Inter-temporal CAPM: An Empirical Test with Brazilian Market Data

(CAPM Intertemporal: Um Teste Empírico Utilizando Dados Brasileiros)

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Abstract

This paper examines the empirical validity of the Inter-temporal Capital Asset Pricing Model (ICAPM) with Brazilian market data. The Bali & Engle (2010) methodology is used with the estimation of conditional covariances between stock portfolio returns and pricing factors. The covariances are then used as explanatory variables in the pricing equation. The results validate the model for the 1988 to 2012 period. The estimated risk aversion coefficient is positive and significant, and the relevant pricing factors are interest rates, inflation and gold prices; the reverse is true in the case of the exchange rate. Breaking up the sample period into sub-periods indicates that major events (changes in economic regimes and the 2008 crisis) are capable of modifying the associations observed and reducing the model's validity.

Keywords: inter-temporal CAPM; conditional correlations; risk aversion coefficient.

JEL codes: C32; G12.

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Resumo

O artigo examina a validade empírica do Inter-temporal Capital Asset Pricing Model (ICAPM) com o uso de dados do mercado brasileiro. É empregada a metodologia de Bali e Engle (2010), com a estimação de covariâncias condicionais entre retornos de carteiras de ações e fatores de precificação. As covariâncias são a seguir utilizadas como variáveis explicativas na equação de precificação. Os resultados confirmam o modelo para o período de 1988 a 2012. O coeficiente de aversão a risco é positivo e significativo, e os fatores relevantes são taxas de juros e de inflação, além do preço do ouro. O oposto ocorre com a taxa de câmbio. A quebra do período de estudo em subperíodos indica que eventos importantes, como mudanças de regime econômico e a crise de 2008 têm o poder de modificar as associações encontradas e enfraquecer a validade do modelo.

Palavras-chave: CAPM intertemporal; correlações condicionais; coeficiente de aversão a risco.

1. Introduction

The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965) and Mossin (1966) is one of the most important asset pricing models (Fama & French, 2004). It says that the risk premium for any asset is a linear function of the covariance of its rate of return with a portfolio containing all the assets in the economy, known as the “market portfolio”. However, the Sharpe-Lintner-Mossin version of the CAPM is a single period model, and does not account for the possibility of changes in the investment opportunity set, represented by all possible combinations along the capital market line, that is, the risk free asset and the risky asset portfolio with the maximum Sharpe ratio.

Because it is an extremely simple model, the CAPM is widely used both by academics and finance professionals. Simplicity, however, entails costs: the CAPM is rejected in most of its empirical tests. Various papers show that the rate of return on an asset cannot be determined solely by its covariance with the rate of return on the market^{1, 2}.

¹The pioneering tests leading to the rejection of the CAPM are Black *et al.* (1972) and Blume & Friend (1973). A comprehensive review of the literature on this topic may be found in Fama & French (2004).

²According to Bodie *et al.* (2009), “The Capital Asset Pricing Model, almost always referred to as the CAPM, is a center-piece of modern financial economics.” (p. 279) The authors add that “Although the CAPM does not fully withstand empirical tests, it is widely used because of the insight it offers and because its accuracy is deemed acceptable for important applications.” (p. 279)

As a consequence of the less than convincing results in empirical tests of the CAPM, several authors have proposed a series of multifactor models. The best known are the Fama & French (1992) three-factor model and the Arbitrage Pricing Theory (APT) by Ross (1976). While those models offer good empirical results, Campbell & Vuolteenaho (2004) claim that those models are not capable of providing micro-foundations for investors' decisions and expectations.

Reconciling the existence of more than one factor in the pricing of assets with the utility maximization idea, Merton (1973) proposes the Inter-temporal Capital Asset Pricing Model (ICAPM). The model is a generalization of the CAPM with the relaxation of the assumption of a single period investment horizon. The model's basic contribution is the recognition of the strong possibility of changes in the investment opportunity set. As previously mentioned, the investment opportunity set consists of all possible combinations between the risk free asset and the equilibrium market portfolio of all risky assets. However, there is no reason for assuming that the risk free rate and the (maximum) Sharpe ratio for the market portfolio will not change over time. It stands to reason that whatever changes occur will be caused by changes in so-called state variables. For example, changes in interest rates affect investors' future consumption plans. For a saver, a lower interest rate implies a lower level of future consumption, given a certain wealth level. That means that investors will demand assets that can protect them against such adverse changes; in other words, there will be demand for hedging.

Therefore, the ICAPM predicts that an asset's risk premium will depend not only on the covariance of its rate of return with the return on the market portfolio, but also on covariances with state variables.

Although the ICAPM is an elegant solution for problems with the CAPM, not all of its theoretical implications have been empirically confirmed. The ICAPM predicts a positive relationship between risk and return, but there is no consensus in the literature for the observed sign of that relationship.

It seems intuitive that, in a given moment, investors will only take on additional risk if they are compensated with higher returns. However, when one approaches the problem with a multi-period model, this relationship does not seem that evident. It may be argued, for example, that in high volatility regimes investors decide to save more, refraining from demanding higher premiums. In addition, if all assets used for transferring wealth to the

future are risky, the prices of risky assets should go up, causing a substantial reduction in risk premiums (Glosten *et al.*, 1993).

The objective of the present paper is to validate the ICAPM predictions empirically, using Brazilian market data. The model is tested with monthly data for the 1988 to 2012 period using the Bali & Engle (2010) methodology. As explained below, that methodology permits the use of conditional covariance series as explanatory variables in the model's pricing equation. Therefore, in contrast with the representation of the pricing equation with the use of betas, the methodology used herein does not ignore the inter-temporal variation in assets' covariances with the pricing factors. The conditional covariances are estimated with the Engle (2002) DCC model.

The paper uses stock portfolios that are ranked by market value and organized into deciles. The pricing factors considered include: the rate of return on the Ibovespa, changes in interest rates, changes in gold prices, inflation rates and changes in the exchange rate.

This is a contribution to a relatively modest literature on empirical tests of the ICAPM for Brazil. Almeida (2010) is the only study that attempts to test the ICAPM using Brazilian market data. The author, however, estimates the ICAPM using its representation with betas. In that type of representation, one loses the inter-temporal variation of covariances between portfolio returns and state variables. In that sense, by adopting a methodology that makes it possible to resort to conditional covariances in the pricing equation, the present paper is the first of its kind for Brazil.

In addition, the use of long time series allows for testing the model for different sub-periods, something that is original in the Brazilian literature. It is believed, then, that the results are a contribution to the empirical literature.

The paper's main conclusion is the confirmation of the ICAPM's empirical validity for the period under analysis. The estimated risk premium is positive and significant, implying a positive association between risk and return in a multi-period investment environment. Further, the coefficients for all the state variables are significant, with the exception of the exchange rate. The validity of the model is also tested using a scatter plot between actual mean returns and expected mean returns, following Cochrane (2005). The scatter plot fits well with a 45° degree line, except for one portfolio, supporting the validity of the model.

Furthermore, because we are dealing with a long sample period, three sub-periods are analyzed: the period preceding the Real Plan (from 1988 to

February, 1994); the period following the introduction of the Real Plan and before currency devaluation (from March, 1994 to December, 1998); and the period following currency devaluation (from January, 1999 to September, 2012). The results are sensitive to the period under analysis; in particular, the risk aversion coefficient is negative and significant for the post-currency devaluation period.

Finally, the impact of the 2008 crisis is examined, with the breaking up of the complete sample period into two sub-samples (before and after the crisis). Once more, the results seem to be period-sensitive. In particular, the risk aversion coefficient is positive and significant before the crisis, but negative and non-significant afterwards.

Those results indicate that care must be taken before assuming that the relationship uncovered by the ICAPM is constant: adverse conditions in financial markets can cause preference shocks that alter the relationships between the variables in a significant fashion.

The paper is organized in five sections, including this introduction. Section 2 contains a review of the literature. Section 3 describes the model and the Bali & Engle (2010) methodology, in addition to the variables and data used. In Section 4 the main results are presented and discussed, and the last section concludes.

2. Review of Literature

There is a large volume of studies in the finance literature dedicated to understanding the inter-temporal relationship between risk and return. It is interesting to note that there is no consensus in terms of empirical results. This can be mostly explained by the various alternative ways available for testing that relationship.

Bollerslev *et al.* (1988) estimate a model in which conditional returns depend on the conditional variance in a multivariate GARCH-in-mean model, and conclude that the inter-temporal relationship between risk and return is positive.

Glosten *et al.* (1993) also resort to GARCH models in order to test that relationship. However, they account for asymmetric volatility, that is, changes in the behavior of volatilities depending on whether past returns are positive or negative.³ The authors obtain a negative and significant relationship between risk and return.

³The model proposed by the authors is known as Threshold ARCH (TARCH).

Campbell (1996) proposes an inter-temporal model in which an asset's risk premium is a function of the market portfolio's risk premium, of variables used for predicting future returns, and the return on human capital.⁴ He uses vector autoregressive (VAR) models and finds a positive relationship between risk and return.

Brandt & Kang (2004) also estimate a VAR model that is capable of identifying both the conditional risk-return relationship, and the non-conditional relationship. The authors conclude that, although the non-conditional relationship is positive, the conditional relationship is negative and statistically significant. In their view, the difference between the conditional and the non-conditional relationships may explain the divergence of results found in the literature.

Ghysels *et al.* (2005) find a positive and significant relationship using a different estimator for the covariance matrix: Mixed Data Sampling (MIDAS). They argue that it is a more powerful estimator than the GARCH methodology. In fact, using GARCH models the authors do not find statistical significance in the risk-return trade off.

Other papers that obtain a positive inter-temporal relationship between risk and return, with similar methods include: Scruggs (1998), Garcia & Bonomo (2001), Chen (2002), Guo & Whitelaw (2006), Bali & Peng (2006), Aquino (2006), Lundblad (2007), and Bali (2008). Other examples of papers that obtain a negative relationship are: Campbell (1987), Nelson (1991), and Whitelaw (1994).

The following exhibit summarizes the main papers in the empirical literature. The data frequency, the sample used, the methodology employed and the results obtained are highlighted.

⁴The author argues that this variable must be included because, even though the market portfolio contains all financial assets, it does not capture the wealth generated by human capital as a production factor.

Table 1

Exhibit 1 – Summary of the empirical literature

Authors	Market	Frequency	Sample	Method	Risk-Return Relationship
Bollerslev <i>et al.</i> (1988)	USA	Quarterly	1959-1984	GARCH-M	Positive and significant
Campbell (1987)	USA	Monthly	1959-1979	Instrumental variables	Negative and significant
Nelson (1991)	USA	Daily	1962-1987	Asymmetric GARCH-M	Negative and significant
Glosten <i>et al.</i> (1993)	USA	Monthly	1951-1989	Asymmetric GARCH-M	Negative and significant
Whitelaw (1994)	USA	Monthly	1953-1989	VAR	Negative and significant
Campbell (1996)	USA	Monthly and anual	1952-1990 1871-1990	VAR	Positive and significant
Scruggs (1998)	USA	Monthly	1950-1994	Asymmetric GARCH-M	Positive and significant
Brandt & Kang (2004)	USA	Monthly	1946-1998	VAR	Negative and significant
Ghysels <i>et al.</i> (2005)	USA	Monthly	1928-2000	MIDAS GARCH-M	Positive and significant Negative and significant
Guo & Whitelaw (2006)	USA	Monthly	1983-1995 1959-2000	Instrumental variables	Positive and significant Positive and non-significant
Bali & Peng (2006)	USA	High frequency	1962-2002	GARCH	Positive and significant
Lundblad (2007)	USA	Monthly	1836-2003	Asymmetric GARCH-M	Positive and significant
Bali (2008)	USA	Monthly	1926-2002	GARCH	Positive and significant

Most of the papers previously cited are concerned with the risk-return relationship in a broader framework, and are not specific tests of the validity of the ICAPM itself. However, a few studies involve such specific tests.

Nanisetty *et al.* (1996) run an ICAPM vs. CAPM test with the methodology originally used in Fama & Macbeth (1973). The authors argue that, if the ICAPM is the correct model, then a pricing equation that is identical to that of the CAPM, but includes idiosyncratic risk as an explanatory variable, must contain a statistically significant coefficient for that variable.⁵ Using monthly U.S. market data for the period from 1926 to 1988, the authors reject the static model's assumption in favor of the ICAPM, by setting up different industry and size portfolios. In addition, they find a positive and significant relationship between risk and return in all of their specifications.

Faff & Chan (1998) test the ICAPM with monthly Australian market data for the 1975 to 1994 period, and use gold prices and an interest rate as state variables. The authors estimate the ICAPM in its beta representation, and conclude that the model fits Australian data adequately.

Aquino (2006) tests the ICAPM for Philippine market data in the post-Asian crisis period (with monthly data from 1997 to 2001). The author argues that the change in the exchange rate must be used as a state variable in the model, since investors use the currency market as protection against adverse changes in the investment opportunity set. The test consists in the comparison of portfolio variances. The author concludes that the ICAPM with exchange rates is an adequate model for Philippine data in the post-crisis period; however, he finds a negative covariance between the portfolios' risk premiums and the market portfolio's risk premium.

Using monthly data for the U.S. market in the 1953 to period, Chen (2002) tests for the significance of the Fama & French (1992) factors (size and book-to-market ratio), in addition to momentum as possible state variables. The idea is that, if they are relevant state variables, they would be capable of helping to predict market returns and the volatility of market returns. The author concludes, however, that the returns on book-to-market and momentum portfolios are too high to be considered as proper compensations for exposures to adverse changes in the opportunity set.

Bali & Engle (2010) is arguably the most complete test of the ICAPM in the literature. The authors test the model estimating the covariances of

⁵Fama & Macbeth (1973) do not propose to test the CAPM against the ICAPM. However, their methodology is a benchmark in the finance literature. They also conclude for the rejection of the CAPM.

returns of several portfolios with the market portfolio using the dynamic conditional correlation model (DCC) proposed by Engle (2002). Next, the authors estimate the ICAPM pricing equation using a system of regression equations. The measures of changes in the investment opportunity set are changes in macroeconomic and financial variables and measures of return volatility. In particular, the macroeconomic state variables used include: changes in the Fed Funds Rate, in yield curve (term) spreads and in the default spread for corporate bonds. Their main result indicates that the ICAPM fits the daily U.S. data well for the period from 1972 to 2009. In addition, the relationship between risk and return is positive and significant in all specifications. Lastly, the macroeconomic state variables are always relevant and their coefficients have the expected signs.

Garcia & Bonomo (2001) examine the validity of conditional CAPM for the Brazilian market. Using monthly data for the 1976 to 1992 period, they test the significance of inflation risk, proxied by the difference between the 30-day certificate of deposit (CDB) rate and the overnight rate, and find that inflation is a significant risk factor, leading us to the consideration of inflation as a candidate state variable. In their results, the inflation betas are negative for all size portfolios, and our results, reported in Section 4, confirm this.

Almeida (2010) is the only study, so far, that attempts to test the ICAPM using Brazilian market data. The author compares the ICAPM to the static CAPM and the Fama & French (1992) three-factor model. It covers the June 2004 to June 2009 period, using both monthly and weekly returns for several stock portfolios. The state variables are changes in the real interest rate and the market portfolio's Sharpe ratio. Using the beta representation of the ICAPM, the author concludes that the model appears to fit the data better than the CAPM. The ICAPM intercepts are not significant in almost all specifications and the risk-return relationship is positive and significant for most specifications. However, the results with the inter-temporal model are not superior to those for the Fama & French (1992) model, based on a comparison of the regressions' coefficients of determination.

The present paper represents an attempt at adapting the Bali & Engle (2010) methodology in the estimation of the ICAPM with Brazilian market data. We use monthly data for the March 1988-September 2012 period. The main difference between our approach and Almeida (2010) is the pricing equation estimation methodology. As mentioned in the introduction, it is necessary to point out that Almeida (2010) – as well as most of the existing

empirical work – estimate the ICAPM in its beta representation. When that representation is adopted, the temporal variation of covariances between returns on an asset and pricing factors is ignored, since only returns are used as explanatory variables in the pricing equation, as explained later.

3. Methodology

3.1 The Inter-temporal Capital Asset Pricing Model (ICAPM)

Merton's (1973) ICAPM results from the solution of an inter-temporal utility-of-consumption maximization problem, in an environment in which assets are continuously traded. With the solution of the dynamic problem, it is possible to find the asset demand function and the equilibrium pricing equation. According to Bali & Engle (2010), the pricing equation can be written in its discrete version:

$$E_t(r_{t+1}) - r_t^f = ACov_t(r_{t+1}, r_{t+1}^m) + Cov_t(r_{t+1}, Z_{t+1}')B \quad (1)$$

where,

$E_t(r_{t+1})$ is the $(nx1)$ vector of expected returns for the “ n ” assets (portfolios) on date $t + 1$;

r_t^f is the $(nx1)$ vector containing the return on the risk free asset;

A is a scalar representing the average investor's risk aversion coefficient;

r_{t+1}^m is the return on the market portfolio at $t + 1$;

$Cov_t(r_{t+1}, r_{t+1}^m)$ is an $(nx1)$ vector containing the conditional covariances at t between the vector of returns for the “ n ” assets (portfolios) and the return on the market portfolio at $t + 1$;

Z_{t+1} is a $(kx1)$ vector containing all state variables responsible for causing changes in the investment opportunity set at $t + 1$;

$Cov_t(r_{t+1}, Z_{t+1}')$ is a (nxk) matrix whose $j - th$ column contains the conditional covariances, at t , between asset returns and the $j - th$ state variable; B is a $(kx1)$ vector in which the j -th element is the risk premium associated with the $j - th$ state variable.

Thus, equation (1) shows that the investor must be compensated not only by the market portfolio's systematic risk, but also by the risk of adverse changes in the investment opportunity set.

In order to interpret the coefficients estimated, we must consider the logic of the ICAPM. According to Merton, investors will always try to protect themselves against adverse changes in the opportunity set. Such

adverse changes are those in the Z_j state variable that would cause a fall in consumption. Therefore, if the correlation between consumption and the state variable is negative, a positive change in Z_j would represent an adverse change.

Now, consider that a positive change in the state variable is observed and, therefore, there is a fall in consumption. An asset whose return is positively correlated with changes in Z_j will have its rate of return increased exactly at a time of low consumption. Hence, the demand for that asset will be high, its price will increase, and its expected return will decrease. According to that logic, it is expected that the risk premium associated with Z_j will be negative, since assets that are highly correlated with changes in Z_j will protect the investor against reductions in the level of consumption.

Alternatively, if there is positive correlation between consumption and the state variable, an asset whose return is negatively correlated with changes in Z_j will be in high demand, since a fall in Z_j implies consumption losses, but increases in the returns on that asset. Since the demand for that asset is strong, its expected return declines. Hence, the lower (the more negative) the correlation between the asset and changes in Z_j , the lower its expected return, and the coefficient associated with Z_j will be positive.

The Bali & Engle (2010) estimation and testing methodology involves two steps: (i) the covariance of the stock portfolio returns with the market portfolio returns and the covariance of the stock portfolio returns with the state variables are estimated using Engle's (2002) bivariate DCC; (ii) model (1) is estimated with the use of a stacked-regression system, in which the covariances obtained in the previous step are used as explanatory variables. The system is estimated with the Seemingly Unrelated Regressions (SUR) method.

This methodology has two clear advantages over the other methods found in the literature. The first advantage is the estimation of equation (1) without the beta representation so that the temporal variation of covariances is not lost. Recall that any asset j 's beta in the market portfolio is defined as:

$$\beta_j = \frac{Cov(r_j, r_m)}{\sigma_m^2} \quad (2)$$

where,

$Cov(r_j, r_m)$ is the covariance of asset j 's return with the return on the market portfolio;

σ_m^2 is the variance of the returns on the market portfolio.

Hence, when a pricing equation is estimated with the beta representation, only the non-conditional covariance between returns on an asset and returns on the market portfolio is used. However, thanks to the Bali & Engle (2010) methodology, one of the regressors in the pricing equation is the series of conditional covariances between asset and market portfolio returns, estimated by DCC. The pricing equation, therefore, is more realistic, since it allows for time-varying covariances between asset returns and pricing factors.

The Bali & Engle (2010) methodology, however, does not allow for time variation in the risk aversion coefficient or the risk premiums. Such a methodological refinement will have to wait for advances in the literature. In addition, since conditional covariances are estimated at a stage preceding the estimation of the pricing equation, there is the possibility of measurement errors in the regressors, as pointed out in Bali & Engle (2010). A solution would be the joint estimation of a GARCH-in-mean model with time-varying covariances. However, computational difficulties in the estimation of such models for a large number of assets limit the feasibility of such an approach. In that sense, the Bali & Engle (2010) two-stage estimation is powerful because of its computational simplicity.

The second advantage is the estimation of the pricing equation by the SUR method. This method permits the stacking of several pricing equations, without the imposition of any restriction on the error covariance matrix. In the case of a pricing model, this means that idiosyncratic noise for any two assets may be correlated, and that is a much less restrictive assumption. If the errors are in fact correlated, SUR estimators will be more efficient than ordinary least squares (OLS). The use of SUR as a consistent estimator, however, requires the stronger assumption that explanatory variables in each and every equation be uncorrelated with the errors in all system equations.

3.2 Dynamic Conditional Correlation model (DCC)

The first step in the Bali & Engle (2010) methodology is the estimation of covariances between stock portfolio returns and market portfolio returns, and between those returns and the state variables. This involves the use of Engle's (2002) bivariate DCC model. Its foremost advantage is ease of computation, when compared to other models such as multivariate GARCH-in-mean (Bali & Engle, 2010). DCC models the conditional

correlation of two standardized series (i.e., with zero means and unit variances). Assuming that r_{1t} and r_{2t} are two return time series, one must, first of all, remove any structure in the conditional means and variances with ARMA and GARCH models. The assumed structure for the mean is autoregressive process of order 1 (AR(1)), and for the variance a GARCH (1,1)⁶ process, or:

$$\begin{aligned} r_{1t} &= \mu_1 + \delta_1 r_{1t-1} + a_{1t} & r_{2t} &= \mu_2 + \delta_2 r_{2t-1} + a_{2t} \\ a_{1t} &= \sigma_{1t} \epsilon_{1t} & a_{2t} &= \sigma_{2t} \epsilon_{2t} \\ \sigma_{1t}^2 &= \theta_1 + \lambda_1 \epsilon_{1t-1}^2 + \psi_1 \sigma_{1t-1}^2 & \sigma_{2t}^2 &= \theta_2 + \lambda_2 \epsilon_{2t-1}^2 + \psi_2 \sigma_{2t-1}^2 \end{aligned} \quad (3)$$

where,

r_{jt} denotes the returns on asset j at time t ;

a_{jt} is the error term in the estimation of the conditional mean for the return series through an AR(1) process;

μ_j, δ_j are constants, with $|\delta_j| < 1, j = 1, 2$;

σ_{jt} is the conditional volatility of returns for asset j , at time t , modeled by a GARCH (1,1) process;

ϵ_{jt} is a normally distributed IID variable, representing the standardized errors in the equation for the mean at time t ;

$\theta_j, \lambda_j, \psi_j$ are positive constants, with $\lambda_j + \psi_j < 1, j = 1, 2$.

Next, the DCC model is estimated for the standardized error series, ϵ_{1t} and ϵ_{2t} following:

$$\rho_{12,t} = \omega + \alpha \epsilon_{1,t-1} \epsilon_{2,t-1} + \beta \rho_{12,t-1} \quad (4)$$

where,

$\rho_{12,t}$ is the conditional correlation between returns r_{1t} and r_{2t} at time t ;

ω, α and β are positive constants, with $\alpha + \beta < 1$.

The covariance, in turn, can be retrieved from:

$$Cov_t(r_{1t}, r_{2t}) = \rho_{12,t} \times \sigma_{1t} \times \sigma_{2t} \quad (5)$$

where $Cov_t(r_{1t}, r_{2t})$ is the covariance between the return series r_{1t} and r_{2t} .

This model's likelihood function is constructed under the assumption that ϵ_{1t} and ϵ_{2t} are normally distributed. However, it is possible to generalize estimation to other distributions.

⁶For a few series, an alternative functional form for the variance was used, such as IGARCH(1,1) or ARCH(1).

3.3 Pricing equation

Following the calculation of covariances, the pricing equation (1) is estimated by SUR, with a system in which each regression equation corresponds to the pricing of a particular portfolio i . In addition, in accordance with the Bali & Engle (2010) strategy, equation (1) is estimated under the restrictions that A and B are common parameters to all system equations, that is, that they are common slopes.⁷ Hence, the equation to be estimated for a particular portfolio i can be written as follows:

$$R_{i,t+1} = C_i + ACov_t(R_{i,t+1}, R_{m,t+1}) + \sum_{j=1}^k B_j Cov_t(R_{i,t+1}, Z_{j,t+1}) + e_{i,t+1} \quad (6)$$

where,

$R_{i,t+1}$ is portfolio i 's excess return at time $t + 1$;

C_i is the intercept term for portfolio i 's returns;

A is the average risk aversion coefficient, common to all portfolios in the same regression system;

$R_{m,t+1}$ is the excess return on the market portfolio at time $t + 1$;

$Cov_t(R_{i,t+1}, R_{m,t+1})$ is the expected covariance between $R_{i,t+1}$ and $R_{m,t+1}$ at time t ;

B_j is the risk premium associated with the j -th state variable ($j = 1, 2, \dots, k$), common to all portfolios;

$Z_{j,t+1}$ is the j -th state variable ($j = 1, 2, \dots, k$) at time $t + 1$;

$Cov_t(R_{i,t+1}, Z_{j,t+1})$ is the expected covariance between $R_{i,t+1}$ and $Z_{j,t+1}$ at time t ;

$e_{i,t+1}$ is the error term in the i^{th} regression equation.

Once the parameters of (6) are estimated, the next step is testing the ICAPM's empirical validity. In equilibrium, the model predicts that the expected return for any asset must be fully explained by the covariance with the expected return on the market portfolio and the covariances with the state variables. Hence, in equilibrium the intercept terms in (6) must be equal to zero. This means that it is necessary to verify whether the C_i are jointly equal to zero for all portfolios. This is performed with a multivariate

⁷According to Bali & Engle (2010), the ICAPM's internal consistency requires that A be the same for all assets, since this parameter represents the average risk aversion coefficient.

Wald test, with the following null hypothesis: $H_0 : C_1 = C_2 = \dots = C_n = 0$.

In addition, in equilibrium the risk aversion coefficient, A , must be significant, since investors should be compensated for the systematic risk they take on. Here, since the slope coefficient is common to all system regression equations, a t test is performed, with the null hypothesis $H_0 : A = 0$.

One concludes for the ICAPM's empirical validity if there is no evidence for rejecting both the Wald test's null hypothesis and the null hypothesis for the risk aversion coefficient.

The significance of each B_j is also tested. This is important for checking whether any particular variable is seen as *hedging* instrument by investors. If the coefficient associated with a given state variable is not significant, the evidence shows that such a variable is not an asset pricing factor. Again, this leads to a t test for each coefficient associated with the state variables. Therefore, the test's null hypothesis is $H_0 : B_j = 0, (j = 1, 2, \dots, k)$.

Finally, a validity check based on the scatter plot between actual excess returns and predicted excess returns for each portfolio is conducted, following the procedure in Cochrane (2005). Although this test is not pursued in Bali & Engle (2010), it is relatively straightforward and provides useful insights on the validity of ICAPM. The test consists of constructing a scatter plot between average actual excess returns and predicted mean excess returns for each portfolio i . If the model is correct, the points should lie on a 45° line.

3.4 Variables and data used

Merton's (1973) ICAPM is a continuous time model. However, in Bali & Engle (2010), as well as in this paper, a discrete version is used. We opted for using monthly data in order to minimize the noise eventually associated with daily data. The sample period runs from January 1988 to September 2012, for a total of 297 observations. In the estimation of conditional covariances, two observations are lost in each series, meaning that the pricing equation is estimated with 295 observations. The use of a relatively long sample period with monthly data is necessary to ensure the convergence of the DCC model parameters.

The ICAPM is estimated for a set of different portfolios, including only stocks. The use of stocks only and no other type of asset, such as shares in fixed income or equity funds is attributed to the fact that fund shares are not

based only on actual market prices. The sample includes both the companies that went private before 2012 and the companies that went public after 1988 in order to avoid survivorship bias.

The set of stock portfolios comprises 10 portfolios segregated according to market value. The portfolios are constructed as follows: in the beginning of each year, stocks are ranked by market value in increasing order. Next, they are classified into deciles, so that the first decile contains the smallest 10% companies listed at the BM&FBovespa, whereas the last decile contains the largest 10%. The returns of each decile are then computed (each decile corresponds to a portfolio) for each month of the year. Stocks in the portfolio are weighted according to the share of each stock in the value of its portfolio. At the beginning of each year, the ranking of companies by market value is repeated and, thus, the composition of each of the ten market value portfolios is modified.

For the sample period, on average 190 stocks were analyzed each year, meaning that each portfolio (decile) contains, on average, 19 stocks. Companies for which market values were not available in the sample period were excluded. We also excluded stocks for which the number of months of trading was less than 60% of the number of months in the sample period. This prevents the use of illiquid assets, whose returns may distort portfolio returns. All closing prices were collected, in Reais, and were adjusted for stock splits and dividends. The Economática data base was used.

As is common practice, the Ibovespa was used as a proxy for the market portfolio. Its log returns with closing prices are calculated for the estimation of the pricing equation. The index series was also collected from Economática.

The CDI rate was used as the risk free rate, given that it faithfully represents a riskless rate known *ex ante* by investors in Brazil. The use of longer-maturity government bonds was not considered, since those securities are highly volatile during the period as a whole, particularly during the 1999 crisis and in 2003. The CDI rate was collected in the Economática data base.

Finally, there is a need for choosing the state variables to be used in the estimation of the ICAPM. According to Bali & Engle (2010) and Merton's (1973) own suggestion, the economy's nominal interest rate should be included as a state variable, since many investors buy fixed income securities that are indexed to that rate. The CDI rate was then used as a proxy for the interest rate.

In terms of other candidates for state variables, we considered changes in the Real/U.S. dollar exchange rate, in the price of gold, and changes in the inflation rate for the Brazilian economy, those variables having been examined in the literature.

Aquino (2006) uses Philippine stock market returns to test the significance of exchange rates as a critical variable following the 1997 Asian crisis. He finds for the significance of exchange rates as a state variable, and, specifically, that negative risk premiums were consistent with assuming that investors are mean-variance optimizers, provided that they are also concerned with hedging against changes in the investment opportunity set, proxied by exchange rate changes. At times of currency volatility, as in 2008, investors may use foreign currencies as protection against adverse changes in the investment opportunity set.

The exchange rate series was constructed with the use of relative Purchasing Power Parity (PPP): the change in the exchange rate is equal to the inflation rate difference between Brazil and the United States.⁸ The necessary data were collected from Ipeadata and Federal Reserve Economic Data (FRED).

The change in the consumer price index (IPCA) was used as another state variable, representing the inflation rate, proxied by the change in the IPCA, instead of the interest rate spread adopted by Garcia & Bonomo (2001). The use of that spread was justified by them with the argument that changes in the short-term real interest rate would be of second order. The Brazilian economy went through a hyperinflationary period before the Real Plan was implemented. However, the magnitude of price changes has remained significant. Hence, it is only natural that inflation be treated as a state variable: in inflationary periods, investors may demand assets whose returns are highly and positively correlated with inflation, in order to protect themselves against purchasing power losses. Hence, there would be a negative premium for inflation risk.

Faff & Chan (1998) considered the change in the price of gold as a potential state variable. The authors, in a test of the ICAPM for the Australian market covering the 1975 to 1994 period, found that gold price was

⁸The actual exchange rate time series was also considered. However, log-likelihood estimation was not feasible due to the occurrence of extreme returns at times when the domestic currency was devalued, specifically in January 1989, August 1993 and July 1994. To overcome this, the average between the previous and the following value around those dates was used to replace the actual values. However, due to the highly questionable use of this method, results are not reported.

a significant state variable. However, since the results were of opposite signs depending on which industry portfolio (resources versus industry) was considered, and although the null hypothesis that gold price risk exposures were jointly equal to zero across portfolios was rejected, overall they could not conclude that their two-factor ICAPM was a satisfactory explanation for Australian cross-sectional returns. In our paper, the price of gold, in U. S. dollars, was collected from Bloomberg, and its log return was then computed.

Thus, the expected results seem to be an empirical issue, given that the role of any candidate state variable, in the first place, may vary with a particular domestic market's characteristics, and even over time in that market, as pointed out, in general, by Faff & Chan (1998) for the role of gold as a hedging instrument:

“Gold has traditionally had a hedging role in investment decisions, but the considerable upheaval in the world's financial markets over the past decade or so may well have diminished its hedging role.” (p. 177)

Given our review of the empirical literature, on the role of the three candidate state variables, one would expect negative risk premiums for both exchange rate and inflation risk. For gold price risk, one would also expect a negative risk premium, but such a premium was not obtained by Faff & Chan (1998).

4. Results

4.1 DCC results

Initially, the methodology requires that dynamic correlations be estimated with the DCC model. For our ten portfolios, equations (3)-(5) were estimated. Table 2 displays the means and standard deviations (in parentheses) of the correlations of each portfolio with the state variables.

Table 2

Means (standard deviations) of correlations between portfolio returns and state variables, 1988 to 2012

	Ibovespa	Interest Rate	Exchange Rate	Inflation	Gold
Dec1	0.2808 (0.1210)	0.4862 (0.0807)	-0.0635 (0.0495)	-0.0780 (0.0544)	-0.0368 (0.0656)
Dec2	0.3977 (0.0837)	0.6195 (0.0278)	-0.0685 (0.0402)	-0.0648 (0.0351)	0.0631 (0.1420)
Dec3	0.4102 (0.0783)	0.6359 (0.0379)	-0.0447 (0.0498)	-0.0419 (0.0459)	0.0098 (0.0191)
Dec4	0.4580 (0.1371)	0.6525 (0.0813)	-0.0898 (0.0679)	-0.0625 (0.0446)	0.0995 (0.0070)
Dec5	0.4744 (0.1227)	0.7357 (0.0563)	-0.1047 (0.0510)	-0.1018 (0.0505)	0.1602 (0.0330)
Dec6	0.4894 (0.1236)	0.7496 (0.0301)	-0.1256 (0.0578)	-0.1013 (0.0414)	0.1236 (0.0510)
Dec7	0.5680 (0.1351)	0.8030 (0.0468)	-0.1228 (0.0420)	-0.0969 (0.0434)	0.0662 (0.0810)
Dec8	0.5157 (0.1643)	0.8157 (0.0376)	-0.1390 (0.0543)	-0.1327 (0.0569)	0.1153 (0.0329)
Dec9	0.5492 (0.1753)	0.8651 (0.0463)	-0.1401 (0.0593)	-0.1223 (0.0602)	0.1445 (0.1145)
Dec10	0.5824 (0.2176)	0.9870 (0.0026)	-0.1574 (0.0534)	-0.1338 (0.0584)	0.1561 (0.0867)

Source: prepared by the authors.

Table 2 indicates the following about the estimated correlations over the full period (1988 to 2012): on average, the portfolio return correlations with the Ibovespa are positive. In addition, the higher the decile, the higher the correlation appears to be. This is as expected, since the higher market value companies are mostly those whose securities are included in the Ibovespa portfolio. It should also be noted that the variability of correlations with the Ibovespa appears to be higher than that for the other state variables.

The portfolio return correlations with the interest rate are also positive. This is as expected, since increases in interest rates are associated with declines in prices of other assets, which is equivalent to increases in their expected returns.

The correlations with the exchange rate are negative, and they seem even more negative as we consider higher market value companies. This result means that currency depreciation (increases in the exchange rate) is associated with declines in the expected return on stocks, often as a result of the exit of foreign investors from the Brazilian market.

Correlations with the inflation rate are negative, with similar magni-

tudes as the correlations with the exchange rate. This means that increases in prices are associated with declines in the expected returns on stocks. One could argue that increases in prices lead to higher demand of financial assets as a way to protect current wealth, and the resulting increase in the prices of financial assets is equivalent to a decrease in expected returns.

Finally, correlations with changes in gold prices are predominantly positive: increases in gold prices are usually accompanied by increases in the assets' expected returns, as observed also in the case of interest rate changes. The argument here could be that when investors change the composition of their portfolios towards gold, gold price increases and the prices of other assets fall (or equivalently, their expected returns increase).

It should also be pointed out that the series of estimated correlations vary substantially over time.⁹ This observation reinforces the need for this methodology, which uses conditional covariances in the estimation of the pricing equation.

4.2 Pricing equation results

The second stage entails the estimation of (6) by SUR. After the estimation, one tests whether the A and $B_j (j = 1, 2, 3, 4)$ parameters are statistically significant.

In addition, in order to ensure that the model corresponds to an equilibrium relationship, all intercept terms must be equal to zero; in other words, it is necessary to ascertain whether C_i is jointly equal to zero for all estimated equations.

Table 3 presents the results of the system estimated for the full sample, that is, from March 1988 to September 2012.¹⁰ The last column contains both the statistic and the p-value associated with a multivariate Wald test for the intercepts of each equation. That is, we test $H_0 : C_1 = C_2 = \dots = C_{10} = 0$. A failure to reject the null hypothesis means that the expected returns of our stock portfolios would not be fully explained by the pricing factors.¹¹

⁹Exhibits with the series of estimated correlations between portfolio returns and state variables are available with the authors.

¹⁰As previously mentioned, the SUR estimator is more efficient than OLS in the presence of serial correlation between equation errors. A serial correlation test was performed with the residuals, and the hypothesis of absence of correlation was rejected at the 1% level for several numbers of lags.

¹¹Results are robust when changing the measurement of the exchange rate, both in terms of magnitude and sign.

Table 3

Estimated pricing equation, by SUR, for the full sample period

Dependent Variable	Independent Variables					Wald Test
	IBOV	Δ CDI	Δ E	INFL	Δ GOLD	
Portfolio Excess	2.387	-3.099	2.026	-3.816	23.016	13.291
Return	(0.286)	(0.345)	(1.792)	(1.960)	(5.751)	[0.208]
	[<0.001]	[<0.001]	[0.258]	[0.052]	[<0.001]	

Number of observations = 295.

Standard errors in parentheses; p-values in square brackets.

Source: prepared by the authors.

The risk aversion coefficient is estimated at 2.387, and it is highly significant. This is in accordance with Merton's (1973) prediction and with the results obtained by Bali & Engle (2010), among others.

Given an understanding of the model's logic, a few interesting remarks may be made about the results shown in Table 3. First, the risk premium associated with changes in the interest rate is negative and significant. This means that, the higher the correlation between returns on a portfolio with changes in the interest rate, the lower its expected return will be (in other words, the stronger will be the demand for that portfolio). This means that the correlation between consumption and the interest rate is negative. If that is the case, then increases in the interest rate are associated with reductions in the level of consumption (adverse changes in the investment opportunity set). This means that assets whose returns are positively correlated with the interest rate will be highly valued, since they will provide higher returns when consumption is low, and their risk premium will be negative.

Second, the risk premium associated with inflation is also negative and significant (at the 10% level). The interpretation is similar to that for the interest rate: consumption is negatively correlated with inflation. Increases in inflation are associated with consumption losses and, thus, assets whose returns are highly correlated with inflation are used as hedging instruments by investors. Those assets, therefore, will have lower expected returns the higher their correlation with inflation. This confirms the results expected with the empirical work by Garcia & Bonomo (2001).

The risk premium associated with the change in gold prices is positive and significant. This leads to the opposite interpretation: the lower the correlation between portfolio returns and gold price changes, the lower a portfolio's expected returns. Assets whose returns are negatively correlated with gold price changes will be in high demand, implying that the correlation between consumption and gold prices is positive. Reductions in the

price of gold are associated with reductions in the level of consumption by individuals. Hence, assets with returns highly correlated with gold prices will not be good hedging instruments. The demand for such assets will be weak, and their expected returns will be higher, implying a positive risk premium.

In addition, the exchange rate does not appear to be a relevant state variable for the period under analysis, since its risk premium is not significantly different from zero. Therefore, there does not seem to be demand for hedging against changes in the exchange rate; in other words, changes in the exchange rate do not seem to be correlated with consumption losses.

The joint tests for the intercepts of the various equations lead to the conclusion that the expected returns are, in equilibrium, fully explained by their covariance with the market portfolio and the state variables. This means that there is strong evidence for the empirical validity of the ICAPM for the full sample period.

Finally, the validity of the model is tested constructing a scatter plot between actual average excess returns and predicted average excess returns for each of the 10 portfolios, as suggested in Cochrane (2005). If the model is correct, the points should all lie on a 45° degree line, indicating that on average the returns predicted by the ICAPM coincide with actual returns. The results are depicted in Figure 1.

Source: prepared by the authors.

Figure 1

Scatter plot between actual average excess returns and predicted average excess returns

Figure 1 provides additional support towards validation of the ICAPM with Brazilian data. Almost all points in the scatter plot fit reasonably well to a 45° degree line. The only exception is the point in the upper-left corner which corresponds to portfolio 1, which contains the bottom 10% stocks by market value. The model underestimates the actual returns for stocks in this portfolio. This suggests that the model may not be adequate for stocks with low market value. Based on the estimated ICAPM, such stocks would perform poorly as compared to their actual returns. However, the model fits well for all other points.

Since the sample period is very long and the Brazilian economy went through several changes in that period, it would be enlightening if the sample period could be broken up into sub-periods, and the ICAPM were exam-

ined for each one. Two important mileposts were considered: the Real Plan and the 1999 currency depreciation, leading to the consideration of three sub-periods: from March 1988 to February 1994, the period preceding the Real Plan; March 1994 to December 1998, the period after the implementation of the Real Plan, but during which the exchange rate was strictly managed; January 1999 to September 2012, under the Real Plan with floating exchange rates. Table 4 provides the results for all three sub-periods.

Table 4

Estimated pricing equation, by SUR, for sub-periods

Panel A: March 1988 to February 1994 (before the Real Plan)

Dependent variable	IBOV	Δ CDI	Δ E	INFL	Δ GOLD	Wald Test
Portfolio	3.146	-2.000	3.155	0.244	20.868	23.668
excess returns	(0.517)	(0.642)	(2.944)	(3.313)	(13.748)	[0.009]
	[<0.001]	[0.002]	[0.284]	[0.941]	[0.129]	
Number of observations = 72						

Panel B: March 1994 to December 1998 (Real Plan/before depreciation)

Dependent Variable	Independent Variables					Wald Test
	IBOV	Δ CDI	Δ E	INFL	Δ GOLD	
Portfolio	2.366	-7.453	-5.334	-22.116	6.939	10.752
excess returns	(0.869)	(0.822)	(8.140)	(8.248)	(16.369)	[0.377]
	[0.007]	[<0.001]	[0.513]	[0.008]	[0.672]	
Number of observations = 58						

Panel C: January 1999 to September 2012 (after depreciation)

Dependent Variable	Independent Variables					Wald Test
	IBOV	Δ CDI	Δ E	INFL	Δ GOLD	
Portfolio	-1.977	4.190	-202.845	-53.805	10.067	19.170
excess returns	(0.692)	(0.999)	(83.655)	(121.605)	(7.430)	[0.038]
	[0.004]	[<0.001]	[0.015]	[0.658]	[0.176]	
Number of observations = 165						

Standard errors in parentheses; p-values in square brackets.

Source: prepared by the authors.

The Wald test results indicate that expected returns are not entirely explained by the variables included in the pricing equation for the period before the Real Plan. The ICAPM seems to fail when this sub-period is considered. In this case, other pricing factors would have to be considered. Since we are examining a period of hyperinflation, variables such as the long term interest rate (TJLP) and the yield on savings accounts would be candidates as state variables. For the other periods, it can be concluded that

the intercept terms are jointly equal to zero at the 1% level.¹²

The risk aversion coefficient is significant in all sub-periods, but is negative in the January 1999-September 2012 period. This result is counterintuitive, but it could be interpreted as a structural break caused by a change in economic regime.

Evidence in support of that explanation is the change in the sign of the risk premium associated with the interest rate, which becomes positive for the 1999-2012 period. This indicates that the correlation between changes in consumption and in the interest rate is now positive. The exchange rate is a relevant state variable only for that sub-period, and its risk premium is negative: assets with returns highly correlated with the changes in the exchange rate are highly demanded (implying lower expected returns), since consumption tends to be lower in currency depreciation periods, and such assets are then used as hedging instruments.

It is noteworthy that inflation ceases to be a relevant state variable in the 1999-2012 period, meaning that changes in inflation do not significantly affect the investment opportunity set. This may be attributed to the control over inflation implemented with the Real Plan.

It is also enlightening to examine the behavior of coefficients before and after the 2008 crisis. Another exercise was performed, with the breaking up of the full sample period into two sub-periods: the first runs from March 1988 to September 2008, and the second from October 2008 to September 2012.¹³ Table 5 contains the results for both sub-periods.

¹²For the third period, however, the Wald test leads to rejection at the 5% level.

¹³An alternative dating for the 2008 crisis was contemplated, with the inception of the crisis in June 2008, because that was the moment at which the Ibovespa reached its peak level (on May 20, 2008). However, the results are not significantly different.

Table 5

Estimated pricing equation, by SUR, before and after the 2008 crisis

Panel A: March 1988 to September 2008 (before the financial crisis)

Dependent Variable	Independent Variables					Wald Test
	IBOV	Δ CDI	Δ E	INFL	Δ GOLD	
Portfolio	2.429	-3.124	2.033	-3.860	21.570	10.013
excess returns	(0.309)	(0.375)	(1.933)	(2.117)	(6.684)	[0.439]
	[<0.001]	[<0.001]	[0.293]	[0.068]	[0.001]	

Number of observations = 247

Panel B: October 2008 to September 2012 (after the financial crisis)

Dependent Variable	Independent Variables					Wald Test
	IBOV	Δ CDI	Δ E	INFL	Δ GOLD	
Portfolio	-2.679	3.843	-478.964	-1522.105	3.915	21.748
excess returns	(2.756)	(2.682)	(145.030)	(441.483)	(9.486)	[0.016]
	[0.331]	[0.153]	[0.001]	[<0.001]	[0.680]	

Number of observations = 48

Standard errors in parentheses; p-values in square brackets.

Source: prepared by the authors.

Table 5 shows that the risk aversion coefficient is positive and significant in the pre-crisis period, and negative but not significant in the post-crisis period. Nevertheless, the conclusion for the joint test of the model intercepts should be made with care: at the 5% significance level, one concludes that there would be other relevant asset pricing factors; however, at the 1% level the conclusion is that the state variables used are sufficient for explaining expected returns. Thus, the ICAPM's empirical validity in the post-crisis period is may be questioned.

It may be noted also that the interest rate and the price of gold are no longer relevant state variables in the second sub-period, i.e., changes in interest rates and gold prices are not significantly correlated with consumption, implying that they are not used as protection instruments. In the case of gold, in particular, this is an unexpected result. During the 2008 crisis, gold prices increased substantially, due to uncertainty in financial markets and the resulting migration by investors to safer assets, including gold (the so-called flight to quality).

Another interesting result is that which indicates that, whereas the exchange rate was not an inter-temporal hedging instrument in the pre-crisis period, it became one from October 2008 onward. The argument, in this case, is similar to that used for gold prices: it may be surmised that Brazilian investors started to use the currency market to construct their inter-temporal hedges in anticipation of an exit by foreign investors from emerg-

ing economies. Since the risk premium is negative, assets highly correlated with the exchange rate are more highly valued, since they protect the individual from consumption losses caused by currency depreciation. This result confirms those obtained by Aquino (2006), whose focus was on the role played by this state variable in a crisis situation.

The decomposition into sub-periods reveals that, although the ICAPM has a good fit when the full sample period is considered, care should be taken so as to not assume that the estimated coefficients and relationships are valid for any period one could be interested in.

5. Conclusion

The objective of this paper was to assess the empirical validity of the Inter-temporal Capital Asset Pricing Model (ICAPM) for the Brazilian market. According to that model, expected returns do not depend only on the covariance with the market portfolio, but also on the covariances with other variables, the so-called “state variables”. Such variables produce changes in the agents’ investment opportunity set, in such a way that, if consumption is negatively correlated with a state variable, then an asset’s expected return will be lower when the correlation of its returns with changes in that variable (negative risk premium) is higher. Alternatively, if consumption is positively correlated with changes in the state variable, an asset’s expected return will be higher when the correlation of its returns with changes in the state variable is higher, i.e., there is a positive risk premium.

In order to test the ICAPM, the Bali & Engle (2010) methodology was used. This involves a two-stage procedure: in the first stage, dynamic correlations between stock portfolio returns and the state variables are estimated with the DCC model. In the second stage, such covariances are used as explanatory variables in a system of pricing equations estimated by SUR. The advantage from using that methodology in the pricing equation over its better known representation (which uses returns as explanatory variables), is the possibility of taking into account the temporal variation of covariances.

The state variables employed in this paper were the rate of return on the Ibovespa, changes in the CDI interest rate, changes in the Real/U.S. dollar exchange rate, inflation and gold price changes. The sample period runs from March 1988 to September 2012, and monthly data are used. Stock portfolios were constructed after ordering the listed companies by their market capitalization, and each portfolio corresponded to a decile of such ordering.

When the conditional correlations of each decile (portfolio) with state variables were computed, it was observed that the correlations with Ibovespa returns are positive, and the same was true for changes in the interest rate and the price of gold. The correlation between returns and the rate of inflation is negative, and the same is true for changes in the exchange rate.

The estimation of the pricing equation system produced results favorable to the ICAPM for the full sample period. The risk aversion coefficient was estimated at close to 2.40 and was highly significant. This result indicates that the risk-return relationship in Brazil is a positive one.

In addition, it was observed that assets whose returns are highly correlated with changes in the interest rate and inflation must have lower expected returns (negative risk premium). This means that investors will demand assets whose returns are positively correlated with such variables, because positive changes in inflation and the interest rate cause adverse changes in the opportunity set, i.e., consumption is negatively correlated with the two variables. In the case of gold prices, the opposite was found: assets that are highly correlated with gold price changes must have higher expected returns, since consumption is positively correlated with gold price changes. When the price of gold falls, consumption declines; hence, assets with returns highly correlated with gold price changes will not be highly demanded and will have higher expected returns. Hence, the observed risk premium is positive. The exchange rate does not seem to be a relevant state variable for the full period, since its risk premium is not significantly different from zero.

Furthermore, it is concluded that the intercept terms are jointly equal to zero; therefore, the estimated relationship may be considered as an equilibrium relationship.

A validity check with a scatter plot of actual average excess returns against predicted average excess returns was conducted. All points fit reasonably well with a 45° degree line, pointing towards the validity of ICAPM. This is not true, however, for portfolio 1. The model appears to underestimate the actual returns for stocks with low market value.

Next, the sample period was broken up into three sub-periods arising from different and important phases of the Brazilian economy, in terms of economic policy, currency regimes and inflation: the pre-Real Plan period (March 1988 to February 1994); the post-Real Plan and pre-currency depreciation period (March 1994 to December 1998); and the post-Real Plan

and post-currency depreciation period, with the current set of currency and monetary policies (January 1999 to September 2012). The analysis of these sub-periods weakens the empirical validity of the ICAPM. Expected returns are not entirely explained by state variables in the 1988-1994 sub-period and the risk aversion coefficient is negative and significant in the 1999-2012 sub-period, contrary to expectations. The sign of the interest rate risk premium also changes, becoming positive for the 1999-2012 sub-period. In addition, the exchange rate is now a relevant state variable in that sub-period, while the inflation rate risk premium turns out to be non-significant.

The last analysis consisted in assessing the impact of the 2008 crisis on the results for the ICAPM tests. The sample period was once more broken up, this time into two sub-periods: pre-crisis (March 1988 to September 2008) and post-crisis (October 2008 to September 2012). The estimated risk aversion coefficient is negative, albeit not significant, for the post-crisis period. The test of the intercept terms shows that other variables may explain expected portfolio returns. Hence, there is little favorable evidence in the post-crisis sub-period. In this same sub-period, the interest rate and gold price are no longer relevant state variables, since their respective risk premiums are not significant. However, the exchange rate is now relevant and its risk premium is negative. This result may be attributed to the possibility that the Brazilian investor began using the exchange rate to construct her inter-temporal hedge in anticipation of an exit by foreign investors from emerging markets.

The analysis points to the empirical validity of the ICAPM in the Brazilian market when the full sample period is considered (1988 to 2012). A positive risk-return relationship was also observed, as indicated by the risk aversion coefficient.

In addition, interest rate, inflation and gold prices are variables that produce changes in the investment opportunity set. The first two variables are negatively correlated with consumption and, hence, have negative risk premiums, while the third variable has a positive risk premium. The analysis for sub-periods, both in terms of economic policy regimes and the global financial crisis, indicates that care must be taken when examining the results, since major shocks affecting financial markets may cause changes in investor preferences and alter the prevailing relationships.

Future studies may offer contributions when they uncover new pricing factors for the periods in which the ICAPM seems not to be valid: 1988-1994 and 2008-2012. In addition, given the change in the risk aversion

coefficient, another possibility, raised in Bali & Engle (2010), would consist in treating that coefficient as a time-varying parameter.

References

- Almeida, Leandro O. 2010. *Estimação Do CAPM Intertemporal Com Ações Da BOVESPA*. Faculdade de Economia, Administração e Contabilidade, Universidade de São Paulo, São Paulo. Dissertação de Mestrado.
- Aquino, Rodolfo. 2006. A Variance Equality Test of the ICAPM on Phillipine Stocks: Post-Asian Financial Crisis Period. *Applied Economics*, **38**, 353–362.
- Bali, Turan G. 2008. The Intertemporal Relation Between Expected Returns and Risk. *Journal of Financial Economics*, **87**, 101–131.
- Bali, Turan G., & Engle, Robert F. 2010. The Intertemporal Capital Asset Pricing Model with Dynamic Conditional Correlations. *Journal of Monetary Economics*, **57**, 377–390.
- Bali, Turan G., & Peng, Lin. 2006. Is There a Risk-Return Tradeoff? Evidence from High Frequency Data. *Journal of Applied Econometrics*, **21**, 1169–1198.
- Black, Fischer S., Jensen, Michael C., & Scholes, Myron S. 1972. The Capital Asset Pricing Model: Some Empirical Tests. *Pages 79–121 of: Jensen, Michael C. (ed), Studies in the Theory of Capital Markets*. New York: Praeger.
- Blume, Marshall, & Friend, Irwin. 1973. A New Look at the Capital Asset Pricing Model. *Journal of Finance*, **28**, 19–33.
- Bodie, Zvi, Kane, Alex, & Marcus, Alan J. 2009. *Investments*. 8th edn. Irwin, New York: McGraw-Hill.
- Bollerslev, Tim, Engle, Robert F., & Wooldridge, Jeffrey M. 1988. A Capital Asset Pricing Model with Time-Varying Covariances. *The Journal of Political Economy*, **96**, 116–131.
- Brandt, Michael W., & Kang, Qiang. 2004. On the Relationship Between the Conditional Mean and Volatility of Stock Returns: A Latent VAR Approach. *Journal of Financial Economics*, **72**, 217–257.

- Campbell, John Y. 1987. Stock Returns and the Term Structure. *Journal of Financial Economics*, **18**, 373–399.
- Campbell, John Y. 1996. Understanding Risk and Return. *Journal of Political Economy*, **104**, 298–345.
- Campbell, John Y., & Vuolteenaho, Tuomo. 2004. Bad Beta, Good Beta. *American Economic Review*, **94**, 1249–1275.
- Chen, Joseph. 2002. *Intertemporal CAPM and the Cross-Section of Stock Returns*. Working Paper, University of Southern California.
- Cochrane, John H. 2005. *Asset Pricing*. Revised edn. Princeton: Princeton University Press.
- Engle, Robert F. 2002. Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models. *Journal of Business and Economic Statistics*, **20**, 339–350.
- Faff, Robert, & Chan, Howard. 1998. A Test of the Intertemporal CAPM in the Australian Equity Market. *Journal of International Financial Markets, Institutions and Money*, **8**, 175–188.
- Fama, Eugene F., & French, Kenneth R. 1992. The Cross-Section of Expected Stock Returns. *Journal of Finance*, **47**, 427–465.
- Fama, Eugene F., & French, Kenneth R. 2004. The Capital Asset Pricing Model: Theory and Evidence. *Journal of Economic Perspectives*, **18**, 25–46.
- Fama, Eugene F., & Macbeth, James D. 1973. Risk, Return and Equilibrium: Empirical Tests. *The Journal of Political Economy*, **81**, 607–636.
- Garcia, René, & Bonomo, Marco A. C. 2001. Tests of Conditional Asset Pricing Models in the Brazilian Stock Market. *Journal of International Money and Finance*, **20**, 71–90.
- Ghysels, Eric, Santa-Clara, Pedro, & Valkanov, Rossen. 2005. There is a Risk-Return Tradeoff After All. *Journal of Financial Economics*, **76**, 508–548.

- Glosten, Lawrence R., Jagannathan, Ravi, & Runkle, David E. 1993. On the Relation Between Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance*, **48**, 1779–1801.
- Guo, Hui, & Whitelaw, Robert F. 2006. Uncovering the Risk-Return Relation in the Stock Market. *Journal of Finance*, **61**, 1433–1463.
- Lintner, John V. 1965. Security Prices, Risk and Maximal Gains from Diversification. *Journal of Finance*, **20**, 587–616.
- Lundblad, Christian. 2007. The Risk Return Tradeoff in the Long Run: 1836–2003. *Journal of Financial Economics*, **85**, 123–150.
- Merton, Robert C. 1973. An Intertemporal Capital Asset Pricing Model. *Econometrica*, **41**, 867–887.
- Mossin, Jan. 1966. Equilibrium in a Capital Asset Market. *Econometrica*, **34**, 768–783.
- Nanisetty, Prasad, Bharati, Rakesh, & Gupta, Manoj. 1996. The Effect of Time-Varying Covariances on Asset Risk Premia. *Review of Quantitative Finance and Accounting*, **7**, 205–220.
- Nelson, Daniel B. 1991. Conditional Heteroskedasticity is Asset Returns: A New Approach. *Econometrica*, **59**, 347–370.
- Ross, Stephen A. 1976. The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory*, **13**, 341–360.
- Scruggs, John T. 1998. Resolving the Puzzling Intertemporal Relation Between the Market Risk Premium and Conditional Market Variance: A Two-Factor Approach. *Journal of Finance*, **53**, 575–603.
- Sharpe, William F. 1964. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance*, **19**, 425–442.
- Whitelaw, Robert F. 1994. Time Variations and Covariations in the Expectation and Volatility of Stock Market Returns. *Journal of Finance*, **49**, 515–541.