Measuring the Cyclical Component of a Time Series: Proposal of a New Methodology*

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Abstract

The objective of this paper is to present a simple technique to estimate the true cycle of a time series. The robustness of the technique was tested under Monte Carlo simulations using several specifications, including deterministic and stochastic switching trends. The results showed that the proposed technique is able to explain the true cycles based on several statistical tests. As an alternative, we used the Hodrick-Prescott (HP) filter to obtain the cyclical component of the simulated series. This technique produced cycles that were different from the true cycles in some cases. Several tests indicate that this technique is greatly influenced by the effect of the error component on the data-generating processes. Furthermore, we applied our technique to the real Brazilian GDP series. According to our methodology, the Brazilian economy showed business cycles generated by short-run economic policies up to 1994. After this period business cycles were no longer the influential factor, with predominance of the long-run growth component.

Keywords: Brazilian Business Cycles, Hodrick-Prescott Filter, Time Series Models.

JEL Codes: E32, C22.

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1. Introduction

In the study of univariate time series, one of the main concerns has to do with the seasonal, trend, cyclical, and error components of the time series. After seasonality and trend are eliminated from a time series, the remaining component is normally referred to as the cyclical component in the signal extraction literature. However, the extraction of the error component present in the cyclical component has not received much attention in the literature so far. In this paper, we propose a technique that goes a step further in determining the components of the series by extracting the error component from it.

In order to accomplish that, we combine three strands of the literature. The first strand develops techniques that separate trend and cyclical components using specific filters, while the second one identifies trend breaks and stochastic trends that may be present in any time series. Our proposed technique consists in combining these two strands of the literature to estimate trend breaks and stochastic trends plus the one in which such components can be written as ARIMA $(p, 1, q)$ models. Therefore, these combinations enable us to develop a new simple way of estimating the error component distribution. This new technique innovates by leaving the cyclical component out of any series.

The underlying motivation for the proposed methodology is the fact that business cycles should not necessarily contain an error component. This component is normally random and sometimes may account for other sources of disturbances not necessarily related to cycles. For instance, one possibility is that this error could represent the first impact of shocks, such as that of a sudden jump and, therefore, it would not be related to any cyclical behavior at all. To show how the true cycle can be misunderstood by having the error component together with the cyclical component, we run Monte Carlo simulations showing that the application of the commonly used filter technique usually fails to identify true cycles. Therefore, if one accepts that the error of a series is not necessarily part of its cyclical component, then the use of any technique that samples just the trend might not be a good tool to identify the true cycle of the variable. This is especially true when we deal with less stable time series.

As we will learn from the following review of the literature, the most widely used filter tends to transform some of the cyclical component into trend behavior. Moreover, filter methodologies are similar in the sense that they pay little attention to the error component of the series. As pointed out by Maravall (1986) and Plosser and Schwert (1978), the real issue is not in differentiating the trend, but rather in understanding the role of the error component in the series. Hence, our paper follows this line by contributing to a better understanding of the role played by the error component and how misleading the interpretation of the cyclical components can be when it includes the error term.

To show how our proposed methodology works, we generate three time series from models widely used in the literature. Such models capture potential problems

related to the trend behavior. The first model has a trend with a stochastic behavior. The second model assumes that the trend has a stochastic switching behavior. And the third model introduces a structural break in the stochastic trend. These models represent the most commonly expected behavior in any time series. Thus, by applying our technique to this data-generating process (DGP) we want to see how much of the true generated cyclical component can be recovered. This Monte Carlo procedure enables us to test our simple methodology for these models and compare the true and the estimated cycle. As a corollary, we test the Hodrick and Prescott's (1997) technique (HP filter) for the same series. The HP filter exercises provide an insight into the role played by the error component in the true cycle of the variables. As a last exercise, we apply our methodology to the Brazilian gross domestic product (GDP) series as a way of obtaining the business cycle dating process.

In sum, our main proposition is to present a simple technique that enables us to extract the error component from the trend and cyclical components of a time series. As a result, we are able to infer from the extracted true business cycle series if the predominant influences are from short- or long-run economic behavior.

In the coming section – Section 2, we focus on the trend-cycle component literature, especially on the signal extraction one. In Section 3, we present our simple technique and its application to the generated series. In Section 4, the subject matter is mainly the component, and the purpose is to learn about its relationship with business cycles. In Section 5, our technique is applied to the Brazilian GDP series. Finally, the last section concludes.

2. Time Series: The Cyclical Component

Business cycles have been a major topic of discussion, especially in periods of large fluctuations in aggregate variables of industrialized economies. The main reason why researchers devote time to learning about fluctuations is to understand their consequences for the economy, especially considering their potential costs. The economic cost of business cycles once considered negligible by Lucas Jr (1987) proved to be very high under endogenous growth by Barlevy (2004). Although studying business cycle cost is quite important, our focus here is on estimating the true cycle that is a precedent condition for measuring its impact on the economy.

The seminal paper by Burns and Mitchell (1946) introduced the business cycle as a research field. Since then, techniques to determine business cycles improved substantially. The authors' proposed methodology for sampling the trend was based on a simple moving average process. Following them, but also determining the nature of the long-term trends, Hamilton (1989) presented some detrending techniques that became important to the analyses of business cycles. Also in the same line of work we have the paper of Harvey and Jaeger (1993).

Hamilton's (1989) paper stands out because it went a step further by developing a nonlinear iterative filter based on an algorithm that is able to explain regime

shifts in the growth rate. These shifts, according to the author, mimic business cycle fluctuations, and, therefore, dating them would be equivalent to finding the business cycle turning points in the series. The main hypothesis of the technique is that a nonstationary series can have discrete shifts in its mean growth rate. As the author pointed out, the proposed algorithm is a statistical identification of “turning points” of a time series. This technique distinguishes itself from a rather arbitrary technique by specifying the “turning point” as a structural event that is inherent to the data generating process (DGP). Therefore, the shifts are represented in a trend model with certain probability of switching regime through a Markov process. So, the Markov switching process is used as method for dating business cycles. This technique became widespread with the application that includes the one conducted by Chauvet (2002) to the Brazilian GDP series.

However, the most popular filter methodology used was developed by Hodrick and Prescott (1980, 1997). The methodology samples the trend using a nonlinear trend process as main hypothesis. In their case, the trend is estimated through a penalty equation. The proposed penalty parameter (λ) balances the trade-off between lack of smoothness and poor fit of the trend. Due to the fact that this technique is very easy to use, it became widely used to extract the trend-cycle component from any time series.

A thorough review of the HP filter made by Maravall (1999) showed that there are two problems associated with its application. First, it may generate spurious results. Second, it can also underidentify or overidentify a component. A third problem was revealed by Kaiser and Maravall (2002) – the considerable error behavior of the cyclical signal. In this paper, we show some of these problems.

The first authors to report a problem associated with the HP filter were Harvey and Jaeger (1993). The mechanical detrending process by the HP filter, according to them, can lead investigators to report spurious cyclical behavior. This is especially true, again according to the authors, when the HP filter is applied to economic series other than the US economy, especially to more volatile series. For instance, the authors found this to be the case when the filter is applied to the GDP series of the Austrian economy, a more unstable series. According to them, this was due to the error component present in the Austrian data. The same may happen to autoregressive integrated moving average (ARIMA) models with relatively small sample sizes. When the sample is not large enough to allow the researcher to observe real facts of the series, one could specify a process as being integrated of order 1 when, actually, it might be integrated of order 2 (Harvey and Jaeger, 1993, p. 246). These discussions generated the field known today as business cycle extraction.

Research into business cycle signal extraction, which includes the HP filter, has turned its focus towards a solution to the problems reported above. For instance, to solve the problem with endpoints, Baxter and King (1999) and Christiano and Fitzgerald (2005) concentrated on long- and medium-term business cycles (between

two and 32 quarters) by using a band-pass filter technique.

The other approach to signal extraction is to run an HP filter in combination with another technique. For instance, Toledo Neto (2004) compared HP and the band-pass filters applied to Brazilian data. The author showed that the HP filter is qualitatively inferior to the band-pass in extracting Brazilian business cycles. The inferiority was found under Monte Carlo simulations with either stationary or nonstationary series. However, according to Comin and Gertler (2006), both filters have problems. These filters, especially the HP, exhibit considerable variation around a linear trend. This variation reflects the presence of significant cyclical activity at medium frequencies (between two and 200 quarters), which are not captured by the filters.

Another way of solving unstable endpoints of any estimates of a business cycle component is the combination of the HP filter with the model-based one. For instance, King and Rebelo (1993) showed that the HP filter can be derived from a model-based framework using minimum mean square error (MMSE). Therefore, the cycle can be estimated by the Kalman filter technique as in Harvey and Jaeger (1993) and in Koopman and Harvey (2003). A further development of the HP model-based method was done by Kaiser and Maravall (2001). The authors showed that the HP filter can also be estimated from an unobserved component model using MMSE. As a result, the authors proposed the modified HP filter (MHP), which combines HP and ARIMA models for endpoint forecasts.

However, the problem with extracting the business cycle component of the series remains, according to Kaiser and Maravall (2002). The MMSE underestimates the variance of the cyclical component. This loss of variance affects mostly the lower frequencies. The consequence is that the estimator inflates the relative importance of higher frequencies. Thus, the improvement obtained with the MHP is more related to the stability of the cyclical signal and to the reduction of spurious results.

The signal extraction literature also evolved by using a matrix formula approach. For instance, Bell and Hillmer (1988) presented matrix formulas for MMSE optimal time-varying filters. McElroy and Sutcliffe (2006) drew attention to two drawbacks of the proposed technique. First, it requires separate estimates of the initial values. Second, the formulas are very difficult to be implemented. McElroy (2008) provided general matrix formulas for the MMSE which, in his view, solve both problems. The major contribution is to solve the finite-sample problem of the MMSE. Given the fact that this is a very recent technique, its advantage or employability remains to be fully tested. Our approach is similar in scope by estimating the error signal matrix.

Although in the same field, another area of investigation was exclusively related to the trend component behavior. This literature is mostly concerned with the pure trend identification. The research in this field was accomplished by Phillips (1987), Perron (1988), Phillips and Perron (1988), Perron and Vogelsang (1992), Vogel-

sang (1997), among others. Their main focus consisted in developing techniques that identify trend processes as being stochastic, stationary and/or verify whether they have breaks, in addition to dating them. In this respect, they have further developed Hamilton's turning point dating process. The further developments consisted mainly in presenting tests that identify series trend breaks.

In our view, the methodologies for estimation and identification of breaks in time series with possible changes in intercepts and/or slope around a trend can be very helpful in the cycle extraction studies of any series. The reason for this is that business cycles are in ultimate instance changes in slope and/or intercepts of the trend component in the variable representing the economic activity. As a matter of fact, we employ the MMSE techniques to detect trend and intercept breaks before cycle extraction. This generates a matrix formula; this matrix contains the estimates of the cyclical and error components of the series. In a subsequent step, we propose a new way of separating the error component from the remaining series by using an empirical distribution of this error component. The error component is separated by using a general result of an ARIMA $(p, 1, q)$ model. By decomposing the two remaining components, the business cycle component is found to be very close to the true one according to Monte Carlo simulations. Hence, this is the main contribution of this paper to the overall literature.

Therefore, this paper combines three strands of the literature: the identification of intercept/trend breaks proposed by Vogelsang (1997), the McElroy (2008) matrix formula, and the identification of short- and long-run components of an ARIMA model by Beveridge and Nelson (1981). However, the most important aspect of the paper, in addition to the proposed technique, is its contribution to the better understanding of signal extraction.

3. The Proposed New Technique

This section intends to explain our proposed new technique and how it accounts for the trend, cyclical and error or irregular components of the variable. In order to concentrate on these three components, we assume that the variable of study has been cleared out of seasonality. We make this assumption since our main purpose is not to present a new seasonality technique. Therefore, our DGP series will not have a seasonal component.

3.1 Time series

In explaining what we mean by each component in the series, we will make use of a model known in the literature, found in Maddala and Kim (1998). These authors use this model to generate data series to show the effect of each component on series behavior. They develop an ARIMA $(0,1,1)$ model with the following specification:

$$\Delta y_t = \alpha + e_t + \gamma e_{t-1} \quad (1)$$

where $\Delta y_t = y_t - y_{t-1}$ is the series of interest and e_t is the error component that is assumed to be *independently and identically distributed (i.i.d.)*. Let $y_0 = e_0 = 0$ so that we can write y_t by successive substitution as

$$y_t = \alpha t + \sum_{i=1}^t e_i + \gamma \sum_{j=1}^{t-1} e_j \quad (2)$$

By adding and subtracting γe_t on the right hand side of the above equation, we decompose the components into deterministic trend (DT_t), stochastic trend (ST_t) and cycle (CL_t). More specifically, we have:

$$y_t = \alpha t + (1 + \gamma) \sum_{i=1}^t e_i - \gamma e_t \quad (3)$$

In the above equation, it can be easily seen that $DT_t = \alpha t$, $ST_t = (1 + \gamma) \sum_{i=1}^t e_i$ and $CL_t = -\gamma e_t$.

So, any innovation represented by e_t will be split into cyclical and stochastic behavior at time t . This can generate what is known as a mean-reverting process and can clearly hinder the identification of the trend stationary (TS) and difference stationary (DS) components of any series. Beveridge and Nelson (1981) show that any ARIMA $(p, 1, q)$ model can be represented by the ST_t and CL_t components.¹ However, the most important result for our technique is that the cycle has a negative coefficient for this ARIMA $(0, 1, 1)$ or IMA $(1, 1)$ process. The idea is to see if this negative coefficient persists in a more general ARIMA $(p, 1, q)$ process.

We follow Hatanaka's (1996) demonstration that any ARIMA $(p, 1, q)$ model can be written as a function of the stochastic trend and business cycle component. We start by first differentiating the ARIMA $(p, 1, q)$ model which becomes an ARMA (p, q) process. So, let us assume the stationary ARMA process to be the following.

$$(1 - L)A(L)y_t = A(L)\Delta y_t = B(L)e_t \text{ or} \quad (4)$$

$$\Delta y_t = [A(L)]^{-1}B(L)e_t \quad (5)$$

where $A(L)$ and $B(L)$ are polynomials in the lag operator L of orders p and q , respectively. The error component is assumed to be *iid* as before and $A(L)$ can be inverted. By setting $b(L) = [A(L)]^{-1}B(L)$, equation (5) may be written as

$$\Delta y_t = b(L)e_t = b_0e_t + b_1e_{t-1} + b_2e_{t-2} + \dots \text{ and } b_0 = 1 \quad (6)$$

Let us also assume that $\{b_j\}$ is bounded by a decaying exponential where there is a constant that is positive and less than unity or $1 > c > 0$ and $|b_j| < c^j$ for

¹See Maddala and Kim (1998) for a detailed demonstration.

$j = 0, 1, 2, 3, \dots$. Thus, the roots of the equation, $z^p + a_1 z^{p-1} + a_2 z^{p-2} + \dots + a_p = 0$ all have absolute values smaller than unity, which guarantees convergence. In this case, equation (6) can be simplified by setting $b(1) = \sum_0^\infty b_i$. Thus, we have the following:

$$b(L) - b(1) = (1 - L)b^*(L) \quad (7)$$

where

$$b^*(L) = -b_0^* - b_1^* L - b_2^* L^2 - \dots \text{ and} \quad (8)$$

$$b_0^* = \sum_{i=1}^{\infty} b_i, b_1^* = \sum_{i=2}^{\infty} b_i, b_2^* = \sum_{i=3}^{\infty} b_i, \dots \quad (9)$$

We use equations (7) and (8) to rewrite equation (6) as

$$\Delta y_t = b(L)e_t = b(1)e_t + (1 - L)b^*(L) \quad (10)$$

As a final result, for $t = 0$, we have

$$y_t = b(1) \sum_{s=1}^t e_s - \left(\sum_{i=1}^{\infty} b_i e_t + \sum_{i=2}^{\infty} b_i e_{t-1} + \dots \right) \text{ or} \quad (11)$$

$$y_t = b(1) \sum_{s=1}^t e_s - b^*(L)e_t \quad (12)$$

where y_t is decomposed into the long-run component $ST_t = b(1) \sum_{s=1}^t e_s$ and the short-run component $CL_t = b^*(L)e_t$. Clearly, from equations (8) and (9), the coefficients of the cycle component of equation (12) are negative. It is easy to introduce a simple deterministic trend into equation (12) by adding an αt to the right hand side. Thus, the general solution becomes

$$y_t = \alpha t + b(1) \sum_{s=1}^t e_s - b^*(L)e_t \quad (13)$$

Again, in this general specification, the CL_t coefficients represented by $b^*(L)e_t$ are preceded by the negative signal in order to guarantee that the ARIMA will converge. Hence, our main focus is on finding the coefficient distribution that generates the cycle component or the short-run component of the series, $b^*(L)e_t$.

In what follows, we explore three Monte Carlo experiments. In each of these experiments, the trend, cycle, and irregular term will be known. Therefore, we can easily compare the estimated cycle proposed by our technique with the true generated one. Moreover, in order to test the robustness of our proposed technique, each of these experiments also considers the occurrence of deterministic and stochastic

trends as well as behavior change in the error term. The generated series in all of the three models have 100 observations drawn from 1,000 replications.

Additionally, we will apply the HP filter to the generated series. The idea is to estimate the trend of the series and compare the remaining series with the true cycle in the generated series. Recall that the remaining series after the HP filter contains the irregular and the true cycle components together. This will enable us to better view the effects of the irregular component that might be present in business cycle studies.

3.2 Model I

For the sake of simplification, we adopted names for the variables that closely resemble their description. Our initial data generating process (DGP) model has the following features:

$$\begin{aligned} Y_{1t} &= DT_{1t} + ST_{1t} + CL_{1t} + ER_{1t} \\ DT_{1t} &= (1 + 0.5)^*T, \\ ST_{1t} &= T^*ER_{1t}, \\ CL_{1t} &= -0.5^*ER_{1t} \\ ER_t &\sim N(0, 0.5^2) \end{aligned} \tag{14}$$

where Y_{1t} is the final series; DT_{1t} is the deterministic trend; ST is the stochastic trend; CL_{1t} represents the true cycle; T is a trend variable; and ER_{1t} is the irregular component. The last equation tells us that ER_{1t} has normal distribution with zero mean and variance 0.5.

As one may notice, the series generated in this Monte Carlo procedure did not contain seasonal components. If one believes that seasonality is important, then our technique should be applied after excluding seasonality.

In estimating the true model represented by equation (14), we will use the following general model based on Vogelsang (1997):

$$y_t = \theta_t + f(t) + \delta y_{t-1} + \sum_{i=2}^{n=1} y_{i-1} + e_t \tag{15}$$

where y_t is a time series; θ_t is the parameter representing the intercept of the variable; and $f(t)$ is the trend function of the variable. To capture potential breaks in the intercept and trend, we follow Vogelsang (1997) to specify the functions θ_t and $f(t)$ as follows:

$$\theta_t = \alpha_0 + \alpha_1 d_1 \tag{16}$$

$$f(t) = \beta_1 t + \beta_2 \bar{t} \tag{17}$$

where d_1 and $\bar{t}$ are indicator variables that capture possible changes in the intercept and slope at some point in time, respectively. We start with observation $n = 2$ and finish with observation $n - 1$ or $n = 99$. Thus for $n = 2$, d_1 assumes $[0, 1, 1, \dots, 1, 1]$, for $n = 3$, $d_1 = [0, 0, 1, \dots, 1, 1]$ and for $n = 99$, $d_1 = [0, 0, 0, \dots, 1]$. The t in $f(t)$ is a normal trend variable $t = [1, 2, 3, \dots, 98, 99]$; however, $\bar{t}$ assumes $[0, 1, 2, \dots, 98]$ for $n = 2$ and $\bar{t} = [0, 0, 1, \dots, 96, 97]$ for $n = 3$ and for $n = 99$, $\bar{t} = [0, 0, 0, \dots, 0, 1]$. For $n = 99$ d_1 and $\bar{t}$ are the same, so we just consider one of them.

Using the MMSE in a recursive manner, we estimate 100 times equation (15) subjected to the different specifications of equations (16) and (17). Each regression produced a set of residuals that are then gathered into a 100×99 matrix. More specifically, the following steps were done:

- i. First, equation (15) was estimated considering d_1 and $\bar{t}$ as being equal to zero;
- ii. Second, the residuals from the above equation were saved, forming the first column of our residual matrix, e_1 . Where e_1 is a (1×100) vector;
- iii. Third, equation (15) was estimated using the definition of d_1 and $f(t)$ for $n = 2$, thus generating the vector of residuals $e_2(1 \times 100)$;
- iv. Fourth, equation (15) was estimated using the definition of d_1 and $f(t)$ for $n = 3$, thus generating the vector of residuals $e_3(1 \times 100)$, and so on;
- v. Last, equation (15) was estimated using the definition of d_1 and $f(t)$ for $n = 99$, thus generating the vector of residuals $e_{99}(1 \times 100)$.

The resulting matrix of residuals (100×99) was made symmetrical by dropping the first line of the residual matrix. Keeping the (99×99) matrix of residuals or the short one (92×92) does not change the results, since the last four lines were identical in our case. Thus, each observation had an estimated residual that accounts for potential changes in intercept and slope all the way to the $n = 92$ observation. To make things easier, we used quarterly data in our series for the 1977:1-1999:4 period.

Using the (92×92) matrix of residuals, we computed the mean and the standard deviation of the errors for each horizontal line, μ_e and σ_e . This simple procedure allows us to construct a distribution that represents the estimated true cycle component ($b^*e_t = ECL_t$) of the series in the following way:

$$b^*(L)e_t = ECL_t = (\mu_e + \sigma_e) \quad (18)$$

where μ_e is the mean and σ_e is the standard deviation of the estimated distribution. Thus, the final vector $(1 \times 92)ECL_t$ is just the simple sum of the mean and the standard deviations of the residuals of each line of the (92×92) matrix. As expected, the estimated equation (18) enters as negative sign obeying the general

results of equations (3) and (12) of an ARIMA $(p, 1, q)$ process, thus guaranteeing convergence of the series.

Figure 1 displays the results. There, one can see that the estimated true cycle (Est. Cycle1) mimics the true cycle (Cycle1) of this Monte Carlo model for most part of the sample. There is an exception only for the 1978-1981 period. The HP filter (HP-Cycle1), however, predicts an opposite cycle for the series throughout the sample. This technique presents the cycle of the variable in such a way that it leads to a complete opposite cycle analysis. Later on, we will show why this technique may lead one not to see this opposite prediction.

Figure 1
Model I Cycle Estimation

Now, we move on to the test statistics for equality of the Cycle1 and Est. Cycle1 series, obtained by our technique. The correlation coefficient between the two series is very high, 99.4% under the linear hypothesis. However, to measure potential departure from non-linearity, we follow Conover (1980) and estimate the nonparametric Cramer's V and contingency coefficients. The two correlation coefficients are 89.0% and 84.0%, respectively. As we can see, all the statistics support a strong relationship between the two series.

The tests of equality of mean, median, and variance reported in Table 1 are described in detail in Conover (1980) and Brown and Forsythe (1974). The mean test's null hypothesis is that the mean of the two series is the same. This test

result is reported for t -test and F -test statistics. Both produce probabilities of accepting the null hypothesis of mean equality.

The three median tests are as follows. The first is the Wilcoxon-Mann-Whitney test, which examines the comparability of median ranks across subgroups. This test is based upon the values of the classification variable instead of on the value of the observation relative to the median. The idea behind this test is to rank the series from the smallest (rank 1) to the largest value, and to compare the sum of the ranks of subgroup 1 to the sum of the ranks of subgroup 2. If the groups have the same median, the values should be similar. The second is the median chi square test, sometimes referred to as the median test. This is a rank-based analysis of variance (ANOVA), which compares the number of observations above and below the overall median in each subgroup. The third is the Kruskal-Wallis test, which is a one-way ANOVA by ranks. This is a generalization of the Wilcoxon-Mann-Whitney test to more than two subgroups. In Table 1, we see that all of the three reported tests produce acceptable probabilities of being equal to the median of the two series.

Table 1
Test of Equality of Model I Series

Measures of Association:	Value	Probability
Correlation Coefficient	0.99	
Cramer's V	0.89	
Contingency Coefficient	0.84	
Mean Tests:		
t -test	0.81	0.42
Anova F -statistic	0.66	0.42
Median Tests:		
Wilcoxon-Mann-Whitney	0.79	0.43
Median Chi-square	0.78	0.38
Kruskal-Wallis	0.62	0.43
Variance Tests:		
F -test	1.02	0.91
Siegel-Tukey	0.51	0.61
Bartlett	0.01	0.91
Levene	0.11	0.75
Brown-Forsythe	0.11	0.74
Test of Independence:		
Pearson	216.45	0.00
Likelihood Ratio	161.01	0.00

Source: Calculations made by the authors.

The variance tests in the table above are subdivided into four tests. The first is the traditional F -test of equality of variance of the two series. The second is the Siegel-Tukey test. The test's null hypothesis assumes that the two series are independent and have the underlying hypothesis of equal median. The third is the Bartlett test. This test compares the logarithm of the weighted average variance with the weighted sum of the logarithms of the variances. It computes under the joint null hypothesis that the two series variances are equal and that the sample is normally distributed. The fourth test is the Levene test. This test is based on an analysis of variance (ANOVA) of the absolute difference from the mean. The last test is the Brown-Forsythe test (modified Levene test). This test replaces the absolute mean difference with the absolute median difference and appears to be a superior test in terms of robustness and power. The Table 1 results show very high probabilities of equal variance between the two series.

The last two tests reported in Table 1 are the independence tests. The Pearson and likelihood ratio tests report the chi-square statistics. These are tests for overall independence between the two series. The null hypothesis of independence is rejected for the two series on both tests. This result is equivalent to saying that both series are from the same distribution.

Overall, our tests support the equality between the generated true cycle series and the cycle estimated by our technique.

3.3 Model II

Model II was generated using stochastic trend as the main cause for the variable behavior change. The main objective was to test the procedure under a stochastic switching trend. This is an extreme case in the sense that the stochastic process captures external shocks that switch the trend over time.²

Again, the time series (Y_{2t}) has $N = 100$ observations obtained from 1,000 replications each. The names of the components are the same as the previous ones. The series was generated according to the following set of equations:

$$\begin{aligned} Y_{2t} &= DT_{2t} + ST_{2t} + CL_{2t} + ER_{2t}, \\ DT_{2t} &= (1 + 0.5)^*T, \\ ST_{2t} &= [1 + (0.5^*T^*ER_{2t})] / [1 + (0.5^*T)], \\ CL_{2t} &= -0.25^*ER_{2t}, \\ ER_{2t} &\sim N(0, 0.5^2) \end{aligned} \tag{19}$$

Notice that the stochastic trend (ST_{2t}) is more complex than that of the first model. The purpose here is to find out if the cyclical component of a relatively more erratic variable could still be obtained by our technique.

²An interesting demonstration of this mean-reverting case can be found in Hatanaka (1996, chapter 2).

We apply the technique described by equations (15)-(17) to the generated series Y_{2t} . Again, we computed the estimated cyclical component by adding the mean and the variance of the residuals for each line of the (92×92) matrix. In sum, we apply equation (18) to the residuals estimated from Model II equations.

Figure 2 shows the resulting cycles of this model. It is worth mentioning that the estimate of this model is not as accurate as that of Model I. Nonetheless, the smoothing technique seems to give a good approximation of the true cycle. In Figure 2 below, the true cycle (Cycle2) represents the CL_{2t} series; the estimated true cycle variable is Est. Cycle2; and the HP-Cycle2 is obtained using the HP filter technique.

Figure 2
Model II Cycle Estimation

As we may see on the graph above, our estimated cycle series (Est. Cycle2) follows closely the true cycle series (Cycle2), but without reproducing exactly the expected results. Thus, stochastic trend change influences our estimate of the true cycle by increasing its variance. Under this extreme case, Hatanaka (1996, chapter 8) has also shown that discriminating DT and ST using any statistical inference technique lacks precision.

Again, the HP filter was not very good at estimating the true cyclical component. It predicts a behavior that is almost contrary to the true one.

Table 2 reports the tests of mean, median, variance and independence of the series. The first group – the measurement of coefficients of association – indicates

a very high association between the two series (above 80%). The tests of equality of the mean do not reject the null hypothesis of equality of the mean. The same is true for the three tests of equality of the median. However, all five variance tests reject the hypothesis of both series having the same variance. The Pearson and the likelihood tests show that the two series come from the same distribution.

In sum, all tests support the fact that the two series are highly related and follow the same path, but produce a different variance behavior. This is an expected result since this model has a switching stochastic trend, which we predict to be very difficult to capture by any method.

Table 2
Test of Equality of Model II Series

Measures of Association:	Value	Probability
Correlation Coefficient	0.99	
Cramer's V	0.81	
Contingency Coefficient	0.81	
Mean Tests:		
<i>t</i> -test	1.21	0.22
Anova <i>F</i> -statistic	1.48	0.22
Median Tests:		
Wilcoxon-Mann-Whitney	0.97	0.33
Median Chi-square	0.35	0.55
Kruskal-Wallis	0.96	0.33
Variance Tests:		
<i>F</i> -test	7.25	0.00
Siegel-Tukey	6.50	0.00
Bartlett	77.19	0.00
Levene	65.92	0.00
Brown-Forsythe	65.21	0.00
Test of Independence:		
Pearson	179.37	0.00
Likelihood Ratio	158.96	0.00

Source: Calculations made by the authors.

3.4 Model III

In this model, we constructed a variable with a unique and permanent break in the deterministic and stochastic trend. The objective was to test the robustness of our methodology with respect to the estimation of the cyclical component of the variable under trend breaks, as well as of deterministic and stochastic components. The procedure to estimate the cycle was the same as before; however, the variable

(Y_3) differs substantially.

$$\begin{aligned}
 Y_{3t} &= DT_{3t} + ST_{3t} + CL_{3t} + ER_{3t}, \\
 DT_{3t} &+ 2 + 2*(0.5*T), \text{ for } t = 1, 2, \dots, 50, \\
 DT_{3t} &= 1 + 2*(0.5*T), \text{ for } t = 51, \dots, 100, \\
 ST_{3t} &= 1 + (0.5*T), \text{ for } t = 1, 2, \dots, 50, \\
 ST_{3t} &= 0.5*T, \text{ for } t = 51, \dots, 100, \\
 CL_{3t} &= -0.25*ER_{3t}, \\
 ER_{3t} &\sim N(0, 0.5^2)
 \end{aligned} \tag{20}$$

In line with this framework, we applied the regression analysis to equations (15)-(17). Again, we use equation (18) to generate an estimate of the true cycle of this model.

Figure 3
Model III Cycle Estimation

The graphical representation of the estimated cycle for this model is shown in Figure 3. There, the estimated cycle (Est. Cycle3), according to the proposed methodology, seems to follow closely the true cycle (Cycle3). The HP filter (HP-Cycle3) also performs poorly in this model.

We follow the previous two test procedures and report the results in Table 3. The correlation coefficients are high, but comparably smaller than the previous two models. The mean and median tests have acceptable levels of equality.

Table 3
Test of Equality of Model III Series

Measures of Association:	Value	Probability
Correlation Coefficient	0.79	
Cramer's V	0.52	
Contingency Coefficient	0.62	
Mean Tests:		
<i>t</i> -test	1.42	0.16
Anova <i>F</i> -statistic	2.00	0.16
Median Tests:		
Wilcoxon-Mann-Whitney	1.30	0.19
Median Chi-square	3.13	0.08
Kruskal-Wallis	1.70	0.19
Variance Tests:		
<i>F</i> -test	14.87	0.00
Siegel-Tukey	6.66	0.00
Bartlett	130.62	0.00
Levene	58.57	0.00
Brown-Forsythe	57.85	0.00
Test of Independence:		
Pearson	74.37	0.00
Likelihood Ratio	53.21	0.00

Source: Calculations made by the authors.

Nonetheless, we have to stress that their results are slightly above the acceptable minimum of 10%. Again, all variance tests reject the equality between the two series. These are expected results since the break in the deterministic and stochastic trends generates a complex series whose variance is very hard to emulate. In this case, the error component influences the result, and also the behavior of the deterministic and stochastic trends. In this sense, our simple technique succeeded in obtaining series that have a good correlation coefficient and potential equality of mean and median.

Given the poor performance of the HP filter in predicting the series true cycle in the three models, we try to provide a possible explanation in the following section. It should be recalled that the HP filter technique takes the trend out of the series, leaving what is supposed to be the error component.

4. Learning about the Error Component

In this section, the objective is to learn about the influence of the error component on HP estimates of the cycle of the series. We should recall that if the error component of the series is indeed the main cause for the poor performance of the HP filter in predicting the cycle, then we might not hold it as neutral. Thus, our aim in this section is to compare the estimated true cycle of the models by the HP filter with the true cycle plus the error component. If the series are the same, then we know that the HP filter has a poor performance thanks to the error component behavior. Thus, in order to compare the resulting HP cycle with the cycle generated by our Monte Carlo procedure, we create a new variable. This new series is equivalent to the sum of the two variables CL_t and ER_t in the models, which we will label CTE. This new variable represents the true cycle plus the true error component of Models I-III. The variable that contains both business cycle and error components in our technique is just the mean of the error, μ_e , which we label mean-cycle.

4.1 Model I – Business cycle and error components

According to Figure 4, one may observe that both estimates of the cycle for Model I's HP-cycle1 and the Mean-cycle1 are close to cycle plus error component $CTE1 = CL_{1t} + ER_{1t}$. However, testing the robustness of CTE1 requires additional investigation through tests of mean, median, and variance.

The Mean-cycle1 variable and the HP-Cycle1 filter mimic quite well the true cycle component when added to the error component of Model I ($CTE1 = CL_{1t} + ER_{1t}$). Thus, the HP filter series (HP-cycle1) seems to be greatly influenced by the error component of the series. To see how close the above series are, we carried out some additional tests.

The F test statistics support the null hypothesis that the estimated series have the same mean as CTE1, yielding p -values of 81%. The same test of equality of

Figure 4
Comparing Model I

medians between the three series provided probabilities equivalent to 100%.³

The Wilcoxon, Mann and Whitney test proposes the null hypothesis that the three series are independent samples from the same general distribution. In this case, the probabilities are 83% and 94% when compared to the true series (CTE1).⁴

The tests of equality of variances between the three series produced the probability of equality of 87% and 47% for the Mean-cycle1 and HP-cycle1 F tests, respectively. The F -test took the series with larger variance (L) and divided it by the series with the lowest variance.

The Bartlett test compares the logarithm of the weighted average variance with the weighted sum of the logarithms of the variances.⁵ For this test, the adjusted statistic reported for the Mean-cycle1 series was 0.026 with probability of 87% while that of the HP-cycle1 series was 47%.

The Levene test performs an analysis of variance (ANOVA) of the absolute difference from the mean.⁶ This test indicates that the probability of the variances of the Mean-cycle1 and CTE1 series to be equal is 78% while for the HP-cycle1 it is 47%.

In addition to these results, we also performed another variance test among the series, as follows:

$$\begin{aligned} VAR(CTE1) &= (0.138452)^2, \\ VAR(Mean-cycle1) &= (0.136123)^2, \text{ and} \\ VAR(HP-cycle1) &= (0.128497)^2 \end{aligned} \quad (21)$$

In what follows, we compute how close the measurement of the cycle (mean-cycle1 or HP-cycle1) is to the cycle plus error component ($CTE1 = CL_{1t} + ER_{1t}$), resulting from the Monte Carlo experiment. Therefore,

$$\left(\frac{0.136123}{0.138452} \right)^2 > \left(\frac{0.128497}{0.138452} \right)^2 \Rightarrow 97\% > 86\% \quad (22)$$

Thus, the mean-cycle1 method mimics the true cycle plus error component better than the HP-cycle1, outperforming it by 11%.

4.2 Model II

The estimations of Model II showed to be similar to those of Model I. Figure 5 suggests that mean-cycle2 is closer to $CTE2 = CL_{2t} + ER_{2t}$ than to HP-cycle2. However, the F -tests of equality of mean imply that the probability of Mean-cycle2, HP-cycle2 and $CTE2$ having the same mean is about 92%.

³See the description of the F -tests in Greene (2000).

⁴See references for these tests in Conover (1980).

⁵See Greene (2000) for a full description.

⁶This test was fully developed as a test of equality of variance by Brown and Forsythe (1974).

Figure 5
Comparing Model II

In Model II, the probability of Mean-cycle2 having the same general distribution as CTE2 goes from 88% to 100% for the F and chi-square tests. The highest p -value comes from the median chi-square statistics. Again, the HP-cycle2 passes the same tests showing p -values that range from 77% to 92% with the highest one from the median chi-square.

The tests of equality of variance show lower probabilities than those of Model I. Nevertheless, the results for the Mean-cycle2 series are significantly higher than the ones calculated for the HP-cycle2 series. In Model II, the probability of Mean-cycle2 having the same variance as CTE2 ranges from 36% to 49%. If one considers the HP-cycle2 series, its probability to have the same variance as CTE2 ranges from 8% to 11%. This last result suggests the rejection of the null hypothesis of same variance for HP-cycle2 and CTE2.

Despite the above results, we apply our own test procedure to the variance of the series.

$$\begin{aligned} VAR(CTE2) &= (0.207678)^2, \\ VAR(Mean-cycle2) &= (0.188464)^2, \text{ and} \\ VAR(HP-cycle2) &= (0.176366)^2 \end{aligned} \quad (23)$$

By comparing the variances, we have the following:

$$\left(\frac{0.188464}{0.207678}\right)^2 > \left(\frac{0.176366}{0.207678}\right)^2 \Rightarrow 82\% > 72\% \quad (24)$$

The result of the comparison of the variances indicates that Mean-cycle2 is 10% closer to the movements of CTE2 than is HP-cycle2.

4.3 Model III

Now, turning our attention to the third model we see that the previous results do not hold. Figure 6 suggests that the resulting HP-cycle3 is closer to the true cycle plus error or $CTE3 = CL_{3t} + ER_{3t}$ than is Mean-cycle3. This seems to be true especially for the period after the break (50th observation).

Figure 6
Comparing Model III

Comparing the F -tests of equality of means, the p -value for equality between the means for the CTE3 and Mean-cycle3 series is approximately 97%. For HP-cycle3, it has a 93% probability of being equal to that of CTE3, showing that both are highly acceptable.

Moreover, the F -tests of equality of medians of the two series indicate that the probability of Mean-cycle3 having the same distribution as CTE3 ranges from 77% to 88%. The best p -value is given by the adjusted median chi-square statistic, which is a continuity correction. The same statistic produced p -values between

77% and 91% for the HP-cycle3 series, indicating a high probability of belonging to the same general distribution as CTE3.

In addition, the F -tests of equality of variances indicate that the probability of Mean-cycle3 and CTE3 having the same variance can reach as much as 70%. The HP-cycle3 has a higher probability of having the same variance as CTE3 (90%); however it also gives a low p -value to the F -test (27%). The possible explanation for HP-cycle3 having a better performance under breakpoints might be related to the OLS estimation of Mean-cycle3.

We also apply our method to compare the two variances.

$$\begin{aligned} VAR(CTE3) &= (0.207678)^2, \\ VAR(Mean-cycle3) &= (0.268027)^2, \text{ and} \\ VAR(HP-cycle3) &= (0.232822)^2 \end{aligned} \quad (25)$$

Then, these variances imply that:

$$\left(\frac{0.268027}{0.207678}\right)^2 > \left(\frac{0.232822}{0.207678}\right)^2 \Rightarrow 166\% > 126\% \quad (26)$$

In this case, the Mean-cycle3 variance overestimates the true variance of the cycle by 66%, and then the estimated Mean-cycle3 series has a relatively higher amplitude than CTE3. The HP methodology also overestimates the real cycle, but at a smaller rate (26%).

Overall, these tests do confirm our suspicion that the error component strongly influences the estimate of the true cycle of the variables. The HP filter appeared to be a very poor technique for obtaining true cycles in the presence of stochastic and deterministic trend changes.

5. An Application to the Brazilian GDP

In this section, we apply the proposed new technique to the Brazilian real GDP. The sampling period goes from 1981:Q1 to 2010:Q1. The data source is IPEA (Brazilian Institute of Applied Economic Research). The purpose is to compare the resulting cyclical component of our technique with the HP Filter series. First, we start by testing the real GDP series for unit root. The objective is to check if the series is $I(1)$ and therefore if it can be modeled by the $ARIMA(p, 1, q)$ process. The performed tests are augmented Dickey-Fuller (ADF), Phillips-Perron (PP), and Elliot-Lothman-Rothemberg-Stock (DF-GLS). All the tests were highly significant in first differences $I(0)$. The results indicate that the series is stationary upon differentiating it.⁷

⁷The ADF reports a t -statistic of -9.22 , PP statistic of -9.19 and DF-GLS statistic of -8.47 . They are all significant at 1%.

We apply the HP filter and our technique to the real GDP series. After that we smooth the series using the Holt-Winter filter with alpha and beta parameters set to 0.5. The resulting two series were labeled HP-FILTER-SM and MEAN-CYCLEBR. The MEAN-CYCLEBR is the estimated true cycle plus the error component. We chose these two series to show how closely they are to the ones estimated by Chauvet (2002), Chauvet and Piger (2002), and Toledo Neto (2004). The authors found that there were structural breaks in the Brazilian real GDP series. Chauvet uses a sample for the period between 1980:Q1 and 2000:Q4 and Toledo Neto for the period between 1970:Q1 and 2002:Q4. We will concentrate on the period between 1980 and 2000 to be more accurate with the previous authors' analysis.

Figure 7

Comparing the HP Cycle and the Mean-Cycle for Brazil – 1981:Q1-2010:Q1

We start by analyzing the first decade, 1980-1990. The smoothed result of the HP filter (HP-CYCLEBR-SM) is close to the one found by Toledo Neto. The first slowdown started in 1981:Q2 and lasted up to 1983:Q3. Then the economy entered a period of recovery that lasted up to 1987:Q1. Chauvet's (2002) results are very close to those of the Mean-cycle series (MEAN-CYCLEBR-SM), starting with a slowdown and an unstable period from 1981:Q1 to 1983:Q1. After that, the econ-

omy starts its longest period of expansion, 1983:Q2-1987:Q1. As to the 1987-1992 period, both authors found it to be very unstable.⁸ For instance, Chauvet (2002) reports five severe recessions in this period and Toledo Neto, four severe recessions. The two series above reproduce exactly their number of recession forecasts. From 1993 to 1995 there was a fast economic growth, followed by a slowdown up to the year 2000. To show how closely related these series are we conducted some tests of equality.

Table 4
Tests of Equality between the Mean-CycleBR and HP-CycleBR Series

Measures of Association:	Value	Probability
Correlation Coefficient	0.68	
Cramer's V	0.48	
Contingency Coefficient	0.64	
Mean Tests:		
<i>t</i> -test	0.26	0.79
Anova <i>F</i> -statistic	0.07	0.79
Median Tests:		
Wilcoxon-Mann-Whitney	0.48	0.63
Median Chi-square	0.66	0.41
Kruskal-Wallis	0.23	0.63
Variance Tests:		
<i>F</i> -test	1.55	0.06
Siegel-Tukey	2.75	0.00
Bartlett	3.48	0.00
Levene	5.89	0.02
Brown-Forsythe	5.55	0.02
Test of Independence:		
Pearson	51.64	0.00
Likelihood Ratio	31.71	0.00

Source: Calculations made by the authors.

The coefficients that measure the association between the two series range between 48.0% and 68.0%. The mean equality *t*-test and *F*-test both support the null hypothesis of equality at a very high level. The three median tests also show probability between 41.0% and 63.0%, which is way above the standard minimum of 10%. The variance tests, on the other hand, all reject equality. Thus, the variance tests do confirm that the two variables are indeed different in their

⁸It is worth mentioning the work done by Ellery Jr et al. (2002). They did not find an unstable period using the HP filter because the parameter used to smooth the trend was set to 100, which is not a consensus in the literature, as they emphasize.

variances. Even though they seem to be following the same path, the same is not true regarding their amplitude movements.

Table 4 also shows, in addition to the above tests, two other pieces of information, the Pearson chi-Square and the likelihood ratio test. Both tests reject the null hypothesis of independence of the two variables.

In sum, both series seem to follow the same path, but produce different variance (oscillation) along the path. The most important aspect, however, is the fact that these tests corroborate the hypothesis that HP-CYCLEBR-SM and MEAN-CYCLEBR-SM are greatly influenced by the error component. As a matter of fact, the CYCLEBR variable obtained through equation (18) as an estimate of the Brazilian true cycle shows a somewhat different behavior from the other two variables. Figure 8 shows that more clearly.

Figure 8
True Business Cycle Estimated for Brazil

The question is now how to explain the difference in behavior. First, let us recall the Beveridge and Nelson's (1981) interpretation of any ARIMA $(p, 1, q)$ process. According to them, it can be split into two components. The first is the long-run component (ST-Stochastic Trend) and the second is the short-run component (CL-Cyclical Component). In accordance with the growth theory, the

long-run component is mostly influenced by factor accumulation, institutions, or other exogenous factors that influence the domestic economy. In contrast, the short-run component is guided mostly by fiscal and monetary economic policies and international and/or domestic shocks. Thus, the technique laid out in this paper allows determining more clearly the contribution of each component to the economic process.

As an example, let us assume that we observe a long-run growth period in the economy. If in the same period we find that the short-run component is very stable (near zero line), then the growth can be attributed exclusively to the long-run component. Moreover, if the short-run component is negative, the long-run component more than compensates for this lack of contribution to the economic policies. The opposite happens when we observe a negative growth period. If the short-run component shows an upward trend or is above the zero line, then the growth is clearly led by short-run economic policies. In sum, our technique can better show the contributions of the short- and long-run components to the economic cycles.

Figure 8 shows that the short-run component (CLYCLEBR) is clearly negative between 1981:Q1 and 1982:Q2 and positive from 1982:Q2 to 1983:Q1. According to our data, the economy grew at a yearly average of -2.14% . This result indicates that some economic policies were adopted to pull the economy out of recession as indicated by the CYCLEBR series. However, it seems that this was not enough to overcome the exogenous shock that generated the recession period in the economy.

The longest period of expansion between 1983 and 1992, when the economy grew at a yearly average of 2.06% according to our data, was also reported by Chauvet and Toledo Neto. As we may see in Figure 8, this was also the most unstable period.⁹ By splitting this period into two subperiods, note that the first one started in 1983:Q1 and ended in 1987:Q2 whereas the second one started in 1987:Q3 and ended in 1992:Q2. The two subperiods have different growth rates. During the first subperiod the economy grew at a yearly average of 4.28% and during the second one it was barely positive (0.47%). Figure 8 also shows that the short-run component (CYCLEBR) had a negative contribution during the larger growth period and a positive contribution to the cycle during the smaller growth period. In other words, short-run economic policies and domestic shocks seem to cause cyclical behavior contrary to the long-run growth component during this period. This opposite behavior might explain why the economy experienced such an unstable period.

Fast growth occurred between 1993 and 1995, according to our data. The yearly average growth was 4.86% . The short-run component was no longer the cause of this growth, according to Figure 8. The business cycle measured by the CYCLEBR

⁹During this period the economy experienced six stabilization plans, “Plano Cruzado” (1986:2), “Plano Bresser” (1987:6), “Plano Verão” (1989:1), “Plano Collor I” (1990:3), “Plano Collor II” (1991:1), followed by “Plano Marcílio” (1991:5) in the same year.

series shows a decline in 1994:Q3 and 1994:Q4. This negative contribution of the short-run factor was compensated for by the positive contribution of the year 1995. After this year, the cycle amplitude decreased, signaling that the business cycle did not have much power over economic expansion. Hence, the long-run component was the main predominant cause for the growth process.

From 1996 to 2007, the Brazilian economy grew at a yearly average of 2.84%. The most important aspect of this period according to Figure 8 is the fact that the short-run component was close to the zero line. In other words, the long-run component was dominant and cycles due to any short-run economic policy were very small. The stabilization process showed to be the main objective of the economic policies and this caused business cycles to lower their influence on the economy. The stabilization of the economy with the “Real plan” led families to restore their purchasing power, which means a structural change.¹⁰ As a matter of fact, long-run decisions seem to be the major difference between the period starting in 1995 and the previous period. Hence, it seems that the economic policies change their focus to make the economic cycle more stable, leaving the long-run component to act in the economy.

The recent fast growth period between 2007 and 2010 with a yearly average of 4.25% could have been even higher if 2009 had not grown at a negative rate of -0.17%. The CYCLEBR shows a negative cycle resulting from short-run components since 2006 with great influence on 2008. However, the bad outcomes obtained for 2009 were not caused by short-run or domestic factors. The domestic factor was pushing the economy up, but the international crisis made the economy slump. In fact, in Figure 8, the domestic factor is clearly positive and helped the economy to come out of the international negative influence during the 2007-2008 period. In other words, the previous growth was led by long-run causes and the recovery process of 2009 by short-run policies.

Overall, our technique shows that if we interpret the short-run component as a consequence of short-run policies, the business cycle in the Brazilian economy since 1995 would have been minimized. The short-run economic policies were used either to cause less impact on the growth period or to leave the economy out of recession in bad times. Hence, this technique enables us to better qualify the economic policy being adopted.

On the technical side, it seems that the Brazilian series has an error component and that deterministic and stochastic trends are close to those of the models in Section 3. This result is corroborated by Chauvet (2002), Chauvet and Piger (2002) and Toledo Neto (2004). According to the authors, the Brazilian GDP tends to be more characterized by stochastic trends and log-linear trends with breaks. The breaks seem to be caused by exogenous factors and they are very hard to anticipate. However, business cycles resulting from economic policies and domestic shocks become less unstable and more predictable. If this is the case,

¹⁰See Cyrillo et al. (1997) , Barros et al. (2004).

then our technique produced better estimates of the Brazilian true cycle.

6. Conclusion

We developed a simple methodology that enables us to learn about the true cycle of the time series. This technique consists of a sequence of applied OLS regressions using indicator variables that consider either change in constant or slope. From each of these estimates, we produced an error series. Thus, by constructing a matrix of the error components, we were able to estimate the true cycle by using a simple formula applied to the residuals of the regressions. Using this technique to compare the series with the HP filter method, we learned that the error component greatly influences the HP cycle series. It causes the HP Cycle series to behave almost in an opposite way to the true cycle.

We apply this technique to the Brazilian real GDP series to learn about the cycle behavior of this variable. The result seems to corroborate the prediction made by our technique. For dating the Brazilian real cycle, we use the Holt-Winters exponential smoothing process as guideline. The Brazilian economy had the following two distinct periods. The first one started in 1980 and went up to 1994, during which time short-run factors were the dominant aspect, causing a very unstable business cycle. From 1995 on, the short-run factor was minimized and long-run factors predominated. The business cycle became less volatile and the quality of the economic policies could be seen by our technique as being less influential on periods of economic growth and more incisive during recessions. In sum, the business cycle component of the Brazilian GDP had a lesser impact on the overall behavior of the economy after 1994. Is it due to more stable economic policies after 1994? Probably yes.

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