run strategy is always much more attractive for type-2 people is evidence that partial runs (in which only type 2 runs) can exist in economies where perfect monitoring would eliminate it. In this sense, a new source of instability emerges. We have seen that insurance level is the essence of run existence in the PM case. Now, a run would exist in economies with not so high insurance level, but with weaker monitoring capacity.

Increasing q from zero to 2% always decreases willingness to run since the relative payoff in telling the truth increases in all rows when changing columns from the left to the right. The reason is that the more probable is type 2, more distortion is necessary to incentive double-access patient agent to tell the truth. Such distortions reduce insurance level and, therefore, the incentive to run.

3.6 Final remarks

In this chapter, we have taken the Peck-Shell (2003) setup as a benchmark for studying the occurrence of runs, and have identified the important role played by the provision of insurance/liquidity on financial stability, with particular emphasis on the size of the population. We have also presented a stronger version of the limit result of chapter 1 in the following sense. We use a generalization of the no-run-constraint concept due to Peck and Shell (2003) and extend the recursive method due to Ennis and Keister (2009) to incorporate the constraint in the case of homothetic preferences. We then show that runs can be eliminated in the standard model at a low cost when the population is small, and at a negligible cost when the population is increased, without appealing to disclosure of information or taking the limit as the population size grows to infinity as in chapter 1.

Having showed that runs are pervasive, and that the most efficient way to avoid them may require a careful computation of the effects of mixed strategies, we also discuss the effects of persistence on the generation of types along the queue, as well as the effects of insolvency. We find that persistence increases the cost of avoiding runs by more than an order of magnitude, and that for small populations the costs are substantially higher in comparison to the independence case. We then show that a small change in the model may produce insolvency. While the level of liquidly insurance falls down as a result of the need to avoid insolvency, bank fragility can still be generated as before. This possibility suggests that the concept of no-run constraints should be taken seriously in a broad sense.

The recursive method has proven very tractable with active constraints. Its construction highlights a simple and enduring message for all experiments: bank stability requires a high interest rate. This golden-rule kind of principle is in contrast with the discussion presented by Wallace (1988) of the main features of the Diamond-Dybvig model. There it is argued that intermediaries would compete to supply contracts with implicit returns re-

sembling those simulated here under weak implementation. The need to assure uniqueness of equilibrium, which was not addressed by Wallace (1988), now tell us that the interest rates supported by this kind of competition might be too low to convince people to focus on the future, a need strongly emphasized by Thornton among other early thinkers.

Appendix A

The monetary-economy algorithm

Recursive planner problem defined in section 2.4 can be solved using standard Bellman operator if we know set $\Gamma = \{(\theta, r) \in \Theta \times \mathbb{R}^2 : r \in \Gamma(\theta)\}$. However, few information about it is available a priori. We can only assure that

$$\Gamma(\theta) \subseteq \mathbb{C} = \left\{ r \in \mathbb{R}^2_+ : \max_i r_i \leq \frac{u(\bar{y})}{K(1-\beta)} \right\} \quad \text{for all} \quad \theta \in \Theta.$$

Returns cannot be negative since this would incentive either monitored people to deviate to become non-monitored or non-monitored with money to dispose off their monetary holding. Upper bound is given by the expected utility a person would get if she consumes $\bar{y}$ in every period she meets a producer and produces nothing when she meets a consumer.

A.1 The set operator

Inspired in the set operator defined by Cavalcanti and Monteiro (2006) in their recursivity result, we construct a sequence of sets intended to converge to Γ. We initially guess that Γ equals $\Theta \times \mathbb{C}$, i.e., $\Gamma_0(\theta) = \mathbb{C}$ for all $\theta \in \Theta$. Guesses are then updated by eliminating from $\Gamma_i(\theta)$, for each $\theta \in \Theta$, return r such that planner problem is unfeasible when current state is (θ, r) and (θ_+, r') is required to satisfy $\Gamma_i(\theta_+)$. Such operator implies a decreasing sequence of sets whose limit contains only sustainable states.

Computationally, we actually calculate an approximation of such sequence of sets, since a finite subset (grid of points) of $\mathbb{C}$ is used to build the initial guess to Γ. In addition, constraints (2.4) and (2.5) in the planner's recursive problem are replaced by

$$r_m \leq (1/K) \sum_{s'} \theta^{s'} \Big[\Pi^p_m(s') + \Pi^c_m(s') \Big] + \beta r'_m \tag{A.1}$$

$$r_n \leq (1/K) \sum_{s'} \theta^{s'} \Big[\Pi^p_n(s') + \Pi^c_n(s') \Big] + \beta(1-\xi) r'_n. \tag{A.2}$$

That way, we allow planner to pay more return than it has promised to individuals last period. Of course, the allocation derived from the solution to this problem coincides to the optimal allocation only if (2.4) and (2.5) are verified in its path.

A return $r \in \Gamma(\theta)$ is excluded only and only if there is not (y, ξ, r', θ_+) which satisfies $r' \in \Gamma(\theta_+)$, incentive constraints (2.2-2.3), distribution dynamics (2.6) and relaxed Markov constraints (A.1-A.2). The following example shows how we search for such a point. Let $\tilde{r}_m$ denote the rhs of (A.1), $\tilde{r}_m$ the rhs of (A.2), and $\tilde{r} = (\tilde{r}_m, \tilde{r}_n)$. For a given transition $((\theta, r), (\theta_+, r'))$ and inflation ξ satisfying $r' \in \Gamma(\theta_+)$ and (2.6), we search for an implementable y which which satisfies (A.1-A.2). Figure A.1 illustrates this search.

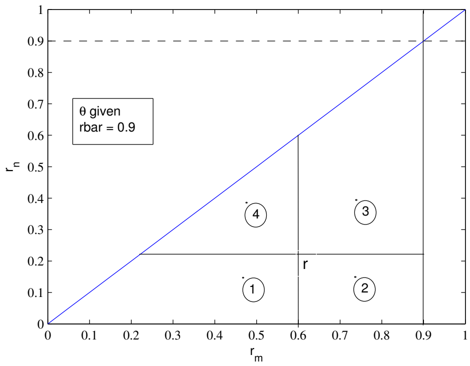

Figure A.1: Feasibility test

- First, we choose an implementable production in order to maximize $\tilde{r}_n$.

If resulting $\tilde{r}$ is in region 1 or 2 indicated in the figure, we are able to conclude that current transition is unfeasible. If implied $\tilde{r}$ is in region 3, then a feasible transition to (θ, r) has been found. In the remaining case (region 4), result is inconclusive and the next step is necessary.

- Second, choose an implementable y to maximize $\tilde{r}_m$ subject to $\tilde{r}_n \geq r_n$.

From previous step, we know that this problem is feasible since we have already found a point in region 4. If maximum is attained in region 4, current transition is unfeasible. Otherwise, it is feasible. After test all possible transitions, we are able to conclude if current state should be excluded or not from current guess.

An attractive feature of this test is that it involves virtually no computational effort, since required optimizations are trivial. For each production, other than $y^{m0,n1}$, monotonic pattern is the same in both $\tilde{r}_m$ and $\tilde{r}_n$. In particular, both returns are maximized when producer constraint binds in meetings $(n0, n1)$ and $(n0, m0)$, and consumer constraint binds in meeting $(m0, n0)$. For the other production, we have that $\tilde{r}_n$ is invariant to production in meeting $(m0, m0)$ and $\tilde{r}_m$ is maximized when $y^{m0,m0} = \min\{y^*, c^{-1}(\beta\partial'_m)\}$. It follows that two-step feasibility test described above resumes to choosing production for meeting $(m0, n1)$.

Because production in meeting $(m0, n1)$ is an expected cost to monitored people and an expected benefit to non-monitored people with money, return $\tilde{r}_n$ is maximized when producer constraint binds, and $\tilde{r}_m$ is maximized when consumer constraint binds. Therefore, the point to be tested in the first step is

$$\begin{bmatrix} y^{01} \\ y^{0m} \\ y^{m0} \\ y^{m1} \\ y^{mm} \end{bmatrix} = \begin{bmatrix} \min\{\bar{y}, \beta(1-\xi)r'_n\} \\ \min\{\bar{y}, \beta(1-\xi)r'_n\} \\ 0 \\ \min\{\bar{y}, \beta r'_m\} \\ \min\{y^*, \beta r'_m\} \end{bmatrix}$$

The point for second step is chosen by decreasing $y^{m0,n1}$ until return constraint for r_n or consumer constraint in this meeting binds, keeping all other production fixed. Simple algebra can be done to show that this production is given by the maximum value between $u^{-1}(\beta(1-\xi)r'_n)$ and

$$u^{-1}\left(\frac{K}{\alpha}r_n - \frac{K-1-\alpha}{\alpha}\beta(1-\xi)r'_n - \sum_{s'\in S}\frac{\theta^{s'}}{\alpha}\left[I_{[s'\neq n0]}y^{n0,s'} + I_{[s'=n0]}u(y^{n0,n1})\right]\right)$$

where $I_{[exp]}$ equals 1 if logical expression exp is true and zero otherwise.

Therefore, feasibility test for each transition consists of logical tests of at most two points. Furthermore, once feasibility of a transition is established, remaining future states do not need to be studied.

A.2 Optimality

Limiting set of the sequence constructed by the set operator described above is used to solve recursive planner problem. It consists of a finite set of points contained in Γ, and is denoted by Γ_∞. Define Bellman operator T which maps function $W : \Gamma_\infty \to \Gamma_\infty$ in function $TW : \Gamma_\infty \to \Gamma_\infty$ given by

$$\max_{(y,r',\xi)}\left\{\frac{1}{K}\sum_{s,s'}\theta^s\theta^{s'}\left[u(y^{ss'}) - y^{ss'}\right] + \beta W(\theta_+, r')\right\}$$

subject to incentive constraints (2.2-2.3), the distribution dynamics (2.6), and $r' \in \Gamma_\infty(\theta_+)$. True value function W^* is the fixed point of T, i.e., $TW^* = W^*$. After value function convergence, we choose initial state (θ_0, r_0) by maximizing $W(\theta, r)$, from which we can use policy function calculated to construct optimal allocation.

Consistent to our strategy to approximate state space, choice over (θ_+, r') is done by grid optimization after an optimal $y(\theta, r)$ is chosen. Fortunately, the choice of production

is not complex. Let $y_p^{ss'}$ denote the maximum production producer is willing to deliver in meeting (s, s'), and $y_c^{ss'}$ denote the minimum consumption consumer is willing to buy in the same meeting. Then optimal production in meeting (ss') for a given pair of lagrange multipliers $\eta = (\eta_m, \eta_n)$ to the Markov constraints (A.1-A.2) equals

$$\max\left\{y_c^{ss'}, \min\left(y_p^{ss'}, y^{ss'}(\eta)\right)\right\} \tag{A.3}$$

where $y^{ss'}(\eta)$ is the production which equals $\frac{1}{u'(y)}$ to the quantity presented in the following table for each meeting,

meeting	$(n0, n1)$	$(n0, m0)$	$(m0, n0)$	$(m0, n1)$	$(m0, m0)$
	$\frac{1+\eta_n/\theta^{n1}}{1-(\eta_n+\eta_m)/\theta^{n0}}$	$\frac{1+\eta_m/\alpha}{1-(\eta_n+\eta_m)/\theta^{n0}}$	$\frac{1-(\eta_n+\eta_m)/\theta^{n0}}{1+\eta_m/\alpha}$	$\frac{1+\eta_n/\theta^{n1}}{1+\eta_m/\alpha}$	1

True lagrange multipliers are found using penalty function method. We use optimal production (A.3), current transition $((\theta, r), (\theta_+, r'))$, and corresponding inflation ξ to calculate the implied return $\tilde{r}$. Multiplier η_m is increased if constraint (A.1) is violated and decreased if it is satisfied. Similarly, η_n is increased if constraint (A.2) is violated and decreased otherwise. Magnitude of such adjustments is given by

$$\eta'_m = \eta_m \exp(r_m - \tilde{r}_m)$$

$$\eta'_n = \eta_n \exp(r_n - \tilde{r}_n).$$

Experience has shown that a good strategy is to first test $\eta = (0, 0)$, then search for one of the multipliers, keeping the other equal to zero, and only after that search for both multipliers at the same time.

Appendix B

The imperfect-monitoring algorithm

As in the perfect monitoring case, we guess the true value for λ, compute optimal solution relative to this guess, and then update multiplier guess in a penalty function fashion. We still calculate relative optimal solution recursively, but now there is an additional procedure in doing so. It is motivated by the non-separability in date 1 consumption in type-2 truth-telling constraint. In what follows, we present how recursive computation of optimal lagrangian value is modified by this non-separability.

First, observe that if we define

$$w_1(x,y) \equiv E\left\{\frac{1}{N}\sum_{i=1}^{N}\left[u\Big(y_i\left(\omega_{-i},1\right)\Big) - u\Big(x_i\left(\omega_i^-,0\right)\Big)\right]\right\}$$

and

$$w_2(x,z) \equiv E\left\{\frac{1}{N}\sum_{i=1}^{N}\left[u\Big(z_i\left(\omega_{-i},2\right)\Big) - u\Big(x_i\left(\omega_i^-,0\right) + x_{i+1}\left(\omega_i^-,0,0\right)\Big)\right\}\right.$$

then the lagrangian in the imperfect monitoring case is

$$\mathcal{L}(x,y,z) = U(x,y,z) + \lambda_1 w_1(x,y) + \lambda_2 w_2(x,z).$$

Proposition 15 *Suppose that we know the set $\theta = \{\theta_1, ..., \theta_N\}$, where $\theta_n(\omega^{n-1})$ is the ratio $x_n(\omega^{n-1},0)/x_{n+1}(\omega^{n-1},0,0)$ at the optimal solution relative to the current guess for λ. Then the optimum value for the lagrangian relative to λ is*

$$\mathcal{L}(\lambda) = \frac{1}{N}u(Y)\left(f_1^{\mathbf{0}}(1)\right)^{\delta}$$

where $f_n^{\boldsymbol{j}}(t)$ is defined as

$$\left\{p_0\left[\frac{(\alpha_n^t)^{1-\delta}[a^\delta - \lambda_2(p_2/p_0)(1+\theta_n^{-1})^{1-\delta}] + f_{n+1}^{\boldsymbol{j}}(0)}{(\alpha_n^t + f_{n+1}^{\boldsymbol{j}}(0))^{1-\delta}}\right] + \sum_{k=1}^{2}p_k\left\{f_{n+1}^{\boldsymbol{j}_k^+}(k)\right\}^{\delta}\right\}^{\frac{1}{\delta}}$$

in which $\alpha \equiv \left(A - \lambda_1 \frac{p_1}{p_0}\right)^{\frac{1}{\delta}}$ *and* $\alpha_n^t \equiv \left(\alpha^\delta - \lambda_2 \frac{p_2}{p_0}(1+\theta_n^{-1})^\delta - \lambda_2 \frac{p_2}{p_0^2} \frac{I_{[t=0]}}{(1+\theta_{n-1})^\delta}\right)^{1/\delta}$, *and the condition*

$$f_{N+1}^{(j_1,j_2)}(\omega) \equiv R^{1/\delta - 1} \sum_k j_k (1+\lambda_k)^{1/\delta}$$

Proof. Collecting terms for date 2 consumptions in history ω, and using equal treatment among type 1 and among type 2, we have that planner must choose how to allocate an amount $a(\omega)R$ between patient types in date 2

$$\max \left\{ \sum_t (1+\lambda_t)|\omega|_t u\left(\frac{\phi_t a(\omega)R}{|\omega|_t}\right); \phi_1 + \phi_2 \leq 1 \right\} =$$
$$u(a(\omega)R) \min \left\{ \sum_t (1+\lambda_t)(|\omega|_t)^\delta (\phi_t)^{1-\delta}; \phi_1 + \phi_2 \leq 1 \right\}$$

where $|\omega|_t = \sum_i I_{[\omega_i = t]}$. The solution is easily seen to be $\phi_t = \frac{|\omega|_t (1+\lambda_t)^{1/\delta}}{\sum_k |\omega|_k (1+\lambda_k)^{1/\delta}}$, which produces optimal value

$$v_{N+1}(\omega) \equiv u(a(\omega)R) \left\{ \sum_k |\omega|_k (1+\lambda_k)^{1/\delta} \right\}^\delta = u(a(\omega)) \left\{ f_{N+1}^{(|\omega|_1, |\omega|_2)} \right\}^\delta$$

Consider planner choice before the last position and after meeting j_1 patient agents of type 1 and j_2 patient agents of type 2. If $a = a(\omega^{N-2}, t)$ denotes the amount in the bank at this moment, then

$$v_N^j(\omega^{N-2}, t) = \max_{x_N} \left\{ p_0 \left[\alpha^\delta u(x_N) - \lambda_2 \frac{p_2}{p_0} u(x_N + x_{N+1}) + \left\{ f_{N+1}^{\boldsymbol{j}} \right\}^\delta u(a - x_N) \right] \right.$$
$$\left. + \sum_k p_k \left[\left\{ f_{N+1}^{\boldsymbol{j}_k^+} \right\}^\delta u(a) \right] - I_{[t=0]} \lambda_2 \frac{p_2}{p_0} V(x_{N-1} + x_N) \right\}$$

where $\boldsymbol{j} = (j_1, j_2)$, $\boldsymbol{j}_1^+ = \boldsymbol{j} + (1,0)$, and $\boldsymbol{j}_2^+ = \boldsymbol{j} + (0,1)$. Observe that the type of the last agent to access the bank, t, is relevant to determine current consumption. The solution satisfies

$$\left(\frac{a}{x_N} - 1\right)^\delta \left[(\alpha_N^1)^\delta - \lambda_2 \frac{p_2}{p_0^2} \frac{I_{[t=0]}}{(x_{N-1}/x_N + 1)^\delta} \right] = (f_{N+1}^{\boldsymbol{j}})^\delta$$

If we use the ratio θ_{N-1}, the solution is $x_N = \left(1 + f_{N+1}^{\boldsymbol{j}} / \alpha_N^t\right)^{-1} a$. Now, plugging these solutions in the objective function we have

$$v_N(\omega^{N-2}, t) = u(a)(f_N^{\boldsymbol{j}}(t))^\delta - I_{[t=0]} \lambda_2 \frac{p_2}{p_0} u(x_{N-1} + x_N^*)$$

Now, consider planner choice before the last two positions. If we define $v_{N-1}(\omega^{N-3}, t)$ as

$$\max_x \left\{ p_0 \left[\alpha^\delta u(x)\right] + \sum_{k=0}^{2} p_k v_N(\omega^{N-2}, k) - I_{[t=0]} \lambda_2 \frac{p_2}{p_0} u(x_{N-2} + x) \right\}$$
$$= \max_x \left\{ p_0 \left[\alpha^\delta u(x) + (f_N^{\boldsymbol{j}}(0))^\delta u(a-x) - \lambda_2 \frac{p_2}{p_0} u(x + x_N^*) \right] + \right.$$
$$\left. + u(a) \sum_{k=1}^{2} p_k \left\{ f_N^{\boldsymbol{j}_k^+}(k) \right\}^\delta - I_{[t=0]} \lambda_2 \frac{p_2}{p_0} u(x_{N-2} + x) \right\}$$

Necessary first-order condition is

$$\left(\frac{\alpha}{x}\right)^\delta - \lambda_2 \frac{p_2}{p_0^2} \frac{I_{[t=0]}}{(x_{N-2} + x)^\delta} - \lambda_2 \frac{p_2}{p_0} \frac{1}{(x + x_N^*)^\delta} = \frac{(f_N^{\boldsymbol{j}}(0))^\delta}{(a-x)^\delta}$$

Using the ratios $(\theta_{N-1}, \theta_{N-2})$ and a_{N-1}^t, we can rewrite it as

$$\left(\frac{\alpha_{N-1}}{x}\right)^\delta - \lambda_2 \frac{p_2}{p_0} \frac{I_{[t=0]}}{(x_{N-2} + x)^\delta} = \frac{(f_N^{\boldsymbol{j}}(0))^\delta}{(a-x)^\delta}$$

whose solution is $x_{N-1} = (1 + f_N^{\boldsymbol{j}}(0)/\alpha_{N-1}^t)^{-1} s$. Plugging this solution in the objective function, we have

$$v_{N-1}(\omega^{N-3}, t) = u(a) \left\{ f_{N-1}^{\boldsymbol{j}}(t) \right\}^\delta - I_{[t=0]} \lambda_2 \frac{p_2}{p_0} u(x_{N-2} + x_{N-1}^*)$$

Similar algebra can be used to get the result

$$v_n(\omega^{n-2}, t) = u(a) \left\{ f_n^{\boldsymbol{j}}(t) \right\}^\delta - I_{[t=0]} \lambda_2 \frac{p_2}{p_0} u(x_{n-1} + x_n^*)$$

for all $n < N - 1$. The last step is to note that $v_1(\emptyset)$ equals the maximum lagrangian value relative to λ. ■

The previous result shows that if we know θ we are able to compute optimal solution relative to λ by iterating object f_n. However, we generally do not know θ. What we do is to is to guess this set. After computing a solution relative to such guess, we verify if the ratios defined by the current solution and this set agree. If not, we use this new set of ratios as a guess for the next step. This procedure is repeated until convergence. After convergence, we evaluate violation in truth-telling constraints and update multipliers if necessary.

For completeness, we report recursive relation on truth-telling constraints. We have

$w_t = \frac{1}{N}u(a)g_1^0(t)$, where $g_1^0(t)$ can be recursively computed using

$$g_n^j(t) = p_0\left[\frac{g_{n+1}^j(t)(f_{n+1}^{\boldsymbol{j}}(0))^{1-\delta} - \Theta_n(t)(\alpha_n^t)^{1-\delta}}{(\alpha_n^t + f_{n+1}^{\boldsymbol{j}}(0))^{1-\delta}}\right] + \sum_{k=1}^{2} p_k\left[g_{n+1}^{j_k^+}(t)\right]$$

and condition $g_{N+1}^j(t) \equiv j_t\left(\frac{R(1+\lambda_t)^{\frac{1}{\delta}}}{\sum_k j_k(1+\lambda_k)^{\frac{1}{\delta}}}\right)^{1-\delta}$, where $\Theta_n(t) = \frac{p_t}{p_0}\left(1 + \frac{I_{[t=2]}}{\theta_n}\right)^{1-\delta}$. This relation can be obtained by just plugging optimal solution in w_t, from the end of the queue to its beginning.

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