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Dedication

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Introduction

The main lesson from the recent experience with auctions in the allocation of goods (e.g., privatization) and licenses (e.g., the spectrum auctions) is that auction design matters. The notion that the specific auction format is not important does not find support in auction theory. Distinct auction formats differ in their ability to raise revenue and to allocate the item to the bidder who values it the most. They also differ in its vulnerability to collusion, entry deterrence and predatory behavior, traditional concerns of competition policy that are amongst the most important issues in the agenda of auction designers.

Auctions were one of the favored sale instruments in the privatization process and major restructuring of infrastructure sectors (such as electricity, gas distribution, and telecommunications) that was caused by the change in the perceived role of government that took place in the last decades of the twentieth century. Auction-like markets were instrumental for the introduction of competition in most restructuring processes. Through auction theory it is possible to select or design a mechanism appropriate to the sale of commodities for which there is no well established markets.

Given the existing knowledge about the different features of standard auction designs, practitioners and theorists alike started focusing their attention on hybrid auction designs that combine features of different auction formats. Klemperer

(2001)[1] for example discusses the Anglo-Dutch auction - a hybrid of the sealed bid and ascending auctions - that may perform better in terms of the traditional concerns of competition policy such as preventing collusive, predatory and entry deterring behavior.

This dissertation consists of three essays on hybrid auctions. In the first chapter we examine the relationship between auction design and some of the main concerns of competition policy. In particular, we postulate that hybrid auctions might be viable alternatives to avoid common problems. We also examine the implementation of hybrid auctions in Brazil, such as the ones used in the sale of Telebrás (the government owned Telecom holding in Brazil), the privatization auctions of Banestado and Banespa and also the recent spectrum auctions.

In the second chapter we examine the revenue properties of one hybrid auction that combines a sealed bid first price auction with an ascending auction. This particular mechanism has been used, for example, in the sale of the companies constituted through the partial division of the Telebras System, and in the privatization of Banestado and Banespa.

This hybrid auction works as follows. Each buyer submits a sealed bid. Once the highest bid is known, the bidder who submitted it is declared the winner if her bid is higher than the second highest bid by more than a predetermined amount or percentage. If at least one more bidder submitted a bid sufficiently

[1] Klemperer, P., (2001), "What Really Matters in Auction Design," Working Paper, Nuffield College, Oxford University. Available at www.nuff.ox.ac.uk/economics/people/klemperer.htm.

close to the highest (that is, if the difference between this bid and the highest bid is smaller than the predetermined amount or percentage) the qualified buyers compete in an open ascending auction that has the highest bid of the first stage as the reserve price. The qualified bidders include the one who tops the first price sealed bid auction and those who bid sufficiently close to her.

We develop a model that captures some of the features of this hybrid auction. We model a situation where risk neutral bidders compete in a two stage auction. The first stage is a sealed bid first price auction. This is followed by a Vickrey auction as a second stage when there are bids sufficiently close to the highest one in the sealed bid auction. We consider a model in which potential buyers' values have both a private and a common component. Of course, special cases include the independent private values model and the pure common values case. For the case of a discrete distribution, we show that the hybrid auction generates more revenue than any standard auction. This conclusion is robust to relaxing the assumptions of risk neutrality and symmetry assumptions. The reason is that one may view this hybrid auction as a Vickrey auction with a reserve price set endogenously at the first stage. Moreover, this hybrid auction is ex-post efficient.

In the third chapter we report the results of an experiment designed to test this proposition for the case of independent private values. We conducted an experimental session composed of three parts, with 45 subjects recruited from undergraduate economic courses. During the experiment the subjects were allocated in groups of three participants according to a predetermined rule. Each

subject participated in 18 auctions. In the first six auctions the subjects bid for a fictitious commodity sold through a standard first price sealed bid auction. In the next twelve auctions, a hybrid Dutch-Vickrey auction was used.

Several conclusions emerged from this experimental study. Firstly, ex-post efficiency was achieved overwhelmingly by the hybrid auctions. Secondly, overbidding (with respect to the risk-neutral Bayesian Nash equilibrium) was a regular feature of individual behavior in first-price auctions. Overbidding, however, was less prominent in hybrid auctions. Finally, we compared the revenue generated by the hybrid auction with that generated by a standard first-price sealed-bid auction and the results were ambiguous.

Chapter 1

1 Hybrid Auctions and Auction Design

The change in the perceived role of the government in the last decades of the twentieth century brought about a worldwide privatization process and major restructuring of infrastructure sectors such as electricity, gas distribution, and telecommunications. Auctions were one of the favored sale instruments in the privatization process and auction-like markets were instrumental for the introduction of competition in most restructuring processes.

The FCC (Federal Communications Commission) spectrum auctions have shown that using an auction to allocate scarce resources is far superior from the prior methods: comparative hearings and lotteries. There is now considerable agreement amongst economists that auctions are the best way to assign scarce spectrum resources (McMillan, 1995). A well designed auction will allocate the licenses to the parties that values them the most, allowing the Treasury to raise revenues in the process. These auctions have fostered innovation and competition in wireless communication services. Additionally, taxpayers, companies and consumers have benefited from the auctions. The FCC adopted an innovative design, improved since them to address particular problems that emerged during its implementation.

Following the US leadership with the FCC spectrum auctions, several countries became interested in designing auctions seeking to best assign spectrum licenses.

Although game theory cannot handle all the complex settings that arise in real economies, the use of insights from auction theory and known properties

of the standard auction formats, like the English and the Dutch among other auctions, made possible to design auction formats to deal with specific concerns. Chiefly among these concerns is the prevention of collusive, predatory and entry deterring behavior. Other concerns include achieving efficiency (for example, by allowing positive synergies to be realized) and maximizing the number of potential competitors.

Trying to deal with concerns arising in the implementation of the designed auctions, new auction formats are being developed, combining distinct standard auctions in the hope to prevent eventual problems. As an example, Klemperer (2001) proposed the Anglo Dutch auction, a combination of an oral ascending auction followed by a first price sealed bid auction. This mechanism was initially proposed and experimentally implemented to the auctioning of the UK spectrum.

In Brazil hybrid auctions have been extensively used by the government in the privatization process and in the spectrum allocation. Examples of this use are the sale of the Telebrás (the telecommunication monopoly holding company) assets, the privatization of some state owned banks, as Bemge, Banespa and Banestado, and the auctioning of the licenses for the Mobile Personal Service (SMP).

Here we examine the relationship between auction design and some of the main concerns of competition policy. We also discuss hybrid auction, which combine distinct standard auction formats proposed as alternatives to avoid common problems. In the fourth section we examine the implementation of hybrid auctions in Brazil, such as the ones used in the sale of Telebrás (the government

owned Telecom holding in Brazil), the privatization auctions of Banestado and Banespa and also the recent spectrum auctions. However it is possible that the design and choice of particular hybrid auctions in Brazil be merely incidental, without particular attention to major concerns of competition policy and its relation with auction design. To our knowledge this design did not take advantages of the worldwide experience. This can be confirmed by the choice of the hybrid auction used in the SMP auctions that have not taken advantages of the ascending mechanisms claimed to be consensually better for spectrum allocations worldwide.

1.1 Auction Design and Competition Policy

We commence with a distinction between auction formats. In the context of auctions of a single object it is possible to distinguish between sealed bid and oral auctions. The most usual sealed bid auctions are the first price sealed bid auction (when the bidder with the highest bid wins and pays her bid), and the Vickrey or second price sealed bid auction (when the participant with the highest bid wins but pays the second highest bid). The most common form of oral auction is the English or oral ascending auction, in which the auctioneer raises prices monotonically till just one bidder remains. In this case the winner is the bidder who last announced her intention to keep on bidding and the price is the last announced bid. Another common oral auction is the Dutch or oral descending

auction. In this auction the auctioneer decreases prices till one bidder announces her intention to take the object at the current price.

We now discuss the relationship between auction design and some of the traditional concerns of competition policy, such as collusion, entry deterrence and predatory behavior.

1.1.1 Collusion

It is now well known[2] that multiple unit ascending and uniform price auctions[3] may be more susceptible to collusive behavior than discriminatory auctions. Even when explicit collusion does not take place, tacit collusion frequently does. In a multi unit ascending auction bidders can use early stages, when prices are still low, to signal their preferences concerning the desirable allocation. As a result, the auction may end earlier, with price levels lower than the ones that would be reached without collusion: once the preferred allocation is reached, bidders stop pushing prices up. In turn, the same coordination may not be easily achieved in a simultaneous sealed bid auction. Once bidders have to make simultaneously a best and final offer, there is no opportunity to signal other players.[4]

Cramton and Schwarz (2000) investigate the extent to which collusive be-

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[2]See Klemperer (2001).

[3]Common formats of simultaneous auctions are uniform and discriminatory price auctions. In the uniform price auction all items are sold at the (same) price that equates supply and demand. In a discriminatory auction bidders pay the amount they bid for the quantity won. In sequential auctions units are sold sequentially. For further references about multiple unit auctions see Klemperer (1999).

[4]See, for example, Menezes (1996).

havior occurred in the FCC (Federal Communications Commission) auction for broadband frequency blocks D, E and F.[5] They identify two signaling techniques: code bidding (when one bidder encodes a meaningful market number in the trailing digits of its bid); and reflexive code bidding (when this code is used together with a punishing code bid).[6] Other types of signaling techniques include retaliating bid (punishing bid without a code), "jump bidding"(discontinuously raising the bidding level to intimidate opponents into quitting the auction) and withdrawals from the auction of an object desired by an opponent as an incentive to get the rival quit competing on another (desired) market.

Cramton and Schwarz (2000) find evidence that bidders who actually used punishment strategies (by their algorithm's criteria) achieved favorable prices relative to bidders who did not use signaling. They conclude that the simultaneous ascending auction is vulnerable to revenue reducing strategies when competition is weak. Even if code bidding is prevented by the auction rules, retaliating bids may be as effective as code bids in getting the prices low relative to what would be expected in the absence of collusive behavior. In turn, in a sealed bid auction there is no opportunity to retaliate.

In uniform price auctions, in which agents bid demand functions for multiple

[5] The FCC auction was a simultaneous ascending auction. For details see Milgrom (2000) and McAfee and McMillan (1996).

[6] Cramton and Schwarz (2000) report one example of collusive bidding in the FCC spectrum auctions. U.S.West was bidding aggressively against McLeod for Rochester (license 378). Although bids usually ended in rounded million dollars, U.S.West bid U$313.378 for Waterloo, IA, in round 59, and U$62.378 for Marshaltown, IA, in round 64 – two licences for which McLeod was the standing high bidder – apparently punishing McLeod for bidding in Rochester, MN. Then McLeod withdrew from Rochester.

units of a homogeneous good (such as Treasury bills or electricity), the price is set by the lowest winning bid. In this case, bidders may tacitly agree to share the market and bid functions can be used as costless threats that only set prices if at least one bidder deviates from an implicitly agreed market division. The proposed market division can be reached by each player bidding very aggressively for a market share smaller than the one agreed.

Back and Zender (1993) characterize uniform-price auction equilibria that can be supported as collusive behavior. In their setting a single seller auctions a quantity Q of a perfectly divisible good to n risk neutral bidders with heterogeneous information concerning the actual value of the good. A comparison between the uniform price auction and the discriminatory auction shows that the former may be more susceptible to collusion, implying a lower revenue to the seller. There are equilibria of the uniform price auction such that the seller could do better by fixing the price of the good at the lowest possible signal of the bidders and discarding the auction.[7] In turn, discriminatory auctions are not sensitive to such agreements, once each bidder pays her bid for the quantity won.

Besides tacit collusion, illegal behavior is also a concern. An extreme situation is when just one bidder needs to be withdrawn to end the auction. In this case

[7] Wilson (1979) analyses share auctions – auctions such that each bidder submits a schedule specifying a price for each possible fraction of the item. In the model there are collusive equilibria in the uniform price auction, meaning that the resulting prices are much lower than the prices that would result if the item is sold undivided. The intuition is that bidders can implicitly agree to share the item at a lower price by bidding aggressively for a quantity smaller tan the equilibrium share. In this sense other bidders may be deterred from entry.

an auctioneer's threaten to punish an illegal action may not be credible. [8]

The determination of a reserve price is also a critical instrument the auctioneer can use. An inadequately set reserve price can facilitate collusion by increasing the potential gains from a tacit agreement. A lower reserve price can induce an advantaged bidder (a bidder with a high valuation or a better informed bidder) to collude instead of pushing prices up in the hope of driving weak bidders out of the auction. As a result, the auctioneer's revenue may be severely depressed. However when some factors prevent effective competition, an adequate reserve price may avoid a complete failure of an auction poorly designed.

1.1.2 Entry Deterrence and Predatory Behavior

The second major concern of competition policy in auction design is to attract bidders. An auction with few bidders may not be profitable for the seller and is potentially inefficient.

Bullow and Klemperer (1996) stress the value of attracting additional bidders relative to other concerns in auction design. Under reasonable assumptions it is shown that an auction with no reserve price but with additional bidders is

[8] Klemperer (2001) reports the example of the August 2000 3G spectrum auction in Germany. In the German 3G auction twelve blocks of the spectrum were offered. Competitors could aggregate the blocks in (six) licenses of two blocks or (four) licenses of three large blocks. Seven bidders participated in the simultaneous auction. Among them, Debitel was perceived as a weak bidder. MobilCom let Debitel know by the media that it would allow Debitel to become a virtual network operator using MobilCom's network without having to pay an up-front license fee (in case Debitel fail to win a license in the auction). This worked as a side-payment offer for Debitel leaving the auction. But the government did not punish MobilCom, once it could imply ending the auction at relatively low prices (comparing to the price reached). Although Debitel did not quit early, it did withdraw when prices were still low.

preferable to the seller when signals are independent. Under some restrictions this conclusion extends for affiliated signals.[9] This result suggests that the value of negotiating skills is small relative to the value of additional competition; that is, it will often be preferable for a seller to devote resources to expand the market rather than collecting the information required to design the best mechanism.

Distinct auction formats show different properties in terms of efficiency and revenue maximization. Ascending auctions can allow some bidders to deter entry or depress the rivals' bidding, since the presumption that a strong bidder will win an ascending auction inhibits entry, specially if there are positive costs of bidding. In a first price sealed bid auction the outcome is less certain. Once each bidder is restricted to a single and final offer, even an advantaged bidder will not follow the same strategy used in an ascending auction. Because it wants to profit through her bid, she will not bid her value to the item. Foreseeing a chance to win, potential entrants are more willing to enter a sealed bid auction than an ascending auction.[10]

In the common values model, asymmetries may imply that in oral ascending auctions the entry is made more difficult when compared to sealed bid auctions. When bidders have the same value for the prize, but different information about

[9] Let z and z' be points in $\Re^{m+n}$and $z \vee z'$ and $z \wedge z'$ denote the component wise maximum and minimum of z and z', respectively. Two variables are said to be affiliated if for all z and z' $f(z \wedge z') f(z \wedge z') \geq f(z) f(z')$. For auction theory in the context of affiliated values see Milgrom and Weber (1982).

[10] Vickrey (1961) offers a simple example in which a bidder who would surely loose an ascending auction makes positive expected profit in a first price auction.

it, each player must bid cautiously, considering that her probability of winning is higher the more she overestimates the object's actual value. Beating a rival that is perceived as advantaged reinforces this belief, inducing a bidding behavior even more cautious. Knowing this, a strong (or better informed) bidder does not have to be so careful, since winning the auction does not necessarily imply that the real value is overestimated. But then weak bidders have further incentives to depress their bid, since beating the advantaged bidder means her signal was extremely bad. Then weak players bid very conservatively if they bid at all. As a consequence of the reduced entry, the strong bidder is more likely to pay a low price in the ascending auction.

In an asymmetric first price sealed bid auction the winner's curse problem is not so severe. As the strong bidder wants to profit she will not bid the maximum she is willing to pay. This creates incentives to a disadvantaged bidder to participate and also to bid more aggressively. As a consequence the seller can obtain higher prices even for a given number of bidders. Additionally, comparing to the oral ascending auction one can observe a positive effect on the entry in the contest.

Bulow and Klemperer (1999) model a situation where partial ownership of a takeover target – a "toehold" – induces an aggressive bidding behavior in an ascending auction since offers are at the same time bids for the remaining shares of the company and asks for the bidder's own holdings. In a common value model with asymmetric toeholds the owner of the large toehold bids more aggressively

in an ascending auction, causing the smaller toeholder to behave in a more conservative way on average. Since the price is determined by the lower of two bids (probably submitted by the owner of the smaller toehold) prices in an ascending auction will be lower on average relative to first price sealed bid auctions. Bulow and Klemperer suggest that the sale price may be increased by allowing a second bidder to buy a second toehold on favorable terms or by running a sealed bid auction.

In sum, considering that outcomes in ascending auctions are so influenced even by small perceived advantages, there are incentives for a bidder to invest resources to create a reputation for being strong, what can restrain entry and bidders' participation. This effect is particularly important when there is only one bidder who needs to be withdrawn from the auction. Klemperer (2001) reports one episode from the July 2000 3G spectrum auction in Netherlands. One bidder (Telfort) threaten the other (Versatel). Although Versatel complained to the government, no action was taken. If it were, the auction could have ended immediately.

Even though predatory behavior of this sort can develop in any auction format, ascending auctions are more vulnerable since the predator can assess the success of her strategy during the contest. In ascending auctions the belief that the object goes to the high value bidder with certainty blocks weak bidders' entry encouraging a joint bidding that would not take place in first price sealed bid auctions. In a first price sealed bid auction such behavior is not adequate. It may

simply attract other firms in the hope of beating the consortium. Thus, strong bidders are induced to bid independently, what stimulates competition.

Discriminatory price auctions, however, are not flawless. Because bidders have to pay their own bids, the information requirements concerning rivals' costs are larger in order to bid properly. In this context smaller bidders may find it not worthy incurring the costs to obtain this information. As a result, they may be deterred from entering the auction. Even if it is possible to pay larger intermediaries to aggregate the necessary information, a bidder who has good information only about her own value may prefer an ascending or a uniform price auction (where bidding one's value is the best strategy and payments are determined by the non winning bidders).

In sum, the main concerns in auction design are related to monopsony and oligopsony power: auction design is not that important when there are a lot of potential bidders for whom entry is easy.

1.2 Hybrid Auctions

Both auction formats have desirable and undesirable properties. Ascending auctions are more susceptible to collusive behavior, but have the desirable property of allocative efficiency.[11] A high value bidder always have the opportunity to top a bidder with a lower value who may have bid aggressively in early stages. Also in

[11] Remember that bidding one's value is an equilibrium strategy either in the private values or in the commom values model.

the multi unit context, when the objects are complementaries ascending formats are more likely to allow an efficient aggregation of the multiple commodities. [12] If collusion and predation are absent and information is affiliated, an ascending auction allows bidders to learn about each other's valuations in the process of bidding.

Hybrid auction formats are then becoming increasingly popular as allocation mechanisms as a way to combine desirable properties of different auction formats.

1.2.1 The Anglo-Dutch Auction

Klemperer (2001) proposes an Anglo Dutch auction that works as follows. Assume, for simplicity, that a single object is to be auctioned. In the Anglo Dutch auction the auctioneer begins by running an ascending auction, raising the price continuously till only two bidders remain. The remaining bidders then dispute the object in a first price sealed bid auction, through a best and final offer no lower than the current asking price. The winner is the bidder who bid higher and she pays her bid.

Klemperer (2001) argues that this mechanism is particularly suitable to situations in which one bidder is thought to be stronger then her rivals.[13] Although

[12] Cramton, Gibbons and Klemperer (1987) demonstrate that allowing resale does not substitute for an efficient initial allocation, once resale is generally inneficient. Under incomplete information no trading mechanism can guarantee that an object to be traded will be allocated to the person who values it the most.

[13] Examples are the incumbent operator of a license to be reauctioned and the SMP spectrum auction in Brazil, where incumbent operators were allowed to dispute licenses for portions of the spectrum.

the incumbent bidder may be perceived as a winner, the early stages of the ascending auction may work towards increasing the price of the commodity, once entry is not discouraged. The incumbent's rivals may perceive the final sealed stage as a chance to outbid the advantaged bidder. This works as an incentive to entry. Also, the two finalists have a chance to learn something about each other's valuation during the ascending auction of the first stage.

By eliminating the final stage of the ascending auction, the Anglo Dutch auction eliminates the stage vulnerable to collusive and predatory behavior. The incentives to form consortia are also reduced, once this would only attract new firms seeking to beat the consortium in the final stage.

Klemperer (2001) argues that the Anglo Dutch would do the most in terms of making bidders comfortable with their own valuation in the ascending stage, implying increased revenue when information is affiliated, and raising revenue in the first price sealed bid auction relative to what would be expected from a pure ascending auction (due to the effects of risk aversion, budget constraints and asymmetries). [14]

Klemperer (2001) conjectures that all these benefits of the Anglo Dutch auction would apply even if no further entry occur in the bidding. The conjectures extend to a multi unit object context when it would also help to avoid tacit collusion (relatively to the pure ascending auction), once the sealed bid stage allows

[14]Under risk aversion and some types of asymmetries the sealed bid auction increases the seller's revenue relative to ascending auctions. For details in the independent private values setting see Maskin and Riley (1985) and (2000).

bidders to renege on tacit deals without fear retaliation. Additionally, under complementarity the ascending stage would facilitate an efficient aggregation of bundles.

1.2.2 The Dutch Anglo Auction

Hybrid auction mechanisms have been used extensively in the Brazilian privatization process and in the radio spectrum auction.[15] One particular hybrid auction mechanism was used in the sale of the companies constituted through the partial division of the Telebras System - the government-owned Telecom holding in Brazil. This mechanism combines a sealed bid auction and an oral ascending auction but in an order reverse to the Anglo-Dutch auction proposed by Klemperer (2001).

In the Dutch Anglo auction each bidder submits a sealed bid. Once the highest bid is known, the bidder who submitted it is declared the winner if her bid is higher than the second highest bid by more than a predetermined amount or percentage. If at least one more bidder bids close enough to the highest bid (that is, if the difference between this bid and the highest bid is smaller than a predetermined amount or percentage) the qualified bidders dispute the good in an ascending auction that has the highest bid of the sealed bid auction as a reserve price. The qualified bidders are the one who top the first price sealed bid

[15] The next section discusses some apllications of hybrid auction mechanisms.

auction and those who bid sufficiently close to her.

The first price auction of the first stage of this auction mechanism creates incentives for the entry of potential bidders.[16] As the ascending auction is contingent, a weak bidder may follow an aggressive bidding behavior, trying to top an advantaged bidder in the first price sealed bid auction of the first stage. Considering this, the strong bidder have incentives not to shade (bid under the expected value for the object) her bid too much. As a result, the reserve price of the contingent second stage is increased relative to what one would have in a pure ascending auction.

In sum, the first price sealed bid auction of the first stage prevents collusive and predatory behavior. By using a second price auction the Dutch Anglo auction retains the desirable allocative efficiency property.

One could argue, in line with Klemperer's (2001) reasoning, that the last stage of the ascending auction, when only two bidders remain and that is more susceptible to collusive behavior, is preserved. But the reserve price that will emerge from the first price auction may be higher, compensating the tendency for reduced revenue for the seller as a result of collusion.

In the second chapter we develop a model that captures some properties of this hybrid auction format, modelling a situation where bidders dispute a single object in a two stage auction. In the contingent second stage if there are bids

[16]The desire to profit and the belief she will eventually win induces an advantaged player not to bid as high as in a pure ascending auction. This preserves low value bidders' incentives to entry.

sufficiently close to the highest bid, that is, if the difference between the highest bid and these bid is smaller than a predetermined amount z, the single object is allocated through a Vickrey auction.

In a model in which potential bidders have private values or even a common value for the single object we show that this auction format can generate higher revenue to the seller than any standard auction mechanism. The reason, we believe, is that this auction can be seen as a Vickrey auction with an endogenously set reserve price. Additionally, contrary to the optimal auction[17], this mechanism is ex post efficient.[18] In the third chapter we report the results of an experiment designed to examine the properties of the Dutch Vickrey auction.

1.2.3 Other Hybrid Auctions

To our knowledge the only other theoretical analysis of a hybrid auction mechanism is in Perry, Wolfstetter and Zamir (2000). Their model of a hybrid auction rationalizes the idea of two round auctions in the context of the symmetric affiliated values model. In the model all risk neutral agents simultaneously submit sealed bids in the first stage. The two highest bidders them dispute the object in a second round through a second price auction where each one is bounded below by her first round bid. In round 2 the auctioneer publicly announces the bids rejected in round 1 and no selected bidder is allowed to withdraw. The rel-

[17] For refferences see Myerson (1981) and Riley (1981).

[18] For a definition of ex post efficiency see Cramton, Gibbons and Klemperer (1987).

evant equilibrium concept in the model is that of perfect Bayesian equilibrium. Compared to the English (oral ascending) auction, the model implies reduced opportunity to bidders signaling via bids, yielding as much expected revenue to the seller as the English auction,[19] than in the affiliated values setting implies higher expected revenue to the seller that the second price sealed bid auction an the first private values auction.[20] [21]

Perry, Wolfstetter and Zamir (2000) claim that two stage auctions has been used frequently in takeover and merger acquisition procedures. One argument in favor of this choice is the high cost of bidding and the vulnerability of open ascending auction to collusive behavior.

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[19]Recall that with only two bidders the second price sealed bid auction and the oral ascending (English) auction are equivalent even with common values and affiliation. In the open auction the player withdraws once the price reaches her value for the item conditional on the other bidder having the same signal; in the sealed bid version a player bids her value conditional on the other player having the same signal.

[20]For the ranking of the standard auctions in terms of expected revenue when values are affiliated see Milgrom and Weber (1982).

[21]By restricting participation in the second round to only two bidders, Perry, Wolfstetter and Zamir(2000) are able to show that the unique symmetric strict equilibrium is payoff equivalent to the symmetric equilibrium of the English auction. In fact the English button auction (Milgrom and Weber, 1982) amounts to an ascending auction where the two last remaining bidders dispute the object in a Vickrey auction. Then the hybrid auction mechanism analysed by Perry, Wolfstetter and Zamir (2000) mimics the English auction in Milgrom and Weber (1982).

1.3 Hybrid Auctions in Brazil

1.3.1 The Telebras Auction

As a key part of the restructuring of the Brazilian telecommunications system, the telecommunication monopoly holding company, Telebras, was privatized on July 29, 1998. The Telebras system, previously composed of 27 operating subsidiaries, had been the sole telecommunications provider in Brazil since 1972.

On May 22, 1998 a shareholders meeting approved the partial breakup of the company. The resulting 12 companies, three fixed line operators, eight cellular and one long-distance carriers, were auctioned in three groups according to a pre-determined order.[22] The qualified bidders[23] submitted bids in sealed envelopes. All bids for the same group were delivered together, color coded. When the top bids for a certain asset differed by 5% or less, the bidders who submit these bids were invited to an open outcry. The winners were the bidders who offered the highest premium over the minimum price.

Even though an investor could bid for more than one asset per group, she was not allowed to acquire relevant participation in more than one company of the same group.[24]

[22] The companies were arranged into three groups, and sold in the following order:
Group A: Telesp, Tele Centro-Sul, Tele Norte-Leste, Embratel; Group B: Telesp Celular, Tele Sudeste Celular, Telemig Celular, Tele Celular Sul; Group C: Tele Nordeste Celular, Tele Leste Celular, Tele Centro-Norte Celular, Tele-Norte Celular.

[23] Pre-identified investors, reunited in consortium or not, that met the eligibility requirements of technical capability, managerial experience and regularity of fiscal situation.

[24] Relevant participation was defined as direct or indirect control over 20% of the voting capital of the asset being sold.

	B-Band Auction				A-Band Auction			
Area	Minimum Price	Winners	Bids	Premium	Minimum Price	Winners	Bids	Premium
1	R$600	BCP	R$2,647	441%	R$1,100	Portugal Tel	R$3,588	226%
2	R$600	Tess	R$1,326	221%	N/A	N/A	N/A	N/A
3	R$500	Algar	R$1,508	302%	R$570	TISA	R$1,360	139%
4	R$400	Vicunha	R$520	130%	R$230	Telpart	R$756	229%
5	R$330	Global	R$773	235%	R$230	UGB/Intel	R$700	204%
6	R$330	Telet	R$334	101%	N/A	N/A	N/A	N/A
7	R$270	Americel	R$338	103%	R$230	BID/Splice	R$442	92%
8	N/A	N/A	N/A	N/A	R$90	Telpart	R$188	109%
9	R$230	Vicunha	R$250	109%	R$125	Iberdrola	R$428	242%
10	R$230	BSE	R$555	242%	R$225	Globopar	R$660	193%

Notes: (1) Areas one and two, for which two separate license were sold in the B-Band auctions, were sold as one area in the A-Band auction.

(2) Area 6 was not included in the Telebras auction, since the newly privatized CRT operates the A-Band.

(3) Amounts expressed in Million Reais.

Figure 1: Mirror Licenses(B-Band) and Wireless Companies(A-Band) Auctions.

In preparation for the auction, data rooms were opened to investors interested in the Telebras assets. International operators were eventually represented. Most of the Telebras bidders submit bids through consortia, and most of the winners are groups made up of a mix of local and foreign partners, these ones generally being the majority stake holders.

The Telebras auction raised revenue that amounts to a 63.74% premium over minimum bid price, significantly higher than the 40% forecasted by market analysts. However in general the premium in the A band auction was significantly lower than the one in the B band auction, the mirror licenses for cellular operators.[25] Table 1 and Figure 1 presents the results of the auction.

[25] Considering that the incumbent in the market generally retains a market share between 50 and 55% of the market even after entry of new operators (*Valor Econômico*, February 15. 2001), one would expect that prices in the A band would be higher than the corresponding licenses in the B band. .

Company	Minimum Price	Winners	Bids	Premium
Telesp	R$3,520	Telefonica S/A	R5,783	64.29%
Tele Centro Sul	R$1,950	Telecom Italia	R2,070	6.15%
Tele Norte Leste	R$3,400	Andrade Gutierrez	R3,454	1%
Embratel	R$1,800	MCI	R2,650	47.22%

Table 1: Telebrás Auction Results: Fixed Line Operators and Embratel

Even though the auction was considered successful in terms of raising revenue, its efficiency remains to be investigated. Raising revenue was not the sole concern of Brazilian government in the Telebras auction. In fact, it may be argued that it was not the major concern. The ranking of the fixed line operators, the most attractive companies, in the same group reflects a government intention to avoid cream skimming.[26] The breakup of the fixed-line business in three companies is one of the main points of criticism once the acquisition of each company required large amounts of capital. A larger number of companies responsible for smaller geographic extension would have reduced capital requirements inducing more competition and increasing prices.

Efficiency requires that no bidder regrets winning. However in the Telebras auction some investors regret winning. The grouping of companies influenced the results in the auction once every time an investor won a company, her bids to the remaining companies in the same group were destroyed without being revealed. This created a problem because some investors ended up winning a company they did not really want by a price significantly lower.

[26]See Laffont and Tirole (2000) for an analysis of cream skimming in the telecommunications sector.

There are potential problems that stand in the way of an efficient allocation. The efficiency results from single unit auctions do not carry forward to multiple unit settings. There is a consensus in the auction literature that in multiple unit auction settings efficiency is not trivially obtained.[27] Two phenomena limit efficiency and revenue in multiple unit auctions: demand reduction[28] and collusive bidding.

In an ascending auction for a single object a bidder has a dominant strategy: keep on bidding until bids reach her private value. The object is then awarded to the bidder who values it most. On the contrary, if two identical items are sold in a simultaneous ascending action, a bidder has an incentive to withdraw from the auction before its marginal value is reached. If she kept on bidding, she would raise the price for the first object. As a consequence, a bidder who wants just one object may bump a bidder with the highest value for the second item.[29]

Ausubel and Cramton (1998) show that in multiple unit uniform-price auction typically every equilibrium is inefficient.[30] Small bidders tend to inefficiently win

[27] Ausubel (1997) proposes a new ascending auction that is efficient. However, it may imply an equilibrium with lower revenue to the seller.

[28] Cramton (2001) defines demand reduction as a tendency for a bidder to reduce demand knowing that this will reduce prices.

[29] Cramton (2001) provides an illustration of the increased incentives for demand reduction and collusive bidding in an ascending version of a uniform-price auction. The example shows that in settings with few bidders and few objects bidders have incentives to reduce demand, a phenomenon observed in the FCC auction. See Cramton (2001) for details.

[30] Cramton (2001) contains an example with two risk neutral bidders and two identical goods where each bidder draws her marginal value from a uniform distribution. In the sealed bid uniform-price auction there are two equilibria: a sincere fully revealing one and another equilibrium demand reducing and inefficient. But in the ascending version of the auction only the demand reducing equilibrium is obtained.

licenses that would be won by large value bidders. The incentive to shade bids increases with the quantity demanded. Thus large bidders tend to shade more their values.

Simultaneous ascending auctions, in turn, may render higher revenues by revealing information in the auction process, reducing bidder's uncertainty, inducing a more aggressive behavior. However they have the disadvantage of inducing collusive behavior.

Sequential auctions limit informations available to bidders and limit the way bidders can respond to information. If objects are interdependent, incorrect guesses about future bids may result in an inefficient assignment. As a bidder cannot switch back to an earlier bundle of objects, a lot of strategies are eliminated. Bidders may regret either their purchases or the prices or both. Strategies are more complex considering the great deal of guesses about future auction, what tends to reduce the efficiency of sequential auctions.

Considering the difficulty in achieving an efficient result in the multiple units context, one question remains. Did the Dutch-Anglo auction imply greater revenue and improved efficiency in the Telebras privatization? Two of the twelve companies required an open outcry bid. In both cases the winner of the auction was not the bidder who top the first price sealed bid auction of the first stage. In this sense one can argue that the Dutch Anglo auction restored efficiency and that the first stage may have contributed to increase revenue, working as a reserve price setting mechanism in the English auction of the second stage. But

assessments like this need further investigation.

Several conclusions can be reached. Firstly, the order of the auction affected results, which indicated that efficiency was not achieved. Second, revenues were higher than initially forecasted, but considering the auctioning of the mirror licenses in the SMC they were not as high as one would expect considering also that the assets being sold were from incumbent firms. In general these are the firms who keep a greater market share even after some entry in the market.

It is worth noting that the Telebras auction is a multiple unit auction where items are heterogeneous. The auctions were sequential and not independent. Bidders were asymmetric with a private value as well as a common value to the object and also with different budget constraints. They did not have single demands and also there were probably positive synergies[31] involved. In sum, it is too complex a setting.

The analysis of the privatization auctions in the banking sector is probably a more appropriate example of the Dutch- Anglo auction mechanism that is the focus of the present thesis, once these are examples of single object auctions. The two next subsections report the results of two privatization auctions in the banking sector: the Banestado and Banespa auctions.

[31]Menezes and Monteiro (1999) define positive synergies as a situation where an individual's value for a bundle of objects is higher than the sum of the objects' values when considered separately.

Company	Bids (Millions)	Open Outcry Bids	Premium
Itaú	$R\$1,515$	$R\$1,625$	303%
Unibanco	$R\$1,251$	$R\$1,580$	
Bradesco	$R\$710$	–	

Table 2: Results of the Banestado Auction

1.3.2 The Banestado auction

Banestado was privatized in October 2000 through a Dutch Anglo auction in which the contingent second stage would take place if there were bids smaller than the highest bid by 20% or less.[32]

The three largest private banks in Brazil, Bradesco, Itaú and Unibanco, bid in the first price auction of the first stage. As the difference between the two highest bids was smaller than 20% (see Table 2) the allocation was decided through an English auction where Itaú bumped Unibanco. The final bid amounts to a 303% premium over the minimum price of $R\$403,417$ million reais.

The acquisition of Banestado gave Itaú a better position in the ranking. The auction mechanism succeeded in raising revenue (far above the minimum price) and attracting bidders. For Itaú there was a private value component[33] associated with its strategic position besides the common component (the resale value of the firm).

[32]A Dutch Anglo auction was also used in the privatization of Bemge. There was not open outcry bids, but the auction was considered a success. (*Radiobrás*, September 15.1998, captured at http://www.radiobras.gov.br/anteriores/1998/sinopses_1509.htm).

[33]Expected profits differ because of fiscal credits, synergies involved in the operation and market share.

1.3.3 The Banespa Auction

In November of 2000 Banespa, a state owned bank, was privatized. The allocation mechanism used was a Dutch-Anglo auction. The contingent second stage would occur if there were bids smaller than the highest bid by 20% or less. Bradesco and Itaú, the two largest private banks in Brazil, were perceived as the two most interested players. For Itaú buying Banespa would imply reversing its position in the ranking of Brazilian private banks.

At first nine bidders were expected, but only six confirmed participation in the auction: Bradesco, Itaú, Unibanco, Safra and two foreign institutions, the Spanish Santander and the English HSBC. From Bradesco and Itaú's perspective, winning the auction was not so critical as loosing it to each other, once the winner would be the leader of the ranking. But winning had some disadvantages: both institutions already had a good penetration in Banespa's business area of concentration, São Paulo. Also, the large amount of money involved would lower shareholders' profits.

What was not realized at first was the great incentives Santander had in the dispute. Buying Banespa would give it leadership in Latin America ranking, anticipating a goal to be achieved by the year 2003. Also for Santander the superposition problem was absent; on the contrary, there was a desire to expand business in São Paulo.

The auction ended in the first stage, with no open outcry bids. Santander

Bidder	Bids (Millions)	Premium
Santander	$R\$7,050$	285%
Unibanco	$R\$2,010$	
Bradesco	$R\$1,860$	

Table 3: Results of the Banespa Auction

won the auction with a bid 2.83 times larger than the minimum price. At first the other competitors said Santander had bid too high, considering that the second highest submitted[34] bid amounts to a 8.65% premium over the minimum price of $R\$1.85$ billion reais. (See Table 3).

One can argue that such a high bid was submitted as a (successful) trial to end the auction in the first stage. Realizing the interests involved and the importance of Banespa to Itaú and Bradesco, Santander updated its information considering the price resulting from the Banestado auction. Santander gave Banespa a strategic value much higher than expected, trying to increase its market share.

It can be argued that in the Banestado and Banespa's auctions the Dutch-Anglo auction achieved efficiency (allocated the object to the high value bidder) succeeding also in raising revenue. This conforms to the Chapter 2 analysis of a Dutch-Vickrey auction, an auction model that captures some of the features of this hybrid auction.

In an electronic version of a first price sealed bid auction Bradesco would

[34]When Bradesco realized that Itau would not show up, it changed envelopes, probably choosing a smaller bid. In the SMP auction, by its turn, all bidders are obliged to hand the envelopes. A qualified bidder who had decided not to participate would submit an empty envelope.

not have know that Itaú was not bidding, what could have induced Bradesco to bid higher. In particular, if winning for Bradesco when Itaú was competing had a value larger than Santander's actual bid, then the revenue would have been higher. Similarly, the use of an ascending auction could also result in a different revenue: Itaú and Bradesco could have dropped out from the auction earlier on resulting in having Santander to pay the runner-up's bid, that was probably lower than Santander's actual bid. The argument here is again that the auction design may matter.

1.3.4 The SMP Auction

In the beginning of the year 2001 Anatel, the Brazilian telecommunications regulator, held auctions to allocate nine licenses for the Mobile Personal Service (SMP) auction which launched the new C, D and E bands for mobile telephony. The SMP, which will replace the Mobile Cellular Service (SMC) will operate in bands C, D and E at the 1.8 GHz frequency. The Agency tried not only to allocate new portions of the radiospectrum, but to redefine the regulatory setting in the telecommunications sector through the auctioning of these licenses.

The allocation mechanism used works as follow. Bidders submit sealed bids for all the licenses in the same day, although the auctions happen in different dates. In the day of the auction, the sealed bids are open and revealed. If at least one more bidder submit a bid that is equal or higher than 80% of the highest

price bid, then the qualified bidders are asked to submit replacement bids. The qualified bidders are the player who bid highest and those who bid close enough to her. Even if no player bid sufficiently close to the highest bidder, the two highest bidders are asked to submit replacement bids no lower than the top one added by 5%. This process is repeated until all but one bidder decline from the right to place a substitutive proposal. The remaining bidder wins and pays a price equal to the standing high bid.

Several rules restrained participation in the auctions. Fixed-line operators are not allowed in the C band. The SMC operators are induced to migrate to the SMP. Those who do will win the right to own an additional license to operate as long distance carriers. But to migrate new contracts must be signed, changing the regime from a concession (valid up to 2005 for the A band and 2012 for the B band) to an authorization regime.[35] Some players understand this change of rules as a breakup of contract. For the fixed-line operators, participation in the D and E bands (with operations starting 6 months after the C band operations) requires the anticipation of the expansion and universalization[36] goals from December 2003

[35]Holders of concessions are required to invest significantly in rapid line growth in order to meet the universal provision of service guidelines before 2002. They inherit the brand name of the operator as well as its user base and start loosing market share when mirror licensees start operations. In turn, authorizations holders will invest significantly in infrastructure building prior to starting-up. They need to build up brand name and customer base from scratch and are not subject to the same universal services guidelines(see Note A) as incumbents. Finally, they are likely to pay a lower price for the authorizations than their corresponding concessionaires.

[36][Note A]: The Universal Services Mandate (Plano de Metas para a Universalização) by Anatel holds that every person has the right to access a fixed-line telephone line, independent of place of residence and economic situation. Therefore, it mandates that a certain number of telephone lines be installed by companies operating public service. Additionally, there will be a mandate to install a number of terminals for the public in general and for handicaps use.

Band/Region	Winner	Minimum Price	Price	Premium
D-I	Tele NorteLeste PCS S.A. (Telemar)	940	1,102	17.23%
D-II	Blucel (Italia Telecom)	540	543	0.56%
D-III	Starcel (Italia Teelcom)	710	997	40.42%
E-I	Unicel (Italia Telecom)	940	990	5,31%

Table 4: Results of the SMP Auction

to December 2001.

The Agency did not have success in the C band auction: no consortium submit bids. Table 4 presents the results of the *SMP* auction.[37] In the D band auction licenses' prices were not much higher than the reserve price: no license was sold by more than 41% above the reserve price. The E band auction was also not a success: two licenses remain unsold.[38]

Several factors in the regulation (auction rules) contributed to decrease participation in the *SMP* auction: fixed-line carriers were not allowed to participate in the C band auction, what reduced entry in the contest; operators in the *SMC* that win licenses to *SMP* have to opt (through controlled and controller companies) for one of the licenses; and also there will be a change in the interconnection pricing rules.

Two interrelated factors contribute to increase calling prices in the *SMP*, depressing demand relatively to the demand new entrants would face under the present regulatory setting. Firstly, while in the *SMC* the mobile company is

[37]Values are expressed in million reais.

[38]A new auction for the licenses II and III of the E band is scheduled to May 2001.

responsible for all callings started by its customers, even if they are out of its concession region, in the new regulation the SMP operation is restricted to callings inside a wireless company registration area.[39] Thus a lot of calls that used to be completed only by the mobile operator will have to pay interconnection fees to a long distance carrier under the SMP.[40] Secondly, the interconnection rules are different: in the SMC the mobile companies pay the fixed line carriers a fixed amount regulated by the Agency, whilst in the SMP firms have to negotiate prices.[41] [42]

Several other reasons contributed to decrease revenue in the SMP auction. Amongst them, the international scenario was not favorable.[43] The reserve prices were perceived as too high considering the decrease in value in the telephonic company's shares.[44] There is also much uncertainty concerning the technology and the regulation of the upgrade to the "third generation" mobile phone networks.

[39] In the SMP the national territory is divided into 67 registration areas. The mobile carrier's license authorizes operation in a set of registration areas.

[40] As in the fixed-line, the customer will be able to choose a preferred company by dialing its code every time she makes a call.

[41] Mobile companies who charge local tariffs for callings between different cities in its concession area will have, in the SMP, to pass their callings through a long distance company. (*Gazeta Mercantil*, September 8, 2000).

[42] Trying to answer to some of this critiques the Agency decided to include in the authorizations of the SMP licenses to operate a long distance service.

[43] (*Gazeta Mercantil*, January 1, 2001).

[44] Gruber (2000) develops an oligopoly model with endogenous sunk cost that illustrates the trade-off between ex ante extraction of oligopoly rents and market entry of firms. In the model the higher the license fee the lower the number of firms sustained by the market. A tension emerges between the purpose of extracting the most value from spectrum allocation and having as many firms as possible in the industry. If excessive entry fees (fees that exceed the level consistent with non negative profits in the industry with an exogenously given number of firms) are paid, firms may be threatened with bankrupticity. In Gruber's model up front license fees may become a way of letting endogenous sunk costs determine the market structure in a way that could contrast with the policy makers ex ante objectives for market structure.

The high level of debt of the international telecommunication companies due also to the investments in "third generation" projects left them in a fragile position to make new investments. Global investors became more concerned with factors such as income distribution: a high income inequality did not render high profits since average bills and the consumption of services of high aggregate value are low. Another reason of concern is the great extension of the licenses' areas that increases investment requirements. Also, according to a research sponsored by Merril Lynch in six European markets (Britain, Netherlands, Italy, Spain, Greek and Portugal) the incumbent company keeps an average market share between 50 and 55% even after some entry in the market.[45] Hence new operators face decreased opportunities. As a consequence of all these factors entry in the SMP became less profitable, what reduced competition in the auction.

According to Cramton (2001), there are two steps in making spectrum available to companies: allocation and assignment. The first step, the allocation of the spectrum for licensing, defines the license: the frequency band, the geographic area, the time period and restrictions on use. In the second step the licenses are assigned to particular companies. Often the allocation step is of greater relevance. The greatest economic gains will come from a better allocation of spectrum, rather from improved methods of assigning it, once current spectrum auctions are already highly efficient. The spectrum allocation determination involves complex political, engineering and economic factors and can also have a

[45](*Valor Econômico*, February 15, 2001).

pronounced effect on the success of a particular auction design. In the SMP auction the spectrum allocation was critical to the auction performance.[46]

In sum, some regulatory decisions restrained participation in the SMP auction, contributing to some of the licenses remaining unsold and to a revenues decrease of others.

1.4 Conclusion

In this paper we analyzed the relation between major concerns of competition policy and auction design.

When the number of potential bidders is limited, the auction design must be robust against collusion and entry deterrence behavior. Ascending auctions are more likely to allocate the object to the high value bidder. This auction format also allows a bidder to aggregate information during the bidding process. If the objects are complementary, ascending auctions allow an efficient bundle aggregation. However they are more susceptible to collusion. First price sealed bid auctions are more robust against such behavior, attracting bidders even under

[46] In the UK UMTS auction, auctioning five licenses rather than four was critical to stimulating competition. As there were four incumbents in the market (Vodafone, BT, Orange and One2One), four license would make an ascending auction vulnerable to collusive behavior, once each of them was determined to win one license. An ascending auction may be inappropriate in situations where competition is weak. Thus Klemperer (2001) proposed a hybrid auction mechanism, the Anglo-Dutch auction. This auction was experimentally tested showing good performance in terms of efficiency and revenue maximization. But when it became possible to auction five licenses an ascending auction became appropriate: as one license would necessarily be awarded to an entrant, competition would be guaranteed.

asymmetry. If bidders are risk averse it tends to imply higher revenue for the seller.

Hybrid auctions are thus proposed as a way to capture desirable properties of standard auction formats. However auction design is very sensitive to the details of the environment, and the Brazilian experience with hybrid auctions confirms this. There are failures and successes. In the banking sector the privatization auctions succeeded in raising revenue and attributing the assets to the high value bidder. In the Telebrás auction even though too many factor prevent a deeper analysis, there are evidences that the allocation mechanism fail to give the assets to the high value bidders and that revenues could have been higher. In the SMP auction our understanding is that the allocation (i.e., the definition of the license: the frequency band, the geographic area, the time period and restrictions on use), rather than its assignment (i.e., the attribution of a license to a particular company), was an important determinant of the lack of competition that resulted in some the licenses remaining unsold and in low prices for the licenses that were sold.

Chapter 2

2 A Hybrid Auction Model

Hybrid auctions, which combine features of different auction formats, are becoming increasingly popular as allocation mechanisms. Klemperer [24] for example discusses the Anglo-Dutch auction - a hybrid of the sealed bid and ascending auctions - that may perform better in terms of the traditional concerns of competition policy such as preventing collusive, predatory and entry deterring behavior.

In this paper we examine the revenue properties of another hybrid auction, one that combines a sealed bid first price auction with an ascending auction. This particular mechanism has been used, for example, in the sale of the companies constituted through the partial division of the Telebras System (The Brazilian Telecom). The sale represented a major step towards the restructuring of the telecommunications sector in the country and it raised in excess of R$ 22 billion.

This hybrid auction works as follows. Each buyer submits a sealed bid. Once the highest bid is known, the bidder who submitted it is declared the winner if her bid is higher than the second highest bid by more than a predetermined amount or percentage. If at least one more bidder submitted a bid sufficiently close to the highest (that is, if the difference between this bid and the highest bid is smaller than the predetermined amount or percentage) the qualified buyers compete in an open ascending auction that has the highest bid of the first stage as the reserve price. The qualified bidders include the one who tops the first price sealed bid auction and those who bid sufficiently close to her.

We develop a model that captures some of the features of this hybrid auction. We model a situation where three risk neutral bidders compete in a two stage

auction. The first stage is a sealed bid first price auction. This is followed by a Vickrey auction as a second stage when there are bids sufficiently close to the highest one in the first price sealed bid auction. We consider a model in which potential buyers' values have both a private and a common component. Of course, special cases include the independent private values model and the pure common values case. For the case of a discrete distribution, we show that the hybrid auction generates more revenue than any standard auction. In sections 3 and 4 we show that this conclusion survives the relaxing of the risk neutrality and, under one condition, the symmetry assumptions, respectively. The reason is that one may view this hybrid auction as a Vickrey auction with a reserve price set endogenously at the first stage. Moreover, this hybrid auction is ex-post efficient.

2.1 The Basic Model

Suppose that three risk neutral bidders ($i = 1, 2, 3$) compete for a single object. Each bidder's valuation to the object is a function of a private value, specific to the agent, and of an unknown common value. The individual's private and common value components are independently distributed. *s_i, the vector of the signals observed by the bidder has two elements: $s_i = (s_{i1}, s_{i2})$, where s_{i1} is the private value, and s_{i2} is a signal sent by experts that represents an estimate of the common value component of the object.* The realized value of the object to

bidder i, gross of her expected payment, is given by

$$v_i = u_i(s_i, v) = s_{i1} + v \qquad i = 1, 2, 3 \tag{1}$$

where v is the common value component. The private value, s_{i1}, may take one of two values,

$$s_{i1} = \{x_0, x_1\} \qquad x_0 < x_1.$$

We suppose further that each player i, $i = 1, 2, 3$, knows her own private value, but knows only that her opponents' values are x_1 with probability p or x_0 with probability $q = (1 - p)$. This structure is common knowledge among players.

The common value, v, is not directly observable by the bidders. In this setting v has one of two possible values. Without loss of generality, suppose that the common value can be either $V_0 = 0$ or $V > 0$. Let p_0 be the probability that the common value is V_0 and p_v the probability that it is V. Even though the bidder does not know the common value component, at the beginning of the first period she has access to an expert's appraisal of it. Therefore, she observes simultaneously both her private value and the signal sent by the experts, s_{i2}. This signal can also take two values, $s_{i2} = \{L, H\}$, indicating an unfavorable result, L, or a favorable result, H, from the experts estimates of the common value factor of the object. Given this information structure, there is a positive probability that the common value is mistakenly estimated. Let $q_{L|0}$ be the probability that the

common value is 0 when the bidder receives a low signal and $q_{H|0}$ the probability that it has been mistakenly evaluated. In turn, let $q_{H|V}$ and $q_{L|V}$ be, respectively, the probability that the bidder receives a favorable or an unfavorable result when the item has a positive common value.

Given symmetry, we restrict attention to the problem faced by one of the bidders, say Bidder 1. Her goal is to choose a bid $b(s_{i1}, s_{i2})$ that maximizes her expected payoff. Let $b^{(t)}$ represent the t^{th} highest bid. Conditional on winning the hybrid auction, the expected profit of Bidder 1 who receives simultaneously signals s_{i1} and s_{i2} and bids b is given by

$$E_j\left[\left(u\left(s_{11}, v\right) - b\left(s_{11}, s_{12}\right)\right) 1_{\left\{b^{(2)}(s_{j1}, s_{j2}) + z < b(s_{11}, s_{12})\right\}} \left|s_1, s_2, s_3\right.\right] + \tag{2}$$
$$+ E_j\left[\left(u\left(s_{11}, v\right) - \beta^{(2)}\left(s_{j1}, s_{j2}\right)\right) 1_{\left\{b^{(2)}(s_{j1}, s_{j2}) + z > b(s_{11}, s_{12})\right\}} \left|s_1, s_2, s_3\right.\right].$$

where $\beta(.)$ stands for the bid in the contingent second stage. From now on z denotes the cutoff value that determines the occurrence of the contingent second stage; that is, if there are bids that are lower than the top bid by an amount smaller than or equal to z, thus the winner is decided in a Vickrey stage.

An auction is said to be *efficient* if in equilibrium, for all signal values $s_i = (s_1, s_2, s_3)$, the winner is buyer i such that $v_i(s_i, v) \geq v_j(s_j, v)$, $\forall j \neq i$. To aid our intuition, we start our analysis by considering the special case where an individual's value for the object is determined only by her private signal.

2.1.1 Independent Private Values

In the independent private values setting the agent's value for the object is a function of its private value only. The item has no intrinsic value that is common to all bidders. In the general framework of the last section it amounts to set $v = V = V_0 = 0$ and $v_i(s_1, s_2, s_3) = s_{i1}$. Without loss of generality assume that the seller's evaluation, v_s, is equal to zero. Let $s_{jt}^{(2)}$ be the second highest s_{jt} signal. Conditional on winning the auction, the expected return to the bidder 1 that observes signal s_{i1} and bids b is given by

$$\begin{aligned} \pi(s_{i1}, b(s_{i1})) &= E_j\left[(s_{i1} - b(s_{i1}))\, 1_{\{b(s_{i1})>b(s_{j1})+z\}}\right] + \\ &\quad + E_j\left[\left(s_{i1} - s_{j1}^{(2)}\right) 1_{\left\{s_{j1}^{(2)}<b(s_{i1})<b(s_{j1})+z\right\}}\right], \end{aligned} \tag{3}$$

where z is the cutoff value that *implies the occurrence of the contingent second stage*. In this auction game the proper equilibrium notion is that of a Bayesian Nash equilibrium. This concept extends the Nash equilibrium notion to static games of incomplete information. Each player's action is a best response to other players' actions, that is, each individual chooses a strategy that maximizes her expected payoff given that the other players are also choosing strategies to maximize their expected payoff. A strategy for player i in a Bayesian game is defined as a function from her set of types into her set of actions.[47]

We will focus on symmetric equilibrium, an equilibrium in which all bidders

[47] See Myerson [38].

choose the same bidding function. Given the discrete nature of the model, we will characterize a mixed strategy equilibrium for this game consisting, for each possible type of bidder, of a support to the strategies, that in the present setting correspond to equilibrium bidding functions, and of the associated distribution functions. Upon learning that her private value is x_0, the lower type bidder never bids higher than her value. By playing a mixed strategy that randomizes in a variety of bids her expected profit would be negative. We will characterize an equilibrium such that, for each player, a bidder observing x_0 bids so as to earn 0 expected return and a bidder having a private value x_1 randomizes according to a continuous distribution function $F(b)$ in $[\underline{b}, \overline{b}]$. The equilibrium existence is guaranteed by Maskin and Riley [28] who show that with a finite number of types and private values there is an equilibrium to the first price auction when ties are solved through a Vickrey auction. To characterize the equilibrium it is convenient to consider two separate cases.

$x_1 - x_0 > z$ If a bidder has a value x_0, she only wins the auction when all the other bidders have the same value.

$$U(x_0) = \frac{1}{3}(x_0 - b(x_0))(1-p)^2$$

Her equilibrium bid must guarantee her a zero expected return, that is

$$b(x_0) = x_0.$$

Note that in this case the second stage Vickrey auction will always occur. However at this stage no bidder will raise her bid, otherwise she would pay more than her value, earning a negative expected return. Ties in this stage will be resolved through a random mechanism that assigns the same probability to all participants.

If Bidder 1 has a private value x_1, she wins the first price auction when $b_1(x_1) - b_j(s_{j1}) > z$, $\forall j \neq 1$. She may also win the auction in the second stage. But if at least one of the other bidders has the same private value her expected payoff from the Vickrey auction is zero once the equilibrium bidding function in the second stage is to bid one's value, that is, $\beta^*(s_{i1}) = s_{i1}$, where $\beta^*(\cdot)$ stands for the second stage equilibrium bidding function. Therefore, the expected return of a bidder who bids b when she has value x_1 for the item is

$$U_1(x_1, b) = (x_1 - b)\left[(1-p) + pF(b-z)\right]^2 \tag{4}$$

As there can be no gaps in the support of the equilibrium bids distribution, it is possible to show that $\underline{b} = b(x_0) = x_0$. In a mixed strategy equilibrium, the player must be indifferent to all bids in the support of her bids distribution. Given that

$F(x_0) = 0$, one can find the expected return of a x_1-type bidder who bids b.

$$\overline{U}_1 = U_1(x_1) = (x_1 - x_0 - z)(1-p)^2 \quad (5)$$

Using the fact that $F(\overline{b}) = 1$ in (4) we are able to solve for $\overline{b}$.

$$\overline{b} = \left(1 - (1-p)^2\right)(x_1 - z) + (1-p)^2 x_0$$

The equilibrium bid strategies for the first price auction with a Vickrey auction as second stage when there are bids that are sufficiently close to the higher bid are:

$$b(s_{i1}) = \begin{cases} \cdot \quad x_0 \quad if \quad s_{i1} = x_0; \\ \cdot \quad \text{bid randomly in the interval } [x_0, \overline{b}] \\ \text{according to the bid distribution function} \\ \cdot \\ F(b) \quad if \quad s_{i1} = x_1. \end{cases} .$$

Proposition 1 *The sealed bid first price auction with a Vickrey auction as the second stage when there are bids that are suficiently close to the highest bid implies higher expected revenue to the seller relatively to standard auction institutions.*

Proof. The seller's expected revenue, ER, is the difference between the expected social surplus,

$$x_0(1-p)^3 + x_1\left(1 - (1-p)^3\right) \quad (6)$$

and the bidders expected return,

$$(1-p)\,U_l + pU_h = 0 + p\,(x_1 - x_0 - z)\,(1-p)^2$$

where U_l is the expected return to the low value bidder and U_h denotes the expected return to the bidder observing the high private value. Thus

$$ER = x_0\,(1-p)^3 + x_1\left(1-(1-p)^3\right) - 3p\,(x_1 - x_0 - z)\,(1-p)^2.$$

■

The effect of z is to reduce the expected return to the high type bidder and, therefore, to increase the seller's expected revenue. The reason is that the hybrid auction may be viewed as a Vickrey auction with an endogenously determined reserve price. This generates more revenue than any standard auction with a reserve price set at zero (that is, equal to the seller's value).

Recall from the optimal auction literature (e.g., Myerson [39] and Riley and Samuelson [43]) that the auction that maximizes the seller's expected revenue may be implemented by a Vickrey auction with an optimally chosen reserve price. This of course implies that the optimal auction is ex-post inefficient – that is, there is a positive probability that the object is not sold although there is at least one bidder with a value greater than the seller's value. In contrast, the hybrid auction is ex-post efficient as the outcome of the first stage has produced at least

one bidder who is willing to pay the highest bid in that stage.

$x_1 - x_0 < z$ We claim that when $x_1 - x_0 < z$,

$$b(s_{i1}) = x_0, \qquad i = 1, 2, 3,$$

is an equilibrium of the proposed auction mechanism. Furthermore, it implies the same expected return to the seller as standard auction institutions.

When $x_1 - x_0 < z$ the second stage always occur and the hybrid mechanism is equivalent to a Vickrey auction. The bidder equilibrium bidding function in the second stage is $\beta^*(s_{i1})$. The bidder's expected return is given by

$$U(x_1) = (x_1 - x_0)(1 - p)^2$$

. .

and the seller's expected revenue is then

$$R = x_0 (1 - p)^3 + x_1 \left(1 - (1 - p)^3\right) - 3 (x_1 - x_0) p (1 - p)^2$$

In sum, the revenue equivalence theorem still holds in this setting.[48]

[48]From now on we do not analyze the equilibrium when bidders' valuations differ by a magnitude smaller than z because in this case the hybrid auction is equivalent to a Vickrey auction. This situation is analyzed in Maskin and Riley [29] and in Milgrom and Weber [37].

2.2 Risk Aversion

In this section we drop the risk neutrality assumption retaining the assumption that the item has no intrinsic value that is common to all bidders. In particular, we extend the hybrid auction model to the case in which a buyer i who values the commodity at v_i and purchases a single item at a price ρ receives a Von Neumann utility $u(v_i - \rho)$, where $u(\cdot)$ is a strictly concave utility function normalized to satisfy $u(0) = 0$. Conditional on winning the auction, the expected payoff of Bidder 1 who observes a private value v_i and bids b in the first price auction of the first stage is given by:

$$\begin{aligned} \pi(v_1, b(v_1)) &= E_j\left[u(v_1 - b(v_1))\, 1_{\{b(v_1) > b(v_j) + z; j \neq 1\}}\right] + \\ & \quad E_j\left[u\left(v_1 - v^{(2)}\right) 1_{\left\{v^{(2)} < b(v_1) < b\left(v^{(2)}\right) + z\right\}}\right] \end{aligned} \tag{7}$$

considering that in the contingent Vickrey Stage bidding one's value remains a bidder's best response even under risk aversion.

In the hybrid auction x_0 bidders continue bidding x_0. Once again the equilibrium is a mixed strategy one. If $F_R(\cdot)$ is the cumulative distribution function through which x_1 bidders randomize in equilibrium, it must satisfy

$$u(x_1, b) = u(x_1 - b)\left[(1-p)^2 + 2(1-p)pF_R(b-z) + p^2 F_R(b-z)^2\right].$$

The same reasoning of the previous section allow us to determine the expected

payoff to a bidder observing the high value, $\overline{U}_1$, that is the same for all bids in the support of the mixed strategy equilibrium. This also allows us to determine the supports in which bidders randomize and the cumulative distribution function of bids, F_R, for $b \in \left[\underline{b}, \overline{b}_R\right]$.

$$\overline{U}_1 = u\left(x_1, \underline{b} + z\right) = u\left(x_1 - \underline{b} - z\right)(1-p)^2$$

$$F_R\left(b - z\right) = \left(\frac{1-p}{p}\right)\left[-1 + \left(1 - \frac{u\left(x_1 - z\right)}{u\left(x_1 - b\right)}\right)^{0.5}\right]$$

The equilibrium bid strategies for the hybrid auction with symmetric risk averse bidders are:

$$b\left(s_{i1}\right) = \begin{cases} \cdot \quad x_0 \quad if \quad s_{i1} = x_0; \\ \cdot \quad \text{bid randomly in the interval } \left[x_0, \overline{b}_R\right] \\ \text{according to the bid distribution function} \\ F_R\left(b\right) \quad if \quad s_{i1} = x_1; \end{cases}$$

where

$$\overline{b}_R = \left(x_1 - z\right) - \varphi\left(u\left(x_1 - z\right)(1-p)^2\right), \qquad \varphi\left(\cdot\right) = u\left(\cdot\right)^{-1}.$$

The strict concavity of u implies that F_R stochastically dominates F.[49] Con-

[49] This can be seen by comparing the expression valid to the risk averse case, $u\left(x_1 - b\right)\left[(1-p)^2 + 2p(1-p)F_R\left(b-z\right) + p^2 F_R\left(b-z\right)^2\right] = u\left(x_1 - z\right)(1-p)^2$, to the expression valid to the risk neutral case, $\left(x_1 - b\right)\left[(1-p)^2 + 2p(1-p)F\left(b-z\right) + p^2 F\left(b-z\right)^2\right] = \left(x_1 - z\right)(1-p)^2$.

sidering that when bidders are risk averse the first price auction implies higher expected revenue to the seller than the oral ascending auction, we have the following

Proposition 2 *When bidders are risk averse, the first price auction with a Vickrey auction as second stage when there are bids sufficiently close to the top bid implies higher expected revenue to the seller than standard auction mechanisms.*

Proof. Again bidder's expected utility is reduced by z. ■

2.3 Asymmetry

In the present section we retain the risk neutrality assumption, while dropping the assumption that bidders' values are identically distributed. We also assume that the item has no intrinsic value that is common to all bidders. Different from the other sections, we handle the two bidders case. Suppose now that two bidders dispute an indivisible item in a hybrid auction. Two cases of asymmetry are considered: in the first both buyers have the same probability of observing the high value, although this high value is different for each bidder; in the second buyers have differing probabilities of observing the high value.

2.3.1 When Bidders Have Distinct High Values

Without loss of generality suppose that bidder 1, the "strong" bidder, can have values $s_{s1} = \{x_0, x_2\}$, $x_2 > x_1$, while the "weak" bidder continues to observe either x_0 or x_1. The probabilities of observing the low and the high values are common to both bidders; that is, $pr[s_{i1} = x_0] = (1-p)$, $i = s, w$.

Once again we have a mixed strategy equilibrium that is completely characterized by the cumulative distribution functions of bids of the strong and of the weak bidders, F_s and F_w, respectively, and by the relevant supports. A bidder with a low value bids as to have zero expected payoff in equilibrium. This implies that $b(x_0) = x_0$. The expected payoff to the strong bidder (U_s) who has a high value of the commodity is given by:

$$U_s(x_2, b) = (x_2 - b)\left[(1-p) + pF_w(b-z)\right] + p(x_2 - x_1)(1 - F_w(b-z)) \quad (8)$$

The second term in the right hand side of equation (8) is the payoff to the strong bidder when a contingent Vickrey stage happens, that is, when buyers' bids in the first price auction are close enough. In turn, the expected profit to a weak bidder who values the item at x_1 and bids b is given by:

$$U_w(x_1, b) = (x_1 - b)\left[(1-p) + pF_s(b-z)\right]. \quad (9)$$

In this context the optimal response from a weak bidder with a high value to a

strong bidder's equilibrium strategy is to bid higher, implying that $0 = F_w(\underline{b})$, where F_w stands for the weak buyer's cumulative distribution function of bids. This allows us to determine the expected payoff to a strong bidder with a high value when she randomizes in the range $[\underline{b}, \overline{b}_s]$.

$$\overline{U}_s = U_s(x_2, \underline{b} + z) = (x_2 - \underline{b} - z)(1-p) + (x_2 - x_1)p \tag{10}$$

Substituting (10) into (8), we can determine the equilibrium bids distribution of the weak bidder. Once in a mixed strategy equilibrium all bids in the support of the winning bids distribution must guarantee equal payoff to the bidder, including the maximum bid $\overline{b}_i, i = s, w$.

$$F_w(b - z) = \left(\frac{1-p}{p}\right)\left(\frac{b-z}{x_1 - b}\right)$$

$$\overline{b}_w = \left(\frac{p}{1-p}\right)(x_1 - z) \tag{11}$$

Suppose the weak buyer is bidding according to her equilibrium mixed strategy. If her maximum bid is $\overline{b}_w$, the strong buyer has no incentive to bid higher. If she were to do so, she would do better lowering her bid by an infinitesimal amount, to $\overline{b}_s - \epsilon$. This would increase her expected payoff, because her probability of winning would not change, but her winning bid would be lower. This implies that $\overline{b}_w = \overline{b}_s \equiv \overline{b}_A$, *that is, the maximum equilibrium bid is the same to the strong and to the weak buyer*. Analogously, one can show that the lower point in the

winning bids support, $\underline{b}_A$, is common to both bidders. In fact, as there are no gaps in the equilibrium bids distribution, $\underline{b}_A = b(x_0)$ .By substituting (11) into (9) one can determine the expected payoff to the weak bidder and then her bids cumulative distribution function, that is the same as the one of the strong bidder.

$$\overline{U}_w = U_w\left(x_1, \overline{b}_A + z\right) = (x_1 - z)(1 - p)$$

In sum, when bidders have distinct high values, but the same probability of being high the equilibrium bid strategies for the hybrid auction are

$$b(s_{i1}) = \begin{cases} \cdot \quad x_0 \qquad if \qquad s_{i1} = x_0; \\ \cdot \quad \text{bid randomly in the interval } \left[x_0, \overline{b}_A\right] \\ \text{according to the bid distribution} \\ \text{function } F_w(b) \quad if \quad s_{i1} = x_1,\ x_2. \end{cases}$$

Compared to the first price auction, the effect of z is to increase the expected revenue to the seller.

Proposition 3 *In the two bidders asymmetric case, when bidders have distinct high values to the item, but the same probability of being high, the first price auction with a Vickrey auction as second stage when there are bids that are sufficiently close to the top bid implies higher expected revenue to the seller than standard auction mechanisms.*

Proof. Again z reduces bidders' expected utility. Considering that a first price auction implies a greater revenue than the oral ascending auction,[50] under the present kind of asymmetry these auctions can be ranked as $ER^{HA} > ER^{FPA} > ER^{OAA}$. ■

2.3.2 Different Probabilities of Observing the High Value

Suppose now that both bidders have values in the same set, $s_{i1} = \{x_0, x_1\}$, but different probabilities of observing the high value. The strong bidder $(i = s)$ has a value x_1 with a higher probability, p, whilst the weak buyer $(i = w)$ has a value x_1 with probability $\widetilde{q} < p$. In equilibrium a bidder with a value x_0 bids her own value, earning zero expected payoff, which implies $b(x_0) = x_0$. In turn, a high value bidder bids randomly in the interval $\left[\underline{b}_i, \overline{b}_i\right]$, $i = w, s$.

The equilibrium characterization is completed through the cumulative distribution functions, and the interval in which bidders randomize. The expected payoff to the bidder s who bids b when $s_{i1} = x_1$ is given by:

$$U_s(x_1, b) = (x_1 - b)\left[(1 - \widetilde{q}) + \widetilde{q}F_w(b - z)\right]. \tag{12}$$

In turn, the expected payoff to the weak bidder when $s_w = x_1$ is given by

$$U_w(x_1, b) = (x_1 - b)\left[(1 - p) + pF_s(b - z)\right]. \tag{13}$$

[50] See Maskin and Riley [29].

As there are no gaps in the equilibrium bids distribution, $\underline{b}$, the minimum bid by a x_1 bidder, is equal to $b(x_0)$. In a mixed strategy equilibrium the expected payoffs to the bidders are

$$\overline{U}_s(x_1, \underline{b} + z) = (x_1 - \underline{b} - z)\left[(1 - \widetilde{q}) + \widetilde{q}F_w(\underline{b})\right] \tag{14}$$

and

$$\overline{U}_w(x_1, \underline{b} + z) = (x_1 - \underline{b} - z)\left[(1 - p) + pF_s(\underline{b})\right]. \tag{15}$$

By the same reasoning of the previous subsection, the maximum bid by a high bidder, $\overline{b}$, must be common to both bidders; that is, $F_w(\overline{b}) = 1 = F_s(\overline{b})$. From (12) and (13) in equilibrium both bidders have the same expected return, once $\overline{U}_s(x_1, \overline{b} + z) = \overline{U}_w(x_1, \overline{b} + z)$. As $p > \widetilde{q}$ from (14) and (15) we see that one cannot have both $F_w(\underline{b})$ and $F_s(\underline{b})$ equal to zero. Instead we have $F_s(\underline{b}) \geq F_w(\underline{b}) = 0$. The optimal response from a weak buyer when $s_{i1} = x_1$ is to bid more aggressively. This allows us to determine $\overline{U}_s = (x_1 - z)(1 - \widetilde{q})$ and the cumulative distribution function of bids to the weak buyer with a high value.

$$F_w(b - z) = \left(\frac{1 - \widetilde{q}}{\widetilde{q}}\right)\left(\frac{b - z}{x_1 - b}\right)$$

Substituting the expression $\overline{U}_w$ into equation (13) we determine the cumulative

distribution function of bids of a strong buyer with a high value.

$$F_S(b-z) = \left(\frac{1}{p}\right)\left[(1-\widetilde{q})\left(\frac{x_1-z}{x_1-b}\right)-(1-p)\right]$$

In sum, when both bidders have the same high values but different probabilities of being high, the equilibrium strategy in the hybrid auction is

$$b(s_{i1}) = \begin{cases} \cdot \quad x_0 \qquad if \qquad s_{i1} = x_0; \\ \cdot \quad \text{bid randomly in the interval } \left[x_0, \overline{b}_{\hat{A}}\right] \\ \text{according to the bid distribution function} \\ F_i(b) \quad if \quad s_{i1} = x_1, \ i = s, w, \end{cases}$$

where $\overline{b}_{\hat{A}} = (1-\widetilde{q})\,\underline{b} + \widetilde{q}\,(x_1 - z)$. Once we have $\overline{U}_s = (x_1-z)(1-\widetilde{q}) = \overline{U}_w$,the expected revenue to the seller in the hybrid auction is higher than in the first price auction.

$$ER^{HA} = [1-(1-p)(1-\widetilde{q})]\,x_1 - (p+\widetilde{q})(x_1-z)(1-\widetilde{q})$$

In the present setting the equilibrium strategy in the oral ascending auction is to bid one's value. So the seller's expected revenue is equal to $p\widetilde{q}x_1$. The difference between these auction mechanisms in terms of expected revenue is given by

$$\widetilde{\Delta} = ER^{HA} - ER^{OAA} = -\widetilde{q}\,(p-\widetilde{q}) + (p+\widetilde{q})(1-\widetilde{q})\,z.$$

If z is large enough, that is, if $\widetilde{\Delta} > 0$, these auction mechanisms are ranked as $ER^{HA} > ER^{OAA} > ER^{FPA}$ in terms of expected revenue .

2.4 Mixed Values

In the section 2 we examined the independent private value case and showed that the hybrid auction generates more revenue than any standard auction. We show that this is also true outside the independent private values paradigm.[51] Now each bidder can observe one of four possible combinations of signals: $s^0 = (x_0, L)$, $s^1 = (x_0, H) = (x_1, L)$, $s^2 = (x_1, H)$. For simplicity, we assume that bidders who observe the intermediate signals have a similar pattern of bidding.

We look for an equilibrium in mixed strategies such that a bidder 1 who receives signals $s^0 = (x_0, L)$ wins the auction only when bidders 2 and 3 observe the same signals. The expected payoff of one such bidder if she bids b is given by

$$U\left((x_0, L), b\right) = \frac{1}{3}\left[x_0 + \pi_0 - b\right] p^2 \pi_{(L,L|L)}$$

where

$\pi_{(x,y|w)} = \Pr\left\{s_{22} = x, s_{32} = y | s_{12} = w\right\}$;

s_{i2}, $i = 1, 2, 3$, is bidder i's observed signal that represents the experts' appraisal of the common value factor;

and π_t, $t = 0, 1, 2, 3$, is the expected value of the common factor given that t

[51] Bidders are assumed to be symmetric and risk neutral.

buyers observe a H signal.

Once again to determine her bid we can use the fact that in equilibrium the lower type bidder earns zero expected return.

$$\underline{b} = b(x_0, L) = x_0 + \pi_0$$

The other bidders' types, those who observe signals s^1 and s^2, bid probabilistically. The monotonicity and continuity properties[52] allow us to determine the equilibrium returns of those buyers.[53] The monotonicity property implies that the set of possible bids in mixed strategy to a bidder who observes signal s^{t+1} must be at least as great as the set of possible bids when she observes signal s^t. Let $\overline{b}_1$ be the largest possible bid to a s^1-type buyer. By continuity there can be no gaps in the winning bids distribution so $\overline{b}_1$ is also the lowest possible bid of an agent who observes signal s^2.

The equilibrium of one such auction consists of supports $[\underline{b}, \overline{b}_1]$ and $[\overline{b}_1, \overline{b}_2]$ to bidders observing s^1 and s^2, respectively, and the associated distribution functions. Let the winning bids distribution function of the s^1-type bidders be G_1 with support $[\underline{b}, \overline{b}_1]$. The expected return to a bidder who observes signal s^1 and

[52]By continuity we mean that there can be neither mass points (points of strictly positive probability) nor gaps in the distribution of the equilibrium bids.

[53]We begin by assuming that these two properties hold and then verify that they are in fact satisfied in equilibrium.

bids b is given by:

$$U_1\left(\left(x_1, L\right), b\right) = \left(x_1 + \pi_0 - b\right) p^2 \pi_{(L,L|L)} + \left[2\left(x_1 + \pi_1 - b\right) p^2 \pi_{(L,H|L)} + \right. \tag{16}$$
$$\left. 2\left(x_1 + \pi_0 - b\right) pq\pi_{(L,L|L)}\right] G_1\left(b - z\right) + \left[\left(x_1 + \pi_2 - b\right) p^2 \pi_{(H,H|L)} + \right.$$
$$\left. 2\left(x_1 + \pi_1 - b\right) pq\pi_{(L,H|L)} + \left(x_1 + \pi_0 - b\right) q^2 \pi_{(L,L|L)}\right] G_1^2\left(b - z\right).$$

Using the fact that $G_1\left(\underline{b}\right) = 0$, one may define the expected payoff of a s^0-type bidder.

$$\overline{U}_1 = U_1\left(\underline{b} + z\right) = \left(x_1 + \pi_0 - \underline{b} - z\right) p^2 \pi_{(L,L|L)} \tag{17}$$

Equating (16) and (17), considering that $G_1\left(\overline{b}_1\right) = 1$, it is possible to find $\overline{b}_1$.

$$\begin{aligned} \overline{b}_1 &= \left(\underline{b}p^2\pi_{(L,L|L)} + \left(x_1 + \pi_0 - z\right)\left(1 - p^2\right)\pi_{(L,L|L)} + \right. \\ &\quad \left. 2\left(x_1 + \pi_1 - z\right) p\pi_{(L,H|L)} + \left(x_1 + \pi_2 - z\right) p^2 \pi_{(H,H|L)}\right) / \\ &\quad \left(\pi_{(L,L|L)} + 2p\pi_{(L,H|L)} + p^2\pi_{(H,H|L)}\right) \end{aligned}$$

The winning bids' distribution function for a buyer observing s^1, $G_1\left(.\right)$, can then be completely determined through equations (16) and (17). Its expressions is provided in the appendix. In turn, a s^2-type buyer bids in the range $\left[\overline{b}_1, \overline{b}_2\right]$. As all the strategies played with positive probability in a mixed strategy equilibrium must guarantee her equal expected payoff, it is possible to determine the expected return of one such buyer using the fact that when bidding $\overline{b}_1$ a bidder observing s^2

only wins with positive probability when all her opponents observe lower signals. If she bids b, her expected return is

$$U_2\left(\left(x_1, H\right), b\right) = \left(x_1 + \pi_1 - b\right) p^2 \pi_{(L,L|H)} + [2\left(x_1 + \pi_2 - b\right) p^2 \pi_{(L,H|H)} + \quad (18)$$
$$2\left(x_1 + \pi_1 - b\right) pq\pi_{(L,L|H)}] G_1\left(b - z\right) + [\left(x_1 + \pi_3 - b\right) p^2 \pi_{(H,H|H)} +$$
$$2\left(x_1 + \pi_2 - b\right) pq\pi_{(L,H|H)} + \left(x_1 + \pi_1 - b\right) q^2 \pi_{(L,L|H)}] G_1^2\left(b - z\right) +$$
$$\{\left(1 - G_1\left(b - z\right)\right) \left[\left(x_1 + \pi_1 - \beta\left(s^1\right)\right)\left(1 - p^2\right) \pi_{(L,L|H)} +\right.$$
$$\left.\left(x_1 + \pi_2 - \beta\left(s^1\right)\right) 2p\pi_{(L,H|H)} + \left(x_1 + \pi_3 - \beta\left(s^1\right)\right) p^2 \pi_{(H,H|H)}\right]\}.$$

The expected payoff to a s^2-type buyer that bids b allows us to rewrite (18) as

$$U_2\left(b\right) = \overline{U}_1 + K + \psi G_1\left(b - z\right) + \theta G_1^2\left(b - z\right), \quad \psi < 0,$$

where

$$\psi = 2\left(\pi_3 - \pi_2\right) p\pi_{(L,H|H)} - \left(\pi_1 - \pi_0\right)\left(1 - p^2\right) \pi_{(L,L|L)} - \left(x_1 - x_0\right) p^2 \pi_{(H,H|H)}$$

and

$$\theta = \left(x_1 - x_0\right)\left(q^2 \pi_{(L,L|L)} + 2pq\pi_{(L,H|H)} + p^2 \pi_{(H,H|H)}\right).$$

Ordinarily the monotonicity property would imply that the expected payoff function to a bidder observing signal s^{t+1} would increase monotonically in the support of the winning bids distribution function of an agent observing signal s^t. This

would imply that the lowest possible bid for an s^{t+1} buyer would be the largest possible bid for an s^t agent. In turn, in the present setup a s^2-type bidder that bids in the range $\left[\bar{b}_1, Es^1\right]$ competing with at least one s^1 bidder and no s^2 buyer has a chance to go to a second stage once her opponents bid close enough. If this happens, in the Vickrey auction she raises her bid to $\beta\left(s^2\right) = Es^2$ whilst her opponents raise their bids to their conditional expected value for the item, that is $\beta\left(s^t\right) = Es^t$. As a result, the s^2 bidder wins the auction earning a payoff of $\left(x_1 + \pi_1 - \beta\left(s^1\right)\right)$. This set of events is expressed enclosed in braces in equation (18). In summary, the expected return function to a bidder observing s^2 does not increase monotonically in the range $\left[\underline{b}, \bar{b}_1\right]$ when three is the number of possible types. In this range the expected payoff function is a convex function if condition

$$\left(\psi G_1\left(b - z\right) + \theta G_1^2\left(b - z\right)\right)\left(\bar{b}_1 - \underline{b}\right) \leq \left(b - \underline{b}\right)\left(\psi + \theta\right) \tag{19}$$

is satisfied.[54] Considering the expected return of a s^2-type bidder in both limits of these support, it is possible to exclude $\underline{b}$ as an equilibrium. If this were the bid to one such buyer, a s^1 bidder could beat a s^2 bidder in the first stage of the mechanism through a bid $b\left(s_1\right) = \underline{b} + z$. So,

$$b\left(s_i\right) = \begin{cases} \underline{b} & if \quad s_i = s^0, s^2 \\ \underline{b} + z & if \quad s_i = s^1 \end{cases}$$

[54]The convexity condition stems from $\frac{U_2(b) - U_2(\underline{b})}{b - \underline{b}} \leq \frac{U_2\left(\bar{b}_1\right) - U_2(\underline{b})}{\bar{b}_1 - \underline{b}}$.

cannot be an equilibrium, as it would imply an expected return of

$$U_2(b,(x_1,H)) = (x_1 + \pi_1 - \underline{b})\, p^2 \pi_{(L,L|H)}$$

to a bidder observing (x_1, H). Note that this expected return is strictly lower than the one earned by the buyer in the proposed equilibrium. We can then conclude that if condition (19) is satisfied, a bidder observing s^2 strictly prefers bidding $\overline{b}_1$ than any lower value. So the monotonicity property holds in equilibrium.[55] The expected return to a s^2-type bidder is then

$$\begin{aligned} U_2 = {} & \left(x_1 + \pi_1 - \overline{b}_1 - z\right) p^2 \pi_{(L,L|H)} + 2\left(x_1 + \pi_2 - \overline{b}_1 - z\right) p^2 \pi_{(L,H|H)} + \\ & + 2\left(x_1 + \pi_1 - \overline{b}_1 - z\right) pq \pi_{(L,L|H)} + \left(x_1 + \pi_3 - \overline{b}_1 - z\right) p^2 \pi_{(H,H|H)} + \\ & + 2\left(x_1 + \pi_2 - \overline{b}_1 - z\right) pq \pi_{(L,H|H)} + \left(x_1 + \pi_1 - \overline{b}_1 - z\right) q^2 \pi_{(L,L|H)}. \end{aligned} \tag{20}$$

The equilibrium bid strategies for the hybrid auction with symmetric risk neutral

[55] It is not hard to verify that the continuity property also holds in equilibrium.

buyers who have mixed values for a single indivisible object are:

$$b(s_{i1}, s_{i2}) = \begin{cases} \cdot \quad \underline{b} \quad if \quad s_i = s^0; \\ \cdot \quad \text{bid randomly in the interval } [\underline{b}, \overline{b}_1] \\ \text{according to the bid distribution function} \\ G_1(b) \quad if \quad s_i = s^1; \\ \cdot \quad \text{bid randomly in the interval } [\overline{b}_1, \overline{b_2}] \\ \text{according to the bid distribution function} \\ G_2(b) \quad if \quad s_i = s^2; \end{cases}$$

where

$$\begin{aligned} \overline{b}_2 &= \left[\left(\pi_{(L,L|H)} + 2p\pi_{(L,H|H)} + p^2\pi_{(H,H|H)}\right)\overline{b}_1 + 2(x_1 + \pi_2 - z)(1-p)\pi_{(L,H|H)} + \right. \\ &\left. (x_1 + \pi_3 - z)\left(1 - p^2\right)\pi_{(H,H|H)}\right] / \left(\pi_{(L,L|H)} + 2p\pi_{(L,H|H)} + p^2\pi_{(H,H|H)}\right). \end{aligned}$$

Assuming that the convexity condition is satisfied,[56] we have

Proposition 4 *The first price auction with a Vickrey auction as a second stage when there are bids sufficiently close to the top bid guarantees a higher expected revenue to the seller as compared to standard auction mechanisms.*

Proof: z decreases the expected return of the bidders that observe signals s^1

[56] The winning bids distribution for a buyer observing s^1, $G_1(.)$ and s^2, $G_2(.)$, are provided in the Appendix A.

and s^2, as can be seen by expressions (17) and (20) but the expected social value does not change with z.

2.5 Conclusion

We have demonstrated that the hybrid auction generates more revenue than any standard auction and that it is ex-post efficient. Additional research is needed to study the properties of such mechanisms in terms of the traditional concerns of competition policy such as preventing collusive, predatory and entry deterring behavior. For example, the sealed bid stage may help to deter tacit collusion, a common phenomenon in ascending auctions (see, for example, Menezes [34]). On the other hand, the Vickrey auction stage may work towards increasing revenue. In the Brazilian Telecom auctions, the average number of bidders was equal to four bidders. Of the twelve auctions, only two were followed by a second stage. Additionally, in both cases when there was a second stage, the winner of the second stage was the bidder who submitted the second highest bid in the first price auction!

Chapter 3

3 Experimental Evidence

There was a change in the perceived role of government in the last decades of the twentieth century. This change brought about a worldwide privatization process and major restructuring of infrastructure sectors such as electricity, gas distribution, and telecommunications. Auctions were one of the favored sale instruments in the privatization process and auction-like markets were instrumental for the introduction of competition in most restructuring processes.

Although auctions have been used since time immemorial, they have never been used on such a scale before. By using insights from auction theory and known properties of standard auction formats, such as the first-price, English and Dutch auctions, several new auction formats were designed to deal with specific concerns such as how to prevent collusive, predatory and entry deterring behavior, to maximize efficiency (for example by allowing synergies to be realized) or to maximize the number of potential competitors.

As an example, with asymmetric players, a sealed bid auction may attract more bidders than an English auction as weaker bidders have a higher chance of winning in the former than in the latter auction format. English auctions, however, allow information to be revealed about other players' signals. Klemperer [24], for instance, follows along these lines to suggest an Anglo-Dutch auction, a hybrid of the ascending and sealed bid auctions, that may perform better in terms of the traditional concerns of competition policy.

In this paper we examine the properties of another hybrid auction - a Dutch-Vickrey auction, that combines a sealed bid first-price auction with a sealed bid

second-price auction. This auction mechanism shares some important features with the Dutch-Anglo mechanism used in the sale of the companies constituted through the partial division of the Telebras System - the government-owned Telecom holding in Brazil. The sale represented a major step towards the restructuring of the telecommunications sector in the country and it raised in excess of R$22 billion.

The Dutch-Anglo auction works as follows. Each buyer submits a sealed bid. Once the highest bid is known, the bidder who submitted it wins if her bid is higher than the second highest bid by more than a predetermined amount or percentage. If at least one bidder submits a bid sufficiently close to the highest bid (that is, if the difference between this bid and the highest one is smaller than the predetermined amount or percentage) the qualified bidders compete in an open ascending bid auction that has the highest bid of the first stage as the reserve price. The qualified bidders include the one with the highest bid in the first stage of the auction and those who bid close enough to her.

In Chapter 2 we develop a model that captures some of the features of this hybrid auction. We model a situation where risk neutral bidders compete in a two stage auction: a first price auction followed by a Vickrey auction as a second stage when there are bids sufficiently close to the highest bid in the first stage. We consider a model in which potential buyers' values have both a private and a common component. Special cases include the independent private values model and the pure common values model. Of course, with private independent values

the Vickrey and the Ascending (English) auctions are strategically equivalent and, consequently, so are the Dutch-Vickrey and the Dutch-Anglo auctions.

For the case of a discrete distribution, we show that the hybrid auction generates more revenue than any standard auction. The reason is that one may view this hybrid auction as a Vickrey auction with an endogenously set reserve price at the first stage. Although this hybrid auction generates (weakly) less revenue than the optimal auction, it has the advantage of being ex-post efficient. In contrast, the optimal auction may not be ex-post efficient.

In this paper we report the results of an experiment designed to verify this proposition for the case of independent private values. We conducted an experimental session composed of three parts, with 45 subjects recruited from undergraduate economic courses. During the experiment the subjects were allocated in groups of three participants according to a predetermined rule. Each subject participated in 18 auctions. In the first six auctions the subjects bid for a fictitious commodity sold through a standard first price sealed bid auction. In the next twelve auctions, a hybrid Dutch-Vickrey auction was used.

Several conclusions emerged from this experimental study. Firstly, ex-post efficiency was achieved overwhelmingly by the hybrid auctions. Secondly, overbidding (with respect to the risk-neutral Bayesian Nash equilibrium) was a regular feature of individual behavior in first-price auctions. Overbidding, however, was less prominent in hybrid auctions. Finally, we compared the revenue generated by the hybrid auction with that generated by a standard first-price sealed-bid

auction and the results were ambiguous.

The paper is organized as follows. In the next section we describe our theoretical model and predictions of equilibrium behavior in the Dutch-Vickrey auction. The complete instructions given to all subjects are presented in the Appendix. Owing to its length we summarize in Section 3 the key points developed in the instructions, together with other significant design features. Section 4 discusses the results and in section 5 we comment on issues that need further investigation.

3.1 The Dutch-Vickrey Auction

Suppose that three risk neutral bidders compete for a single object, so that each bidder i receives a signal v_i and her final valuation is equal to $u(v_i) = v_i$. The private values may take one of three values:

$$v_i = \{x_0, x_1, x_2\}, \qquad i = 1, 2, 3.$$

We suppose further that each player i knows her own private value, but knows only that her opponents' values are x_0 with probability p_0, x_1 with probability p_1 and x_2 with probability p_2. This structure is common knowledge among players. The seller's value for the single object is equal to $0 \leq x_0$.

Given symmetry, we can restrict attention to the problem faced by one of the bidders, say bidder 1. Her goal is to choose a bid $b(v_1)$ that maximizes her expected payoff. Let $v^{(t)}$ represent the t^{th} highest signal. Conditional on winning

the hybrid auction the expected profit of Bidder 1 who receives signal v_1 and bids b is given by

$$\begin{aligned} \pi\left(v_1, b\left(v_1\right)\right) &= E_j\left[\left(v_1 - b\left(v_1\right)\right) 1_{\{b(v_1) > b(v_j) + z; j \neq 1\}}\right] + \\ &+ E_j\left[\left(v_1 - v^{(2)}\right) 1_{\left\{v^{(2)} < b(v_1) < b\left(v^{(2)}\right) + z\right\}}\right]. \end{aligned} \tag{21}$$

where z stands for the cutoff value. If at least one more bidder bid sufficiently close to the highest bidder (that is, if she submits a bid that is less than the top bid by up to an amount z), the winner of the hybrid auction is chosen in a Vickrey auction that takes place in a second stage.[57] This is expressed in the second expectation term in the right hand side of equation (21).

We focus on the symmetric equilibrium. Given the discrete nature of the model, we characterize a mixed strategy equilibrium for this game; that is, for each individual's possible value, we compute the support and the associated distribution of the equilibrium bidding functions.

Proposition 5 *The symmetric equilibrium bid strategies for the Dutch-Vickrey*

[57] The subjects allowed in the Vickrey auction are the top bidder and those who bid sufficiently close to her.

auction are as follows:

$$b(v_1) = \begin{cases} \cdot \quad x_0 \quad if \quad v_1 = x_0; \\ \cdot \quad \textit{bid randomly in the interval } \left[x_0, \overline{b}_1\right] \\ \textit{according to the bid distribution function} \\ G_1(b) \quad if \quad v_1 = x_1; \\ \cdot \quad \textit{bid randomly in the interval } \left[\overline{b}_1, \overline{b}_2\right] \\ \textit{according to the bid distribution function} \\ G_2(b) \quad if \quad v_1 = x_2; \end{cases}$$

where the expressions for $G_1(b), G_2(b), \overline{b}_1$ and $\overline{b}_2$ can be found in the appendix. [58]

These equilibrium bidding strategies imply the following

Proposition 6 *The Dutch-Vickrey auction generates more expected revenue than any standard auction with a reserve price equal to the seller's valuation.*

Sketch of the Proof. The seller's expected revenue is the difference between the expected social value and the bidder's expected return. The effect of z is to reduce the expected return to the bidders who have a value other than the lowest and, therefore, to increase the seller's expected revenue. The reason is that the hybrid auction may be seen as a Vickrey auction with an endogenously set reserve

[58] A complete proof can be found in the previous chapter. The exact values that hold for the experimental session are presented in section 4.

price. This generates more revenue than any standard auction with a reserve price set at zero (that is, equal to the seller's value). ■

3.2 The Design of the Experiment

Bidder *i's* valuation for the object consists of a private value v_i only. At the beginning of the period (auction) the buyer receives an envelope with an enclosed bidding form that contains her private value to the fictitious commodity in the current period. The subject is also informed that in each and every period her opponents' values to the item are independent draws from a fixed discrete distribution. In particular, in each period each agent's private value, v_i, is the result of a lottery that with probability 0.4 is R\$0.00, with probability 0.3 is R\$3.00 and with probability 0.3 is R\$6.00. When bidder i wins the auction, she receives a net amount of $v_i - p$, where p stand for the commodity price at the period. The subjects' profits in the experiment are expressed in monetary values ("reais", the local currency).[59]

The recruiter accepted subscriptions of 50 students in order to make a provision for bankruptcies or no shows. After all subjects arrived, the 45 participants were distributed in 15 groups, according to a preestablished order. In order to minimize interactions we implemented a fixed rotation rule such that no in-

[59] Kagel [22] points to a procedure called the binary lottery, introduced by Becker, Degroot and Marschak [4], which controls for risk aversion. However, application of the binary lottery procedure to induce risk neutral bidding in first price auctions has met with mixed results in the literature, so we decided to express subjects receipts in monetary units.

dividual would be matched with any opponent more than twice in the whole experiment. In each group a graduate student previously selected and trained was responsible for the conduct of the auctions, acting as a monitor.

The instructions were divided into three parts, one for each part of the experiment. After reading the instructions relative to each part, the subjects were required to answer some questions in order to verify their understanding of the experiment rules. Monitors evaluated subjects' understanding of the game rules by checking their answers to these hypothetical questions. Once all subjects finished reading the instructions and all questions were properly answered the experimental session started.

The complete experiment consisted of three parts.

Part 1. In each of the 6 auctions that composed the first part of the experiment a fictitious commodity was sold through a first-price sealed bid auction. This is of course equivalent to a hybrid auction with $z = 0$. At the beginning of a period each subject knew only her value of the commodity - which was printed on the bidding form - and the process by which others' values were generated – that is, through the lottery. The private values signals were independently and identically distributed across subjects and periods and the procedure for generating them was common knowledge.

There were no restrictions on bids values. In order to bid the subjects had a $R\$10.00$ credit that could be used during the experimental session.[60] Once each

[60]For comparison purposes, R$10 is about two times what a typical student would pay for

participant's cumulative losses surpassed this amount, they would refrain from bidding. During the first part of the experiment the subject who submitted the highest sealed bid won. In the event of a tie, the winner was chosen randomly with equal probability. At the end of a period, the monitor in charge revealed only the final price and the identity of the winner so that the winner could update his or her earnings in an enclosed worksheet. At the end of the experimental session the subjects received their monetary rewards in cash.

Part 2. The second part of the experiment lasted from period 7 through 12. In every period of this session the commodity was sold through a hybrid of first-price sealed bid auction and Vickrey auction: the subject who submitted the highest bid won if her bid was higher than the second highest bid by more than R$1.00. If one or more subjects submitted a bid smaller than the highest bid by up to *R*$1.00, they were qualified, together with the subject who submitted the highest bid in the first price auction, to compete in a Vickrey auction that had the highest bid of the first price auction as the reserve price. The winner of the Vickrey auction was declared the winner, earning a profit equal to the difference between her value and the second highest bid in the Vickrey auction.[61]

Part 3. The third part of the experiment lasted from period 13 to 18. In this part a second stage occurred if one or more subjects submitted a bid smaller than

his lunch. It is about the same as the cost of a movie ticket.

[61]The reason for ordering the treatment in the order $z = 0, 1$ and 1.5 was a practical one. Since we wanted to shift people around to avoid repated interaction, this was the most efficient way to design the experiment.

the highest bid by up to R$1.50. Then the qualified bidders played a Vickrey auction.

Subjects and Bankruptcies. We conducted the experiment with inexperienced subjects recruited at undergraduate economic courses. The experimental session finished within two hours and subjects earned R$ 6.50 on average plus a R$10.00 show up fee. Their starting capital of R$10.00 provided some buffer against bankruptcies; a subject would become bankrupt if her initial cash balance of R$10.00 was depleted. Only one subject went bankrupt. Even though subjects were told that once their losses surpassed R$10.00 they would not be allowed to bid, we did not enforce this rule.[62] Instead we let the other subjects that were matched with the bankrupt subject continue playing (only) to give them the opportunity to make some money. However, the data from these auctions were discarded.

3.3 Results

In this session we report on subjects' bidding behavior on both the first-price and the hybrid auctions. We also provide a comparison of the actual revenue generated by the different auction formats and report on the overwhelming evidence of the ex-post efficiency property of the hybrid auctions.

[62] We believe this procedure did not significantly affect our results since only one subject went bankrupt by the end of the third part and her opponents did not know this.

3.3.1 Equilibrium

According to our equilibrium predictions subjects with a null private value should bid 0, that is $b(x_0) = \underline{b} = x_0 = 0$ in all auctions mechanisms. Subjects receiving a value x_1 bid in the range $[\underline{b}, \bar{b}_1]$ while subjects with a x_2 value bid in the range $[\bar{b}_1, \bar{b}_2]$. From now on we designate the hybrid auction with a $R\$1.00$ cutoff by hybrid-1 and the analogous auction with a $R\$1.50$ cutoff by hybrid-2. Table 5 presents the exact values for the experimental session.

z	$\underline{b}$	$\bar{b}_1$	$\bar{b}_2$	U_1	U_2	ER
0	0	2.02	4.05	0.48	1.95	2.59
1.0	0	1.35	3.21	0.32	1.79	2.88
1.5	0	1.01	2.79	0.24	1.71	3.02

Table 5: Exact Values Valid to the Experimental Session

Table 5 also presents the expected payoff per auction under equilibrium play. U_1 designates the expected profit to a winner with a value $x_1 = R\$3.00$ while U_2 is the expected payoff when the winner's value is $R\$6.00$.

Figure 2 presents period-by-period bids for the whole experimental session averaged over groups. When the value is $R\$0.00$, there is occasional bidding above value, a phenomenon reported in the experimental literature. However, looking at the data at an individual level one can verify that this behavior is restricted to a subgroup of subjects: 15% of the subjects occasionally did not bid 0, their dominant strategy, when their private value was $R\$0.00$. Nearly half of these subjects bid higher than their dominant strategy only in the first-price

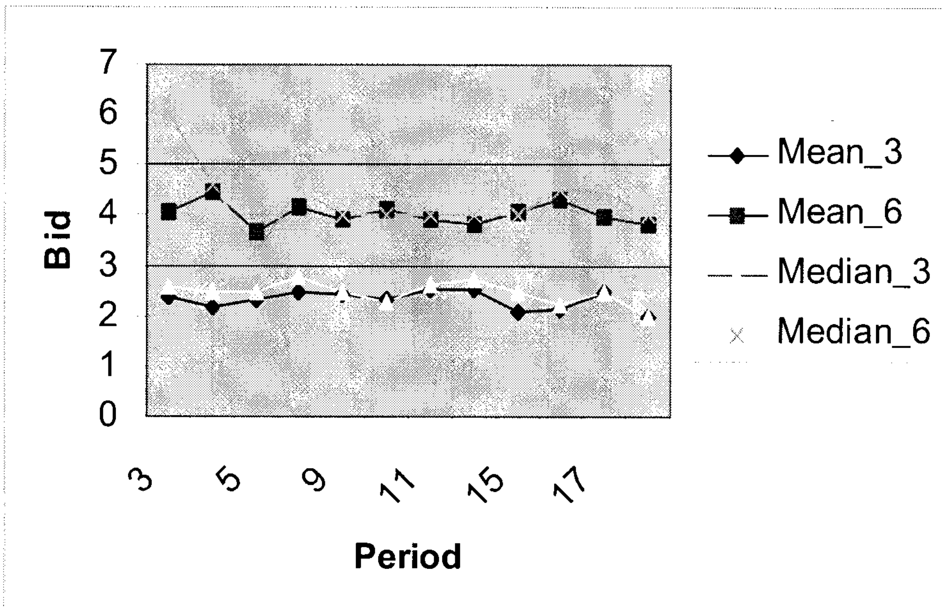

Figure 2: Mean and Median Bids

sealed bid auctions of the first part.

One concern is the properness of the equilibrium assumptions in the presence of discernible time trends in the actual data. The existence of a time trend could prevent beliefs from "settling down". As suggested by Goeree, Holt and Palfrey [17] we fitted a regression of average bids on time, pooled over all private values to verify the possible existence of time trends in bid data. The time coefficient was insignificant at a 10% level. Using the reciprocal of time, as suggested by Kagel [22], we also estimated the coefficient of $1/t$. We were unable to reject the null hypothesis that mean bids were equal at a 10% significance level. In addition, we estimated separate equations for each private value. The time coefficients were

all statistically insignificant.

In the analysis of the experimental results we discarded the data relative to the first two auctions from each part as the subjects were inexperienced. For consistency, we also investigated the existence of a time trend in the final four periods of each part. Again we found no evidence of a significant time trend.

An individual bidder observes a private value and then computes a bid to compete against the aggregate bid distribution that contains all the relevant strategic information about her opponents' bids. Using a Kolmogorov-Smirnov statistic we tested the null hypothesis that the data were generated by our proposed theoretical distribution. We decided to aggregate data over groups. For every period we tested the null that for a given private value the empirical distribution of a set of sample values (the observed bids in the period controlled by the subject's value) agrees with the corresponding theoretical distribution, as specified in section 2.

Table 6 presents the results of the Kolmogorov-Smirnov tests applied to the restricted data set.[63] Observe that the fit is relatively worse in the first price auctions and in the hybrid-1 auction when subject's value is equal to R3.00.

[63]The critical values are presented in Siegel [44].

Kolmogorov-Smirnov One Sample Test				
Auction	KS statistic [a] (v=3)	N	KS statistic [a] (v=6)	N
3	0.71***	15	0.50***	14
4	0.75***	18	0.60***	14
5	0.83***	12	0.26	10
6	0.76***	13	0.53***	14
9	0.54***	16	0.15	9
10	0.38*	13	0.37**	16
11	0.50***	14	0.32*	16
12	0.61***	13	0.32	10
15	0.33**	16	0.31	13
16	0.40**	14	0.42**	12
17	0.44***	13	0.50***	18
18	0.35*	14	0.25	16

[a] *The null hypothesis is that the empirical distribution agrees with the theoretical distribution of bids.*
**** indicates that the null hypothesis can be rejected with only a 0.01 chance of committing a type one error.*
*** indicates that the null hypothesis can be rejected with only a 0.05 chance of committing a type one error.*
** indicates that the null hypothesis can be rejected with only a 0.10 chance of committing a type one error.*

Table 6: Analysis of empirical distribution of bids under risk neutrality

It is a well documented fact in the experimental literature that market prices from first price private value auctions typically exceed the risk neutral Nash Equilibrium (RNNE) prediction irrespective of the number of bidders in the auction or the research group conducting the investigation.[64]. A naive plot of the bids points to a general overbidding pattern relative to the equilibrium prediction, detailed in Table 5, as can be seen from Figures 4 and 5. This is consistent with risk averse behavior. Some facts reinforce the explanation of this behavior in terms of risk aversion. Bidding the dominant strategy in second price auctions is independent of attitudes towards risk. Even though in the experimental session subjects were not restricted to bid up to their private value, in general they did. Plots of the bids in the contingent Vickrey stage present some adherence of subjects' bids to

[64]Cox,J.C., B.Roberson and V.L.Smith [9]; Kagel, J.H. and A.E. Roth. [23]; and Goeree, Jacob K., Charles A. Holt and Thomas R. Palfrey, [17].

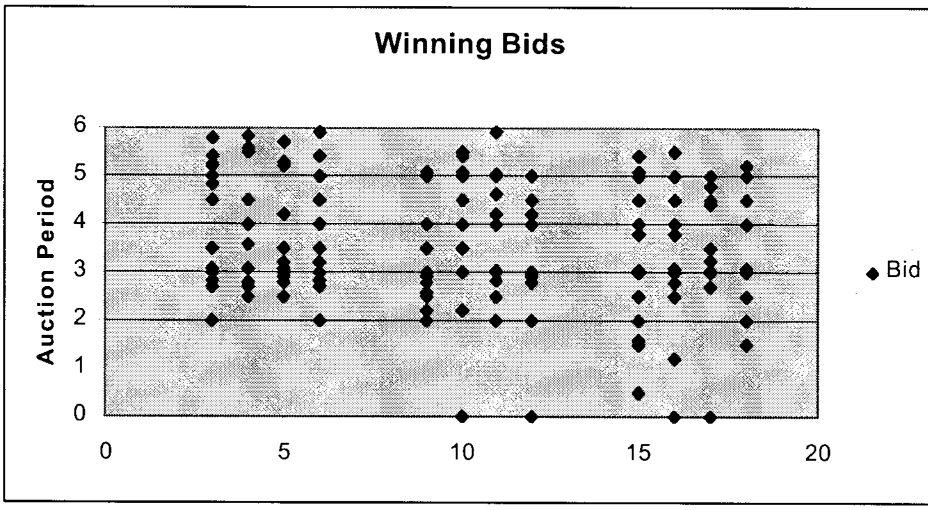

Figure 3: Plot of the Winning Bids

the dominant strategy, as can be seen by Figures 6 and 7. Additionally, we were unable to reject the null hypothesis that mean bids were equal (using a simple test of equality of means) at a 5% significance level. This suggests that subjects follow their dominant.strategy in the contingent Vickrey stage.

The examination of these figures and of Table 6 allows one to infer that deviations from this dominant strategy behavior are relatively more frequent for low value as compared to high value bidders. This supports Kagel and Roth's [23] examination of Harrison's critique. Harrison [20] presents a methodological critique of the evidence Cox, Smith and Walker [10] employ to reject RNNE bidding theory. He argues that under the typical payoff values employed the expected cost of deviations from the RNNE is quite small, so that in terms of expected monetary payoffs many subjects had little to loose by deviating from

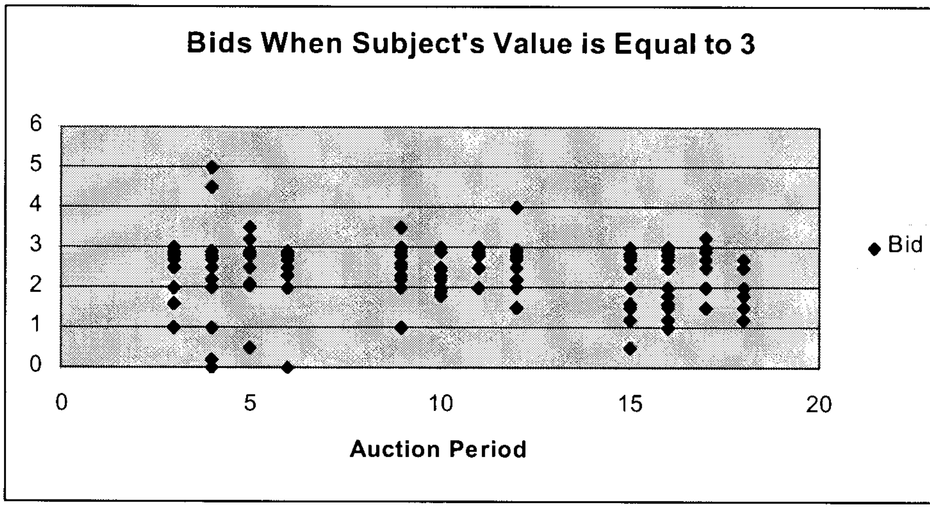

Figure 4: First-price auction bids when subject's value equals R$3.00

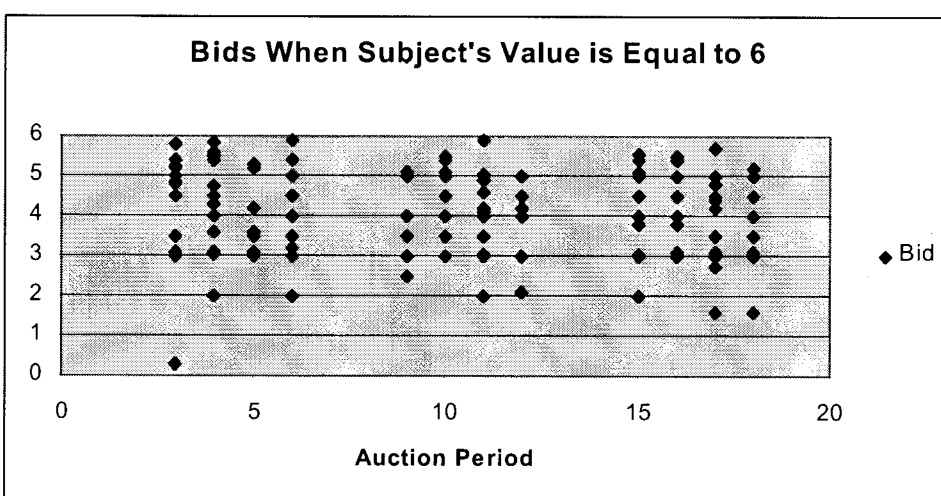

Figure 5: First-price auction bids when subject's value equals R$6.00

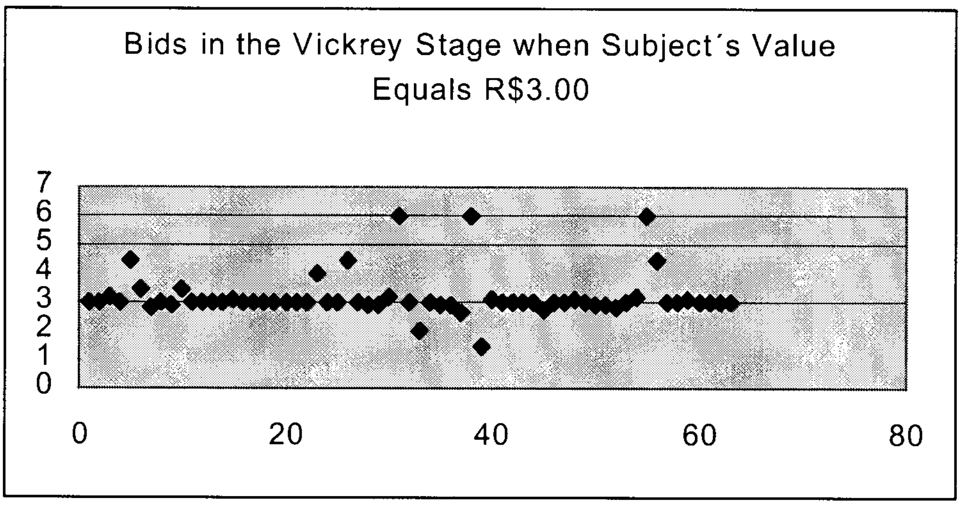

Figure 6: Bids in the Vickrey stage when subject's value equals R$3.00

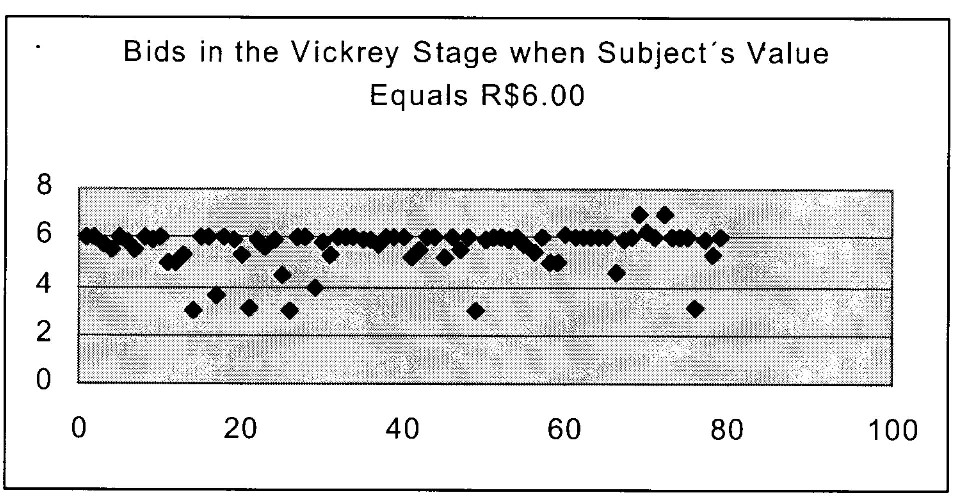

Figure 7: Bids in the Vickrey stage when subject's value equals R$6.00

the RNNE strategy (that is, the payoff function around the maximum is flat).

Kagel and Roth [23] have examined some implications of Harrison's critique. They present evidence that the greatest proportionate deviations from the RNNE predictions are made by bidders who draw the lowest valuations by computing the simple correlation coefficient between private values and the absolute value size of the deviation from RNNE bidding relative to the underlying private valuation. As these subjects have relatively smaller chances of winning the auction, their expected cost of deviating from the Nash equilibrium is lower either if they are risk neutral or risk averse. Our results are compatible with deviations from the Nash equilibrium guided by risk aversion and low expected costs of deviating in terms of expected payoff.[65]

3.3.2 Revenue

Table 7 presents the expected payoff under equilibrium play as well as the mean payoff observed in the experimental session controlled by the group private values' configuration. Given that values are drawn independently in each period and, therefore, the composition of people having the same configuration of values is changing at each round, there is no *a priori* reason to suspect that the independence hypothesis has been violated.

[65]By stating that our results are consistent with Harrison's critique we open the question of whether the incentives offered to the bidders were sufficiently strong. However, the evidence we have is that incentives were appropriate. We delay this discussion concerning the possible motivations to the observed bidding pattern in the first-price auctions to subsection 4.3.

Simple tests of equality of means were conducted in the data to verify the adherence of observed to predicted revenue. These results are also presented in Table 7 As can be inferred from the data, in going from the second to the third part, that is from the hybrid-1 to the hybrid-2 auction format, the observed mean revenue increases, as expected. However, in the first price auctions the mean observed revenue is higher than predicted for some values' configurations. One could argue that in the initial periods subjects could be bidding too high due to their inexperience. According to Kagel [22], one must find an explanation that works consistently across different treatment conditions before one could relate the overbidding pattern in the first price auctions to risk aversion. We have already verified that in the second price auctions of the contingent second stage the bidding pattern was consistent with risk aversion. In the next subsection we investigate the explanatory power of the risk aversion assumption in the alternative conditions of our experiment.

Expected Revenue						
Format	First Price		Hybrid-1		Hybrid-2	
Private Values	Observed (P-values)	Predicted	Observed (P-values)	Predicted	Observed (P-values)	Predicted
(6,6,6)	5.40 (n.a.)	4.05	5.90 (n.a.)	4.21	5.45 (n.a.)	4.29
(6,6,3)	5.67 (0.00)	4.05	4.85 (0.43)	4.21	5.46 (0.00)	4.29
(6,6,0)	5.24 (0.00)	4.05	4.54 (0.31)	4.21	5.57 (0.00)	4.29
(6,3,3)	5.10 (0.02)	4.05	3.83 (0.40)	4.21	3.90 (0.52)	4.29
(6,3,0)	3.95 (0.65)	4.05	4.01 (0.38)	4.21	4.13 (0.49)	4.29
(6,0,0)	4.13 (0.86)	4.05	4.13 (0.87)	4.21	4.26 (0.93)	4.29
(3,3,3)	2.65 (0.55)	2.52	3.00 (n.a.)	2.68	3.01 (n.a.)	2.76
(3,3,0)	2.40 (0.63)	2.52	3.00 (n.a.)	2.68	3.00 (0.00)	2.76
(3,0,0)	2.73 (0.06)	2.52	2.36 (0.02)	2.68	2.00 (0.03)	2.76
(0,0,0)	0.10	0.00	0.19	0.00	0.30	0.00

Table 7: Observed and predicted revenue under risk neutrality

3.3.3 Risk Aversion

As the tendency for subjects to "overbid" in first price auctions is commonly rationalized in terms of risk aversion (as in Cox, Smith and Walker, [11]), we extended the hybrid auction model to the case in which a buyer i who values the commodity at v_i and purchases a single item at a price p receives a Von Neumann utility $u(v_i - p)$, normalized so that $u(0) = 0$. Conditional on winning the auction, the expected payoff of Bidder 1 who observes a private value v_1 and bids b in the first price auction of the first stage is given by:

$$\begin{aligned} \pi(v_1, b(v_1)) &= E_j\left[u(v_1 - b(v_1))\, 1_{\{b(v_1)>b(v_j)+z; j\neq 1\}}\right] + \\ & \quad E_j\left[u\left(v_1 - v^{(2)}\right) 1_{\left\{v^{(2)}<b(v_1)<b\left(v^{(2)}\right)+z\right\}}\right] \end{aligned} \tag{22}$$

considering that in the contingent Vickrey Stage bidding one's value remains a bidder's best response even under risk aversion. The property of an increase in the expected revenue relative to standard auction institutions extends to the present setting.[66]

It is a well known fact in the auction literature that under risk aversion the expected revenue in the first price auction is higher than in the oral auction in the private values case. It turns out that under risk aversion the equilibrium bids distributions in the hybrid auction stochastically dominate the corresponding distributions in the risk neutral case.[67] Intuitively, in the present setting risk aversion induces a bidder to bid higher in the first price auction stage. Given our interpretation that the hybrid auction amounts to a Vickrey auction with a reserve price endogenously set, the effect of risk aversion is to increase this reserve price. Comparing Tables 8 and 9 one can see that risk aversion implies a larger increase in the expected revenue from the first price auction relative to the hybrid auction.

We fitted alternative models corresponding to CARA (constant absolute risk aversion) and CRRA (constant relative risk aversion) utility functions with different risk aversion parameter values. The best fit corresponds to the utility function $u(x) = x^{1-s}$, when $s = 0.45$.[68] The fact that the overbidding is rel-

[66]See Chapter 2 for details.

[67]See Chapter 2 for details.

[68]We fitted the model with the utility functions $u(x) = 1 - \exp[-ax]$, $u(x) = (1/(1-a))(1+x)^{1-a}$ and $u(x) = x^{1-s}$ for different values of the a and s parameters. The results can be obtained upon request.

atively more pronounced between low value bidders corroborates our choice of a utility function showing decreasing absolute risk aversion. This is compatible with the experimental literature.

According to McAfee and Vincent [32] there is a general acceptance that increasing absolute risk aversion is an unsatisfactory characterization of attitudes towards risk. Goeree, Holt and Palfrey [17] report a risk aversion parameter estimate of 0.52 to their data set. Harrison [19] presents a risk aversion parameter estimate of $r_i = 0.45$ for the utility function $u_i(y_i) = y_i^r$ that implies "virtual equality" between observed and predicted behavior in the data from a four subjects experiment from Cox, Roberson and Smith [9].

Goeree, Holt and Palfrey [17] argue that there is some consistency in bidding behavior across many auctions with varying number of bidders, and value structures, and that behavior is consistent with a simple model of risk aversion.[69]

Table 8 presents the expected payoff under equilibrium play for the preferred specification. The adherence of observed to predicted revenue is examined using tests of equality of mean. Table 9 reports the Kolmogorov Smirnov statistic for the model with risk averse bidders. The agreement between the empirical distribution of bids and the equilibrium bids distribution denotes some improvement when confronted to the risk neutral case. That is, the evidence we have is that the model with risk aversion has a better fit to the experimental data.

[69] Further references can be found in Chen and Plott [7], Cox and Oaxaca [8] and Holt and Sherman [21].

Expected Revenue						
Format	First Price		Hybrid-1		Hybrid-2	
Private Values	Observed (P-values)	Predicted	Observed (P-values)	Predicted	Observed (P-values)	Predicted
(6,6,6)	5.40 (n.a.)	5.04	5.90 (n.a.)	5.06	5.45 (n.a.)	5.07
(6,6,3)	5.67 (0.03)	5.04	4.85 (0.78)	5.06	5.46 (0.06)	5.07
(6,6,0)	5.24 (0.23)	5.04	4.54 (0.12)	5.06	5.57 (0.08)	5.07
(6,3,3)	5.10 (0.83)	5.04	3.83 (0.04)	5.06	3.90 (0.12)	5.07
(6,3,0)	3.95 (0.00)	5.04	4.01 (0.00)	5.06	4.13 (0.00)	5.07
(6,0,0)	4.13 (0.06)	5.04	4.13 (0.12)	5.06	4.26 (0.20)	5.07
(3,3,3)	2.65 (0.76)	2.71	3.00 (n.a.)	2.77	3.01 (n.a.)	2.80
(3,3,0)	2.40 (0.28)	2.71	3.00 (n.a.)	2.77	3.00 (0.01)	2.80
(3,0,0)	2.73 (0.83)	2.71	2.36 (0.00)	2.77	2.00 (0.02)	2.80
(0,0,0)	0.10	0.00	0.19	0.00	0.30	0.00

Table 8: Observed and Predicted Revenue under Risk Aversion - $(u(w) = w^{0.55})$

However, like Goeree, Holt and Palfrey [17], we share Friedman's [16] belief that one has to be careful when specifying a utility function parameter that has the effect of pushing predictions in the direction of the observed data. Our results imply only that in the experimental session subjects behaved "as if" they were risk averse.

Kolmogorov-Smirnov One Sample Test				
Auction	KS statistic [a] (*v=3*)	N	KS statistic [a] (*v=6*)	N
3	0.40**	15	0.25	14
4	0.39***	18	0.36**	14
5	0.43**	12	0.47**	10
6	0.46***	13	0.22	14
9	0.31*	16	0.44**	9
10	0.23	13	0.25	16
11	0.35**	14	0.31*	16
12	0.46***	13	0.34	10
15	0.35**	16	0.26	13
16	0.34*	14	0.19	12
17	0.15	13	0.28*	18
18	0.49***	14	0.33**	16

[a] *The null hypothesis is that the empirical distribution agrees with the theoretical distribution of bids.*

**** indicates that the null hypothesis can be rejected with only a 0.01 chance of committing a type one error.*

*** indicates that the null hypothesis can be rejected with only a 0.05 chance of committing a type one error.*

** indicates that the null hypothesis can be rejected with only a 0.10 chance of committing a type one error.*

Table 9: Analysis of empirical distribution of bids under risk aversion

3.4 Hybrid Auctions

From auctions 7 to 18 a hybrid auction was played in each group in every period, totalling 180 auctions. Of these, 84 required a Vickrey stage. In 27 of these auctions the winner in the Vickrey stage was not the subject who submitted the highest bid in the first price auction of the first stage. We suspect that one type of behavior that could emerge in hybrid auctions is that, motivated by a conservative bid from the high value subject, a bidder with a value different from the highest value may have incentives to outbid a high value subject, hereby winning the auction in the first stage. This happened in only 7 auctions. Within these 7 auctions, 5 showed the highest value bidder winning the auction - as predicted - in the second stage. In all 27 Vickrey stage auctions, the second stage

implied an increase in the seller's revenue. Table 10 presents the total number of auctions and the frequency with which a Vickrey stage took place, for periods 9 to 12 and 15 through 18, grouped according to private values' configurations .

	Auction Format	Hybrid -1				Total	Hybrid 2				Total
Private Values	**Auction**	9	10	11	12		15	16	17	18	
(6,6,6)	Vickrey Stage			1		1			1	1	2
	Number of Auctions			1		1			1	1	2
(6,6,3)	Vickrey Stage		2	1		3	1	2	2	1	6
	Number of Auctions		2	1		3	1	2	3	1	7
(6,6,0)	Vickrey Stage	1	1	2	1	5	0	1	1	2	4
	Number of Auctions	3	2	3	2	10	2	1	2	2	7
(6,3,3)	Vickrey Stage	0	2	0	1	3	1		1	1	3
	Number of Auctions	1	2	1	1	5	2		2	1	5
(6,3,0)	Vickrey Stage	0	1	1	0	2	3	2	1	2	8
	Number of Auctions	2	5	3	3	13	4	4	2	6	16
(3,3,3)	Vickrey Stage			1		1		2			2
	Number of Auctions			1		1		2			2
(3,3,0)	Vickrey Stage	3	1	1	2	7	2	3	0	1	6
	Number of Auctions	3	1	1	3	8	2	3	1	1	7
(6,0,0)	Vickrey Stage		0	0	0	0	0		0		0
	Number of Auctions		1	1	2	4	1		1		2
(3,0,0)	Vickrey Stage	1		0	0	1	1		0	0	1
	Number of Auctions	6		3	2	11	3		2	3	8
(0,0,0)	Vickrey Stage		2		2	4		3	1		4
	Number of Auctions		2		2	4		3	1		4
	Vickrey Stage	5	9	7	6	27	8	13	7	8	36
	Number of Auctions	15	15	15	15	60	15	15	15	15	60

Table 10: Frequency of Ocurrence of Vickrey Stage

Traditional concerns of competition policy, such as entry deterring behavior, are among the most prominent questions of auction design. There is a strong presumption in ascending auctions that the bidder who values the commodity the most will win the auction. Even if she is outbid in early stages, she can eventually reverse this scenario. So other bidders, who believe themselves not having a chance to win the object, might have reduced incentives to enter the bidding.

Median Bid and Prices by Auction Format				
Private Values		FPA	Hybrid-1	Hybrid-2
(6,6,6)	Bid	5.50	5.90	3.03
	Price	5.50	5.90	5.27
(6,6,3)	Bid	5.60	4.50	4.51
	Price	5.60	5.00	5.62
(6,6,0)	Bid	5.35	4.35	4.51
	Price	5.35	4.80	5.90
(6,3,3)	Bid	5.25	3.50	3.01
	Price	5.25	3.50	3.76
(6,3,0)	Bid	3.55	4.01	3.90
	Price	3.55	4.01	4.50
(6,0,0)	Bid	3.5	4.25	4.26
	Price	3.5	4.25	4.26
(3,3,3)	Bid	2.65	2.85	2.50
	Price	2.65	3.00	3.01
(3,3,0)	Bid	2.51	2.90	2.51
	Price	2.51	2.95	2.99
(3,0,0)	Bid	2.80	2.20	2.25
	Price	2.80	2.02	2.25

Table 11: Median Bids and Prices

On the other hand, sealed bid auctions may attract a larger number of bidders as the chances of winning might be higher. However, there is a trade-off in terms of efficiency: a sealed bid auction has a relatively smaller ability to give the commodity to the high value bidder as compared to open ascending bid auctions. The experimental data indicated that this phenomenon did not occur in the hybrid auction mechanism.

3.4.1 Efficiency

One strategic complication introduced by a hybrid auction is the possibility that subjects might reduce their bids in the first stage, as can be inferred from the equilibrium predictions presented in Table 5. This might result in an inefficient outcome: a subject with a private value distinct from the highest one could have

incentives to outbid a higher value bidder winning the auction in the first stage.

Out of the 270 auctions conducted in our experimental session, 9 resulted in an inefficient outcome: 4 first-price auctions, 1 hybrid-1 auction and 4 hybrid-2 auctions. In three of the four hybrid-2 inefficient auctions the winner submitted a bid higher than his value in the Vickrey stage. In the other hybrid-2 inefficient auction and the hybrid-1 inefficient auction, a bid higher than the subject's value was submitted in the first price auction. In 8 out of these 9 inefficient auctions the winner incurred in loss. So only one auction was characterized by an inefficient outcome that can not be interpreted as a mistake. The achieved efficiency level of the experimental session is higher than 96%.

3.5 Conclusion and Directions of Further Research

In this paper we developed and implemented an experimental design to investigate a hybrid auction, similar to that used in the sale of the Brazilian Telecom Company. This hybrid format has achieved efficient outcomes and induced less overbidding (with respect to the risk-neutral Bayesian Nash equilibrium) than first-price auctions. Calibrating the experimental results for risk aversion seems to explain a significant part of the overbidding. Finally, the revenue comparison between the hybrid auction and the standard-first price auctions has proven ambiguous, although an increase in the parameter z has shifted the expected revenue in the predicted direction. These conclusions indicate several questions

that need further investigation. Natural extensions include the introduction of a continuous distribution of types, the use of an open auction in the second stage when valuations are not independent, the introduction of asymmetric players, and the reversal of the auction order.

4 Conclusion

Hybrid auction mechanisms have been increasingly used as allocation mechanisms with several advantages. They allow price discovery what is of utter importance when there are no well established markets. This is particularly important in multiple units contexts, where efficiency is not trivially obtained.[70] They allow a designer to address particular problems of the standard auction mechanisms, combining them in the hope to better attribute the items with increased revenues to the seller. Restraining collusion, entry deterrence and predatory behavior are also concerns that can be addressed through a careful auction design. Such behavior assume greater importance once, as Bullow and Klemperer [6] suggest, the value of additional competition is large relative to negotiation skills. Considering this elements, a careful design is of paramount importance.

This dissertation consists of three essays on hybrid auctions. In the first chapter we examined the relationship between auction design and some of the main concerns of competition policy. In particular, we postulate that hybrid auctions might be viable alternatives to avoid common problems. We also examined the implementation of hybrid auctions in Brazil, such as the ones used in the sale of Telebrás (the government owned Telecom holding in Brazil), the privatization auctions of Banestado and Banespa and also the recent spectrum auctions.

In the second chapter we examined the revenue properties of one hybrid auc-

[70] Ausubel, Lawrence and P. Cramton [2].

tion that combines a sealed bid first price auction with an ascending auction and that capture some features of a particular mechanism that was used, for example, in the sale of the companies constituted through the partial division of the Telebras System, and in the privatization of Banestado and Banespa.

We modelled a situation where risk neutral bidders compete in a two stage auction. The first stage is a sealed bid first price auction. This is followed by a Vickrey auction as a second stage when there are bids sufficiently close to the top one in the sealed bid auction. We consider a model in which potential buyers' values have both a private and a common component. Of course, special cases include the independent private values model and the pure common values case. For the case of a discrete distribution, we showed that the hybrid auction generates more revenue than any standard auction. This conclusion is robust to relaxing the assumptions of risk neutrality and symmetry assumptions. The reason is that one may view this hybrid auction as a Vickrey auction with a reserve price set endogenously at the first stage. Moreover, this hybrid auction is ex-post efficient.

In the third chapter we reported the results of an experiment designed to test this proposition for the case of independent private values. We conducted an experimental session composed of three parts, with 45 subjects recruited from undergraduate economic courses. During the experiment the subjects were allocated in groups of three participants according to a predetermined rule. Each subject participated in 18 auctions. In the first six auctions the subjects bid for

a fictitious commodity sold through a standard first price sealed bid auction. In the next twelve auctions, a hybrid Dutch-Vickrey auction was used.

Several conclusions emerged from this experimental study. Firstly, ex-post efficiency was achieved overwhelmingly by the hybrid auctions. Secondly, overbidding (with respect to the risk-neutral Bayesian Nash equilibrium) was a regular feature of individual behavior in first-price auctions. Overbidding, however, was less prominent in hybrid auctions. Finally, we compared the revenue generated by the hybrid auction with that generated by a standard first-price sealed-bid auction and the results were ambiguous.

These conclusions indicate several questions that need further investigation. Natural extensions include the introduction of a continuous distribution of types, the use of an open auction in the second stage when valuations are not independent, the introduction of asymmetric players, and the reversal of the auction order.

In Brazil hybrid auctions have been used intensively as allocation mechanisms. This auctions have raised huge amount of money, being also critical to the restructuring of important sectors in the economy, as infrastructure. However the major concerns of competition policy may have been overlooked in the process of auction design. It remains for policy makers to devote more resources to this process, once, as these dissertation tried to show, auction design matters.

A Appendix to Chapter 2

A.1 Mixed Values

In the mixed values equilibrium the s^1-type buyers bid randomly in the interval $\left[\underline{b},\overline{b}_1\right]$ according to the winning bids distribution function $G_1(b-z)$.

$$G_1(b-z) = \Big(2\ \big(\ (x_1+\pi_2-b)\,p^2\pi_{(H,H|L)} + 2\,(x_1+\pi_1-b)\,pq\pi_{(L,H|L)} +$$

$$(x_1+\pi_0-b)\,q^2\pi_{(L,L|L)}\ \big)\Big)^{-1}\Big\{\ \ -2\,(x_1+\pi_1-b)\,p^2\pi_{(L,H|L)} +$$

$$-2\,(x_1+\pi_0-b)\,pq\pi_{(L,L|L)}\ \ +\ \Big[\ \big(2\,(x_1+\pi_1-b)\,p^2\pi_{(L,H|L)} +$$

$$2\,(x_1+\pi_0-b)\,pq\pi_{(L,L|L)}\big)^2\ -\ 4\,\big((x_1+\pi_2-b)\,p^2\pi_{(H,H|L)} +$$

$$+2\,(x_1+\pi_1-b)\,pq\pi_{(L,H|L)} + (x_1+\pi_0-b)\,q^2\pi_{(L,L|L)}\,(\underline{b}+z-b)\,p^2\pi_{(L,L|L)}\Big]^{\frac{1}{2}}\Big\}$$

. .

In turn, the s^2-type buyers bid randomly in the range $[\overline{b}_1, \overline{b}_2]$ according to the $G_2(b-z)$ distribution function.

$$G_2(b-z) = \big(2(x_1+\pi_2-b)\,pq\pi_{(L,H|H)}+$$

$$-\big(2Min\{1, G_1(b-z)\}\big((x_1+\pi_2-b)\,q^2\pi_{(L,H|H)}+(x_1+\pi_3-b)\,pq\pi_{(H,H|H)}\big)+$$

$$+\big[\big((x_1+\pi_2-b)\,pq\pi_{(L,H|H)}-2Min\{1, G_1(b-z)\}\big((x_1+\pi_2-b)\,q^2\pi_{(L,H|H)}+$$

$$(x_1+\pi_3-b)\,pq\pi_{(H,H|H)}\big)^2-4(x_1+\pi_3-b)\,q^2\pi_{(H,H|H)}+$$

$$+(x_1+\pi_1-b)\,p^2\pi_{(L,L|H)}+-2(x_1+\pi_1-b)\,pq\pi_{(L,L|H)}+$$

$$-(x_1+\pi_1-b)\,q^2\pi_{(L,L|H)}-2(x_1+\pi_2-b)\,p^2\pi_{(L,H|H)}+$$

$$-(x_1+\pi_2-b)\,pq\pi_{(L,H|H)}-(x_1+\pi_3-b)\,p^2\pi_{(H,H|H)}+$$

$$+2Min\{1, G_1(b-z)\}\left[(x_1+\pi_1-b)\,pq\pi_{(L,L|H)}+\right.$$

$$\left.+(x_1+\pi_2-b)\,p^2\pi_{(L,H|H)}\right]+Min\left\{1, G_1^2(b-z)\right\}\left[(x_1+\pi_1-b)\,q^2\pi_{(L,L|H)}+\right.$$

$$+2(x_1+\pi_2-b)\,pq\pi_{(L,H|H)}+(x_1+\pi_3-b)\,p^2\pi_{(H,H|H)}+$$

$$(1-Min\{1, G_1(b-z)\})\big((x_1+\pi_3-\beta(s^1))\,p^2\pi_{(H,H|H)}+$$

$$2(x_1+\pi_2-\beta(s^1))\,pq\pi_{(L,H|H)}+(x_1+\pi_1-\beta(s^1))\,(2p+q)\,q\pi_{(L,L|H)}+$$

$$+\;\;2(x_1+\pi_2-\beta(s^1))\,p^2\pi_{(L,H|H)}\big)\big)\big)\big]^{\frac{1}{2}}\big)\Big/\big(2(x_1+\pi_3-b)\,q^2\pi_{(H,H|H)}\big)$$

A.2 Two-Bidders Case: Continuous Distribution

In this appendix we show that for the private values model there is no pure bidding strategy equilibrium in the Dutch-Vickrey auction when bidder's values are uniformly distributed on the interval $[0,1]$. We seek a monotone symmetric equilibrium.

Suppose bidder i bids the amount b, and her rival bids according to a monotone increasing equilibrium strategy $b(y)$. Then bidder i wins if her bid is higher than her rival's bid by more than z, obtaining a payoff $(v-b)$. Otherwise both bidders dispute the object in a Vickrey auction, where bidding one's value is a dominant strategy. If she wins, her payoff is $(v-y)$. But if $y<b<b(y)+z$, bidder i's payoff is $(v-b)$.Thus bidder i's problem is to find a bidding functions $b(v)$ that maximizes her expected payoff – that is expressed in equation (23).

$$\pi(v,b,b(y)) = E\left[(v-b)\,1_{\{b>b(y)+z\}} + (v-Max\{b,y\})^{+}\,1_{\{b<b(y)+z\}}\right] \tag{23}$$

Thereby $\lambda(b)$ is the inverse of $b(v)$, which indicates the valuation that leads to bidding b when strategy $b^{*}(v)$ is to be played.

$$= E\left[(v-b)\,1_{\{y<\lambda(b-z)\}} + (v-Max\{b,y\})^{+}\,1_{\{y>\lambda(b-z)\}}\right]$$

$$= (v-b)\,F(\lambda(b-z)) + E\left[(v-Max\{b,y\})^{+}\,1_{\{y>\lambda(b-z)\}}\right]$$

$$= (v-b)\,F(\lambda(b-z)) + (v-b)\int\limits_{\lambda(b-z)}^{b} f(y)\,dy +$$

$$+ \int\limits_{Max\{b,\lambda(b-z)\}}^{1} (v-y)^{+} f(y)\,dy$$

In general there are two cases:

Case 1: $b < \lambda(b-z)$

$$y > \lambda(b-z) \Rightarrow y > b$$

$$\pi(v,b,b(y)) = (v-b)\,F(\lambda(b-z)) + \int\limits_{\lambda(b-z)}^{1} (v-y)^{+} f(y)\,dy$$

Case 2: $b \geq \lambda(b-z)$

$$\pi(v,b,b(y)) = (v-b)\,F(b) + \int\limits_{b}^{1} (v-y)^{+} f(y)\,dy$$

In both cases

$$\pi(v,b,b(y)) = (v-b)\,F(Max\{\lambda(b-z),b\}) + \int\limits_{Max\{b,\lambda(b-z)\}}^{1} (v-y)^{+} f(y)\,dy.$$

Let $g(b) = Max\{\lambda(b-z),b\}$ .Then

$$\frac{\partial\pi}{\partial b} = -F(g(b)) + (v-b)\,f(g(b))\,g'(b) - (v-g(b))^{+} f(g(b))\,g'(b)$$

The first order condition is

$$\left(g\left(b\right)-b\right)g'\left(b\right)=\frac{F\left(g\left(b\right)\right)}{f\left(g\left(b\right)\right)}.$$

For $v_i \sim U\left[0,1\right],$ the first order differential equation reduces to:

$$\left(g\left(b\right)-b\right)g'\left(b\right)=g\left(b\right). \tag{24}$$

Solving the differential equation (24) one obtains

$$g\left(b\right)=b+\sqrt{b^2+c^2}. \tag{25}$$

The initial condition

$$\lambda\left(0\right)=z \tag{26}$$

means that the type z bidder is the one who bids z. Substituting (26) in (25) one can determine the constant c and then obtain the candidate equilibrium bidding function $b^*\left(v\right)$.

We claim that

$$b^*\left(v\right)=\begin{cases}\dfrac{\left(\left(v-z\right)^+\right)^2}{2v} & if \quad v>z\\ 0 \quad otherwise & \end{cases}$$

is not a pure bidding strategy equilibrium in the first stage.

Proof: It suffices to show that by deviating from b^* a player 1 bidder with a private value $v_1 < z$ can earn a positive payoff. Suppose that bidder 1 follows $\widetilde{b}(v) = v/2$. There is a positive probability that the other player has a private value smaller than z also. Assume that $v_2 < v_1 < z$. If bidder 2 follows $\widetilde{b}$, bidder 1 may earn a positive payoff equal to $(v_2 - v_1) > 0$ at the end of the second stage - that in this case is mandatory once $\frac{v_2 - v_1}{2} < v_2 - v_1 < z$.If bidder $2's$ value is larger than z, bidder 1 will loose nothing either if she looses in the first price sealed bid auction, $b^*(v_2) > \widetilde{b}(v_1) + z$, or if she disputes the object in the contingent second stage, that is, if $b^*(v_2) < \widetilde{b}(v_1) + z$. So $\widetilde{b}(.)$ can be a best response to a bidder with a private value lower than z and $b^*(.)$ is not an equilibrium.

A.3 N-Bidders Case: Continuous Distribution

Let $y = \underset{i \neq j}{Max}\{v_j\}$. Then the expected payoff to the bidder who values the item to be auctioned at v and bids b when the maximum bid from her opponents is $b(y)$ is

$$\pi(v, b, b(y)) = E\left[(v-b)\, 1_{\{b > b(y)+z\}} + (v - Max\{b, y\})^+ \, 1_{\{b < b(y)+z\}}\right]$$

$$= (v-b)\, F(Max\{\lambda(b-z), b\})^{n-1} + \int_{Max\{b,\lambda(b-z)\}}^{1} (v-y)^+ F(y)^{n-2} f(y)\, dy.$$

Let $g(b) = Max\{\lambda(b-z), b\}$ .Then

$$\pi(v, b, b(y)) = (v-b) F(g(b))^{n-1} + \int_{g(b)}^{1} (v-y)^{+} F(y)^{n-2} f(y)\, dy.$$

The first order condition is then

$$(n-1)(g(b)-b) g'(b) = \frac{F(g(b))}{f(g(b))}.$$

For $v \sim U[0,1]$,

$$(n-1)(g(b)-b) g'(b) = g(b).$$

To solve the differential equation we multiply both sides of it by $g(b)^{n-2}$ and integrate. We get for some positive c:

$$g(b)^n - \frac{n}{(n-1)} g(b)^{n-1} b = c.$$

Using initial condition $g(z) = z$,

$$(n-1) g(b)^n - n g(b)^{n-1} b + z^n = 0.$$

With the assumption that $\lambda(.)$ is an increasing function, it is easy to see that $g(b) = \lambda(b-z) \geq b$. Thus we get the following equation for $b(v)$:

$$(n-1)v^n - nv^{n-1}b(v) + z) + z^n = 0$$

and therefore

$$b(v) = \frac{(n-1)v^n - nv^{n-1}z + z^n}{nv^{n-1}}.$$

This matches the expression we obtained before for $n = 2$. It remains to show that

$$b^*(v) = \begin{cases} \frac{(n-1)v^n - nv^{n-1}z + z^n}{nv^{n-1}}, & if \quad v > z \\ 0 & if \quad v < z \end{cases}$$

is not a pure strategy equilibrium. This follows from the fact that the bid function cannot contain flat portions in some interval $[v_1, v_2]$ in the support $[0, 1]$. Otherwise a bidder with some valuation in the flat interval could raise her expected payoff by marginally raising her bid once she could always win when the second highest valuation were in the same interval. The same reasoning of the two-bidders case applies.

B Appendix to Chapter 3

In this section we present a version of the instructions the participants received at the beginning of the experimental session. We report only the instructions of the first and the second parts as for the third part the instructions are similar to the ones valid to the second part except that the cutoff value is set at $R\$1.50$.

B.1 INSTRUCTIONS

This is an experiment in the economics of decision making supported by the Getulio Vargas Foundation. You may earn an additional amount of money, to be paid in cash at the end of the experiment, if you read carefully the instructions and make the right choices.

In this experiment we create a market in which the participants act as buyers of a fictitious commodity in a sequence of periods. The experiment is composed of three parts, with six periods each, summing eighteen periods. Before the beginning of any part you will read the proper instructions with examples and questions about the experiment in which you are going to participate. At any period only one unit of this commodity will be auctioned off.

With this Instructions you are also receiving an identification card. Your identification code is composed of two numbers and a letter (A, B or C). This identification letter is important to ease subjects' rotation among groups. At each period you will be allocated to a new group of three participants. Pay attention to the monitor's

instructions regarding the rotation rules.

INSTRUCTIONS TO THE FIRST PART

1. GENERAL INFORMATION

1.1. The first part is composed of six periods. In each period you will be allocated to a group of three subjects. In your group a fictitious commodity will be sold through an auction. Your task is to submit a written bid to this commodity competing with the two other subjects in your group.

1.2. In the beginning of each period you will receive a bidding form in a sealed envelope. The commodity's value to you in the period is written in this form (the process of individuals' values determination is described on item 2).

1.3. You must fill in the bidding form with your identification number, the period that is taking place and your bid to the fictitious commodity.

2. COMMODITY VALUE

2.1 At each period the commodity value to any subject is a draw from a lottery that assigns $R\$0.00$ with probability 0.4, $R\$3.00$ with probability 0.3, and $R\$6.00$ with probability 0.3.

2.2. At the beginning of each period the monitor in charge of your group will hand you an envelope that contains your value to the commodity written in a form. (Each period is identified by one color). Each of the other participants in your group knows only her/his value and the generating process of subjects' values - the lottery.

2.3. Your value to the commodity is a private one, that is, it does not depend on

how other subjects value the commodity. THIS VALUE CANNOT BE REVEALED TO THE OTHER SUBJECTS UNDER PENALTY OF ELIMINATION FROM THE EXPERIMENT.

3. COMMODITY ALLOCATION.

3.1. You must fill your bid in the bidding form, enclose it in the envelope and hand it back to the monitor in charge of your group. The monitor will then open the envelopes and will reveal the winning bid and the commodity price.

3.2. At any period in the current part the winner will be the participant with the highest bid. The commodity price will be equal to the highest bid.

3.3. In the event of a tie the monitor will choose the winner between the tied participants randomly and with equal probability.

4. SUBJECTS' PROFITS IN EACH AUCTION.

4.1. At any period the winner makes a profit equal to the difference between her/his value and her/his bid, according to equation(27).

$$\begin{pmatrix} Commodity \\ Value \end{pmatrix} - \begin{pmatrix} Higher \\ Bid \end{pmatrix} = \begin{pmatrix} Winner's \quad Profit \\ in \quad the \quad Period \end{pmatrix} \tag{27}$$

4.2. If the result of equation (27) is negative, the winner will incur in a loss.

4.3. If your bid is not the highest one, you will earn nothing.

5. TOTAL REVENUE OF THE PARTICIPANT. At the end of the experimental session you will receive in cash your Participation Fee - $R\$10.00$ to subjects showing

up at the site of the experiment with at least 10 minutes in advance and $R\$5.00$ to the others - plus the net balance of your transactions. This balance is the sum of your profits (positive values to equation (27)) over the three parts, deducted eventual losses (negative values to equation (27)) . At any time you may verify your net balance through column (4) of your balance sheet. If at any period you accumulate losses equal or higher to $R\$10.00$, you will not be allowed to bid in later periods, leaving the experiment with the Participation Fee only.

6. <u>ADDITIONAL INFORMATION</u>.

6.1. In each period you will bid in a market together with the two other subjects in your group. Once all components in your group submit their bids in the period, the monitor in charge will open the envelopes and will announce the winning bid and the commodity price.

6.2 You may then update the Balance Sheet you received with this Instructions (Appendix 1). At any period you will keep a record of your value to the commodity and the market price in the line corresponding to the period in course. If you win the object in a given period you will update the corresponding cell. Otherwise you must fill the cell with an asterisk.

6.3. Another period will start, in which you will be allocated to a new group. Your private value to the commodity in one period is independent of your private value in any other period.

Now you must answer some questions that allows us to verify your understanding of the game to be played.

1. Assume your private value for the commodity according to the bidding form you received in the beginning of the period equals $R\$6.00$. If you submit a bid in the amount of $R\$5.30$, what will be your profit if the other subjects in your group submit bids equal to:

Subject 2: $R\$5.40$

Subject 3: $R\$3.20$?

2. How much would you profit if you had submitted a bid of $R\$6.20$ instead, if the other subjects in your group had submitted bids as described in question 1?

Note: Subjects can only pass a question by giving the right answer. After a wrong answer the relevant part in the instructions will be explained anew, and the subject will try again.

ATTENTION. DURING THE EXPERIMENT THE COMMUNICATION BETWEEN SUBJECTS IS PROHIBITED UNDER PENALTY OF BEING ELIMINATED FROM THE EXPERIMENT.

ADDITIONAL INFORMATION FOR THE SECOND PART

The second part is composed of six periods, in which you will act as a buyer of a fictitious commodity, competing with the two other participants in your group. The instructions of the first part remain valid, with the exception of the following items.

1. COMMODITY VALUE. The instructions of the first part remain valid. At each period the commodity value to any subject is a draw of a lottery that with probability 0.4 is equal to $R\$0.00$, with probability 0.3 is equal to $R\$3.00$ and with probability

0.3 is equal to $R\$6.00$.

2. COMMODITY ALLOCATION.

2.1. At any period in the current part the winner will be the participant with the higher bid if this bid is higher than the second highest bid by more than $R\$1.00$. The commodity price will be equal to the higher bid.

2.2. If one or more subjects submit bids smaller than the highest bid by an amount equal or smaller than $R\$1.00$, they will be qualified, together with the participant who submitted the highest bid, to participate in a Second Stage.

2.3 SECOND STAGE

2.3.1. In the beginning of each period you will receive a bidding form in a sealed envelope. Your private value is written in this bidding form. You must fill it with your identification number and your bid for the period.

2.3.2. If a Second Stage takes place in your group the monitor will return to the qualified bidders the envelopes with their bidding forms enclosed.

2.3.3. Before the Second Stage actually takes place the monitor in charge will announce the minimum admissible bid that is equal to the highest bid submitted in the First Stage.

2.3.4. If you are a qualified bidder you must fill in the new bid for the second stage in the line correspondent to the "Bid in the Second Stage," enclose it in the envelope and hand it back to the monitor in charge of your group. The monitor will open the envelopes and will announce the winning bid and the commodity price in the period.

2.3.5. In the event of a tie the monitor will choose the winner between the tied

participants randomly and with equal probability.

3. SUBJECTS' PROFITS IN EACH AUCTION.

3.1. If there is no Second Stage, the winner's profit is calculated according to the instructions of the First Part .

3.2. If a Second Stage actually takes place, the participants' profits in any auction are calculated as follows. Once all the group's participants return their bid forms filled, the monitor will open the envelopes and will declare the participant who bid higher the winner. However, the commodity price in the period will be equal to the second highest bid. The commodity price in the period will be announced, allowing each subject to update her/his records in her/his Balance Sheet and the winner to update her/his profits, according to equation (28) .

$$\cdot \begin{pmatrix} Commodity \\ Value \end{pmatrix} - \begin{pmatrix} Second \quad Highest \quad Bid \\ in \quad the \quad Second \quad Stage \end{pmatrix} \cdot = \begin{pmatrix} Winner's \quad Profit \\ in \quad the \quad Period \end{pmatrix} \tag{28}$$

3.3. If the result of equation (28) is negative, the subject will incur in a loss.

3.4. If your bid is not the highest bid, you will earn nothing.

Now you must answer some questions that allows us to verify your understanding of the game to be played.

3. Assume that your private value for the commodity according to the bidding form you receive in the beginning of the period equals $R\$9.00$. If you submit a bid in the

amount of $R\$4.30$, what will be your profit if the other subjects in your group submit bids equal to:

Subject 2: $R\$6.40$

Subject 3: $R\$3.20$?

4. How much would you profit if you had submitted a bid of $R\$6.60$ instead, if the other subjects in your group had submitted bids as described in question 3? Considering that the difference between your bid and subject's 2 bid is lower than $R\$1.00$, a Second Stage is going to take place in your group. How much would you profit by bidding $R\$7.30$ if subject's 2 bid is equal to $R\$7.10$?

Note: Subjects can only pass a question by giving the right answer. After a wrong answer the relevant part in the instructions will be explained anew, and the subject will try again.

ATTENTION. DURING THE EXPERIMENT THE COMMUNICATION BETWEEN SUBJECTS IS PROHIBITED UNDER PENALTY OF BEING ELIMINATED FROM THE EXPERIMENT.

B.2 Equilibrium expressions valid in the experimental session.

$$\bar{b}_1 = \left((x_1 - z)(p_0 + p_1)^2 - (x_1 - x_0 - z)\, p_0^2\right)(p_0 + p_1)^{-2}$$

$$G_1(b) = +\Big[\left((2(x_1-b)p_0p_1)^2 + 4(x_1-b)(b-x_0-z)(p_0p_1)^2\right)^{0.5}$$
$$-2(x_1-b)p_0p_1\big]\left[2(x_1-b)p_1^2\right]^{-1}$$

$$\overline{b}_2 = (x_2-z) - \left(x_2-\overline{b}_1-z\right)(p_0+p_1)^2$$

$$G_2(b) = \left(2(x_2-b)p_2^2\right)^{-1}\left[-(2(x_2-b)(p_0+p_1)p_2)\right.$$
$$\left[(2(x_2-b)(p_0+p_1)p_2)^2 - 4(x_2-b)p_2^2\left[(x_2-b)-\left(x_2-\overline{b}_1-z\right)\right](p_0+p_1)^2\right]^{0.5}\Big]$$

For further references see [15].

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