NOMINAL INCOME DETERMINATION,
FINANCIAL ASSETS AND LIABILITIES AND FISCAL POLICY*

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RESUMO


ABSTRACT

This paper concentrates attention on the dynamics of income and expenditure determination which are seen to be the logically necessary counterpart of a portfolio adjustment process. The model constitutes a development of the New Cambridge hypothesis discussed in Fetherston and Godley (1978) and also Anyadike-Danes (1982).

* Major contributions have been made by Francis Cripps and Ken Coutts both from the Cambridge University, England
Modern macroeconomics emphasises the influence of portfolio adjustment on income and expenditure flows, and it has become commonplace to include wealth terms in functions representing private consumption or savings decisions. But despite the emphasis placed by Brainard and Tobin as far back as 1968 on the need to consider the full implications of identities which constrain portfolio adjustment, few models have explicitly examined the consistency of expenditure decisions with wealth objectives. Indeed Brainard and Tobin themselves accepted wealth as the outcome of savings decisions without enquiring into the plausibility of the implied accumulation of wealth, an issue eventually taken up by Purvis (1978). Similarly, although monetarist economists have always stressed possible implications of a stable demand function for one class of financial assets - money - they have not integrated this into a fully consistent account of the demand for all financial assets and liabilities.

The main exception is a paper by Tobin and Buitier (1974) which developing the well-known Blinder-Solow (1973) model of fiscal and monetary policy, explicitly regarded savings as a process which adjusts wealth towards some target value relative to income. This enabled them to give a description of the steady state, often adumbrated in earlier literature, where portfolio equilibrium implies that both private savings and the government's budget deficit are reduced to zero. They were also able to examine the stability of adjustment towards the steady state under various disequilibrium assumptions.
1. INCOME FLOWS, EXPENDITURE FLOWS AND STOCKS OF MONEY: THE SIMPLEST CASE

Consider the relationship between flows of income, flows of expenditure on goods and services and stocks of money, assumed initially to be the only financial asset.

Flows are related to changes in stocks by the identity,
\[ PE_t = YP_t - \Delta A_t \]  \hspace{1cm} (1)
where \( PE \) is private expenditure per period, \( YP \) is private disposable income per period and \( A \) is the end period stock of money.

A linear adjustment process towards stock flow equilibrium implies that the stock of money at the end of each period is given by,
\[ A_t = \sum_{i=0}^{n} \alpha_i Y_{t-i} \]  \hspace{1cm} (2)
Putting (2) back into (1) we can derive the relationship between expenditure in each period and income in that and in previous periods,
\[ PE_t = \sum_{i=0}^{n+1} \beta_i Y_{t-i} - \beta_{i+1} \]  \hspace{1cm} (3)
where
\[ \beta_0 = 1 - \alpha_0 \]  \hspace{1cm} (4)
\[ \beta_i = \alpha_{i-1} - \alpha_i \quad i = i \ldots \ldots n \]  \hspace{1cm} (5)
\[ \beta_{n+1} = \alpha_n \]  \hspace{1cm} (6)

Now the mean of the distributed lag of expenditure after income is defined as,
\[ \xi = \sum_{i=0}^{n+1} i \beta_i \]  \hspace{1cm} (7)
\[ = 0.130 + \sum_{i=1}^{n} ((i\beta_1)+(n+1) \beta_{n+1}) \]  \hspace{1cm} (8)
\[ = \sum_{i=0}^{n} \alpha_i \]  \hspace{1cm} (9)
This proves (in view of equation (2)) that the mean lag of the response of expenditure to income is equal to the steady state stock flow ratio irrespective of the time profile of the portfolio adjustment process. In other words it does not matter what value any of the individual $a_i$ take; the mean lag is uniquely determined by their sum.

The essential point may be conveyed graphically. In each of the two charts below we start from a stationary steady state where the stock of money is unchanging and (therefore) where the income flow exactly equals the expenditure flow. The steady state stock flow ratio is assumed to be 0.5. We then postulate a step increase of 50% in income. Chart 1 shows the adjustment of expenditure to its new steady state on the assumption that the stock of money adjusts very slowly.

**CHART 1**

**A SLOW ASSET ADJUSTMENT**
In this chart $Y$ and $E$ are flow variables while $A$ is a stock variable. The rectangles constructed around the $A$ line represent the flow counterpart of $A$ (that is $\Delta A$) in each period which by equation (1) must be equal in area to the gap between income and expenditure immediately above taking each discrete time period as a whole. This gives the key to why the dynamics of the expenditure adjustment process are pinned down by the asset income norm; the total cumulative gap between income and expenditure is entirely governed by - is indeed necessarily equal to - the size of the asset adjustment and has nothing at all to do with the rate at which the adjustment occurs.

As we should now be expecting, the very different asset adjustment process shown in Chart 2 has not made much difference to the expenditure adjustment, the mean lag being half a period in each case.

CHART 2
A FAST ASSET ADJUSTMENT
2. DYNAMIC SOLUTION OF A WHOLE SYSTEM WITH CREDIT MONEY

Now postulate an economy with no government but with a commercial banking system. We still assume that there is no financial asset other than money, all of which is bank money (BD).

In this world the total stock of money is exactly equal, by the balance sheet identity of the banking system, to the stock of loans. There is no logical constraint on the extent to which the banking system can expand its lending operations.

The complete system may now be represented,

\[ E_t = Y_t + \Delta L_t - \Delta BD_t \]  

(10)

Unlike equation (1) equation (10) represents a complete flow system since it implies (with \( \Delta L \equiv \Delta BD \)) that total income equals total expenditure.

Defining loan financed expenditure (*) (LFE),

\[ LFE_t = \Delta L_t \]  

(11)

the national income identity may be expanded

\[ Y_t = LFE_t + YFE_t \]  

(12)

where \( YFE_t \) represents income financed or "endogenous" expenditure.

The aggregate income flow in this model is governed by discretionary expenditure financed by bank loans. We only need to postulate the same "demand for money" function as in section 1,

\[ BD_t = \sum_{i=0}^{n} \alpha_i Y_{t-i} \]  

(13)

(*) This does not imply that loans are always necessarily associated with and additional act of expenditure. In so far as loans are the counterpart of financial portfolio decisions they should, in this model, be deducted from financial assets.
to obtain the period by period solution,

\[ y_t = \{ \Delta L_t + \sum_{i=1}^{n+1} \beta_i y_{t-i} \} / \alpha_0 \]  

The stationary steady state solution of this model, when the stock of loans and money is unchanging (and therefore loan financed expenditure zero) is simply,

\[ y_t = L_t / \Sigma \alpha_i = \beta D_t / \Sigma \alpha_i \]  

The solution of this system as it evolves through time is represented in the following chart (Chart 3).

**CHART 3**

**AN INCREASE IN LOANS IN A CREDIT ECONOMY WITH NO GOVERNMENT**

Taking loan financed expenditure as discretionary or "exogenous", the solution for the dynamic path of the economy depends upon the rate at which people spend the income they receive or what amounts to the same thing the quantity of money they wish to hold relative to their income flow. The system of identities ensures that the three shaded areas are exactly equal to one another period by period,

\[ BD_t \equiv \Delta L_t \equiv LFE_t \equiv Y_t - YFE_t \]  

(16)
An aggregate income (= expenditure) flow must always be generated such that,

\[ BD_t = L_t = \sum_{i=0}^{n} \alpha_i Y_{t-i} \]  

(17)

Note that,

a) While this looks for a moment like a monetarist model, with changes in the money stock preceding equal proportionate changes in nominal income flows, it is changes in loans and therefore loan financed expenditure which are the causal agent generating the whole flow system.

b) \( \Delta L \) has so far been treated as exogenous. But it is pretty obvious that if there were a large response in the adjustment of loan financed expenditure to a change in income the entire system could become highly unstable.

3. A FISCAL SYSTEM AND A BANKING SYSTEM

The model can be made very much more realistic if we combine aspects of the two preceding sections.

The national income identity can be written as,

\[ Y_t = G_t + PE_t \]  

(18)

where \( G \) is government expenditure and the flow of funds identity as,

\[ G_t - T_t = Y_t - T_t - PE_t = \Delta GSFA_t - \Delta L_t \]  

(19)

where \( T \) is the tax yield and GSFA is the gross stock of financial assets. Gross because it combines net private sector lending to the government, identically equal to the budget deficit, with private sector borrowing from the banking system.

To see this more clearly, recall that,

\[ PE_t = YPE_t + LPE_t \]  

(20)
and that,
\[ \Delta L_t = \text{LPE}_t \]  
(21)

we can write,
\[ \Delta \text{GSFA}_t = Y_t - T_t - \text{YFE}_t = (G_t - T_t) + \Delta L_t \]  
(22)

It is evident that the gross acquisition of financial assets is also equal to the gap between private disposable income and income financed expenditure.

The completion of the system of accounts requires the introduction of the national debt and we postulate that the government operates a market in bonds which it sells to both the non-bank public (B_p) and the banks (B_b).

We now have implicit in the system of accounts a complete representation of changes in banks' balance sheets. To see this consider first the financial counterpart of the government's income expenditure flow system,
\[ G_t - T_t = \Delta B_{B_t} + \Delta B_{P_t} \]  
(23)

The financial counterpart of the private sector income expenditure flow system is:
\[ Y_t - T_t - \text{YFE}_t - \text{LFE}_t = \Delta B_{D_t} + \Delta B_{P_t} - \Delta L_t \]  
(24)

Therefore, using the primary flow of funds identity,
\[ G_t - T_t = Y_t - T_t - \text{YFE}_t - \text{LFE}_t \]  
(25)

It follows that,
\[ \Delta B_{B_t} + \Delta B_{P_t} = \Delta B_{D_t} + \Delta B_{P_t} - \Delta L_t \]  
(26)

Which, rearranged gives changes in the banks' balance sheet,
\[ \Delta B_{B_t} + \Delta L_t = \Delta B_{D_t} \]  
(27)
4. HOW DOES THE SYSTEM WORK?

If we postulate an aggregate tax function such that,

$$ T_t = \theta Y_t $$

(28)

and, as before, a linear portfolio adjustment process,

$$ \Delta \text{GSFA}_t = \sum_{i=0}^{n+1} Y_{t-i} (1-\theta) $$

(29)

(where GSFA is defined as the stock of financial assets gross of financial liabilities) the disequilibrium solution for the whole system is given by,

$$ Y_t = \{G_t + \Delta L_t + \sum_{i=1}^{n+1} [1-\theta] Y_{t-i} \} / \{1 + \beta_0 (1-\theta)\} $$

(30)

and the steady state solution, when changes in all stocks are zero, is the familiar result,

$$ Y_t = G_t / \theta $$

(31)

As was shown in an earlier section the mean lag of private (income financed) expenditure behind disposable income is necessarily equal to the steady state ratio of financial assets to income, i.e.,

$$ \sum_{i=0}^{n} \alpha_i $$

The lag between the aggregate national income flow and "exogenous" expenditures \{G_t + LFE_t\} is given by \((1-\theta)/\theta \sum_{i=0}^{n} \alpha_i\)

(*) The proof of this proposition can be found in Godley and Cripps (1983).
The solution of this whole system is represented graphically in the following Chart (see Chart 4).

**AN INCREASE IN GOVERNMENT SPENDING AND BANK LENDING**

What is imagined here is an economic system starting in a stationary steady state. There is then a shock in the form of a once for all increase in government expenditure and in loans.

Note that by identity $\Delta L_t \equiv LFE_t$ in each period and that the gross acquisition of financial assets (by the identity in (22)) represented by the upper shaded area, must be equal in each period to the sum of the budget deficit and loan financed expenditure i.e. the sum of two lower shaded areas. The flow of income financed expenditure is implied in the Chart, since,

$$YFE_t \equiv Y_t - G_t - LFE_t$$
Note finally that although we have postulated so far only two forms of financial asset (money and government bonds) and two forms of financial liability (bank loans and government bonds), no essential difference is made to the model by including other financial assets as part of GSFA (e.g. deposits with non bank financial intermediaries, industrial debentures or new equity issues) together with the financial liabilities implied by these assets and the associated acts of loan financed expenditure.

5. HYPOTHESIS CONCERNING PROFESSOR BENJAMIN FRIEDMAN'S PROPOSITION

In a series of important articles Professor Friedman (*) has drawn attention to the stability in the US over a long period of time in the ratio of total non financial debt (the equivalent in my model of $L_t + B_{pt} + B_{pt}$) to disposable income. The ratio has been pretty constant, Friedman notes, although there has been a marked fall in government debt relative to privately issued debt.

Is it possible that the model outlined above brings some illumination to this problem?

It has been shown that by accounting identity the acquisition of financial assets gross of financial liabilities by the private sector ($\Delta \text{GSFA}_t$) is equal to the sum of the budget deficit and the change in private financial liabilities, i.e., is equal to what Friedman calls total non financial debt.

$$\Delta \text{GSFA}_t \equiv Y_t - T_t - \text{YFB}_t \equiv (G_t - T_t) + \Delta L_t \quad (32)$$

We have also postulated a stable steady state ratio between the gross stock of financial assets and disposable income,

$$\text{GSFA}_t = \sum_{i=0}^{n} \alpha_i Y_{t-i} (1 - \theta) \quad (33)$$

(*) See, for example, Friedman (1982).
The hypothesis (33) and the accounting identity (32) taken together would generate precisely the result noted by Friedman.

Thus the process represented in Chart 4, since GSFA is adjusting so as to stay in a constant ratio to disposable income, must imply by identity that total non financial debt (the total of bank loans and the cumulative budget deficit) is adjusting by identically equal amounts period by period. And if, as has clearly happened in the US, total loans made to the private sector have increased relative to disposable income, it would therefore follow as well that public sector debt outstanding has fallen by an exactly equal amount.

Such a process can be represented graphically by the adaptation of Chart 4 presented in Chart 5.

CHART 5
AN INCREASE IN BANK LENDING
In this example it is postulated that, starting from a stationary steady state, there is a rise in loans outstanding which eventually levels off. Fiscal policy stance does not change, so the tax yield rises in the same proportion as aggregate income and a budget surplus is generated.

Gross acquisition of financial assets, always by identity equal to the change in loans less the budget surplus, first goes positive then goes negative soon after loans level off and then the old stationary steady state \((Y_t = G_t/\theta)\) is eventually established. Between the two steady states the national debt (cumulative \((G - t)\)) must have been retired by an amount which exactly equals the addition to private sector borrowing.

6. SOME PRELIMINARY EMPIRICAL RESULTS

Work has started on a systematic examination of the postulated stability of the asset income ratio using 30 years of post-war U.S. data (1949-1978) and the results so far are quite encouraging. Chart 6 displays the ratio of private financial assets gross of financial liabilities to the annual flow of private disposable income (*) . This ex post ratio has been fairly stable in the U.S. over the whole period, and over the last 20 years or so it has been almost dead level at around 1.2 although there has been, as one would expect, a tendency for the ratio to fall temporarily in periods when income has risen particularly fast. Since stability is, though, a matter of degree the ratio of the money stock (cash plus demand deposits) has been plotted on the same chart to aid comparison. Needless to say the marked degree of relative stability exhibited by the series measuring the gross stock ratio (its coefficient of variation is about one-tenth of that for the money income ratio) would not surprise those familiar with the work of Benjamin Friedman's referred to in the previous section.

(*) The data used in the empirical work reported here are described and presented in the Appendix.
A more exacting test of the consistency with the post-wear U. S. experience of the form of the asset income relationship can be provided by an examination of its predictive accuracy. Although the behavioural proposition advanced above concerned the accumulation of financial assets it can be used to generate predictions of total private expenditure and this has been done here since the latter is a more familiar aggregate, predictions of which will be more easy to assess. The equation used in the predictive exercise combines the asset income relationship.

\[
GSFA_{t} = \sum_{l=0}^{n+1} \alpha_{l} (Y_{t} - T_{t})
\]  

(34)
and the identity linking asset accumulation with private expenditure,

\[ PE_t = (Y_t - T_t) + \Delta L_t - \Delta GSF_A_t \]  \hspace{1cm} (35)

Now the asset income relationship has, so far, been written in a quite general form, but for the purpose of generating predictions a simpler one has been chosen, a first-order partial adjustment mechanism and equation (34) can be re-written as,

\[ \Delta GSF_A_t = \phi(\alpha(Y_t - T_t) - GSF_A_{t-1}) \]  \hspace{1cm} (36)

where \( \alpha \) measures the mean lag between income and expenditure the length of which is, as has already been shown, independent of \( \phi \).

Equation (36) can then be combined with equation (34) to yield the aggregate private expenditure function,

\[ PE_t = \Delta L_t + (1 - \alpha \phi)(Y_t - T_t) + \phi GSF_A_{t-1} \]  \hspace{1cm} (37)

Using this equation and data on private borrowing, disposable income and just the initial lagged value for the gross stock of financial assets a series of "dynamic" predictions (*) for private expenditure can be generated for alternative values of \( \alpha \) and \( \phi \). The consistency of any predictions based on any particular pair of values can be assessed by calculating the sum of squared percentage prediction errors and the smaller the value of this statistic the "better" the pair of values.

\(*)\) The predictions are "dynamic" because the predicted value of private expenditure is then used to generate the next value for the gross stock of financial assets using the identities

\[ \Delta GSF_A_t = (Y_t - T_t) + \Delta L_t - PE_t \]

\[ GSF_A_t = GSF_A_{t-1} + \Delta GSF_A_t \]

The quotation marks surrounding better are used to indicate that the minimization criterion used here is not the only possible criterion for choosing between different pairs of parameter values.
The set of parameter values to be considered is limited by two constraints. The first is that if the asset accumulation process is to be dynamically stable $\phi$ must lie between zero and unity, while the second stems from the recognition that if disposable income is growing then the observed asset income ratio, $\bar{a}$, must be smaller than the parameter $\alpha$. So the relevant parameter space is bounded by the two constraints:

a) $0 < \phi < 1$; and  
b) $\alpha > \bar{a}$

The relationship between the growth of disposable income, the observed asset income ratio, and the parameters $\alpha$ and $\phi$, can, in fact, be made somewhat more precise. It can be shown (*) that if disposable income is growing steadily at some positive rate, $g$, then,

$$\bar{a} = \alpha \left[ \frac{\phi(1+g)}{(g+\phi)} \right]$$  

(38)

Substitution of the average growth of U.S. disposable income between 1949 and 1978 and the average observed asset income ratio into this expression will then give a further indication of the appropriate subset of the parameter space.

The relationship $\alpha$ and $\phi$ implies by the observed values of $g$ and $\bar{a}$ is plotted on Chart 7 which also records the results of the search over the parameter space with $\alpha$ and $\phi$ independently varied by .01 and provides a "contour map" of the parameter space. The contours are constructed by computing the values of the sum of squared percentage prediction errors calculated for different parameter pairs (**). It seems clear that the "best" values (i.e. the values giving rise to the minimum squared prediction error) are $\alpha$ between 1.30 and 1.35 associated with $\phi$ between 0.40 and 0.30. A more precise estimate might be obtained by narrowing the grid but

(*) This relationship is formally derived in the Appendix.

(**) The values actually calculated are tabulated in the Appendix.
since no probability statement can be made about such a point estimate without making an assumption about the stochastic properties of the underlying behavioural relationship there seems little to be gained.

CHART 7
CONTOURS FOR THE SUM OF SQUARED PREDICTION ERRORS

An indication, in more conventional form, of the predictive ability of the aggregate private expenditure function is given in Chart 8 which displays (on a log scale) a series of predictions generated using $\alpha = 1.32$ and $\phi = 0.32$ plotted with the actual values of the series (*). The associated percentage prediction errors are displayed in Chart 9. The recursive prediction mechanism implied by equation (37), despite the fact that it contains neither a constant nor a trend term, evidently "tracks" the evolution of aggregate private expenditure reasonably well since all but three of the prediction errors are less than 2.5% and the average absolute error is about 1.5%.

(*) The implied predictions of the change in the gross stock of financial assets are displayed on a chart in the Appendix.
CHART 8
ACTUAL AND PREDICTED PRIVATE EXPENDITURE

CHART 9
PERCENTAGE PREDICTION ERRORS
It seems, then, that the post-war U.S. experience is reasonably consistent with an aggregate private expenditure function which implies that the mean lag of expenditure behind income is about 1.3 years and, moreover, that about 80 per cent of the adjustment of expenditure to a change in disposable income will have taken place within two years.

7. SOME CONCLUDING REMARKS

The empirical investigation of the integrated approach to the treatment of asset accumulation and income and expenditure determination proposed in this paper has only just begun (*). An obvious next step, for example, involves modelling the determination of loan-financed expenditure.

Even at this stage, though, the implications of the results reported above, if confirmed in subsequent work, seem to be of considerable importance since they illustrate how stock flow norms pin down the flow dynamics of a very large part of the U.S. economy. In particular, the stability of the ratio between the gross stock of financial assets and the flow of private disposable income implies, as a matter of logic, stability in the mean lag in the response of private expenditure to changes in disposable income. For the post-war U.S. this lag seems relatively stable and the mean lag between private disposable income and "income-financed" expenditure (which has been on average (1949-1978) more than 90 per cent of total private expenditure) is evidently less than eighteen months long.

(*) The theoretical framework is at a rather more advanced stage and will be found in Godley and Cripps (1983).
A. DATA SERIES, SOURCES AND DEFINITIONS

The source for all but one of the series used is: Board of Governors of the Federal Reserve System (1979), Flow of Funds Accounts, 1949 - 1978 (Washington, Federal Reserve) denoted in the following by FOF.

The series for private disposable income (Y-T), aggregate private expenditure (PE), and the accumulation of the gross stock of financial assets (GSFA) are derived as follows,

\[(Y-T) \equiv Y - G + GDEF\]

\[PE \equiv (Y-T) - GDEF + FDEF\]

\[\Delta GSFA \equiv (Y-T) - PE + \Delta L\]

where

Y, national income, is calculated as the average of the income and expenditure estimates of GNP (excluding the inventory valuation adjustment). The series for the non-corporate business inventory valuation adjustment is not published in FOF and was obtained from U.S. Department of Commerce, The National Income and Product Accounts of the United States of America, 1929-1974 (Washington, U.S. Government Printing Office) and the various issues of the U.S. Department of Commerce Survey of Current Business, denoted by NIPA. National income is constructed as: FOF, "Income and Product Accounts", line 1; less FOF, "Non Financial Corporate Business", line 69; less NIPA, Table 5.8, line 9; plus half of FOF, "Income and Product Accounts", line 75.

G, public (Federal plus State and Local) purchases of goods and services, from: FOF, "Income and Product Accounts", line 5 plus line 6.

GDEF, public (Federal plus State and Local) sector deficit, from: FOF, "Income and Product Accounts", line 39 plus line 40, sign reversed.

FDEF, current account of balance of payments deficit, from: FOF, "Income and Product Accounts", line 33, sign reversed.

\(\Delta L\), change in net private sector borrowing from the private financial sector (private financial sector excludes insurance companies and pension funds). It is constructed as follows: FOF, "Total Mortgages" line 9 less the sum of lines 17 to 19 and 29 to 32; plus FOF, "Consumer Credit", line 1 less the sum of lines 6, 7, 14 and 15; plus FOF, "Bank Loans Not Elsewhere Classified", line 13 less line 18; plus FOF, "Open..."
Market Paper”, line 3 plus line 9 less line 14 less line 15 less line 22; plus FOF, "Other Loans", line 1 plus line 13 plus line 14.

The series for the Money Stock used in the preparation of Chart 6 is from: FOF, "Demand Deposits and Currency", line 2.

The Table below provides the major series used. The end-1948 value for the Gross Stock of Financial Assets was $ 263.4bn, a value found from the appropriate FOF, "Outstandings" figures.

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<tr>
<td>1975</td>
<td>1188.0</td>
<td>1272.8</td>
<td>42.5</td>
</tr>
<tr>
<td>1976</td>
<td>1353.4</td>
<td>1397.0</td>
<td>112.3</td>
</tr>
<tr>
<td>1977</td>
<td>1534.7</td>
<td>1544.3</td>
<td>185.2</td>
</tr>
<tr>
<td>1978</td>
<td>1733.5</td>
<td>1723.5</td>
<td>205.1</td>
</tr>
</tbody>
</table>
B. THE RELATIONSHIP BETWEEN \( \bar{\alpha} \), \( g \), \( \alpha \) AND \( \phi \)

The relationship set out in equation (38) can be derived from the asset accumulation equation as follows:

\[
\Delta \text{GSFA}_t = \phi(\alpha(Y_t - T_t) - \text{GSFA}_{t-1}) \tag{B.1}
\]

which can be rewritten as,

\[
\text{GSFA}_t = \alpha \phi(Y_t - T_t) + (1-\phi) \text{GSFA}_{t-1} \tag{B.2}
\]

and it is evident that repeated substitution for \( \text{GSFA}_{t-1} \) will yield,

\[
\text{GSFA}_t = \alpha \phi(Y_t - T_t) + (1-\phi)(Y_{t-1} - T_{t-1}) + (1-\phi)^2 (Y_{t-2} - T_{t-2}) + \ldots \tag{B.3}
\]

Now if we assume that disposable income is growing at some steady rate \( g \), i.e.,

\[
(Y_t - T_t) = (1+g)(Y_{t-1} - T_{t-1}) = (1+g)^2 (Y_{t-2} - T_{t-2}) = \ldots \tag{B.4}
\]

Then we can substitute for \( (Y_{t-i} - T_{t-i}) \) in (B.3) from (B.4) to obtain:

\[
\text{GSFA}_t = \alpha \phi(Y_t - T_t) \left[ \frac{1 - \phi}{1+g} + \left( \frac{1-\phi}{1+g} \right)^2 + \ldots \right] \tag{B.5}
\]

and summing to infinity the bracketed geometric progression, assuming that \( \frac{1-\phi}{1+g} \) < 1, and that \( 0 < \phi < 1 \), yields

\[
\text{GSFA}_t = \alpha \phi(Y_t - T_t) \left[ \frac{1+g}{g+\phi} \right] \tag{B.6}
\]

Now if we define,

\[
\bar{\alpha} \equiv \frac{\text{GSFA}_t}{(Y_t - T_t)} \tag{B.7}
\]

then we obtain equation (38) in the text,

\[
\bar{\alpha} = \alpha \phi \left[ \frac{1+g}{g+\phi} \right] \tag{B.8}
\]
### C. TABLE OF SUM OF SQUARED PERCENTAGE PREDICTED ERRORS

Columns correspond to different values of \( \alpha \).

<table>
<thead>
<tr>
<th>Value of ( \alpha )</th>
<th>( \sum ) of Squared Percentage Predicted Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0.1 )</td>
<td>1.90</td>
</tr>
<tr>
<td>( 0.2 )</td>
<td>6.21</td>
</tr>
<tr>
<td>( 0.3 )</td>
<td>15.67</td>
</tr>
<tr>
<td>( 0.4 )</td>
<td>30.14</td>
</tr>
<tr>
<td>( 0.5 )</td>
<td>57.69</td>
</tr>
</tbody>
</table>

The \( \sum \) is given as:

\[
\sum = \frac{100}{1 - \frac{(p - 2)}{n}}
\]

where \( p \) is the number of parameters and \( n \) is the number of observations.
D. ACTUAL AND PREDICTED ASSET ACCUMULATION

Chart D displays actual and predicted changes in the gross stock of financial assets generated as part of the process of predicting the series for aggregate private expenditure displayed in the text Chart 8, i.e. using parameter values $\alpha = 1.32; \phi = 0.32$. 

CHART D
ACTUAL AND PREDICTED ASSET ACCUMULATION

$\text{bn}$

$200$

$150$

$100$

$50$


Actual
Predicted
REFERENCES


