This paper studies the effects of cash transfers to the poor on the labor market. We build a matching model of the labor market with endogenous job destruction in which agents can be in three states: employed, unemployed, or out of the labor force (home production). Workers are heterogenous in their labor market productivity. An idiosyncratic productivity shock arrives at constant instantaneous rate. Depending on this shock, workers might want to leave the labor market and workers out of the labor force might decide to look for a job. We introduce cash transfers to all agents with income below some threshold level. Our analytical results show: (i) The size of cash transfers has a negative effect on the employment rate, but an ambiguous effect on the unemployment rate; and (ii) the coverage of this welfare program has a positive effect on the employment rate, and an ambiguous effect on the unemployment rate. We also provide some numerical simulations.
de transferência de renda considerado no modelo tem como beneficiários todos os agentes com renda abaixo de um determinado nível. Dentre os resultados obtidos, verifica-se que: (i) a dimensão da renda transferida tem efeito negativo sobre a taxa de emprego e um efeito ambíguo sobre a taxa de desemprego; (ii) a cobertura do programa de transferência de renda tem efeito positivo sobre a taxa de emprego e efeito ambíguo sobre a taxa de desemprego. Para dar maior consistência aos resultados apresentados foram realizados alguns procedimentos numéricos.

1. INTRODUCTION

In his study about the growth of social spending in the last three centuries, Peter Lindert posits that: “The first kind of social spending to exceed 1 percent of national product was, and still is, the most controversial kind: direct assistance to the poor” (Lindert, 2004, p. 39). Some of these assistance to the poor are in kind, but there are also transfers in cash.1 Recently, many developing countries have adopted some type of conditional cash transfers to the poor as a mechanism to fight poverty, malnutrition, and inequality. Two examples are the Bolsa Família in Brazil and Progressa in Mexico, but similar programs are widespread around the world, including one in the city of New York (see Currie and Gahvari, 2008). Under Bolsa Família, for instance, families with income less than R$ 60 (roughly US$ 38) per capita receive R$ 62, plus R$ 20 per child (aged 15 and lower) up to three children, and R$ 30 per adolescent (aged 16 and 17) up to two children. Therefore, poor families can receive up to R$ 182, which is about half of a minimum wage in Brazil. The conditionalities of these transfers are the beneficiary families compliance with requirements such as school attendance, vaccine, and pre-natal visits (see Brazilian Ministry of Social Development). The total coverage is large: Roughly 11.1 million families or about 44 million people. In the poorest Brazilian region, i.e., in the Brazilian Northeast about 49 percent of all families is enrolled in this program.2

In this paper we study the effects of such cash transfers program on the labor market:3 What are the effects of cash transfers to the poor on the unemployment rate, employment rate, and participation rate? What are the effects on inequality and productivity? In order to address these questions, we build a matching model of the labor market based on Diamond (1982), Mortensen (1982), and Pissarides (1990) where workers can be in three states: employed, unemployed, or out of the labor force, producing the consumption good at home.4 The model is based on a recent contribution by Garibaldi and Wasmer (2005). There is a matching technology, which makes the number of successful matches at any moment

1In his book Capitalism and Freedom, Milton Friedman (1962) also advocates cash transfers to the poor as a social measure to alleviate poverty.
2In some very poor cities, such as Manari in the state of Pernambuco, 90 percent of all families participates in the program. Estimates show that government spending on this program accounts by only 0.3 percent of total GDP Soares et al. (2006), for instance, show that this cash transfer program is well targeted and it accounts for 28 percent of the fall in the Gini inequality index between 1995 and 2004 and corresponds to about 0.82 percent of the total family income reported at the Brazilian National Household Survey (PNAD). For comparison, they show that pensions equal to the minimum wage account for 32 percent to the fall in the Gini inequality index in the same period, but they make up to 4.5 percent of the total family income. Therefore, the Bolsa Família seems an effective way to fight inequality and poverty.
3Most of the time the goals of these programs are to: (i) reduce poverty and inequality; and (ii) break the inter-generational transmission of poverty by conditioning these transfers on beneficiary compliance with, for instance, school attendance. It is not our goal in this paper to investigate the long run effects of such cash transfer, but instead to study its impact on the labor market.
4Search and matching models are the main tools used in macroeconomics to evaluate the effects of policies on the labor market. See Mortensen and Pissarides (1999) and Rogerson et al. (2005) for a review of the literature on search and matching models. For the effects of unemployment insurance and Welfare-to-Work policies on the labor market, see Pissarides (1990), Ljungqvist
in time a function of the aggregate measures of unemployed workers and vacant jobs. As in Mortensen and Pissarides (1994), job destruction is endogenous. Workers are heterogeneous in their labor market productivity. A new productivity level arrives at constant instantaneous rate. Depending on the shock of their productivity level, workers might want to leave their job and workers out of the labor force might decide to search for a job.

We introduce cash transfers to the poor in this framework. The government provides benefits, $b > 0$, to all workers with income less than $w$. It is important to highlight that differently from unemployment insurance, workers in this welfare program receive cash transfers independently whether they are working, unemployed, or out of the labor force – working in home production. The only requirement to be entitled to this program is to have income less than $w$. We investigate the effects of the size and the coverage of cash transfers on the labor market. We show the following qualitative results:

(i) the size of cash transfers has a negative effect on the employment rate, but an ambiguous effect on the unemployment rate. Participation rate is affected negatively; and

(ii) the coverage of the program has a positive effect on the employment rate, and an ambiguous effect on the unemployment rate.

We also provide some numerical simulations, which allow us to investigate the effects of cash transfers on income inequality, and analyze alternative policies. From our knowledge, we were the first to analyze the impact of cash transfers on the labor market in a matching model with endogenous job destruction and labor market participation.

Besides this introduction, this paper has three more sections: In Section 2 we develop the matching model with cash transfer and labor market participation. Section 3 defines the equilibrium and provides some analytical results. Section 4 provides some numerical simulations. Section 5 contains the concluding remarks.

2. THE MODEL

2.1. Set up

Time is continuous and there are two commodities in this economy: one consumption good and one labor input. The economy is inhabited by a continuum of infinitely lived agents of measure one who are heterogeneous with respect to their labor market productivity, $x$. There is also a continuum of measure one of firms. Each firm has access to a production technology that exhibits constant returns to scale with labor as the only input. Without loss of generality, we assume that each firm employs at most one worker. When employed, a worker produces $yx$ units of the consumption good. Both workers and firms are neutral to risks and discount the future at a constant exogenous rate $r$.

As in Garibaldi and Wasmer (2005), workers can be in three states: Employed and receiving the market wage $w(x)$ in units of the consumption good; unemployed and searching for a job; or working at home producing $h$ units of the consumption good. Worker’s productivity in the market is stochastic and is determined by a general distribution $G(x)$, with support in the unit interval. Productivity $x$ changes over time according to a Poisson process with arrival rate $\psi$. In this way, workers productivity...
can move, with probability $\psi$ to a new productivity value that can be bigger or lower than the previous value.\footnote{Observe that our productivity process implies that conditional on the arrival of a shock, workers’ new productivity is independent of the workers’ current productivity. This implies that, once a shock arrives, a worker with a high productivity value has the same probability of facing a very low productivity shock than a worker that has already a low productivity value. Notice, however, that ex-ante all workers are the same, so our analysis focuses in a particular fraction of the labor market in which workers might be entitled to cash transfers. We therefore believe that our analysis is appropriated to investigate the effects of cash transfers on the labor market of unskilled workers.} Note that workers productivity can be hit by a shock moving workers to a situation of program coverage. Before starting a productive activity, workers and firms are involved in a search process for a productive partner, where $c$ equals search cost for the firm. The number of job matches formed per period is given by a non-negative, concave and homogeneous of degree one function, $m(v,u)$, which is crescent in its two arguments, where $v$ equals vacancy rate and $u$ denotes the fraction of unemployed workers. By the homogeneity assumption, we can write the probability rate of filling a vacancy as:

$$q(\theta) = m(u,v) = m(\frac{v}{u},1),$$

where $\theta = \frac{v}{u}$ denotes the tightness of the labor market. In addition, the rate at which an unemployed worker moves to employment is

$$\theta q(\theta) = m(u,v)\frac{1}{u}.$$

### 2.2. Government sector

There is a government that provides income transfers, $b > 0$, to all workers with income less than $w$. Workers receive income transfers independently whether they are working, unemployed, or out of the labor force, i.e., working in home production. As in Acemoglu et al. (2001), we assume that cash transfers are financed through lump-sum taxes.

### 2.3. Workers

Let $W_{NB}^B(x)$ and $W^B(x)$ be the asset value for a worker with productivity $x$ of being employed receiving or not benefits, respectively. Let $U(x)$ be the asset value of being unemployed. The following Bellman equations describe the problem of a worker with productivity $x$:

\[
(r + \psi)W_{NB}^B(x) = w(x) + \psi \int_0^M \max\{W^B(z),U(z)\}dG(z) \tag{1}
\]

\[
+ \psi \int_0^1 \max\{W_{NB}^B(z),U(z)\}dG(z)
\]

\[
(r + \psi)W^B(x) = w(x) + b + \psi \int_0^M \max\{W^B(z),U(z)\}dG(z) \tag{2}
\]

\[
+ \psi \int_0^1 \max\{W_{NB}^B(z),U(z)\}dG(z)
\]

\[
(r + \psi)U(x) = b + \max\{h,\theta q(\theta)G(M)[W^B(x) - U(x)]
\]

\[
+ \theta q(\theta)(1 - G(M))\max\{W_{NB}^B(x) - U(x)\} + \psi \int_0^1 U(z)dG(z) \tag{3}
\]

Equation (1) implies that employed workers without government benefits receive $w(x)$ flow units of the consumption good in wages, and at instantaneous rate $\psi$ obtain a new value for their productivity, which may lead them to leave the job. Equation (2) is analogous to Equation (1). It implies that
employed workers with income less than $w$ receive flow income $w(x)$ plus benefits $b$, and at rate $\psi$ obtain a new value for their productivity, which might lead them to change their employment status. Observe that $M = w^{-1}(w)$ corresponds to the productivity level such that if $x < M$ workers receive cash transfer $b$, and if $x > M$, workers do not receive government benefits. Equation (3) suggests that workers without a job receive $b$ and they might choose between home activities receiving $h$ from home production or they might choose to search for a job. At rate $\psi$, workers draw a new productivity level, $x$, reevaluating their choice between seeking for a job or remaining in home production. Notice that once a job match is formed, it can only be destroyed endogenously, due to variations related to market productivity.

2.4. Firms

Let $V$ and $J(x)$ describes the asset values for a firm of a vacant and a filled job with a worker with productivity $x$, respectively. They are described by the following Bellman equations:

$$rV = -c + q(\theta)[J^*(x) - V]$$

$$J^*(x) = yx - w(x) + \psi \int_0^1 \max\{J(z), V\} dG(z)$$

A firm with a vacant position spends flow $c$ in searching for a worker and at rate $q(\theta)$ the firm fills this vacancy. Observe that, before a position is filled firms do not know the productivity of the worker or the quality of the match. $J^*(x)$ therefore represents the expected value of filling a vacant position. In addition, a firm with a filled job produces flow $yx$ of the consumption good, paying $w(x)$ in wage. At rate $\psi$, the worker that fills this vacancy obtains a new productivity level, which allows the firm to reevaluate their decision to continue producing with this worker or destroy the match to search for a new one.

Free entry in the labor market implies that no economic rents can be gained from vacancy, i.e., $V = 0$. Therefore, from (4) we have that:

$$J^*(x) = \frac{c}{q(\theta)}$$

There is job creation up to the point where the expected value of a new job match equalizes the cost of occupying a vacancy, expressed in terms of the rate that this position becomes occupied.

2.5. Reservation productivities

Once a job match is formed, there is an economic surplus that must be divided between the worker and the firm. As standard in the literature, we assume that this surplus is divided according to the Generalized Nash Bargaining between firms and workers:

$$\beta[J(x) - V] = (1 - \beta)[W^j(x) - U(x)], j = NB, B$$

where $\beta$ denotes workers’ bargaining power.

---

8As we will show, wages are a function of the labor market productivity.
9Alternatively, we could have added a Bellman equation for being out of the labor force, such that:

$$H = h + b + \psi \int_0^1 \max\{H(z), U(z)\} dG(z).$$

Equation (3) contains already this state.
It is important to define the reservation productivity that makes workers indifferent between working in the market or in home production, $E$, and also the productivity level that makes workers indifferent between searching for a job or working at home, $R$. The surplus generated by a match is:

$$ S^j(x) = J(x) + W^j(x) - U(x), \ j = NB, B $$

(8)

Using equations (1), (2), (3) and (5), evaluating $S^j(x)$ for $j = NB$ and $B$ at $x = E$, and subtracting $S^j(E)$ from $S^j(x)$, we have that:

$$ \beta S^j(x) = \frac{\beta y(x - E^j)}{r + \psi}, \ j = NB, B $$

(9)

The surplus for both cases have the same structure because $b$ affects uniformly the outside option of all workers.

Equation (3) implies that when workers are indifferent between searching for a job or engaging in home activities, then:

$$ h \theta q(\theta) = \beta y(\frac{R^j - E^j}{r + \psi}), \ j = NB, B $$

(10)

The left hand side of (10) represents workers’ expected opportunity search cost. The right hand side stands for the benefits obtained with market participation. Since $h > 0$, then the left hand side of (10) is always positive, and $R^j > E^j$. Moreover, since the left hand side of (10) is the same whether the worker receive or not benefits, then $R_{NB} - E_{NB} = R_B - E_B$.

Following Garibaldi and Wasmer (2005), the market exit dynamics is determined by:

$$ E_{NB} = b + h + \frac{w}{r + \psi} \int_{E_{NB}}^1 G(x)dx $$

$$ E_B = h + \frac{w}{r + \psi} \int_{E_B}^1 G(x)dx $$

(11)  (12)

Proposition 2.1. If $b > 0$, then $E_{NB} > E_B$ and $R_{NB} > R_B$.

Proof: See Appendix A.

Expressions (11) and (12) determine the level of productivity resulting from indifference between working in the market or engaging in home activities. The left hand side of both equations gives the level of productivity that makes workers indifferent between leaving or not the job to home production. The right hand side represents the gain obtained by being employed. Observe that the higher income transfers, $b$, the higher will be the exit rate, since the value of being at home increases. However, the levels of reservation productivity $(E^j, R^j)_{j=NB,B}$ do not depend on productivity level $M = w^{-1}(\bar{w})$ and some of these threshold values might not be binding depending on the coverage of the program.

In an economy without the cash transfer program (e.g., $M = w^{-1}(\bar{w}) = 0$), workers without a job and with productivity in the interval $(E_{NB}, R_{NB})$ are marginally attached to the labor market. If they had a job, they would be working in the labor market, but they are not willing to pay the searching cost (see Garibaldi and Wasmer (2005) and Jones and Riddell (1999)). Employed workers with productivity in this interval at some point in time had productivity above $R_{NB}$ but if they loose their job they will be out of the labor force. When there is a cash transfer program, the measure of marginally attached workers change. We will discuss more about this shortly.
2.6. Job creation

Using equations (6) and (9) yields:

\[ J^e(x) = (1 - \beta)S(x) = \frac{c}{q(\theta)} = \frac{(1 - \beta)g(R^j - E^j)}{r + \psi} \]  

(13)

which determines the dynamics for new job creation. The right hand side of (13) describes the firm benefit of filling a vacant position. The left hand side represents the expected cost related to filling this vacancy. The larger the difference \((R^j - E^j)\), the higher will be \(\theta\), thus the higher will be the new job creation dynamics.

2.7. Wage determination

Using equations (1)-(7), we obtain four wage functions. Firstly, if \(x > M\), then:

\[ w_{1B}^N(x) = \beta yx + (1 - \beta)h, \text{ for } x < R_{NB}^N \]  

(14)

\[ w_{2B}^N(x) = \beta(yx + c\theta) + (1 - \beta)b, \text{ for } x > R_{NB}^N \]  

(15)

Moreover, if \(x < M\), then:

\[ w_1^B(x) = \beta yx + (1 - \beta)h, \text{ for } x < R_B \]  

(16)

\[ w_2^B(x) = \beta(yx + c\theta), \text{ for } x > R_B \]  

(17)

Observe that the four previous equations give us the wage rates considering that once unemployed, the worker either migrates to home production or returns to the labor market to look for a new job opportunity. The wage rates are composed by two terms: one related to workers’ job match productivity and the other to workers’ outside option. The outside option varies whether workers receive or not benefits and whether his productivity is larger than reservation productivity \(R^j, j = NB, B\). It is important to observe that some of these wages might not be binding, depending on the productivity level \(M\) associated with income \(w\). Recall that all workers with productivity \(x < E_B\) will be out of the labor force.

Five cases are possible and we study them below:

Case 1: If \(M = w^{-1}(w) < E_B\), then only unemployed workers and those out of the labor force will receive government income transfer \(b\). In this case, equations (16) and (17) will not be binding. Workers with productivity \(x < E_{NB}^B\) will be out of the labor force. Marginally attached workers are those with productivity in the interval \([E_{NB}^B, R_{NB}^B]\).

Case 2: If \(E_B < M = w^{-1}(w) < R_B\), then only workers unemployed, out of the labor force and those with productivity \(x \in [E_B, M]\) will receive government benefits. Equation (17) will not bind. Interestingly, agents with productivity \(x \in [M, E_{NB}^B]\) will be out of the labor force, despite the fact that workers with lower productivity, \(x \in [E_B, M]\), might be working in the market. This is because if workers with productivity \(x \in [M, E_{NB}^B]\) have a job, they will not receive cash transfers. The measure of marginally attached workers is given by: \(x \in [E_B, M] \cup [E_{NB}^B, R_{NB}^B]\).

Case 3: If \(R_B < M = w^{-1}(w) < E_{NB}^B\), then all equations will bind. Similarly to Case 2, agents with productivity \(x \in [0, E_B] \cup [M, E_{NB}^B]\) will be out of the labor force. The measure of marginally attached workers is: \(x \in [E_B, \min\{M, R_B\}] \cup [E_{NB}^B, R_{NB}^B]\).
3. EQUILIBRIUM AND ANALYTICAL RESULTS

The equations that describe employment and unemployment dynamics are given by:

\[
\dot{e} = \left[ \theta q(\theta) u - \psi G(E^B) e \right] G(M) + \left[ \theta q(\theta) u - \psi G(E^{NB}) e \right] [1 - G(M)]
\]

\[
\dot{u} = \left\{ -\theta q(\theta) + \psi G(R^B) u + \psi [1 - G(R^B)] (1 - e - u) \right\} G(M) + \ldots
\]

\[
\left\{ -\theta q(\theta) + \psi G(R^{NB}) u + \psi [1 - G(R^{NB})] (1 - e - u) \right\} [1 - G(M)]
\]

Notice that in both equations we separate the flow of workers that receive or not benefits. In the employment equation, there are two flows: the first one represents the rate at which unemployed workers migrate to employment \(\theta q(\theta) u\), while the second one equals the rate in which workers move from employment to home production \(\psi G(E^r) e\). In the unemployment equation there are three flows: the flow of unemployed workers who find a job in the market \(\theta q(\theta) u\); the flow of unemployed workers who move to home production \(\psi G(R^r) u\); and the flow of agents who move from home production to unemployment \(\psi [1 - G(R^r)] (1 - e - u)\).

In the steady-state \(\dot{e} = \dot{u} = 0\) and therefore:

\[
e = \frac{\theta q(\theta) u}{\psi \{G(E^B) G(M) + G(E^{NB}) [1 - G(M)]\}}
\]

\[
u = \frac{(1 - e) \psi \{[1 - G(R^B)] G(M) + [1 - G(R^{NB})] [1 - G(M)]\}}{\theta q(\theta) + \psi}
\]

This is the case in which \(E^B < M < E^{NB}\).
Figure 1: The effects of cash transfers on income. Grey solid line: case without cash transfer. Black solid line: case with cash transfer \( b = 0.1y \) and \( M \in (E^B, R^B) \). Black dotted line: case with cash transfer \( b = 0.2y \) and \( M \in (E^B, R^B) \).

Now, we can define the equilibrium for this economy.

**Definition:** Given government policies \((b, w)\), a steady-state equilibrium for this economy is a seven-tuple: 

\((\theta, R^j, E^j, w^j(x), w^j_2(x), u, e)\) \(_{N; B, B}\) such that equations (14)-(17), (10)-(13), (20) and (21) are satisfied.

The equilibrium has a block recursive structure. From equations (11) and (12), we can find \( E^N_B \) and \( E^B \). Using equations (10) and (13) we obtain:

\[
\theta = \frac{(1 - \beta)h}{\beta e}.
\]

Given \( \theta \) and \( E^j \), we can use (10) to find \( R^j \).

Then, using equations (14)-(17) we can find the wage rates. Finally, we can use (20) and (21) to find the equilibrium employment and unemployment rate. Notice that Equation (20) defines a positive relationship between \((e, u)\), while (21) implies in a negative relationship between these two variables and we can show that the locus (20) crosses the (21) locus in only one point. Figure 2 shows the two equations in the \((e, u)\) space for a particular parametrization of the model (see subsection 4.1).
The following proposition establishes the analytical effects of government policies \((b, M = w^{-1}(w))\) on the steady-state level of employment and unemployment.

**Proposition 3.1.** Consider a steady-state equilibrium. Then, for given government policies \((b, M = w^{-1}(w)) \gg 0\), we have that:

i. \(b\) affects \(e\) negatively; and \(b\) affects \(u\) ambiguously;

ii. \(M\) affects \(e\) non-negatively; and \(M\) affects \(u\) ambiguously.

**Proof:** See Appendix B.

When cash transfer, \(b\), increases there are two opposing effects on unemployment: First, for relative high productive workers (i.e., workers with productivity level higher than \(R_{NB}\)) the opportunity cost of searching increases – because if they get a job they will lose benefits, therefore some of them choose to leave the labor market, which decreases unemployment. However, for less productive workers (i.e., with productivity slightly higher than \(E_{NB}\)) the opportunity cost of being employed increases, and some of them choose to leave their job to home production. This decreases employment (and the labor force) and consequently increases the unemployment rate. Figure 2 illustrates the effects of cash transfer \(b\) on the unemployment and employment rates. It shows that while the unemployment rate is almost unchanged, the employment rate decreases as the level of benefits increases.

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The productivity levels \(E_i\) and \(R_i\) for \(i = B\) and \(NB\) are independent of the coverage of the program \(M\). When the coverage of the program increases then there might be an increase in the fraction of workers that might work only because they receive benefits \(x \in [E_B, \min(M, R_B)]\). See also this in Figure 1, which is the interval between the productivity in which workers are indifferent between working in the market or at home and the productivity in which workers become eligible to the program. This would increase the number of employed workers without increasing the number of workers searching for a job. In this case, the employment rate increases and the unemployment rate might decrease. However, if \(M\) increases further such that \(M > R_B\), then there might be some workers \(x \in [R_B, M]\) that might look for a job only because if they find one they would still receive benefits. The unemployment rate will increase.

**4. QUANTITATIVE EXERCISES**

4.1. **Calibration**

In order to quantitatively investigate the main results derived previously, we solve the model numerically. We choose a model economy without government benefits, i.e., \(b = 0\). We first have to calibrate the parameter values such that they are consistent to the empirical observations in the United States. We assume that distribution \(G(x)\) is uniform with support in the unit interval. We normalize output \(y\) to one. We let the model period be a month and set the discount rate \(r = 0.0033\), making the annual interest rate equal to 4 percent. Following Ljungqvist (2002), we let the matching technology be represented by a Cobb-Douglas type function:

\[
m = m(u, v) = u^\alpha v^{1-\alpha}, \quad \text{with} \quad \alpha \in (0, 1)
\]

We assume the same elasticity of the matching technology with respect to each input, i.e., \(\alpha = 0.5\), which was a number estimated by Blanchard and Diamond (1989). We let the worker’s surplus share \(\beta\) to be equal to 0.5, which is also used in the matching literature.

It remains to determine three parameter values: search cost, \(c\), home production parameter, \(h\), and the rate of arrival of a new productivity level, \(\psi\). We choose such parameters such the unemployment...
rate is equal to 3.5 percent, the tightness of the labor market is equal to 0.5, and the extended unemployment rate\textsuperscript{11} is 40 percent larger than the conventional measure (see Garibaldi and Wasmer (2005)). We found the following values for these three parameters: $c = 0.175y$, $h = 0.5c$, and $\psi = 0.11$.

Table 1 lists the value of each parameter and includes a comment on how each was selected.

4.2. Simulations

The first row of Table 2 displays for the calibrated economy ($b = 0$ and $M = w^{-1}(w) = 0$) the unemployment rate, extended unemployment rate, the employment rate, and the income Gini index.\textsuperscript{12} Then, we introduce a cash transfer program in this economy. For the case that $b = 0.1y$ and $M = 0.47$ (i.e., $M \in (E^B,R^B)\right)$, cash transfers correspond to roughly one fifth of the average wage. Observe

\textsuperscript{11}This is the unemployed rate plus the rate of marginally attached employed workers.

\textsuperscript{12}The Gini index measures the level of income inequality. Income in this case corresponds to the labor income of those that are working $y_x$ and cash transfers $b$. Therefore, cash transfer will affect the Gini index directly because of benefits and indirectly through changes in workers’ labor market participation decision.
Table 1: Parameter values, baseline economy.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Comment/Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>1</td>
<td>Value normalized to one</td>
</tr>
<tr>
<td>$r$</td>
<td>0.0033</td>
<td>Discount rate: annual interest rate of 4%</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.5</td>
<td>Worker’s bargaining power</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>Parameter of the matching technology</td>
</tr>
<tr>
<td>$c$</td>
<td>0.175</td>
<td>Search cost: calibrated to match $\theta = 0.5$</td>
</tr>
<tr>
<td>$h$</td>
<td>0.0875</td>
<td>Home production: calibrated to match $u = 3.5$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.11</td>
<td>Calibrated to match extended unemployment rate, $u + e^{ma} = 4.9%$</td>
</tr>
</tbody>
</table>

that as the size of the income transfers increases, employment decreases and unemployment remains roughly constant (see also Figure 2), though our extended unemployment rate measure, which includes the marginally attached employed workers, increases substantially. It increases by 24 percent. Participation rate also decreases and income inequality decreases sharply. Notice that when we increase the coverage of the program, keeping the size of the transfer constant, both employment and participation rate increase. The program seems to have a negligible impact on government finances. Total spending as a share of output\(^{13}\) in the economy with $b = 0.1$ and $M = 0.47$ is about 0.01 percent.

Table 2: Quantitative effects: Cash Transfers

<table>
<thead>
<tr>
<th>$b = 0$, $M = 0$</th>
<th>Unemployment rate, $u$</th>
<th>Extended Unempl. rate, $u + e^{ma}$</th>
<th>Employment rate, $e$</th>
<th>Gini</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b = 0.1y$, $M = 0.47$</td>
<td>3.5%</td>
<td>6.06%</td>
<td>45.03%</td>
<td>0.37</td>
</tr>
<tr>
<td>$b = 0.15y$, $M = 0.47$</td>
<td>3.47%</td>
<td>5.88%</td>
<td>42.93%</td>
<td>0.31</td>
</tr>
<tr>
<td>$b = 0.1y$, $M = 0.52$</td>
<td>3.5%</td>
<td>6.08%</td>
<td>45.38%</td>
<td>0.35</td>
</tr>
<tr>
<td>$b = 0.15y$, $M = 0.52$</td>
<td>3.48%</td>
<td>5.92%</td>
<td>43.44%</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Figure 3 displays the effects of cash transfers on the economy (i.e., unemployment rate, extended unemployment rate, employment rate, Gini index, output, and government spending) for different levels of benefits and the coverage of the program. The square in each graph corresponds to the baseline case without any cash transfer. Observe that when the coverage of the program is small, cash transfers decrease both the unemployment and the employment rates.\(^{14}\) Only when the eligibility of the program increases that the employment rate starts to increase. Some workers would receive benefits if they are working or not.\(^{15}\) Inequality decreases sharply. The Gini index decreases by roughly 30 percent with the introduction of cash transfers. However, inequality has a U-shape, since it starts to increase as more

---

13Total spending on cash transfers is: $(1 - e)b + bG(M)e$.
14This is consistent with empirical findings. See, for instance, Hoynes (1996) who estimate how changes in benefit levels of the program Aid to Families with Dependent Children affect labor supply in the United States.
15The model therefore could explain variation in the employment rate across countries by differences in the level of benefits and in the eligibility of welfare programs. See Ngai and Pissarides (2008) for data on differences in employment rates among OECD countries and a model of how taxes and welfare programs affect such employment rate. See also Rogerson (2007). The model used by Ngai and Pissarides (2008) is completely different from ours. Their focus is on the sectoral allocation of labor.
people become entitled to participate in this welfare program. Notice that output first decreases due
to a decrease in the participation rate. However, as the employment rate increases with the coverage
of the program, output starts to increase.\textsuperscript{16} Government spending as a share of income is roughly constant.

Figure 3: The effects of cash transfers on the economy. Black square: Baseline economy. Black solid line: 
\( b = 0.1y \) and different levels of \( M \). Gray dotted line: \( b = 0.15y \) and different levels of \( M \)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3.png}
\end{figure}

Notice that as the level of benefits increase further the quantitative effects on the economy become
stronger as well as on government finance.\textsuperscript{17} When the eligibility of the program is low, the Gini index
decreases by 15 percent when the level of benefits increases by 50 percent. Output, however, decreases
by about 16 percent and government spending as a share of income in the welfare program increases by
35 percent. If the government target is to decrease inequality, the more efficient policy is to increase the
level of benefits instead of increasing the eligibility of the program. However, such policy comes with
a negative impact on efficiency (unless the majority of workers receive transfers) and on government
finances.

supply and on home production. They use a consumer problem with a standard leisure-labor choice in which agents choose
the allocation of their time endowment in leisure, market hours, and home production.

\textsuperscript{16}As in Acemoglu et al. (2001), we are assuming that the taxes are financed by non-distortionary taxation. Results for aggregate
output might be different once we introduce distortionary taxes.

\textsuperscript{17}An increase in 50 percent in the level of benefits increase government spending as a share of income by roughly 40 percent.
5. CONCLUDING REMARKS

This paper shows the effects of cash transfers on the labor market in a matching model with endogenous job destruction and labor market participation. In our model, the government provides cash transfers, \( b > 0 \), to all workers with income less than \( w \), independently whether they are working, unemployed, or out of the labor force, i.e., working in home production. We also discuss the role of alternative policies, such as transfers conditional to labor market participation.

We show that the size of cash transfers has a negative effect on the employment rate, but an ambiguous effect on the unemployment rate. Participation rate is also affected negatively. In addition, the eligibility of the program has a positive effect on the employment rate, and an ambiguous effect on the unemployment rate. The numerical simulations show that the quantitative effects might be sizeable, specially on income inequality which decreases sharply with cash transfers. In order to avoid the negative effects on participation rate (and productivity) the government could also introduce a labor market participation program providing benefits for agents that are either working or searching for a job.\(^{18}\)

There are some important extensions of this paper. First, it will be interesting, for instance, to investigate the effects of cash transfers on the labor market using an alternative model with risk aversion in which agents accumulate assets, as the equilibrium search model of Alvarez and Veracierto (1999). Risk aversion would increase the welfare effects of cash transfers to the poor when agents cannot insure against idiosyncratic shocks. It will be also interesting to check whether or not the predictions of the model are consistent to the empirical evidence.

BIBLIOGRAPHY


\(^{18}\)If we interpret the home production sector as the informal sector, this labor market program might be an effective mechanism to reduce informality and tax evasion if benefits are conditioned on working in the formal sector. This might be particularly important in developing countries with economies that present a large informal sector and tax evasion (e.g., Antunes and Cavalcanti (2007)).
A. PROOF OF PROPOSITION 2.1

Using the implicit function theorem and Equation (11), we can show that:

\[
\frac{\partial E_{NB}}{\partial b} = \frac{1}{\frac{v}{\rho}} G(E_{NB}) > 0
\]

Therefore, \( E_{NB} > E_B \), since \( E_{NB} = E_B \) when \( b = 0 \). Moreover, \( R_{NB} - R_B = E_{NB} - E_B \), which implies that \( R_{NB} > R_B \).
B. PROOF OF PROPOSITION 3.1

Observe that, in the steady state $\theta$, $E^B$, and $R^B$ do not depend on $b$ and $M$. In addition, $M$ does not also affect $E^{NB}$ and $R^{NB}$, however, from equations (11) and (10), we can show that $b$ affects $E^{NB}$ and $R^{NB}$ positively, i.e., $\frac{\partial E^{NB}}{\partial b} > 0$ and $\frac{\partial R^{NB}}{\partial b} > 0$.

From (20) and (21) we have that:

\begin{align*}
\text{(B-1)} \quad f^1(e,u,b,M) &= e - \frac{\theta q(\theta) u}{\psi(G(E^B)G(M) + G(E^{NB})[1 - G(M)])} = 0 \\
\text{(B-2)} \quad f^2(e,u,b,M) &= u - \frac{(1 - e)\psi([1 - G(R^B)]G(M) + [1 - G(R^{NB})][1 - G(M)])}{\theta q(\theta) + \psi} = 0
\end{align*}

Observe that:

\begin{align*}
f^1_e &= 1, \quad f^1_u = -\frac{\theta q(\theta)}{\psi(G(E^B)G(M) + G(E^{NB})[1 - G(M)])} < 0 \\
f^2_e &= (1 - e)\psi G'(R^{NB})[1 - G(M)] \frac{\partial R^{NB}}{\partial b} \theta q(\theta) + \psi > 0
\end{align*}

Moreover,

\begin{align*}
\text{(B-1)} \quad f^1_b &= \frac{u\theta q(\theta) G'(E^{NB})[1 - G(M)] \frac{\partial E^{NB}}{\partial M}}{\psi(G(E^B)G(M) + G(E^{NB})[1 - G(M)])^2} > 0 \\
\text{(B-2)} \quad f^2_b &= (1 - e)\psi G'(R^{NB})[1 - G(M)] \frac{\partial R^{NB}}{\partial b} \theta q(\theta) + \psi > 0
\end{align*}

and

\begin{align*}
\text{(B-1)} \quad f^1_M &= -\frac{\theta q(\theta) u G'(M) [G(E^{NB}) - G(E^B)]}{\psi(G(E^B)G(M) + G(E^{NB})[1 - G(M)])^2} < 0 \\
\text{(B-2)} \quad f^2_M &= -\frac{(1 - e)\psi G'(M) [G(R^{NB}) - G(R^B)]}{\theta q(\theta) + \psi} < 0
\end{align*}

Since the determinant of the matrix with the derivative of functions $f^1$ and $f^2$ with respect to the endogenous variables $e$ and $u$ is non-singular, we can use the implicit function theorem to show that:

\begin{align*}
\frac{\partial e}{\partial b} < 0, \quad \text{and} \quad \frac{\partial u}{\partial b} > 0 \\
\frac{\partial e}{\partial M} > 0, \quad \text{and} \quad \frac{\partial u}{\partial M} < 0
\end{align*}