1. Introduction

Price dispersion is present in several markets, including the investment fund market. Funds could hardly be considered homogeneous products, since their characteristics such as return, risk and portfolio composition vary. Although the fee variation could solely reflect cost differences, it could also be due to markup differences.

In this paper I analyze the roles of cost differences and second-degree price discrimination to explain the observed fee dispersion in the fund industry. In order to do this I use a model that allows to recover the marginal cost and therefore decompose the fee into the marginal cost and a markup term. A fee decreases mostly due to markup variation, as minimum initial application increases, suggests the presence of price discrimination.

With this purpose I estimate a structural model of consumer and pricing behavior that allows consumer and fund heterogeneity. Consumers value fund characteristics...
and choose the fund that maximizes utility. Consumers are heterogeneous in their valuation of fund characteristics and therefore they choose different funds. Banks are profit maximizing multi-product firms that compete in prices. The results can give a better idea of how the competition is played in this market and thus suggest some guidelines regarding policy.

The existence of fee dispersion in the fund industry has already been studied by other papers, but never—at least to my knowledge—as reflecting bank attempt to price discriminate. Hortaçsu and Syverson (2004) estimate a structural model with consumer heterogeneity in search costs and funds vertically differentiated. They estimate nonparametrically the c.d.f. of the search cost distribution which rationalizes the observed market shares. Iannotta and Navone (2012) controls for funds observable sources of heterogeneity and attribute to search costs the not explained fee dispersion. Some papers study the price discrimination empirically. Cohen (2008) investigates the extent to which quantity discounts for paper towels are consistent with second degree price discrimination. The presence of nonlinear pricing in the specialty coffee market is studied in McManus (2007).

Data were provided by Brazilian Financial and Capital Markets Association (Anbima) and consist of monthly information of Brazilian funds. Anbima classifies funds in mutually exclusive categories such as short term, multimarket, pension, DI referenced, fixed income and shares, which allows different portfolio composition. Among these categories, I only consider fixed income, which is the largest category, including over 1,000 funds and 25% of the 2 trillion Brazilian Reais (BRL) net asset.

With the purpose of access how fees, minimum initial applications and consumer welfare would change in a scenario where banks could neither charge multiple fees nor minimum initial application, I use the model estimates to simulate two counterfactual scenarios. In the first, each bank can charge a unique administration fee for its funds, given the minimum initial application observed in the data. In the second, given a minimum initial application, the bank chooses a unique administration fee. Results indicate that an increase of 100 BRL in the minimum initial application decreases the marginal cost in 0.002408 BRL. Using the parameters estimates I recover the marginal costs and obtain the funds markups, which decrease with larger minimum initial applications and therefore suggest the presence of second-degree price discrimination.

Regarding the counterfactuals, the predicted administration fee decreases monotonically with the minimum initial application. In order to achieve the administration fees charged with the observed minimum initial application (first counterfactual), it would be necessary an initial application around 20,000 BRL for all funds. The change in consumer surplus is negative in the first counterfactual and it is positive in the second counterfactual only for initial applications above 30,000 BRL.

The remainder of this paper is organized as follows. Section 2 describes the Brazilian fund market. Section 3 presents the data used. Section 4 presents the model of consumer and firm behavior, the estimation procedure and discusses the identification. Section 5 shows the results and discusses the possibility of price discrimination. Section 6 presents the counterfactuals and the welfare analysis. Section 7 concludes. Appendix presents the computational procedure in details.
2. Brazilian Fixed Income Fund Market and Price Discrimination

The CVM 409 instruction regulates fund market in Brazil.\(^1\) It provides a mutually exclusive classification of funds in categories such as short term, multimarket, pension, DI referenced, fixed income, shares. Fixed income is the largest fund category regarding net asset and holds around 25% of the 2 trillion BRL existent in this market.

The fund characteristics also need to be specified. For example, if it is open or closed, which investor segment it is interested in and if it has performance, entry or exit fees.\(^2\) A fund is classified as open if its shareholders can request withdrawal before the end of its term.\(^3\) Otherwise it is classified as closed and withdraws can only occur at the end of the term, previously defined.

The administration fee is charged annually as a percentage of the amount invested. While the performance fee is calculated based on the return that exceeds a certain percentage of the benchmark. The performance fee is charged with a predefined periodicity.

Entry and exit fees are charged at the moment of application and withdraw, respectively. Figure 1 presents the administration fee histogram.

Generally, funds set minimum initial applications and therefore consumers can not invest any amount initially. Figure 2 shows the minimum initial application histogram.

Each point at Figure 3 is the administration fee and minimum initial application for a fund in a month. The correlation between these variables is −0.4382.

3. Data

I used two sources to construct the dataset. The information on funds was provided by the Brazilian Financial and Capital Markets Association (Anbima) and consists of monthly information on all existing funds in Brazil from January 2000 to June 2012. The data on consumer characteristics are from the National Household Sample Survey (PNAD).

Anbima classifies funds into categories whose last reclassification was in August 2006. To avoid considering funds which are now in other categories, the dataset starts in 2007. Since funds start and end during this period, the dataset is an unbalanced panel.

In the considered period there was a total of 174 funds offered by 26 banks. Figure 4 shows the market share of the largest banks, which contain almost ninety percent of the total market share.

Since few funds have another fee then the administration fee, I will only consider those that have only the administration fee along being offered by the six largest banks.\(^4\) This leads to a sample with 5,255 observations and 104 funds. Figure 5 and Figure 6 present respectively the evolution of the net asset and number of funds.

Table 1 presents some descriptive statistics of the variables considered. The age of a fund (Age) is the number of months of existence divided by 12. Minimum initial

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\(^{1}\) This can be found at http://www.cvm.gov.br

\(^{2}\) Site http://www.anbima.com.br/mostra.aspx/?op=z&id=112 contains the characteristics and rules.

\(^{3}\) Each fund has to specify maximum periods between the shareholder request, conversion and receipt of a withdrawn.

\(^{4}\) Nine funds have an exit fee, three have performance fee and no one has an entry fee.
Figure 1. Histogram – Administration Fee.

Figure 2. Histogram – Minimum Initial Application.

Figure 3. Administration Fee and Minimum Initial Application.
**Figure 4.** Market Share of the largest 6 banks.

**Figure 5.** Total Net Asset.

**Figure 6.** Number of Funds.
application (Min Init Appl) and minimum additional application (Min Addit Appl) are in tens of thousands of BRL. Market share of the outside good (Mkt Share Out Good) is the market share of not considered funds, ie, those with fees other than administration or that are not offered by the considered banks. Therefore the option of not consuming one of the funds considered means its substitution to one from a non-considered bank or with fees other than the administration fee. I do not intend to explain the portfolio allocation between different kinds of investment and thus restrict the choices available to fixed income funds.

Lastly, PNAD is a survey conducted by the Brazilian Institute of Geography and Statistics (IBGE) and I use the data on consumer income from 2007 to 2011. This data serves to generate dispersion in consumers sensibility to funds characteristics, which will be explained in details next section.

Table 1. Descriptive statistics.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Median</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Admin Fee</td>
<td>2.2</td>
<td>2</td>
<td>1.3</td>
<td>0.4</td>
<td>11</td>
</tr>
<tr>
<td>Age</td>
<td>9.6</td>
<td>10</td>
<td>5.7</td>
<td>0</td>
<td>32.4</td>
</tr>
<tr>
<td>Monthly Return</td>
<td>0.7</td>
<td>0.7</td>
<td>0.4</td>
<td>-4.8</td>
<td>17.4</td>
</tr>
<tr>
<td>Min Init Appl</td>
<td>1.8</td>
<td>0.5</td>
<td>2.8</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>Min Addit Appl</td>
<td>0.05</td>
<td>0.01</td>
<td>0.15</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Number of Funds</td>
<td>78</td>
<td>78</td>
<td>4.2</td>
<td>65</td>
<td>83</td>
</tr>
<tr>
<td>Mkt Share Out Good</td>
<td>14.3</td>
<td>15.2</td>
<td>4.7</td>
<td>7.1</td>
<td>26.1</td>
</tr>
</tbody>
</table>

4. Empirical Framework and Estimation

To assess how much of the price variation is due to changes in the marginal cost, I estimate a structural model of consumer and pricing behavior that allows the recovery of the marginal cost for each fund. Consumers value funds characteristics and choose the one that maximize their utility, while banks are multi-product firms that compete in prices and maximize profits.

I also discuss the estimation procedure and how endogeneity emerges and is corrected in this framework.

4.1 Demand

The demand is similar to the random coefficients logit presented in Berry, Levinsohn, and Pakes (1995). Consumer $i$ chooses fund $j$ (omitting the time subscript $t$ to simplify notation) and obtains utility

$$u_{ij} = x_j \beta_i - \alpha_i p_j + \xi_j + \epsilon_{ij},$$

where $x_j$ is a $k$-dimensional vector of fund observed characteristics, $\xi_j$ is the unobserved (by the econometrician) funds characteristics, which represents quantifiable characteristics that are not available in the data and unquantifiable characteristics such
as prestige and reputation. Administration fee is given by $p_j$ and $\varepsilon_{ij}$ is an idiosyncratic shock to consumer utility.\(^5\)

The observed characteristics considered are the fund administration fee, minimum initial application, age and monthly return.

Parameters $\alpha_i$ and $\beta_i$ capture the consumer $i$ taste for the price and observed characteristics. These have the following functional form:

$$
\begin{pmatrix}
\alpha_i \\
\beta_i
\end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \Pi d_i + \Sigma v_i,
$$

where $d_i$ is a $d$-dimensional vector of individual demographic characteristics and $v_i$ is a $(k + 1)$-dimensional vector of individual shocks. The parameters $\alpha$ and $\beta$ are consumers mean taste over the observed characteristics. While $\Pi$ is a $(k + 1) \times d$ matrix of parameters that captures the effects of demographics on consumer deviates from the mean taste, $\Sigma$ is a diagonal matrix of parameters that captures the effects of individual shocks on tastes. Therefore with $k$ observed characteristics (other than price), there are $2(k + 1) + d(k + 1)$ parameters to be estimated.

Hence the utility obtained from investing in a fund depends on its characteristics and consumer tastes. Furthermore, agents with different tastes may choose different funds.

The utility consumer $i$ receives from good $j$ can also be written as

$$
\begin{aligned}
\delta_j & = x_j \beta - \alpha p_j + \varepsilon_{ij} \\
\mu_{ij} & = \left[ p_j x_j' \right] \cdot \left[ \Pi d_i + \Sigma v_i \right]
\end{aligned}
$$

Therefore with $k$ observed characteristics (other than price), there are $2(k + 1) + d(k + 1)$ parameters to be estimated.

Considering that this is a discrete choice model, each consumer must choose one and only one fund. The outside good $j = 0$ represents consumer option of not buying any of the $J$ funds and has normalized mean utility $\delta_0 = 0$. This can be thought as the consumer ability to substitute to other markets. Note that without the outside good, a homogeneous increase in the price of all funds does not change the market demand.

Fund $j$ market share is the integral over the set of consumer tastes that lead to its choice

$$
\begin{aligned}
s_j(p, x, \xi; \theta) & = \int_{\{d_i, v_i, \varepsilon_i | u_{ij} > u_{ik}, \forall k \neq j\}} dF_d(d) dF_v(v) dF_{\varepsilon}(\varepsilon),
\end{aligned}
$$

where $\theta = (\alpha, \beta, \Pi, \Sigma)$ and $F_d$, $F_v$ and $F_{\varepsilon}$ are the CDF of consumer demographics, shocks and idiosyncratic shocks respectively. Integrating in $\varepsilon$, as shown in McFadden (1974), gives

$$
\begin{aligned}
s_j(p, x, \xi; \theta) & = \int_{d, v} \frac{\exp\{x_j \beta_i - \alpha_i p_j + \xi_j\}}{1 + \sum_k \exp\{x_k \beta_i - \alpha_i p_k + \xi_k\}} dF_d(d) dF_v(v).
\end{aligned}
$$

\(^5\)Where $\varepsilon_{ij}$ iid $\sim$ type I extreme value.
4.2 Supply

F multi-product banks are offering each one some subset \( J_f \) of the \( J \) funds. Banks choose funds administration fees in order to maximize profits considering other banks funds characteristics and administration fees. The profit function for bank \( f \) is given by

\[
\Pi_f = \sum_{j \in J_f} (p_j - mc_j)Ms_j(p,x,\xi;\theta),
\]

where \( p_j \) is the administration fee, \( mc_j \) the marginal cost and \( M \) the market size. The marginal cost has the following functional form

\[
mc_j(x^c_j, \omega_j; \gamma) = x^c_j \gamma + \omega_j,
\]

where \( x^c_j \) and \( \omega_j \) are the observed and unobserved cost characteristics respectively. The observed characteristics that influence marginal cost are the minimum initial and additional application and the net asset. There are also dummies for each bank and also a trend.

4.3 Estimation

I follow Berry et al. (1995) to estimate the demand parameters \( \alpha, \beta, \Pi \) and \( \Sigma \) along with the supply parameter \( \gamma \). This consists in isolating the error terms \( \xi, \omega \) and interact them with the proper set of instrumental variables to form the moment conditions to use a GMM estimator.

Next I present the estimation procedure, while the computational details are in Appendix. For more details see Berry et al. (1995) or Nevo (2000).

From demand subsection, the market share of fund \( j \) is given by the integral in equation (5). This integral does not have a closed form and therefore is integrated using simulation. With a guess of the parameters values \( \Pi, \Sigma \), one vector of products mean utilities \( \delta \) and \( ns \) pseudo random draws of consumer tastes, calculate

\[
s_j(\delta, p, x; \Pi, \Sigma) = \frac{1}{ns} \sum_{i=1}^{ns} \exp\left\{ \delta_j + \mu_{ij}(\Pi, \Sigma) \right\} \frac{1}{1 + \sum_{k=1}^{J} \exp\left\{ \delta_k + \mu_{ik}(\Pi, \Sigma) \right\}}.
\]

I iterate over the operator defined below to obtain the vector of mean values \( \delta \) that equalizes the predicted and the observed market shares

\[
T(\delta) = \delta + \ln(S) - \ln[s(\delta, p, x; \Pi, \Sigma)],
\]

where \( S \) is the vector of observed market shares.\(^6\) The error term can be written as

\[
\xi_j = \delta_j - x_j \beta + a p_j.
\]

For the supply side, it is obtained the FOC of the profit function in equation (6):

\[
s_j(p, x, \xi; \theta) + \sum_{r \in J_f} (p_r - mc_r) \frac{\partial s_r(p, x, \xi; \theta)}{\partial p_j} = 0, \quad \forall j \in J,
\]

\(^6\)In Berry et al. (1995) it is shown the existence and uniqueness of the \( \delta \) vector and that the operator (9) is a contraction. Therefore any initial guess of \( \delta \) will lead to the same fixed point.
and in vector notation is
\[ s(p, x, \xi; \theta) - \Delta(p, x, \xi; \theta)(p - mc) = 0, \]  
with \( \Delta_{jr}(p, x, \xi; \theta) = -\left( \frac{\partial s_j}{\partial p_j} \right) 1_{jr} \), where \( 1_{jr} \) is an indicator function that assumes value 1 if \( j \) and \( r \) are sold by the same bank and 0 otherwise.

Replacing the marginal cost in the FOC and rearranging,
\[ \omega = p - x^c \gamma - \Delta(p, x, \xi; \theta)^{-1}s(p, x, \xi; \theta). \]  

The identification condition also follows Berry et al. (1995) and entails that under appropriate instruments \( Z \), at the true parameters values \( \mathbb{E}(\xi \omega | Z) = 0 \).

Therefore given the appropriate instruments I use a GMM procedure to estimate the parameters, ie, search over the parameters \( \alpha, \beta, \Pi \) and \( \Sigma \) that makes the sample moments as close as possible to zero. The computational details are provided in the Appendix.

### 4.4 Endogeneity

The endogeneity appears since it is expected that some of the observable characteristics \( (x, p) \) are correlated with the unobservable \( \xi \) as well as some elements of \( x^c \) are correlated with \( \omega \).

For demand I suppose that fund characteristics (other than administration fee) \( x \) are exogenous to the unobserved product characteristics \( \xi \). For supply all cost characteristics \( x^c \) are supposed to be exogenous. Therefore I am concerned about the correlation between price \( p \) and unobservables \( \xi \) and \( \omega \).

As in Berry et al. (1995) I assume that supply and demand unobservables are mean independent of both observed product characteristics and cost shifters, i.e.,
\[ \mathbb{E}(\xi_j | x, x^c) = \mathbb{E}(\omega_j | x, x^c) = 0, \quad \forall j, \]
where \( x = (x_1, \ldots, x_J) \) and \( x^c = (x^c_1, \ldots, x^c_J) \).

The instrument matrix for demand and cost shifters are \( z^d = (z^d_1, \ldots, z^d_J)' \) and \( z^c = (z^c_1, \ldots, z^c_J)' \), respectively. Instruments for demand/cost for fund \( j \) are a function of the exogenous observed demand/cost shifters
\[ z^d_j = z^d(x_1, \ldots, x_J) \quad \text{and} \quad z^c_j = z^c(x^c_1, \ldots, x^c_J) \]
with \( \#(z^d_j) \geq \#(x_j) \) and \( \#(z^c_j) \geq \#(x^c_j) \), for all \( j \), to satisfy the necessary order condition for identification.

Let \( z_{jk} \) be the \( k \)th exogenous characteristic of product \( j \) and consider
\[ z_{jk}, \quad \sum_{r \neq j, r \notin f_j} z_{rk}, \quad \sum_{r \neq j} z_{rk}. \]

---

7 The direction of bias in the price coefficient can sometimes be determined intuitively. As exemplified in Train (2003), price coefficient is biased downward (in modulus) if higher prices are associated with desirable attributes, because the estimated price coefficient captures both price effect and desirable unobserved attributes effects. While opposite direction occurs if a price decrease is made jointly with an increase of advertising, for example. The increase in demand comes from both effects and price coefficient is biased upward (in modulus).
The first is the proper \( k \)th characteristic of product \( j \); the second is the sum of this characteristic over the products produced by the same firm; and third is the sum across other firms. Then, if there are \( K - 1 \) exogenous observed characteristics for demand, there are \( 3(K - 1) \) instruments for the price. Cost shifters are used for supply.

5. Results

Results from estimation of demand and supply parameters are shown in Tables 2 and 3, respectively. First column of Table 2 presents the values of mean coefficients \( \alpha \) and \( \beta \). Next columns display the coefficients that capture the heterogeneity in tastes, i.e., the shocks \( \Sigma \), income and logarithm of income \( \Pi \).

The mean coefficient on administration fee has the expected negative sign. Minimum initial application coefficient has a positive sign as well as age. Differently than expected, the mean effect of monthly return is negative.

The positive sign in the coefficients that captures the interaction between administration fee and income indicates that persons with more income are less price sensitive.

Cost parameters estimates indicate that larger minimum initial and additional applications decrease costs and that costs should decrease over time. For the minimum initial application, an increase of 10000 BRL would decrease the marginal cost in 0.2408 BRL. Unexpectedly the coefficient on the net asset is positive, however it is not significant.

Since larger minimum initial application decreases marginal costs, an administration fee negatively correlated with the minimum initial application does not necessarily indicate the presence of second-degree price discrimination.

For the purpose of testing the presence of second-degree price discrimination, I calculated the markup for each fund using the estimates obtained for the marginal cost parameters. Then for each bank I observed how it varies as minimum initial application increases. Since markups decrease with larger minimum initial applications, results suggest the presence of second-degree price discrimination.

6. Counterfactual and Welfare Analysis

Banks offer multiple funds with negatively correlated administration fees and minimum initial applications. Results indicate that differences in marginal cost are not essential to explain differences in administration fees, which evidences a possible second-degree price discrimination by banks.

Since individuals with a lower income should invest a lower amount and therefore pay a higher price, one should be interested in evaluating how prices and therefore consumer welfare would vary if banks could not charge multiple administration fees.

For this purpose I perform two counterfactual exercises. In the first counterfactual, each bank can charge a unique administration fee for its funds, given minimum initial application observed in the data. In the second counterfactual, each bank choose a unique administration fee for its funds, given a hypothetical minimum initial application.

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\[ \text{The markup for fund } j \text{ is given by } \left( p_j - mc_j \right)/mc_j. \]
### Table 2. Demand parameters estimate

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Shocks</th>
<th>log(Income)</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$-4.63^{***}$</td>
<td>$-0.01$</td>
<td>$0.05^{***}$</td>
<td>$-0.01$</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0032)</td>
<td>(0.0009)</td>
<td>(0.0029)</td>
</tr>
<tr>
<td>Admin Fee</td>
<td>$-0.54^{***}$</td>
<td>$-0.02^{***}$</td>
<td>$0.01$</td>
<td>$0.01^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0028)</td>
<td>(0.0009)</td>
<td>(0.0016)</td>
</tr>
<tr>
<td>Min Init Appl</td>
<td>$0.11^{***}$</td>
<td>$-0.01^{***}$</td>
<td>$0.01^{***}$</td>
<td>$0.03^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
<td>(0.0027)</td>
<td>(0.0006)</td>
<td>(0.0012)</td>
</tr>
<tr>
<td>Age</td>
<td>$0.12^{***}$</td>
<td>$0.01^{***}$</td>
<td>$-0.01^{***}$</td>
<td>$-0.01^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0627)</td>
<td>(0.3556)</td>
<td>(0.0726)</td>
<td>(0.3377)</td>
</tr>
<tr>
<td>Monthly Return</td>
<td>$-0.03^{***}$</td>
<td>$0.04^{***}$</td>
<td>$0.01^{***}$</td>
<td>$-0.09^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0015)</td>
<td>(0.0007)</td>
<td>(0.0021)</td>
</tr>
</tbody>
</table>

*Note: Standard Errors are reported in parentheses. ** Significant at 1% level.*

### Table 3. Supply parameter estimates.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Supply parameter estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min Init Appl</td>
<td>0.24^{***} (0.0317)</td>
</tr>
<tr>
<td>Min Addit Appl</td>
<td>1.38^{***} (0.2228)</td>
</tr>
<tr>
<td>Santander</td>
<td>2.40^{***} (0.3986)</td>
</tr>
<tr>
<td>BB DTVM</td>
<td>2.90^{***} (0.2432)</td>
</tr>
<tr>
<td>Bradesco</td>
<td>2.81^{***} (0.0662)</td>
</tr>
<tr>
<td>Caixa</td>
<td>2.94^{***} (0.0877)</td>
</tr>
<tr>
<td>HSBC</td>
<td>2.07^{***} (0.1813)</td>
</tr>
<tr>
<td>Itaú</td>
<td>3.31^{***} (0.2272)</td>
</tr>
<tr>
<td>Trend</td>
<td>$-0.01^{***}$ (0.0012)</td>
</tr>
<tr>
<td>Net Asset</td>
<td>0.001 (0.0017)</td>
</tr>
</tbody>
</table>

*Note: Standard Errors are reported in parentheses. ** Significant at 1% level.*
I allow the values 100, 1,000, 5,000, 10,000, 15,000, 20,000, 25,000, 30,000 and 35,000 BRL for the minimum initial applications.

The bank can charge only one administration fee \( p \), thus its profit function becomes

\[
\Pi_f = \sum_{j \in J_f} (p - mc_j) Ms_j(p, x, \xi; \theta)
\]

with FOCs

\[
\sum_{j \in J_f} \left( s_j(p, x, \xi; \theta) + (p - mc_j) \frac{\partial s_j(p, x, \xi; \theta)}{\partial p} \right) = 0.
\]

Which in vector notation are

\[
\left( s(p, x, \xi; \theta) - \Delta(p, x, \xi; \theta) * [p - mc] \right)' * \mathbf{1}_{|J_f|} = 0,
\]

where \( \Delta_j(p, x, \xi; \theta) = \frac{\partial s_j(p, x, \xi; \theta)}{\partial p} \) and \( \mathbf{1}_{|J_f|} \) is a vector of ones with dimension equal to the number of funds offered by bank \( f \).

To compute the counterfactual predicted prices, I search for the vector of prices \( p \) that solve these FOCs along with the parameters estimates from the structural model and the minimum initial application specified. \(^9\)

The upper part of Table 4 presents the predicted administration fees for each counterfactual. The last column shows the administration fees under the first scenario, which keeps the observed minimum initial application. Figure 7 shows the minimum, maximum, mean and counterfactual administration fee for each bank. As can be seen the predicted fee for each bank lies around its mean.

Other columns show the predicted administration fee under the second counterfactual, which fixes the minimum initial application. These are given in tens of thousands

<table>
<thead>
<tr>
<th>Bank</th>
<th>Minimum Initial Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Santander</td>
<td>2.32 2.31 2.24 2.15 2.06 1.98 1.89 1.80 1.72 1.89</td>
</tr>
<tr>
<td>BB DTVM</td>
<td>2.06 2.04 1.97 1.89 1.80 1.72 1.63 1.54 1.45 1.55</td>
</tr>
<tr>
<td>Bradesco</td>
<td>2.44 2.43 2.36 2.27 2.18 2.10 2.01 1.92 1.84 2.11</td>
</tr>
<tr>
<td>Caixa</td>
<td>1.58 1.57 1.50 1.41 1.32 1.24 1.15 1.06 0.98 1.22</td>
</tr>
<tr>
<td>HSBC</td>
<td>2.06 2.05 1.98 1.89 1.80 1.72 1.63 1.54 1.46 1.52</td>
</tr>
<tr>
<td>Itaú</td>
<td>2.71 2.69 2.62 2.53 2.45 2.36 2.28 2.19 2.11 1.9</td>
</tr>
</tbody>
</table>

### Change in Consumer Surplus

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>−1.15</td>
<td>−0.45</td>
<td>−6.18</td>
</tr>
<tr>
<td>Max</td>
<td>−1.11</td>
<td>−0.29</td>
<td>−5.76</td>
</tr>
<tr>
<td>Min</td>
<td>−0.96</td>
<td>−0.13</td>
<td>−5.33</td>
</tr>
</tbody>
</table>

\(^9\)The system cannot be solved analytically. The operator used was not proved to be a contraction, but I tried from different initial \( p \) which lead to the same fixed point.
Figure 7. Minimum, maximum, mean and counterfactual administration fee for each bank.
Figure 8. Minimum, Maximum and Mean of the Minimum Initial Application for each bank.
of BRL and vary from 100 BRL to 35,000 BRL. Figure 8 presents the observed minimum, maximum and mean of minimum initial application for each bank.

For each bank the administration fee decreases monotonically with a larger initial application. Moreover, the initial application would have to be around 25,000 BRL to reach the administration fee predicted with the observed initial application.

I also compute the variation in consumer surplus for each counterfactual scenario. Consumer surplus is given by the following expression:

\[ CS_i = \int \frac{1}{\alpha_i} \log \sum_j \exp \{ \delta_j(\alpha, \beta) + \mu_{ij}(\beta^s, \beta^d) \} \, dF_v(v) \, dF_d(d) + K. \]  

(17)

Therefore the change in consumer surplus is\(^{10}\)

\[ \Delta CS_i = \int \frac{1}{\alpha_i} \left( \log \sum_j \exp \{ \delta_j(\alpha, \beta) + \mu_{ij}(\beta^s, \beta^d) \} \right. \]

\[ - \log \sum_j \exp \{ \delta_j(\alpha, \beta) + \mu_{ij}(\beta^s, \beta^d) \} \, dF_v(v) \, dF_d(d). \]  

(18)

For this it is supposed that the coefficient \(\alpha_i\) is independent of income, which does not occur in my specification. But as commented in Train (2003) one can use this specification when the change in consumer surplus is small relative to the income.

I compute the change in consumer surplus for each consumer at each counterfactual scenario. The results are in the lower part of Table 4. Given that consumer tastes vary, along with the mean change in consumer surplus I also present the maximum and minimum change in consumer surplus for each scenario.

For lower values of the minimum initial application, the administration fee becomes larger than observed and all consumers are in a worse situation. With a larger initial application, the loss in consumer surplus decreases but the mean variation only becomes positive from 30,000 BRL.

Using the observed minimum initial application, some consumers are better and others worse. However, the mean change is negative. The existence of consumers with positive and others with negative variation in utility seems reasonable as consumers that have a fund with large (low) initial application, have a low(large) administration fee which means that in the counterfactual its administration fee increase (decrease).

These results may give some advice for policy makers. Even though there is evidence that banks price discriminate, which leads consumers who buy a lower amount (and possibly have a lower income) to pay higher fees. The attempt to prevent banks from price discriminate—imposing the same administration fee and (or) minimum initial application—almost makes a decrease in consumer welfare. If the government wants to increase the welfare of those who buy lower amounts, it should seek through other ways.

\(^{10}\)Actually, this integral does not have closed form and is calculated using simulation. With \(n_s\) draws of individuals

\[ \Delta CS_i = \frac{1}{n_s} \sum_{i=1}^{n_s} \frac{1}{\alpha_i} \left( \log \sum_j \exp \{ \delta_j(\alpha, \beta) + \mu_{ij}(\beta^s, \beta^d) \} - \log \sum_j \exp \{ \delta_j(\alpha, \beta) + \mu_{ij}(\beta^s, \beta^d) \} \right). \]
7. Conclusion

This work estimated demand and supply for Brazilian fixed income funds using a structural model of consumer and pricing behavior. As expected, results indicate that administration fee decreases funds market share, higher income agents are less price sensitive, age and minimum initial application increase funds demand. Marginal cost increases over time and decreases with larger minimum initial and additional applications.

In the data administration fee and minimum initial application are negatively correlated. I use the estimates obtained to recover how much of the fee differences could not be cost explained and therefore are evidence of second-degree price discrimination. The results show that most of the fee differences are not cost explained and that markups decrease with larger minimum initial application, suggesting the presence of price discrimination.

I performed two counterfactual exercises to understand how fees would change if banks could neither charge multiple fees nor minimum initial applications. The results indicate that consumers would be worse in almost situations. Therefore any regulation to mitigate bank price discrimination should be viewed with prudence.

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Appendix. Computational Details

The algorithm for estimation of random coefficients logit model with supply consists of an outer loop that guess different values of the parameters \((\Pi, \Sigma)\) and an inner loop that searches for the vector \(\delta\) that equalizes the predicted and the observed vector of market shares.\(^{11}\)

This consists of four steps:

1. Obtain \(ns\) draws of the individual taste vector \((d_i, v_i)\).
2. Guess \(\theta_2\) and calculate:
   a) Predicted market share using \(\theta_2\) and \(\delta\).
   b) The vector of mean values \(\delta\) that equals the predicted and observed market shares using the fixed point contraction.
   c) The vector of \(mc\) from firms FOC.
   d) Optimal \(\theta_1\) and \(\gamma\) using GMM.
   e) Objective function value.
3. Return to item 2.
4. Repeat item 3 until find a global minimum of the objective function.

\(^{11}\)This kind of algorithm is often called a Nested Fixed Point (NFP) algorithm, following the terminology of Rust (1987).