THE HAHN PROCESS WITH QUANTITY CONSTRAINTS*

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ABSTRACT

We define a non-tâtonnement process for an exchange economy in which agents perceive quantity constraints, which are determined by kinks in their conjectural demand curves. In the course of the process nonbinding constraints may be arbitrarily relaxed or tightened. Binding constraints may also be arbitrarily tightened, and may be relaxed, due to price changes, along the corresponding conjectural demand curves. Quasi-stability is proved under a standard Hahn Process assumption, but the limit points of the process cannot be conjectural equilibria, although they may be quantity-constrained equilibria.

* This paper is a revision of the third chapter of my doctoral dissertation at MIT. I am grateful to my advisor Franklin Fisher and to an anonymous referee for many helpful comments. Of course, errors are mine.

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RESUMO

É definido um processo non-tâtonnement para uma economia de trocas, na qual os agentes percebem restrições quantitativas, determinadas por "ângulos" nas suas curvas de demanda conjecturais. Durante o processo, as restrições não ativas podem ser arbitrariamente relaxadas ou apertadas. As restrições ativas também podem ser arbitrariamente apertadas, como podem igualmente ser relaxadas, em função de alterações de preço, ao longo das curvas de demanda conjecturais correspondentes. A quase estabilidade é provada sob uma hipótese clássica do Processo de Hahn, porém os pontos-límites do processo não podem ser equilíbrios conjecturais, embora possam ser equilíbrios com restrições quantitativas.

1. INTRODUCTION

There is a well developed theory of non-tâtonnement processes for competitive economies. A survey of this literature can be found in Fisher (1983). Of course, in such models, only Walrasian demands are considered. This means that, in any given period, agents believe that a Walrasian equilibrium has been attained¹. Also, any perception of monopoly (or monopsony) power is excluded. In a disequilibrium model, the assumption that agents constantly believe that the economy is in a Walrasian equilibrium is certainly awkward. A model in which agents would have disequilibrium awareness would be more appropriate for stability analysis. Such a model can be found in Fisher (1983), where stability is proved under the assumption of "No Favorable Surprise". This assumption precludes the appearance of new exogenous or endogenous opportunities that might allow for increases in the expected utilities or profits of agents. Clearly, in a deterministic model, no stability result can be proved unless exogenous shocks are excluded, but we would like to know more about the conditions under which new opportunities are endogenously created.

There exists an intermediate possibility between the extreme assumptions of belief in the prevalence of a Walrasian equilibrium and of disequilibrium awareness. One can assume that, in each period of time, agents believe that a non-Walrasian equilibrium obtains. Then the analysis of the links between market conditions and expectations creates no problem. But also one cannot expect convergence

¹ By Walrasian demands we mean the utility-maximizing demands of agents acting under the belief that they can perform any budget-feasible trade at the given prices. If agents do not believe that an equilibrium has been attained they will try to speculate and/or will believe that demand and supply curves are not horizontal, so their demands will not be Walrasian.

towards the set of competitive equilibria. Stability models of this type were constructed by Fisher (1978), Franke (undated), and Veendorp (1975). Of these models, the one by Veendorp is a tâtonnement process, while the other two are non-tâtonnement processes with out production and consumption outside of equilibrium. In all three models quantity constraints cannot be manipulated through price offers.

Veendorp's (1975) model contains a gross substitutes assumption. This imposes fairly strong restrictions on utility functions.

Fisher (1978) proves stability under the assumption that the unconstrained excess demand of an agent for a commodity does not differ in sign from the aggregate constrained excess demand for that commodity. As Fisher points out, there are some problems with this assumption. The story used to justify it involves agents perceiving their constraints only after attempting to exercise their unconstrained demands, and thus makes sense only in positions close to Walrasian equilibrium. And even if agents perceive their constraints in that two-step way, the assumption remains restrictive. A detailed discussion of these problems is contained in Fisher (1978). Our proof depends on a related, but much more plausible assumption: individual and aggregate excess demands cannot differ in sign (Hahn process assumption).

Franke (undated) function used in the present paper. Unfortunately, his proof of stability, which also depends on a Hahn process assumption, is incorrect, as it disregards the effects of price changes on quantity constraints. In our model additional assumptions (a version of Fisher's (1983) "No Favorable Surprise") are introduced, and are shown to be sufficient for stability.

In the present paper a model of an exchange economy in which all agents perceive quantity constraints is constructed, and a non-tâtonnement process without production and consumption outside of equilibrium is defined for this economy. We provide a proof of quasi-stability under reasonable assumptions. The extension to the case of an economy with firms but no production, like the one mo-

deled by Fisher (1974) presents no problems. We prefer to avoid the introduction of firms, as it complicates the model and does not bring additional insights.

While in the models of Fisher (1978) and Veendorp (1975) constraints cannot be manipulated through price offers, in our model manipulation is possible, but never optimal. In each period of time agents believe that the economy is in a state of conjectural equilibrium, as defined by Hahn (1978). The distinction between the two kinds of models is relevant insofar as the sets of equilibria towards which these processes converge are concerned. The processes of Fisher (1978) and Veendorp (1975) converge towards sets that contain both Walrasian and non-Walrasian equilibria, and therefore provide some foundation to non-Walrasian equilibrium theory. As shown by Saldanha (1987), Hahn's conjectural equilibria cannot be the limit points of any non-tâtonnement process without production and consumption outside of equilibrium, and therefore if a non-Walrasian equilibrium is a limit point of our process at this equilibrium all constraints must be nonmanipulable. More precisely, our objective is to investigate the stability of the quantity-constrained equilibria whose existence has been shown by Dréze (1975), and of the conjectural equilibria proposed by Hahn (1975). Although our process allows for conjectures of the Hahn type in disequilibrium, and displays quasi-stability, the set of equilibria towards which a trajectory converges contains only quantity constrained equilibria. This is because, as shown by Saldanha (1987), conjectural equilibria cannot be limit points of non-tâtonnement processes without production and consumption outside of equilibrium. It follows that our result provides a disequilibrium foundation for quantity-constrained equilibrium models, but such a foundation for conjectural equilibrium models cannot exist in the framework of non-tâtonnement without production and consumption outside of equilibrium.

The structure of the paper is as follows. In Section 2 we provide a heuristic discussion of the model, which is formally defined in Section 3. Section 4 contains a formal definition of the dynamic process, whose quasi-stability is proved in Section 5. Finally, in Section 6, we present our general conclusions.

2. THE MODEL: DISCUSSION

The most natural foundation for the assumption that agents face quantity constraints is to assume that they face conjectural demand (or supply) curves that are kinked, and are sufficiently inelastic beyond the kink points. Hahn (1978) is the original reference for this formulation. By "sufficiently inelastic" we mean that it is disadvantageous for agents to move along these curves beyond the kink points, when making purchases or sales. See Figure 1, where the consumer has an initial endowment $\bar{x}$ of the commodity, and can buy up to b or sell up to s units at the given price $\bar{p}$.

FIGURE 1

In our model, such conjectural demand curves provide the rationale for the existence of quantity constraints. Of course, price effects can cause these constraints to get looser or tighter². We assume that, if a constraint is binding and a change in price loosens it, then it does so without creating (subjective) opportunities to trade that were previously unavailable. This is again

² Here and elsewhere we say that a quantity constraint is loosened (tightened) if the amount that the agent can trade without making a price offer increases (decreases).

the assumption of "No Favorable Surprise", but here we give it a specific meaning. Figure 2 shows a case in which a consumer is facing a binding constraint on purchases of a commodity in period z_0 . In that period he/she purchases $\bar{x}_1 - \bar{x}_0$ units of that commodity, and the price rises from P_0 to P_1 . The assumption of "No Favorable Surprise" in this case consists in a restriction on the new maximum amount of the commodity available to the consumer at given prices P_1 , $\bar{x}_1 + b_1$: it may not be given by a point to the right of the demand-supply curve D_0S_0 . Notice that there is no restriction on the possibilities of sales at the new given prices. That is, any $s_1 \in [0, \bar{x}_1]$ is permitted.

FIGURE 2

Strictly speaking, only a weaker assumption is needed. It may be the case that even for a demand-supply curve that is more elastic than D_0S_0 at the point $\bar{x}_0 + b_0$ the consumer only wants to buy b_0 at prices P_0 . In this case his constraint on purchases may be allowed to relax along this more elastic demand curve, and $(\bar{x}_1 + b_1, P_1)$ would lie to the right of D_0S_0 . And even if the inelasticity of D_0S_0 at $\bar{x}_0 + b_0$ is just enough to prevent the consumer from offering a price higher than P_0 , it may be the case that, at $\bar{x}_1 + b_1$, D_0S_0 is more inelastic than necessary. In this case D_1S_1 would lie below D_0S_0 to the right of $\bar{x}_1 + b_1$. In both cases

new opportunities to trade would be created, but the consumer would not exploit them, and our stability result would hold.

Our version of the "No Favorable Surprise" assumption is formalized in Assumption IV of Section 4. Its role in the stability proof is as follows. The Hahn Process assumption is usually used to insure welfare. This is enough to establish the existence of a Lyapounov junction in models without quantity constraints, since an argument based on the envelope theorem shows that other effects disappear. However, in the present model, quantity constraints appear as parameters in the maximization problems of households, and hence the welfare effects due to variations in the tightness of these constraints must be taken into account. Of course, there is no problem when a constraint is tightened, since the corresponding welfare effect goes in the appropriate direction. The role of the "No Favorable Surprise" assumption is to rule out the possibility of positive welfare effects due to the relaxation of constraints. Clearly, "Favorable Surprises" do occur in practice. However, it is difficult to conceive a process that would converge even if such surprises continued to take place forever. The "No Favorable Surprise" assumption appears then as a necessary condition for stability. Ideally, we would like to have a proper treatment of expectations and to be able to characterize the conditions under which the assumption does hold. Given the precarious state of the theory of expectations we do not try to undertake this task, but rather limit ourselves to showing that the assumption is also sufficient.

The general framework holds trade among themselves and prices change in the directions given by excess demands, if possible without violating nonnegativity, until equilibrium is reached. Only then does consumption take place. In each period of time the process goes through the following phases, in the indicated order:

- 1) Quantity constraints change.
- 2) Households form their demands.
- 3) There is trade at the given prices.
- 4) Prices change in the directions given by excess demands, if possible without violating nonnegativity.

The whole process is repeated until equilibrium is reached. Then there is consumption.

A Hahn Process assumption is made for all commodities, the numeraire being excepted. This means that if, after trade, an agent has a positive (negative) excess demand for a commodity and this commodity is not the medium of exchange, then no other agent has a negative (positive) excess demand for that commodity. An immediate consequence of this assumption is that the equilibria of the model are autarchic. Indeed, in equilibrium, aggregate excess demands are nonpositive and clearly the same is true of each agent's excess demands, if the Hahn Process assumption holds³. Walras' Law implies that negative excess demands correspond to commodities with zero prices.

The effective demands considered in the paper are Drèze demands as opposed to Clower demands. That is, agents simultaneously take into account all quantity constraints, when forming their demands. On the distinction between Walrasian, Drèze and Clower demands, see Svensson (1980).

3. THE MODEL: FORMAL PRESENTATION

We consider an economy with H households and N commodities. The first commodity is the numeraire. The price vector is $p = (1, p^2, \dots, p^N)$.

Each household has a monotonic, continuously differentiable, quasi-concave utility function U_h , defined on the nonnegative orthant of $\mathbb{R}^N$. Also we assume that the marginal utility of wealth $\partial U_h / \partial x_h^i$ is strictly positive everywhere.

³ Even in the absence of the Hahn Process assumption the equilibria of a non-tâtonnement model in which production and consumption are not allowed to occur outside of equilibrium must be autarchic. For a discussion of this point, see Saldanha (1987).

As discussed in Section 2, households face quantity constraints that are determined by conjectural demand curves. Since we assume that these conjectural demand curves are such that households never want to make price offers in order to violate their quantity constraints, we omit a detailed specification of these demand curves⁴. It is nevertheless necessary to introduce some notation for the left- and right-derivatives of these demand curves at the prevailing price. Referring to Figure 1, we denote by $\bar{x}_h = (\bar{x}_h^1, \bar{x}_h^2, \dots, \bar{x}_h^N)$ the vector of actual stocks with household h , by $x_h = (x_h^1, x_h^2, \dots, x_h^N)$ the vector of its demands, by $b_h = (b_h^1, b_h^2, \dots, b_h^N)$ the vector of its quantity constraints on purchases, and by $s_h = (s_h^1, s_h^2, \dots, s_h^N)$ the vector of its quantity constraints on sales. We assume that the household ignores cross-price effects. Then we can write $p_h^i: \bar{R}_+ \rightarrow \bar{R}_+$ for its conjectural inverse demand curve for commodity i , and define

$$\beta_h^i = \frac{1}{(\partial p_h^i / \partial x_h^i) + (\bar{x}_h^i + b_h^i)}$$

$$\sigma_h^i = \frac{1}{(\partial p_h^i / \partial x_h^i) - (x_h^i - s_h^i)} \quad (1)$$

⁴ Of course, in general there is no reason why the actual evolution of these curves would not induce agents to make price offers. By excluding this possibility we are therefore studying a special case. The objective is to analyze the consistency of the assumption that agents face quantity constraints, in a disequilibrium framework. Such consistency is demonstrated in an equilibrium framework by existence proofs, as in Drèze (1975). Here we assume that the conditions under which agents behave as if they were facing quantity constraints are continuously satisfied, and ask whether, under these conditions, there is convergence to the set of equilibria. It is also worthwhile noticing that, in general, the conditions mentioned above can only be continuously satisfied if the conjectural demand curves of agents change in specific ways. In particular, it is argued below in the text that in a non-Walrasian equilibrium curves must necessarily be of the type shown in Figure 3. On this point see also the discussion in Section 2.

⁵ The left- and right-derivatives defined in (1) are allowed to take infinite values.

Thus, β_h^i and σ_h^i are respectively the rates at which purchases and sales may be increased by making price offers.

In this framework, the household's demand vector x_h must belong to the set of optimal solutions of the following convex program:

$$\text{Max } U_h(x)$$

subject to

$$p \cdot x_h \leq p \cdot \bar{x}_h \quad (2)$$

$$\underline{x}_h \leq \bar{x}_h + b_h \quad (3)$$

$$\underline{x}_h \geq \bar{x}_h - s_h \quad (4)$$

$$x \geq 0.^6$$

We define $\lambda_h \in \mathbb{R}$ as the marginal utility of income for household h . That is, λ_h is the Lagrange multiplier associated to the household's budget constraint (2). We also let $\theta_h = (\theta_h^2, \theta_h^3, \dots, \theta_h^N)$ and $u_h = (u_h^2, u_h^3, \dots, u_h^N)$ be respectively the Lagrange multiplier vectors associated to the household's quantity constraints (3) and (4) on purchases and sales.

Of course, s_h and b_h must be nonnegative. Also, since the household, by assumption, does not want to violate its quantity constraints, β_h and σ_h must satisfy the conditions

$$0 \leq \beta_h^i < +\infty \quad (5)$$

$$\theta_h^i \beta_h^i \leq \lambda_h b_h^i \quad (6)$$

$$0 \leq \sigma_h^i < +\infty \quad (7)$$

⁶ A $\sim$ below a vector indicates that its first component has been deleted.

$$\mu_h^i \sigma_h^i \leq \lambda_h s_h^i. \quad (8)$$

The inequalities (5) simply indicate that the household cannot relax its constraint on the purchases of a commodity by offering a lower price. Since λ_h and b_h are nonnegative, (6) is a restriction only if $\theta_h^i > 0$, that is, only if the i -th constraint on purchases is binding. In this case, (6) states that the benefit $\theta_h^i \beta_h^i dp^i$ obtained from making a marginally higher price offer $p^i + dp^i$ does not exceed the cost of making that offer. When the household offers $p^i + dp^i$ for commodity i its constraint on purchases is relaxed from b_h^i to $b_h^i + \beta_h^i dp^i$. The gain, in terms of utility, is therefore $\theta_h^i \beta_h^i dp^i$. On the other hand, the loss from paying a higher price for b_h^i units is $b_h^i \Delta p^i$, which in terms of utility amounts to $\lambda_h b_h^i \Delta p^i$. As is well known, the effects due to adjustments on other variables vanish, from the Envelope Theorem. The relations (7) and (8) are similarly interpreted.

The excess demand vector of household h is $z_h \equiv x_h - \bar{x}_h$. We define

$$\bar{x} = (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_H)$$

$$x = (x_1, x_2, \dots, x_H)$$

$$z = (z_1, z_2, \dots, z_H)$$

$$Z = \sum_{h=1}^H z_h.$$

Households are assumed to satisfy a "no-swindling" assumption (see Fisher (1974)) that states that the net value of the purchases made by the household in a period equals zero:

$$p \cdot \dot{\bar{x}}_h(t) = 0 \quad \forall t \geq 0. \quad (9)$$

⁷ A dot over a variable indicates its right time derivative, which is assumed to exist.

The closure equations are

$$\sum_{h=1}^H \bar{x}_h(t) = \sum_{h=1}^H \bar{x}_h(0) \quad \forall t \geq 0, \quad (10)$$

and Walras' Law can be written as

$$p \cdot z(t) = 0 \quad \forall t \geq 0. \quad (11)$$

For simplicity, we do not distinguish between target and active excess demands. There is no difficulty in developing the present analysis in the context of an economy in which money plays a role as a medium of exchange.

The Hahn Process assumption is formalized as:

$$\text{For } h = 1, 2, \dots, H, \text{ and } i = 2, 3, \dots, N, \quad z_h^i \neq 0 \rightarrow z_h^i z^i > 0. \quad (\text{HP})$$

The process consists of a dynamical system (Ψ, Ω) defined on the space Ω that consists of the vectors $\omega = (p, \bar{x}, s, b, \sigma, \beta) \in \{1\} \times \mathbb{R}_+^K$, where $K = (5H + 1)N - (4H + 1)$, such that (HP) holds. The process is further specified in the next Section. We now define the concept of equilibrium.

To each $\hat{\omega} = (\hat{p}, \hat{\bar{x}}, \hat{s}, \hat{b}, \hat{\sigma}, \hat{\beta}) \in \Omega$ we associate the set $D(\hat{\omega})$ consisting of all $\omega = (p, \bar{x}, s, b, \sigma, \beta) \in \Omega$ such that

$$\sum_{h=1}^H \bar{x}_h = \sum_{h=1}^H \hat{\bar{x}}_h.$$

We say that $D(\hat{\omega})$ is the economy defined by $\hat{\omega}$, since the states in $D(\hat{\omega})$ are those that can be attained, given the closure equations (10), from the initial conditions ω . We say that $\omega \in D(\hat{\omega})$ is an equilibrium for the economy defined by $\hat{\omega}$ if, at ω , $z \leq 0$. We denote the set of equilibria for the economy defined by $\hat{\omega}$ by $E(\hat{\omega})$.

4. THE TRADING PROCESS

The dynamical system (Ψ, Ω) , where $\Psi: \Omega \times \overline{\mathbb{R}}_+ \rightarrow \Omega$ is continuous, satisfies certain axioms that define the trading process considered here. Before we give these axioms, we introduce some useful definitions.

For any $\omega \in \Omega$, let $\gamma(\omega)$ be the trajectory, or motion, associated to ω ,

$$\gamma(\omega) \equiv \{\Psi(\omega, t) \mid t \geq 0\},$$

and let $\chi(\omega)$ be the set of limit points of the motion associated to ω ,

$$\chi(\omega) \equiv \bigcap_{t \geq 0} \text{cl}(\gamma(\Psi(\omega, t))).^8$$

The process is characterized by the following axioms.

$$\text{I) } \forall \omega \in \Omega, \text{cl}(\gamma(\omega)) \subset D(\omega).$$

$$\text{II) } \forall \omega \in \Omega, \gamma(\omega) \text{ is a bounded set.}$$

$$\text{III) For } i = 2, 3, \dots, N, \text{ and all } t \geq 0$$

$$p^i(t)z^i(t) \geq 0,$$

with the inequality holding strictly if either $z^i(t) > 0$ or $z^i(t) < 0$ and $p^i(t) > 0$.

$$\text{IV) For } h = 1, 2, \dots, H; i = 2, 3, \dots, N; \text{ and all } t \geq 0,$$

⁸ Here and elsewhere closures are taken with respect to the space $\{1\} \times \overline{\mathbb{R}}_+^K$. Of course, $\chi(\omega)$ consists of all limits of sequences $\{\Psi(\omega, t_n)\}$, where $\lim_{n \rightarrow \infty} t_n = +\infty$.

$$\mu_h^i > 0 \rightarrow \dot{\bar{x}}_h^i - \dot{s}_h^i \geq \sigma_h^i \dot{p}^i \quad (12)$$

$$\theta_h^i > 0 \rightarrow \dot{\bar{x}}_h^i + \dot{b}_h^i \leq \beta_h^i \dot{p}^i.$$

V) For $h = 1, 2, \dots, H$; $i = 2, 3, \dots, N$, and all $t \geq 0$, the relations (5) - (8) hold. Also, if $b_h^i > 0$, then (6) holds as a strict inequality, and if $s_h^i > 0$ then (8) holds as a strict inequality.

We now discuss the above axioms.

Axiom I states that, in the closure of the trajectory associated to $\omega \in \Omega$, the Hahn Process assumption (HP) and the closure equations (10) are satisfied. Also, households do not run out of money. The only substantive assumption in II is the assertion that prices are bounded. The reader is referred to Fisher (1974) for a discussion of this issue. Axiom III states that prices change in the direction of the corresponding excess demands, if possible without violating nonnegativity. Axiom IV only restricts the movements of those constraints that are binding. It allows for the relaxation of constraints due to own-price effects, but it imposes that other effects cause constraints to get tighter. Notice that the Hahn Process assumption implies that the own-price effect always determines a relaxation of a constraint. Figure 2 and the related discussion are relevant here. Axiom V defines almost precisely the conditions under which the assumption that agents are quantity-constrained is sensible. These conditions are discussed above in Section 3. But here, in order to prove quasi-stability, we have to assume away the knife-edge cases in which the household expects to be able to trade in the desired direction ($b_h^i > 0$ or $s_h^i > 0$), but finds it (marginally) a matter of indifference whether to violate the constraint or not (the equality holds in (6) or (8)).

5. QUASI-STABILITY

In this Section we prove that the process defined by Axioms I -V is quasi-stable. In what follows we say that household h is in equilibrium in market i if $z_h^i \leq 0$ and $p^i z_h^i = 0$.

Theorem: Let $\omega \in \Omega$. Then $\chi(\omega) \neq \emptyset$ and $\chi(\omega) \subseteq E(\omega)$.

Proof: Since we assume boundedness in Axiom II, it is enough to exhibit a Lyapounov function. We claim that

$$V \equiv \sum_{h=1}^H U_h$$

is such a function.

Using the Envelope Theorem, it is easy to see that, for $h = 1, 2, \dots, H$, and all $t \geq 0$,

$$\dot{U}_h = -\lambda_h \dot{p} \cdot z_h + \mu_h \cdot (\dot{s}_h - \dot{\bar{x}}_h) + \theta_h \cdot (\dot{b}_h - \dot{\bar{x}}_h). \quad (13)$$

From Axiom IV and (13) it follows that

$$\dot{U}_h \leq \sum_{i=2}^N (-\lambda_h z_h^i + \mu_h^i \sigma_h^i + \theta_h^i \beta_h^i) \dot{p}^i. \quad (14)$$

Now, for each $i \in \{2, 3, \dots, N\}$, there are five possibilities:

i) $\mu_h^i = \theta_h^i = 0$. Then, from (HP) and Axiom III, the i -th term on the right-hand side of (14) is nonpositive, and negative if household h is not in equilibrium in the i -th market.

ii) $\theta_h^i > 0$ and $b_h^i > 0$. Then obviously $\mu_h^i = 0$ and $z_h^i = b_h^i$. From (HP) and Axiom III, $\dot{p}^i > 0$. Then Axiom V implies that the i -th term on the right-hand side of (14) is negative.

iii) $\mu_h^i > 0$ and $s_h^i > 0$. This case is similar to ii), and the same conclusion applies.

iv) $\theta_h^i > 0$ and $b_h^i = 0$. Then obviously $\mu_h^i = 0$ and $z_h^i = b_h^i = 0$. Also, (6) must hold as an equality. Therefore the i -th term on the right-hand side of (14) is zero.

v) $\mu_h^i > 0$ and $s_h^i = 0$. This case is similar to iv), and the same conclusion applies.

We conclude that $\dot{U}_h \leq 0$ and $\dot{U}_h < 0$ unless household h is in equilibrium in all markets. Therefore V is decreasing, unless $z \leq 0$ and $p \cdot z_h = 0$ for all h , in which case $\dot{V} = 0$. This means that V is a Lyapounov function, and the proof of the theorem is complete.

Remark: As usual, the Hahn Process assumption is used to show that the target utilities of agents are decreasing outside of equilibrium. In Walrasian models this is the case because the commodities that agents are trying to buy are becoming more expensive, and the commodities that agents are trying to sell are becoming less expensive. This means that price changes have an adverse effect on the welfare of agents. In the present model this is still the case, even though price changes always cause binding constraints to relax. That is, the fact that what households are wanting to buy (sell) is always becoming more (less) expensive has a stronger effect on the welfare of households than the fact that constraints are being relaxed. Indeed, at any point of time, households can cause constraints to relax, by making price offers, but choose not to do so. When the market, by changing the price of a commodity, forces an outcome that was not chosen by a consumer, even though it was feasible, the consumer suffers a welfare loss.

Corollary: A tâtonnement process with consumers having identical preferences, endowments, and quantity constraints, and in which binding constraints are not relaxed, must be stable.

Proof: Such a process would clearly satisfy axioms I-V.

6. CONCLUSIONS

In the process defined in this paper agents are aware that they are not in a Walrasian equilibrium. In each period of time they think that a quantity-constrained non-Walrasian equilibrium will obtain, at the given prices and constraints, after trade. The rationale proposed for the perception of quantity constraints is

that agents have conjectural demand curves that are horizontal on a central segment and have kinks at the quantities that correspond to the constraints. These curves are sufficiently inelastic, when movements that violate the constraints are considered, so that it is disadvantageous for agents to violate the constraints by running along the curves. The process is proved to be quasi-stable.

The case in which all constraints remain fixed is permitted. Moreover, in contrast to the previous literature, the constraints are allowed to get looser. More specifically, a constraint is assumed to change due to: i) an own-price effect, and ii) other effects. The own price effects always loosen the corresponding constraint, and it is required that all the other effects, on balance, do not loosen the constraints.

The set towards which the process converges contains both Walrasian and non-Walrasian equilibria. Also, the equilibria of the process are autarchic. That is, there is no trade at equilibrium ($z \leq 0$). Given this, if a quantity constraint is binding at equilibrium, then that constraint must be determined by a conjectural demand curve that becomes vertical at the constraint point, like the one shown in Figure 3.

FIGURE 3

To see that this must be the case, just notice that, if a constraint is binding at an autarchic equilibrium, then one of the cases iv) and v) considered in the proof of the Theorem must obtain.

In case iv) $\beta_h^i = 0$ (this is the case depicted in Figure 3), and, in case v), $\sigma_h^i = 0$.

We conclude that the conjectural equilibria whose existence has been proved by Hahn (1978) cannot be limit points of the present non-tâtonnement process. On the other hand, our process provides a disequilibrium foundation for those models in which quantity constraints are viewed as absolute bounds for purchases or sales, bounds that cannot be violated by offering higher or lower prices, like the one constructed by Drèze (1975).

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