Evidences of Bull and Bear Markets in the Bovespa index: An application of Markovian regime-switching models with duration dependence

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Abstract
The aim of this paper is to identify bull and bear markets in the Bovespa index. For this, we estimated Markovian regime-switching models that incorporate duration dependence in which the transition probability also depends on the number of periods that the process has been in a particular state. The results indicated a regime with positive returns and low volatility and another with high volatility and negative returns. Moreover, the probability of switching out of the regime declines with the persistence of bull and bear markets. The smoothed probabilities evidenced the success of the methodology in identifying the main market downturns in Brazil. Finally, investment strategies were formulated based on the predicted probabilities of the models and these provided a higher average return and Sharpe ratio than the Bovespa index.

Keywords: Duration Dependence; Markov-Switching; Volatility.

JEL codes: C24; G17; N26.
1. Introduction

Over the past decades the finance literature has studied and documented stylized facts of financial data series. Evidences regarding volatility clustering, leptokurtosis, and the existence of an equity premium have accumulated in the literature (Campbell et al., 1997, see, among others). However, the characteristic that receives most attention outside the specialized literature is the perception that there are periods when stock prices rises and others when these prices decline. In the financial market jargon these situations are known as bull and bear markets, respectively. Despite there being theoretical arguments for the existence of these regimes (St-Amour and Gordon, 2000), more complicated dynamics in the conditional mean returns are frequently ignored. In empirical analysis, the knowledge of these conditional moments can help in predicting not only financial variables but also macroeconomic variables (Chauvet, 1999, Stock and Watson, 2003) and even in carrying out monetary policy (Rigobon and Sack, 2003). An investor that can predict bull and bear markets in advance can avoid unexpected losses or obtain profits by assuming long or short positions and so identifying the state of the market is relevant for portfolio decisions and risk management. In an attempt to predict bull and bear markets, in this paper we implement Markov-switching models to identify bull and bear regimes in the Ibovespa and we test the results in an empirical application based on investment strategies.

The interest in identifying trends for financial assets returns is long-standing (De la Vega, 1668) and at the beginning of the last century it gave rise to what would come to be known as technical analysis (see Murphy, 1999). In the economic literature, the search to measure the state of the economy and to understand the transition between recessions and expansions has been an important topic in empirical research. Algorithms to detect the business cycle phases, including peaks and troughs, have been developed in the last decades (see Bry and Boschan, 1971). More recently, Pagan and Sossounov (2003) and Lunde and Timmermann (2004) have considered various methods inspired by the tradition of Bry and Boschan (1971) to identify turning points in the financial markets. However, these applications are based on the use of ex-post algorithms and the turning points are only identified several periods after their occurrence, which is unfavorable for real-time investment.
decisions. According to Nyberg (2013), the idea of classifying bull and bear stock market states is similar to identifying recession and expansion phases in economic activity. The methods to identify business cycle or stock market states can be divided into two main classes: in the first the turning points are obtained based on parametric models, such as Markovian regime-switching models, and the second group includes the non-parametric identification rules, such as the algorithm from Bry and Boschan (1971) and its variations.

One of the most successful methods for identifying different states in time series was developed by Hamilton (1989), which defines the regime-switching using a first order Markov chain. This approach enables statistical inference not only for the current period, but also of the probability of staying in a particular state in the following period. In his application, Hamilton (1989) sought to characterize economic cycles and showed that GDP in the U.S. could be specified as a process of two states: expansion and recession. After the study by Hamilton (1989) various authors extended the Markov-switching methodology with the aim of capturing the different characteristics of the data series (see, for example Guidolin, 2013, and papers cited there). For example, Durland and McCurdy (1994) developed a Markov-switching model that depends not only on the current state, but also the number of periods the process has been in a particular state. The authors expanded the matrix of transition probabilities to incorporate a higher order Markov chain and introduced the concept of duration dependence. The results showed that as a contraction ages the probability of moving into an expansion increases, i.e., coming out of the recession is more plausible should the crisis be prolonged. In the opposite scenario, the model did not find significant results for duration dependence associated with the probability of a transition out of expansions, but could nicely match NBER business cycle dates.

In the context of identifying market states, Maheu and McCurdy (2000) extended the approach of Durland and McCurdy (1994) for analyzing bull and bear markets in the New York stock exchange. Particularly, the duration-dependent Markov-switching (DDMS) model allows duration to affect the transition probabilities. This means that while the bull market persists, investors could become more optimistic in relation to the future and are expected to invest more. Similarly, a bear market
could persist while the investors pessimism about the future continues. According to expected, the results from Maheu and McCurdy (2000) indicated that the probability of staying in the bull and bear market increases as duration increases. Moreover, the higher returns state presented less volatility when compared to negative returns, confirming the economic intuition that the market is more volatile when the economy is falling or expected to fall compared to periods of expansions. Maheu and McCurdy (2000) also highlight that the regime-switching model with fixed transition probabilities is inadequate for capturing all of the dependency of volatility on financial data. To overcome this drawback, the authors proposed the DDMS-ARCH specification to explore the persistence associated with volatility clustering. In the DDMS-ARCH model the conditional mean and variance can change with duration and also captures ARCH effects.

As previously highlighted, in financial markets, the investors are especially interested in predicting bull and bear periods when making asset allocation decision. Thus, identifying market states can be used by investors to implement investment strategies. Guidolin and Timmermann (2007), Tu (2010), Guidolin and Hyde (2012) and Nyberg (2013), among others, have recently considered asset allocation decisions in regime-switching environment. The existence of bull and bear markets can be interesting for evaluating the risk of possible regime-switching. For example, during the bear market regime, investing in stocks is not very attractive, since stock prices are generally falling. If the future market regime is predictable, investors can obtain better results by changing their investments to risk free assets when the predicted probability of a bear market is high, and vice-versa with a bull market. Therefore, if the regime switches are precisely identified, it will be possible to exploit them in profitable investment strategies. Thus, this paper considers different investment strategies based on the predicted probabilities of the regime switches and uses their performance as an economic indicator of the accuracy of different models for identifying bull and bear market periods.

It is important to highlight that the possibility of exploring the predicted probabilities of bull and bear markets, besides being a way of testing the adjustment of the models to the data, is also an interesting application from a financial perspective. In this paper, we use weekly data from the Ibovespa closing prices from
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July 1994 to December 2014, covering 1,069 observations. Different strategies with
and without leveraging were considered, using Ibovespa futures contracts and a risk
free asset based on the predicted probability of bull and bear markets. The em-
pirical results show that even after taking realistic transaction costs into account
the strategies based on the predicted probabilities presented good performance rel-
ative to the benchmark, i.e., the passive buying strategy for the Ibovespa known
as buy-and-hold. Specifically, during the period from 01/2005 to 12/2014, most of
the strategies based on the predictive probabilities of bear markets using Markov-
switching models outperform the benchmark net of transaction costs. The strategies
present annualized returns of up to 12% and Sharpe ratios between 0.367 and 0.528,
compared with an average annual return of 10.11% and a Sharpe ratio of 0.365
for the benchmark. However, some investment strategies presented a lower average
annual return compared to the benchmark.

A comparison of the results for the period from January 1994 to December 2014
shows that the non-parametric rules-based approach proposed by Lunde and Tim-
mermann (2004) reflects the recent direction of the market, but only the Markov-
switching models take into account the risk-return trade-off, characterizing bull mar-
ket regimes as periods of high average returns, but also low volatility. Periods of low
average returns and high volatility are characterized as bear markets. Thus, some
periods considered as being bull by the rules-based approach are identified as be-
ing bear by the Markov-switching if they present high volatility. These occur when
the regime-switching is included in the volatility; that is, the information contained
in the variance improves the detection of the unobservable regime. However, the
fact that bear markets are associated with high volatility is inferred from the data.
Markovian regime-switching models with four regimes can also be considered to ac-
commodate these correction periods within one same regime (Maheu et al., 2012).
The parametrization used by the DDMS models showed that the probability of
switching out of the regimes decreases with bull and bear market persistence and
the smoothed probabilities revealed the main peaks of instability in the Ibovespa.

Besides this introduction, this article is organized into 4 sections. Section 2 ex-
plorers the concept of duration dependence and the DDMS models. Section 3 presents
the parameters estimates and the turning points analysis. Section 4, an empirical
evaluation is elaborated for the investment strategy involving market states. Finally, the last section offers the concluding remarks.

2. Regime-switching and Duration Dependence

This section describes the parametrization of the duration dependence and the duration-dependent Markov-switching models. Details regarding estimation will be discussed at the end of this section.

2.1 Duration Dependence

In this paper, bull and bear markets are characterized using the state variable $S_t = i$ for $i = 0, 1$, where $t = 1, \ldots, T$. Unlike the traditional regime-switching models, the idea of duration dependence is to capture the temporal dependence of the state variable. In this context, the aim is to make the transition probabilities more dynamic using the $D(S_t)$ variable. This variable portrays memory in the consecutive occurrence of state $S_t$ and can be defined using equation below:

$$
D(S_t) = \min(D(S_{t-1})I(S_t, S_{t-1}) + 1, \tau). \tag{1}
$$

The indicator function $I(S_t, S_{t-1})$ is 1 if $S_t = S_{t-1}$ and 0 otherwise. It is noted that a limit parameter $\tau$ is established in the equation and its value is a discrete number that will established using the log-likelihood.\(^2\)

According to Maheu and McCurdy (2000), one way to parametrize the transition probabilities is by using a logistic function. This guarantees that the probabilities are between 0 and 1. Using $i$ and $d$ to index the occurrence of the states and the duration, where $\gamma_1(i)$ and $\gamma_2(i)$ are the parameters, the transition probability for $i = 0, 1$ is given by:

$$
P(S_t = i | S_{t-1} = i, D(S_{t-1}) = d) = \begin{cases} 
\frac{\exp(\gamma_1(i)+\gamma_2(i)d)}{1+\exp(\gamma_1(i)+\gamma_2(i)d)}, & d \leq \tau \\
\frac{\exp(\gamma_1(i)+\gamma_2(i)\tau)}{1+\exp(\gamma_1(i)+\gamma_2(i)\tau)}, & d > \tau
\end{cases}
$$

\(^1\) $T$ is the sample size.

\(^2\) For more details see section (2.3).
As proposed by Durland and McCurdy (1994), the duration effect enables the transition probability to vary over time until it reaches $\tau$. The $\gamma_2(i)$ parameter summarizes the duration dependence effect: $\gamma_2(i) < 0$ means that a long period in regime $i$ implies a greater probability of switching out of the state; $\gamma_2(i) = 0$ means that the transition probability is independent of the regime duration; and $\gamma_2(i) > 0$ indicates that the longer the duration of regime $i$, the greater the chance of the process being maintained in $i$. It is interesting to note that if there is no duration dependence, $\gamma_2(i) = 0$, the parametrization collapses into the first-order Markov process: $P(S_t = i|S_{t-1} = i, D(S_{t-1}) = d) = P(S_t = i|S_{t-1} = i)$, i.e. the regime-switching will only depend on the most recent past value of the state variable and will be independent of the number of periods in which the process is in a particular state.

### 2.2 DDMS Models

Considering that most of the financial data series present conditional heteroskedasticity, in this article, volatility is incorporated into the duration-dependent Markov-switching models. Initially, the DDMS-ARCH (Duration-Dependent Markov-switching Autoregressive Heteroskedasticity) model, the return follows an autoregressive process given by:

$$R_t = \mu(S_t) + \phi_1\{R_{t-1} - \mu(S_{t-1})\} + \phi_2\{R_{t-2} - \mu(S_{t-2})\} + \cdots + \phi_P\{R_{t-P} - \mu(S_{t-P})\} + \varepsilon_t,$$

where $\phi_1, \ldots, \phi_P$ are parameters, $P$ is the number of lags, and $\varepsilon_t$ is an error term in period $t$. $\mu(S_t)$ is seen as a state variable $S_t$:

$$\mu(S_t) = \mu_0(1 - S_t) + \mu_1 S_t,$$

in which $\mu_0$ and $\mu_1$ are parameters. If $S_t = 0$, then $\mu(S_t) = \mu_0$. If $S_t = 1$, then $\mu(S_t) = \mu_1$. The error term $\varepsilon_t$ of equation (3) is expressed by:

$$\varepsilon_t = \sqrt{h_t} \varepsilon_t,$$
so that \( v_t \) is assumed to follow an independently and identically normal distribution and \( h_t \) is given by:

\[
h_t = m_{st} + \sum_{k=1}^{K} \alpha_k \tilde{\varepsilon}_{t-k}^2,
\]

(6)

where \( K \) denotes the number of lags, and \( \alpha_1, \ldots, \alpha_K \) are parameters. The difference between this model and the standard ARCH model with lag \( K \) is that the DDMS-ARCH depends on a state variable \( m_{st} \). If \( S_t = 0 \), then \( m_{st} = m_0 \), however if \( S_t = 1 \), then \( m_{st} = m_1 \), where, \( m_0 \) and \( m_1 \) are parameters to be estimated. In equation (6), \( \tilde{\varepsilon}_{t-k} \) is the prediction error given by:

\[
\tilde{\varepsilon}_{t-k} = R_{t-k} - E_{t-k-1}[R_{t-k}],
\]

(7)

where \( E_{t-k-1}[R_{t-k}] \) is the expected value of \( R_{t-k} \) given the information set \( \Omega_{t-k-1} = \{R_1, R_2, \ldots, R_{t-k-1}\} \), so that:

\[
E_{t-k-1}[R_{t-k}] = \sum_{s_{t-k}, s_{t-k-1}, \ldots, s_{t-k-P}} E[R_{t-k}|S_{t-k}, S_{t-k-1}, \ldots, S_{t-k-P}, d, \Omega_{t-k-1}] \times P[S_{t-k}, S_{t-k-1}, \ldots, S_{t-k-P}, d|\Omega_{t-k-1}].
\]

(8)

With the aim of capturing the asymmetric effects due to the shocks in the data series, this study extends the article of Maheu and McCurdy (2000) by adapting the EGARCH model in the duration-dependent Markov-switching structure. In the DDMS-EGARCH (Duration-Dependent Markov-switching Exponential Generalized Autoregressive Conditional Heteroskedasticity) model, the conditional volatility is given by:

\[
\ln (h_t) = m_{st} + \sum_{q=1}^{Q} \alpha_q \left( \frac{\tilde{\varepsilon}_{t-q}}{\sqrt{\tilde{h}_{t-q}}} - \sqrt{\frac{2}{\pi}} \right) + \sum_{r=1}^{R} \zeta_r \left( \frac{\tilde{\varepsilon}_{t-r}}{\sqrt{\tilde{h}_{t-r}}} \right) + \sum_{s=1}^{S} \beta_s \ln (\tilde{h}_{t-s}),
\]

(9)

where \( \alpha \) and \( \zeta \) are the parameters related to the dynamic of the shock and \( \beta \) refers to past volatility. The asymmetry is represented by parameter \( \zeta \) and its value is expected to be negative, indicating the fact that an increase in volatility occurs more subsequent to negative shocks than positive shocks. In this parametrization the regime-switching occurs in the level of the process and is given by \( m_{st} \), i.e., similar
to the previous model. It is important to highlight that the recursive structure of
the conditional volatility $\tilde{h}_{t-q}$ is computed by aggregating the conditional variance
of the latent states to each time period and is given by the following equation:

$$
\tilde{h}_{t-q} = E_{t-q-1}[R_{t-q-1}^2] - [E_{t-q-1}[R_{t-q}]]^2,
$$

(10)
in which the terms on the right side are given by:

$$
E_{t-q-1}[R_{t-q}^2] = \sum_{S_t-q, S_{t-q-1}, \ldots, S_{t-q-P}, d} E[R_{t-q}^2|S_t-q, S_{t-q-1}, \ldots, S_{t-q-P}, d, \Omega_{t-q-1}] \times P[S_t-q, S_{t-q-1}, \ldots, S_{t-q-P}, d|\Omega_{t-q-1}].
$$

(11)

In addition to DDMS-ARCH and DDMS-EGARCH, we will estimate 2 additional
models of the DDMS class. The DDMS-1, the regime-switching only occurs in the
unconditional mean of the process:

$$
R_t = \mu(S_t) + \phi_1\{R_{t-1} - \mu(S_{t-1})\} + \phi_2\{R_{t-2} - \mu(S_{t-2})\} + \cdots + \phi_P\{R_{t-P} - \mu(S_{t-P})\} + \sigma v_t,
$$

(12)

where $\sigma$ is the parameter to be estimated. The DDMS-2 is defined as:

$$
R_t = \mu(S_t) + \phi_1\{R_{t-1} - \mu(S_{t-1})\} + \phi_2\{R_{t-2} - \mu(S_{t-2})\} + \cdots + \phi_P\{R_{t-P} - \mu(S_{t-P})\} + \sigma(S_t)v_t.
$$

(13)

In the equation above, both mean and variance (unconditional) incorporate the
regime-switching, i.e., $\mu(S_t) = \mu_0(1 - S_t) + \mu_1 S_t$, and $\sigma(S_t) = \sigma_0(1 - S_t) + \sigma_1 S_t$. In
both models $v_t \sim NID(0,1)$ and $S_t = 0, 1$.

2.3 Estimation

The DDMS models were estimated using the maximum likelihood method, as seen
in Maheu and McCurdy (2000) and this procedure is similar to the approach of
Hamilton (1989). For the model with two states, $S_t = i$ where $i = 0, 1$ and $P$ is the

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3 For more details see Gray (1996).
lag of $R_t$, it is necessary to define a new latent variable $\Upsilon_t$. This variable covers all of the possible historical trajectories from $S_t$ to $S_{t-P}$, together with the respective duration in the current sequence of events. The new state variable is defined as:

\[
\Upsilon_t = 1 \quad \text{if} \quad S_t = 1, S_{t-1} = 0, S_{t-2} = 0, \ldots, S_{t-P} = 0, D(S_t) = 1 \\
\Upsilon_t = 2 \quad \text{if} \quad S_t = 1, S_{t-1} = 1, S_{t-2} = 0, \ldots, S_{t-P} = 0, D(S_t) = 2 \\
\Upsilon_t = 3 \quad \text{if} \quad S_t = 1, S_{t-1} = 1, S_{t-2} = 0, \ldots, S_{t-P} = 1, D(S_t) = 2, \\
\vdots \\
\Upsilon_t = N \quad \text{if} \quad S_t = 0, S_{t-1} = 0, S_{t-2} = 0, \ldots, S_{t-P} = 0, D(S_t) = \tau.
\]

The assumed values for these variables seek to characterize all of the combinations in the dependent structure of returns up to order $\tau$. The total number of combinations is given $N = 2^{P+1} + 2(\tau - P - 1)$. Each of $N$ states is presented as $\Upsilon_t = j, j = 1, \ldots, N$.

Let $\xi_t$ be a random vector of dimension $(N \times 1)$, such that:

\[
\xi_t = \begin{cases} 
(1 \ 0 \ 0 \ \ldots \ \ 0)' & \text{when } \Upsilon_t = 1 \\
(0 \ 1 \ 0 \ \ldots \ \ 0)' & \text{when } \Upsilon_t = 2, \\
(0 \ 0 \ 1 \ \ldots \ \ 0)' & \text{when } \Upsilon_t = 3, \\
\vdots & \\
(0 \ 0 \ 0 \ \ldots \ \ 1)' & \text{when } \Upsilon_t = N.
\end{cases}
\]

Let $\hat{\xi}_{t|t-1}$ be a vector whose $j$th element is $P(\Upsilon_t = j|R_{t-1}, \ldots, R_1, \theta)$, where $\theta$ represents the parameter set and $P(\Upsilon_t = j|\cdot)$ is the prediction probability of state $j$ under the condition ($\cdot$). $\hat{\xi}_{t|t}$ is a vector whose $j$th element is given by $P(\Upsilon_t = j|R_t, \ldots, R_1, \theta)$, that is, the filtered probability.

Let $\eta_t$ be a vector whose $j$th element is the conditional density of $R_t$, $f(R_t|\Upsilon_t = j, R_{t-1}, \ldots, R_1, \theta)$:

\[
\eta_t = \begin{bmatrix} 
  f(R_t|\Upsilon_t = 1, R_{t-1}, \ldots, R_1, \theta) \\
  f(R_t|\Upsilon_t = 2, R_{t-1}, \ldots, R_1, \theta) \\
  \vdots \\
  f(R_t|\Upsilon_t = N, R_{t-1}, \ldots, R_1, \theta)
\end{bmatrix}.
\]
Let $P$ be the transition probability of $\Upsilon_t$, such that:

$$
P = \begin{bmatrix}
p_{11} & p_{21} & \ldots & p_{N1} \\
p_{12} & p_{22} & \ldots & p_{N2} \\
& & \ddots & \\
p_{1N} & p_{2N} & \ldots & p_{NN}
\end{bmatrix},
$$

(17)

where $p_{ja, jb} = P(\Upsilon_t = jb | \Upsilon_{t-1} = ja), j_a, j_b = 1, \ldots, N$.

The filtered probability $\hat{\xi}_{t|t}$ for $t = 1, \ldots, T$ can be obtained using the following equations:

$$
\begin{align*}
\frac{\hat{\xi}_{t|t-1} \odot \eta_t}{1' (\hat{\xi}_{t|t-1} \odot \eta_t)} &= \hat{\xi}_{t|t} \\
\hat{\xi}_{t+1|t} &= P \hat{\xi}_{t|t}.
\end{align*}
$$

(18)

For the DDMS-ARCH, the iteration (18) begins at $t = P + K + 1$ given $R_1, \ldots, R_{P+K}$, in which $\odot$ means multiplication element-by-element between the vectors.\footnote{R_{t-k-1}, \ldots, R_{t-k-P} are needed to construct $\tilde{\varepsilon}_{t-k}$ (term that appears on the right side of equation $h_t = m_{st} + \sum_{k=1}^{K} \alpha_k \tilde{\varepsilon}_{t-k}^2$) and $h_t$ needs $R_{t-k-1}, \ldots, R_{t-k-P}$, because, given $R_1, \ldots, R_{P+K}$, the model is estimated for $t = P + K + 1, \ldots, T$.} To initiate iteration (18), we use the starting value $\hat{\xi}_{P+K+1|P+K}$ which is the unconditional probability, that is, we are considering that the Markov chain is stationary and ergodic.\footnote{For more details see Hamilton (1994).}

Finally, the log-likelihood function is given by the expression:

$$
L(\theta) = \sum_{t=P+K+1}^{T} \ln f(R_t | R_{t-1}, \ldots, R_1, \theta),
$$

(19)

where the term on the right side of the above equation is given by: $f(R_t | R_{t-1}, \ldots, R_1, \theta) = 1' (\hat{\xi}_{t|t-1} \odot \eta_t)$. The specifications DDMS-1, DDMS-2, and DDMS-EGARCH are also estimated in the same way as the DDMS-ARCH model. It is important to highlight that the value for $\tau$ was calibrated using grid search.
from [5, 25] with the log-likelihood values as the criterion as advocated by Maheu
and McCurdy (2000).

3. Empirical Results

This section begins with an introduction regarding the database used in the empiri-
cal analysis and then presents the estimation results. A comparison between the
smoothed probabilities obtained with different regime-switching models and differ-
ent rules-based specifications will also be presented. An economic evaluation using
the model out-of-sample forecasts is explored in Section 4.

3.1 Data

The empirical analysis involves the weekly serie of the Ibovespa. The period of
analysis runs from July 1994 to December 2014, which yields 1,069 observations. 6
The choice of using weekly data takes into consideration a trade-off between the
number of observations available for the analysis and the presence of noise regarding
the bull and bear states. As a bull market (bear market) is a prolonged period of
increasing (decreasing) stock prices, the use of daily data ultimately confuses any
detection of these longer periods with a shorter sequence of rises (falls). For this
reason, Maheu and McCurdy (2000) use monthly data. However, the use of monthly
data for the Brazilian market would considerably reduce the number of observations
both for the in-sample and out-of-sample analysis, respectively.

Table 1 shows some descriptive statistics for the data. It is noted that the kur-
tosis of Ibovespa return is greater than the value of the normal distribution and
its asymmetry is negative, i.e., negative returns tend to occur more frequently. The
non-normality of the serie is shown in the Jarque-Bera test, which rejects the null
hypothesis of normal returns. 7

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6 The Ibovespa series, \( P_t \), was converted in terms of log-returns \( R_t = 100 \times (\ln P_t - \ln P_{t-1}) \),
where the first observation is given by \( R_2 \).

7 The results presented in this section were also calculated with the Ibovespa return relative to
the CDI, however, did not show major differences.
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Table 1
Summary Statistics for Returns

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. of obs.</td>
<td>1069</td>
</tr>
<tr>
<td>Mean</td>
<td>0.232</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>4.542</td>
</tr>
<tr>
<td>Median</td>
<td>0.495</td>
</tr>
<tr>
<td>Asymmetry</td>
<td>−0.537</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>6.216</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>512.07</td>
</tr>
</tbody>
</table>

Note: This table presents descriptive statistics for the log-returns of the Bovespa index. The database is weekly and the sample period runs from July 1994 to December 2014, totaling 1,069 observations. The average return, median, and standard deviation are expressed in %.

3.2 Estimated parameters

First, an autoregressive model was estimated and the BIC criterion used to determine the number of lags. The model that best adjusts to the data was an AR(2) and based on this specification, the duration-dependent Markov-switching models were implemented.

The results in Table 2 show that the estimates of the mean and variance in the AR(2)-DDMS-2 and AR(2)-DDMS-ARCH(5) are statistically significant and different from zero. In the bull market state a positive mean and low variance are noted and in the bear market state the variance is high and the mean is negative. These results are in line with the empirical evidence already observed by other authors (Maheu et al., 2012, Nyberg, 2013, among others). Intuitively, in the bull markets prices mostly follow positive trends and this is reflected in the positive mean of the log-returns. Similarly, in the bear markets, the negative mean portrays the tendency of price falls. The difference in the volatility represents the uncertainty of the predictive ability of the models in each market state. It is expected that in a bear market volatility is higher than in a bull market. The perception that stock prices tend to fall more quickly than they rise is a stylized fact that is well documented in the stock market (see, for example Ait-Sahalia et al., 2013, Nelson, 1991, Engle and Ng, 1993). Usually one of the main explanations for this phenomenon is the
leverage effect (Christie, 1982, French et al., 1987, Guidolin and Timmermann, 2008, among others) and the ripple effect on prices: an initial sell reaction based on some negative news is followed by more selling, bringing prices down more quickly to their fundamental levels (DeLong et al., 1990). Thus, a big and sudden change in spread is more likely to occur during market crises and not during markets with positive or stable trends.

The estimates of the mean parameter of the conditional volatility in each market state in the AR(2)-DDMS-ARCH(5) show a drastic difference. The mean of the conditional volatility in the bear market is almost ten times greater than the value obtained in the bull markets, 63.239 and 7.604, respectively, which may be an indication of misspecification. In this model, the autoregressive parameters of the conditional variance are within the unitary circle: $\sum_{i=1}^{K=5} \alpha_i < 0.34$. Modeling the conditional mean, the parameter estimates $\phi_1$ and $\phi_2$ are significant and similar to the AR(2) model. The estimated values for the parameters $\alpha_i$, $i = 1, \ldots, 5$ indicate a positive relationship between the volatility of the current week and previous ones. The estimation results show that the mean return is positive during bull market periods ($\mu_1 = 0.39\%$, weekly) and negative during bear market periods ($\mu_0 = -2.65\%$, weekly). In both cases the values are 5\% statistically significant. In this model, the value of the log-likelihood is the highest compared to all the others.

The AR(2)-DDMS-EGARCH(1,1,1) model takes the asymmetry in the volatility caused by negative and positive returns. It also captures the difference in the level of the conditional volatility, 2.611 and 1.250, respectively. The parameter estimate $\zeta_1$ indicated a negative value that is statistically different from zero (-0.178), which means that the model captured the perception that positive and negative shocks tend to have different impacts on volatility. The regime-switching in the unconditional mean also captured negative and positive values; however, despite these results the mean for the bear market is not significant at a 90\% confidence level. Therefore, particularly in this specification, bear markets are characterized by null returns.

As mentioned in Section 2.1, the duration effect is captured by the parameters $\gamma_2(i), i = 0, 1$. The parameter estimates $\gamma_2(0)$ and $\gamma_2(1)$ for the AR(2)-DDMS-2, AR(2)-DDMS-ARCH(5), and AR(2)-DDMS-EGARCH(1,1,1) are positive and statistically significant, which indicates that the probability of staying in a particular
Table 2
Results of the Regime-Switching Models

<table>
<thead>
<tr>
<th>Parameters</th>
<th>AR(2)</th>
<th>AR(2)-ARCH(5)</th>
<th>AR(2)-EGARCH(1,1)</th>
<th>AR(2)-MS-2</th>
<th>AR(2)-DDMS-2</th>
<th>AR(2)-MS-ARCH(5)</th>
<th>AR(2)-DDMS-ARCH(5)</th>
<th>AR(2)-MS-EGARCH(1,1)</th>
<th>AR(2)-DDMS-EGARCH(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu, \mu )</td>
<td>0.234***</td>
<td>0.209***</td>
<td>0.219***</td>
<td>-13.09***</td>
<td>-0.854**</td>
<td>-0.878**</td>
<td>-2.486**</td>
<td>-2.655***</td>
<td>-0.628</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.594***</td>
<td>0.437***</td>
<td>0.443***</td>
<td>0.479***</td>
<td>0.393***</td>
<td>0.386***</td>
<td>0.421***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma, m )</td>
<td>4.524***</td>
<td>8.296***</td>
<td>0.118***</td>
<td>3.950***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( m, \sigma )</td>
<td>8.064***</td>
<td>8.278***</td>
<td>58.93***</td>
<td>63.239***</td>
<td>2.710***</td>
<td>2.611***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma(0) )</td>
<td>-1.380***</td>
<td>2.880***</td>
<td>-1.106*</td>
<td>2.207***</td>
<td>-0.968</td>
<td>3.032***</td>
<td>-1.074**</td>
<td></td>
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</tr>
<tr>
<td>( \gamma(1) )</td>
<td>0.675</td>
<td>4.568***</td>
<td>-1.090*</td>
<td>4.282***</td>
<td>-0.731</td>
<td>4.275***</td>
<td>-0.489</td>
<td></td>
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</tr>
<tr>
<td>( \gamma(0) )</td>
<td>0.242</td>
<td>0.623***</td>
<td>0.482***</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>( \gamma(1) )</td>
<td>0.615</td>
<td>0.885***</td>
<td>0.590***</td>
<td>0.462***</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>( \phi_1 )</td>
<td>-0.053**</td>
<td>-0.029</td>
<td>-0.031</td>
<td>-0.076**</td>
<td>-0.098**</td>
<td>-0.066**</td>
<td>-0.076***</td>
<td>-0.065**</td>
<td>-0.041*</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>0.073***</td>
<td>0.100*</td>
<td>0.069**</td>
<td>0.072**</td>
<td>0.066**</td>
<td>0.069***</td>
<td>0.059**</td>
<td>0.059**</td>
<td>0.062**</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>0.088***</td>
<td>0.204***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_2 )</td>
<td>0.037**</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_3 )</td>
<td>0.155***</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>( \sigma_4 )</td>
<td>0.230***</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_5 )</td>
<td>0.105**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.960***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \zeta )</td>
<td>-0.063***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \tau \) = 0.31246, -3031.0, -3041.3, -3055.5, -3014.8, -3012.2, -2984.6, -2981.7, -2999.0, -2993.5

Note: This table presents the estimated parameters for the Markov-switching models with and without duration dependence for the log-returns of the Bovespa index referring to the period from July 1994 to December 2014, making up a total of 1,069 weekly observations. In \( L \) is the value of the log-likelihood. ***, **, * indicate 1%, 5% and 10% significance, respectively.
state increases with the realization of the state. This means that the probability of the Ibovespa remaining in the bull and bear states increases with the number of periods in these states. Panel (a) in Figure (1) shows the results for the AR(2)-DDMS-2 model. The state probability when $d = 1$ is 0.38 for bear markets (dotted line) and 0.44 for bull markets (continuous line). Therefore, in cases in which the duration of the bull or bear market is short, the probability of switching out of the regimes is greater. After the point where the duration extrapolates the limit, $d > \tau$, the probability of remaining in the current state is time independent and constant. In these cases, the probabilities of bull and bear markets are 0.99 and 0.93, respectively. Panel (b) shows the same idea for the AR(2)-DDMS-ARCH(5) model. Unlike the previous model where $\tau = 6$, in this model $\tau = 7$ and the probability of bear market (dotted line) is 0.38 when $d = 1$ and 0.62 for bull markets (continuous line). Modeling the conditional variance led to a greater probability for bull markets. Finally, Panel (c) shows the results of the AR(2)-DDMS-EGARCH(1,1,1) model. In this case we have $\tau = 9$ and the probability of bear market (dotted line) is 0.34 when $d = 1$ and 0.49 for bull market (continuous line). In the case, $d > \tau$, the probability of bear market is 0.94 and 0.97 for bull market.

The different approaches considered lead to different characteristics for the moments of the returns distributions in the bull and bear markets. Table 3 presents the mean and volatility of the two states for the different models considered. In the case of the rules-based approach (LT) two sets of values for the parameters $(\lambda_1, \lambda_2)$ are considered, where the mean is quite different in either state. Considering $\lambda_1 = 20\%$ and $\lambda_2 = 15\%$, the mean of the weekly return during the bull market is 1.19%, and -1.21% for the bear market periods. The volatilities are lower in bull markets than in bear markets, 4.13% and 4.72%, respectively. One of the explanations for the low difference in the standard deviations between these states is that the rules-based approach is based only on the first moment of the price series and does not take care of the conditional variance, i.e, the rules-based approach ignores volatility. Considering the regime-switching models, it is noted that the bull market periods exhibit higher averages and lower volatilities compared with the bear market periods. The volatility difference between states is much more pronounced for the regime-switching models (3.39% for bull compared with 9.75% for bear) than the
rules-based approach, since regime-switching models make inferences based on the two moments, mean and volatility of the returns (Kole and van Dijk, 2013).

Among the regime-switching models, expressive differences are noted between the mean in the two regimes. For example, the model with two states without duration dependence and without conditional heteroskedasticity, the mean value is 0.44% and -0.85% in the bull and bear market states. Maheu and McCurdy (2000) proposed the use of the DDMS model with ARCH to capture conditional heteroskedasticity. In the AR(2)-DDMS-ARCH(5), the values for the mean are 0.39% and -2.66%. For the model with asymmetric volatility, AR(2)-DDMS-EGARCH(1,1,1), the mean value is 0.42% and -0.76%, respectively.
### Table 3
Statistics for Returns in Bull and Bear Markets

<table>
<thead>
<tr>
<th>Regime</th>
<th>LT (20,15)</th>
<th>LT (15,15)</th>
<th>AR(2)-MS-2</th>
<th>AR(2)-DDMS-2</th>
<th>AR(2)-MS-ARCH(5)</th>
<th>AR(2)-DDMS-ARCH(5)</th>
<th>AR(2)-MS-EGARCH(1,1,1)</th>
<th>AR(2)-DDMS-EGARCH(1,1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.19</td>
<td>1.09</td>
<td>0.44</td>
<td>0.44</td>
<td>0.48</td>
<td>0.39</td>
<td>0.38</td>
<td>0.42</td>
</tr>
<tr>
<td>Bull</td>
<td>(0.03)</td>
<td>(0.15)</td>
<td>(0.12)</td>
<td>(0.12)</td>
<td>(0.12)</td>
<td>(0.12)</td>
<td>(0.13)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.13</td>
<td>4.12</td>
<td>3.44</td>
<td>3.36</td>
<td>3.48*</td>
<td>3.39*</td>
<td>3.12*</td>
<td>2.89*</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.11)</td>
<td>(0.09)</td>
<td>(0.11)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>−1.21</td>
<td>−1.59</td>
<td>−0.85</td>
<td>−0.88</td>
<td>−2.49</td>
<td>−2.66</td>
<td>−0.62</td>
<td>−0.76</td>
</tr>
<tr>
<td>Bear</td>
<td>(0.05)</td>
<td>(0.26)</td>
<td>(0.69)</td>
<td>(0.71)</td>
<td>(1.09)</td>
<td>(1.54)</td>
<td>(0.65)</td>
<td>(0.72)</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.72</td>
<td>4.82</td>
<td>8.06</td>
<td>8.28</td>
<td>9.34*</td>
<td>9.75*</td>
<td>8.04*</td>
<td>9.14*</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.18)</td>
<td>(0.54)</td>
<td>(0.63)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table presents the mean and volatility (in % per week) for the different regimes and for the two approaches considered. In the LT approach the identification is based on the peaks and troughs in the price series. In the regime-switching approach, models are estimated with two regimes with and without duration dependence, AR(2)-MS-2 and AR(2)-DDMS-2, respectively. Models are also estimated with two heteroskedastic regimes, AR(2)-MS-ARCH(5), AR(2)-DDMS-ARCH(5), AR(2)-MS-EGARCH(1,1,1), and AR(2)-DDMS-EGARCH(1,1,1). The standard errors are presented in brackets.

*Estimate of unconditional standard deviation.
3.2.1 Residual diagnostics

Due to the latent variable $S_t$, the residuals of the regime-switching models are unobservable. Using the equation below, the Ljung-Box portmanteau test can be applied through the standardized expected residuals. According to Maheu and McCurdy (2000), we have the following expression for the models with conditional volatility (DDMS-ARCH and DDMS-EGARCH):

$$\sum_{S_t, \ldots, S_{t-P}, d}^{R_t - E[R_t|S_t, \ldots, S_{t-P}, d, R_{t-1}, \ldots, R_1]} \frac{\hat{\rho}_j}{\sqrt{h_t(S_t)}} \times P(S_t, \ldots, S_{t-P}, d|R_{t-1}, \ldots, R_1),$$

(20)

where the conditional standard deviation is given by $\sqrt{h_t(S_t)}$, such that:

$$h_t(S_t) = \sum_{S_t, \ldots, S_{t-P}, d} h_t(S_t) P(S_t, \ldots, S_{t-P}, d|Y_T),$$

(21)

the last term to the right of the equation (21) is the smoothed probability. For the models without conditional volatility, DDMS-1 and DDMS-2, $\sqrt{h_t(S_t)}$ in equation (20) is replaced by $\sigma$ and $\sigma(S_t)$, respectively.

The Ljung-Box test statistic for the autocorrelation in the residuals with lag $h$ is given by:

$$Q = n(n + 2) \sum_{j=1}^{h} \frac{\hat{\rho}_j^2}{n - j},$$

(22)

where $\hat{\rho}_j, j = 1, 2, \ldots, h$ is the autocorrelation of the residuals and $n$ is the sample size of the residuals. Under the null hypothesis of no autocorrelation, this statistic follows the chi-square distribution with $h$ degrees of freedom. The statistic test for the DDMS models will be carried out both for the residuals $Q(8)$ and for the squared residuals $Q^2(8)$, respectively. The idea of the test is to verify if they are random or if there are some relationship, where in this case the autocorrelation will be $h = 8$.

The results in Table 4 show that the AR(2)-DDMS-1 model presented a significant persistence for its residuals and evidences of the ARCH effect rejecting the null hypothesis of no autocorrelation. In the case of the AR(2)-DDMS-2 model, there was a surpassing of autocorrelation in the structure of the conditional mean; however, for the squared residuals this result did not remain. The results found show that

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8 For more details see next section.
the best adjustments were obtained with the AR(2)-DDMS-ARCH(5) and AR(2)-DDMS-EGARCH(1,1,1). The hypothesis of no autocorrelation cannot be rejected for the residual nor for the squared residuals, suggesting that conditional variance models present a better adjustment to the data, i.e., these models were able to filter the non-linear structures of the mean and variance.

Table 4
Residuals Test

<table>
<thead>
<tr>
<th>Parameters</th>
<th>AR(2)-DDMS-1</th>
<th>AR(2)-DDMS-2</th>
<th>AR(2)-DDMS-ARCH(5)</th>
<th>AR(2)-DDMS-EGARCH(1,1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q(8)$</td>
<td>39.250***</td>
<td>6.927</td>
<td>6.090</td>
<td>6.132</td>
</tr>
<tr>
<td>$Q^2(8)$</td>
<td>212.477***</td>
<td>22.850***</td>
<td>10.362</td>
<td>7.629</td>
</tr>
</tbody>
</table>

***, **, * reject the null hypothesis of no autocorrelation at the 1%, 5% e 10% levels, respectively.

3.3 Smoothed Probabilities and Turning Points

The different approaches lead to the identification of different characteristics of the bull and bear markets periods, both for mean and variance of returns and for states persistence. For a better understanding of these differences the smoothed probabilities are calculated, which are based on an information set of the returns. In the case of the Markov-switching models the identification and estimation of the model parameters are carried out in a single stage. As for the rules-based approach, the identification is the first step, and the previously discussed characteristics are obtained in a second stage. The identification is important to calculate the number of bull and bear market periods and the average of the duration of these periods.

The smoothed probability associated with $S_t$ in the DDMS-ARCH model was estimated using the Kim algorithm described in Hamilton (1994). $\hat{\xi}_{T|T}$, $\hat{\xi}_{T-1|T-1}$, and $\hat{\xi}_{T|T-1}$ are obtained using the maximum likelihood estimator. $\hat{\xi}_{T-1|T}$ is obtained by:

$$\hat{\xi}_{T-1|T} = \hat{\xi}_{T-1|T-1} \odot \left\{ P' \left[ \hat{\xi}_{T|T}(\div)\hat{\xi}_{T|T-1} \right] \right\},$$  

(23)

where $($ ÷ $)$ means division of element-by-element and $P$ was defined in equation (17).

Taking $t = T - 2, \ldots, K + P + 1$ and the vectors $\hat{\xi}_{t|t}$ and $\hat{\xi}_{t+1|t}$ from the maximum likelihood estimation, such that $\hat{\xi}_{t+1|T}$ comes from the past iteration, the smoothed probability $\hat{\xi}_{t|T}$ is given by:

$$\hat{\xi}_{t|T} = \hat{\xi}_{t|t} \odot \left\{ P' \left[ \hat{\xi}_{t+1|T}(\div)\hat{\xi}_{t+1|t} \right] \right\}.$$

(24)
Evidences of Bull and Bear Markets in the Bovespa index: An application of Markovian regime-switching models with duration dependence

The $j$th element in $\hat{\xi}_t|T$ is the probability of $\Upsilon_t = j$ under the information $R_T, \ldots, R_1$ and $\theta$. For example, in the DDMS-ARCH, this is represented as $P(\Upsilon_t = j|R_T, \ldots, R_1, \theta)$ or $P(S_t, S_{t-1}, \ldots, S_{t-K-P}, D(S_t)|R_T, \ldots, R_1, \theta)$. In this case, the smoothed probability of the market states, $S_t = i$ for $i = 0, 1$, i.e., bull and bear markets is given by summing the states in each period, such that:

$$P(S_t|R_T, \ldots, R_1, \theta) = \sum_{s_{t-1}, \ldots, s_{t-k-P}, D(s_t)} P(S_t, s_{t-1}, \ldots, s_{t-K-P}, D(s_t)|R_T, \ldots, R_1, \theta).$$

(25)

The procedure above is similar to the others DDMS specifications. Figure (2) and Figure (3) show the smoothed probabilities of these models (black lines) and the LT approach (gray bars). The rules-based approach produces a binary series indicating during which weeks the market is in a bull or bear state. Even in the case of the Markov-switching models it is noted that most of the time the probability is close to zero or one, which indicates that the two regimes are quite distinct and that the approach is able to identify the current state. Most weeks, identification using LT or the DDMS models indicates the same thing. However, in some periods the results are different. Periods of low volatility in which prices exhibit a falling trend can be identified as bull by the DDMS models, for example, the periods from March to August of 2010 (average weekly return of -0.1% and volatility of 2.92%) and from February to August of 2013 (average weekly return on -0.49% and volatility of 2.83%). It can also happen that in periods of high volatility but with short price changes are classified as bear markets.

The beginning of the sample, the second half of the 1990s, is a period characterized by several financial turbulences in the international financial markets, especially in the emerging economies. The black line in Figure (2) and Figure (3) shows that the probability of bear market in August 1994 is greater than half, indicating the beginning of the Mexican crisis sparked in December of 1994. According to the news, between 1995 and 1996 the average daily volume traded on the stock exchange did not surpass an annual average of 100 million dollars compared with 400 million dollars in 1994. The probability of a bear market in the following years, 1997 and 1999, also indicated that the models capture the historic lows during the Asian and Russian crisis. The period was marked by various circuit breaker occurrences due to the investor pessimism. The effects of the Nasdaq bubble bursting and the con-
sequences of the twin towers terrorist attack are portrayed by both specifications. Between March and April of 2000 the Bovespa index fell 17%, annulling the valuation that had been built since the Russian crisis. A few days after the September 11th terrorist attack, the index reached its lowest level since August of 1999 (10,038 points).

Widely known by the market as the ”Lula effect” and ”Tango effect”, 2002 was marked by a strong devaluation of stock market assets. The electoral uncertainties and rumors of default in Argentina contributed to the 65% fall between January and October of that year. Figures (2) and (3) show that the probability of bear market in this period is high and extends until the beginning of 2003. The turning

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The general rule for using the circuit-breaker is when the Ibovespa reaches that limit of a 10% fall in relation to the close of the previous day.
Evidences of Bull and Bear Markets in the Bovespa index: An application of Markovian regime-switching models with duration dependence

Figure 3
Bear Market Identification

(a) LT: $\lambda_1 = 0.20$, $\lambda_2 = 0.15 \times \text{AR(2)}$-DDMS-EGARCH(1,1,1)

(b) LT: $\lambda_1 = 0.20$, $\lambda_2 = 0.10 \times \text{AR(2)}$-DDMS-EGARCH(1,1,1)

(c) LT: $\lambda_1 = 0.15$, $\lambda_2 = 0.15 \times \text{AR(2)}$-DDMS-EGARCH(1,1,1)

(d) LT: $\lambda_1 = 0.15$, $\lambda_2 = 0.10 \times \text{AR(2)}$-DDMS-EGARCH(1,1,1)

Note: The figure shows different bear market periods in the Bovespa index from July 1994 until the end of 2014. The smoothed probabilities of the regime-switching models with duration dependence (DDMS) is represented by the black-colored line (y-axis, left). The analysis within the model was also carried out using the non-parametric (rules-based) method proposed by Lunde and Timmermann (2004). For more information regarding this algorithm see the appendix. In this case, the choice of the values of $\lambda_1$ and $\lambda_2$ was based on Maheu et al. (2012). The performance of these algorithms is represented by the area shaded in grey. Finally, the dark blue colored line (y-axis, right) is the Bovespa index in the period.

Point analysis also indicated the excessive optimism that preceded the American real estate crisis, i.e., a strong valuation of the index from 2003 until the end of 2007. Despite the increase in volatility in some specific periods (April 2004 and July 2007), the Bovespa tripled the average volume traded and there was a large number of initial public offerings (IPOs) in this period. In April 2007 the Brazilian stock market reached the historic mark of 50,000 points, representing a valuation of more than 1000% since the Real Plan, when the index varied close to 4,000 points. The following year, the expressive valuation gained even more force and reached the 73,589 point mark in May 2008. However, the gradual increase of the chance of reversal of the situation known up to then as "brigadeiro (chocolate cake) sky" is shown at the beginning of the second semester of 2008. It is interesting to highlight that the model captured the deterioration of the index weeks before
reaching the historic falls in the month of October 2008. The last significant turning point in the bull and bear market analysis is shown by the high volatility and negative return in August 2011. This is due to the negative reaction of the market after the announcement of the downgrade in American sovereign bonds. Trading floor operations were close to being suspended and the index recorded the biggest fall since October 2008. As seen in the figures above, in this period the smoothed probabilities were very close to half.

4. Investment strategies

Investors can use the predictions of the different models and approaches in various ways to construct investment strategies. We apply the predictions from the regime-switching model to make portfolio decisions. This is based on Ibovespa futures, which uses the São Paulo Stock Exchange index (Ibovespa) as an underlying asset and is listed by points with each point equaling a value in Brazilian Reals established by BM&F (for details on Ibovespa futures operations, see, Baptista and Pereira, 2008). Ibovespa futures present a high degree of liquidity as well as enabling short selling transactions to be carried out. Futures have a three month expiry data, always on the closest Wednesday to the 15th calendar day of even months (February, April, June, October, and December), using the Ibovespa value on the expiry data as the underlying asset. The series used was obtained from Bloomberg and it is constructed from the prices of the most traded contracts at a given moment (for more details on Ibovespa futures strategies, see Saffi, 2003, Baptista and Pereira, 2008).

The simplest strategy involves taking binary decisions. If the model predicts a bull market, the investor assumes a long position for the Bovespa index future, if the prediction is for a bear market, he/she invests the whole amount in the risk free asset (considered here as the CDI). In this case three asset allocation strategies are constructed and the performances are compared with the passive buy strategy for the index known as buy and hold (Resnick and Shoesmith, 2002, Fernandez-Perez et al., 2014). Each strategy is started with a $1 investment on January 2005 and the results are reinvested until the end of the period of the sample size i.e., December 2014. Although the rules-based approach has been used to identify market states, it cannot be used for making investment decisions in real time. In this approach,
identification of the turning points between market states only takes place various periods after they occur; that is, it is not even possible making inference about the current market state, nor regarding future regimes. As for the regime-switching models, these enable the probability of the market being in a particular state at the moment to be calculated (filtered probability), as well as calculating the probability of the market being in a particular state in the following period (predictive probability). Thus, these models can be used to construct investment strategies that exploit this knowledge, since it is possible to make assumptions about the probability of the market being bear or bull in the following period.

The first strategy consists of buying Ibovespa futures\textsuperscript{10} if the probability of a bear market 1 step ahead is lower than the threshold ($\lambda$), or investing in the fixed income asset if the probability of a bear market is greater than $\lambda$. In this strategy the positions are assumed in each period and maintained for a month.

- If $\Pr(\widehat{R}_{t+1} = 1) \geq \lambda \leftrightarrow$ invest in the risk free asset.
- If $\Pr(\widehat{R}_{t+1} = 1) < \lambda \leftrightarrow$ invest in the Ibovespa.

where $\Pr(\widehat{R}_{t+1} = 1)$ is the probability of a bear market predicted 1-step ahead. The strategy simulations are implemented with three probability limits, $\lambda = 30\%$, 40\% e 50\%, respectively. Specifically, if the predictive probability is lower than the threshold, it is accepted that the market is in a bull state and a long position is assumed in Ibovespa futures. Therefore, the thresholds, $\lambda$’s, can be thought of as a spectrum representing measures of the risk aversion.

Although the above strategy is simple, it ignores the predictive powers of the models. For example, a predictive probability of a bear market of 0.95, $\Pr(\widehat{R}_{t+1} = 1) = 0.95$, is a stronger sign than a predictive probability of 0.70. As an alternative strategy, the investor could take into account the real predictive probability of a bear market in $t+1$, $\Pr(\widehat{R}_{t+1} = 1)$ assuming a position of $1 - \Pr(\widehat{R}_{t+1} = 1)$ in futures. A correct bull market prediction, $\Pr(\widehat{R}_{t+1} = 1) = 0$ results in total al-

\textsuperscript{10} The alternatives to using futures is implementing the strategy using the EFT BOVA11 or PIBB - Brazil Fund index - 50 - Brazil Tracker, an investment fund for shares that aim to reflect, as faithfully as possible, the performance of one of the main indices of references for the Brazilian stock market, the IBrX 50.
location in futures. If the investor is not so sure about the direction of the market, his/her investment is a fraction proportional to the probability. Another alternative strategy exploits the information on bear market predictions and assumes a short position in futures when the predictive probability of the market switching to a bear state is greater than one threshold ($\lambda_1 = 0.80$) and buy otherwise. Specifically, the leverage strategy known as 130/30, which is widely used by professional investors is employed (Gastineau, 2008, Lo and Patel, 2008). These strategies are known as long-short and are widely used by hedge funds (Fung and Hsieh, 2011, Candelon et al., 2014).

To take into account the transaction costs, each time a position is altered transaction costs are applied of 20 base points on futures trades (0.20%) and 10 base points (0.10%) for fixed income assets (Han et al., 2013). In the case of strategies with short position costs of 40 base points (0.40%) are considered. Finally, the buy and hold strategy is treated as the benchmark and the performance of the strategies is evaluated using the average annual return, the Sharpe ratio, and the final value related to the $1 investment for the whole period outside the sample (ten years).

In Table 5 the average annualized return, the Sharpe ratio, and the final value of the $1 investment for the 10 year period are presented for the three binary investment strategies and the passive buy and hold strategy in the sample period (January 2005 to December 2014). Investing $1 in the passive (buy and hold) strategy results in $1.85 at the end of the period and an annualized Sharpe ratio of 0.365. The performance of the regime-switching models alternated between slightly negative and significantly better than the benchmark. All of the models presented in Table 5 exceed the benchmark in terms of final value and Sharpe ratio, except for the AR-(2)-MS-ARCH(5) model when $\lambda = 0.30$ is used. In all cases the profitability of the strategy increases when the threshold changes from 0.30 to 0.40. The strategy with $\lambda = 0.50$ also consistently exceeds the passive strategy, although in a smaller magnitude than the strategy with $\lambda = 0.40$. When $\lambda = 0.40$ is used, the AR(2)-MS-ARCH(4) model resulted in a final value of $2.38$, which is almost 30% higher than that obtained with the benchmark. The active strategies based on the predictions of the regime-switching models consistently and convincingly exceed the passive
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strategy. Besides the possibility of being used in devising strategies, the predictions can also be used in risk management or in options strategies (Candelon et al., 2014).

The results indicate that active rebalancing based on the predictions generated by the regime-switching models outperforms the passive buy-and-hold strategy. Specifically, the predictions obtained from the AR(2)-DDMS-ARCH(4) and AR(2)-MS-ARCH(3) models present better results, with the $\lambda = 0.40$ probability limit generally leading to optimal results.

Table 5
Economic evaluation of the Bull and Bear Markets predictions

<table>
<thead>
<tr>
<th>Model</th>
<th>&lt; 30%</th>
<th>&lt; 40%</th>
<th>&lt; 50%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Av. ret</td>
<td>Sharpe</td>
<td>Final value</td>
</tr>
<tr>
<td>AR(2)-DDMS-ARCH(5)</td>
<td>9.60</td>
<td>0.419</td>
<td>1.93</td>
</tr>
<tr>
<td>AR(2)-DDMS-ARCH(4)</td>
<td>11.99</td>
<td>0.528</td>
<td><strong>2.40</strong></td>
</tr>
<tr>
<td>AR(2)-DDMS-ARCH(3)</td>
<td>10.81</td>
<td>0.480</td>
<td>2.17</td>
</tr>
<tr>
<td>AR(2)-MS-ARCH(5)</td>
<td>8.86</td>
<td>0.388</td>
<td>1.80</td>
</tr>
<tr>
<td>AR(2)-MS-ARCH(4)</td>
<td>9.34</td>
<td>0.418</td>
<td>1.91</td>
</tr>
<tr>
<td>AR(2)-MS-ARCH(3)</td>
<td>10.68</td>
<td>0.474</td>
<td>2.14</td>
</tr>
<tr>
<td>AR(2)-DDMS-2</td>
<td>10.04</td>
<td>0.447</td>
<td>2.03</td>
</tr>
<tr>
<td>AR(2)-MS-2</td>
<td>9.92</td>
<td>0.441</td>
<td>2.00</td>
</tr>
<tr>
<td>AR(2)-DDMS-EGARCH(1,1,1)</td>
<td>10.50</td>
<td>0.444</td>
<td>2.11</td>
</tr>
<tr>
<td>AR(2)-MS-EGARCH(1,1,1)</td>
<td>10.70</td>
<td>0.451</td>
<td>2.15</td>
</tr>
<tr>
<td><em>Buy &amp; Hold</em></td>
<td>10.11</td>
<td>0.365</td>
<td>1.85</td>
</tr>
</tbody>
</table>

Note: This table presents the results of the binary investment strategies based on the different methods for predicting bull and bear markets. The binary strategies are implemented with bear market probability limits of 30%, 40%, and 50%. The out-of-sample period runs from January 2005 to December 2014. For each strategy and each method the average annualized returns in %, Sharpe ratios, and final values of the $1 investment for the ten year period are presented.

Table 6 presents the results of the proportional and leveraged strategies. With the proportional strategy all of the models result in final values higher than that of the passive strategy and also present higher Sharpe ratios than the benchmark. The AR(2)-DDMS-EGARCH(1,1,1) stands out since it presented an average annualized return of 10.70% and a final value of $2.16. In the case of the leveraged strategy, the AR(2)-DDMS-ARCH(5) model generates a lower final value than the benchmark, the other models present a superior performance, especially the AR(2)-MS-ARCH(4) model. It is noted that with the proportional investment strategy there is less variability in the results between the models. Comparing the different strategies it is noted that the binary strategy with $\lambda = 0.40$ generates the best results. Not even the proportional strategy exceeds the binary strategies. This indicates that the predictive accuracy is limited. Finally, the results show that the regime-switching
models can be useful tools in predicting different investment strategies. The results also have implications for risk management and hedging operations.

Table 6
Economic Evaluation of the different investment strategies

<table>
<thead>
<tr>
<th>Model</th>
<th>Proportional Strategy</th>
<th>Leveraged Strategy (130/30)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Av. ret</td>
<td>Sharpe</td>
</tr>
<tr>
<td>AR(2)-DDMS-ARCH(5)</td>
<td>9.18</td>
<td>0.409</td>
</tr>
<tr>
<td>AR(2)-DDMS-ARCH(4)</td>
<td>9.20</td>
<td>0.416</td>
</tr>
<tr>
<td>AR(2)-DDMS-ARCH(3)</td>
<td>8.94</td>
<td>0.406</td>
</tr>
<tr>
<td>AR(2)-MS-ARCH(5)</td>
<td>9.07</td>
<td>0.412</td>
</tr>
<tr>
<td>AR(2)-MS-ARCH(4)</td>
<td>9.25</td>
<td>0.425</td>
</tr>
<tr>
<td>AR(2)-MS-ARCH(3)</td>
<td>9.85</td>
<td>0.462</td>
</tr>
<tr>
<td>AR(2)-DDMS-2</td>
<td>8.89</td>
<td>0.409</td>
</tr>
<tr>
<td>AR(2)-MS-2</td>
<td>9.29</td>
<td>0.429</td>
</tr>
<tr>
<td>AR(2)-DDMS-EGARCH(1,1,1)</td>
<td>10.70</td>
<td>0.453</td>
</tr>
<tr>
<td>AR(2)-MS-EGARCH(1,1,1)</td>
<td>10.40</td>
<td>0.440</td>
</tr>
<tr>
<td>Buy &amp; Hold</td>
<td>10.11</td>
<td>0.365</td>
</tr>
</tbody>
</table>

Note: This table shows the results of the different investment strategies based on the different methods for predicting bull and bear markets. The proportional strategy assumes a weight of $1 - \Pr(\hat{R}_{t+1} = 1)$ in futures and $\Pr(\hat{R}_{t+1} = 1)$ in the risk free asset. The leveraged strategy is of the 130/30 type, which involves the sale of contracts (30% of the portfolio) if the probability of a bear market is greater than 0.8 and allocates funds in the risk free asset. The out-of-sample period runs from January 2005 to December 2014. For each strategy and method, the average annualized returns, Sharpe ratios, and final $1 investment values for the 10 year period are presented.

5. Conclusion

This article compares the identification and prediction of bull and bear market periods in the Ibovespa based on the Markov-switching models with duration dependence and using rules-based nonparametric methods. These two approaches were able to identify the bull market periods, characterized by a positive mean return and low volatility, and the bear market periods, characterized by a negative mean return and high volatility. These results are similar to those found by Maheu & McCurdy (2000) and Shibata (2011). Unlike the traditional Markov-switching model, the parametrization with duration dependence improves the modeling of the long term dependence on the cyclical pattern of the stock retns. The model captured the duration effect of a bull market, which is associated with more optimistic investors in relation to the future. This positive response means that the probability of staying in the bull market increases as duration increases. Similarly, the duration of a bear market could persist while the investors pessimism about the future con-
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continues. From an identification point of view, it is noted in the case of the rules-based approach that if there is a downturn in prices from the last peak, this is called as bear market. If prices are rising after a trough it will be identified as a bull market. This is different to Markov-switching models that take into account the conditional distribution of returns (mean and variance). In particular cases, some periods can be identified as bull market with low volatility and falling prices. In general, the identification using the two approaches is quite similar.

We need to pay attention when specifying the structure of the financial data only using a first order Markov process, which is related to the low memory of the parameters. The DDMS specification with conditional mean and variance allows a set of intra-state dynamics, improving the smoothed probabilities. The DDMS-ARCH and the DDMS-EGARCH show better in-sample performance. The smoothed probabilities analysis showed that the models captured the main periods of instability in the Brazilian market and characterized the dynamic of the conditional volatility in both market states.

The elaboration of investment strategies showed the applicability of these models compared to a passive strategy. The results of the binary allocation for different thresholds were mostly superior to the benchmark model compared by the final value, in this case the rebalancing based on the predictive probability generates better performance, especially for the AR(2)-DDMS-ARCH(4) and AR(2)-MS-ARCH(3) specifications for the threshold value, 0.30 and 0.50, respectively. By taking the predictive power of the models into account, the proportional strategy presented a higher Sharpe ratio than buy and hold, and similarly, a better result compared by final value. In the case of the leverage, the results are also similar, except the slightly lower result of the AR(2)-DDMS-ARCH(5) specification. By comparing the different strategies, the binary application generated the best overall result by taking a threshold of 0.40 into account.

Acknowledgements

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A. Appendix

A.1 Bull and Bear Markets Dating Algorithms

There are alternative approaches for classifying market states, such as the common ones being the algorithm proposed by Pagan and Sossounov (2003), which is an adaptation of the business cycle dating algorithm from Bry and Boschan (1971). Another approach for classifying stock markets into bull and bear states is the non-parametric methodology proposed by Lunde and Timmermann (2004).

The algorithm of Lunde and Timmermann (2004) defines a binary indicator variable, \( I_t \), which assumes value 1 if the stock market state is bull at time \( t \) and zero if the market state is bear. According to Lunde and Timmermann (2004) and Maheu et al. (2012), it is assumed that the time is measured on a discrete scale and the stock market index (or the price of a specific asset) at the end of period \( t \) is denoted by \( P_t \). Let \( \lambda_1 \) a scalar defining the threshold that defines a change from a bear market to a bull market and \( \lambda_2 \) the threshold for changing from a bull market to a bear market. The application of Lunde and Timmermann (2004) implemented here follows Maheu et al. (2012) and uses an initial 6-month window to establish a local maximum or local minimum point.

Suppose at time \( t_0 \), we have a local maximum \(( I_{t_0} = 1)\), in this case \( P_{t_0} = P_{t_0}^{\text{max}} \). The stopping time variables associated with a bull market are defined as:

\[
\tau_{\text{max}}(P_{t_0}^{\text{max}}, t_0 | I_{t_0} = 1) = \inf \{t_0 + \tau : P_{t_0 + \tau} \geq P_{t_0}^{\text{max}}\} \quad (A.1)
\]

\[
\tau_{\text{min}}(P_{t_0}^{\text{max}}, t_0, \lambda_2 | I_{t_0} = 1) = \inf \{t_0 + \tau : P_{t_0 + \tau} \leq (1 - \lambda_2)P_{t_0}^{\text{max}}\} \quad (A.2)
\]

where \( \tau \geq 1 \). Thus, \( \min\{\tau_{\text{max}}, \tau_{\text{min}}\} \) is the first time that the process that controls the prices changes state: \( \{P_{t_0}^{\text{max}}, (1 - \lambda_2)P_{t_0}^{\text{max}}\} \). One of the following situations occurs:

- If \( \tau_{\text{max}} < \tau_{\text{min}} \), the bull market continues, the new peak value is updated to the value of the local maximum for:

\[
P_{t_0 + \tau_{\text{max}}} = P_{t_0 + \tau_{\text{max}}}.\]

\[11\] For applications of the method proposed by Lunde and Timmermann (2004) see, for example, Chiang et al. (2009), Maheu et al. (2012) and Kole and van Dijk (2013).
and the market remains in the bull state between \( t_0 + 1 \) and \( t_0 + \tau_{\text{max}} \) and set \( I_{t_0+1} = \ldots = I_{t_0+\tau_{\text{max}}} = 1 \). Update \( t_0 = t_0 + \tau_{\text{max}} \) still as a local maximum and continues with equations (26) and (27).

- If \( \tau_{\text{max}} > \tau_{\text{min}} \) a local minimum is found at \( t_0 + \tau_{\text{min}} \) and there is a bear market from \( t_0 + 1 \) to \( t_0 + \tau_{\text{min}} \). Set \( I_{t_0+1} = \ldots = I_{t_0+\tau_{\text{min}}} = 0 \). Record the value:

\[
P_{t_0+\tau_{\text{min}}}^{\text{min}} = P_{t_0+\tau_{\text{min}}},
\]

and update \( t_0 = t_0 + \tau_{\text{min}} \) as a local minimum. Given that now \( t_0 \) is a local minimum, leads to equations (28) and (29) below.

On the other hand, when \( t_0 \) is a local minimum. Bear market stopping times are:

\[
\tau_{\text{min}} \left( P_{t_0}^{\text{min}} \left| t_0, I_{t_0} = 0 \right. \right) = \inf \{ t_0 + \tau : P_{t_0+\tau} \leq P_{t_0}^{\text{min}} \} \tag{A.3}
\]

\[
\tau_{\text{max}} \left( P_{t_0}^{\text{min}} \left| t_0, \lambda_1, I_{t_0} = 0 \right. \right) = \inf \{ t_0 + \tau : P_{t_0+\tau} \geq (1 + \lambda_1) P_{t_0}^{\text{min}} \} \tag{A.4}
\]

One of the following happens:

- If \( \tau_{\text{min}} < \tau_{\text{max}} \), the bear market continues, the new minimum point is updated \( P_{t_0+\tau_{\text{min}}}^{\text{min}} = P_{t_0+\tau_{\text{min}}} \), discard the previous local minimum at time \( t_0 \) and set \( I_{t_0+1} = \ldots = I_{t_0+\tau_{\text{min}}} = 0 \). Update \( t_0+ = t_0 + \tau_{\text{min}} \) and continue with equations (28) and (29).

- If \( \tau_{\text{min}} > \tau_{\text{max}} \) a local maximum is found at \( t_0 + \tau_{\text{max}} \) and there is a bull market from \( t_0 + 1 \) to \( t_0 + \tau_{\text{max}} \). Set \( I_{t_0+1} = \ldots = I_{t_0+\tau_{\text{min}}} = 1 \). Record the value \( P_{t_0+\tau_{\text{max}}}^{\text{max}} = P_{t_0+\tau_{\text{max}}} \) and defined \( t_0+ = t_0 + \tau_{\text{max}} \) as a local maximum. Go to the beginning of the process, equations (26) and (27), since \( t_0 \) is now a local maximum.

The process is repeated until the last observation of the data series. This procedure divides the data series analyzed into mutually exclusive and exhaustive bull and bear subsets based on the sequences of closing times. With this, the resulting indicator function \( I_t \) generates a random variable, \( \Delta \), which measures the duration of the bull and bear states, given as the time between the successive changes in \( I_t \).
It is possible to consider different values for $\lambda_1$ e $\lambda_2$. The lower the values of these parameters, the greater the number of bull and bear markets that are expected to be found. In general, the values of these parameters are defined empirically, testing different combinations. However, it is reasonable to consider $\lambda_1 > \lambda_2$, that is, that the percentage variation needed to characterize a change from bear to bull is greater than the percentage required to identify a change from bull to bear. This parametrization defines a greater prevalence of bull states over time, in line with what is expected to be found in practice.