Are higher-order factors useful in pricing the cross-section of hedge fund returns?

Elaine Fang†
Caio Almeida‡

Abstract

This paper investigates hedge funds' exposures to various risk factors across different investment strategies through models with both linear and second-order factors. We extend the analysis from an augmented linear model based on Fama & French (1993) and Fung & Hsieh (2001) to second-order models that include all quadratic and interaction terms by adopting a novel multistep strategy that combines the variable selection capabilities of the LASSO regression with the Fama & MacBeth (1973) two-step method. We find that, for some strategies, several quadratic and interaction terms are statistically significant. Nonetheless, there is no evidence that the second-order models have more overall explanatory or predictive power than the linear model. Moreover, while both linear and second-order models perform well for directional funds (like emerging markets, event driven and managed futures), missing factors may still remain for semi-directional funds, such as fund of funds, long/short equity hedge and multi-strategy.

***Keywords*:** Hedge Fund Performance; LASSO; Risk Factors; Cross-Section of Returns
JEL Code: C14, C58, G11, G17.

1. Introduction

Hedge fund returns are usually associated with a variety of risk factors, each with a risk premium. Most of the literature that specializes in performance analysis of such funds searches for additional risk factors that could linearly explain their cross-section of expected returns. Standard practice is to choose a set of excess returns representing risk factors and to linearly regress hedge fund excess returns on these factors. Improving on initial models that used the market plus a few linear factors as a benchmark, Fung & Hsieh (2001) argue that hedge funds typically generate option-like returns and propose a seven-factor model that has greater explanatory power than models with only standard asset indices. Fung and Hsieh's seven-factor model is regarded as one of the main benchmarks for modeling hedge fund returns.[1]

Despite a large effort to find new factors, one drawback of these models and their variations is that they constrain the relationship between the risk factors and returns to be linear — and linear models cannot price assets whose payoffs (returns) are nonlinear functions of the risk factors. Several studies address this issue with a nonlinear asset-pricing framework. The pioneering work of Bansal, Hsieh & Viswanathan (1993) extends the arbitrage pricing theory (APT) by allowing returns to be nonlinearly related to risk factors. Harvey & Siddique (2000a, 2000b) propose a model in which returns are related to the quadratic market return. Unfortunately, this strand of literature focuses mainly on standard asset indices and mutual funds, which employ less dynamic trading strategies and have different risk exposures, compared to hedge funds.

In this study, we contribute to the hedge fund literature by extending the analysis from an augmented linear model based on Fung and Hsieh's seven-factor model (hereinafter referred to as the **linear model**) to models

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†Princeton University, Princeton, NJ, USA. E-mail: efang@princeton.edu

‡(Corresponding author) EPGE Escola Brasileira de Economia e Financas - FGV EPGE, Praia de Botafogo, 190, 11o andar, Rio de Janeiro, Brazil. Present Address: Bendheim Center for Finance, Julis Romo Rabinowitz Building, Room 306B, Princeton University, Princeton, NJ, USA. The second author acknowledges financial support from CNPq. This study was partially financed by the Coordenacao de Aperfeicoamento de Pessoal de Nivel Superior - Brasil (CAPES) - Finance Code 001. E-mail: calmeida@fgv.br

[1] An additional classical reference on the hedge fund performance literature is Agarwal & Naik (2004) who show the importance of adopting call and put option returns as additional risk factors, especially when considering equity-oriented hedge funds performance. More recently, Buraschi, Kosowski & Trojani (2014) adopt a novel dataset on correlation swaps and show that most of the so-called market-neutral hedge funds have high exposure to a new correlation risk factor.

that include all second-order terms (**second-order models**). We investigate four models — the Fama-French three-factor model (as a basic sanity check), the linear model, and two second-order models (which include linear, quadratic and interaction terms) — and use a data-driven method to select the final sets of risk factors for each. We then investigate hedge funds' exposures to the factors and examine the performance of each model in explaining and predicting the cross-sectional variation in returns. Including second-order factors increases the flexibility of a model's functional form and decreases the danger of model misspecification. In addition, second-order factors (quadratic and cross terms) may be associated to an attempt to do factor timing.

The aim of this study is to address two major sets of research questions. The first set relates to the performance of the second-order models: (1) Do second-order models provide more explanatory and/or predictive power than linear models based on commonly-used factors, including those proposed by Fama & French (1993) and Fung & Hsieh (2001)? (2) Are the comparison results consistent across all funds? The second set of questions relates to the variation between different hedge fund strategies: (3) Is there a single set of linear and/or second-order factors that can capture the risk dynamics of all hedge fund strategies? (4) Are the hedge fund returns on some investment strategies more difficult to model than others? (5) Do our second-order models still exhibit missing factors for some strategies?

Note that the main goal of this paper is not to search for new factors, but rather to investigate whether second-order factors provide more explanatory and/or predictive power than their linear counterparts, and if they are sufficient for modeling cross-sectional hedge fund returns. Moreover, we are more interested in hedge fund exposure to various risk factors than in the returns performance of the individual funds.

Our linear and second-order models all start with the following set of factors: the three Fama & French (1993) factors representing basic dynamics of the stock market, the momentum factor (Carhart, 1997), five option-like factors (Fung & Hsieh, 2001), the 10-year Treasury bill, and the MSCI World and Emerging Markets indices. Our empirical analysis is based on 10 years of monthly hedge fund returns data, spanning the time period between January 2006 and December 2015. The hedge fund data was obtained from the Lipper Hedge Fund Database (TASS), accessible via the Wharton Research Data Services (WRDS). Figure 1 presents the distribution of hedge funds with complete returns data in TASS by investment strategy (see Table 2 for a more detailed breakdown). This categorization allows us to answer our second set of proposed research questions. As we can see, TASS contains little data for a number of strategies. Thus, we only include the six most represented strategies in our empirical analysis: fund of funds, long/short equity hedge, multi-strategy, managed futures, emerging markets and event-driven (See Appendix A for a description of each strategy).

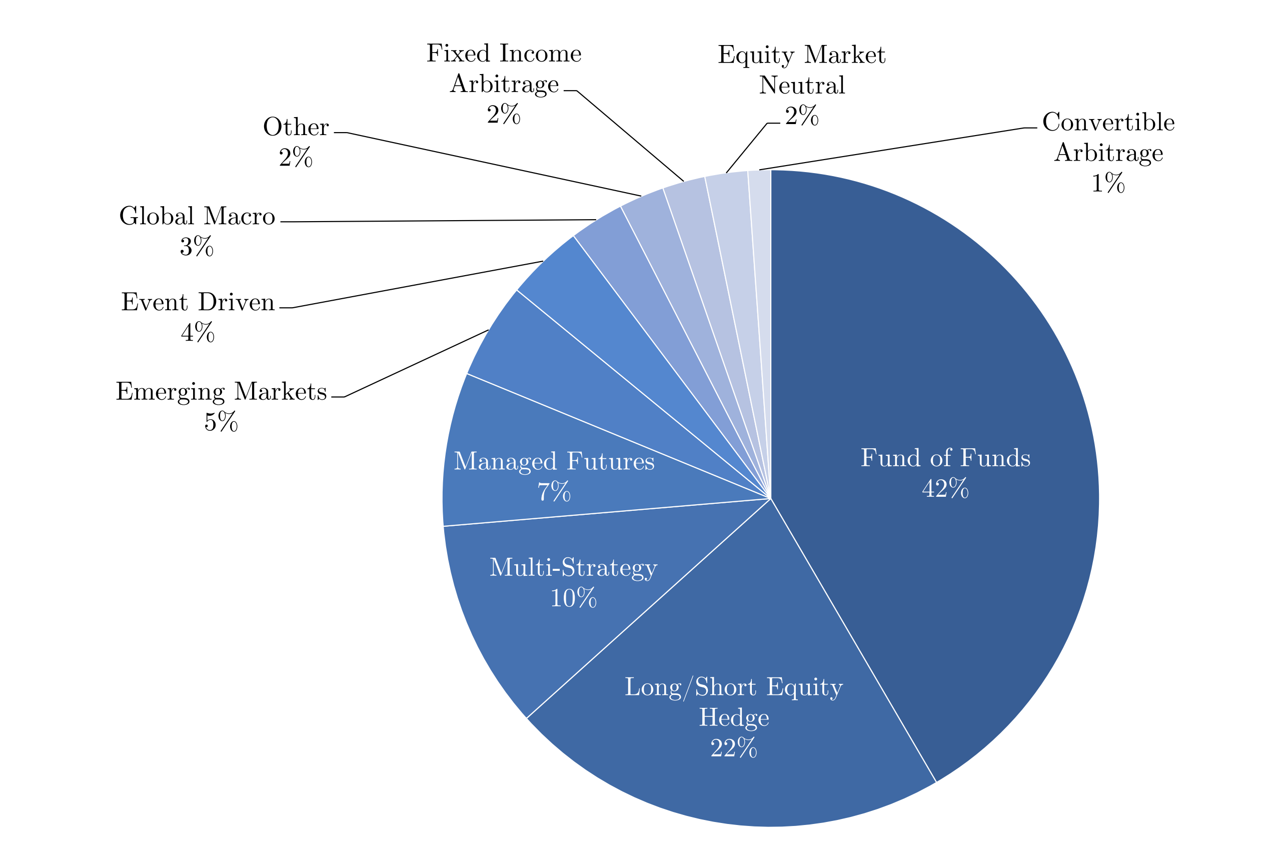
There are some important highlights of this study. First, contrary to most existing literature in which the risk factors are fixed for all funds, we allow the risk factors to be strategy-specific. Different strategies vary in investment style and are hence exposed to distinct sets of risk factors. Following Bali, Brown & Caglayan (2011), we classify emerging markets, event-driven and managed futures as **directional strategies**, and fund of funds, long/short equity hedge and multi-strategy as **semi-directional strategies**.[2] Directional funds willingly take direct market exposure and risk, while semi-directional funds take both long and short positions to try to diversify the market risk. Due to the diverse set of strategies used in this study, we include only Fama and French's three factors in all of the final models as a set of factors representative of the stock market.[3] We select all other factors separately for each strategy based on a data-driven method that makes use of a combination of LASSO to select factors on a first stage, with Fama-Macbeth regressions to identify factors' risk-premium on a second stage.[4] Using strategy-specific sets of factors allows us to address the second set of proposed research

[2] Bali et al. (2011) classify hedge funds into three categories based on their investment styles. The third category is non-directional, which includes convertible arbitrage, equity market neutral and fixed income arbitrage. Funds using a non-directional strategy try to minimize market risk altogether and generate returns that are less dependent on the performance of the market.

[3] In fact, Brysgalova (2016) uses a novel robust procedure to estimate risk premium in linear factor models and identifies the Fama and French factors among a small set of truly robust factors that explain the cross-section of expected stock returns.

[4] We use LASSO to help in reducing large factor dimensionality introduced by quadratic and interaction terms. However LASSO is criticized in settings were there is reasonably large correlation between factors or there exist weak/useless factors. That is, a traditional LASSO, when used in a model selection procedure, is known for generating biased coefficients when there exist too many correlated factors or when there exist factors weakly correlated to test assets returns or useless factors. In any of these cases, this bias should affect risk premium estimation in the second stage of our procedure (Fama-MacBeth). A way of correcting/mitigating these effects is

Figure 1
Number of Hedge Funds by Strategy (Jan. 2006 – Dec. 2015)



This figure shows the distribution of hedge funds by strategy that have no missing returns data in TASS over the 10-year period January 2006 to December 2015 (see Appendix A for a description of each strategy). 1,526 total funds are represented. The dedicated short bias and options strategies are not represented due to an insignificant number of funds. A more detailed discussion of the data used in this study is presented in Section 3.

questions.

We first examine the extent to which aggregate risk factors can explain the time series of hedge fund returns across investment strategies. Similar to Bali, Brown & Caglayan (2012), we quantify the risk hedge funds face by dividing the total risk into its systematic and fund-specific components. This decomposition provides micro-level explanations for different hedge fund strategies. We then investigate how well hedge funds' risk exposures can predict the cross-sectional variation in returns using Fama & MacBeth's (1973) cross-sectional regressions of one-month-ahead excess returns on the factor betas. If the slope coefficient for a certain risk factor is significant, we can conclude that the risk factor's beta has significant predictive power over future hedge fund returns.[5]

Our analysis provides the following findings regarding our proposed research questions. First, although several quadratic and interaction terms are statistically significant (for some strategies), there is no statistical

to adopt the Ordered and Weighted L_1 regularizer suggested by Sun (2018) which is robust to both factor correlation as well as to the existence of weak and/or spurious factors. Alternatively, Brysgalova (2016) suggests in the context of Fama-MacBeth regressions, a nice way of using LASSO on the first stage to penalize the risk-premium coefficients in the second stage, with penalty depending on the inverse of the exposures (Betas) obtained in the first stage. This allows the procedure to restore correct identification of strong factors' risk-premium even in the presence of useless and/or weak factors within the linear models. We abstracted from these potential issues when using LASSO, but refer to these two above-mentioned papers as potential ways of directly addressing those issues.

[5] While many hedge fund studies adopt the Fama and MacBeth method (e.g., see Teo (2009) and Bali et al. (2011)), they mainly focus on linear models. None of these studies, to our knowledge, has analyzed the statistical significance of factor loadings in predicting future hedge fund performance based on models with a complete set of second-order factors.

evidence that the second-order models have more overall explanatory or predictive power than the linear model. Second, evidence strongly indicates that the set of risk factors should be strategy-specific. Our analysis suggests that both the linear and second-order models can effectively explain the predictive power for directional funds. On the other hand, they fail to explain the predictability of semi-directional funds, implying that missing factors may still remain. Ultimately, our results imply that searching for additional factors may be more effective than adding higher-order or interaction terms.

The rest of this paper is organized as follows. Section 2 reviews the relevant hedge fund and asset pricing literature. Section 3 describes the hedge fund returns and factor data used in our analysis and Section 4 presents our multistep estimation procedure and model performance measures. Finally, we present and discuss our empirical analysis results in Section 5 and conclude in Section 6.

2. Literature Review

Asset pricing theory is a central topic of discussion in financial literature. While many methods of pricing different securities have emerged, a universally-accepted model describing cross-sectional hedge fund returns does not currently exist.

2.1 CAPM and Fama-French Factor Models

The Capital Asset Pricing Model (CAPM), introduced by William Sharpe (1964) and John Lintner (1965), is the foundation of most modern asset pricing theories. When applied to funds, CAPM describes the following relationship between the systematic risk and expected return of fund i:

$$R_{it} - r_{ft} = \alpha_i + \beta_i(R_{mt} - r_{ft}) + \varepsilon_{it}, \quad t = 1, \ldots, T, \tag{1}$$

where R_{it} is the return of fund i at time t, r_{ft} is the risk-free rate at time t, the intercept α_i is the fund's excess return after accounting for market risk, the slope β_i is the fund's "beta," and R_{mt} is the expected market return (commonly defined as the S&P 500 return) at time t.

In particular, the expected return of a portfolio depends on the risk-free return and the fund's risk premium, which is derived from its beta. The risk-free rate compensates for the time-value of money.

As we can see, CAPM prices only the market (or systematic) risk. Eugene Fama and Kenneth French (1993) made great strides in the asset pricing literature by introducing additional factors that capture other risks priced in the cross-section of U.S. stock returns. In their three-factor model (equation 2), Fama and French build upon the CAPM by adding two factors based on size (SMB – small minus big) and value (HML – high minus low book-value ratio):

$$R_{it} - r_{ft} = \alpha_i + \beta_i(R_{mt} - r_{ft}) + \beta_{SMB,i}SMB_t + \beta_{HML,i}HML_t + \varepsilon_{it}, \quad t = 1, \ldots, T, \tag{2}$$

where the three βs are the exposures to their corresponding risk factors. Size and value are added based on the assumptions that small-cap stocks regularly outperform large-cap stocks and value stocks regularly outperform growth stocks.[6]

Carhart (1997) extends the Fama-French three-factor model to a four-factor model by introducing momentum. The momentum of a stock describes the tendency for its price to continue in the direction it is currently trending, and is calculated as the difference between the equal weighted averages of the lowest-performing firms and highest-performing firms, lagged one month. More recently, Fama & French (2015) introduce a five-factor model with additional factors based on profitability and investment. While their five-factor model has greater explanatory power than their three-factor model, the profitability factor is highly correlated with Carhart's momentum factor.

[6] Similar to the CAPM, APT also assumes that fund returns are influenced by several factors. However, as the name implies, APT is used to describe a mechanism investors use to identify incorrectly priced assets. Since our approach in this study is mainly built upon the CAPM, we will not review APT. Please see Ross (1976) for more information on APT.

Ultimately, these factor models can be expanded to accommodate any number of risk factors, with the following general form:

$$R_{it} - r_{ft} = \alpha_i + \sum_{j=1}^{K} (\beta_{ij} \times F_{jt}) + \varepsilon_{it}, \quad t = 1, \ldots, T, \tag{3}$$

where $F_{1t}, F_{2t}, \ldots, F_{Kt}$ are the values of the K risk factors at time t. Economically, the intercept α_i is interpreted as the over- or under-performance of fund i relative to the K risk factors, while β_{ij} is interpreted as fund i's exposure to risk factor F_j. If the risk factors accurately capture all of the cross-sectional variation in fund i's returns (in other words, the model is correct), α_i should be zero.

Although the βs provide information on a fund's exposure to various risk factors, fund managers care more about the economic implications – the reward or payoff associated with each risk exposure. Fama & MacBeth (1973) develop a two-step procedure to quantify this payoff. Technical details of the Fama-MacBeth two-step method are provided in Appendix B.

2.2 Nonlinear Options-Based Factors

Though prevalent in the literature, CAPM, the Fama-French factor models, and many of their derivatives have a major drawback: they only consider linear factors. While a model with only linear factors may effectively describe the returns on mutual funds as they largely rely on buy and hold strategies, hedge funds employ more dynamic trading strategies, resulting in payoffs that exhibit nonlinear structures. A large branch of the literature accounts for these nonlinearities through option-like factors.

Fung & Hsieh (2001) show that cross-sectional hedge fund returns exhibit option-like features and attempt to capture these features using option-like factors. They introduce five trend-following factors that describe portfolios of lookback straddles derived from exchange-traded options.[7] Ultimately, they find that including these five factors increases the explanatory power of their seven-factor model, which also includes the market and size factors.

Agarwal & Naik (2004) also confirm the nonlinear nature of hedge fund payoffs, finding that the betas in up-market versus down-market conditions are asymmetric. The authors first confirm the importance of Fama and French's size and value factors and Carhart's momentum factor. Then, they use a piecewise linear model that includes call and put options on the S&P 500 index. They use options as options are liquid, frequently-traded assets with payoffs that have a nonlinear relation with the market return. Ultimately, Agarwal & Naik (2004) rely on the F-statistic to determine the robustness of the models.

Building upon Agarwal and Naik's work, Diez de los Rios & Garcia (2011) use options on whichever benchmark portfolio they determine best captures the characteristics of each hedge fund strategy, rather than just the S&P 500 index. They estimate 10 piecewise linear regressions, one for each major strategy. Unlike Agarwal & Naik (2004), Diez de los Rios & Garcia (2011) find statistical evidence of nonlinearities in returns for only a few hedge fund strategies and for one-fifth of individual funds. They also assess the returns performance of the funds in each strategy. They determine that only funds with emerging markets, dedicated short bias and managed futures strategies do not exhibit positive valuations.

There is much evidence that cross-sectional hedge fund returns are related to the factors proposed by Fung & Hsieh (2001). Thus, we consider these five options-based factors in our study, in addition to Fama and French and Carhart's linear factors described in Section 2.1.

2.3 Factors Constructed from Higher Moments

While the literature generally recognizes the existence of the nonlinear features of hedge fund returns, the model's functional form is still debated. Despite lack of consensus on the model's functional form, most research presumes that average cross-sectional hedge fund returns are linearly related to economic risk factors. Ample evidence, however, suggests that this is not necessarily the case.

[7] A lookback straddle consists of a lookback call and lookback put option. A lookback call (put) option provides the owner with the option to retroactively buy (sell) the underlying asset at its minimum (maximum) price during a previously-defined lookback period.

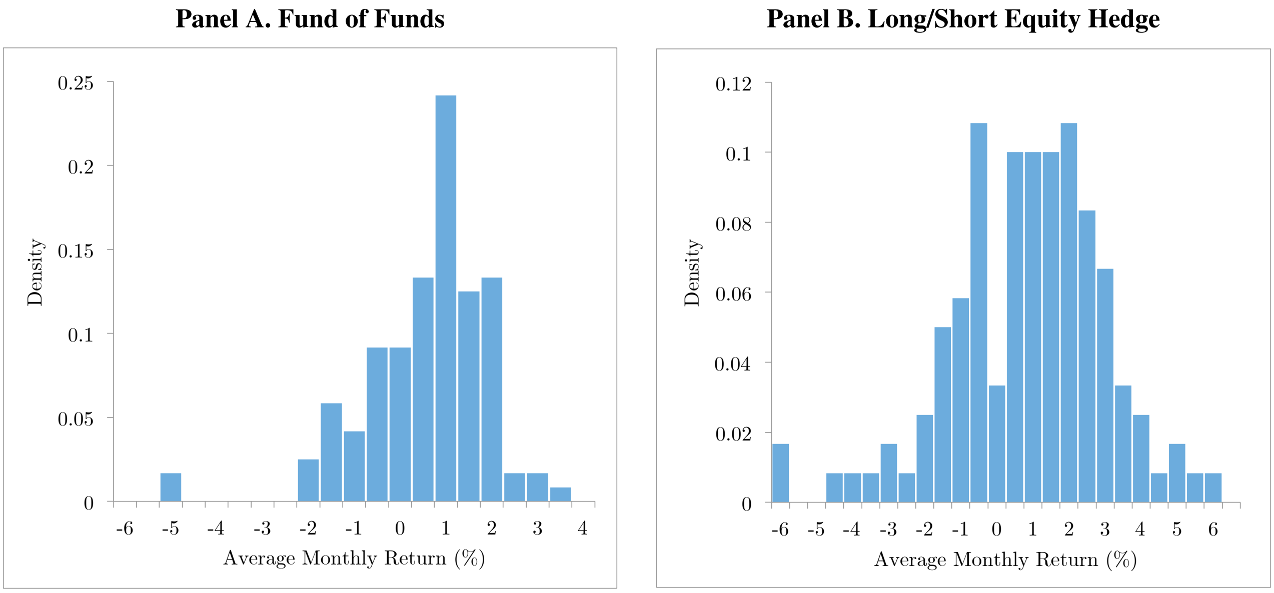
The linear theoretical framework restricts the risk-return tradeoff to a simple mean-variance relationship. This relationship fails if hedge fund returns do not follow a normal distribution. To demonstrate the invalidity of the normality assumption, we present the returns distributions of the two most popular hedge fund strategies in Figure 2: fund of funds (Panel A) and long/short equity hedge (Panel B). The distributions are based on the average monthly returns on all funds for each strategy during the 10-year time period between January 2006 and December 2015.

As Figure 2 illustrates, the distributions of both fund of funds and long/short equity hedge fund returns are skewed to the left (with skewness -1.29 and -0.59, respectively), suggesting that a nonlinear model and/or the assumption of non-Gaussian returns may be needed to capture the dynamics of hedge fund returns. This is consistent with past literature that has shown hedge funds to exhibit significant left-tail risk.

To account for the asymmetric nature of hedge fund returns, Bansal et al. (1993) allow returns to be nonlinearly related to risk factors. Harvey & Siddique (2000a, 2000b) augment the Fama-French three-factor model with an additional factor derived from skewness. Barone-Adesi, Gagliardini & Urga (2004) propose a model that includes co-skewness based on the observation that portfolios of small (large) firms exhibit negative (positive) co-skewness with the market. Moving a step further, Ranaldo & Favre (2005) propose models including quadratic and cubic market factors to account for the co-skewness (quadratic CAPM) and co-kurtosis (cubic CAPM) between hedge fund returns and the market return.

Motivated by these studies, we extend the models with simple higher moments to models based on the factor space spanned by all second-order factors. Thus, our models include linear factors, their quadratic terms and all the interaction terms between them. Analyzing models with a complete set of second-order factors allows us to empirically address our proposed research questions within a much less restricted framework that has not previously been investigated.

Figure 2
Returns Distributions by Fund Strategy (Jan. 2006 – Dec. 2015)



These figures show the distributions of the 120 average monthly returns over the period January 2006 to December 2015 across all TASS funds categorized as funds of funds (Panel A) and long/short equity funds (Panel B). There are 633 funds of funds and 331 long/short equity funds represented. See Tables 2 and 5 in Section 3 for more detail.

3. Data

Sections 3.1 and 3.2 describe the database containing the hedge fund returns and risk factor data we utilize in this study. Sections 3.3 and 3.4 describe how we filter the returns data and prepare the initial set of factors. Section 3.5 presents a descriptive summary and summary statistics of the final data used in our empirical analysis.

3.1 Hedge Fund Returns Data

In this study, we utilize data from the Lipper Hedge Fund Database (TASS), accessible via the Wharton Research Data Services (WRDS), due to its relative completeness and accuracy (Getmansky, Lee & Lo, 2015).[8]

This study sorts the target hedge funds by strategy and focuses primarily on their monthly returns. Before using the data, however, it is important to note several well-known biases associated with hedge fund data. The first major bias is selection bias. Unlike mutual funds, hedge funds are not obligated to report their financial information – all provided data in TASS is voluntarily reported, often with the intent of marketing and advertising the fund's abilities. As a result, the data may be biased upwards if only the top-performing funds choose to report their superior returns, while funds that perform poorly choose not to report their returns.

This problem is further exacerbated by backfill bias. When funds decide to start reporting their returns, the database automatically backfills the fund's historical returns as well. Thus, some funds only decide to start reporting once they have experienced multiple months of high performance.

A final major bias is survival bias. The life of hedge funds can be very short due to the dynamic nature of the industry, and funds experiencing low returns may dissolve or simply choose to stop reporting, once again biasing the returns data upwards.

Taking these biases into account, we further filter the hedge fund returns data in Section 3.3. In the analysis that follows, we include both live and graveyard funds to ensure that our estimation results are robust to survival bias.[9] In addition, hedge fund returns exhibit considerable cross-sectional correlation (Fama & MacBeth, 1973). The second step of the Fama-MacBeth method is used to circumvent this issue.

As a final remark, we note that there is no easy way to combat selection and backfill bias. Teo (2009) argues that dropping the first 12 months of returns data from each fund can help mitigate these two biases. As our analysis requires data from a sufficiently long time period, however, we decide not to remove the first 12 months of return data, rather to keep a homogeneous dataset for all the funds included when estimating Fama-MacBeth regressions based on five-year rolling windows.

3.2 Factor Data

We first establish our initial set of factors, to be used as the basis for our quadratic and interaction terms. Following existing hedge fund literature, we compile a list of 10 commonly-used factors: three factors proposed by Fama & French (1993), Carhart (1997) momentum factor, five PTFS trend-following risk factors from Fung & Hsieh (2001), and the 10-Year Treasury Constant Maturity Rate. We choose to use the Fama-French three-factor model over the Fama-French five-factor model as our base case, as it is the most consistently-used standard in existing hedge fund literature. Additionally, the profitability factor is highly correlated with momentum, which we include in our analysis (Fama & French, 2015).

We also consider the MSCI World and Emerging Markets indices (*MSCIW* and *MSCIE*, respectively) to capture the dynamics of firm equity outside the U.S. The inclusion of *MSCIW* and *MSCIE* is important, as many hedge funds invest heavily in non-U.S. equity markets (e.g., funds with an emerging markets strategy).

[8]TASS is one of the most comprehensive commercial hedge fund databases used in hedge fund literature. In addition to containing monthly returns data, the database categorizes each fund as using one of 14 investment strategies, including "other" and "undefined" (see Appendix A for a description of each strategy).

[9]TASS classifies each fund as "live" or "graveyard." Live (graveyard) funds are funds that were active (inactive) at the time of the most recent database update. In other words, funds that did not report a return for the latest month are classified as "graveyard." Graveyard funds may include the following: (1) funds that shut down due to various reasons, such as poor performance, and (2) funds that reached maximum capacity and stopped pursuing new investments.

Our initial factors, obtained from various sources, are described in Table 1.[10]

Table 1
Initial Risk Factors Considered

	factor	description
1	*MKT*	market return (Market Index) is derived as the value-weighted return of all firms incorporated in the U.S. with ordinary common shares listed on the NYSE, AMEX or NASDAQ stock exchanges.[11] This weighted average minus the risk-free rate (the 1-month Treasury bill rate in our analysis) gives us the excess market return (*MKT*)
2	*SMB*	difference between the average returns on three small portfolios and three big portfolios: *SMB* = 1/3 (Small Value + Small Neutral + Small Growth) - 1/3 (Big Value + Big Neutral + Big Growth)
3	*HML*	difference between the average returns on two value portfolios and two growth portfolios: *HML* = 1/2 (Small Value + Big Value) - 1/2 (Small Growth + Big Growth)
4	*MOM*	Carhart's momentum factor is calculated as the difference between the equal weighted averages of the lowest-performing firms and highest-performing firms, lagged one month
5	*PTFSBD*	return on PTFS Bond Lookback Straddle
6	*PTFSFX*	return on PTFS Currency Lookback Straddle
7	*PTFSCOM*	return on PTFS Commodity Lookback Straddle
8	*PTFSIR*	return on PTFS Short Term Interest Rate Lookback Straddle
9	*PTFSSTK*	return on PTFS Short Term Interest Rate Lookback Straddle
10	*GS10*	the yield of the U.S. 10-year Treasury Bill
11	*MSCIW*	the MSCI World Index describes the performance of large and mid-cap equity assets across 23 countries with developed markets (note that this does not include emerging markets)
12	*MSCIE*	the MSCI Emerging Markets Index describes the performance of 10% of the world's market capitalization, spanning 24 countries. These indices account for nearly 85% of the free float-adjusted market capitalization in each country

3.3 Filtering TASS Data

The sample data in our analysis consists of monthly hedge fund returns from January 2006 to December 2015. TASS contains data for 16,943 funds during this 10-year period. To utilize the data, we remove all funds with any missing returns from our analysis, following Fang (2017). The result is a set of funds that each includes a full set of returns for every month of the 10-year period.

We select January 2006 to December 2015 as our analysis period as it is the most recent 10-year period with a substantial number of funds after our filtering procedure. If we include the year 2016, the last year for which TASS contained a full year of data at the time of this study, only 149 funds remain after removing those with missing returns.

Table 2 summarizes the availability of returns data for our analysis period. After filtering the original 16,943 funds, we are left with 1,526 funds (including both live and graveyard) that reported a return every month between January 2006 and December 2015. As a number of strategies do not have sufficient data to

[10] We also considered three additional factors to account for the nonlinear features of hedge fund returns: the 1-month constant maturity time series of Hellinger variance, skewness and kurtosis, computed at mid prices. However, the first principal component of these three factors, which captures 95% of the total variance, does not contribute significantly to the analysis, and is thus excluded from the final analysis.

[11] These are firms that have a CRSP (Center for Research in Security Prices) share code of 10 or 11; see http://www.crsp.com/products/documentation/name-history-array-codes.

Table 2
Summary of TASS Funds (Jan. 2006 – Dec. 2015)

	no. of funds			no. of funds w/o missing data		
strategy	live	graveyard	subtotal	live	graveyard	subtotal
convertible arbitrage	37	178	215	17	-	17
dedicated short bias	3	35	38	1	2	3
emerging markets	233	255	488	71	1	72
equity market neutral	122	457	579	31	1	32
event driven	163	520	683	56	2	58
fixed income arbitrage	113	305	418	32	-	32
fund of funds	1,642	4,615	6,257	585	48	633
global macro	241	692	933	39	2	41
long/short equity hedge	874	2,472	3,346	308	23	331
managed futures	302	570	872	113	2	115
multi-strategy	850	1,530	2,380	153	4	157
options strategy	21	46	67	1	-	1
other	1	662	663	32	2	34
undefined	2	2	4	-	-	-
subtotal	4,604	12,339	16,943	1,439	87	1,526

ensure the statistical validity of the second step in the Fama-MacBeth method, we will only analyze the six strategies with at least 50 funds (highlighted in gray in Table 2).

3.4 Factor Selection and Construction

As discussed in Section 4.1.2, we start with 12 risk factors compiled from previous studies. First, we must ensure that none of the factors are strongly correlated. A correlation matrix for all 12 original factors is presented in Panel A of Table 3. As we can see, there is a severe multicollinearity problem. Specifically, the three equity indices *MKT*, *MSCIW* and *MSCIE* are highly correlated. The correlation between *MSCIW* and *MKT* (*MSCIE*) is 0.954 (0.906). Figure 3 graphically illustrates their similar dynamics during the 10-year analysis period. It is evident from the figure that the three indices generally rise and fall together.

To better understand the relationship between these three indices, we conduct a PCA analysis, the results of which are summarized in Table 4. The first principal component captures 92% of the total variance, while the first two together capture 99% (Panel A). These results imply that the first two principal components are sufficient for explaining the variance in the data. We will thus reduce the information from all three indices into two factors, with the following considerations in mind: first, our two new factors should be able to explain nearly all the variance in the data without being highly correlated; second, we would like to keep the excess market return (*MKT*) as a factor, as it is the standard base factor in nearly all literature.

Panel B of Table 4 presents the coefficients of the linear combinations that make up each principal component. The first principal component is nearly an equal-weighted average of all three factors, implying that the use of one of the three factors (i.e., *MKT*) is reasonable. The second principal component is a scaled difference between *MSCIE* and the sum of *MSCIW* and *MKT*. We will thus use the difference between *MSCIW* and *MSCIE* as our second factor. Economically, the difference *MSCIE–MSCIW* (hereinafter denoted as the "*MSCI*" factor) represents the extra reward obtained from investing in the emerging markets index over the world index.

Panel B of Table 3 provides the correlation matrix for our 11 adjusted factors, and confirms that no strong correlations remain. Thus, we will use these 11 final factors in our analysis and as the basis for our second-order factors.

Table 3
Correlation Analysis of Factors

Panel A. Original 12 Factors

	MKT	*SMB*	*HML*	*MOM*	*PTFSBD*	*PTFSFX*	*PTFSCOM*	*PTFSIR*	*PTFSSTK*	*GS10*	*MSCIW*	*MSCIE*
MKT	1.000											
SMB	0.396	1.000										
HML	0.336	0.289	1.000									
MOM	-0.348	-0.169	-0.442	1.000								
PTFSBD	-0.367	-0.132	-0.145	0.113	1.000							
PTFSFX	-0.327	-0.107	-0.123	0.133	0.462	1.000						
PTFSCOM	-0.244	-0.126	-0.102	0.185	0.240	0.422	1.000					
PTFSIR	-0.342	-0.087	0.007	0.021	0.203	0.298	0.239	1.000				
PTFSSTK	-0.294	-0.032	0.176	-0.094	0.206	0.278	0.235	0.385	1.000			
GS10	-0.100	0.026	0.058	0.033	-0.119	0.088	0.043	0.247	0.054	1.000		
MSCIW	**0.954**	0.300	0.277	-0.369	-0.378	-0.322	-0.224	-0.369	-0.327	-0.016	1.000	
MSCIE	**0.788**	0.280	0.162	-0.361	-0.333	-0.271	-0.213	-0.365	-0.317	0.074	**0.906**	1.000

This table shows the correlation matrix for the 12 original factors considered in this study (see Table 1).

Panel B. Final 11 Factors

	MKT	*SMB*	*HML*	*MOM*	*PTFSBD*	*PTFSFX*	*PTFSCOM*	*PTFSIR*	*PTFSSTK*	*GS10*	*MSCI*
MKT	1.000										
SMB	0.396	1.000									
HML	0.336	0.289	1.000								
MOM	-0.348	-0.169	-0.442	1.000							
PTFSBD	-0.367	-0.132	-0.145	0.113	1.000						
PTFSFX	-0.327	-0.107	-0.123	0.133	0.462	1.000					
PTFSCOM	-0.244	-0.126	-0.102	0.185	0.240	0.422	1.000				
PTFSIR	-0.342	-0.087	0.007	0.021	0.203	0.298	0.239	1.000			
PTFSSTK	-0.294	-0.032	0.176	-0.094	0.206	0.278	0.235	0.385	1.000		
GS10	-0.100	0.026	0.058	0.033	-0.119	0.088	0.043	0.247	0.054	1.000	
MSCI	0.228	0.141	-0.078	-0.209	-0.135	-0.087	-0.112	-0.217	-0.178	0.184	1.000

This table shows the correlation matrix for the 11 final factors we will use in our empirical analysis.

Table 4
Summary of Principal Component Analysis

Panel A. PCA Summary

measure	PC1	PC2	PC3
standard deviation	1.68	0.48	0.15
proportion of variance	0.92	0.08	0.01
cumulative proportion	0.92	0.99	1.00

Panel B. Variable Factor Loadings

factor	PC1	PC2	PC3
MKT	-0.58	-0.56	0.60
MSCIW	-0.60	-0.21	-0.78
MSCIE	-0.56	0.80	0.21

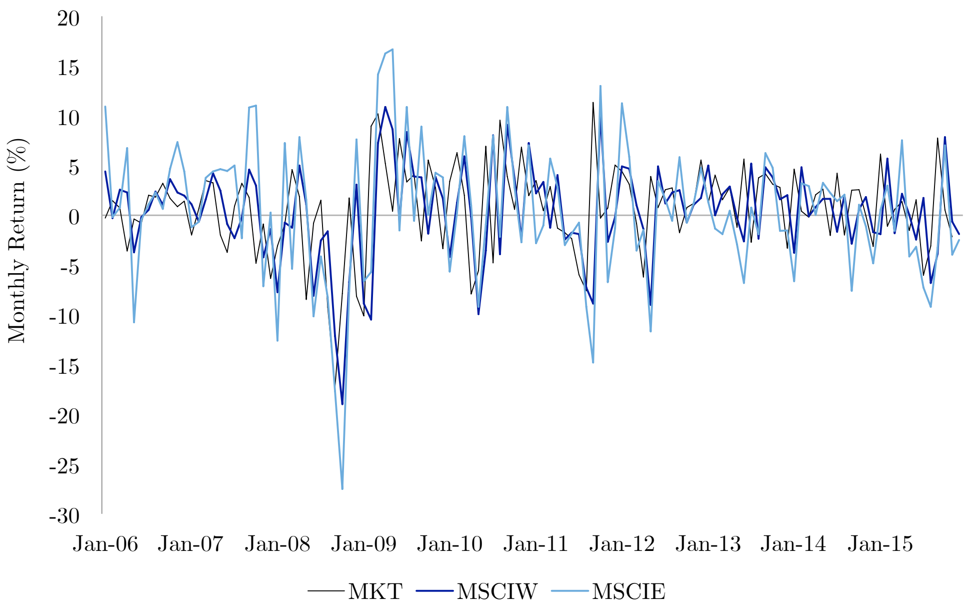
This table shows a summary of the PCA analysis for *MKT*, *MSCIW*, and *MSCIE*. Panel A summarizes how much variance each principal component captures, while Panel B displays the coefficients of the linear combinations that make up each principal component.

3.5 Data Summary

Table 5 presents summary statistics for the prepared hedge fund data by strategy (Panel A) and final 11 risk factors (Panel B).

As shown in Panel A, all strategies have positive average monthly returns, ranging from 0.317% (fund of funds) to 0.739% (emerging markets and multi-strategy). Additionally, emerging markets and options strategy funds have considerably higher standard deviations (4.412% and 4.073%, respectively). We also report the median, skewness (skew.), kurtosis (kurt.) and extreme values for each strategy.

Figure 3
Monthly Returns on Three Equity Indices



This figure displays the 120 monthly returns on the market factor (*MKT*) and MSCI indices (*MSCIW* and *MSCIE*) during our analysis period January 2006 to December 2015.

Table 5
Summary Statistics of Filtered Returns and Final Factors

Panel A. Monthly TASS Hedge Fund Returns (%)

strategy	mean	median	sd	skew.	kurt.	min.	max.
convertible arbitrage	0.559	0.590	3.149	-2.032	14.065	-18.594	11.932
dedicated short bias	0.401	0.783	3.519	-1.718	7.650	-19.069	9.041
emerging markets	0.739	1.041	4.412	-0.593	3.332	-18.465	15.514
equity market neutral	0.512	0.607	1.196	-2.327	11.943	-6.877	2.534
event driven	0.362	0.432	2.251	-1.138	4.993	-10.378	6.506
fixed income arbitrage	0.585	0.721	1.164	-3.729	25.392	-7.925	3.083
fund of funds	0.317	0.599	1.362	-1.290	3.639	-5.388	3.068
global macro	0.656	0.711	1.464	0.276	0.718	-3.260	5.672
long/short equity hedge	0.587	0.830	2.157	-0.586	1.061	-6.615	5.617
managed futures	0.629	0.671	3.215	0.199	-0.563	-5.345	9.275
multi-strategy	0.739	0.848	1.298	-1.294	5.499	-5.528	4.045
options strategy	0.542	0.980	4.073	-1.017	7.303	-20.690	15.410
other	0.724	0.755	1.722	-0.902	7.003	-8.058	7.145
undefined	-	-	-	-	-	-	-
average	0.566	0.736	2.383	-1.242	7.080	-10.476	7.603

This table presents summary statistics of the equally-weighted monthly returns for our filtered hedge funds during the 10-year sample period January 2006 to December 2015. Note that there are no funds classified as "undefined" that have a complete set of returns.

Panel B. Risk Factors

factor	mean	median	sd	skew.	kurt.	min.	max.
MKT	0.609	1.235	4.458	-0.696	1.698	-17.230	11.350
SMB	0.086	0.065	2.386	0.252	-0.112	-4.760	6.710
HML	-0.142	-0.245	2.606	-0.199	2.889	-11.100	7.760
MOM	0.089	0.340	4.983	-2.942	19.303	-34.580	12.450
PTFSBD	-2.842	-5.745	15.238	1.320	1.828	-26.630	50.500
PTFSFX	-1.421	-6.200	19.965	1.463	2.440	-27.940	69.220
PTFSCOM	0.080	-3.730	15.587	0.778	0.040	-24.650	42.870
PTFSIR	-1.926	-8.370	32.346	4.324	24.315	-35.130	221.920
PTFSSTK	-4.183	-7.020	15.113	1.982	6.278	-26.900	66.620
GS10	3.117	2.885	1.025	0.398	-1.015	1.530	5.110
MSCI	0.001	-0.050	3.165	0.177	0.013	-8.030	7.210
Average	-0.594	-2.440	10.625	5.243	0.623	-19.584	45.611

This table presents summary statistics of each risk factor during the 10-year sample period January 2006 to December 2015.

Several strategies display significant skewness and kurtosis. All strategies except global macro and managed futures display negative skewness; in other words, their return distributions are skewed to the left. The kurtosis of most strategies (except global macro, long/short equity hedge and managed futures) is greater than 3, indicating that the returns distributions are leptokurtic, or more concentrated around the mean than the normal distribution. The mixed features of negative skewness and large positive kurtosis provide strong evidence that hedge fund returns are not normally distributed.

Panel B of Table 5 provides summary statistics for the risk factor data. Although most of the risk factors are standard in the recent literature, the dynamics of several factors are worth noting (Panel B). *MOM* has significantly negative skewness and positive kurtosis, and *PTFSIR* has significantly positive skewness and kurtosis. In general, the PTFS options-based factors display large negative mean values and extremely high standard deviations.

4. Estimation Methodology

In this section, we describe the four models we analyze in this paper, our estimation procedure based on an augmented Fama-MacBeth method, and the two measures we use to evaluate model performance.[12]

4.1 Our Models

In our analysis, we investigate four models. The first is the Fama-French three-factor model (hereinafter FFM). FFM is used as our base case for comparative purposes, consistent with most asset pricing literature. The second model is the linear model (LM), which consists of all 11 factors selected in Section 3.4, including Fama and French's three factors. Next, we add the second moment of each factor in our quadratic model (QM), bringing the total number of factors to 22. Finally, we augment QM with all the interaction terms between factors in our complete second-order model (CM); these 55 cross-terms bring the total number of factors to 77. As QM and CM are complex, due to the large number of risk factors, we use a LASSO regression step to reduce the dimensionality and denote the two resulting models as QM-LASSO and CM-LASSO, respectively.

Table 6 summarizes these four models (hereinafter referred to by their abbreviations), indicating which types of factors and the maximum number of factors each model includes. Factors in the four models are nested and build upon each other. However, although the maximum number of factors increases with each model, the actual number of factors used in our analysis for QM-LASSO and CM-LASSO depends on the LASSO dimensionality reduction step.

[12]See Appendix B for a detailed review of the econometric methods we use.

Table 6
Maximum Number of Factors in Each Model

model	Fama-French three factors (3)	other linear factors (8)	quadratic factors (11)	interaction terms (55)	maximum no. of factors
FFM	✓	–	–	–	3
LM	✓	✓	–	–	11
QM-LASSO	✓	✓	✓	–	22
CM-LASSO	✓	✓	✓	✓	77

Note. For QM-LASSO and CM-LASSO, the number of actual risk factors used is reduced through a dimensionality reduction step (see Step 0 in Section 5.2).

4.2 Estimation Procedure

For FFM and LM, we directly implement the Fama-MacBeth two-step method by skipping the factor selection step, as the number of factors in these two models is relatively small. For QM-LASSO and CM-LASSO, we introduce an extra dimensionality reduction step based on the LASSO regression (denoted as Step 0), prior to Fama-MacBeth's two steps.

Note that we will not penalize the three Fama-French factors to ensure that the LASSO regression chooses them. They serve as the control factors in our analysis and allow us to directly compare our models with the FFM benchmark (our base case). Additionally, we set the λ penalty factor separately for each QM-LASSO and CM-LASSO regression in Step 0 to guarantee that each model/strategy combination chooses approximately 11 factors, on average. Thus, both models will be consistent with LM, in terms of the number of factors. See Table 17 in Appendix C for the specific lambda values used in the analysis.

Finally, we use a five-year rolling window in the Fama-MacBeth method (i.e., $m = 60$). Only 10 years of monthly data are available in this study. Hence, there is a tradeoff between how much data is used in the Step 1 time series regressions and in the Step 2 cross-sectional regressions. A five-year window is chosen to ensure a sufficient amount of sample data for each step.

Our final three-step estimation procedure for each of the four models is described in detail as follows:

- Let n_s be the number of funds of strategy s (refer to the rightmost column of Table 2). Additionally, denote each of the 120 months in our 10-year sample period as $t = 1, 2, \ldots, 120$. We are interested in the first 60 five-year windows in the sample period.[13] For each five-year window, starting with January 2006 to December 2010 ($t = 1$ to 60), complete Steps 0-2. In other words, roll the window forward by one month after each iteration, until reaching the window of December 2010 to November 2015 ($t = 60$ to 119).

- **Step 0: Factor selection** – For each strategy, estimate the following LASSO regression over the given five-year window to determine which risk factors to move forward with:

$$\overline{R}_s - r_f = \alpha_s 1 + F\beta_s + \varepsilon_s \quad \text{for each strategy } s, \tag{4}$$

  where $\overline{R}_s$ is the 60×1 vector of average monthly returns across all funds of strategy s for the current five-year window, r_f is the 60×1 vector of corresponding risk-free rates, α_s is the (scalar) intercept, 1 is the 60×1 vector of 1s, F is the $60 \times K$ vector of risk factors and K is the maximum number of factors in the given model (see Table 6), β_s is the $K \times 1$ coefficient vector, and ε_s is the 60×1 vector of residuals.

  We run our estimation procedure for each strategy separately, as we assume that each strategy has different risk exposures to the factors. Denote the number of selected risk factors for strategy s as k_s. Note that this step is only performed for QM-LASSO and CM-LASSO, which each have 360 regressions (60 five-year windows for each of the six strategies). Hence, a total of 720 regressions are estimated in Step 0.

- **Step 1: Time series regressions** – For each strategy, estimate the following OLS regression for each individual fund using the factors chosen by the LASSO regression in Step 0:[14]

$$R_i - r_f = \alpha_i 1 + F_s\beta_i + \varepsilon_i \quad \text{for each fund } i, \tag{5}$$

  where R_i is the 60×1 vector of monthly returns on fund i for the current five-year window, r_f is the 60×1 vector of corresponding risk-free rates, α_i is the (scalar) intercept, 1 is the 60×1 vector of 1s, F_s

[13] Theoretically, there are 61 five-year windows in the 10-year period. However, we ignore the last window that ends with December 2015 ($t = 120$), as there is no $(t+1)$ month, which is required for Step 2.

[14] While LASSO has variable selection capabilities, the regression method eliminates the economic interpretation of the magnitude of the coefficients, as it shrinks the coefficients towards zero. Thus, we use LASSO only for its dimensionality reduction capabilities in Step 0 and rely on OLS to obtain our β and γ coefficients in the latter two steps.

is the $60 \times k_s$ vector of the risk factors selected for strategy s in Step 0, β_i is the $k_s \times 1$ coefficient vector, and ε_i is the 60×1 vector of residuals.

For each fund, 240 regressions are estimated (60 five-year windows for each of the four models), resulting in 327,840 total estimated regressions in Step 1.

- **Step 2: Cross-sectional regressions** – For each strategy, estimate the following cross-sectional regression using the βs derived from Step 1 through month t as the new predictor variables for month $(t+1)$:

$$R_s^* - r_f^* 1 = \beta_s^* \gamma_s + \varepsilon_s^* \quad \text{for each strategy } s, \tag{6}$$

where R_s^* is the $n_s \times 1$ vector of monthly returns for month $(t+1)$ of the n_s funds of strategy s, r_f^* is the (scalar) risk-free rate for month $(t+1)$, 1 is the $n_s \times 1$ vector of 1s, β_s^* is the $n_s \times k_s$ vector containing the k_s β coefficients through month t for each fund from Step 1, γ_s is the $k_s \times 1$ coefficient vector, and ε_i^* is the $n_s \times 1$ vector of residuals for month $(t+1)$.

For each model and strategy, equation 6 produces a set of γ coefficients for each of the 60 rolling windows. Hence, there are a total of 1,440 $(4 \times 6 \times 60)$ regressions estimated in Step 2.

Table 7 summarizes the total number of regressions estimated in our analysis using this procedure.

Table 7
Total Number of Regressions

model	step 0	step 1	step 2	subtotal
FFM	–	$60 \times 1{,}366$	6×60	82,320
LM	–	$60 \times 1{,}366$	6×60	82,320
QM-LASSO	6×60	$60 \times 1{,}366$	6×60	82,680
CM-LASSO	6×60	$60 \times 1{,}366$	6×60	82,680
subtotal	720	327,840	1,440	330,000

Notes. 6 is the total number of strategies considered in this study; 60 is the total number of 5-year windows in our 10-year sample period; 1,366 is the total number of funds across the six strategies (see Table 2 in Section 3.3 for a detailed breakdown).

4.3 Measures of Model Performance

Finally, we introduce the two measures we use to evaluate and compare the performance of the four models described in Section 5.1.

The variation in hedge fund returns can be broken into its systematic and residual risk components. The systematic (residual) risk is the risk that can (cannot) be explained by the model's factors. Like Bali et al. (2012), we use this decomposition to explore micro-level explanations for different hedge fund strategies. The comparison of results between models with and without second-order terms provides a direct measure of the explanatory power of the second-order terms.

Our first performance measure is the coefficient of determination (R^2), which quantifies the ratio of systematic to residual risk. The higher the R^2, the stronger the overall performance of the model. Interpretation of R^2 is different for each step in our analysis. In Step 1, the R^2s of the time series regressions provide a measure of the explanatory power of the risk factors. A high R^2 suggests that the fund's returns are closely linked to the factors in the model. On the other hand, the R^2s of the cross-sectional regressions in Step 2 measure the predictive power of the factor betas. More specifically, they quantify the factors' performance in predicting the cross-sectional variation in one-month-ahead hedge fund returns.

The second performance measure involves examining the distributions of the residuals from the cross-sectional regressions in Step 2. Theoretically, the residuals ε_s^* in Equation 6 should not be significantly different from zero if the risk factors are sufficient for explaining the predictive power of the models.

These two performance measures complement each other. As R^2 is an increasing function of the number of explanatory variables, it rewards models with a higher number of factors. However, a model with too many factors induces overfitting. On the other hand, the distribution of ε_s^* allows us to determine whether factors are still missing, regardless of the number of factors already present.

5. Empirical Results

In this section, we present empirical results based on our three-step estimation procedure. We first present the LASSO factor selection results from Step 0 in Section 5.1. Next, we report each strategy's exposures to risk factors, as measured by the explanatory power of the Step 1 time series regressions (Section 5.2) and predictive power of the Step 2 cross-sectional regressions (Section 5.3). The remaining sections further analyze the results from Step 2. In Section 5.4, we present the significance of the γ coefficients. In Section 5.5, we analyze whether the risk factors considered are sufficient. In Section 5.6, we perform an additional analysis to account for intercept bias. The main findings are summarized in Section 5.7. The econometric analyses were programmed and implemented in R.

5.1 Risk Factors Selected by Lasso Regression

We first discuss the factor selection results in Step 0 for each model (equation 4). Table 8 summarizes the total number of times each factor is selected in QM-LASSO (Panel A) and CM-LASSO (Panel B), compiled across all six strategies and five-year time windows. Note that a darker shade of orange corresponds to a higher number, for easier comparison. As discussed in the estimation procedure detailed in Section 4.2, each of the 720 LASSO regressions in Step 0 chooses 11 factors, on average, with the three Fama-French factors guaranteed to be chosen for consistency with LM.

There are four important findings based on the results in Table 8. First, each of the quadratic risk factors are selected in QM-LASSO for over 20% of the 360 regressions, ranging from 74 times for SMB^2 to 254 times for MOM^2. Moreover, several quadratic factors are chosen more often than their linear counterparts: MOM^2, $PTFSCOM^2$, $PTFSSTK^2$ and $MSCI^2$.

Second, the majority of the interaction terms are selected in CM-LASSO. In fact, the following interaction terms are chosen significantly more often than their linear or quadratic counterparts: $PTFSBD \times PTFSSTK$ (201 times), $HML \times MOM$ (179), $HML \times PTFSCOM$ (157), $PTFSFX \times PTFSCOM$ (148), $SMB \times MSCI$ (142), and $MKT \times HML$ (130).

Third, the chances of the linear factors being selected decline dramatically as the number of potential factors increases, as evidenced by the significant difference between the results reported in the first row of Panel A (QM-LASSO with 22 potential factors) and those in the first row of Panel B (CM-LASSO with 77 potential factors). As more potential factors are included in the model, some linear factors are replaced by their second-order terms.

Finally, the two second-order models reveal that some factors seem to be more important than others. For example, QM-LASSO selects *PTFSSTK* and its quadratic term 202 and 208 times, respectively; its interaction term with *PTFSBD* is selected by CM-LASSO 201 times. *MOM* is also selected frequently. QM-LASSO selects the linear *MOM* factor 217 times and its quadratic term 254 times (the highest of all quadratic terms). CM-LASSO selects $HML \times MOM$ 179 times (the second highest of all interaction terms).

The frequent selection of quadratic and interaction terms in the LASSO regressions is not sufficient to prove their importance, however. In Sections 5.2 and 5.3, we examine how the selected factors affect the explanatory and predictive power of the models, respectively. More importantly, we report whether the selected factors produce significant γ coefficients in Section 5.4.

As a final remark, we have only presented and compared the factor selection results for different models thus far. In Section 5.4, we will break down these results by strategy and present them along with the γ significance

Table 8
LASSO Variable Selection by Model – Step 0

Panel A. QM-Lasso

	MKT	SMB	HML	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	GS10	MSCI
Linear	360	360	360	217	233	194	153	173	202	103	103
Quadratic	161	74	101	254	137	184	187	124	208	79	157

Panel B. CM-Lasso

	MKT	SMB	HML	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	GS10	MSCI
Linear	360	360	360	26	22	45	45	61	62	20	28
Quadratic	-	7	11	5	-	-	35	15	31	24	30

Interaction	MKT	SMB	HML	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	GS10	MSCI
MKT											
SMB	14										
HML	130	39									
MOM	-	-	179								
PTFSBD	12	25	9	7							
PTFSFX	-	109	59	58	-						
PTFSCOM	-	51	157	35	34	148					
PTFSIR	34	22	46	14	75	11	18				
PTFSSTK	14	1	29	38	201	-	-	5			
GS10	-	-	-	68	84	93	88	47	18		
MSCI	10	142	26	44	28	-	33	37	11	40	

This table summarizes the total number of times each factor is chosen in Step 0 (equation 4) for QM-LASSO (Panel A) and CM-LASSO (Panel B). Note that there are 360 total LASSO regressions run in Step 0 for each model. A dash "-" indicates that the factor was not chosen in any of the regressions. A darker shade of orange corresponds to a higher number.

results.

5.2 Explanatory Power of Time Series Regressions

To further investigate the importance of the quadratic and interaction terms, we examine the overall explanatory power of QM-LASSO and CM-LASSO compared to FFM and LM, as measured by the R^2s from Step 1 (equation 5). Table 9 presents the average R^2 across the 327,840 time series regressions run in Step 1 (see Table 7) for each model and strategy. Figure 4 presents boxplot representations of the data to provide information on the variability of the R^2 distributions.

Table 9
R^2 (adj. R^2) Estimates – Step 1

	model			
strategy	FFM	LM	QM-LASSO	CM-LASSO
emerging markets	0.344 (0.327)	0.572 (0.528)	0.520 (0.471)	0.502 (0.451)
event driven	0.390 (0.374)	0.529 (0.481)	0.536 (0.489)	0.533 (0.485)
fund of funds	0.375 (0.359)	0.533 (0.485)	0.517 (0.468)	0.509 (0.459)
long/short equity hedge	0.371 (0.355)	0.509 (0.459)	0.514 (0.465)	0.497 (0.446)
managed futures	0.105 (0.082)	0.349 (0.283)	0.340 (0.273)	0.259 (0.184)
multi-strategy	0.242 (0.222)	0.445 (0.388)	0.433 (0.375)	0.414 (0.354)
average	0.305 (0.287)	0.490 (0.438)	0.477 (0.423)	0.452 (0.397)

This table summarizes the R^2 values of all the regressions in Step 1, averaged across each model and strategy. The parenthetical values are the adjusted R^2 values.

As Table 9 shows, LM, QM-LASSO and CM-LASSO have much higher average R^2s than FFM for all strategies, even after accounting for the number of factors (adjusted R^2). This result is consistent with those reported previously in the literature: the three factors in FFM are not sufficient to fully explain average cross-sectional hedge fund returns. However, we find no evidence that QM-LASSO or CM-LASSO have more overall explanatory power than LM.[15]

Perhaps surprisingly, CM-LASSO yields a lower average R^2 than LM and QM-LASSO, which have similar R^2 distributions (Figure 4). In theory, CM-LASSO should outperform the other models since both LM and QM-LASSO are nested within CM-LASSO. Our interpretation of this result is that the inclusion of 55 interaction terms greatly increases the estimation complexity, making it more difficult for LASSO to choose just 8 factors (in addition to the three Fama-French factors). With limited returns data, the estimation efficiency of CM-LASSO declines, consistent with the greater variability of its R^2 distribution. Hence, CM-LASSO may not necessarily provide greater explanatory power, given the 10 years of monthly returns in this study.

Interestingly, as Figure 4 illustrates, more variation in average R^2s exists between strategies than between models. We believe that this difference suggests two points: (1) the relatively small variation between models demonstrates that our procedure in Step 0 does a relatively good job selecting the most important factors and (2) the relatively large variation between strategies indicates that the returns on different strategies are highly heteroscedastic.

[15] Note that LM, QM-LASSO and CM-LASSO all have, on average, approximately 11 factors. Thus, we can directly interpret a model with a higher R^2 as having greater explanatory power.

Table 10
R^2 (adj. R^2) Estimates – Step 2

	model			
strategy	FFM	LM	QM-LASSO	CM-LASSO
emerging markets	0.405	0.687	0.699	0.667
	(0.390)	(0.655)	(0.668)	(0.633)
event driven	0.367	0.736	0.734	0.684
	(0.351)	(0.709)	(0.707)	(0.652)
fund of funds	0.283	0.422	0.404	0.396
	(0.264)	(0.363)	(0.343)	(0.334)
long/short equity hedge	0.232	0.361	0.358	0.342
	(0.212)	(0.296)	(0.293)	(0.275)
managed futures	0.459	0.676	0.688	0.684
	(0.445)	(0.643)	(0.656)	(0.652)
multi-strategy	0.317	0.506	0.497	0.486
	(0.299)	(0.456)	(0.446)	(0.434)
average	0.344	0.565	0.563	0.537
	(0.327)	(0.520)	(0.519)	(0.497)

This table summarizes the R^2 values of all the regressions in Step 2, averaged across each model and strategy. The parenthetical values are the adjusted R^2 values.

Finally, R^2 can be interpreted as a measure of systematic risk. A large R^2 value implies that the model's factors have largely captured the variation in returns. The difference between total and systematic variation is the residual risk, which is not explained by the model. Hence, the large variation between strategies implies a large variation in residual risks, suggesting that the level of difficulty of modeling returns varies across different strategies.

5.3 Predictive Power of Cross-Sectional Regressions

As discussed previously, the average cross-sectional hedge fund returns in month $(t+1)$ are regressed on the βs (through time t) obtained in Step 1. Hence, we can use the R^2s from the Step 2 cross-sectional regressions (equation 6) to assess the predictive power of the various models. Table 10 presents the average R^2 of the 1,440 cross-sectional regressions run in Step 2 (see Table 7) for each model and strategy. Again, Figure 5 provides boxplot representations of the data.

Similar to those of the Step 1 time series regressions, the R^2s of the Step 2 cross-sectional regressions for LM, QM-LASSO and CM-LASSO are significantly higher than those for FFM for all the considered strategies. The average R^2s for LM, QM-LASSO and CM-LASSO are 0.565, 0.563 and 0.537, respectively. The difference between any two of these three models is not statistically significant, implying a lack of evidence that second-order models have greater predictive power than LM. This result strongly corroborates the findings on explanatory power reported in Section 5.2.

Our results also indicate that the models' predictive powers vary across strategies. In general, the average R^2 for funds based on directional strategies (emerging markets, event-driven and managed futures) is higher than those based on semi-directional strategies (fund of funds, long/short equity hedge and multi-strategy). For example, the average R^2 for all event-driven hedge funds when using LM is 0.736. In contrast, the average R^2 for long/short equity hedge funds when using LM is only 0.361. We can thus conclude that the predictive power for directional funds is higher than for semi-directional funds. These results are consistent with the R^2 values from Step 1 and with our hypotheses.

Figure 4
R^2 Distributions – Step 1

Panel A. By Model **Panel B. By Strategy**

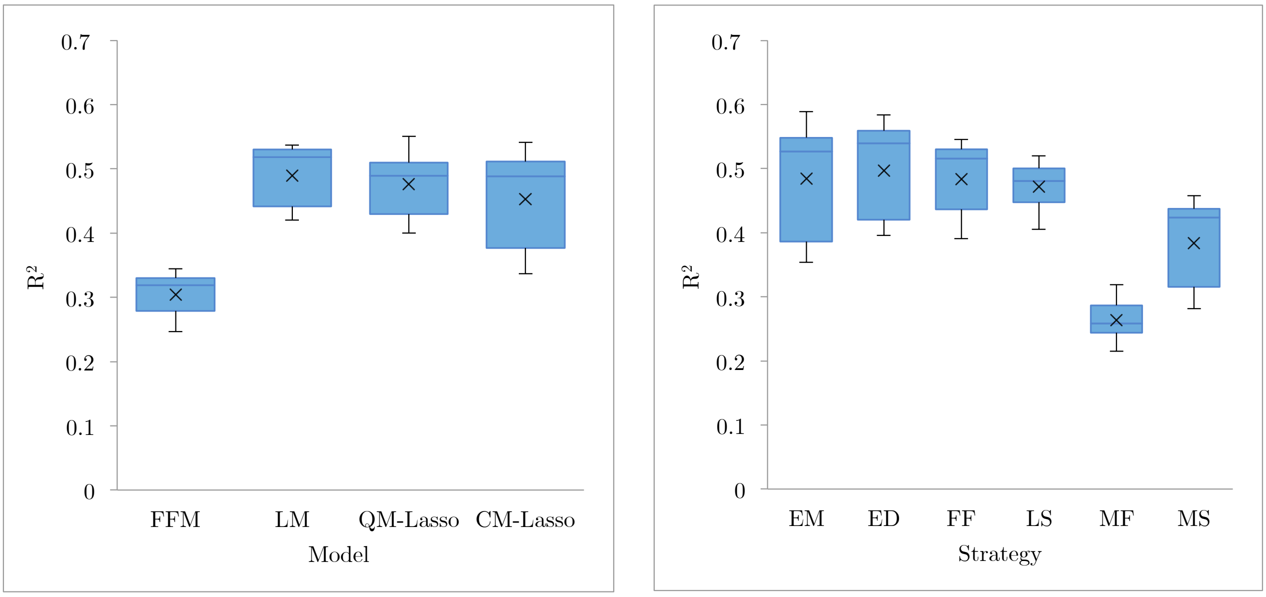
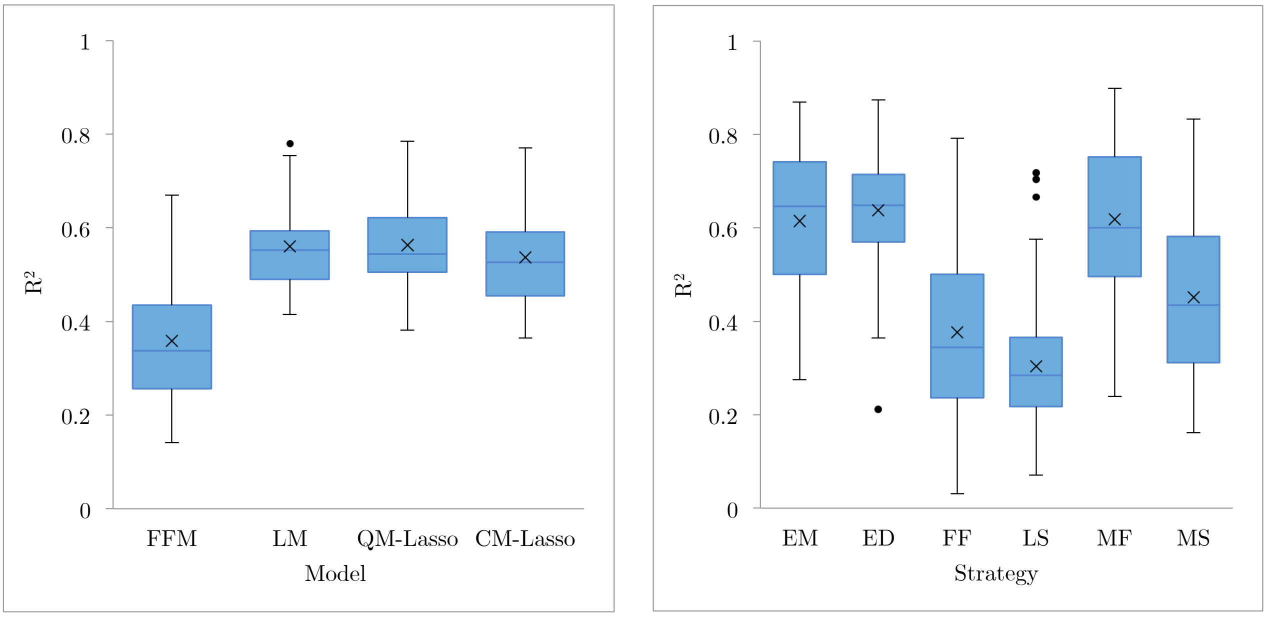


Figure 5
R^2 Distributions – Step 2

Panel A. By Model **Panel B. By Strategy**



5.4 Factors Important In Explaining Hedge Fund Risk Premia

In this section, we further investigate the predictive power of the models by examining which β factors (γ coefficients) are statistically significant in the Step 2 cross-sectional regressions (equation 6). A significant γ suggests a link between the fund's risk exposure to that factor and its one-month-ahead cross-sectional return. We first present the results for each model. Then, we break down the results by strategy.

Table 11 presents the percentage of γ coefficients that are significant for each model at the 95% significance level, compiled across all six strategies. Again, a darker shade of orange corresponds to a higher percentage. The percentage is calculated using the number of times each factor is chosen in Step 0 as the denominator (see Table 8), rather than the total number of regressions run in Step 2 (360 for each model).

Our first observation is that *MKT* is overall the most important factor. *MKT*, which is present in all 360 regressions, has the greatest proportion of significant γ coefficients among all factors in FFM, LM and QM-LASSO, and also produces a high percentage of significant γs in CM-LASSO (64.2%). Second, *SMB*, *HML*, *MOM*, *GS10* and *MSCI* are shown to be the next most important factors. These factors (including their quadratic and interaction terms) each produce at least one γ coefficient that is significant in over 50% of the regressions they are selected in (in either FFM, LM, QM-LASSO and/or CM-LASSO). Third, we find that a number of quadratic and interaction terms affect one-month-ahead cross-sectional hedge fund returns. Among them, factors involving *PTFSSTK* and *PTFSIR* are the most notable (particularly in CM-LASSO).

We also note that the interaction terms seem to be more important than the quadratic factors when included. In CM-LASSO, while several quadratic factors have a large percentage of significant γs (up to 57.1% for SMB^2), there is always an interaction term involving the same factor that has a higher percentage of significant γs (e.g., 67% for $SMB \times PTFSCOM$), with the exception of *MKT*.

As noted at the beginning of this subsection, the percentage of significant γs reported in Table 11 depends on how many times that factor is selected by the LASSO regression in Step 0. To provide further insight, we would like to see which factors are chosen the most frequently in Step 0 *and* produce a large percentage of significant γ coefficients in Step 2. To this end, Figure 6 graphically combines the factor selection results from Table 8 (represented by the height on the y-axis of each bubble) and γ significance results from Table 11 (represented by the size of each bubble). For comparison purposes, we also summarize the most important factors from Figure 6 in Table 12. Note that we exclude the three linear Fama-French factors in Table 12 as they are not penalized in the LASSO regression.

The following observations emerge. First, *MKT* appears to be the most important factor among Fama and French's three factors (*SMB* and *HML* produce a similar percentage of significant γs). Second, *MSCI* and *GS10* seem to be the most important factors in LM, while *MOM* and the PTFS options-based factors are the most important in LM and QM-LASSO. Third, only the interaction terms are both chosen a considerable percentage of the time and consistently produce significant γs in CM-LASSO, indicating their relative importance.

To address the strategy-specific set of proposed research questions, Figure 7 breaks down the results in Figure 6 by strategy.[16] Since this study focuses on the importance of the second-order terms, we only report the results for QM-LASSO and CM-LASSO for easy comparison. Note that the three Fama-French factors — *MKT, SMB* and *HML* — are included in all of the final models and consequently selected exactly 60 times for every strategy.

As Figure 7 illustrates, there are no two strategies that select the same set of factors. For example, QM-LASSO selects the linear *GS10* term only 8 out of 60 times for managed futures funds, which is much lower than the proportion for event-driven and long/short equity hedge funds. Similarly, QM-LASSO selects the linear *MSCI* term 50% of the time (30 times) for multi-strategy funds, but only 7 times for event-driven and long/short equity hedge funds. These results are well-expected and consistent with all other observations thus far.

There are several discernible patterns, however. For example, of the 8 linear factors that are penalized in Step 0, *PTFSBD* and *MOM* are selected with high frequencies in QM-LASSO. *PTFSBD* is the most frequently selected linear factor for emerging markets (50 out of 60 times) and event-driven funds (38 times), and tied

[16] The numerical values are provided in Table 18 in Appendix C.

Table 11
Percentage of Significant Gamma (γ) Coefficients by Model – Step 2

Panel A. FFM

	MKT	SMB	HML
Linear	73.9%	59.4%	55.3%

Panel B. LM

	MKT	SMB	HML	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	GS10	MSCI
Linear	65.0%	43.9%	41.1%	42.2%	44.7%	43.6%	41.7%	37.5%	38.9%	45.3%	50.3%

Panel C. QM-Lasso

	MKT	SMB	HML	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	GS10	MSCI
Linear	61.9%	39.4%	41.9%	47.6%	44.3%	43.2%	45.0%	35.7%	44.5%	50.4%	50.7%
Quadratic	36.1%	29.1%	37.0%	35.3%	44.6%	56.2%	36.3%	39.5%	40.2%	40.5%	38.5%

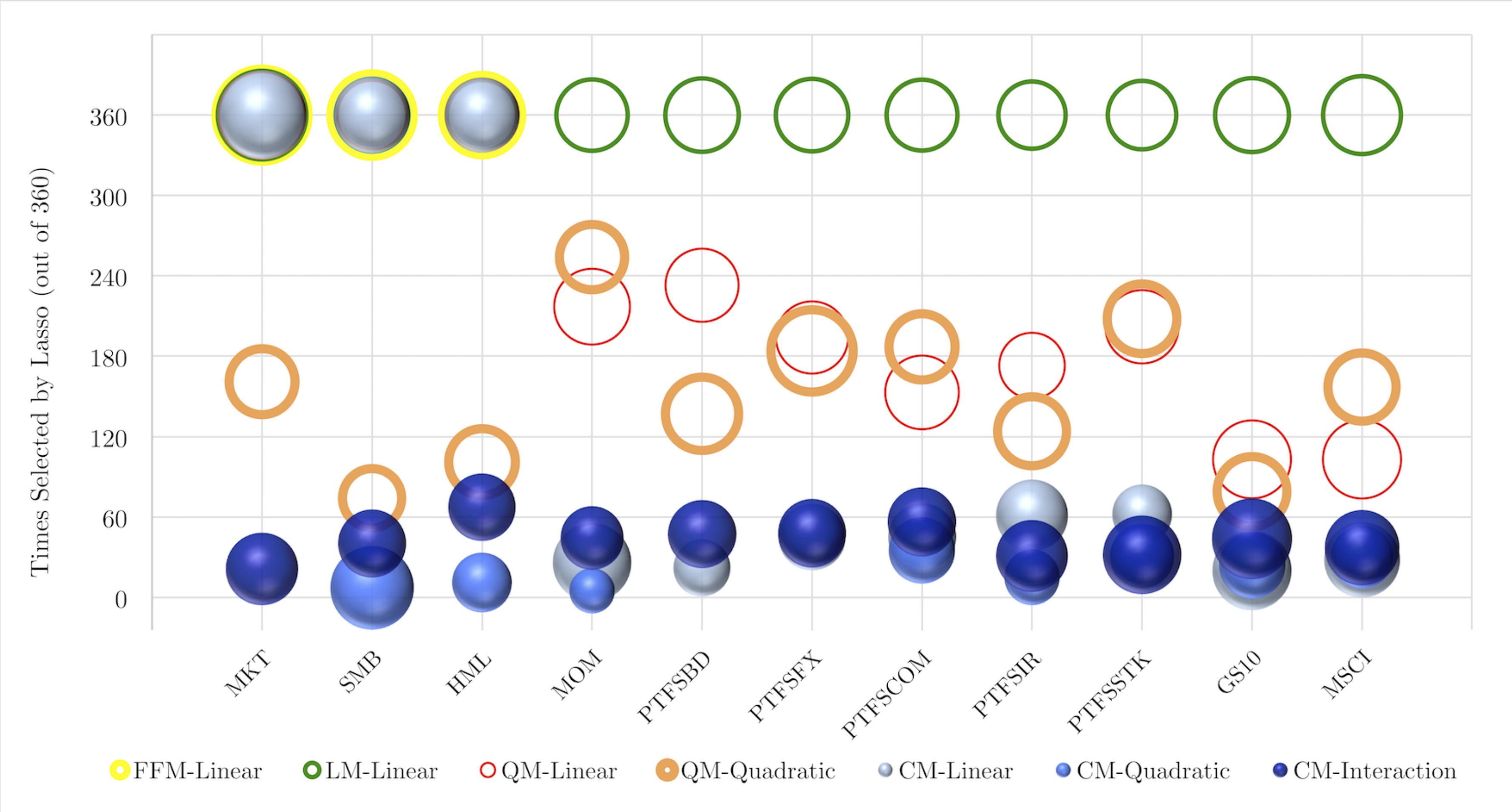
Panel D. CM-Lasso

	MKT	SMB	HML	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	GS10	MSCI
Linear	64.2%	47.8%	45.6%	50.2%	26.2%	36.5%	37.8%	42.4%	29.4%	51.7%	47.0%
Quadratic	-	57.1%	29.2%	16.7%	-	-	36.0%	25.0%	33.7%	36.0%	32.2%

Interaction	MKT	SMB	HML	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	GS10	MSCI
MKT											
SMB	35.7%										
HML	36.8%	58.4%									
MOM	-	-	34.7%								
PTFSBD	58.3%	29.0%	42.5%	27.8%							
PTFSFX	-	40.9%	34.4%	24.0%	-						
PTFSCOM	-	67.0%	30.3%	24.6%	41.2%	41.8%					
PTFSIR	58.7%	28.0%	27.8%	3.8%	39.2%	36.4%	22.9%				
PTFSSTK	47.6%	0.0%	29.4%	66.7%	37.7%	-	-	80.0%			
GS10	-	-	-	46.1%	36.2%	51.6%	43.7%	65.5%	66.7%		
MSCI	20.8%	45.4%	41.6%	34.7%	36.6%	-	38.4%	59.5%	75.0%	61.1%	

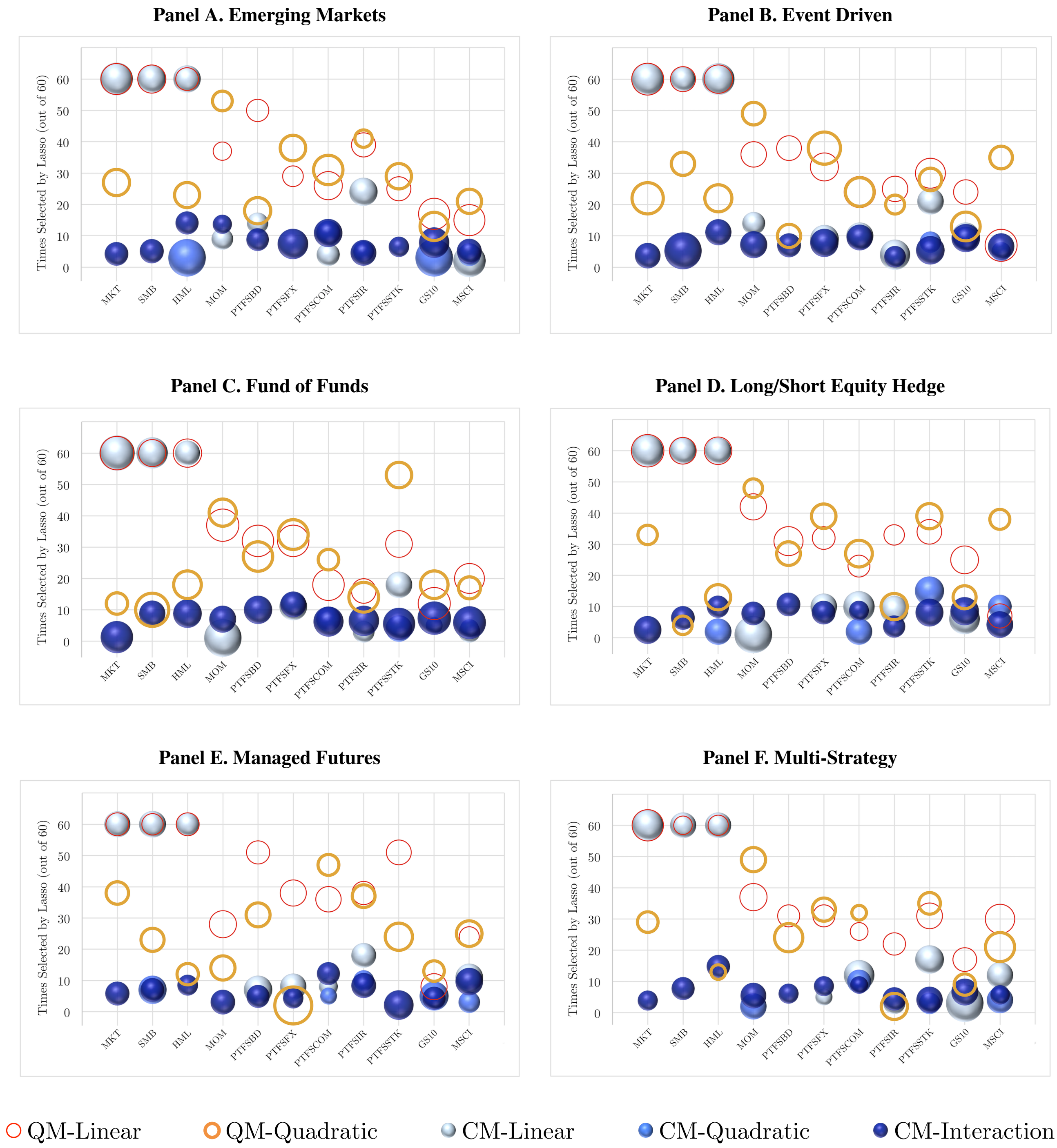
This table shows the percentage of γ coefficients that are significant at the 95% level in the Step 2 cross-sectional regressions (equation 6), of the number of times each factor is chosen in Step 0, averaged across all 6 strategies. Note that a darker shade of orange corresponds to a higher percentage.

Figure 6
Lasso and Gamma (γ) Significance Results by Model – Steps 0 & 2



This figure displays the LASSO factor selection results (Table 8) and γ significance results (Table 11) for all four models, compiled across all six strategies. The higher the bubble, the more times the factor was chosen. The larger the bubble, the higher the percentage of significant γs the factor produced. "-Linear" and "-Quadratic" represent the 11 linear and quadratic factors in each model, respectively; "CM-Interaction" represents the average over the 10 interaction terms involving the given factor, standardized on a scale of 360 on the y-axis for comparison purposes. Note that the figure summarizes a total of 360 Step 0/2 regressions (equations 4 and 6).

Figure 7
LASSO and Gamma (γ) Significance Results by Strategy – Steps 0 & 2



This figure displays the LASSO factor selection results and γ significance results for QM-LASSO and CM-LASSO for each strategy. The higher the bubble, the more times the factor was chosen. The larger the bubble, the higher the percentage of significant γ coefficients the factor produced. "-Linear" represents the 11 linear factors in the respective models; "-Quadratic" represents the 11 quadratic factors in the respective models; "CM-Interaction" represents the average over the 10 interaction terms involving the given factor, standardized on a scale of 60 on the y-axis for comparison purposes. Note that each graph summarizes a total of 60 Step 0/2 regressions (equations 4 and 6). The numerical values are provided in Table 18 in Appendix C.

with *PTFSSTK* as the most frequently selected for managed futures funds (51 times). On the other hand, *MOM* has the highest selection rate for the three semi-directional strategies: long/short equity hedge (42 times), fund of funds (37) and multi-strategy (37). This particular division emphasizes the differences between directional and semi-directional funds. Regarding the quadratic terms, MOM^2 is chosen the most frequently for four of the strategies – emerging markets (53 times), event-driven (49), multi-strategy (49) and long/short equity hedge (48) – while $PTFSSTK^2$ is the most frequently selected quadratic term for funds of funds (53 times) and $PTFSCOM^2$ is the most frequently selected for managed futures funds (47 times).

One final point of interest in Figure 7 is that both the linear and quadratic terms are rarely selected in CM-LASSO for all six strategies. This result implies that for all strategies, interaction terms play an important role. In particular, $HML \times MOM$ and $PTFSBD \times PTFSSTK$ are the two most frequently selected terms overall (including linear and quadratic terms) for emerging markets, event-driven and long/short equity funds by a considerable margin. Moreover, $HML \times MOM$ is also the most frequently selected term for multi-strategy funds and $PTFSBD \times PTFSSTK$ is the most frequently selected for funds of funds. The only strategy for which neither of these two factors is the most popular is managed futures. The frequency and consistency with which $HML \times MOM$ and $PTFSBD \times PTFSSTK$ are chosen underscore their significance.

5.5 Mean Residuals: Do We Need More Factors?

In this section we assess whether our risk factors can fully explain the predictive power documented in Section 5.3 by conducting significance tests on the residuals (ε_s^*) of each of the cross-sectional regressions in Step 2 (equation 6). As discussed in Section 4.3, if the factors in the model can fully explain the predictive power, the mean of these residuals should be zero.

For each model and strategy, the Step 2 cross-sectional regressions produce 60 sets of ε_s^*, each with n_s values (see Table 2). For each set of ε_s^*, we determine if the mean residual is significant at the 95% level. Table 13 presents the average residuals of the cross-sectional regressions for each model and strategy, as well as the percentage of mean residuals that are significantly non-zero.

With regards to the different models, LM, QM-LASSO and CM-LASSO perform better than FFM for all strategies except event-driven (for which the percentage of significant non-zero mean residuals is the same for all four models) and multi-strategy (for which there is a slight increase in the percentage for LM, QM-LASSO and CM-LASSO).

With regards to the different strategies, there are once again large variations. All of the models excel at explaining the predictive power for directional hedge funds. In particular, fewer than 5% of emerging markets and event-driven funds have mean residuals that differ significantly from zero. Although managed futures funds do not do as well, the percentage of significant non-zero mean residuals is relatively low (ranging from 8.3% to 11.7%) if we disregard FFM, whose factors we previously determined to be insufficient. In contrast, the models

Table 12
Frequently Chosen and Significant Factors by Model – Steps 0 & 2

LM		QM-LASSO		CM-LASSO	
factor	(% Sig.)	factor	# selected (% sig.)	factor	# selected (% sig.)
MOM	(42.2%)	*MOM*	217 (47.6%)	$SMB \times PTFSFX$	109 (40.9%)
PTFSBD	(44.7%)	*PTFSBD*	233 (44.3%)	$SMB \times MSCI$	142 (45.4%)
PTFSFX	(43.6%)	*PTFSSTK*	202 (44.5%)	$PTFSFX \times GS10$	93 (51.6%)
PTFSCOM	(41.7%)	$PTFSSTK^2$	208 (40.2%)	$PTFSFX \times PTFSCOM$	148 (41.8%)
GS10	(45.3%)			$PTFSCOM \times GS10$	88 (43.7%)
MSCI	(50.3%)				

This table shows which factors are chosen > 200 times in QM-LASSO or > 80 times for CM-LASSO in the Step 0 LASSO regressions *and* produce > 40% significant γ coefficients in Step 2 (parenthetical values) for LM, QM-LASSO and CM-LASSO. Note that the three linear Fama-French factors are excluded and that all factors are chosen exactly 360 times in LM.

do quite poorly for the three semi-directional strategies. In particular, multi-strategy funds have the highest percentages of significant non-zero mean residuals (about 70%) for all four models, by a large margin.

We believe that the failure to explain the predictive power of the cross-sectional regressions is partly due to missing factors. Although LM includes the standard hedge fund risk factors, our results strongly suggest that LM, QM-LASSO and CM-LASSO are all insufficient to explain the cross-sectional returns on hedge funds that utilize multiple investment styles and positions.

5.6 Further Analysis Accounting For Intercept Bias

The large percentage of non-zero mean residuals reported in Section 5.5 for certain strategies implies that the assumption of a non-zero intercept in Step 2 may be invalid. To investigate the issue, we add an intercept (α_s^*) to equation 6 and estimate the following modified version of the cross-sectional regressions in Step 2:

$$R_s^* - r_f^* 1 = \alpha_s^* 1 + \beta_s^* \gamma_s + \varepsilon_s^* \quad \text{for each strategy } s. \tag{7}$$

We first discuss the estimation results of the intercept (α_s^*), and then examine the impact of its addition on R^2. Note that the estimated γ coefficients are largely unchanged, and thus not reported.

Table 14 reports the mean intercept estimates (and percentage of significant intercepts at the 95% level) for each model and strategy. The results are consistent with those reported in previous sections. There is clear evidence that the intercept may be non-zero for the three semi-directional strategies. In particular, funds of funds and multi-strategy funds have extremely high percentages of significant intercepts (above 70% for all four models), while long/short equity hedge funds have the third-highest average percentage. A significant intercept implies that the variation in cross-sectional returns cannot be explained by the risk factors, supporting our earlier conclusion that there may be missing factors for these three strategies.

Next, we investigate if the previously-reported R^2 values from the original cross-sectional regressions are similar after accounting for intercept bias. Table 15 presents the new average R^2 (adj. R^2) values from Equation 7 for each model and strategy.

Table 13
Residual Estimates – Step 2

	model			
strategy	FFM	LM	QM-LASSO	CM-LASSO
emerging markets	0.196 (3.3%)	0.054 (1.7%)	0.074 (1.7%)	0.012 (0.0%)
event driven	0.149 (1.7%)	0.043 (1.7%)	0.085 (1.7%)	0.074 (1.7%)
fund of funds	0.107 (63.3%)	0.083 (53.3%)	0.083 (58.3%)	0.081 (53.3%)
long/short equity hedge	0.175 (33.3%)	0.145 (23.3%)	0.123 (23.3%)	0.121 (21.7%)
managed futures	0.106 (30.0%)	0.109 (11.7%)	0.108 (8.3%)	0.120 (11.7%)
multi-strategy	0.395 (68.3%)	0.338 (70.0%)	0.340 (70.0%)	0.331 (71.7%)
average	0.188 (33.3%)	0.129 (26.9%)	0.136 (27.2%)	0.123 (26.7%)

This table shows the average residual values (ε_s^*) of all the cross-sectional regressions in Step 2 (equation 6). The parenthetical value below each average residual is the percentage of significant non-zero mean residuals for that given model and strategy (at the 95% level). Note that each entry summarizes the residuals from 60 regressions.

It is well-known that the R^2 of a regression without an intercept is typically higher than that of the same regression with an intercept.[17] Compared to the R^2 values reported in Table 10, the new average R^2s are indeed lower for every model and strategy. Of interest is that the decreases in R^2 are mild for LM, QM-LASSO and CM-LASSO (average decreases of approximately 12.1%, 11.5% and 13.5%, respectively). In contrast, the average R^2 for FFM drops by 33.9%. Overall, the average R^2s of the semi-directional funds remain considerably lower than those of the directional funds for all four models.

5.7 Summary and Discussion

In this section, we summarize our main empirical findings, along with their implications, corresponding to the two sets of research questions proposed in Section 1.

Our first main finding is that both the linear and second-order models have significantly greater explanatory and predictive power than FFM. Additionally, several quadratic and interaction terms play an important role and produce significant γ coefficients for most strategies – in particular, MOM^2, $HML{\times}MOM$ and $PTFSBD{\times}PTFSSTK$. However, there is no evidence that the second-order models have more overall explanatory or predictive power than the linear model, as evidenced by the fairly similar average R^2 values and mean residuals. These results are consistent across all six strategies considered. This suggests that although hedge fund returns may not be a linear function of the risk factors, expanding the set of factors with quadratic and interaction terms is not extremely effective. Note that this result applies only to the average cross-sectional returns on all funds in each strategy and may not hold for individual hedge funds. Nevertheless, an important practical implication is that our augmented 11-factor linear model (LM) based on Fama and French's three factors and Fung and Hsieh's options-based factors generally performs well and is difficult to beat. The results imply that searching for additional factors may be more effective than adding higher moments or interaction terms to improve model performance.

Our second main finding is that there is no single set of risk factors that can capture the dynamics of all

[17] As regressions typically have a non-zero intercept, the explained sum of squares (ESS) of the dependent variable accounted for by the intercept is not included in the total sum of squares (TSS). When the intercept is left out, TSS increases. Although this tends to inflate both the ESS and residual sum of squares (RSS), ESS increases more relative to RSS, leading to an increase in R^2.

Table 14
Intercept Estimates – Step 2 with Intercept

	model			
strategy	FFM	LM	QM-LASSO	CM-LASSO
emerging markets	0.690	0.249	0.313	0.002
	(18.3%)	(13.3%)	(11.7%)	(13.3%)
event driven	0.508	0.227	0.450	0.363
	(33.3%)	(11.7%)	(25.0%)	(25.0%)
fund of funds	0.288	0.281	0.290	0.291
	(83.3%)	(81.7%)	(78.3%)	(75.0%)
long/short equity hedge	0.413	0.380	0.334	0.336
	(43.3%)	(41.7%)	(41.7%)	(38.3%)
managed futures	0.243	0.307	0.316	0.330
	(45.0%)	(30.0%)	(40.0%)	(31.7%)
multi-strategy	0.633	0.665	0.683	0.657
	(73.3%)	(76.7%)	(81.7%)	(78.3%)
average	0.462	0.351	0.398	0.330
	(49.4%)	(42.5%)	(46.4%)	(43.6%)

This table shows the average intercept values for each model and strategy combination (60 regressions each). The parenthetical values represent what percentage of the 60 intercepts are significant at the 95% level.

Table 15
R^2 (adj. R^2) Estimates – Step 2 with Intercept

	model			
strategy	FFM	LM	QM-LASSO	CM-LASSO
emerging markets	0.312	0.632	0.644	0.609
	(0.294)	(0.595)	(0.608)	(0.569)
event driven	0.287	0.686	0.696	0.636
	(0.299)	(0.654)	(0.665)	(0.599)
fund of funds	0.187	0.335	0.315	0.303
	(0.200)	(0.267)	(0.245)	(0.232)
long/short equity hedge	0.136	0.278	0.272	0.253
	(0.114)	(0.204)	(0.198)	(0.177)
managed futures	0.339	0.603	0.618	0.567
	(0.322)	(0.563)	(0.579)	(0.523)
multi-strategy	0.283	0.491	0.485	0.468
	(0.264)	(0.439)	(0.433)	(0.414)
average	0.257	0.504	0.505	0.473
	(0.238)	(0.453)	(0.455)	(0.419)

This table summarizes the R^2 values of all the modified Step 2 regressions (Equation 7), averaged across each model and strategy. The parenthetical values are the adjusted R^2 values.

strategies. The linear and second-order factors can explain most of the predictive power for directional hedge funds, such as emerging markets, event-driven and managed futures funds. However, our results show that the factors fail to explain the predictability of semi-directional funds, including fund of funds, long/short equity hedge and multi-strategy funds. The level of difficulty of modeling hedge fund returns varies considerably across different strategies, and missing factors may still remain for semi-directional funds. On the whole, this finding suggests that although several factors are widely shared by all strategies, funds in each strategy are exposed to additional strategy-specific risk factors.

The two most important practical implications of this study are the following. First, it is difficult to build a model that can be applied to all hedge funds, as researchers generally have little sample data relative to the large number of potential factors. Analyzing models with strategy-specific factors may be more effective. Second, the linear and second-order models used in this study may be sufficient for directional funds; however, it is less clear if these models should be used for semi-directional funds, which utilize more diversified investment styles and positions. Understanding the risk exposures of funds using these strategies is an important area for further research.

6. Conclusion

Most existing asset pricing studies rely on linear models, which constrain the relationship between risk factors and returns to be linear. This paper investigates hedge funds' exposures to various risk factors across different investment strategies through models with both linear and second-order factors. In the hedge fund literature, this is the first analysis of the predictive power of complete second-order models for monthly cross-sectional hedge fund returns, categorized by strategy. The main aim of this paper is to investigate the importance of second-order factors, rather than to analyze the returns performance of individual funds.

We adopt a three-step technique for our empirical analysis. To reduce the dimension of the factor space, we use LASSO regression in Step 0 to select the most relevant risk factors from the large set of candidates. After Step 0, we estimate sets of time series and cross-sectional regressions using a two-step method adopted from Fama & MacBeth (1973). Our multistep approach has the following advantage: it is an operationally-convenient method that concurrently allows us to deal with a large number of factors and to evaluate the contributions of the second-order terms. Our measures of model performance are based on both the explanatory power of the time series regressions and predictive power of the cross-sectional regressions.

Our results tell an interesting story. Although there is evidence that certain quadratic and interaction terms are statistically significant for some hedge fund strategies, the second-order models do not improve either the overall explanatory or predictive power for any of the strategies. We also find that no single set of factors spans the entire risk space of all funds. The level of difficulty of modeling hedge fund returns varies considerably across different strategies — specifically, the cross-sectional returns on directional hedge funds (emerging markets, event-driven, managed futures) are easier to model than those of semi-directional hedge funds (fund of funds, long/short equity hedge, multi-strategy). Our results suggest that missing factors may still remain for the latter groups of funds.

However, one must also keep in mind three main limitations of this study, corresponding to the TASS data, our empirical methodology, and evaluation methods. The data is limited in that the ratio of the number of months of full returns data to the number of factors (which reaches up to 77 in this study) is small. Overfitting is a common issue that arises when using a complicated model with a large number of explanatory variables. Ideally, we would have a greater number of months of full returns data. One could argue that using a much longer time series of returns may increase the performance of the second-order models. While this may be true, it is a difficult hypothesis to test. Obtaining consistent and reliable returns data for the same hedge funds over a long time period is extremely difficult due to the highly dynamic nature of the hedge fund industry and voluntary returns reporting.[18] For example, as discussed in Section 4.2.1, there are a total of 1,526 live and graveyard funds without missing data during the 10-year time period used in this study (2006 to 2015). If we extend the time period to 2016, there are fewer than 200 live funds without missing data during the 11-year time period (2006 to 2016).

In terms of our empirical methodology, an alternative method to our dimensionality reduction step is to use elastic net, a regression model that combines the features of ridge and LASSO, instead of LASSO in Step 0. If there are any highly-correlated explanatory variables with similar predictive power, LASSO simply omits all but one of the variables. Elastic net, on the other hand, forms a group for each set of correlated variables. If any one of the variables in a group is a strong predictor, elastic net includes the entire group in the model. This alteration addresses the issue of potentially lost information that comes from simply omitting correlated variables. While elastic net has its benefits, the inclusion of an entire group of factors if even one is significant increases the factor space, which may cause complications in the second step of the Fama-MacBeth method. However, elastic net may be a stronger choice for studies with a larger amount of available data.

The final limitation relates to the parametric evaluation methods adopted in this paper, which are based on the models and estimation techniques used. An alternate line of research instead develops nonparametric performance evaluation methods. Chen & Knez (1996) develop a framework for evaluating portfolio performance independently of the models. They establish a minimum set of conditions that any performance measure must satisfy, and apply them to the evaluation of mutual funds. Almeida, Ardison & Garcia (2019) extend Chen and Knez's work by proposing a novel class of performance measures particularly suited to evaluating the performance of hedge funds. The authors use individual hedge fund returns data to confirm that the cross-section of hedge fund alphas (derived using OLS) is significantly related to their co-skewness and co-kurtosis with the market return. They additionally link a hedge fund's performance to its higher-order mixed moments with nonlinear stochastic discount factor-based benchmarks. While beneficial, this method is beyond the technical scope of this paper.

In summary, we conclude that it is unlikely that the explanatory or predictive power for average cross-sectional hedge fund returns will increase significantly by implementing second-order models; we suspect this limitation may hold for higher-order models as well. Instead, a search for missing factors may be more promising for future research. Missing factors do not seem to be a problem for directional hedge fund strategies, such as emerging markets, event-driven and managed futures. However, we find strong evidence that unidentified factors exist for semi-directional strategies, including fund of funds, long/short equity hedge and multi-strategy.

[18] In contrast, mutual funds are required to report their returns.

Hence, our results suggest that an effective way to address the problem of missing factors is to focus on funds with these three strategies. Moreover, it is essential to consider each strategy separately in any future hedge fund studies, due to their drastically different risk profiles.

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Appendix

A. TASS Fund Category Definitions[19]

Convertible arbitrage: This strategy is identified by hedge investing in the convertible securities of a company. A typical investment is to be long the convertible bond and short the common stock of the same company. Positions are designed to generate profits from the fixed income security as well as the short sale of stock, while protecting the principal from market moves.

Dedicated short bias: Short biased managers take short positions mostly in equities and derivatives. The short bias of a manager's portfolio must be constantly greater than zero to be classified in this category.

Emerging markets: This strategy involves equity or fixed income investing in emerging markets around the world. As many emerging markets do not allow short selling, nor do they offer viable futures or other derivative products with which to hedge, emerging markets investing often employs a long-only strategy.

Equity market neutral: This investment strategy is designed to exploit equity and/or fixed income market inefficiencies and usually involves being simultaneously long and short matched market portfolios of the same size within a country. Market neutral portfolios are designed to be either beta or currency neutral, or both.

[19] Getmansky et al., 2015; Thomson Reuters Lipper, 2016.

Event driven: This strategy is defined as “special situations” investing designed to capture price movement generated by a significant pending corporate event, such as a merger, corporate restructuring, liquidation, bankruptcy, or reorganization. Three popular subcategories of event-driven strategies include risk (merger) arbitrage, distressed/high yield securities, and Regulation D.

Fixed income arbitrage: This strategy refers to funds that attempt to limit volatility and generate profits from price anomalies between related fixed income securities. Most managers trade globally with a goal of generating steady returns with low volatility. This category includes interest rate swap arbitrage, U.S. and non-U.S. government bond arbitrage, forward yield curve arbitrage, and mortgage-backed securities arbitrage. The mortgage-backed market is primarily U.S.-based and over-the-counter.
Fund of funds: A “multimanager” fund employs the services of two or more trading advisers or hedge funds that are allocated cash by the trading manager to trade on behalf of the fund.

Global macro: Global macro managers carry long and short positions in any of the world’s major capital or derivative markets. These positions reflect their views on overall market direction as influenced by major economic trends and/or events. The portfolios of these funds can include stocks, bonds, currencies, and commodities in the form of cash or derivatives instruments. Most funds invest globally in both developed and emerging markets.

Long/short equity hedge: This strategy involves equity-oriented investing on both the long and short sides of the market. The objective is not to be market neutral. Managers have the ability to shift from value to growth, from small to medium to large capitalization stocks, and from a net long position to a net short position. Managers may use futures and options to hedge. The focus may be regional, such as long/short US or European equity, or sector-specific, such as long and short technology or health care stocks. Long/short equity funds tend to build and hold portfolios that are substantially more concentrated than those of traditional stock funds.

Managed futures: This strategy invests in listed financial and commodity futures markets and currency markets around the world. The managers are usually referred to as commodity trading advisors (CTAs). Trading disciplines are generally systematic or discretionary. Systematic traders tend to use price- and market-specific information (often technical) to make trading decisions, while discretionary managers use a judgmental approach.

Multi-strategy: This strategy describes hedge funds that use several strategies within the same pool of assets. Hedge funds can use quantitative and fundamental techniques, including strategies that are broadly diversified or narrowly focused on specific sectors, and have different levels of net exposure, leverage employed, holding period, market capitalization, and valuation techniques.

Options strategy: This strategy seeks to capture “the spread” between similar options through inefficiencies in the market. Options arbitrage indicates a loosely-defined category describing any manager who focuses on options.

B. Econometrics Background

This appendix reviews three important econometric techniques used in our analysis. The first two, principal component analysis and LASSO, are dimensionality-reduction techniques, as our models deal with a large number of second-order factors. We use principal component analysis to construct a new factor in Section 3.4 to account for multicollinearity and reduce the dimensionality of the factor space. We use LASSO to select the most relevant risk factors for each strategy, which serves as the first step of our three-step estimation procedure described in Section 4.2. Finally, we review the Fama-MacBeth two-step method, which serves as the last two steps of our estimation procedure.

Principal Component Analysis

The goal of principal component analysis (PCA), a popular dimensionality-reduction technique, is to reduce the number of predictor variables in a high-dimensional dataset and eliminate any multicollinearity problems, while still capturing most of the information.

The principal components are derived from orthogonal transformations in the directions of highest variance. The first principal component is the linear combination of original factors that captures the maximum amount of information, as measured by variance, in the dataset. Each successive principal component is a different linear combination that captures most of the remaining variance, but is uncorrelated with all of the components before it. The result is a set of uncorrelated variables (linear combinations of the original factors) that captures most of the variance in the dataset. Depending on the required analysis and resulting proportions of variance, a lower-dimensional set of variables can be chosen. PCA is performed without regard to the dependent variable.

LASSO Regression

There are three major regression techniques we are interested in: ordinary least squares (OLS), ridge and LASSO (least absolute shrinkage and selection operator). OLS, which minimizes the mean squared error (MSE) of a model, is the most commonly-used regression method. In comparison, ridge and LASSO both add a penalty term on the size of the regression coefficients, the magnitude of which is determined by the parameter λ. Ridge uses the L_2-norm to regularize the coefficients, while LASSO uses the L_1-norm. Mathematically, the penalty shrinks the coefficients towards zero and allows for a tradeoff between the bias and variance of the estimator – ridge and LASSO reduce the variance of the estimator, but sacrifice the unbiasedness property of the OLS estimator. The bias (variance) of the ridge and LASSO estimators increases (decreases) as a monotonic function of λ, which is ordinarily chosen through cross-validation.[20] The properties of these three regressions are summarized in Table 16.

Table 16
Summary of Regression Methods

regression method	tuning parameter	loss function *(MSE + penalty factor)*
OLS	–	$\lVert Y - X\beta \rVert_2$
ridge	λ	$\lVert Y - X\beta \rVert_2 + \lambda \lVert \beta \rVert_2$
LASSO	λ	$\lVert Y - X\beta \rVert_2 + \lambda \mid \beta \mid$

[20] In l-fold cross-validation, the data is split into l equal parts. The model is trained on all but the l-th part and then validated on the l-th part, for $l = 1, 2, \ldots$ The procedure is repeated for various λs and the λ with the best overall performance (lowest sum of validation errors) is chosen. In this paper, however, we do not use cross-validation. Instead, we use a data-driven approach to set λ manually, to improve comparability between models. See Table 17 in Appendix C for the specific λ values used.

Although these three methods look similar, they serve different purposes. We are particularly interested in the variable selection capabilities of the LASSO regression. OLS becomes difficult to use in high-dimensional cases when the sample size is small, relative to the number of factors. Both ridge and LASSO reduce the complexity of the model to combat large factor spaces. However, while the L_2-norm of the ridge regression simply shrinks all of the coefficients towards zero (produces non-sparse coefficients), the L_1-norm of the LASSO regression sets a subset of coefficients to exactly zero (produces sparse coefficients), depending on their predictive power. As λ increases, LASSO sets more coefficients to zero and the shrinkage of the non-zero coefficients increases. Thus, LASSO can also be used as a dimensionality reduction technique.

We use both LASSO and PCA for their dimensionality reduction properties in our analysis. Each technique has its advantages. PCA allows us to combine factors while retaining all the information (variance) in the dataset. LASSO, on the other hand, takes into account both the predictor and dependent variables and retains each variable in its original form, which maintains the qualitative interpretation of the coefficients. Thus, we use PCA in the factor construction process and LASSO to select the final factors in our estimation procedure.

Fama-MacBeth Two-Step Method

The Fama-MacBeth method is a two-step regression method commonly used in financial literature to study the relationship between the cross-sectional returns and risk factor betas of hedge funds. In the first step, the betas are estimated from a time series regression using a sub-period of the data through time t. In the second step, average cross-sectional hedge fund returns at time $(t+1)$ are regressed on the betas that were derived in the first step. This method was designed for two important reasons. First, it provides additional economic information regarding a risk factor's premium: the payoff expected from the factor's beta exposure. Second, from a statistical standpoint, it corrects the standard errors for cross-sectional correlation.[21]

When applied to our study, the Fama-MacBeth method has a key additional benefit: it provides meaningful economic interpretations for models with non-traded and non-return-based risk factors. This benefit is especially important for this paper, as our analysis involves models with second-order factors that are neither tradable nor return-based. For most traditional linear asset pricing models, the α (intercept) and βs (slope coefficients) in the first-step time series regression represent the fund's excess return relative to the risk factors and the fund's exposures to each risk factor, respectively. However, while the regression is still statistically valid, the α and βs lose their economic interpretation when the risk factors are neither tradable nor return-based. By estimating the relationship between cross-sectional hedge fund returns at time $(t+1)$ and betas through time t in the second step, the Fama-MacBeth method provides economic interpretations for the α and βs that are lost in the first step.

In this study, we adopt the Fama-MacBeth method and implement it with a rolling time window. Using a rolling window consisting of the most recent m months ensures that only past (known) information is used to estimate the model's parameters and to make predictions. The adopted Fama-MacBeth two-step method is thus as follows:

- Start with a window of the first m months of data and perform both steps below. After each iteration, roll the time period forward one month and repeat until the last m-month window in the sample period.

- **Step 1:** Time series regressions – Estimate an OLS regression for each fund over the m-month window to obtain the risk factor exposures β_js; denote the last month of the window as month t. As the errors from these regressions may be correlated with each other, we perform a second regression in the next step, in which these estimated β coefficients become the explanatory variables.

- **Step 2:** Cross-sectional regressions – Estimate a cross-sectional regression using the fund returns in month $(t+1)$ as the dependent variable and the β_js derived in Step 1 as the explanatory variables.

We describe our implementation of the Fama-MacBeth two-step method in detail in Section 4.2.

[21] See, for example, Bailer & Martin (2007) for more discussion on the Fama-MacBeth method and how it is used.

C. Additional Data

Table 17
Summary of Lambda (λ) Values – Step 0

Panel A. Lambda (λ) Values Used in LASSO

	strategy					
model	EM	ED	FF	LS	MF	MS
QM-LASSO	0.25	0.15	0.09	0.15	0.15	0.09
CM-LASSO	0.60	0.35	0.20	0.35	0.45	0.20

This table shows the specific λ values used in the Step 0 LASSO regressions (equation 4) for each strategy in QM-LASSO and CM-LASSO. These values were chosen to ensure that each strategy chooses approximately 11 factors (Panel B), on average, in Step 0.

Panel B. Average Number of Factors Chosen in Step 0

	strategy					
model	EM	ED	FF	LS	MF	MS
QM-LASSO	11.92	11.50	10.80	11.50	11.50	11.50
CM-LASSO	11.75	10.90	10.55	10.70	10.30	10.63

This table shows the number of factors chosen in Step 0, averaged across all the LASSO regressions for each strategy and model. The λ values (Panel A) were chosen to keep these average values as close to 11 as possible to be consistent with LM.

Table 18
Lasso and Gamma (γ) Significance Values – Steps 0 & 2

Panel A. Emerging Markets

	MKT	SMB	HML	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	GS10	MSCI
QM-Linear	60	60	60	37	50	29	26	39	25	17	15
	(46.7%)	(38.3%)	(25.0%)	(16.2%)	(24.0%)	(20.7%)	(38.5%)	(28.2%)	(28.0%)	(47.1%)	(46.7%)
QM-Quadratic	27	3	23	53	18	38	31	41	29	13	21
	(33.3%)	(0.0%)	(30.4%)	(18.9%)	(33.3%)	(31.6%)	(41.9%)	(14.6%)	(31.0%)	(38.5%)	(28.6%)
CM-Linear	60	60	60	9	14	2	4	24	0	0	2
	(51.7%)	(36.7%)	(35.0%)	(22.2%)	(21.4%)	(0.0%)	(25.0%)	(37.5%)	-	-	(50.0%)
CM-Quadratic	0	0	3	2	0	0	11	5	1	3	4
	-	-	(66.7%)	(0.0%)	-	-	(27.3%)	(20.0%)	(0.0%)	(66.7%)	(25.0%)
CM-Interaction	4.2	5.1	14.1	13.7	8.8	7.4	10.9	4.6	6.5	7.8	5.1
	(26.5%)	(27.6%)	(24.8%)	(17.4%)	(24.3%)	(44.3%)	(38.4%)	(31.7%)	(19.7%)	(44.3%)	(29.5%)

Panel B. Event Driven

	MKT	SMB	HML	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	GS10	MSCI
QM-Linear	60	60	60	36	38	32	24	25	30	24	7
	(56.7%)	(35.0%)	(45.0%)	(36.1%)	(34.2%)	(43.8%)	(54.2%)	(36.0%)	(50.0%)	(33.3%)	(57.1%)
QM-Quadratic	22	33	22	49	10	38	24	20	28	13	35
	(54.5%)	(30.3%)	(40.9%)	(28.6%)	(30.0%)	(57.9%)	(45.8%)	(20.0%)	(28.6%)	(46.2%)	(28.6%)
CM-Linear	60	60	60	14	0	9	10	4	21	10	0
	(58.3%)	(31.7%)	(56.7%)	(28.6%)	-	(44.4%)	(40.0%)	(50.0%)	(38.1%)	(40.0%)	-
CM-Quadratic	0	0	1	1	0	0	0	0	8	9	5
	-	-	(0.0%)	(0.0%)	-	-	-	-	(25.0%)	(33.3%)	(20.0%)
CM-Interaction	3.7	5.3	11.2	7.2	7	7.8	9.5	3.4	5.4	9.3	6.6
	(36.0%)	(76.9%)	(37.4%)	(39.2%)	(31.0%)	(43.7%)	(37.0%)	(25.6%)	(44.8%)	(44.4%)	(38.0%)

Panel C. Fund of Funds

	MKT	SMB	HML	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	GS10	MSCI
QM-Linear	60	60	60	37	32	32	18	16	31	12	20
	(81.7%)	(53.3%)	(58.3%)	(75.7%)	(71.9%)	(71.9%)	(72.2%)	(43.8%)	(51.6%)	(75.0%)	(65.0%)
QM-Quadratic	12	10	18	41	27	34	26	14	53	18	17
	(33.3%)	(80.0%)	(55.6%)	(53.7%)	(63.0%)	(64.7%)	(30.8%)	(64.3%)	(47.2%)	(55.6%)	(35.3%)
CM-Linear	60	60	60	1	0	11	1	3	18	1	3
	(88.3%)	(70.0%)	(45.0%)	(100.0%)	-	(54.5%)	(0.0%)	(33.3%)	(50.0%)	(0.0%)	(33.3%)
CM-Quadratic	0	0	0	0	0	0	7	0	4	5	4
	-	-	-	-	-	-	(42.9%)	-	(50.0%)	(20.0%)	(25.0%)
CM-Interaction	1.3	8.8	8.8	7	10	11.7	6.2	6.5	5.5	7.3	5.9
	(75.0%)	(49.3%)	(58.2%)	(51.5%)	(57.0%)	(47.2%)	(64.7%)	(65.9%)	(73.7%)	(80.0%)	(78.1%)

(Continued on next page)

This table shows the total number of times each factor is chosen in Step 0 (equation 4) for QM-LASSO and CM-LASSO. The parenthetical values are the percentage of corresponding γs that are significant at the 95% level in Step 2 (Equation 6). "-Linear" and "-Quadratic" represent the 11 linear and quadratic factors in each model, respectively. "CM-Interaction" represents the average over the 10 interaction terms involving the given factor, standardized on a scale of 60. These values are used to build Figure 7.

Table 18: (continued)

Panel D. Long/Short Equity Hedge

	MKT	SMB	HML	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	GS10	MSCI
QM-Linear	60	60	60	42	31	32	23	33	34	25	7
	(76.7%)	(51.7%)	(55.0%)	(50.0%)	(61.3%)	(37.5%)	(34.8%)	(30.3%)	(44.1%)	(56.0%)	(42.9%)
QM-Quadratic	33	4	13	48	27	39	27	10	39	13	38
	(27.3%)	(25.0%)	(46.2%)	(25.0%)	(40.7%)	(43.6%)	(51.9%)	(50.0%)	(46.2%)	(38.5%)	(28.9%)
CM-Linear	60	60	60	1	1	10	10	9	1	6	0
	(70.0%)	(48.3%)	(48.3%)	(100.0%)	(0.0%)	(50.0%)	(70.0%)	(55.6%)	(0.0%)	(66.7%)	-
CM-Quadratic	0	0	2	0	0	0	2	0	15	2	10
	-	-	(50.0%)	-	-	-	(50.0%)	-	(60.0%)	(0.0%)	(40.0%)
CM-Interaction	2.5	6.4	10	7.8	10.7	8.1	8.7	3.6	8.1	8.5	4.2
	(54.5%)	(40.0%)	(35.0%)	(39.0%)	(40.1%)	(39.7%)	(28.2%)	(35.3%)	(55.5%)	(57.4%)	(50.7%)

Panel E. Managed Futures

	MKT	SMB	HML	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	GS10	MSCI
QM-Linear	60	60	60	28	51	38	36	38	51	8	24
	(38.3%)	(33.3%)	(38.3%)	(53.6%)	(39.2%)	(50.0%)	(47.2%)	(39.5%)	(45.1%)	(50.0%)	(29.2%)
QM-Quadratic	38	23	12	14	31	2	47	37	24	13	25
	(36.8%)	(39.1%)	(33.3%)	(42.9%)	(41.9%)	(100.0%)	(31.9%)	(37.8%)	(54.2%)	(30.8%)	(48.0%)
CM-Linear	60	60	60	0	7	8	8	18	5	0	11
	(48.3%)	(51.7%)	(40.0%)	-	(57.1%)	(50.0%)	(25.0%)	(44.4%)	(0.0%)	-	(54.5%)
CM-Quadratic	0	7	0	0	0	0	5	10	0	5	3
	-	(57.1%)	-	-	-	-	(20.0%)	(30.0%)	-	(60.0%)	(33.3%)
CM-Interaction	5.8	7	8.5	3.1	4.9	4.3	12.2	8.4	2.1	4.3	9.6
	(41.7%)	(39.9%)	(30.7%)	(43.6%)	(37.4%)	(29.9%)	(37.8%)	(44.4%)	(62.7%)	(38.3%)	(54.2%)

Panel F. Multi-Strategy

	MKT	SMB	HML	MOM	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK	GS10	MSCI
QM-Linear	60	60	60	37	31	31	26	22	31	17	30
	(71.7%)	(25.0%)	(30.0%)	(54.1%)	(35.5%)	(35.5%)	(23.1%)	(36.4%)	(48.4%)	(41.2%)	(63.3%)
QM-Quadratic	29	1	13	49	24	33	32	2	35	9	21
	(31.0%)	(0.0%)	(15.4%)	(42.9%)	(58.3%)	(39.4%)	(15.6%)	(50.0%)	(34.3%)	(33.3%)	(61.9%)
CM-Linear	60	60	60	1	0	5	12	3	17	3	12
	(68.3%)	(48.3%)	(48.3%)	(0.0%)	-	(20.0%)	(66.7%)	(33.3%)	(58.8%)	(100.0%)	(50.0%)
CM-Quadratic	0	0	5	2	0	0	10	0	3	0	4
	-	-	(0.0%)	(50.0%)	-	-	(40.0%)	-	(33.3%)	-	(50.0%)
CM-Interaction	3.9	7.7	14.8	5.5	6.1	8.5	8.9	4.4	4.1	6.6	5.7
	(28.5%)	(38.0%)	(38.6%)	(47.3%)	(28.6%)	(28.4%)	(22.8%)	(39.7%)	(50.3%)	(52.4%)	(26.9%)

This table shows the total number of times each factor is chosen in Step 0 (Equation 4) for QM-LASSO and CM-LASSO. The parenthetical values are the percentage of corresponding γs that are significant at the 95% level in Step 2 (equation 6). "-Linear" and "-Quadratic" represent the 11 linear and quadratic factors in each model, respectively. "CM-Interaction" represents the average over the 10 interaction terms involving the given factor, standardized on a scale of 60. These values are used to build Figure 7.