Assessing Drawdown-at-Risk in Brazilian Real Foreign Exchange Rates

Vinicius Ratton Brandi*
Beatriz Vaz de Melo Mendes**

Abstract
The investigation of the stochastic behavior of financial series has become widespread over the literature. There is empirical and theoretical evidence that the total stock price change over a long period is usually concentrated in a few hectic runs of trading days. The drawdown is a random variable which provides information on alternative characteristics of market behavior during these periods. In this work, we use distributions from extreme value theory to model the severity of drawdowns and drawups. We illustrate using nine currency exchange rates and gold.

Resumo
A investigação do comportamento estocástico das séries financeiras tornou-se bastante difundida na literatura acadêmica. Há evidências empíricas e teóricas de que as variações nos preços dos ativos observadas em um longo período, normalmente, encontram-se concentradas em poucos e turbulentos dias. O drawdown é uma variável aleatória que fornece informação adicional a respeito das características de mercado durante estes dias. Neste trabalho, usamos distribuições de valores extremos para modelar a severidade dos drawdowns e dos drawups obtidos a partir das séries históricas de nove taxas de câmbio e do ouro.

Keywords: foreign exchange risk; drawdowns; drawdown-at-risk.

JEL codes: C52; G10; G15.

1. Introduction

The stochastic behavior of financial series has been widely investigated since the seminal paper of Bachelier (1900). Thereafter, several other studies have evidenced the presence of serial correlation, heteroscedasticity and long memory as well-established stylized facts of financial assets returns.

In another noteworthy work, Mandelbrot (1963) argues that price changes frequently show sharp discontinuities. For a given data set, it is noticed that the total stock price change is usually concentrated in a few random sequences of trading days.
days, suggesting that the price movement may be related to another subordinated process.

Clark (1973) was the first to notice that the distribution of increments of subordinated process provides a natural mechanism for fat tails. He argues that a time dependent subordinated process, such as transaction volume and number of trades, would explain the occurrence of turbulent cascades in stock market.

Such complex behavior asks for careful modeling. Empirical evidence that levels of price activity last randomly and are not uniformly distributed over time suggest new modeling procedures where physical time is no longer a deterministic variable, but described as a stochastic process subordinated to another random variable.

Theoretical research on this area has led to the concept of fractals (Mandelbrot, 1997), initially developed by Mandelbrot (1972). This alternative approach consists of disregarding conventional physical time standards by analyzing financial series on different scales which are determined by an exogenous variable, named yardsticks, and comparing further the results.

Müller et al. (1995) contend that the success of the above mentioned approach would support the hypothesis that the financial market is fractal itself, with heterogeneous trading behavior, where market participants behave quite differently, presenting different perception and responding-time in reacting to historical events and immediate information.

The most regular type of fractal is named self-similar fractal (Mandelbrot, 1983), corresponding to an object (a financial time series, for instance) which statistical properties remain unchanged, except for a scaling factor represented by a power function of the size of the yardstick. Examining foreign exchange markets, Müller et al. (1995) observe that, as long as the distribution function of the returns becomes increasingly fat-tailed with higher time resolution (smaller yardsticks), currency rates are not self-similar fractals.

Gensay et al. (1998) evaluate the statistical properties of currency rates time series by using a real-time trading model based on technical analysis. Their results indicate that excess returns are not spurious, suggesting the presence of short-range serial correlation in the data.

In order to address those issues Johansen and Sornette (2001) propose using drawdowns as a downside risk benchmark. They remind that one-point statistics are insufficient for characterizing financial market moves and two-point statistics, even though those modeled by AR-GARCH families, although offering important complementary information, are still limited. Therefore, the short-cut would be to realize that fixed time scale statistics are not adapted to real dynamics of price moves and, according to them, relatively low-order statistics with suitable adjustments to the relevant intrinsic-time (also called market-time or elastic-time) scales, as drawdowns, could lead to a better characterization of the information flow and the rhythm of trading in the financial markets.

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1See also Geman and Ané (1996).
A drawdown is defined as the percentage price change between a local maximum and the following local minimum. Formally, assume that $P_t$ is the price of an asset at day $t$ and $r_t = (P_t / P_{t-1}) - 1$ is the return. $P_t$ is a local maximum when $P_{t-1} < P_t > P_{t+1}$ and a local minimum when $P_{t-1} > P_t < P_{t+1}$. Considering a sequence of price drops $P_t > P_{t+1} > \cdots > P_{t+d}$, where $P_t$ is a local maximum and $P_{t+d}$ is a local minimum, the drawdown, with duration $D$ equal to $d$ days, has a severity defined as $Y = (P_{t+d} / P_t) - 1$, which is equivalent to the product of a random sequence of return factors:

$$Y = \prod_{j=1}^{D} (1 + r_{t+j}) - 1 \quad (1)$$

By analogy, the same concept is applied to drawups, which is computed on a sequence of price rises.

A broader definition of drawdown can be found in Johansen (2003). It is called $\epsilon$-drawdown and is calculated as the sum of consecutive negative log-returns, but ignoring contrary movements smaller than $\epsilon$. The choice of $\epsilon$ may be quite subjective and whenever a supposed $\epsilon$-drawdown shows up as a positive variation, the author suggest discarding this observation. Once again, $\epsilon$-drawups can be obtained in an analogous way.

The estimation of the probability distribution of the drawdowns of a particular investment provides a new insight of its risk profile. Since it is related to cumulative losses of an investment, it represents a rather relevant risk measure and may be quite useful not only for investors assessing downside risk of their portfolio but also for financial system regulators, aiming to mitigate systemic risk through capital requirement policies.

Additionally, once drawdowns are calculated from negative consecutive returns, they are supposed to embody short-range subtle dependence, capturing the way successive drops can influence each other. Moreover, as they incorporate higher order correlations, they may offer a better measure of real market risks than other based on fixed time scale. In this work, we examine foreign exchange risk in Brazilian markets through the modeling of drawdowns and drawups. As well as the widespread used Value-at-Risk (VaR), the Drawdown-at-risk (DdaR) and the Drawup-at-Risk (DuaR) are computed as, respectively, the quantiles of the distribution of the severities of the drawdowns and drawups, according to a specified exceedance probability. The difference between those measures relies on the time horizon. While VaR represents a measure for a fixed and predetermined period of

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2It is important to note that, eventually, there may be observed a sequence of equal prices. In those cases, in order to define the local maximums and minimums, the sequence of equal prices will be considered as a single observation. Alternatively, when zero returns are observed inside a sequence of negative (positive) returns, they are considered to belong to this drawdown (drawup). Also, to compute the duration of drawdowns and drawups whenever a local minimum or a local maximum is represented by a sequence of equal prices, we consider the last observation as the local maximum or minimum.

3Lag 1 serial correlation in the closing prices of US dollar is positive 1% statistically significant.
time, the \( \text{DdaR} \) and the \( \text{DuaR} \) are computed on stochastic-time basis.

Additionally, we investigate the effect of diversification in currency portfolios and, in order to assess overnight and intraday risks, we propose to use particular measures that we define as overnight and intraday drawdowns.

Drawdowns and drawups can also be incorporated into risk-return analysis. Checklov et al. (2000), for example, propose the use of Conditional Drawdown-at-Risk (\( \text{CDaR} \)) in portfolio optimization algorithm. The \( \text{CDaR} \) is defined as the average excess drawdown over a specified threshold or a significance level, similarly to Conditional \( \text{VaR} \), sometimes referred to as mean (expected) shortfall.

The rest of the paper is organized as follows. In section 2 we present the statistical models and section 3 presents the inference methods. In section 4 we fit the Modified Generalized Pareto distribution to the drawdowns and drawups of currency exchange rates and, finally, section 5 summarizes the results and presents our conclusions.

2. Modeling Drawdowns

The finite sample distribution of the severities of drawdowns is typically right skewed and long tailed. Johansen and Sornette (2001) suggested the use of the Weibull (Stretched Exponential) distribution to model drawdown’s severities from stock indexes, commodities and currencies. Mendes and Brandi (2004) found the Modified Generalized Pareto distribution (MGPD), from extreme value theory (EVT),\(^4\) to be the appropriate parameterization for modeling drawdown’s and drawup’s severities from stock market indexes.

The MGPD is a generalization of the generalized Pareto distribution (GPD) proposed by Anderson and Dancy (1992) to model aggregated excesses over thresholds. In that paper, asymptotic arguments are used to show that this distribution is suitable to model the sum of a random string of variables presenting local dependence. These results motivated applying the MGPD to model the long tailed right skewed drawdown data.

Unlike the strictly decreasing GPD density, the MGPD allows for shapes with a mode greater than zero. This greater flexibility is achieved by the inclusion of an extra parameter \( \theta \). The MGPD cumulative distribution function is given by:

\[
G_\xi(y) = \begin{cases} 
1 - \left(1 + \frac{x^\theta}{\varphi}\right)^{-1/\xi} & , \text{ if } \xi \neq 0 \\
1 - e^{-\frac{y}{\varphi}} & , \text{ if } \xi = 0 
\end{cases}
\]

for \( \theta \in \mathbb{R}, \xi \in \mathbb{R} \) and where \( \varphi > 0 \) is a scale parameter. The shape parameter \( \xi \), also known as the “tail index” parameter, has the same interpretation as in the GPD.

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case and determines the rate that the distribution initially falls off. For a given \( \theta \), higher \( \xi \) and \( \varphi \) will represent a distribution with fatter tail.

Observe that GPD is a special case of MGPD, when \( \theta \) equals to 1. In short, the following three constrained models are obtained from the full model MGPD \( (\theta, \xi, \varphi) \): the Stretched Exponential (or Weibull distribution): MGPD \( (\theta, 0, \varphi) \); the GPD: MGPD \( (1, \xi, \varphi) \); and the unit exponential distribution: MGPD \( (1, 0, \varphi) \).

\[
\text{MGPD}
\]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Density functions of the MGPD with \( \xi = 0.8, \varphi = 2.5 \), and \( \theta = 0.5, 1.0 \) and 1.5}
\end{figure}

Figure 1 illustrates the flexibility of the MGPD distribution and its density function for \( \xi = 0.8, \varphi = 2.5 \) and varying values of \( \theta = 0.5, 1.0 \) and 1.5. The MGPD density is given by:

\[
g(\theta, \xi, \varphi; y) = \begin{cases} 
(1 + \xi \varphi^{-1} y^{\theta})^{-\frac{1-\xi}{\theta}} \varphi^{-1} y^{\theta-1}, & \text{if } \xi \neq 0 \\
\exp\left\{ -\varphi^{-1} y^{\theta} \right\} \varphi^{-1} y^{\theta-1}, & \text{if } \xi = 0 
\end{cases}
\]

(3)

3. Estimation and Statistical Tests

We use a two-step methodology to obtain the MGPD parameters estimates. First, we estimate the GPD shape \( \xi \) and scale \( \varphi \) parameters using robust L-moments estimators. Hosking and Wallis (1997) demonstrated the efficiency of L-moments estimators at small samples and at a wide range of distributions. Their main properties are small bias, small variance and robustness. The GPD L-moments estimates are obtained from:
\[ \xi = \frac{-l_1}{l_2} + 2 \]  \hspace{1cm} (4)

\[ \varphi = (1 - x_i) l_1 \]  \hspace{1cm} (5)

where \( l_1 \) and \( l_2 \) are sample estimates of population L-moments \( \lambda_n, n = 1, 2 \).

L-moments estimators, introduced by Hosking (1990), are linear combinations of probability weighted moments (PWM)\(^5\) and may represent characteristics of a distribution, such as location, dispersion and shape. The first four L-moments are:

\[ \lambda_1 = \beta_0; \lambda_2 = 2 \beta_1 \beta_0; \lambda_3 = 6 \beta_2 \beta_1 + \beta_0; \lambda_4 = 20 \beta_3 - 30 \beta_2 + 12 \beta_1 - \beta_0 \]  \hspace{1cm} (6)

where \( \beta_k \) are the population probability weighted moments. Sample estimates of \( \beta_k \) are obtained as follows:

\[ b_0 = \frac{\sum_{i=1}^{n} X_i}{n} \]  \hspace{1cm} (7)

and

\[ b_r = \frac{\sum_{i=r+1}^{n} \frac{(i-1)(i-2)\ldots(i-r)}{(n-1)(n-2)\ldots(n-r)} X_i}{n} \]  \hspace{1cm} (8)

Then, we obtain the maximum likelihood estimates of \( \xi, \varphi \) and \( \theta \) by using the L-moments estimates as initial values and maximizing (3). Maximum likelihood is chosen due to their good asymptotic properties including their well known statistical tests. We obtain standard errors and confidence intervals using the asymptotic properties of the maximum likelihood estimators. The significance of the parameters estimates is assessed by Student’s \( t \) test at the 5% significance level. The goodness of fit is assessed graphically, through QQ-plots, and formally, through the Kolmogorov test.

4. Empirical Analysis

In this section, we investigate the behavior of drawdowns and drawups from ten exchange rates, quoted in terms of Brazilian real (BRL) against foreign assets.\(^6\)

Nine of them are currency rates and the remaining is the gold exchange rate. Currencies may be grouped in two classes, five from developed markets and four from

\(^5\)See Greenwood et al. (1979) for details.

\(^6\)US dollar, euro, G.B. pound, Japanese yen and Swiss franc were chosen because they represent the most traded currencies in the foreign exchange market. Also, we decided to include currencies from emerging markets that are quite relevant in the international capital market. Gold was also included because it is often seen as an international currency by foreign exchange investors.
emerging markets. Our data set comprises series of 1466 daily log returns from 03/1999 to 12/2004, after the implementation of the floating exchange rate regime in Brazil.

### 4.1 Drawdowns and drawups

The exchange rate drawdowns and drawups are collected from each daily return series and some of their simple statistics are shown in Table 1. We can observe that the severities of the three largest drawups are, in general, higher than those of the three smallest drawdowns, revealing some asymmetry in the tails of consecutive losses and consecutive gains distributions.

#### Table 1

<table>
<thead>
<tr>
<th>Currency</th>
<th>Code</th>
<th>N DD</th>
<th>3 smallest DDs (%)</th>
<th>N DU</th>
<th>3 largest DUs (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Dollar</td>
<td>USD</td>
<td>352</td>
<td>-14.22 -10.61 -10.06</td>
<td>352</td>
<td>21.54 19.22 11.62</td>
</tr>
<tr>
<td>Japanese Yen</td>
<td>JPY</td>
<td>374</td>
<td>-13.52 -12.11 -10.08</td>
<td>373</td>
<td>18.57 15.12 12.77</td>
</tr>
<tr>
<td>Gold</td>
<td>AUX</td>
<td>371</td>
<td>-12.58 -10.51 -9.73</td>
<td>371</td>
<td>22.94 20.72 15.53</td>
</tr>
<tr>
<td>Mexican Peso</td>
<td>MXN</td>
<td>375</td>
<td>-12.56 -10.02 -9.56</td>
<td>374</td>
<td>18.07 15.15 9.34</td>
</tr>
<tr>
<td>Russian Rouble</td>
<td>RUB</td>
<td>351</td>
<td>-16.29 -14.59 -10.68</td>
<td>351</td>
<td>21.52 19.23 11.71</td>
</tr>
<tr>
<td>Chinese Yuan</td>
<td>CNY</td>
<td>349</td>
<td>-14.20 -10.60 -10.07</td>
<td>349</td>
<td>21.53 19.23 11.66</td>
</tr>
<tr>
<td>S. Korean Won</td>
<td>KRW</td>
<td>368</td>
<td>-14.48 -10.08 -10.01</td>
<td>367</td>
<td>16.53 16.31 12.47</td>
</tr>
</tbody>
</table>

To assess asymmetry in the empirical distribution of drawdown’s and drawup’s durations we compute the difference of cumulative duration frequencies, as shown in Figure 2. It is observed that the distribution of the durations of the drawdowns are generally more concentrated in the shorter durations when compared to the drawup’s distributions. These results may suggest that short-term serial correlation is stronger in consecutive rises than in consecutive drops. The highest drawdown duration, fifteen days, was observed for the South Korean won consecutive losses, and the longest drawup duration correspond to fifteen consecutive daily gains of the US dollar, the Mexican peso, the Chinese yuan and the South Korean won.
Table 2 presents the maximum likelihood parameters estimates, their standard errors and the log-likelihood values obtained from the MGPD fit to the severity of the drawdowns and drawups. We observe that while euro drawdowns and drawups can be best fitted by the MGPD distribution, the US dollar risk measures can be adjusted by the simpler GPD distribution. Some other currencies, such as the G.B. pound, the Japanese yen, gold and the Russian ruble demand different distributions for drawdowns and drawups modeling.

Also, we can see that most of the shape parameters of the drawup’s distribution are significantly positive, showing that drawup’s distributions have, in general, fatter tails than drawdown’s. That is the case for US dollar and euro, for instance, what is consistent with historical Brazilian real devaluation against strong currencies since 1999 and empirical evidence that huge positive returns (Brazilian real devaluation), which may be caused by stress events, show higher correlation than a sequence of Brazilian real appreciation. Figure 3 shows QQ-Plots based on the US dollar and the Mexican peso drawdown’s and drawup’s fits. Despite the quite adequate fit, typically less than 1.0% of the largest drawdowns and drawups are not
properly fitted by the model. In this case, we can clearly see one and two outliers at the US dollar and the Mexican peso drawup’s fits, respectively.

Table 2
Parameters estimates, standard errors and log-likelihood (LL) of the MGPD distribution fitted to the severity of drawdowns and drawups. (The standard errors are shown in parenthesis)

<table>
<thead>
<tr>
<th>Currency</th>
<th>( \xi )</th>
<th>( \varphi )</th>
<th>( \theta )</th>
<th>LL</th>
<th>( \xi )</th>
<th>( \varphi )</th>
<th>( \theta )</th>
<th>LL</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD</td>
<td>0.09</td>
<td>1.53</td>
<td>1.01</td>
<td>-528.37</td>
<td>0.19*</td>
<td>1.52</td>
<td>1.04</td>
<td>-552.57</td>
</tr>
<tr>
<td>(0.06)</td>
<td>(0.12)</td>
<td>(0.04)</td>
<td>(0.06)</td>
<td>(0.12)</td>
<td>(0.04)</td>
<td>(0.06)</td>
<td>(0.12)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>XEU</td>
<td>0.09</td>
<td>2.00</td>
<td>1.14**</td>
<td>-623.79</td>
<td>0.19*</td>
<td>2.02</td>
<td>1.14**</td>
<td>-662.56</td>
</tr>
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<td>(0.05)</td>
<td>(0.15)</td>
<td>(0.03)</td>
<td>(0.06)</td>
<td>(0.16)</td>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.15)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>GBP</td>
<td>0.11*</td>
<td>1.62</td>
<td>1.05</td>
<td>-596.66</td>
<td>0.16*</td>
<td>1.80</td>
<td>1.10**</td>
<td>-634.98</td>
</tr>
<tr>
<td>(0.05)</td>
<td>(0.12)</td>
<td>(0.03)</td>
<td>(0.06)</td>
<td>(0.14)</td>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.12)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>JPY</td>
<td>0.27*</td>
<td>1.94</td>
<td>1.31**</td>
<td>-595.93</td>
<td>0.11</td>
<td>1.86</td>
<td>1.02</td>
<td>-635.08</td>
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<td>(0.14)</td>
<td>(0.03)</td>
<td>(0.07)</td>
<td>(0.16)</td>
<td>(0.04)</td>
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<tr>
<td>CHF</td>
<td>0.12*</td>
<td>2.02</td>
<td>1.17**</td>
<td>-631.88</td>
<td>0.19*</td>
<td>2.05</td>
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<tr>
<td>(0.06)</td>
<td>(0.15)</td>
<td>(0.03)</td>
<td>(0.06)</td>
<td>(0.16)</td>
<td>(0.03)</td>
<td>(0.06)</td>
<td>(0.15)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>AUX</td>
<td>-0.05</td>
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<td>1.00</td>
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<td>-676.99</td>
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<td>(0.05)</td>
<td>(0.15)</td>
<td>(0.03)</td>
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<td>MXN</td>
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<td>(0.07)</td>
<td>(0.12)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>RUB</td>
<td>0.24*</td>
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<td>1.11**</td>
<td>-552.50</td>
<td>0.15*</td>
<td>1.70</td>
<td>1.05</td>
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<td>(0.07)</td>
<td>(0.13)</td>
<td>(0.04)</td>
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<td>CNY</td>
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<td>0.98</td>
<td>-523.39</td>
<td>0.15*</td>
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<tr>
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<td>(0.12)</td>
<td>(0.04)</td>
<td>(0.06)</td>
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<td>(0.04)</td>
<td>(0.06)</td>
<td>(0.12)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>KRW</td>
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<td>1.88</td>
<td>1.08**</td>
<td>-594.21</td>
<td>0.17*</td>
<td>2.12</td>
<td>1.17**</td>
<td>-628.25</td>
</tr>
<tr>
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<td>(0.14)</td>
<td>(0.03)</td>
<td>(0.06)</td>
<td>(0.17)</td>
<td>(0.03)</td>
<td>(0.05)</td>
<td>(0.14)</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>

1/ Values with * represent significantly positive \( \xi \) estimates at the 5% significance level.
2/ Values with ** represent \( \theta \) estimates significantly greater than 1.0 at the 5% significance level.
3/ All \( \varphi \) estimates are significantly positive at the 5% significance level.

Table 3 presents the drawdown-at-risk (DdaR) and drawup-at-risk (DuaR) based on MGPD fitted model. These measures are the quantiles of the distributions of the severities of the drawdowns and drawups considering different exceedance probabilities (5.0%, 2.5%, 1.0% and 0.1%). According to previous interpretations, drawup’s risk measures are higher that drawdown’s. In most cases this may be explained by the higher values of the shape and scale parameters. The opposite case is the Japanese yen distribution, which the tail quantile is influenced by the \( \theta \) parameter.
With the purpose of examining the effect of diversification in drawdown’s and drawup’s profiles, and especially in the quantiles of their distributions, we create three equally weighted portfolios composed of the following assets:
$P_1$: all currencies in Table 1 and gold.

$P_2$: US dollar, euro, British pound, Japanese yen, Swiss franc and gold.

$P_3$: Mexican peso, Russian ruble, Chinese yuan and S. Korean won.

Table 4 presents the estimates for the MGPD parameters, their standard errors and the log-likelihood values obtained from the MGPD fit to the severity of the drawdowns and drawups. Table 5 shows the quantiles (DdaR and DuaR) of the parametric distributions of the severities of the drawdowns and drawups obtained from the three portfolios.

Table 4
Parameters estimates, standard errors and log-likelihood (LL) of the MGPD distribution fitted to the severity of maximum drawdowns (DD) and drawups (DU)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$\xi$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>LL</th>
<th>$\xi$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>LL</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
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<td>1.41</td>
<td>0.97</td>
<td>-557.90</td>
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<td>1.45</td>
<td>1.06</td>
<td>-591.55</td>
</tr>
<tr>
<td></td>
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<td>(0.15)</td>
<td>(0.03)</td>
<td>(0.06)</td>
<td>(0.11)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td>0.04</td>
<td>1.52</td>
<td>0.97</td>
<td>-578.60</td>
<td>0.21*</td>
<td>1.59</td>
<td>1.09**</td>
<td>-618.44</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.11)</td>
<td>(0.04)</td>
<td>(0.06)</td>
<td>(0.12)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>P3</td>
<td>0.01</td>
<td>1.45</td>
<td>0.91</td>
<td>-546.35</td>
<td>0.17*</td>
<td>1.47</td>
<td>1.03</td>
<td>-572.48</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.10)</td>
<td>(0.01)</td>
<td>(0.06)</td>
<td>(0.12)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td></td>
</tr>
</tbody>
</table>

1/ Values with * represent significantly positive $\xi$ estimates at the 5% significance level.
2/ Values with ** represent $\theta$ estimates significantly greater than 1.0 at the 5% significance level.
3/ All $\varphi$ estimates are significantly positive at the 5% significance level.

Comparing the outputs of Table 3 and Table 5, we note that, in general, the diversification reduces risk, by observing that the quantiles of the portfolios are close to the lowest quantile of the currencies that compose it. These results are quite similar to the reduction of the volatility in a Gaussian distribution, and the risk diversification is expected to be more intense in the more extreme quantiles. That is not the case only for the drawups’s quantiles of Portfolio 2 at the 0.1% significance level, what may suggest that the strong currencies are strongly correlated when it is observed a sequence of huge positive returns against the Brazilian real. As observed individually for each currency, the portfolio’s drawups represent higher risk than drawdowns’.

Table 5
Drawdowns at risk (DdaR) and Drawups at risk (DuaR) for drawdowns (DD) and drawups (DU) of three portfolios composed by equally weighted exposures on currencies and gold

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Drawdowns</th>
<th>Drawups</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.0%</td>
<td>2.5%</td>
</tr>
<tr>
<td>P2</td>
<td>5.10</td>
<td>6.41</td>
</tr>
<tr>
<td>P3</td>
<td>5.09</td>
<td>6.42</td>
</tr>
</tbody>
</table>

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4.2 Overnight and intraday preliminary analysis

Over the last decade we have observed an increasing interest in the investigation of financial time series using high frequency data. It is expected that intraday analysis may offer valuable additional information that cannot be obtained by higher scale measures. Therefore, we try to analyze particular drawdowns and drawups that represent a worst-case scenario since a previous determined observation on a daily basis. We believe this measure is able to capture overnight and intraday idiosyncrasies and to provide better understanding of one-day risk of an investment.

Let $C_t$ denote the closing price and $L_t$ the lowest price of an asset at the day $t$, the overnight drawdown $Y_{OVN}$ is defined as:

$$Y_{OVN} = \frac{L_t}{C_{t-1}} - 1, \text{ if } L_t < C_{t-1}$$

(9)

The intraday drawdown, is defined as:

$$Y_{ID} = \frac{L_t}{O_t} - 1, \text{ if } L_t < O_t$$

(10)

where $O_t$ denotes the opening price of an asset at the day $t$.

The concept here is quite similar to the range approach presented by Chou (2005) as an alternative to the modeling of financial volatilities. By adjusting a dynamic structure for the conditional expectation of the range Chou (2004) suggests that the Conditional Autoregressive Range (CARR) model is a more powerful forecasting methodology when compared to the usual GARCH models. With the aim of assessing the asymmetry of downward and upward price movements he proposes the ACARR (Asymmetric CARR) methodology, which independently models the dynamic dependence structure among positive and negative returns.

Nevertheless, according to the tests described in subsection 4.3, we have verified that only the observations of overnight drawdowns are considered to be independent. As well as our approach is based on independence assumption, we conclude that intraday investigation of drawdowns must incorporate conditional autoregressive structure, as proposed by Chou (2004).

4.3 Investigation of serial correlation

The estimation methods used assume a sample of independent and identically distributed (i.i.d.) observations. The drawdowns are random variable that aggregates (fixed-time) daily return observations possessing short-range (local) dependence. It seems very reasonable to assume that the drawdowns observations are, therefore, i.i.d. Also, we remind that every two observations of drawdowns, by construction, must present an opposite price movement in between, represented by the drawups.

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Resnick (1997) reminds the importance of realizing that confidence intervals drawn by typical statistical packages are based on the assumption that the data is Gaussian or at least has finite fourth moment, which is totally inappropriate for heavy tailed data. Therefore, considering data independence is a very useful assumption for modeling heavy tailed phenomena, he outlines several tests to check whether an i.i.d. assumption is adequate and whether it is necessary to fit a stationary time series with dependencies to the data set.

We decided to use non-parametric standard tests of randomness (Brockwell and Davis (1991) *apud* Resnick (1997)). The i.i.d. hypothesis will be rejected at the 5% significance level whenever any of the standardized variables fall out of the interval [-1.96, 1.96]. The tests are described below:

(i) Turning point test: If $T$ is the number of turning points among $X_1, \ldots, X_n$ then under the null hypothesis that the random variables are i.i.d. we have:

$$T \sim \mathcal{N}(2(n - 2)/3, (16n - 29)/90)$$  \hspace{1cm} (11)

(ii) Difference-sign test: Let denote $S$ as the number of $i = 2, \ldots, n$ such that $X_i - X_{i-1}$ is positive. Under the null hypothesis that the random variables $X_1, \ldots, X_n$ are i.i.d. we have:

$$S \sim \mathcal{N}((n - 1)/2, (n + 1)/12)$$  \hspace{1cm} (12)

(iii) Rank test: Assume $P$ is the number of pairs $(i, j)$ such that $X_j > X_i$, for $j > i$ and $i = 1, \ldots, n - 1$. Under the null hypothesis that the random variables $X_1, \ldots, X_n$ are i.i.d. we have:

$$P \sim \mathcal{N}(n(n - 1)/4, n(n - 1)(2n + 5)/8)$$  \hspace{1cm} (13)

Concerning the ten exchange rates, the turning point test rejects the independence in US dollar drawdowns and the Difference-sign test rejects it in the Mexican peso data. For the remaining drawdowns data and drawups series, the three tests fail to reject the null hypothesis of randomness, suggesting independence in the data. Considering the overnight and intraday analysis, as mentioned in the previous section, the tests fail to reject only the overnight drawdowns. That is additional evidence that the drawdowns incorporate short-term serial correlation in time series observations.

Moreover, we go further and suggest adding some graphical analyses to address this issue in a quite trivial form. In that case, we may informally assess serial correlation by examining lagged plots, as shown in Figure 4 for the case of U.S. dollar drawups.
Figure 4
Lag Plots of dollar drawups. (Lags from 1 to 4)

5. Concluding Remarks

This paper addresses concerns on how to properly assess financial time series behavior. It is rather consensual that one-point statistics are insufficient for characterizing financial market moves and two-point statistics, even though those modeled by AR-GARCH structures, although offering important complementary information, are still limited. As a consequence, a better characterization of the dynamics of price moves may be provided by using relatively low-order statistics with suitable adjustments to the relevant market-time scales, such as drawdowns and drawups. This implies new modeling procedures where physical time is described as a stochastic process dependent on a subordinator variable.

In this work, we estimate by maximum likelihood the extreme value distributions of the severity of drawdowns and drawups for nine currency exchange rates and gold. It is shown that risk measures based on the drawdowns may be more relevant, since they are related to cumulative losses of an investment, providing a new insight of its risk patterns. Formal goodness of fit tests were carried out, as well as graphical analysis.

The results indicate the distribution of the durations of the drawdowns are generally more concentrated in the shorter durations when compared to the drawup’s distributions, what may suggest that short-term serial correlation is stronger in consecutive rises than in consecutive drops. The highest drawdown’s and drawup’s duration is fifteen days. Concerning the distributions of severities, we observe that
drawup’s distributions have, in general, fatter tails than drawdown’s, showing consistency with historical Brazilian real devaluation against strong currencies since 1999 and empirical evidence that huge positive returns (Brazilian real devaluation), which may be caused by stress events, show higher correlation than a sequence of Brazilian real appreciation.

Also, the portfolio analysis permitted to note risk reduction in the drawdown’s and drawup’s measures promoted by diversification. As expected, risk diversification is more intense in the more extreme quantiles.

Non-parametric standard tests of randomness are used in order to assess serial correlation. The tests reject independence assumption only in US dollar drawdowns and Mexican peso data.

References


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