# Long-run Implications of the Brazilian Capital Stock and Income Estimates*

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**Abstract**

The present study aims to analyze the empirical as well as theoretical implications related to the possible inconsistencies between the Brazilian capital stock estimate and its associated investment decision. The common practice of using the country's accumulated (depreciated) fixed capital formation data as a proxy for the capital stock series generates a set of incompatible facts with dynamic models built on balanced growth and on aggregate production functions. Moreover, a related issue on the Brazilian capital income is considered in our analysis. According to the country's National Accounts, the participation of capital income reaches about half of the aggregate income which is an unusual high share compared to international standards. It is shown that this problem can also be solved using alternative methods that lead to a more suitable capital stock series to be used in recursive equilibrium models. Finally, the long-run impacts of using the proposed capital stock series is studied using a modified basic growth model calibrated to reproduce some Brazilian empirical facts.

*Keywords:* Investment, Capital Accumulation, Functional Distribution of Income.

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## 1. Introduction

Capital stock constitutes one of the key variables to study economic growth and cycles. Nevertheless, there is no consensus in the literature on its definition as well as the appropriate method to measure it. In less developed countries as Brazil the problem is aggravated due to the lack of official data on capital stock.

Generally, in the macroeconomic literature, the lack of capital series data is solved using the perpetual inventory method (PIM), namely through the accumulation of depreciated investment.[1] Nonetheless, this method presents some theoretical as well as measurement problems.

From the theoretical point of view, the PIM assumes that the value of capital must be equal to the value of the associated investment decision. However, as Hicks (1974) and, more recently, Prichett (2000) argue, this assumption is not necessarily true. The implicit assumption of this method according to which the entire value assigned to investment turns out to be next period's capital stock cannot be sustained if part of the resources allocated into investment is wasted. This could be often the case, in particular for public investment, for subsidized private investment, or whenever the associated capital stock is not fully operational due to technological reasons as in capital vintage models.[2]

Furthermore, from a measurement point of view, the obtained series strongly depends on the depreciation rate and the initial capital stock, which are not easily observable. Moreover, relative price changes can also change the capital stock value, as in the example presented by Prichett (2000) of a reduction of import barriers to capital goods.

Indirectly associated with the above measurement problem, the capital income share in aggregate income has to be considered. Similarly to the capital stock measurement problem, shortcomings related to its quantification are more serious for less developed countries. As pointed out by Young (1995) and Gollin (2002), this measure is rather confusing in less developed countries. In particular, the latter observes that this parameter value can vary from 20% to 80% among different countries. This measurement problem generates, in turn, serious limitations for economic growth analysis. As Young (1995) argues, in terms of total factor productivity (TFP), the economic performance analysis can be completely different depending on that capital income share in aggregate income.

The present study introduces a friction into the dynamics of capital stock accumulation, assuming that part of the current investment does not turn into capital next period. In other words, it is herein assumed that the value of the current capital stock does not correspond to the value of the last period investment. Hence, the law of motion for capital accumulation is redefined. This new dynamics can be modeled including a waste component into the capital accumulation process.

[1] For a careful estimation of the capital stock series based on the PIM, refer to Morandi (1998).

[2] For this consideration, refer to Parente (1994).

Then, a recursive general equilibrium model is used to quantitatively assess the long-run implications of the existence of this waste associated with the investment decision of the economy. Furthermore, the quantitative impact on the steady state equilibrium derived from the utilization of an alternative capital stock series will be discussed. To this end, alternative imputation methods, proposed by Young (1995) and Gollin (2002), to derive the capital income share into aggregate income will be introduced into the analysis.

The present study is organized as follows. Section 2 presents the theoretical as well as empirical justification to introduce a waste factor into the capital accumulation dynamics for the Brazilian economy. Section 3 describes the model economy and the recursive equilibrium definition to be numerically computed. Section 4 explains the calibration of the model economy to mimic some of the main Brazilian empirical evidences. Section 5 analyzes the long-run behavior of the model economy. Finally, section 6 concludes pointing out some suggestions for future research.

## 2. Stock of Capital and Investment

The issue of the capital stock measurement of a given economy has important consequences on macroeconomic analysis. Hicks (1974) defined two different ways to measure the value of capital. The first one regards the acquisition cost as the value of the capital stock available in the economy and, the second one defines it as the sum of all goods that could be produced in the future using that capital.[3]

Hence, the practice of evaluating the capital stock as the accumulated sum of depreciated investment is clearly associated to the materialist view, for the emphasis is given to the cost associated to produce the capital good regardless of the value of future goods it could produce. The same author presented an example of the problem associated with this view taken from Hayek: a machine can be used to produce a given good which is very trendy in only a given period. Thus, this capital good remains intact after its utilization. In this example, the materialist view considers the value of the machine in the second period to be defined as its value in the previous year subtracting the corresponding depreciation,[4] whereas according to the second view, the machine would not have any value in the second period for it would not be used to produce any future goods.

One way to reconcile both approaches is to regard investment as a result of an optimal decision such that its cost equals the present value of future goods the associated capital stock could produce. Then, with this hypothesis in hand, the practice of using the accumulated investment as proxy for the capital stock value

[3]Hicks named the former view as "materialist" and the latter as "fundist", as the first one regards the capital input as a physical good and the second as a fund to finance the production of future goods.

[4]This is exactly the procedure adopted when evaluating the capital as the accumulated sum of depreciated investment.

could be justified. Therefore, changes in the capital accumulation rule will be needed such that the two views could be simultaneously taken into consideration.

However, before introducing the above modification in the capital accumulation rule, the question on the non-optimal investment decision has to be dealt with. There is one important reason to believe that the cost of a given investment decision would be different to the present value of the future goods the associated capital stock could produce. The argument, as suggested by Prichett (2000), is based on the idea that not all agents in the economy, in particular the government sector, behave optimally trying to maximize the return from its investment decision.

In order to justify the application of the above line of argument for Brazil, some empirical studies could be mentioned. In particular, Cândido-Jr. (2001) argues that public expenses in the country are not productive. Defining an efficient public resource allocation as the one in which the generated public good equals the associated social benefit, the author concludes that the Brazilian public expenses are not efficiently used. Moreover, this study shows that the productivity of the Brazilian public sector is only about 60% the productivity of the private sector. Thus, this study clearly shows that the Brazilian public sector does not observe optimality conditions for decisions on public expenses. If the government is not an optimizing agent it turns out to be hard to assume the same agent will be optimizing when taking public investment decisions.

Moreover, as Brazilian public investment reaches about 14% of total investment for the period 1970-1998, distortions in public investment decisions have had important implications both in a macroeconomic and sector-wise sense. An extreme example of this sub-optimal public investment can be easily observed in the country from huge public projects that were never accomplished, or were concluded and never used, as the so-called "*Steel Railway*", the nuclear power installation in Rio de Janeiro, and the "*Transamazonian Highway",* among others. Other forms of waste in public investment can also be easily observed in public projects that are overvalued or take a longer time than originally planned. This argument should be also applied to the investments of special development agencies such as the former Brazilian SUDAM and SUDENE,[5] financed with public funds.

Furthermore, it is possible that a difference between the investment value and the present value of its flow of returns can also arise when markets are not competitive. Gollin et al. (2000) argue that, for instance, the presence of barriers to entry can artificially push up the investment cost or relative price of capital goods. This could also be the case when high import tariffs are applied to capital goods or whenever monopoly power to use a given technology is considered. Parente and Prescott (2000) based on data from the Penn World Tables show that the investment goods prices compared to consumption goods prices are substantially

[5]Superintendência de Desenvolvimento da Amazônia and Superintendência de Desenvolvimento do Nordeste respectively.

high in poorer countries. In order to take into account this friction, the former authors used a capital accumulation rule where part of the investment is lost.[6]

Along with the above arguments to justify the proposed alternative method used to construct the estimated capital stock series for the Brazilian economy, the observed empirical evidence of the country appears to be even stronger to point out the inconsistency in the capital stock series. The increase in the capital output ratio, specially after 1974, suggested by some authors as Morandi (1998), among others, could not be consistent with various facts observed during the same period. Namely, the investment rate also showed a remarkable increase. Figure 1 below shows the behavior of the Brazilian capital output ratio for the period 1950-1998. The proposed interpretation of this inconsistency is that the increase in investment did not turn out into new productive capital stock for the economy, reflecting only an accounting increase of the capital output ratio. For if this increased investment were actually transformed into capital stock, the rate of increase of capital supply higher than the rate of labor supply would have driven a reduction of the productivity of capital due to the law of decreasing returns and, consequently, a reduction of capital income as well as an increase of real wages. Neither of these expected outcomes can be supported by the Brazilian empirical evidence of the period. As pointed out by Ferreira and Araújo (1999) the economy presented an increase in total factor productivity only from 1990 on.

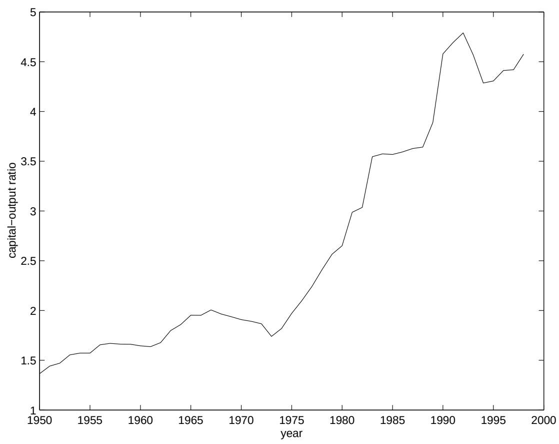


Figure 1
Capital-output ratio series (Brazil 1950–1998)

[6]Although the motivation and aim of the analysis differ, the present study also introduces the modified capital accumulation rule in the model similar to the one proposed by Gollin et al. (2000), in order to capture the waste associated to the Brazilian aggregate investment decision.

The proposed approach tries to solve this problem assuming that part of the current investment does not turn into capital next period, regardless of the reasons behind this dynamics, which could be attributed to corruption, low government productivity and/or existence of entry barriers. Taking into account the existence of this waste in the investment decision, the capital accumulation process will not produce an increasing capital output ratio as the data plotted in Figure 1 above shows. Formally, the following law will be introduced into the model economy:

$$K_{t+1} = (1-\delta)K_t + \frac{1}{\phi}I_t \qquad (1)$$

where $K$ represents the aggregate capital stock, $\delta$ the depreciation rate, $\phi$ a parameter related to the waste and $I$ the aggregate investment defined as the sum of gross fixed capital formation and consumption of durable goods.[7]

As an example of a possible implication of the existence of a waste in investment, consider the wealth output relation for the Brazilian economy in 1970 of approximately 2.2 which is compatible with the values reported by IPEADATA and by Morandi (1998). Taking a depreciation rate of 10%, a population and per capita GDP growth rates of 2% and 2.6%, respectively, as reported in the National Accounts statistics, the following algorithm can be implemented in order to obtain a waste rate for the Brazilian economy.

1. Give an arbitrary initial investment capital relation.
2. According to the modified law of motion for capital accumulation given above, at steady state the waste rate $\phi$ will be given by:

   $$\phi = \frac{I/K}{\delta + (1+x)(1+n) - 1} \qquad (2)$$

   where $x$ and $n$ represent the growth rate of per capita GDP and the population growth rate, respectively.
3. With the above value for the parameter $\phi$ compute the capital stock series.
4. Compute the investment/capital ratio using the series of capital stock obtained above. If this value is equal to the initially given relation, stop the algorithm. Otherwise, continue with the second step.

Using this algorithm the obtained value for the parameter $\phi$ is 1.26 implying an investment waste of approximately 20.6%.[8] According to this modified rule, the mean capital output ratio turns out to be 1.8, reaching its maximum value of 2.17

[7] The series of consumption of durable goods for the Brazilian economy were obtained from Ellery-Jr. et al. (2002).

[8] $1-(1/1.26) = 0.2063$.

in 1990 and 1991, which were exceptionally poor years in terms of the country's GDP performance due to the "*Collor Plan*" policies. If these two exceptional years are excluded, the mean value of the capital output ratio goes down to 1.75. Figure 2 below depicts the obtained capital output ratio series together with the original series reported in Figure 1 above.

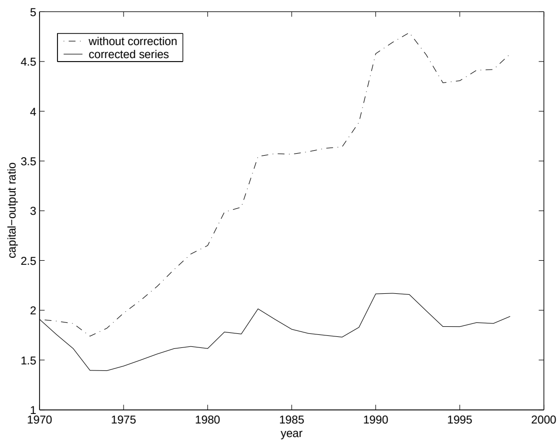


Figure 2
New capital-output ratio series with waste

Figure 2 shows the same declining pattern for both series during the early 1970s. This pattern could be enhanced using the proposed algorithm due to a possible over-estimation of the waste parameter for the period 1970-1973.[9] For the remaining years, the relative stability of the obtained capital output ratio series compared to the original series turns out to be apparent.

The obtained stable capital output ratio series is crucial in order to use macroeconomic growth models due to the fact that this relationship constitutes a key hypothesis of models dealing with steady state equilibrium or with constant growth rates for the main aggregate variables. Otherwise, if the existence of waste in investment is not considered, the only possible way to overcome the observed inconsistency between the increasing Brazilian investment with a stable capital output ratio would be to assume an extremely high capital stock depreciation rate. [10]

[9] A time-varying estimation of the parameter could solve this problem, but this extension will be left as a future exercise.

[10] Ellery-Jr. et al. (2002) suggest a depreciation rate as high as 17% to obtain a stable capital output ratio for the Brazilian evidence on increasing investment during the same period under study.

## 2.1 Transitional dynamics

The argument presented above is based on the assumption that the economy is at its steady state. However, it could be argued that the Brazilian economy had been undergoing a dynamics of transition towards a more capital-intensive steady state, hence, justifying the increasing capital output ratio.

In order to incorporate the transition, equation (1) should be expressed in unit of efficiency as:

$$(1+\eta)(1+\gamma)k_{t+1} = (1-\delta)k_t + \frac{1}{\phi}i_t \tag{3}$$

where $\eta$ represents the population growth rate, $\gamma$ the per capita aggregate output growth rate, $\delta$ the depreciation rate, $k_t$ the capital stock at time $t$ and $i_t$ time $t$ investment. For an economy in a balanced growth path, the parameter $\gamma$ represents the rate of growth of per capita aggregate product and of per capita capital stock as well. Whereas for an economy in transition the rate of growth of per capita output is different than the productivity growth rate, the latter being one of the components of the former.[11]

In a balanced growth path, the variables measured in efficiency units do not grow because they are at steady state. This fact was taken into consideration when the algorithm with a waste in investment was developed, for it was assumed that $k_t = k_{t+1}$ in equation (3). This assumption does not hold for an economy in transition. Therefore, the rate of growth of per capita capital stock measured in efficiency units, i.e. $k_{t+1}/k_t$, needs to be determined. This implies that the per capita output growth rate has to be expressed as the sum of the productivity growth rate and the growth rate induced by the increase in per capita capital stock.

The above decomposition can be implemented by means of a growth accounting exercise using a particular aggregate production function. To this end, the following Cobb-Douglas production function is assumed to characterize the available technology in the economy.

$$Y_t = f(K_t, A_tH_t) = K_t^{\theta}((1+\gamma)^t H_t)^{(1-\theta)} \tag{4}$$

where $Y_t$ refers to output, $H_t$ to labor, both at time $t$ [12] and $\theta$ to the capital share parameter.

Expressing the above equation in per capita units the following equation is obtained.

$$\hat{y}_t = A_t^{(1-\theta)}\hat{k}_t^{\theta} = (1+\gamma)^{t(1-\theta)}\hat{k}_t^{\theta} \tag{5}$$

and taking logarithms, it is possible to express the rate of growth of per capita output $\Gamma_{haty_t}$ as the weighted sum of the rate of labor augmenting productivity

[11] The other components refer to the contribution of factors, capital and labor inputs to the product's growth.

[12] In this study this variable is assumed to be equal to the population.

growth $\Gamma_A$ and the rate of per capita capital stock growth $\Gamma_{\hat{k}_t}$ as follows.

$$\Gamma_{\hat{y}} = (1-\theta)\Gamma_A + \theta\Gamma_{\hat{k}} \tag{6}$$

where $\Gamma_{\hat{y}}$ is the growth rate of per capita output, $\Gamma_A$ the growth rate of labor-augmented productivity, and $\Gamma_{\hat{k}}$ the growth rate of per capita capital. According to the Brazilian National Accounts $\Gamma_{\hat{y}_t}$ is 2.6% per year for the period 1947-1998. Therefore, we have one equation with two unknowns. To solve this problem, we assume the growth rate of U.S. TFP as a proxy for the Brazilian TFP; in other words, we assumed that the Brazilian productivity grows at the same rate as the American aggregate productivity at the maximum.[13]

According to Penn World Table, the American productivity growth rate, in terms of per capita output, is 1.7% for the postwar period. Then, for assumption, this is the highest rate the Brazilian TFP can attain. In other words, it is assumed that the Brazilian productivity can grow at most at the same rate of the leading economy.[14] With these parameters values the rate of growth of the per capita capital stock is computed as shown below:[15]

$$\Gamma_{\hat{k}} = \frac{1}{\theta}\left[\Gamma_{\hat{y}} - (1-\theta)\Gamma_A\right] = \frac{1}{0{,}35}\left[2{,}60\% - 1{,}70\%\right] = 2{,}00\%$$

Then, since by definition $k_t = K/AH = k/A$, the rate of growth of per capita capital in terms of efficiency units can be derived as:

$$\Gamma_{k_t} = \Gamma_{\hat{k}_t} - \Gamma_A = 0.30\%$$

and, with the above value in hand, the parameter related to the waste in investment can be obtained in turn using equation (3) above, i.e.

$$\phi = \frac{I/K}{\delta + (1+x)(1+n)(1+\Gamma_k) - 1} \tag{7}$$

Using the above equation instead of equation (2) in the proposed algorithm, the corresponding series for the capital stock output ratio along the dynamics of transition can be obtained.

Figure 3 below shows the obtained series, which depicts the behavior of the capital output ratio during the transitional dynamics (solid line) and the series computed at steady state (dashed line). Comparing both series we can observe that

[13] This assumption means that the Brazilian economy can undergo a transitional dynamics towards a richer economy only by means of capital accumulation. This assumption is the traditional view that the transitional dynamics can only use capital accumulation (see King and Rebelo (1993)), and diverge from the recent literature where catching up relies on the productivity side – see for example Parente and Prescott (2000), and Lucas-Jr. (2002).

[14] This assumption is equivalent to assume that the American economy is in steady state.

[15] The value of the parameter $\theta$ will be discussed below in subsection 4.1.

when a ransitional dynamics for the Brazilian economy is considered, the capital output ratio increases but remains lower than the one in an economy without any investment waste. The former presents an average ratio of 1.91, with a maximum value of 2.36 and an associated investment waste of approximately 13.4%, i.e. $\phi = 1.15$.

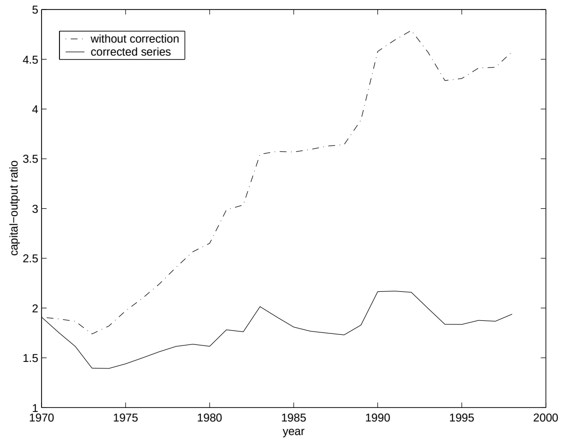


Figure 3
Capital-output ratio series – transition and steady state

The above obtained results suggest that, even taking into consideration the possibility of a transition path, the observed increase in the capital output ratio for the Brazilian economy during the period under study appears to be inconsistent with the theoretically expected result. In other words, the assumption of a waste in the aggregate investment seems to be successful in explaining the discrepancy between the observed data and the analytically expected result.

The next section will introduce an extended basic growth model, introducing the waste factor into the law of motion for the capital accumulation of the artificial economy. Then, following Cooley and Prescott (1995), the impact of this assumption on the steady state recursive general equilibrium properties of the model economy will be evaluated.

## 3. The Model

The model economy consists of a large number of identical agents and firms participating in perfectly competitive factors, capital and labor, and single good markets. Hence, the model is a standard growth model, where population grows at a rate given by $\eta$ and productivity is assumed to grow at a rate given by $\gamma$.

Therefore, in equilibrium, aggregate variables grow at a rate given by $(1+\eta)(1+\gamma)$. Herein, all considered variables will be measured in units of labor, such that the corresponding dynamic programming problem can be properly defined.

However, the artificial economy differs from the standard model in regard to the capital stock accumulation rule. The modified law of motion explicitly considers the assumption of a waste of a fraction of the investment optimally decided by the agents. Taking this friction into account, the problems of the representative agent and firm are introduced below along with the definition of the recursive equilibrium for the model economy.

### 3.1 The representative agent's problem

The identical agents will be modeled as in the standard growth model, so that in every period the representative agent (RA) will choose how much to consume and to save for next period.

Furthermore, the RA has preferences over stochastic sequences of consumption and leisure characterized by the equation below.

$$U = E\left\{\sum_{t=0}^{\infty} \beta^t (1+\eta)^t u(c_t, l_t)\right\} \tag{8}$$

where $c_t$ represents time $t$ consumption, $\beta$ the inter-temporal discount factor, $l_t$ time $t$ leisure, and $u(.)$ the instantaneous utility function assumed to take the explicit form $u(c,l) = ln(c_t) + aln(h)$ and, in every period $t$, the RA faces a budget constraint given by:

$$c_t + i_t = w_t h_t + r_t k_t \tag{9}$$

such that all time $t$ available income given by the sum of current labor income and capital income, i.e. $w_t h_t + r_t k_t$, will be entirely allocated into current consumption $c_t$ and investment $i_t$. The RA is also time-restricted such that in every period the allocation of time between work, $h_t$, and leisure, $l_t$, must be equal to the total available time, i.e. $h_t + l_t = 1$.

Therefore, the RA problem can be summarized as to maximize his/her expected discounted utility (8) over a stochastic sequence of consumption and leisure, subject to the per period budget and time constraint (9), given the initial capital stock and the (modified) dynamics of capital accumulation:

$$(1+\eta)(1+\gamma)k_{t+1} = (1-\delta)k_t + \frac{1}{\phi} i_t \tag{10}$$

where $\delta$ denotes the depreciation rate, and $\phi$ a constant related to the waste of resources allocated to investment. This equation means that if $i_t$ units of current production are allocated into investment only $i_t/\phi$ units will turn out into capital stock next period. Therefore, the amount of the waste will be given by $(1-(1/\phi))$.

### 3.2 Technology

The constant return to scale technology available to the representative firm (RF) is characterized by the following production function:

$$Y_t = z_t f(k_t, A_t h_t) = z_t(1+\gamma)^{(1-\theta)t} k_t^{\theta} h_t^{1-\theta} \tag{11}$$

where $k_t$ denotes the employed capital stock per worker at time $t$, $h_t$ worked hours per unit of labor input, $A_t$ a labor augmenting technical progress and, $z_t$ a random variable capturing the exogenous shock to time $t$ production which evolves according to the following stochastic rule.

$$z_{t+1} = 1 - \rho + \rho z_t + \varepsilon_t \tag{12}$$

where $\epsilon_t$ denotes time $t$ innovation component of the stochastic process, normal, independent and identically distributed with zero mean and finite and constant variance $\sigma_{\varepsilon}^2$. The competitive firms optimally choose the quantity of capital and labor at every period, such that profits are maximized setting their respective marginal products to their returns given in the competitive markets, i.e.

$$w_t = z_t(1+\gamma)^{(1-\theta)}(1-\theta)K^{\theta}H_t^{-\theta} \quad \text{and} \quad r_t = z_t(1+\gamma)^{(1-\theta)}\theta K_t^{\theta-1}H_t^{1-\theta} \tag{13}$$

### 3.3 Dynamic programming problem and recursive competitive equilibrium (RCE)

With the above RA and RF characterization in hand, the following Bellman equation describes the dynamic problem faced by the agents of the model economy.

$$V(z,k,K) = \max_{\{h,k'\}} \{u(c, 1-h) + \beta(1+\eta)EV(z',k',K')\} \tag{14}$$

subject to (9), given the law of motion of the state variables (12) and (10) and the initial capital stock available in the economy. Then, we can define the recursive competitive equilibrium as follows.

**Definition** (*Equilibrium*): *A recursive competitive equilibrium is a value function $V(z,k,K)$; a set of decision rules: $c(z,k,K)$, $h(z,k,K)$ and $i(z,k,K)$; a set of decision rules: $C(z,K)$, $H(z,K)$ and $I(z,K)$; and factor price function $w(z,K)$ and $r(z,K)$, such that these functions satisfy:*

1. *the household's problem, (14):*

$$V(k,z) = \max_{\{h,k'\}} \{u(c,1-h) + \beta(1+\eta)EV(k',z')\}$$

   *subject to (9), given (12), (10), and $k_0$.*

2. *the firm's problem, (13)*

3. *the consistency of individual and aggregate decisions, i.e.,* $Nc(z,k,K) = C(z,K)$*;* $Nh(z,k,K) = H(z,K)$ *and* $Ni(z,k,K) = I(z,K)$*, where* $N$ *is the number of individuals.*

4. *the aggregate resource constraint:*

$$C(z,K) + I(z,K) = Y(z,K)$$

In order to numerically compute the above defined RCE a linear quadratic approximation of the return function around the steady state was implemented following Hansen and Prescott (1995) and Ljungqvist and Sargent (2000, chap. 4).

## 4. Calibration

The calibration of the artificial economy followed Cooley and Prescott (1995). Accordingly, the parameter values have to be set in such a way that the model economy could reproduce, at steady state, the main facts observed in the Brazilian economy. Therefore, the characterization of the Brazilian economy at steady state with a waste of investment is needed.

In order to proceed with this characterization, the below modified first-order condition corresponding to the maximization problem (14) is used.[16]

$$\begin{aligned} &\frac{\phi(1+\gamma)(1+\eta)}{c_{t+1}} = \frac{\beta(1+\eta)\left[\theta k_{t+1}^{\theta-1} h_{t+1}^{(1-\theta)}\right]}{c_{t+1}} \\ &\frac{(1-\theta)k_t^{\theta} h_t^{-\theta}}{k_t^{\theta} h_t^{1-\theta} - \phi k_t[(1+\gamma)(1+\eta)-(1-\delta)]} = \frac{a}{1-h_t} \end{aligned} \tag{15}$$

The corresponding equations used for calibration purposes were obtained from (15) and the capital stock accumulation rule (10) evaluated at steady state such that:

$$\begin{aligned} &\frac{\phi(1+\gamma)}{\beta} - \phi(1-\delta) = \theta\frac{y}{k} \\ &(1-\theta)\frac{y}{c} = \frac{ah}{1-h} \\ &i = \phi\left[(1+\gamma)(1+\eta) - (1-\delta)\right] \end{aligned} \tag{16}$$

The above three equations allowed, in turn, to set the corresponding values for the discount factor $\beta$, the elasticity of substitution between consumption and labor $a$ and the investment waste $\phi$, all of them as function of the other remaining parameters.

[16] For a detailed justification of the choices for the specific utility and production functions used in the model, refer to Ellery-Jr. et al. (2002).

The deprecation rate $\delta$ and its relationship with the waste parameter $\phi$ were discussed in section 2 above and will be analyzed in section 5 below. The value for $\delta$ will be set at 10% for the benchmark case, thus implying $\phi = 1.26$. The parameter value corresponding to the capital income share in aggregate income $\theta$ will be carefully discussed below. The other two values needed for the calibration procedure are the population growth rate, $\eta$, and the rate of growth of per capita GDP, $\gamma$. Based on the National Account statistics, the respective values for these parameters were set at 2% and 2.6%, which are the approximate values observed for the Brazilian economy for the period 1947-1998.

Finally, the model economy should be able to reproduce the capital output ratio, corrected by the investment waste, of approximately 1.8 as well as the worked hours $h_t$. According to PNAD[17] data $h_t$ is approximately 0.31, meaning that a typical Brazilian worker allocates approximately one third of his/her available time to labor market activities.[18]

Given the values adopted above, the calibration procedure for the complete determination of the set of parameter values of our model economy depends solely on the value of the capital income share in aggregate income, $\theta$. The next subsection will be devoted to describe carefully the problems associated to the measurement of this parameter for the Brazilian economy.[19]

### 4.1 Capital and labor income share in aggregate income

The majority of the dynamic macroeconomic studies applied to the Brazilian economy assume that the capital income share is about 50% of aggregate income.[20] However, this share is too high compared to international standards of about 30% as shown by Gollin (2002) and Parente and Prescott (2000). This comparison naturally brings about the question of why the capital income share in Brazil is so high.

In fact, different alternative official National Accounts data show considerable variability regarding this parameter value, which varies from 20% up to 80%. Gollin (2002) suggests that this measurement discrepancy arises from a common national account procedure, particularly in poorer countries, which treat self-employment remuneration as capital income. The author argues out that this measurement problem is associated to an accounting practice, which considers the

[17] Pesquisa Nacional de Amostra Domiciliar (National Household Survey).

[18] See Ellery-Jr. et al. (2002) for further details.

[19] The parameters characterizing the technology shock are calibrated as follows. First, the productivity series are regressed on a time trend and the residual is identified as the estimated technology shock process, $z_t$. This residual series is consistently estimated in turn with an $AR(1)$ process, from which we derive the corresponding persistence parameter value of $\rho = 0.589$. This characterization is used to define a set of innovations in technology, setting the standard deviation of the process at 0.0446. For more details, refer to Prescott (1986).

[20] See for instance Barreto and Oliveira (1995), Ferreira and Araújo (1999) and Ellery-Jr. et al. (2002).

labor share in aggregate income as the ratio between "employment remuneration" and "national product". In fact, this procedure leads to a poor measure of labor income, specially in poorer countries, for it does not take into consideration the self-employment remuneration which is residually accounted as capital income. For instance, for countries as Ghana, Bangladesh and Nigeria, the percentage of self-employed workers in manufacturing accounts for as much as 80% of total employed workers, whereas for the USA this number drops to approximately 2%.

Based on the above line of argument, the author suggests three alternative procedures in order to correct the usual measurement problem related to the self-employed labor income share. The first one, herein referred as G1, consists of treating the operational surplus of non-incorporated private enterprises as labor income. The second G2, divides this non-incorporated income by the ratio between the employment compensation and the aggregate income. The third alternative procedure, G3, imputes employment compensation to all self-employed workers.[21]

Furthermore, the same author shows that applying the commonly used procedure, the average labor income share among different countries reaches 0.479 with a standard deviation of 0.135. Whereas when his alternative methods are applied, the average labor income share increases to 0.745 (G1), 0.675 (G2) and 0.654 (G3) and the respective standard deviations drops to 0.087 (G1), 0.089 (G2) and 0.103 (G3) respectively.

The measures of the Brazilian average share of labor income in aggregate income for the period 1990-1998 period, obtained through the usual procedure (Naive) and the (G1), (G2) and (G3) procedures proposed by Gollin (2002) were computed and the results are shown in Table (1). This Table clearly shows that this share increases from 0.47 (Naive) to 0.55 (G1), 0.5 (G2) and 0.53 (G3).[22] Comparing these results with the ones obtained by Gollin (2002), which suggest an increase in the share of labor income of approximately 0.7, two questions can be formulated: Does (or does not) the Brazilian economy behave as the others? Are the Gollin's procedures suitable to measure the Brazilian labor income share?

It is important to point out that Gollin's main concern was to produce an efficient method to be used in comparative, cross-section studies among different countries. The same author suggests that perhaps a better method would be the one proposed by Young (1995), which is based on micro-data, although it could only be applied to few countries due to the availability of data set. Young's method imputes worker's wages to the self-employed workers and employers, according to their respective production sector, gender, age and education.

In fact, Gollin's (G3) and Young's procedures are similar in terms of the motivation. Both of them impute the average wage to the self-employed worker. Along this line of argument, if these procedures are to be applied for the Brazilian economy, a crucial factor must be observed. The average labor compensation for the

[21] i.e. ((employment compensation/employed workers) working-age population)/GDP.

[22] Data from PEA/PNAD and National Accounts were used in this procedure.

Brazilian employed workers was R$ 5,480.00 in 1998 according to the National Accounts data. However, the imputation of this income to self-employed workers cannot be suitable due to the fact that, as studied by Barros et al. (2001), the Brazilian aggregate income distribution is highly concentrated. Empirical evidence from the 1998 PNAD census shows that the annual average remuneration of employees (R$ 6,024) is similar to the average income of self-employed workers (R$ 5, 268), but much lower than the employers' remuneration (R$ 20,736). Therefore, the imputation of the average wage to the self-employed can clearly bias the share of labor income in aggregate income.

Aiming to capture the income of self-employed workers and employers, we use the above 1998 PNAD census information to compute the average income for different types of workers as follows. First, an average income, weighted for the respective number of workers, of self-employed workers and employers was computed. Second, the ratio of this number and the average income of employees were calculated for 1992, 1993, 1995 and 1998, imputing the corresponding average income of these years for 1990, 1991 and 1994. The obtained value of 1.26 clearly suggests that the Brazilian total labor income is in fact 26% higher than the share shown in the country's National Accounts.

With the above information, a corrective factor of 1.26 is applied to the officially reported total labor income and, the mean value of per worker labor income is obtained by dividing this number by the total number of employed workers. Finally, the share of labor income in aggregate income is obtained multiplying this corrected average labor income by the economically active population and dividing in turn this result by the country's aggregate value added. The obtained result of 0.67 is also introduced in the last column of Table 1 under the DR (difference in remuneration) denomination.

Two characteristics on the obtained labor share derived from the above estimation procedure are noteworthy. From a quantitative point of view, the obtained measure is highly similar to the average international evidence of 0.7 presented by Gollin (2002). Furthermore, the obtained estimates are rather stable along the considered period, contrary to the commonly used (Naive) estimates which present a declining tendency along the same period. This aspect is crucial from a theoretical point of view, for the new stable estimates of labor income share can, in turn, empirically support the utilization of an aggregate Cobb-Douglas production function for our artificial economy.

Table 1
Labor share, 1990-1998

| | Naive | G1 | G2 | G3 | DR |
|---|---|---|---|---|---|
| Mean Labor Share | 0,47 | 0,55 | 0,50 | 0,53 | 0,67 |

Setting the alternative labor income shares, based on the alternative procedures described above – Naive, G1, G2, G3 and DR shown in Table 1 –, we are able to assign the corresponding values to the capital income share parameter, $\theta$, the discount factor, $\beta$, and the elasticity of substitution between consumption and labor, $a$. Table 2 below shows the calibrated values for this set of parameters, recalling that the depreciation rate is set to 10%, the investment waste factor to 1.26, the population growth rate to 2% and, the rate of growth of labor productivity to 2%.

Table 2
Calibration

| | Naive | G1 | G2 | G3 | DR |
|---|---|---|---|---|---|
| $\theta$ | 0,53 | 0,45 | 0,50 | 0,47 | 0,33 |
| $\beta$ | 0,9050 | 0,9341 | 0,9157 | 0,9266 | 0,9813 |
| $a$ | 1,96 | 1,83 | 1,67 | 1,77 | 2,28 |

## 5. Long-run Implications

With the above calibration for the model economy in hand, the steady-state (long-run) implications of the assumption about the existence of a waste factor on the optimal investment decision are analyzed. In other words, the effects of this assumption on the steady state equilibrium is studied.

To this end, following the suggestion of Parente and Prescott (2000), the per capita income[23] associated to those steady state equilibria with alternative investment waste parameter values, $\phi$, are compared to each other rather than the associated growth rates.

Furthermore, it is important to mention the implications of changing the value of capital income share in aggregate income, $\theta$, before evaluating the long-run impacts of a waste in investment resources. It was argued, in the previous section, how the measurement problems present in the National Accounts data lead to an overestimation of this share relative to the labor income share in aggregate income. Alternative procedures to deal with this problem were proposed, and a corresponding value for the parameter $\theta$ was derived.

In terms of the setup of the model economy, higher values of $\theta$ are related to higher values of capital stock and to smaller values for worked hours. Since the calibration of the model imposes the worked hours to be approximately one third of the available time, i.e. $h = 0.31$, the effect on the dynamics of capital accumulation will be the only relevant impact of changing the value of the parameter $\theta$.

Therefore, higher values for this parameter would simply imply higher values of per capita income in the artificial economy. Figure 4 below sketches this relationship taking alternative values for $\theta$ between 0.33 and 0.53.

[23] According to the authors, "...we came to the conclusion that relative income levels rather than growth rates are key to understanding the problem of development...".

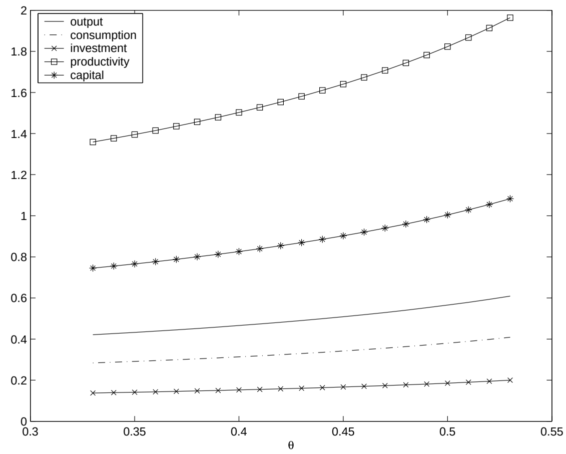


Figure 4
Effects of $\theta$ over the steady state

This figure shows a strictly and monotonically increasing relationship between the capital income share parameter value $\theta$ and the other variables of the model. Hence, it was chosen to set an intermediate value for this parameter, i.e. $\theta = 0.45$, corresponding to the value obtained with the (G1) procedure, and compute the steady state RCE for different values of the investment waste factor $\phi$.

Recall that section 2 above presents a careful discussion of the relationship between the depreciation rate, $\delta$, and the investment waste factor, $\phi$. It is shown there that for each value for the former it is possible to obtain a value for the latter such that a stable capital output relationship could be derived. The same procedure is implemented in this section in order to attribute the values for the waste parameter $\phi$ consistent with a depreciation rate between 4% and 12.5%. Table 3 below presents the results of this computation.

Table 3
Values of $\phi$ and $\delta$

| $\delta$ | $\phi$ | $\delta$ | $\phi$ | $\delta$ | $\phi$ |
|---|---|---|---|---|---|
| 4,0% | 2,28 | 7,0% | 1,64 | 10,0% | 1,26 |
| 4,5% | 2,14 | 7,5% | 1,56 | 10,5% | 1,22 |
| 5,0% | 2,02 | 8,0% | 1,49 | 11,0% | 1,18 |
| 5,5% | 1,91 | 8,5% | 1,43 | 11,5% | 1,13 |
| 6,0% | 1,81 | 9,0% | 1,37 | 12,0% | 1,09 |
| 6,5% | 1,72 | 9,5% | 1,32 | 12,5% | 1,06 |

The above values, consistent with a stable capital output ratio, allow us to quantify the changes in per capita income, at steady state, as a result of a waste of resources optimally allocated to investment. The simulation results suggest that the long-run per capita income of the artificial economy calibrated for the Brazilian economy could be 20% higher with a waste of 5.66%, i.e. $\phi = 1.06$, than for an economy with a waste of 56%, i.e. $\phi = 2.28$. Figure 5 below illustrates the steady state per capita income of the model economy for each considered waste factor $\phi$. As expected, this figure shows that the higher this parameter value or, equivalently, the higher the waste of resources allocated to investment, given by the term $(1 - (1/\phi))$, the lower the per capita income in the long-run.

Another exercise aims to compare the steady state equilibrium of an economy calibrated with $\beta = 0.9341$, $\theta = 0.45$, $\delta = 0.10$, $a = 1.8336$, $\gamma = 0.026$, $\eta = 0.02$ with a waste of $\phi = 1.26$ with the one obtained for an economy without any waste of resources, i.e. $\phi = 1$. This simulation shows that an investment waste of approximately 20.63% causes a reduction of 20% in per capita income and, the capital output ratio shrinks from 2.2 to 1.8 at the long-run equilibrium.

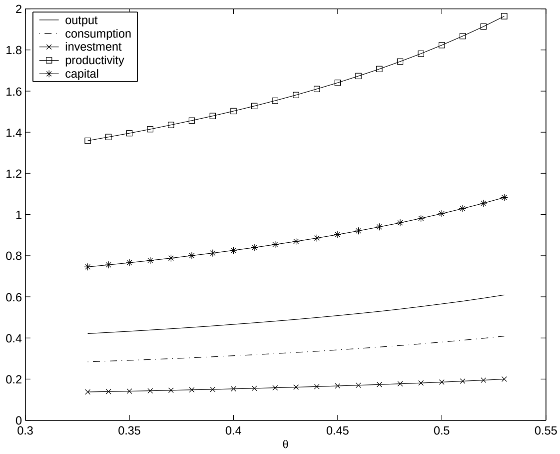


Figure 5
Impact of $\phi$ on per-capita income at steady state

The analysis of this section suggests that a sharp reduction of as much as 20% in per capita income at steady state can be derived from a model economy calibrated to reproduce the main empirical facts of the Brazilian economy, which also includes an empirically observable friction into the dynamics of capital accumulation. In the present study that friction is captured by a waste of investment resources of 20.63%.

Although this effect appears to be insufficient to fully explain the differences in per capita income among Brazil and other more developed countries,[24] clearly shows the relevance of the proposed argument in order to (partially) identify the causes of a lower per capita income in the country compared to more developed economies.

## 6. Conclusion

This study aims to explicitly introduce a friction given by the presence of a waste in the capital accumulation process and to analyze its long-run effects on the performance of the economy.

The above hypothesis is empirically justified. Looking at the Brazilian data (in particular the capital output ratio series), the presence of an investment waste in the Brazilian economy turns out to be apparent.

Moreover, the utilization of balanced growth models as well as of steady state models would be theoretically jeopardized for the Brazilian economy. Aggregate models cannot be of any use to an economy where capital stock grows faster than aggregate output and effective labor. Therefore, the introduction of the proposed assumption reconciles the theoretical requirements and the observed facts in the Brazilian economy.

The effects of such an assumption on the economy are analyzed by means of numerical simulations of a standard growth model calibrated for the Brazilian economy, taking different possible values for the waste factor. The long-run analysis suggests that the presence of an investment waste could be responsible for a reduction of as much as 20% in per capita income.

Finally, it is worthy to point out the need to further study the public and private composition of the Brazilian investment series. This careful analysis is important due to the fact that public sector investment has had a crucial role on the country's growth experience. Moreover, as some studies suggest, the public sector could present a higher waste than the private one.

Along this line of argument, the setup of a two-sector model economy in which one of the sectors presents a higher waste, due for instance to the presence of monopoly rights, also explicitly considering the resource allocation mechanism for investment financing, such as taxes and/or subsidies, could improve the understanding of the impact of such a waste on the economic performance of the Brazilian economy.

[24]In fact, it would be inappropriate to justify the underdevelopment of any economy solely based on the existence of a waste of investment resources.

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