PRICE DISCRIMINATION AND THE PEAK LOAD PROBLEM

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SUMMARY

The pricing policy of a public utility that faces a demand which varies cyclically is examined with respect to its welfare effects. The conventional first best, marginal cost pricing principle is assumed to have prohibitive administrative costs. Two second best alternatives — a "maximum demand tariff" and "price discrimination according to demand peakiness" are examined for their optimality with regard to maximising producers' and consumers' surplus. The optimal maximum demand tariff is shown to yield more total welfare and to be a Pareto improvement on the other.

RESUMO

Examinam-se os efeitos sobre o bem-estar de uma política de preço de um serviço público, que enfrenta uma demanda com variação cíclica. Supõe-se que o princípio convencional da fixação de preço com base no custo marginal tem custo administrativo proibitivo. Assim, duas alternativas são analisadas — "tarifa de máxima demanda" e "discriminação de preços de acordo com o peakiness de demanda". Busca-se a otimilidade com respeito à maximização do excedente dos produtores e consumidores. Mostra-se que o esquema de "tarifa de demanda máxima" gera maior bem-estar total e constitui um melhamento paretiano em relação ao outro.

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1. INTRODUCTION

This paper is concerned with the pricing policy of a public utility that faces a demand that varies over time. The standard solution to this problem is to set peak and off-peak prices according to marginal cost pricing principles (see Williamson (1966), Rees (1976), for example). However, this first best solution may not be attainable (see Craven (1971)) if the demand varies continuously with time, rather than discretely as in the Williamson model. As we shall see, the first best solution in this case requires that the price varies continuously over at least some of the cycle of demand, and this may lead to prohibitive administrative costs.

In this paper we consider some alternative ways of meeting the peak load problem. In particular we examine the possibility of setting a different price (that does not vary over time) to each consumer.

Intuitively, it seems reasonable to charge a higher price to a consumer whose demand exhibits greater peakiness (technically, who has a lower load factor of demand, see below), because he demands capacity that is not used for much of the cycle of demand. We shall see that this is indeed the optimal form of discrimination: the optimal prices vary inversely with the load factor. The same is true if the consumers are partitioned into a finite number of classes, so that consumers in a particular class pay the same price; in this case, the price varies with the average load factor of consumers in the class.

The second form of pricing that we consider is a maximum demand tariff, in which consumers pay a fixed price per unit and a charge related to their maximum instantaneous demand. We shall establish the optimal maximum demand tariff, and show that it is
better for all consumers, and no worse for the public utility, than the optimal price discrimination case. This is an important result, because it establishes that price discrimination is not as good as the maximum demand tariff (neither is first best), even though the price discrimination case allows the utility to charge more to those who add most to the problem of peak demand.

2. THE MODEL

The utility produces a single, non-storable good over a cycle of demand \([0, 1]\). The maximum output at each instant in \([0, 1]\) is defined by the capacity limit \(X\). For simplicity (and following Williamson and others), we assume that the utility has a constant average and marginal operating cost \(c\), and a constant average and marginal capacity cost \(B\).

A consumer's load factor is his average demand in \([0, 1]\) divided by his peak demand in \([0, 1]\). Consumers differ according to a single parameter \(S\) which is inversely related to their load factors. Consumers are distributed according to the density function \(g(S)\) in \([0, 1]\). Without loss of generality,

\[
\int_{\beta=0}^{1} g(\beta) d\beta = 1
\]

Faced with a price \(p(t, \beta)\) at time \(t\), a consumer of type \(\beta\) demands \(D(p(t, \beta), t, \beta)\) units of the good. We assume that \(\partial D/\partial p < 0\) for all \(t\) and \(\beta\), and for all \(p\) up to the price for which \(D = 0\).

If there are no administrative costs involved in changing prices, the price set by the utility can vary with \(t\) and \(\beta\) (not necessarily continuously in either case), and the total demand at time \(t\) is

\[
\int_{\beta=0}^{1} g(\beta) D(p(t, \beta), t, \beta) d\beta
\]
We assume that this integral exists.

The utility chooses its capacity level \( X \) and the prices \( p(t,\beta) \) subject to specified constraints on the frequency of price changes and the extent of price discrimination, to maximise \( TS \), which is the total of consumers' and producer's surplus:

\[
\int_{t=0}^{1} \int_{\beta=0}^{1} g(\beta) \left[ \int_{z=0}^{\infty} D(p(t,\beta),t,\beta) E(z,t,\beta) dz - cD(p(t,\beta),t,\beta) \right] d\beta dt - B X
\]

subject to

\[
\int_{\beta=0}^{1} g(\beta) D(p(t,\beta),t,\beta) d\beta \leq X \text{ for all } t \text{ in } [0, 1]
\]

where \( E(\ ) \) is the inverse demand function:

\[
D(E(z,t,\beta),t,\beta) = z \text{ for all } z, t, \beta
\]

For simplicity in our analysis, we assume that the demand function has the special form:

\[
D(p,t,\beta) = [1 + k(\beta)h(t)]f(p)
\]

where \( k(0) = 0, k(1) = 1, k'(\beta) > 0 \text{ for } 0 < \beta < 1, h(0) = 0 = h(1), h'(t) > 0 \text{ for } 0 < t < m, h'(t) < 0 \text{ for } m < t < 1.

In this simplification, \( h(\ ) \) represents the time-variability of the demand of the consumers with \( \beta = 1 \), who are those with the lowest load factor of demand, and \( k(\ ) \) represents the fraction of this time-variability associated with consumers of type \( \beta \).

We shall write

\[
G(\beta) = \int_{z=0}^{\beta} g(z) dz, \quad G(W) = \int_{z \in W} g(z) dz
\]
The load factor of demand of a consumer of type \( \beta \) is given by

\[
L(\beta) = \int_{t=0}^{1} \frac{[1 + k(\beta) h(t)] dt}{[1 + k(\beta) h(m)]} = \frac{1 + k(\beta) H(1)}{1 + k(\beta) h(m)}
\]

so that

\[
\frac{\partial L}{\partial \beta} = \frac{[H(1) - h(m)] k'(\beta)}{[1 + k(\beta) h(m)]^2}
\]

which is nonpositive because

\[
H(1) = \int_{t=0}^{1} h(m) dt = h(m)
\]

The equality sign holds only for \( \beta = 0 \), and so the load factor \( L(\beta) \) falls as \( \beta \) rises above zero.

The average load factor for a subset \( W \) of the consumers, that is, the average demand in \( W \) as a fraction of peak demand in \( W \), is

\[
L(W) = \frac{[G(W) + H(1) K(W)]}{[G(W) + h(m) K(W)]}
\]

and so, if \( W = [0, 1] \), we have the average load factor of all consumers

\[
L(All) = \frac{[1 + H(1) K(1)]}{[1 + h(m) K(1)]}
\]
Lemma 1 If $D(p,t,\beta) = [1 + k(\beta)h(t)]f(p)$, then

$$
\int_{z=0}^{D(p,t,\beta)} E(z,t,\beta)dz = [1 + h(t)k(\beta)]J(p)
$$

Proof If $D(p,t,\beta) = [1 + k(\beta)h(t)]f(p)$, then

$$
f^{-1}(u) = E(z,t,\beta)
$$

where $u = z/[1 + k(\beta)h(t)]$. So

$$
\int_{z=0}^{D(p,t,\beta)} E(z,t,\beta)dz = \int_{z=0}^{D(p,t,\beta)} f^{-1}(u)dz = \int_{u=0}^{f(p)} f^{-1}(u)du [1 + h(t)k(\beta)]
$$

3. THE FIRST BEST SOLUTION

If there is no restriction on the variability of $p$ with or $\beta$, because there are no administrative costs of price changes, then the utility chooses $p(t,\beta)$ to maximise total surplus, subject to the capacity constraint. The first order conditions for this maximisation are that there is $\mu(t) \geq 0$, defined for all $t$ in $[0, 1]$, such that

$$
p(t,\beta) = c + \mu(t) \quad \text{for all } t \text{ in } [0, 1]
$$

$$
\mu(t) [X - \int_{\beta=0}^{1} (1+h(t)k(\beta))f(p(t,\beta))d\beta] = 0 \quad \text{for all } t \text{ in } [0,1]
$$

and

$$
\int_{t=0}^{1} \mu(t)dt = B
$$
So \( p(t,\beta) \) is independent of \( \beta \):

**Result 1**  The first-best prices are the same for all consumers.

We should expect this result, as the standard marginal cost pricing rules can be derived from knowledge of the total demand, without information on the division of demand between consumers with different load factors.

4. **UNIFORM PRICING**

The logical antithesis of infinitely variable pricing is that the utility sets a single price \( \pi \) at all times and for all consumers. There is no peak load pricing and no price discrimination. In these circumstances, the utility maximises surplus with \( p(t,\beta) = \pi \) for all \( t \) and all \( \beta \); that is, the utility maximises

\[
[1 + H(1)K(1)]J(\pi) - B[1 + h(m)K(1)]f(\pi)
\]

The first order condition for the optimal price \( \pi^* \) is

\[
\pi^* = c + B[1 + h(m)K(1)]/[1 + H(1)K(1)]\]

\[= c + B/L(All)\]

So we have

**Result 2**  The optimal uniform price equals marginal operating cost plus a charge equal to the marginal capacity cost multiplied by the inverse of the average load factor for all consumers.

Since total output in the cycle of demand is \([1+H(1)K(1)]f(p)\), and the capacity required is \([1 + h(m)K(1)]f(p)\), the utility breaks even when the optimal uniform price is charged.
5. CONTINUOUS PRICE DISCRIMINATION

If the utility can charge a different price to each type of consumer, but cannot change any price during the cycle of demand, the utility chooses a function \( p(\beta) \) to maximise

\[
\int_{\beta=0}^{1} g(\beta) \left\{ \int_{t=0}^{1} \left[ 1 + h(t)k(\beta) \right] \left[ J(p(\beta)) - cf(p(\beta)) \right] dt \right\} d\beta

- B \left[ 1 + h(m)k(\beta) \right] f(p(\beta))
\]

The first order condition for the optimal prices \( p^*(\beta) \) is

\[
p^*(\beta) = c + B \left[ 1 + h(m)k(\beta) \right] /[1 + H(1)k(\beta)]
\]

\[
= c + B/L(\beta)
\]

So we have

Result 3 When there is continuous price discrimination, each consumer pays a price which exceeds marginal operating cost by the product of the marginal capacity cost and the inverse of the consumer's load factor.

This implies that consumers with lower load factors, who have a greater capacity requirement compared to their average demand, pay a higher price: that is

\[
\frac{dp^*(\beta)}{d\beta} > 0 \quad \text{for} \quad \beta > 0
\]

Once again, it can be shown that the producer's surplus of the utility is zero.

6. FINITE DISCRIMINATION

An intermediate possibility is that the utility can charge a number of different prices, but that it cannot charge a different
price for every type of consumer. For example, an energy producer may be able to charge a different price to domestic consumers than to industrial and/or commercial consumers, but he may not be able to discriminate between domestic consumers, even though they have differing load factors. We consider the case in which the utility divides the consumers into two classes, W and V. Consumers in W pay an unchanging price p throughout the cycle of demand; those in V pay q throughout the cycle. The utility maximises

$$[G(W) + H(1)K(W)][J(p) - cf(p)] + [G(V) + H(1)K(V)][J(p) - cf(q)]$$

$$- B\{[G(W) + h(m)K(W)]f(p) + [G(V) + h(m)K(V)]f(q)\}$$

The first order conditions for the optimal prices p* and q* are

$$p^* = c + B[G(W) + K(W)h(m)]/[G(W) + K(W)H(1)] = c + B/L(W)$$

$$q^* = c + B[G(V) + K(V)h(m)]/[G(V) + K(V)H(1)] = c + B/L(V)$$

and so we have

Result 4 When two prices are charged, the optimal price to each group exceeds marginal operating cost by the product of marginal capacity cost and the inverse of the load factor that group.

Once again, it can be shown that the utility breaks even, and we note that, because the two-price case contains uniform pricing as a special case, consumers are, on average, better off in the two price case than with uniform pricing. A group gains or loses compared to the uniform pricing case according to whether its average load factor exceeds or is less than the average load factor for the population as a whole.

7. THE MAXIMUM DEMAND TARIFF

A straightforward method of meeting the peak load problem is a maximum demand tariff. Each consumer faces a charge e per unit of consumption and a charge M per unit of maximum instantaneous demand.
A surplus-maximising consumer of type $\beta$ therefore chooses a consumption level $v(t,\beta)$ and a maximum demand $C(\beta)$ to maximise $CS(\beta) = \int_{t=0}^{1} \left\{ \int_{z=0}^{v(t,\beta)} f^{-1}(z)dz - e(t,v(t,\beta)) \right\} dt - MC(\beta)$ subject to the constraint $v(t,\beta) \leq C(\beta)$ for all $t$ in $[0, 1]$.

The first order conditions are that there is $\mu(t,\beta) \geq 0$ defined for all $t$ in $[0, 1]$, such that

$$\mu(t,\beta)(C(\beta) - v(t,\beta)) = 0 \text{ for all } t \text{ in } [0, 1],$$

$$\int_{t=0}^{1} \mu(t,\beta)dt = M$$

and so $v(t,\beta) < C(\beta)$ for all $t$ between 0 and some $u(\beta)$, and between $w(\beta)$ and 1. Between $u(\beta)$ and $w(\beta)$, $v(t,\beta) = C(\beta)$. Therefore, $CS(\beta)$ equals

$$\left[ \int_{t=0}^{w(\beta)} \mu(t,\beta)dt + \int_{t=w(\beta)}^{1} \left\{ (1+h(t)b(\beta)) \left[ f^{-1}(v(t,\beta)) - e - \mu(t,\beta) \right] = 0 \text{ for all } t \text{ in } [0, 1] \right\} \right] dt$$

$$-ef(e/(1+h(t)b(\beta))]$$

$$+ \int_{t=u(\beta)}^{w(\beta)} \left\{ (1+h(t)b(\beta)) \int_{z=0}^{C(\beta)} f^{-1}(z)dz - C(\beta) \left[e(w(\beta)) - u(\beta) \right] \right\} dt$$

The first order conditions for the maximum of $CS(\beta)$ can be written as

$$\int_{t=u(\beta)}^{w(\beta)} (1+h(t)b(\beta)) dt f^{-1}(C(\beta)) = [w(\beta) - u(\beta)]e + M$$

$$[1+k(\beta)h(u(\beta))]f^{-1}(C(\beta)) = e$$

$$[1+k(\beta)h(w(\beta))]f^{-1}(C(\beta)) = e$$
and from these, we can show that $\partial C(\beta) / \partial \beta > 0$, $\partial u(\beta) / \partial \beta > 0$, $\partial w(\beta) / \partial \beta < 0$ so that

**Result 5** Consumers with lower load factors choose higher maximum instantaneous consumption levels, reach their maximum demand level later, and maintain it for less time.

**Proof** From the first order conditions with $Q = f^{-1}(C(\beta)) / dC(\beta)$

$$
\begin{bmatrix}
\int_{w}^{W} (1+h(t)k)Qdt & 0 & 0 \\
(1+h(u)k)Q & h(u)k f^{-1}(C) & 0 \\
(1+h(w)k)Q & 0 & h(w)k f^{-1}(C)
\end{bmatrix}
\begin{bmatrix}
dC(\beta) \\
du(\beta) \\
dw(\beta)
\end{bmatrix}
$$

So

$$
dC(\beta) / \partial \beta = - \int (1+hk')f^{-1}(C)dt / \int (1+hk)Qdt > 0
$$

because $Q < 0$. Similar analysis yields the result for $du/\partial \beta$ and for $dw/\partial \beta$.

The utility makes a producer's surplus of $\pi(\beta)$ from supplying a consumer of type $\beta$, where $\pi(\beta)$ is given by

$$
\left[ \int_{0}^{u(\beta)} + \int_{t=0}^{1} \left[ (1+h(t)k(\beta)) [f(e/(1+h(t)k(\beta))] (e-c) dt \\
(\beta - u(\beta))C(\beta)(e-c) + C(\beta)(M - B)
\right]
\right]
$$

and so the total surplus generated TS is

$$
\int_{\beta=0}^{1} (C(\beta) + \pi(\beta)) g(\beta) d\beta
$$
The utility chooses \( h \) and \( M \) to maximise \( TS \).

**Result 6** The optimal maximum demand tariff has a charge per unit \( e \) equal to the marginal operating cost \( c \), and a charge per unit of maximum instantaneous demand \( M \) equal to marginal capacity cost \( B \).

**Proof** From the envelope theorem, \( \frac{dCS(\beta)}{de} = \frac{\partial CS(\beta)}{\partial e} \) and \( \frac{dCS(\beta)}{dM} = \frac{\partial CS(\beta)}{\partial M} \). Also, \( \frac{\partial \pi}{\partial u(\beta)} = 0 \), \( \frac{\partial \pi}{\partial w(\beta)} = 0 \), and so the first order conditions for choosing \( e \) and \( M \) to maximise \( TS \) are

\[
\int_{\beta=0}^{1} \left[ (e-c)(1+h(t)k(\beta))f'(e/(1+h(t)k(\beta))) + [(e-c)(w(\beta)-u(\beta) - (M-B)) \frac{dC(\beta)}{de}] g(\beta) \, d\beta \right.
\]

\[
\int_{\beta=0}^{1} \left[ (M-B) \frac{dC(\beta)}{dM} \right] g(\beta) \, d\beta
\]

These are satisfied if \( e = c \) and \( M = B \).

This result and the definition of \( \pi(\beta) \) imply that the utility breaks even when it charges the optimal maximum demand tariff.

8. WELFARE CONCLUSIONS

The comparison of the welfare gained by each consumer and by the producer often yields no clear cut results in second best situations. However, we can show (result 7) that every individual is better off with the maximum demand tariff than with continuous price discrimination. Since the utility breaks even in both cases, the maximum demand tariff is a Pareto improvement over the case with continuous price discrimination. This is an important conclusion, in that the maximum demand tariff involves only two charges, \( e \) and \( M \),
whereas the continuous price discrimination case involves a different price for each type \( \beta \) of consumer. The maximum demand tariff is therefore superior from a welfare point of view, and, where it is feasible to use it (for example in energy supply), it is likely to be administratively cheaper than a large amount of price discrimination.

**Result 7** The maximum demand tariff case is a Pareto improvement on the continuous price discrimination case.

The proof of this result requires lemma 2, which demonstrates that, for a given intertemporal weighted average or prices, a constant price gives the lowest surplus to a consumer of any type. [This can be demonstrated diagrammatically in the discrete case, because the first derivative of consumer's surplus with respect to price is negative, and the second derivative is positive as in Figure 1, where]

\[
\alpha S(p) + (1-\alpha)S(q) > S(\alpha p + (1-\alpha)q)
\]
Lemma 2  The constant price \( p \) minimises
\[
\int_{t=0}^{1} \left[ (1+h(t)k(\beta)) (J(p(t)) - p(t)f(p(t)) \right] dt
\]
subject to
\[
\int_{t=0}^{1} (1+h(t)k(\beta))p(t)dt \leq (1+H(1)k(\beta))p^0
\]

Proof  The first order condition for the minimisation is that the constraint holds with equality, and that \( f(p(t)) = \mu \), where \( \mu \) is the multiplier associated with the constraint. So \( p(t) \) is constant and equal to \( p^0 \). The second order condition follows from the sign of \( df/dp \).

Proof of result 7  With continuous price discrimination, a consumer of type \( \beta \) pays a price \( p(\beta) = c + B/L(\beta) \). With a maximum demand tariff, a consumer of type \( \beta \) effectively pays a price of \( c + \mu(t) \) at time \( t \), where \( (1+h(t)k(\beta))f(c+\mu(t)) \) \([u(\beta), w(\beta)]\), and \( \mu(t) = 0 \) outside this time interval. Also,
\[
\int_{t=0}^{1} (1+h(t)k(\beta))(c+\mu(t))dt
\]
\[
= c(1+H(1)k(\beta)) + \int_{t=0}^{1} (1+h(t)k(\beta))\mu(t)dt
\]
\[
= c(1+H(1)k(\beta)) + (1+k(\beta)h(m)) \int_{t=0}^{1} \mu(t)dt
\]
\[
= (1+H(1)k(\beta))(c+B/L(\beta))
\]
The result follows from lemma 2.
9. CONCLUSIONS

This paper has investigated various aspects of the peak load problem that are not covered by the conventional literature of peak load pricing. If the conventional first-best marginal cost pricing principle is, for some reason, inapplicable, price discrimination and/or the maximum demand tariff may be considered. This paper establishes that optimal discrimination is closely related to consumer's load factors, and that the optimal maximum demand tariff is based on marginal costs. If both forms of second best pricing are available, the maximum demand tariff is superior, both in terms of total welfare, and, perhaps less obviously, the maximum demand tariff is a Pareto improvement on the optimal discrimination. This result is not typical in the second best literature, where it is rarely possible to make such conclusive welfare judgements.
REFERENCES


