ON SRAFFA'S "STANDARD COMMODITY" AND THE RATE OF PROFIT

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ABSTRACT

This note proves the inconsistency of Sraffa's Standard Commodity with the Marxian approach. It also shows to be wrong his deduction that non-basic sectors have no effect on the determination of prices and the rate of profit.

RESUMO

Nesta nota procura-se provar a inconsistência da MercadoriaPadrão de Sraffa, com relação à abordagem Marxista. Mostra também ser equivocado se inferir que os setores não-básicos não afetam a determinação dos preços e taxa de lucro.

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1. NON-BASICS AND THE RATE OF PROFIT

Sraffas position on the influence of non-basic products upon the rate of profit, is well known as opposed to Marx's contention that non-basics affect the rate of profit through the average organic composition of capital.

Before Sraffa, Bortkiewicz (1907), through his three sector model that deals with the "transformation problem" offered a mathematical "demonstration" of Marx's supposedly erroneous position on this point. (See Sweezy 1942, Winternitz 1948, May 1948, Seton 1957, Samuelson 1971 and Dobb 1973). But Bortkiewicz's apparent demonstration is based on a subtle algebraic mistake: indeed, he found out that the value of the rate of profit could be expressed as a function of the parameters of sector's I and II ("basics") without any explicit reference to the parameters of Sector III ("non-basic"). Surprisingly he was unaware that simultaneously the basic sectors' parameters could be expressed as functions of the non-basic sector's parameters, through the parameters of the system as whole, so that his demonstration led precisely to the opposite result.(1). With regard to this point May (1948) pointed out that the relevant parameters "are not made independent merely by our failure to write down some explicit relations between them". And May concluded: "As it stands, (Bortkiewicz's and Winternitz's) conclusion is merely the result of an implicit assumption that variables are independent until proved otherwise".

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(1) For instance, constant capital used in sector I, C1, can be expressed also as total constant capital used in the system as a whole, C, less constant capital used in sectors II and III, C1 = C - C2 - C3, and so on, with the result that the rate of profit appears as a function of the explicit parameters of the non-basic sector - III.
It can be said at the outset that Sraffa's demonstration, widely accepted today and similar to Bortkiewicz's, is also based on a logical mistake. However, the exposition of Sraffa's mistake is not as simple, because his model is more general and complex, and because it is not based on values as parameters. But, first of all, let's us recall Sraffa's demonstration:

"Since by the same act (eliminating from the system the equation representing the production of a 'luxury' good) we eliminate an unknown (the price of that good) which only appears in that equation, the remaining equations still form a determinate system which will be satisfied by the solutions of the larger system. On the other hand, if we eliminated one of the other, non-luxury, equations, the number of unknowns would not thereby be diminished since the commodity in question appears among the means of production in the other equations and the system would become indeterminate". (Sraffa 1960, section 6).

"It is to be noted that the absence of the non-basic industries from the Standard System does not prevent the latter from being equivalent in its effects to the original System since, as we have seen (section 6), their presence or absence makes no difference to the determination of prices and the rate of profits". (Sraffa 1960, sect. 35).

It is clear from the outset that Sraffa's demonstration is based, similarly to Bortkiewicz's, on that implicit assumption that "basics" prices are independent of "non-basics" prices because his equation system does not exhibit an explicit relationship in this direction. But, as May remarked regarding Bortkiewicz's and Winternitz's models, variables "are not made independent merely by our failure to write down some explicit relation between them". Thus, if the influence of "non-basics" prices on "basics" prices is demonstrated, following Sraffa's own logical development, it is also demonstrated that Sraffa's argument is not a demonstration at all but a mere erroneous assumption, an unjustifiable assumption which contradicts the very internal logic of the own model, as will be shown below.
At the root of Sraffa's fundamental theoretical construction and conclusions, one finds the concept of a "balancing proportion" (or "critical proportion") of labor "means of production". "Sraffa 1960, Chap. III). For the argument under consideration this concept is essential, so we recall Sraffa's own definition of it:

"Starting from the situation in which the whole of the national income goes to labour, we imagine wages to be reduced: a rate of profits will thereby arise".

"Suppose that prices did remain unchanged when the wage was reduced and a rate of profits emerged. Since in any one industry what was saved by the wage-reduction would depend on the number of men employed, while what was needed for paying profits as a uniform rate would depend on the aggregate value of the means of production used, industries with a sufficiently low proportion of labour to means of production would have a deficit, while industries with a sufficiently high proportion would have a surplus, on their payments for wages and profits".

"There would be a 'critical proportion' of labour to means of production which marked the watershed between 'deficit' and 'surplus' industries. An industry which employed that particular 'proportion' would show an even balance - the proceeds of the wage-reduction would provide exactly what was required for the payment of profits at the general rate". (Sraffa 1960, Sect. 15, 16 and 17).

Thus, if \( \Delta V_i \) is the absolute value of the wage-reduction within any industry \( i \), and \( \Delta V_b \) is the absolute value of the wage-reduction within any industry with a "balancing" (or "critical") proportion of labor to means of production, starting from a situation of zero profits and without any change in prices, according to Sraffa

\[
\Delta V_i = P_i
\]

and

\[
\Delta V_b = P_b
\]
Where $P_i$ stands for the amount of profits in any sector $i$ and $P_b$ is the amount of profits in any sector with a "balancing" proportion.

Now, if $K_i$ is the value of the means of production of any sector $i$, and $K_b$ is the respective figure for any sector with a "balancing" proportion, then

$$\frac{P_i}{K_i} = \frac{\Delta V_i}{K_i} = r_i \quad \text{and} \quad \frac{P_b}{K_b} = \frac{\Delta V_b}{K_b} = r_b \quad (2)$$

where $r_i$ and $r_b$ are the respective rates of profit.

But, any "balancing" sector has the property, also according to Sraffa, that (without any change in prices) "the proceeds of the wage-reduction would provide exactly what was required for the payment of profits at the general rate". That is,

$$r_b = r \quad (3)$$

where $r$ is the general rate of profits.

On the other hand, according to its definition, and according to Sraffa's own system of equations, the general rate of profit can be written as total sum of profits / total sum of values of the means of production (total capital), that is:

$$r = \frac{\sum_{i=1}^{n} P_i}{\sum_{i=1}^{n} K_i} \quad (4)$$

From (1) and (4),

$$r = \frac{\sum_{i=1}^{n} \Delta V_i}{\sum_{i=1}^{n} K_i} \quad (5)$$

And, from (2), (3) and (5),

$$\frac{\sum_{i=1}^{n} \Delta V_i}{\sum_{i=1}^{n} K_i} = \frac{\Delta V_b}{K_b} \quad (6)$$
However, Sraffa defines (1960, chap II) the total value of wages paid within each sector \( V_i \) as the product of the wagerate and the quantity of labor employed within each sector \( w \text{ and } L_i \), respectively, in his own notation.

That is,

\[
V_i = wL_i \text{ and } V_b = wL_b
\]  

Therefore, from (6) and (7)

\[
\frac{\sum_1^n \Delta(wL_i)}{\sum_1^n L_i} = \frac{\Delta(wL_b)}{K_b}
\]

Because \( L_i \) remains constant for every \( i \) (the issue is examined "on the assumption that the methods of production remain unchanged", Sraffa 1960, Sect. 13.), equation (8) can also be written as

\[
\frac{\Delta w}{\sum_1^n L_i} = \frac{\Delta w}{\sum_1^n L_b}{\frac{L_b}{K_b}}
\]

or

\[
\frac{\sum_1^n L_i}{\sum_1^n L_b} = \frac{L_b}{K_b}
\]

Thus, Sraffa's definition of a "balancing" or "critical" proportion boils down to an identity with the proportion of labor to means of production of the economy as a whole, \( L/K \) for short.

But, because \( \sum_1^n L_i \) as well as \( \sum_1^n K_i \) are obtained from the addition of all sectors, \( i = 1 \ldots n \), including both "basics" and "non-basics", it is obvious that \( L/K \) cannot remain unchanged when "non-basics" are excluded from the system (except in the extremely special case that every "non-basic" happens to have "a proportion" identical to the original \( L/K \)).

Therefore, if a "basic" sector \( a \) has a "balancing" (or "critical") proportion in the original system (before any non-basic is
excluded from the system) it cannot have a "balancing" proportion in the resultant system (after some or all basics are excluded from the original system). Because that basic sector's proportion in question has necessarily remained constant (both Li and Ki are constant) while L/K has forcibly changed.

If for the basic sector \( a \) in question "the proceeds of the wage-reduction would provide exactly what was required for the payment of profits at the general rate", with prices unchanged, in the original system, these proceeds cannot accomplish the same fate in the resultant system precisely because, in the latter, sector \( a \) is not a "balancing" sector. In other words, in order to grant the payment of profits at the general rate, the price of sector \( a \) can remain constant in the original system but it would have to change in the resultant system. Thus, the price of basic sector \( a \) cannot be the same within the original system and within the resultant system. That is, the price of basic sector \( a \) must change when "non-basic" sectors are excluded from the original system.

Then, Sraffa's deduction that "their (non-basics') presence or absence makes no difference to the determination of prices" is wrong. And because the exclusion of non-basics changes the prices of basics, and because the rate of profit depends explicitly upon the latter, it is obvious that the exclusion of non-basics from the original system also changes the rate of profit.

Now, it is clear that there may not exist even one basic sector whose proportion of labor to means of production is exactly equal to L/K. But this fact has no effect whatsoever on the above conclusion because, in general, the change in L/K only means that the position of every sector with respect to the hypothetical "balancing" proportion (which is L/K itself in practice) is not the same within the original system and within the resultant system. And this is the only essential fact for the argument under consideration. In other words, and generally speaking, the magnitude (and even the direction) of the necessary change in the price of every sector to bring its rate of profits to equality with the general rate is not the same within the original system and within the resultant system.
When sectoral interconnections of price movements are allowed into the analysis, the above conclusion is even strange. Because the position of every sector with respect to the "balancing" proportion will depend now not only upon its own proportion of labor to means of production, but also upon the proportion of labor to means of production of each of the means of production used by every sector, upon "the proportion" of each of the means of production used to produce the above means of production, and so on, as Sraffa himself explains. So, when L/K changes, the position of every sector with respect to the "balancing" proportion will change not only because of its own "proportion" but also because of these other "proportions", with the result that even a sector a having a La/Ka higher than Lc/Kc of another sector c in the original system, can exhibit La/Ka lower than Lc/Kc in the resultant system.

2. AN INVARIABLE MEASURE OF VALUE AND THE STANDARD COMMODITY

In the best tradition of the classical economists, Sraffa undertakes the explicit search for an invariable measure of value (real income, see Sect. 5). He starts from Ricardo's demonstration that no individual commodity is capable of fulfilling this role. But unlike Smith, Ricardo, Marx and even Keynes, instead of turning his sight towards labor (or employment) he attempts to demonstrate that the required properties, which any individual commodity cannot exhibit, can be found in a Special Composite Commodity he calls the 'Standard Commodity.'

The first peculiarity of the 'standard commodity' is its "balancing" or "critical" proportion of labor to means of production. But this is not enough for a composite commodity to become a standard commodity. It is also required that the means of production used to produce such commodity have a "balancing" proportion, that the means of production of these means of production have also a "balancing" proportion, and so on..." in all successive layers of the industry's aggregate means of production without limit". This is called by Sraffa the "recurrence" condition. (Sraffa 1960, sect. 21).
Sraffa himself explains that "the perfect composite commodity, in which the requirements are fulfilled to the letter, is one which consists of the same commodities (combined in the same proportions) as does the aggregate of its own means of production — in other words, such that both product and means of production are quantities of the self-same composite commodity". (1960, Sect. 24). Immediately, Sraffa proceeds to demonstrate that from any actual system, a standard commodity can always be constructed by chopping the unwanted parts from the former.

It becomes obvious, then, that the standard commodity contains only "basics", excluding every "non-basic". But Sraffa, based on what he thought to be a demonstration but shown like to be a simple erroneous assumption about the role of the non-basics, went directly to the wrong conclusion that "the absence of the non-basic industries from the standard system does not prevent the latter from being equivalent in its effects to the original system since, as we have seen, their presence or absence makes to difference to the determination of prices and the rate of profits". (1960, sect 35).

Actually, as was demonstrated in section 1, the exclusion of "non-basic" industries indeed changes prices and the rate of profits. Thus, the right conclusion is exactly the opposite: the standard system cannot be equivalent in its effects to the original system.

Specifically, because L/K is not the same within the original system and within the standard system, any commodity (or set of commodities) with a "balancing" or "critical" proportion (of labor to means of production) within the standard system cannot have a "balancing" proportion within the original system. Thus, the standard commodity, and the means of production with which the standard commodity is produced, and the means of production with which those means of production are produced, etc., have by necessity a "balancing" proportion within the standard system, but this does not mean they have a "balancing" proportion within the original system. In other words, the standard commodity has an "invariable" price and thus is an invariable standard of value within the standard
system but it has a "variable" price and thus is not an invariable standard of value within the original system. In other words, because no actual system fulfills the extreme peculiarities required by a standard system, Sraffa has adopted as an invariable standard of value for the analysis of the actual system something which is not an invariable standard of value at all.

3. CONCLUSION

It goes without saying that any conclusion about the actual system, obtained from the properties of the standard system and the simultaneous assumption that this system is equivalent in its effects to the actual system, becomes equally invalidated. Among these, Sraffa's conclusion that a linear relationship between the wage-rate and the rate of profits must be underlined (1960, sect. 30). When the error under consideration is corrected, this linear relationship disappears because the rate of profits becomes a function of both the wage-rate and L/K (which changes with any change of "basics" prices in the actual system but not in the standard system).

Then, quite aside from Sraffa's other great contributions to economic theory, it must be concluded that with his standard commodity he failed to provide an invariable standard of value different from the labor theory of value's long lasting invariable standard.

2 It is also obvious that the proportions of the basics (with each other) are not the same within the original system and the standard system. This suffices, even if the actual system has no "non-basics" whatsoever, for a "balancing" commodity within the standard system to become a "non-balancing" commodity within the original system. Indeed, this is the most general case. But it is not strange that Sraffa did not analyze this case because his essential error was to lose sight of the connection his model makes between L/K and his own definition of a "balancing" proportion.
REFERENCES


