Towards a Truthful Land Taxation Mechanism in Brazil

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Abstract

This paper shows that the asymmetric information present in the relationship between the government and agricultural producers has led to persistent problems in the application of land taxes (Imposto Territorial Rural - ITR) in Brazil. The main result is that, when the asymmetric information is taken into account, some use of output taxes in the optimal tax scheme may be more desirable than a pure land tax regime. Herein, we consider a model of optimal taxation in which the government maximizes the expected tax revenue less the farmer's yields from a non-agricultural activity. There is a continuum of farmers using land to produce a homogeneous agricultural output and for non-productive ends. They have private information on the parameters of both activities. Also, there is no land rental market, and the harvested area cannot exceed the farm size. We show that a pure land tax regime is optimal only if there is complete information or there is no idle land in equilibrium. A composite tax mechanism, which simultaneously considers taxes on land and output, can be used to implement the optimal scheme in the case of incomplete information. Nowadays, the ITR rate is a function of the degree of use, which is not observed by the government, and farm size. Our study implies that ITR rate should depend on farm size and the ICMS (Value-added tax on sales and services) per hectare, which is a reliable proxy for land use.

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Resumo

O objetivo do artigo é mostrar que a assimetria de informação presente na relação entre governo e produtores agropecuários pode constituir a origem dos problemas que ainda persistem na aplicação do Imposto Territorial no Brasil. Através da construção de um modelo teórico simples, que se baseia no problema de taxação ótima sob informação assimétrica, é possível analisar limitações inerentes à aplicação do Imposto Territorial Rural que ainda não se incorporaram à análise da taxação de terras. Diante de uma situação onde há terra ociosa, como ocorre no Brasil, o modelo mostra que o uso do ITR como único instrumento tributário não é capaz de implementar o esquema ótimo. A solução apontada pelo modelo envolve a utilização de um esquema misto que considera o Imposto sobre a Circulação de Mercadorias e Serviços (ICMS) e o ITR. Atualmente, a alíquota do ITR é uma função do grau de utilização e do tamanho do estabelecimento agrícola. Como conseqüência da análise, sugerimos que a alíquota do ITR dependa do tamanho do estabelecimento e do ICMS por hectare, que constitui uma medida aproximada e observável do grau de utilização das terras.

1. Introduction.

Rural Property Tax (ITR) in Brazil, since its creation through the Land Statute of 1964, has been used to support public policies for land redistribution. Nevertheless, there is a high level of evasion and default that hinders its efficiency as an instrument of landholding policy. Two large-scale reforms of ITR were carried out, in 1979 and 1996, but the results have not sufficed to overcome the associated problems.

The objective of this paper is to demonstrate that the asymmetry of information between the government and agricultural producers (farmers and ranchers) causes problems that still persist in the application of the tax. Faced with a situation of idle land as in Brazil, the theoretical model developed shows that the use of ITR alone is not
capable of implementing the optimal scheme. The solution pointed out by the model involves the use of a mixed scheme based on an output tax (Value-added Tax on Sales and Services - ICMS) along with the land tax (ITR).

The main contribution of this work is the construction of a theoretical model that concentrates on an analysis of the problem of optimal taxation under asymmetric information, considering specific aspects of agriculture, particularly attuned to the Brazilian situation. Agricultural producers are sorted by the use of land and the land rental market is imperfect (to simplify, it does not exist).

The model presented here attempts to fill a gap among optimal taxation models under asymmetric information and traditional land taxation models. On the one hand, optimal taxation models basically involve taxes on consumption and income, ignoring issues that are relevant to agricultural activity [Mirrlees (1971, 1986)]. On the other hand, papers dealing with land taxation normally concentrate on other questions, described briefly below.

Henry George (1839-1897) was the first to establish an economic rationale for land taxes, in Progress and Poverty, published in 1879. He attributed unemployment and low wages to an artificial dearth of land and the poor operation of the market. This artificial scarcity was the result of unequal land distribution and speculative activities. In this context, George proposed land taxation to make the land market more dynamic so as to induce full soil use without distorting marginal incentives. Arnott and Stiglitz (1979) analyzed the general case of what has become known as the Henry George theorem, becoming a classic reference work in this sense.

Other authors have also pointed out the advantages inherent in the use of taxes on land as a source of government revenue [Deininger (1998) and Skinner (1991b)]. Land taxation does not distort the allocation of resources and constitutes one of the few examples of a
lump-sum tax in aggregate terms that can ensure a minimum level of revenue collection, since the supply of land is inelastic. In addition, the farm size is observable, mainly in regions where land ownership is individual and there is accessible and reliable information on the size of holdings.

Hoff (1991), on the other hand, qualifies the use of land taxation. The author argues that in an uncertain activity such as agriculture, in which producers are risk-averse, the exclusive use of land taxation promotes an inefficient allocation of risk. In Hoff’s (1991) model, the parallel use of an output tax is shown to be Pareto-superior. The optimal composition of output and land tax is determined by the tradeoff between distortion (introduced by output tax) and risk sharing.

Carter and Mesbah (1993), using a model of multiple equilibria, show that the use of a land tax is inefficient in overcoming the “accumulation barrier”. This barrier is established by the critical size of the farm that determines whether the producer will be a small or large landholder in equilibrium. However, this result arises from the linear scheme for land tax adopted by these authors. Rules with progressive rates, as in the case of ITR, can significantly affect the accumulation barrier in their model.

Skinner (1991a) emphasizes the informational costs required to manage this type of tax. Nevertheless, despite considering the possibility of different types of producers, he does not deal with the problem of mechanism design faced by the government. In the relationship between the government and producers, only the behavior of the latter is strategic. The government has only a positive probability of incorrectly appraising land values.

Another question addressed by Skinner (1991a) establishes that the loss of capital resulting from the application of the tax is transitory, affecting only the current landowners. When the agents have
access to other assets, a non-arbitrage condition ensures that the tax is completely absorbed by a reduction in the price of land.

The model developed in this paper is closely tied to the Brazilian situation and the literature mentioned above. The main Brazilian features emphasized in the model are the differentiation among agricultural producers and the absence of land rental market. There is some evidence about an inverse relationship between profitability and farm size in Brazil [Barros et al. (2000), Guanziroli and Cardim (2000)]. One possible explanation to this regularity is that, although small farmers use land only for agriculture, the large landholders can obtain non-agricultural revenues from their land titles. In such economy, there is a role for public policies and the land tax can be used to avoid non-productive use of land.

The model shows that the problem of informational costs raised by Skinner (1991a), present in the Brazilian case, can be at least partially resolved by using an output tax. The employment of this tax, besides transferring less risk to producers [as in the model of Hoff (1991)], constitutes an essential instrument to obtain reliable declarations of productivity parameters and the extent of cultivated land. In a sense, this result is an application of Hoff’s (1991) idea to truthful revelation instead of risk sharing. Output taxes can be used not only to determine a better risk sharing, but also to avoid default and underreporting in an asymmetric information context.

Land ownership in the model has two basic purposes. It can be used for agricultural production or for other non-productive purposes. In the latter case, the extra revenues can arise from several sources: a collateral in an economy with imperfect credit market [Hoff, Braverman and Stiglitz (1993)]; a hedge against macroeconomic instabilities, particularly in periods of high inflation [Brandão and Rezende (1992), Feldstein (1980)]; or to expropriate monoplycum-monopsony rents, where a single landlord can experience market
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power but he is restricted to establish a uniform wage and rental rate to the market [Conning (2001)]. As long as these extra benefits from land are appropriated only by a fraction of the population, a land tax can be an important instrument to move the economy towards a more efficient resource allocation.

The model considers a continuum of types of producers who are differentiated by their proclivity for non-productive activity and/or agricultural productivity. These producers determine the extent of their holdings and the area under cultivation. Each producer’s type constitutes private information and there is no rental market, i.e., the amount of land under cultivation cannot surpass the farm size.

The government maximizes its utility function, which considers the tax receipts and the extra benefits of land, observing the farm size and the total amount of output. The assumption that the amount of production is observed by the government tries to include in the model the fact that collection of ICMS is much more efficient than that of ITR in Brazil. Also, for the sake of simplicity, the model assumes that the ICMS collection can be inferred by the observation of output and vice-versa.

The basic results are: relying only on land tax (ITR) is optimal when the government’s information is complete or when in equilibrium there is no idle land; otherwise, the optimal scheme is a linear combination of output and land taxes. The results obtained in the paper can be used to improve the accuracy of land taxation in Brazil. We suggest that the ITR rate should depend on the collection of ICMS per hectare instead of degree of use.

In other words, if there is idle land in equilibrium, there are no tax rates capable of implementing an optimal tax system solely with ITR. But for small producers who operate in equilibrium without idle land, ITR can be implemented, which is compatible with some empirical evidence indicated in Section II.
The article is organized in seven sections. Section II presents a brief overview of the application of ITR in Brazil. The theoretical model is laid out in Section III, which establishes the basic notation for Sections IV and V. Sections IV and V analyze cases of complete and asymmetric information, respectively, and present the basic results of the model in heuristic form. The formal version of the arguments is contained in the Appendix. Section VI presents a discussion about the assumptions adopted, the generality of the results and implications for public policies. Finally, the main implications and contributions of the paper are summarized in the Conclusion.

2. The Brazilian Experience with ITR.

Land taxation was instituted in Brazil by the Republican Constitution of 1891, delegating competence to the state governments. The responsibility of the states for administering and collecting the tax was maintained by the subsequent Constitutions of 1934, 1937 and 1946. In 1961, with the enactment of Constitutional Amendment No. 5, ITR was transferred to the municipalities, and in 1964, under Amendment No. 10, it was again transferred, this time to the federal government, where it has remained (confirmed in the 1988 Constitution). The passage of the Land Statute in 1964 imposed extra fiscal functions on land taxation, which in principle took on the job of assisting public land redistribution policies. [For more information, see Oliveira (1993) and Reydon et al. (2000)].

Below, there is a brief description of the three successive phases after the implementation of the Land Statute. This history is important to explain the reasons for the main modifications, and insofar as possible, the scope of the solutions adopted. At the end of this section, the reader should be convinced that despite these series of changes, there is still a chronic problem of tax implementation that hinders the attainment of the desired revenue level. This is the cen-
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tral objective of the theoretical model contained in Section III.


The Land Statute (Law No. 4504 of November 30, 1964) made collection of ITR the responsibility of INCRA (National Land Reform Agency). As described by Oliveira (1993), the basic rate was 0.2%, corrected by coefficients related to size (A), location (B), social conditions (C) and productivity (D), which altogether determined a tax burden given by:

\[
ITR = (0.002 \times A.B.C.D.)VTN
\]

where \(VTN\) represents the value of the unimproved land. Given the intervals for variation of each coefficient, the effective tax rate ranged from 0.24% to 3.456%.

Nevertheless, the original objectives of the tax were not achieved. Oliveira and Costa (1979), cited by Oliveira (1993), concluded that ITR had never constituted a good source of revenue and further, hardly managed to achieve any of the desired changes in the rural environment. The authors’ main conclusions were:

1. “Given the small impact of ITR (and parallel taxes) on the profit and rate of return of rural properties, and given the tax evasion by a large majority of owners, it can be inferred that the referred tax has not contributed, and will likely not contribute, to changing the socioeconomic relations in Brazilian agriculture.

2. From the standpoint of real estate categories, ITR is in many respects incoherent because it falls more heavily on small rather than large landowners and, in many cases, treats rural companies more rigorously than large landowners. The reason for such a regressive outcome is the system for calculating the tax, which does
not discriminate between taxpayers according to their category of holding (smallholder, rural company, and large landholder).

3. The categorization of rural properties adopted by INCRA to define smallholdings, rural companies and large holdings does not jibe with reality.

4. The intended variation in the legally set rates is not observed. This is due to the fact that the coefficients for size, location, social conditions and productivity are not adequate to the reality of the Brazilian rural structure.

5. The problem of evasion is widespread and serious.

6. The system of updating raw land values in the years between reassessments, according to a monetary restatement (inflation) index, does not reflect the real behavior of land values over time."

Hence, the situation in the 1970s was, as summarized above, set by a host of problems with the way ITR was implemented. In this period, due to the importance of operational problems (responsible for huge distortions), questions of a more structural nature were not the focus of debate. It was believed - and this feeling still persists - that the problems involving ITR were of a purely operational nature.


The complications pointed out above led to the first important reform of ITR legislation. The most significant changes for questions related to this work involved Article 49 of the Land Statute, according to which “the general rules for setting Rural Property Tax shall henceforth obey progressive and regressive criteria, taking into account the following factors: the raw land value; the size of the rural property; the degree of use for farming, ranching or forestry exploitation; the level of efficiency obtained from the various uses; the total area country-wide of rural properties held by the same owner; the
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land classifications and their form of use and profitability."

Relying again on the description of Oliveira (1993), this overhaul still maintained VTN as part of the tax basis. The applicable tax rate became a function also of the degree of land use (GUT) and the level of efficiency (GEE), in such a way that:

\[ ITR = [t(GUT, GEE)] VTN. \]

According to this scheme, the rate would vary from 0.2% to 3.5% (for properties above 100 fiscal modules). Figure 1.1 – ITR Revenues (1972-1991)

![](image)

Source: Oliveira (1993)

\[1\] The definition of the fiscal module of the municipality considers the following factors: type of predominant economic activity (truck farming, permanent cultivation, temporary cultivation, herding and forestry), productivity per crop or activity, and a consideration for smallholders/family farmers.
The data presented by Oliveira (1993) show that efforts to increase tax collections were frustrated. The levels rose in the years immediately after the reform, but by 1983 they had returned to their previous levels, as shown in Figure 1.1. Even in 1990, when there was another peak, the level of revenue collected was an insignificant US$ 20.30 per rural property. In January 1992, revenues corresponded to about 25% of the minimum monthly wage.

According to the Presidential Press Office, the percentage of VTN declared in relation to the real price of land in the 1980s varied from 20% for properties of less than 10 hectares\(^2\) down to only 1.2% for large properties of over 10 thousand hectares. The area reported as usable was far short of the true figure as well, with large landowners declaring around 50% and small ones 94% of the real measure. This declaration of utilization was even more unrealistic in some cases accepted by INCRA in which the actual productivity was more than tenfold the expected value, as calculated by the Brazilian Bureau of Geography and Statistics (IBGE).

The different impact of the ITR collection scheme on small versus large landholders can be seen in the context of the model in Section III. In this model, systems that use only ITR have the desired effects on small producers. Large landholders with idle land have to resort to an additional instrument, ICMS.

Despite the problems of underreporting and evasion encountered, the question of asymmetric information between the government and landowners still was not incorporated into the analysis. To the contrary, analyses such as that of Sayad (1982) considered hypotheses that summarily dismissed this fundamental aspect of the problem of taxation on the land market. Among the hypotheses enunciated by this author, a standout was that: “farmers and non-farmers alike

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\(^2\) A hectare equals 10,000 square meters or about 2.47 acres.
have the same expectation of value, i.e., both are equally optimistic or pessimistic with relation to the future behavior of prices for rural real estate, and there is no growing of crops just as "cultura de vitrine", i.e., planting just enough for show in order to minimize the tax burden."

Again the debate focused on operational questions, mainly the complex calculation criteria of the tax and difficulty of administrative control. The high levels of evasion were attributed to inefficient collection by the tax authorities. As a consequence of these ideas, the administration of ITR was transferred to the Federal Revenue Secretariat (SRF) in 1990.

Third Phase: Post-1996.

In response to the continuing problems, reform was again undertaken in December 1996, including, among others, the following changes:

- An increase in the tax rate for large and unproductive holdings - the maximum limit of 4.5% for properties larger than 15 thousand hectares rose to 20% on properties above only 5 thousand hectares;
- A reduction in the number of tax-rate brackets from 12 to 6;
- The end of regional differences in rates;
- The value declared by the owner for ITR purposes would henceforth be considered the appraised value to be paid in the event of expropriation.

The rates differ only by degree of utilization and total area, as shown in Table 1.1. There are sharply progressive rates in relation to area and regressive ones for land use, so that productive properties are benefited.
Table 1.1 – Rates for Calculation of ITR

<table>
<thead>
<tr>
<th>Total area (in hectares)</th>
<th>Degree of Use(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&gt;80</td>
</tr>
<tr>
<td>Up to 50</td>
<td>0.03</td>
</tr>
<tr>
<td>50 to 200</td>
<td>0.07</td>
</tr>
<tr>
<td>200 to 500</td>
<td>0.10</td>
</tr>
<tr>
<td>500 to 1000</td>
<td>0.15</td>
</tr>
<tr>
<td>1000 to 5000</td>
<td>0.30</td>
</tr>
<tr>
<td>Over 5000</td>
<td>0.45</td>
</tr>
</tbody>
</table>


Reydon et al. (2000) point out the discontinuity in the rates adopted, observing that a property of 50.1 ha with 80.1% of the land utilized can pay 13 times the tax of one of 50.0 ha and 80.0% degree of use. The solution proposed by many authors is the use of reduction factors, as occurs with income tax.
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On top of that, Reydon et al. (2000) show that despite administrative and legal improvements, the high hopes for the reform have not been met. The main reasons are associated with the difficulty of accurately appraising the value of raw land and the imprecise nature of the concept of area utilized. Figure 1.2 shows on the one hand the improvement obtained with the 1996 reform and, on the other hand, the low level of collection. The area (right axis) represents the evolution of ITR collection and the solid line shows the participation of ITR collection in the total administered by the SRF. Even doubling its collection the ITR has been an insignificant source of funds to SRF.

According to calculations by Oliveira (1993), the potential revenue from ITR would range between 1.4 and 2.8 billion dollars per year if rates between 0.5% and 1.0% were used. Although these calculations do not consider the effect of the effective application of these rates on landowners’ decisions, the magnitude of the estimates makes it clear that much can be done to increase ITR revenues.

Thus, even after ameliorating a series of operational problems, ITR continues to be largely ineffective. The description in this section is indicative of the incapacity of the Brazilian government to correctly apply this tax scheme and reduce the high level of evasion and undertaxation. The provided data also point to the fact that this incapacity is more chronic for larger properties.

3. Basic Structure of the Model.

The model concentrates on the problem of a government that maximizes its utility by designing a taxation mechanism based on the quantity produced and the farm size. This mechanism is capable of implementing the system of equilibrium transfers to the government. The observation of the amount of output is considered, for purposes
of simplification, as the observed ICMS itself, since evasion of ICMS is much lower than that of ITR and not significant to our purposes.

There are non-agricultural uses for land that are considered in ad hoc form, as a reduced form from other models in which land is used for non-productive ends. The basic arguments can be grouped into three categories.

The literature of credit rationing [Stiglitz and Weiss (1981)] under asymmetric information points out the use of collateral. It may be optimal for banks to ration the volume of loans instead of rising the lending rate, as would be predicted by classical economic analysis. The use of collateral may improve the equilibrium of such economy. And land property has been used as reliable collateral [Deininger and Feder (1998), Hoff, Braverman and Stiglitz (1993)].

The second argument reflects an imperfection in the financial markets due to the non-existence of a financial asset that perfectly replicates the features of land property. Then, especially in periods of high inflation, land has been used as a hedge against macroeconomic instabilities [Brando and Rezende (1992), Feldstein (1980)].

Recently, Conning (2001) formalized the idea of a monopoly-cum-monopsony in an agrarian economy. In his model, landholding provides monopsony and monopoly power to landlords. Landlords may choose to withhold land from lease market in an effort to drive up rental rate and hence to decrease the demand for workers, exercising the monopsony power. In other words, the land property provides extra rents, derived from a ‘monopsony’ situation, to large landholders. The structure of markets for land and labor in this case matters for the efficiency of agricultural production.

The model considers, therefore, an economy in which the types of rural producers are indexed by \((\theta, \eta) \in \Theta\), where \(\Theta = [\underline{\theta}, \overline{\theta}] \times [\underline{\eta}, \overline{\eta}]\), distributed according to the distribution function \(F_{\theta \eta}\) with support
in the entire rectangle. The parameters $\theta$ and $\eta$ refer respectively to agricultural productivity and benefits from the non-productive activity.

The analysis is carried out in a partial equilibrium environment. The price obtained for agricultural output is normalized at 1, an unlimited quantity of land is available at price $r$ and each planted hectare costs\(^3\) $w$. Both production and land are perfectly homogeneous.

A farmer of type $(\theta, \eta)$ who buys a property of size $T$, grows $A$ and pays a transfer $t$ to the government, obtains profits given by:

$$\Pi = \theta Q(A) - wA + \eta \phi(T) - rT - t$$

where $Q$ and $\phi$ are respectively the agricultural production function and the non-agricultural rents, with $Q' > 0, Q'' < 0, Q'(0) = \infty, Q'(\infty) = 0, \phi' > 0, \phi'' < 0, \phi'(0) = \infty$ and $\phi'(\infty) = 0$. The government, which can condition it on the amount produced and the size of the holding, determines this transfer, which is observable. It can be noted that the greater $\eta$ is the larger the non-productive benefit, and likewise, as $\theta$ increases, so does agricultural productivity.

Assume that there is no rental market and hence the choice of each producer must respect the scarcity condition $A \leq T$. In this fashion, based on a taxation scheme $t$, farmers are faced with the program:

$$\max_{A,T} \Pi \text{ s.t. } A \leq T. \quad (P)$$

The government’s utility is dependent on tax revenues and the non-productive use of land. With $\lambda \in [0, 1]$ being the “shadow price”\(^3\)

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\(^3\)Implicitly, to simplify, the productive technology includes fixed proportions, in which $w$ represents the costs of labor and intermediate inputs per hectare.
attributed to speculative activity, the government utility function is defined by:

$$U = t - \lambda \eta \phi (T).$$

Therefore, the government decides to oppose speculative activity, which appears to be the Brazilian case.

Opposition to non-productive land use, incorporated in ad hoc form, is justified by market imperfections that determine an inefficient resource allocation. Commonly, $\lambda > 0$ reflects the existence of agents obtaining non-agricultural payoffs from land and an imperfect market for land leasing.

In this case, if land is a scarce resource, its price is above the expected present value of future agricultural profits. Without a land rental market, those restricted agricultural producers access a smaller plot than would be socially desirable, reducing the agricultural output. Therefore, the land distribution becomes not only unequal but also inefficient. To avoid this inefficiency, one possibility to the government is to discourage the profits from the non-productive activity.

The presence of tax revenue in the government’s utility function can be justified in different ways. A possible reason would be that individuals in the economy demand public goods, which are not provided optimally by the private sector. So the government, motivated by its desire to remain in power, has an incentive to maximize its tax receipts in case the provision of resources involves a sufficiently large volume.

To simplify the analysis and bring the problem closer to Brazilian reality, only the case where there is a deterministic relationship between the types $\theta$ and $\eta$ is considered, i.e., $\theta = \theta (\eta)$, with $\theta(.)$ continuous and differentiable. In this form, the producers can be completely specified by the parameter $\eta$. Assume that the distribution of $\eta$ is given by the distribution function $F$ and density $f$. 

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The next step is to present the assumptions used to prove the results. These assumptions aim both at tying the analysis to the Brazilian reality and at simplifying technical matters.

Assumptions

- **(Monotone Likehood Ratio):** the distribution function is such that:

\[
\frac{d}{d\eta} \left( \frac{1 - F(\eta)}{f(\eta)} \right) < 0
\]  

(A1)

- **(Technologies):** the functions \(\theta, \phi\) and \(Q\) are such that, for \(A = T\) and \(k\) sufficiently high\(^4\),

\[
\dot{\theta} < 0 \text{ and } \ddot{\theta} = 0,
\]  

(A2)

\[
\dot{\theta}Q + \phi > 0,
\]  

(A3a)

\[
k\dot{\theta}Q' + \phi' > 0
\]  

(A3b)

\[
\frac{\frac{d}{d\eta} \left( \frac{1 - F}{f} \right)}{[1 - \frac{d}{d\eta} \left( \frac{1 - F}{f} \right)] \dot{\theta}Q'} > 1.
\]  

(A3c)

Assumption 1 is usual in the literature of mechanism design to avoid technical complications in the characterization of the equilibrium [see Salanié (1998)]. It does not qualitatively alter the obtained results and it is satisfied for several probability distributions such as uniform, normal, logistic, exponential, etc.

\(^4\)In the proof of proposition 3' in the Appendix, a lower bound to \(k\) is determined.
Assumption A2 determines that those who use land for non-productive ends have a lower agricultural productivity, which is observed in Brazil [Barros et al. (2000), Guanziroli and Cardim (2000)]. In most cases, especially those with labor-intensive technologies, these producers are only part-time agricultural producers and, given the imperfections of labor markets due to moral hazard problems, become less efficient [Deininger and Feder (1998)].

The other conditions are technical and ensure an easier characterization of the optimal solution. Note that, given (A3a) and (A3b), the assumption (A1) and (A3c) are satisfied to uniform distribution.

Assumption (A3a) establishes that a marginal increase in $\eta$ determines an increase in the level of the non-productive payoff strictly greater than the increase of the agricultural revenue. Condition (A3b) has the same implication to the marginal returns, but with a larger difference. Note that, given (A3a) and (A3b) with $k = 2$, (A3) is satisfied for the uniform case.

Assumptions (A2) and (A3) refer to the diversity of landholders in the economy, emphasizing a well-known stylized fact [Deininger and Feder (1998), Barros et al. (2000), Guanziroli and Cardim (2000)]. Specifically, they imply a negative relationship between farm size and productivity.

The results presented in the next two sections are rigorously enunciated and demonstrated in the Appendix. To illustrate these results graphically, a numerical example is considered. In this numerical example, $Q = \log(A)$, $\phi = k\log(T)$, $\theta = m - \eta$ and $\eta \sim U[0,1]$, where $k$ and $m$ are constants as well as $w$, $r$ and $\lambda$, chosen in a convenient way$^5$.

$^5k=4, m=5, w=0.045, r=0.01$ and $\lambda=0.05$. 
4. Results with Complete Information.

Initially, the taxation mechanism is designed in an environment of complete information, i.e., the government can exactly observe the type of agents and manages to establish tax collection rules that consider the willingness of each producer to use land for agricultural production or not. This is an initial analysis to establish the benchmark situation.

Under complete information, the government can determine the allocations for each producer via a punitive taxation scheme. The sole conditions that restrict the government’s choice are those of scarcity (SC) and participation (IR) for each producer. The producers accept any government taxation system that produces a non-negative profit level in equilibrium. The first-best program (P.FB) in this case defines the optimal taxation mechanism for a producer of type $\eta$:

$$\max_{t,A,T} U$$

subject to

$$\Pi \geq 0, \quad (IR)$$

$$A \leq T. \quad (SC)$$

The solution to this program is characterized by the following result.

**Proposition 1:** Under (A2), the optimal taxation mechanism for (P.FB) can involve two categories of producers: (i) those who operate without idle land and are restricted by (SC): $\eta \in \Theta_R = [\eta, \eta^*]$; (ii) those for whom the restriction (SC) is not active: $\eta \in \Theta_I = (\eta^*, \eta]$.

a) In $\Theta_R$ the equilibrium allocation is such that the cultivated area is equal to the property size. Both are determined by the equality between the total marginal benefit $[\theta Q'(A^*) + \eta \phi'(T^*_\eta)]$ and the marginal cost of each cultivated hectare $[w + r + \lambda \eta \phi'(T^*_\eta)]$. 
b) In $\theta_I$ the producers are not restricted by (SC). The cultivated area is determined by the equality between the marginal benefit $[\theta Q' (A^*_\eta)]$ and the marginal cost of each cultivated hectare $[w]$; The size of the holding is such that it equalizes the marginal benefit from speculative activity $[\eta \phi' (T^*_\eta)]$ to its marginal social cost $[r + \lambda \eta \phi' (T^*_\eta)]$;

c) The government appropriates all the producers’ profit.

Note that the marginal social cost differs from the marginal individual cost by $\lambda \eta \phi' (T^*_\eta)$, which suggests a natural interpretation for $\lambda$. When $\lambda = 0$, the government is not bothered by the non-productive benefit of land ownership and the levels of $A$ and $T$ determined by the government’s design of the optimal taxation scheme (P.FB) coincide with those determined by the agents (P). In this case, the shadow price of speculation is zero and the government maximizes the individual profit of each producer, which is fully appropriated. On the other hand, if $\lambda = 1$, the government completely inhibits idle land since the shadow price of the scarcity restriction becomes constant and equal to $r$ for all $\eta \in \Theta$.

The transfers required by the government represent all the producers’ profits, i.e., in the model with complete information, the constraint (IR) is binding in equilibrium.

Figure 4.1 qualitatively illustrates the shape of the allocations associated with the optimal taxation scheme.
The result that follows shows that this scheme can be implemented by a system analogous to ITR.

**Proposition 2:** Under \((A2)\), the solution to the optimal taxation problem \((P.FB)\) can be decentralized by a menu of linear taxes of the form:

\[
t^*_\eta = \beta^*_\eta T + \gamma^*_\eta
\]

where \(\beta^*_\eta\) corresponds to the difference between the marginal social and individual costs of land ownership and \(\gamma^*_\eta\) is a fixed part that adjusts the level of tax collection.

In this implementation result, the government offers a pair \((\beta^*_\eta, \gamma^*_\eta)\) to a producer of type \(\eta\). Solving \((P)\), each farmer chooses the amount of \(A\) and \(T\) which is determined by the government’s optimal solution. Note that there is a direct analogy between ITR for performing its function on land distribution and the parameter
the willingness of the farmer to use his land unproductively, which is measured by $\eta$. If $\lambda = 0$, the government does not distort the choice of producers by not taxing ownership of land.

In $\Theta_I$, we have $\beta_\eta^* = \frac{\lambda}{1 - \lambda}$ and, therefore, the ITR rate does not vary according to the type of producer. Thus, the model shows that in a context of complete information, one may use a single rate for all producers operating with properties above a determined size. Figure 4.2 illustrates the shape that the optimal ITR rates must have in the case of complete information.

Figure 4.2 – Optimal Rates with Complete Information
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This result shows that if the government could precisely observe landowner productivity parameters, there would be rates capable of implementing an optimal taxation scheme. Therefore, ITR would be able to implement this optimal solution. And in this solution, if \( \lambda > 0 \), the government discourages the speculative use of land, and with \( \lambda = 1 \), there would be no idle land in equilibrium.

5. Results with Asymmetric Information.

Consider a more realistic case in which there is asymmetric information about productivity and the benefits of land speculation. The problem of choosing the optimal taxation scheme then becomes a problem of typical mechanism design.

By the Revelation Principle, it is sufficient for the government to concentrate on mechanisms that directly reveal the truth [Mirrlees (1971)]. In this fashion, besides considering the constraints of scarcity (SC) and participation (IR) for each type of producer, the tax scheme is restricted by the incentive compatibility (IC) constraints.

The (IC) constraints for each type of producer are given by

\[
\eta \in \arg\max \Pi (\hat{\eta}|\eta),
\]

where \( \Pi (\hat{\eta}|\eta) \) is the profit of a producer of type \( \eta \) who declares to be type \( \hat{\eta} \). This is to say, we can focus in an optimal taxation mechanism in which a producer of type \( \eta \) prefers the allocation \((t_\eta, T_\eta, A_\eta)\) to all others.

The government through resolution of the following second-best program determines the optimal taxation mechanism under asymmetric information:

\[
\max_{\{t_\eta, A_\eta, T_\eta\}_{\eta \in [\eta, \bar{\eta}]}} \int_{\eta}^{\bar{\eta}} [t_\eta - \lambda \eta \phi (T_\eta)] dF (\eta) \tag{P.SB}
\]
subject to, for any \((\eta, \hat{\eta}) \in [\eta, \bar{\eta}]^2\),

\[
\Pi (\eta | \eta) \geq 0, \quad (IR_\eta)
\]

\[
\Pi (\eta | \eta) \geq \Pi (\hat{\eta} | \eta), \quad (IC_\eta)
\]

\[
A_\eta \leq T_\eta. \quad (SC_\eta)
\]

Note that the government’s maximization program is written in the variables \(t, A\) and \(T\), withstanding that in the model the government observes only \(T\) and \(\theta Q\) in determining \(t\). Nevertheless, as will be shown in the demonstration of proposition 4’ (in the Appendix), a mechanism based on \(t, A\) and \(T\) can be implemented by a mechanism in \(t, T\) and \(\theta Q\), and vice-versa. In choosing an optimal mechanism, the government incorporates the restrictions \((SC_\eta)\) in its decision, since the restrictions of scarcity and individual rationality of the agents are common knowledge.

The solution of the model with asymmetric information is given by the following proposition.

**Proposition 3**: Assume \((A1)-(A3)\). The optimum tax scheme \((\hat{t}_\eta, \hat{A}_\eta, \hat{T}_\eta)\) of the program \((PSB)\) has the following properties:

i) The are most two categories of producers: \(\tilde{\Theta}_R = [\eta, \bar{\eta}]\) and \(\tilde{\Theta}_I = (\tilde{\eta}, \bar{\eta}]\) such that \(\hat{A}_\eta = \hat{T}_\eta\) for all \(\eta \in \tilde{\Theta}_R\). There is an increase in the number of producers restricted by \((SC)\), i.e., \(\bar{\eta} > \eta^*\);

ii) In \(\tilde{\Theta}_R\), \((\hat{A}_\eta, \hat{T}_\eta)\) are determined by the equality between the marginal benefit and marginal “virtual” cost of each cultivated hectare, which corresponds to the marginal social cost added to the marginal informational rent:

\[
\left(\frac{1 - F(\eta)}{f(\eta)} \left(\theta Q' (\hat{A}_\eta) + \phi (\hat{T}_\eta)\right)\right),
\]

but still, \(\hat{A}_\eta = \hat{T}_\eta > 0\);
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iii) In $\tilde{\Theta}_I$, $\left(\tilde{A}_\eta, \tilde{T}_\eta\right)$ are determined by the equality between the marginal benefit and marginal "virtual" cost of each cultivated hectare and each hectare of the total land: $\frac{1-F(\eta)}{f(\eta)}\tilde{\theta}Q'\left(\tilde{A}_\eta\right)$ and $\frac{1-F(\eta)}{f(\eta)}\phi'\left(\tilde{T}_\eta\right)$, respectively. Also, $\tilde{T}_\eta > 0$ and $\tilde{A}_\eta < 0$;

iv) Because of asymmetric information, the transfers to the government are deducted from the informational rent obtained by each agent;

v) Only agents of type $\eta$ fail to receive informational rent; for the others, the constraints (IR) are not active.

This result demonstrates the effect of asymmetric information on the choice of the amount of land used productively and the one left idle. Even if $\lambda = 1$, there may exist $\tilde{\eta} \in (\eta, \bar{\eta})$ in such a way that the optimal tax scheme of the government cannot fully inhibit idle land.

Figure 5.1 – Optimal Allocation with Complete and Incomplete Information
Even for producers without idle land, $\eta \in [\overline{\eta}, \overline{\eta}]$, there is a distortion in the size of the enterprise with respect to the case with complete information. For these producers, this distortion means cultivating smaller plots. Figure 5.1 shows a comparison between the allocations obtained in the cases of complete and incomplete information. Note that except for those producers of type $\overline{\eta}$, the choices of cultivated area and establishment size are distorted.

The implementation of the optimal allocation via a menu of linear taxes is presented in proposition 4. The total tax is constituted by a three-part tariff, an output tax, a land tax and a fixed part. The fixed part does not alter the choices of the cultivated area $A$ and farm size $T$ - it simply adjusts the total tax to a maximum level for each producer. Allocations are completely determined by the output tax and land tax.

**Proposition 4:** Under $(A1)-(A3)$, the solution to the optimal tax-ation problem (P.SB) can be decentralized by a menu of linear taxes of the form

$$\alpha_A \theta Q(A) + \beta_T T + \gamma(A,T),$$

where $\alpha_A = 0$ for all $A \in [\tilde{A}_\eta, \tilde{A}_\eta]$. In this scheme, the government offers $(\alpha_A, \beta_T, \gamma(A,T))$ observing production $\theta Q(A)$ and farm size $T$. Owners, maximizing their profit, determine $A$ and $T$ in accordance with the conditions of proposition 3.

The above propositions show that the scheme put forward as a solution for the model with complete information cannot be implemented under asymmetric information. This inability to implement ITR as a solution to the problem with asymmetric information gives theoretical support to what the government found in the 1980s by comparing declared against actual data. It is also consistent with

---

6Formally, $\frac{\partial}{\partial A} \gamma(A,T) = \frac{\partial}{\partial T} \gamma(A,T) = 0.
the fact that small producers, in general with no idle land, are more likely to correctly declare their land use.

Figure 5.2 – Optimal Rates of ICMS and ITR

In an economy such as Brazil’s, where producers operate with idle land and the government is often in the dark as to the true parameters of the productive and non-productive activities available to various landholders, proposition 4 shows that there are no tax rates able to make ITR an optimal taxation scheme. The use of ICMS becomes necessary so that producers with better access to non-productive activities will pay their fair share of the tax burden.

Figure 5.2 shows the format for optimal ICMS and ITR rates to implement the optimal taxation scheme. ICMS is zero for producers who are restricted by (SC), operating without idle land. As the level of idle land rises, the ICMS rate rises as well, while the ITR rate decreases. Even though the tax on output causes a distortion in resource allocation, its use is justified by its ability to compose an implementable (or self-revealing) taxation mechanism.
6. Discussion.

Basic Assumptions and the Generality of the Argument

The results presented in the previous section are formally dependent on assumption (A1)-(A3). These assumptions simplify the characterization of the solution of the mechanism design problem and bring the problem closer to the Brazilian reality. However, the basic argument about the power of ITR as the exclusive tax instrument can be extended to a more general context.

The first issue is about the government’s utility function. Suppose a more general case in which \( U(t, A, T) = t - \psi(A, T) \), where \( \psi \) is strictly decreasing in \( A \) and increasing in \( T \). In other words, the government wants to promote agricultural production and to avoid landholdings\(^7\). In this case, the use of output tax is also required in a linear implementation under complete information, such as in proposition 2. Therefore, relying only on land tax (ITR) would not be optimal even if the productivity parameters were publicly observed.

On the other hand, the utility function used in the previous sections shows that the government’s attempt to use ITR as the unique tax instrument can be rationalized in a particular economic environment. Specifically, propositions 2 and 4 show that the economic environment in which the use of ITR as an optimal scheme is such that the productivity parameters are publicly observed. In this case, the government can observe them under complete information or precisely infer them when there is no idle land and the observation of \( T \) and \( A \) are equivalent.

The second issue is about the perfect correlation assumption between \( \theta \) and \( \eta \), i.e., \( \theta = \theta(\eta) \). The two-dimensional general case is

---

\(^7\)This functional form includes the cases in which the government combats idle land, measured as \( T/A \) or \( T - A \).
technically much more complex, and its solution usually involves the adoption of a sorting that would not provide any additional insight [see Rochet and Chone (1997)]. If \( \hat{\theta} > 0 \), the major difference would occur in the use of the output tax, used as a subsidy in certain situations. However, as mentioned above, the case defined by (A2) is the most interesting case to the Brazilian experience.

Assumptions (A3a)-(A3c) are used for the sake of simplification. Again, these assumptions do not significantly alter the results. For example, (A3a) establishes that those producers with higher non-productive benefits have an incentive to self declare as producers with a lower \( \eta \). Informational rents are increasing in \( \eta \) and, therefore, the participation constraint is binding in equilibrium at type \( \eta \). Otherwise, the solution of (P.SB) could be analyzed only with specific functional forms.

Assumption (A3b) determines that, with a marginal increase in \( \eta \), the increase in the non-productive benefits is sufficient to cover the \( k \)-times decrease in the agricultural productivity. For any \( A \) and \( T \), the total profits are increasing in \( \eta \). As already mentioned, this assumption supports the Brazilian case. It implies that the second order condition of the IC constraint in (P.SB) is not binding. Otherwise, the "ironing principle" would be used in the solution. Assumptions (A3b) and (A3c) provide a simple characterization of the solution without using specific functional forms.

Given (A1), (A3) is a sufficient condition to the multiplier of (SC) which will decrease while profits increase in \( \eta \). This is the key behind the simplifications obtained in the characterization results. If it were not the case, the set of producers with or without idle land would be non-connected subsets of \( \Theta \).
Implementation

The result enunciated in proposition 4 is the primary contribution of this paper to the debate about land taxation in Brazil. Now, we discuss how to implement this result in practice.

In proposition 4, the total tax is composed by three parts: the output tax (ICMS), the land tax (ITR) and a fixed payment. The fixed part does not affect the allocation of resources and it is used to adjust the level of collection to the maximum values. In the following analysis of the implementation, this term will be ignored. Actually, the discussion will be centered in the basic arguments implied by the model and in the easiest ways to implement them.

At least two major concerns arise when we try to translate the obtained results into straight prescriptions for a policy reform. The first one is that the relationship between $\theta$ and $\eta$ is deterministic. Therefore the model implies that all farms in the economy can be perfectly described by the farm size in equilibrium. In equilibrium, a farm with size $\hat{T}_{\eta}$ is choosing to cultivate an area $\hat{A}_{\eta}$ and so on. Although it is a natural and reasonable approximation for theoretical purposes, it does not seem to be the most appropriate description of the Brazilian reality. For practical analysis, it is better to consider that for each farm size, there is an interval with positive measure of possible allocations. Other sources of technological differences among farmers may produce a similar effect.

The second issue is about the use of the output tax (ICMS). Brazil already has a well-defined ICMS structure that should be treated as given. A major reform involving both taxes may be desirable as a possible benchmark to the future. However, as long as any tax collection reform has significant political (and therefore social)
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costs, we will concentrate on offering suggestions for a modification in ITR collection, given the ICMS structure.

A natural way to implement the result of proposition 4 is to consider the tax rate a function of the farm size and the total ICMS collected per hectare. It implies only a straightforward change in the columns of table 1.1. However, given the remarks above, the way the rates vary with ICMS and the farm size will be slightly modified in light of proposition 4. The degree of use, which is a non-observable variable, should be replaced with the amount of ICMS collected per hectare, which is an adequate proxy to the agricultural productivity in the model.

This kind of implementation is presented in Table 6.1. Although the number of ranges has only an illustrative role in the table, the general pattern is a slightly different but feasible translation of the properties induced by proposition 4. The tax rates should be such that

\[ t_{i1} \geq t_{i2} \geq t_{i3} \geq t_{i4} \geq t_{i5}, i = 1, \ldots, 6 \]

and

\[ t_{1j} \leq t_{2j} \leq t_{3j} \leq t_{4j} \leq t_{5j} \leq t_{6j}, j = 1, \ldots, 5. \]

\[ ^{\text{More precisely, proposition 4 implies } t_{ij} \text{'s are constant in } j \text{ for low values of } i \text{ and vice-versa. Moreover, the marginal increase is eventually non-monotonic in } i \text{ for high values of } i \text{ and } j. \text{ But this is the case in which the ICMS structure is not taken as given.}} \]
This manner of implementation has a great potential to improve not only the application of ITR but also the collection of ICMS. The scheme proposed implies an additional cost to the evasion and default of ICMS given by the increase in the ITR tax rate. Therefore, this new reformulation constitutes a new step toward more reliable tax institutions. Note that only the information about the IS required in this scheme. Hence, there is no need for large modifications in the existent institutions and mechanisms.

However, the determination of the specific parameters of this new formulation involves a deep and multidisciplinary investigation of tax rates, exceptions, regional differentiation, formal definitions, etc. If the new rates become effective, then the impact on the sustainability of the agricultural sector should be analyzed carefully. For example, tax rates varying from 0.03% to 20% may not be adequate, and they may produce a reverse in the agriculture. Besides the legal issues, the tax instrument should be viewed as an important mechanism to improve agricultural development, not only as another source to finance public expenditures.
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7. Conclusion.

The need to evaluate taxation on land arises from its importance as an instrument for land reform, especially in Brazil.

Agriculture in developing countries like Brazil is characterized by a series of market imperfections that make the price of land higher than the discounted value of the revenue flows generated by agricultural activity and with farm size artificially high. This situation leads to equilibria in which large unproductive landholdings contrast starkly with artificially high land prices that seriously limit the ability of poor family farmers to reach their full productive potential.

Without serious efforts to combat the unproductive use of cultivable land, land reform (via redistribution or land banks) will only have transitory effects. ITR constitutes an important instrument for land redistribution, besides financing other rural anti-poverty programs. In this sense, intensified land reform programs are justified by the need to catalyze the adequate functioning of market forces.

The model presented shows that because ITR is a declaratory tax, the optimal taxation scheme in a context of asymmetric information involves a combination of taxes on land and production. The current rules, even after the 1996 reformulation, would be implementable only if there were complete information or no idle land. Even without problems in administering the tax, the model demonstrates the inability of ITR to combat the chronic high levels of evasion and underreporting.

Hence, the discussion of the effectiveness of ITR needs to be redirected. Besides considering the operational aspects (vital to obtain the desired results), the scope of the instrument in an environment
such as Brazilian agriculture also needs to be questioned. This paper attempts to contribute to expanding the debate on land taxation in Brazil, in complement to the other studies mentioned in Section II.

Furthermore, the suggestion of using a mixed system of land and production taxes perfectly fits the discussion of tax reform in which Brazil is currently engaged. The proposal to return competence for administering ITR to the states could facilitate a cross checking of data regarding the two taxes, in complement to the other studies mentioned in Section I.

There is one closing caveat. The analysis presented here uses a simple theoretical structure that is adequate only to establish the limits of applying a scheme such as ITR. But, faced with the prospect of using ICMS as an important source of information, discussion of the operational aspects again comes to the forefront. Only a careful and detailed empirical investigation can determine the parameters for a new reformulation of rural taxation policy.

Appendix.

This appendix contains the formal arguments presented in sections IV and V. Propositions 1 to 4 are proved rigorously using the notation introduced in section II.

Complete Information

The solution of (P.FB) is characterized by proposition 1' which is a formal version of proposition 1.

Proposition 1': Under (A2), the optimal taxation mechanism of (P.FB) can involve two categories of producers, i.e., there is at most one \( \eta^* \) such that \( \Theta_R = [\eta, \eta^*] \) and \( \Theta_I = (\eta^*, \overline{\eta}] \):
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a) small farmers (without idle land): for every type $\eta \in \Theta_R$, $A^*_\eta$ and $T^*_\eta$ are defined by:

$$A^*_\eta = T^*_\eta,$$

and

$$\theta Q'(A^*_\eta) + (1 - \lambda) \eta \phi'(T^*_\eta) = w + r.$$

b) large landholders (with idle land): for every type $\eta \in \Theta_I$, $A^*_\eta$ and $T^*_\eta$ are defined by:

$$\theta Q'(A^*_\eta) = w,$$

and

$$(1 - \lambda) \eta \phi'(T^*_\eta) = r.$$

c) transfers: every type $\eta \in \Theta$, $t^*_\eta$ is defined by:

$$t^*_\eta = \theta Q(A^*_\eta) - w A^*_\eta + \eta \phi(T^*_\eta) - r T^*_\eta.$$ 

Proof. Initially, note that the (IR) is always binding. If it is not the case for $\eta \in \Theta$, the government would increase tax collection, $t^*_\eta$, satisfying the constraints in (P.FB). Solving (IR) for $t$ and substituting in the objective function, (P.FB) can be written, for each $\eta \in \Theta$, as

$$\max \theta Q(A_\eta) - w A_\eta + (1 - \lambda) \eta \phi(T_\eta) - r T_\eta \quad \text{s.t.} \quad A_\eta \leq T_\eta.$$ 

The first order condition is given by:

$$\theta Q'(A_\eta) = w + \mu_\eta,$$

and

$$(1 - \lambda) \eta \phi'(T_\eta) = r - \mu_\eta,$$

$$\mu_\eta (A_\eta - T_\eta) = 0, \mu_\eta \geq 0.$$
By differentiating \( \mu_\eta \), we get

\[
\dot{\mu}_\eta = \dot{\theta} Q' + \theta Q'' \dot{A}_\eta
\]

\[
= \frac{(1 - \lambda) \left[ \eta \dot{\theta} Q' \phi'' - \theta Q' \phi' \right]}{\theta Q'' + (1 - \lambda) \eta \phi''} < 0
\]

and under (A2), there is at most one \( \eta^* \) such that \( \mu_{\eta^*} = 0 \). Thus, depending on the technologies of agricultural and non-productive activities, we can identify the mentioned categories of producers. (FB\(_R\)) and (FB\(_U\)) give the first-order conditions for each group, respectively.

The next result shows that the optimal scheme can be implemented by a system analogous to ITR under complete information.

**Proposition 2':** Under (A2), a menu of linear taxes can decentralize the solution to the optimal taxation problem (P.FB):

\[
t^*_\eta = \beta^*_\eta T + \gamma^*_\eta,
\]

where \( \beta^*_\eta = \lambda \eta \phi' (T^*_\eta) \) and \( \gamma^*_\eta = t^*_\eta - \beta^*_\eta T^*_\eta \).

**Proof.** Consider \( t_\eta = \beta^*_\eta T + \gamma^*_\eta \), where \( \beta^*_\eta = \lambda \eta \phi' (T^*_\eta) \) and \( \gamma^*_\eta = t^*_\eta - \beta^*_\eta T^*_\eta \). The first-order conditions can be written as:

\[
\theta Q' (A) = w + \mu_\eta
\]

\[
\eta \phi' (T) = r + \beta^*_\eta - \mu_\eta.
\]

Using the definition of \( \beta^*_\eta \) we note that the system above is equivalent to that constituted by the first-order conditions of (P.FB). The value of \( \gamma^*_\eta \) adjusts the level of the transfers.
Incomplete Information

Let $\Pi (\hat{\eta}|\eta)$ denote the profits of producer $\eta$ that declares being $\hat{\eta}$, i.e.,

$$\Pi (\hat{\eta}|\eta) = \theta Q (A_{\hat{\eta}}) - w A_{\hat{\eta}} + \eta \phi (T_{\hat{\eta}}) - r T_{\hat{\eta}} - t_{\hat{\eta}}, \ \forall (\eta, \hat{\eta}) \in [\eta, \bar{\eta}]^2.$$

Thus, we can define:

**Definition:** An allocation $(t_\eta, A_\eta, T_\eta)$ is implementable if, and only if

$$\Pi (\eta|\eta) \geq \Pi (\hat{\eta}|\eta) \ \forall (\hat{\eta}|\eta) \in [\eta, \bar{\eta}]^2. \quad (IC)$$

The next lemma relies on Guesnerie and Laffont (1984) and establishes sufficient conditions for implementability.

**Lemma:** If the allocation $(t_\eta, A_\eta, T_\eta)$ is implementable and piecewise $C^1$ then:

$$\frac{d}{d\eta} \pi (\eta|\eta) = \dot{\theta} Q (A_\eta) + \phi (T_\eta), \quad (IC^1)$$

and

$$\dot{\theta} Q' (A_\eta) \dot{A}_\eta + \phi' (T_\eta) \dot{T}_\eta \geq 0, \text{ a.s. in } [\eta, \bar{\eta}] . \quad (IC^2)$$

**Proof.** Let $(t_\eta, A_\eta, T_\eta)$ be an implementable allocation piecewise $C^1$. Thus, for every $\eta \in [\eta, \bar{\eta}]$,

$$\Pi (\eta|\eta) \geq \Pi (\hat{\eta}|\eta) = \Pi (\hat{\eta}|\hat{\eta}) + \left( \theta - \dot{\theta} \right) Q (A_{\hat{\eta}}) + (\eta - \hat{\eta}) \phi (T_{\hat{\eta}})$$

where $\dot{\theta} \equiv \theta (\hat{\eta})$. Hence,

$$\Pi (\eta|\eta) - \Pi (\hat{\eta}|\hat{\eta}) \geq \left( \theta - \dot{\theta} \right) Q (A_{\hat{\eta}}) + (\eta - \hat{\eta}) \phi (T_{\hat{\eta}})$$
and, switching $\eta$ and $\hat{\eta}$, we get

$$
(\theta - \hat{\theta}) Q(A_\eta) + (\eta - \hat{\eta}) \phi(T_\eta) \geq \Pi(\eta|\eta) - \Pi(\hat{\eta}|\hat{\eta}) \geq (\theta - \hat{\theta}) Q(A_{\hat{\eta}}) + (\eta - \hat{\eta}) \phi(T_{\hat{\eta}}).
$$

Dividing by $(\eta - \hat{\eta})$ and taking the limit $\hat{\eta} \to \eta$, the function $\Pi$ is proved to be piecewise $C^1$ and

$$
\frac{d}{d\eta} \Pi(\eta|\eta) = \hat{\theta} Q(A_\eta) + \phi(T_\eta) \quad a.s.
$$

and, doing the same to $(\eta - \hat{\eta})^2$, we get

$$
\hat{\theta} Q'(A_\eta) \dot{A}_\eta + \phi'(T_\eta) \dot{T}_\eta \geq 0 \quad a.s.
$$

Under (A3a), $\Pi(\eta|\eta)$ is increasing in $\eta$ and only the (IR$\eta$) is biding in equilibrium, which determines that $\Pi(\eta|\eta) = 0$. Integrating (IC$^1$) by parts and substituting into the objective function, the maximization program can be rewritten as:

$$
\max_{\{t_\eta,A_\eta,T_\eta\}} \int_{\eta}^{\bar{\eta}} \left[ \hat{\theta} Q(A_\eta) - wA_\eta + (1-\lambda)\eta \phi(T_\eta) - rT_\eta \\ - \frac{1-F(\eta)}{f(\eta)} (\hat{\theta} Q(A_\eta) + \phi(T_\eta)) \right] dF(\eta) \quad \text{(P.SB)}
$$

subject to

$$
\hat{\theta} Q'(A_\eta) \dot{A}_\eta + \phi'(T_\eta) \dot{T}_\eta \geq 0, \quad \text{(IC$^2$)}
$$

$$
A_\eta \leq T_\eta, \eta \in [\eta, \bar{\eta}], \quad \text{(SC$\eta$)}
$$
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The solution of (P.SB) is characterized by the following proposition.

**Proposition 3’**: Under (A1)-(A3) the optimum taxation scheme \((\tilde{t}_\eta, \tilde{A}_\eta, \tilde{T}_\eta)\) of the program (P.SB), ignoring the \((IC^2)\) constraint, has the following characteristics:

i) there are at most two categories of producers: \(\tilde{\Theta}_R = [\eta, \bar{\eta}]\) and \(\tilde{\Theta}_I = (\bar{\eta}, \bar{\eta}]\);

ii) in \(\tilde{\Theta}_R, (\tilde{A}_\eta, \tilde{T}_\eta)\) is determined by

\[
\tilde{A}_\eta = \tilde{T}_\eta,
\]

\[
\theta Q' (\tilde{A}_\eta) + (1 - \lambda) \eta \phi' (\tilde{T}_\eta) = w + r + \frac{1 - F (\eta)}{f (\eta)} \left( \theta Q' (\tilde{A}_\eta) + \phi' (\tilde{T}_\eta) \right)
\]

\(\text{with } \tilde{A}_\eta = \tilde{T}_\eta > 0;\)

iii) in \(\tilde{\Theta}_I, (\tilde{A}_\eta, \tilde{T}_\eta)\) is determined by

\[
\theta Q' (\tilde{A}_\eta) = w + \frac{1 - F (\eta)}{f (\eta)} \theta Q' (\tilde{A}_\eta),
\]

\[
(1 - \lambda) \eta \phi' (\tilde{T}_\eta) = r + \frac{1 - F (\eta)}{f (\eta)} \phi' (\tilde{T}_\eta)
\]

\(\text{with } \tilde{T}_\eta > 0 \text{ and } \tilde{A}_\eta < 0;\)

iv) \(\tilde{t}_\eta = \theta Q (\tilde{A}_\eta) - w \tilde{A}_\eta + \eta \phi (\tilde{T}_\eta) - r \tilde{T}_\eta - \int_{\eta}^{\bar{\eta}} \left[ \theta Q (\tilde{A}_\eta) + \phi (\tilde{T}_\eta) \right] d\eta;\)
v) $\Pi(\eta) = \int_\eta^\eta \left[ \dot{\theta}Q\left(\tilde{A}_\eta\right) + \phi\left(\tilde{T}_\eta\right) \right] d\eta$;

vi) for every $\eta \in [\eta, \eta^*]$, $\tilde{A}_\eta < A^*_\eta$ and $\tilde{\eta} > \eta^*$, i.e., because of asymmetric information, there is an increase in the proportion of bidding producers.

**Proof.** The first-order conditions of $(P.SB)$, ignoring $(IC^2)$, are given by:

$$\theta Q'(A_\eta) = w + \frac{1 - F(\eta)}{f(\eta)} \dot{\theta}Q'(A_\eta) + \frac{\mu_\eta}{f(\eta)},$$

$$(1 - \lambda) \eta \phi'(T_\eta) = r + \frac{1 - F(\eta)}{f(\eta)} \phi'(T_\eta) - \frac{\mu_\eta}{f(\eta)},$$

$$\mu_\eta (A_\eta - T_\eta) = 0, A_\eta \leq T_\eta.$$

Define $\Theta_R = \{\eta : \mu_\eta > 0\}$. Hence, for every $\eta \in \Theta_R$, $\tilde{A}_\eta = \tilde{T}_\eta$ and the first-order conditions can be written as

$$\theta Q' (\tilde{A}_\eta) + (1 - \lambda) \eta \phi' (\tilde{T}_\eta) = w + r + \frac{1 - F(\eta)}{f(\eta)} [\dot{\theta}Q' (\tilde{A}_\eta) + \phi' (\tilde{T}_\eta)],$$

$$\frac{\mu_\eta}{f(\eta)} = \theta Q' (\tilde{A}_\eta) - w - \frac{1 - F(\eta)}{f(\eta)} \dot{\theta}Q' (\tilde{A}_\eta).$$

The differentiation of the first condition with respect to $\eta$ implies that:

$$\dot{A} = \dot{T} = \frac{\frac{d}{d\eta} \left( \frac{1 - F}{f} \right) - 1}{\theta - \frac{1 - F}{f} \dot{\theta}} Q'' + \frac{\frac{d}{d\eta} \left( \frac{1 - F}{f} \right) - (1 - \lambda)}{(1 - \lambda) \eta - \frac{1 - F}{f} \phi''} > 0.$$
under (A1), (A2) and (A3), since the second-order condition guarantees that the denominator is negative. The value of $k$ refereed in (A3b) is given by

$$k = \max_{\eta} \frac{\frac{d}{d\eta} \left( \frac{1-F}{f} \right) - 1}{\frac{d}{d\eta} \left( \frac{1-F}{f} \right) - (1-\lambda)}.$$ 

Note that, if $\lambda = 0$ then $k = 1$.

Differentiating $\mu_\eta$, the assumptions imply

$$\left( \frac{\dot{\mu}_\eta}{f(\eta)} \right) = \left( \theta - \frac{1-F}{f} \dot{\theta} \right) Q'' \dot{A} + \left( 1 - \frac{d}{d\eta} \left( \frac{1-F}{f} \right) \right) \dot{\theta} Q' < 0,$$

which demonstrates the parts (i) and (ii).

Let $\Theta_I = \Theta - \Theta_R$. For every $\eta \in \Theta_I$, the first-order conditions become:

$$\theta Q' (\tilde{A}_\eta) = w + \frac{1-F(\eta)}{f(\eta)} \dot{\theta} Q' (\tilde{A}_\eta),$$

$$(1-\lambda) \eta \phi' (\tilde{T}_\eta) = r + \frac{1-F(\eta)}{f(\eta)} \phi' (\tilde{T}_\eta).$$

Thus, the item (iii) is demonstrated with:

$$\begin{align*}
\dot{A}_\eta &= - \frac{1 - \frac{d}{d\eta} \left( \frac{1-F}{f} \right)}{\left( \theta - \frac{1-F}{f} \dot{\theta} \right) Q''} \dot{\theta} Q' < 0, \\
\dot{T}_\eta &= - \frac{\left( (1-\lambda) - \frac{d}{d\eta} \left( \frac{1-F}{f} \right) \right) \phi'}{\left[ (1-\lambda) \eta - \frac{1-F}{f} \right] \phi''} > 0.
\end{align*}$$
The item (iv) and (v) can be easily derived from \( (IC^1) \).

The item (vi) involves a comparison between the conditions that determine \( A_\eta^* \) and \( \tilde{A}_\eta \) in \( \Theta_R \) and \( \tilde{\Theta}_R \), respectively. Under (A2), \( \tilde{\theta} < 0 \) and, then, \( A_\eta^* > \tilde{A}_\eta \) in \( [\eta, \min(\eta^*, \tilde{\eta})] \). Considering this inequality in the conditions that characterize \( \Theta_R \) and \( \tilde{\Theta}_R \), we conclude that \( \Theta_R \subseteq \tilde{\Theta}_R \).

**Corollary:** Under (A1) – (A3) the optimum taxation scheme \( (\tilde{t}_\eta, \tilde{A}_\eta, \tilde{T}_\eta) \) is implementable, i.e., the \( (IC^2) \) is satisfied.

**Proof.** To prove that \( (\tilde{t}_\eta, \tilde{A}_\eta, \tilde{T}_\eta) \) is implementable, we start by noting that

\[
\Pi(\eta|\eta) - \Pi(\tilde{\eta}|\eta) = \int_{\tilde{\eta}}^{\eta} [\tilde{\theta} Q(\tilde{A}_\eta) + \phi(\tilde{T}_\eta)] d\tilde{\eta} + (\tilde{\theta} - \theta) Q(\tilde{A}_\eta) + (\tilde{\eta} - \eta) \phi(\tilde{T}_\eta)
\]

\[
= \int_{\tilde{\eta}}^{\eta} \left[ \tilde{\theta} (Q(\tilde{A}_\eta) - Q(A_\eta)) + \phi(\tilde{T}_\eta) - \phi(T_\eta) \right] d\tilde{\eta}
\]

\[
= \int_{\tilde{\eta}}^{\eta} \left[ \int_{\tilde{A}_\eta}^{\tilde{A}_\eta} \tilde{\theta} Q' (A) dA + \int_{\tilde{T}_\eta}^{\tilde{T}_\eta} \phi'(T) dT \right] d\tilde{\eta},
\]

where the last equality results from (A2). Considering \( \tilde{\eta} < \eta \), there are three possibilities:

- \( \eta, \tilde{\eta} \in \Theta_R \): \( \tilde{A}_\eta = \tilde{T}_\eta \) and then

\[
\Pi(\eta|\eta) - \Pi(\tilde{\eta}|\eta) = \int_{\tilde{\eta}}^{\eta} \int_{\tilde{A}_\eta}^{\tilde{A}_\eta} [\tilde{\theta} Q' (A) + \phi'(A)] dAd\tilde{\eta}
\]

which is positive under (A3b);

- \( \eta, \tilde{\eta} \in \Theta_I \): the condition (*) implies that \( Q(\tilde{A}_\eta) < (\tilde{A}_\eta) \) and \( \phi(\tilde{T}_\eta) > \phi(\tilde{T}_\eta) \), thus, \( \Pi(\eta|\eta) > \Pi(\tilde{\eta}|\eta) \), for every \( \tilde{\eta} \in [\tilde{\eta}, \eta] \).
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- \( \hat{\eta} \in \theta_R, \eta \in \Theta_I \): in this case, we have

\[
\Pi(\eta|\eta) - \Pi(\hat{\eta}|\eta) = \int_{\hat{\eta}}^{\eta} \left[ \int_{\hat{A}_{\hat{\eta}}}^{\hat{\eta}} \theta Q'(A) dA + \int_{\hat{T}_{\hat{\eta}}}^{\hat{T}_{\hat{\eta}}} \phi'(T) dT \right] d\hat{\eta}
\]

\[
= \int_{\hat{\eta}}^{\eta} \int_{\hat{A}_{\hat{\eta}}}^{\hat{\eta}} \left[ \theta Q'(A) + \phi(A) dA \right] d\hat{\eta}
\]

\[
+ \int_{\eta}^{\eta^*} \left[ \theta \left( Q\left(\hat{A}_{\hat{\eta}}\right) - Q\left(\hat{A}_{\eta^*}\right)\right) + \left(\phi\left(\hat{T}_{\hat{\eta}}\right) - \phi\left(\hat{T}_{\eta^*}\right)\right) \right] d\hat{\eta} \geq 0,
\]

since, using the same arguments above, the two integrals are non-negative.

The cases in which \( \hat{\eta} > \eta \) are analogous.

**Proposition 4':** Assume (A1)-(A3). A menu of linear taxes can decentralize the solution to the optimal taxation problem (P.SB):

\[
\alpha_A \theta Q(A) + \beta_T T + \gamma(A,T)
\]

where

\[
\alpha_A = \begin{cases} 
0 & \text{if } A \in \left[\tilde{A}_2, \tilde{A}_\eta\right] \\
-\frac{1}{Q(A)} \int_{A}^{\tilde{A}_\eta} \left[ \frac{1-F(\eta(A))}{f(\eta(A))} \frac{d}{dA} Q'(A) \right] dA & \text{if } A \in \left[\tilde{A}_\eta, \tilde{A}_\eta\right]
\end{cases}
\]

\[
\beta_T = \begin{cases} 
\frac{\hat{T}_{\hat{T}}}{f(\eta(T))} + \frac{1}{f(\eta(T))} \int_{\hat{T}_{\hat{T}}}^{T} \left[ \lambda(\hat{T}) \phi'(\hat{T}) + \frac{1-F(\eta(\hat{T}))}{f(\eta(\hat{T}))} \left( \hat{\theta} Q'(\hat{T}) + \phi'(\hat{T}) \right) \right] d\hat{T} & \text{if } T \in \left[\hat{T}_{\hat{T}}, \hat{T}_{\hat{T}}\right] \\
\frac{\hat{T}_{\hat{T}}}{f(\eta(T))} + \frac{1}{f(\eta(T))} \int_{\hat{T}_{\hat{T}}}^{T} \left[ \lambda(\hat{T}) \phi'(\hat{T}) + \frac{1-F(\eta(\hat{T}))}{f(\eta(\hat{T}))} \right] d\hat{T} & \text{if } T \in \left[\hat{T}_{\hat{T}}, \hat{T}_{\hat{T}}\right]
\end{cases}
\]
and $\eta(T)$ and $\eta(A)$ are the inverses of $\tilde{T}_\eta$ and $\tilde{A}_\eta$, respectively.

In this scheme the government offers $(\alpha_A, \beta_T, \gamma(A,T))$ observing the agricultural output $\theta Q(A)$ and the farm size $T$. Producers, maximizing their profit, determine $A$ and $T$ in accordance with (SB$_1$) and (SB$_2$).

Proof. Define $q = \theta Q(A)$. Initially, observe that the mechanisms in $(q, T, t)$ and $(A, T, t)$ are equivalent. Since $Q' > 0$, the inverse $A = Q^{-1}(\frac{q}{\theta})$ is well defined and, designing a mechanism in $(q, T, t)$, the maximization program faced by the government is given by

$$
\max_{\{t_n, q_n, T_n\}_{n \in [2, \bar{n}]}} \int_\eta^\bar{\eta} \left[ q_n - w Q^{-1} \left( \frac{q_n}{\theta} \right) + (1 - \lambda) \eta \phi (T_\eta) - r T_\eta \right. \\
\left. - \frac{1 - F(\eta)}{f(\eta)} \left( \frac{\dot{\theta}}{\theta} q_n + \phi (T_\eta) \right) \right] dF(\eta)
$$

subject to

$$Q^{-1} \left( \frac{q_n}{\theta} \right) \leq T_\eta, \eta \in [\eta, \bar{\eta}].$$

The first-order condition to $q$ is

$$1 - w \frac{1}{Q'(A_\eta)} \frac{1}{\theta} - \frac{1 - F(\eta)}{f(\eta)} \frac{\dot{\theta}}{\theta} - \frac{\mu_\eta}{f(\eta) Q'(A_\eta)} \frac{1}{\theta} = 0$$

which can be arranged to

$$\theta Q'(A_\eta) = w - \frac{F(\eta)}{f(\eta)} \frac{\dot{\theta} Q'(A_\eta)}{f(\eta)} + \frac{\mu_\eta}{f(\eta)}.$$

Thus, we can focus on mechanisms written in $(A, T, t)$ which are algebraically simpler. Indeed, considering $q$ or $A$ is a convenience matter, once there is a well-defined inverse of $Q$. 

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Consider a mechanism in which, for each \((A, T)\) announced by the producer, the tax collection is defined by

\[
t(A, T) = \alpha_A \theta Q(A) + \beta_T T + \gamma(A, T),
\]

where \(\alpha_A \equiv \alpha(A), \beta_T \equiv \beta(T)\) and \(\gamma(A, T) \equiv \gamma(A, T)\) give the scheme designed by the government. In this scheme, \(\alpha_A\) guarantees \(A = \tilde{A}_\eta\) (or \(q = \theta Q(\tilde{A}_\eta)\)), \(\beta_T\) is such that \(T = \tilde{T}_\eta\) and \(\gamma(A, T)\) is determined by

\[
\gamma(A, T) = \begin{cases} 
\left(\theta(\hat{\eta}(T)) - \alpha_{A, \hat{\eta}(T)} \theta\right) Q(\tilde{A}_{\hat{\eta}(T)}) + \hat{\eta}(T) \phi(T) - w \tilde{A}_{\hat{\eta}(T)} - (r + \beta_T) T \\
+ \int_{\tilde{T}_{\hat{\eta}(T)}}^{\tilde{T}_{\hat{\eta}(T)}} \left[\theta Q(\tilde{A}_{\hat{\eta}}) + \phi(\tilde{T}_{\hat{\eta}})\right] d\tilde{\eta} - \Pi(\tilde{\eta}), & \text{if } T = \tilde{T}_{\hat{\eta}} \text{ and } A = \tilde{A}_{\hat{\eta}} \text{ for some } \hat{\eta} \in [\eta, \overline{\eta}]; \\
\infty, & \text{otherwise};
\end{cases}
\]

where \(\hat{\eta}(T)\) is the inverse of \(\tilde{T}_{\hat{\eta}}\) (well-defined under (A1)-(A3)).

Facing this scheme, the producer chooses \((A, T)\) in order to maximize his profits, following the program

\[
\max_{A, T} (1 - \alpha_A) \theta Q(A) + \eta \phi(T) - w A - (r + \beta_T) T - \gamma(A, T)
\]

subject to

\[A \leq T.\]

Denoting by \(\mu\) the Lagrange multiplier, the first-order conditions are given by

\[(1 - \alpha_A) \theta Q'(A) = w + \left(\frac{\delta}{\delta A} \alpha_A\right) \theta Q(A) + \frac{\delta}{\delta A} \gamma(A, T) + \mu\]
and
\[ \eta \phi' (T) = r + \beta T + \left( \frac{\delta}{\delta T} \beta T \right) T + \frac{\delta}{\delta T} \gamma(A, T) - \mu. \]

Suppose, for a moment, that \( \frac{\delta}{\delta A} \gamma(A, T) = \frac{\delta}{\delta T} \gamma(A, T) = 0 \) in the curve \( \hat{\eta} \rightarrow (\hat{A}_\eta, \hat{T}_\eta) \).

a) Case \( \mu > 0 \): \( A = T \) and the first-order conditions imply

\[
(1 - \alpha_A) \theta Q'(A) + \eta \phi'(T) = w + \left( \frac{\delta}{\delta A} \alpha_A \right) \theta Q(A) + r + \beta T + \left( \frac{\delta}{\delta T} \beta T \right) T + \frac{\delta}{\delta A} \gamma(A, T) + \frac{\delta}{\delta T} \gamma(A, T)
\]

and \( A = T = \tilde{A}_\eta = \tilde{T}_\eta \) whether \( \alpha_A = 0 \)

and
\[
\left( \frac{\delta}{\delta T} \beta T \right) \tilde{T}_\eta + \beta T = -\frac{F(\eta)}{f(\eta)} \left[ \theta Q'(\tilde{A}_\eta) + \phi(\tilde{T}_\eta) \right].
\]

Or, integrating the last condition and considering \( \beta_{T_n} = \beta \),

\[
\beta_T = \beta \frac{\tilde{T}_\eta}{T} + \frac{1}{T} \int_{T_n}^T \left[ \lambda \eta \phi'(\tilde{T}) + \frac{1 - F(\eta(\tilde{T}))}{f(\eta(\tilde{T}))} \left( \theta Q'(\tilde{T}) + \phi'(\tilde{T}) \right) \right] d\tilde{T}.
\]

b) Case \( \mu = 0 \): to ensure that \( A = \tilde{A}_\eta \) and \( T = \tilde{T}_\eta \)

\[
\alpha_A Q'(\tilde{A}_\eta) + \left( \frac{\delta}{\delta A} \alpha_A \right) Q(\tilde{A}_\eta) = \frac{1 - F(\eta)}{f(\eta)} \frac{\dot{\theta}}{\theta} Q'(\tilde{A}_\eta)
\]
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and

\[ \left( \frac{\delta}{\delta T} \beta_T, \beta_T \right) \tilde{T}_\eta + \beta_T = \lambda \eta \phi' (\tilde{T}_\eta) + \frac{1 - F(\eta)}{f(\eta)} \phi' (\tilde{T}_\eta). \]

Or, integrating and using \( \alpha_{\tilde{A}_\eta} = 0 \) and \( \beta_{\tilde{A}_\eta} \) given by the case \( \mu > 0 \), we have

\[ \alpha_A = \frac{1}{Q(A)} \int_{A}^{\tilde{A}_\eta} \left[ \frac{F(\eta(\tilde{T}))}{f(\eta(\tilde{T}))} \frac{\theta Q'(\tilde{T})}{\theta Q'(\tilde{A})} \right] d\tilde{T}, \]

and

\[ \beta_T = \frac{\beta_{\tilde{T}_\eta}}{T} + \frac{1}{T} \int_{\tilde{T}_\eta}^{T} \left[ \lambda \eta(\tilde{T}) \phi'(\tilde{T}) + \frac{1 - F(\eta(\tilde{T}))}{f(\eta(\tilde{T}))} \phi'(\tilde{T}) \right] d\tilde{T}, \]

where \( \eta \) can be expressed in terms of \( T \) or \( A \).

Finally, we have to show that \( \frac{\delta}{\delta \eta} \gamma(\tilde{A}_\eta, \tilde{T}_\eta) = \frac{\delta}{\delta T} \gamma(\tilde{A}_\eta, \tilde{T}_\eta) = 0 \) in the curve \( \hat{\eta} \rightarrow (\tilde{A}_\eta, \tilde{T}_\eta) \) in both cases. This is a consequence of the Envelope Theorem. Since there is a bijection between \( \eta \) and \( A \), and between \( \eta \) and \( T \), in each case, it is enough to show that \( \frac{\delta}{\delta \eta} \gamma(\tilde{A}_\eta, \tilde{T}_\eta) = 0 \). Differentiating \( \gamma \), we obtain

\[ \frac{\delta}{\delta \eta} \gamma(\tilde{A}_\eta, T_\eta) = \left[ \theta Q' - w - \theta \frac{\delta}{\delta A} (\alpha_A Q) \right] \dot{A} + \left[ \eta \phi' - r - \frac{\delta}{\delta T} (\beta_T T) \right] \dot{T}. \]

And in the points \( A = \tilde{A}_\eta \) and \( T = \tilde{T}_\eta \) the two brackets vanish.

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