A NOMINAL THEORY OF THE NOMINAL RATE OF INTEREST AND THE PRICE LEVEL WITH THE INTEGRATION OF BOTH THE FISHER EFFECT AND THE GIBSON PARADOX*

Marco Antonio Campos Martins**

Sumário

Este trabalho desenvolve uma teoria simples de determinação de renda e juros nominais sob três pressupostos principais. Em primeiro lugar, a única distinção relevante entre a moeda e títulos do governo reside nos prazos de retenção. Segundo, os indivíduos levam em conta a restrição do orçamento público, não se preocupando em descontar futuras obrigações fiscais associadas à emissão de títulos do governo. Finalmente, o modelo incorpora um mercado de crédito privado para empréstimos e tomada de empréstimos para e pelos indivíduos. De acordo com esta teoria, a taxa nominal de juros é determinada por dois fatores distintos: a relação título/moeda, e a taxa de expansão da base monetária.

A primeira destas influências pode gerar uma correlação positiva entre taxas nominais de juros e níveis de preços; a segunda pode levar a uma relação íntima entre o nível destas taxas nominais e as taxas de expansão do nível de preços. Portanto, esta teoria aplica-se, de maneira coerente, tanto ao Paradoxo Gibson como ao efeito Fisher.

Abstract

This paper develops a simple theory of nominal income interest determination under three key assumptions: firstly, the only relevant distinction between money and government bonds lies in their holding periods. Secondly, individuals take full account of the government budget constraint and do not concern themselves with discounting future tax liabilities associated with the issue of government bonds. Finally, the model incorporates a private credit market for individuals' borrowings and lendings. According to this theory, the nominal rate of interest is determined by two separable influences,

* A previous version of this paper was presented at the 7th Latin American Meeting of the Econometric Society, São Paulo, August 4/7, 1987.

** Senado Federal.

the bond/money ratio and the rate of expansion of the money supply. The first of these influences can give rise to a positive correlation between nominal interest rates and price levels and the second can generate a close relationship between the level of these nominal rate and the rates of expansion of the price level. Therefore, this theory coherently accounts for both the Gibson Paradox and the Fisher-effect.

1. Introduction

This paper presents a simple neoclassical monetary theory which is general enough to simulate both the Fisher-effect and the Gibson Paradox, in the context of one single model. That is, a theory that encompasses both the standard quantity-theoretic laws in clear, exact form, and a result which is considered as an important qualification to these laws, but which is not viewed as an integral part of them yet, as has been emphasized by Lucas (1980, p. 1005-14).

The model that follows has a very sharp solution. According to it, the nominal rate of interest is basically determined by a combination of two clearly separable influences. One is the rate of change in the quantity of money; it accounts for Fisherian interest-inflation relationships. The other is the bond/money ratio; it opens the possibility for simulating Gibsonian positive correlations between nominal interest rates and price levels. As a result, the interest variable may exhibit examples of a pure Fisher-effect, a pure Gibson Paradox, or a mixture of them, depending upon the way the government chooses the time paths of money and bonds alike.

The theory behind this result stems from our own original model (Martins 1975 and 1980, p. 174-85) and embeds all its key assumptions. In particular, it relies on the three-periods version of Samuelson's 1958 pure consumption-loan model of interest as the framework of analysis (Samuelson 1958, p. 467-82). To summarize, it assumes (i) that the only theoretically relevant distinction between money and government bonds lies in the difference between their holding periods; and (ii) that there is no predetermined pattern of real rate of return on capital goods, which imposes its own way on the rest of the model. Otherwise, the bond rate would become inexorably locked to this predetermined pattern, and then it would be impossible to adequately study the nature of the Gibson Paradox. This occurred, for instance, when Preston Miller (1981) imposed such an artificial constraint on our original model.

The novelty is that we now incorporate a private credit market for individuals' borrowings and lendings into the model. This fulfils two specific and closely related analytical goals. Firstly, it brings a

clear-cut concept of real capital to the front of the stage, i. e., the real quantities of the non-storable consumption good which are borrowed and lent at each point in time. In this set-up, borrowers and lenders will make contracts in real terms, according to the real rates of interest observed in the credit market. Secondly, these real quantities of loanable funds and these real rates of interest are not fixed or predetermined. They can vary, and in general depend upon the economic policy and the state of the economy. Moreover, it suffices to remember that the Fisher-effect is a phenomenon associated with the prevalence of a constant real rate of interest over time and that the Gibson Paradox is related to a changing one. Therefore, the inflationary or deflationary economic policy that keeps the real rate of interest constant will bring about Fisherian interest-inflation relationships. Otherwise, it will induce Gibsonian and other types of interest behavior.

In the next section we present the theoretical framework of analysis. It features government, borrowers and lenders in the context of Samuelson's 1958 overlapping generations model (Samuelson, *op. cit.*). The third section is dedicated to the solution of the lender's consumption and investment problems. The key characteristic of the model is that the government bonds and the borrower's issued liabilities enter the lender's portfolio as perfect substitutes. The result is a joint demand for these two types of securities and the notion that their supplies by the government, and the borrower tend to crowd each other out. In the fourth section we solve the model for the borrower's side of the problem and derive his supply of securities (his demand for loanable funds) as a function of his nominal income endowments and of the nominal rate of interest. The next section presents the market solution of the model. It is a more general framework of analysis, for it encompasses both the standard Fisherian interest-inflation relationships and Gibsonian type of phenomenon. Finally, we conclude by comparing the theory in this paper to the one exposed by Preston J. Miller (Miller 1981).

2. The Theoretical Framework

The theory in this paper features government, borrowers and lenders. They all live in the context of a three-periods version of the well known Samuelson's 1958 overlapping generations model (Samuelson, *op. cit.*).

The government issues fiat money and two-periods bonds. They both are essentially pieces of paper which are trustingly held by the public of its own free will. As we have done elsewhere (Martins 1980, p. 178), we assume that the cost to convert bonds into money before the retirement period, and that the cost for conversion of one two-

-periods bond into two one-period bonds, are economically prohibitive. There are no taxes.

Generations overlap. An individual's life only covers three periods. There are two distinct groups of people in the population: borrowers and lenders. At each point in time t , exactly one member of each generation of borrowers, and exactly one member of each generation of lenders, are alive simultaneously. Hence, there are six people in the population at each point in time t .

Each borrower earns W units of output in the first period of his life, nothing in the second, and $1-W$ in the last, being $0 < W < 1$. The state of the economy is such that he always finds that his initial endowment, W , is "too small" to provide for his consumption needs during the first two periods of his existence. Therefore, he borrows. To borrow, he issues liabilities against his future income, $1-W$, and sells them in the credit market. Clearly, he cannot increase the flow of his own real income by selling debt, as for instance the government may do by selling unbacked fiat bonds. To simplify, he borrows only once and does so at the beginning of his life. Moreover, these liabilities are a promise to pay one unit of nominal income two periods later, as are the government bonds. The cost to convert them into money before the retirement period, and the cost for converting them into one-period liabilities, are also prohibitive. Therefore, the market price of the government bonds and of these two-periods private securities must be the same, in equilibrium. The consumption good that borrowers and lenders alike receive as income endowments cannot be stored; it melts away in one period. So borrowers must accumulate money in order to consume in the second period of their lives. For reasons to be made clear later on, the borrower never accumulates bonds. Its total stock is then held by lenders alone.

Each lender earns one unit of the same non-storable output in the beginning of his life, nothing afterwards. Thus, he accumulates securities to provide for future consumption. However, he cannot use two-periods bonds and two-periods private liabilities accumulated in any period to buy goods one period later. Just like the borrower, he also has to accumulate money in period t to provide for consumption in period $t+1$. Nevertheless, he can use both the money left over in period $t+1$ and the bonds and the private securities accumulated in t to buy consumption in $t+2$.

Finally, a "representative" member of the t^{th} generation of borrowers, and of the t^{th} generation of lenders, are assumed to value their consumption plan according to the value of a "regularly shaped" utility function of nonnegative consumptions. This utility indicator is a twice differentiable, strictly concave, strictly monotonic increasing function, and the marginal utility of consumption in any period goes to infinity as the consumption level in that period goes to zero. This

clear-cut concept of real capital to the front of the stage, i. e., the real quantities of the non-storable consumption good which are borrowed and lent at each point in time. In this set-up, borrowers and lenders will make contracts in real terms, according to the real rates of interest observed in the credit market. Secondly, these real quantities of loanable funds and these real rates of interest are not fixed or predetermined. They can vary, and in general depend upon the economic policy and the state of the economy. Moreover, it suffices to remember that the Fisher-effect is a phenomenon associated with the prevalence of a constant real rate of interest over time and that the Gibson Paradox is related to a changing one. Therefore, the inflationary or deflationary economic policy that keeps the real rate of interest constant will bring about Fisherian interest-inflation relationships. Otherwise, it will induce Gibsonian and other types of interest behavior.

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last condition guarantees that, if income is positive, consumption in any period is positive. To simplify, the value of the utility indicator is given by the sum of the logarithms of the t^{th} generation consumptions.

The goal of the next section is to state and solve, from the formal point of view, the lender's part of this theoretical framework.

3. The Lender

The t^{th} lender maximizes

$$u_t^L = \log C_t^L(t) + \log C_t^L(t+1) + \log C_t^L(t+2)$$

with respect to

$$[m_t^L(t+1), m_t^L(t+2), B_t, L_t],$$

subject to

$$P_t C_t^L(t) + m_t^L(t+1) + \frac{B_t + L_t}{1 + i_t} = P_t + G_t \quad (1a)$$

$$P_{t+1} C_t^L(t+1) + m_t^L(t+2) = m_t^L(t+1) \quad (1b)$$

$$P_{t+2} C_t^L(t+2) = m_t^L(t+2) + B_t + L_t \quad (1c)$$

where $C_t^L(i)$ stands for the i^{th} period of life consumption by the member of the t^{th} generation of lenders; $m_t^L(t+1)$ and $m_t^L(t+2)$ are the nominal quantities of money carried over periods $t+1$ and $t+2$; B_t is the nominal quantity of government bonds carried over period $t+2$; L_t is the nominal quantities of borrower liabilities carried over period $t+2$ by the lender; P_t, P_{t+1}, P_{t+2} are the nominal prices of consumption in periods $t, t+1, t+2$; and i_t is the two-periods nominal rate of return on bonds. Unbacked government bonds and backed borrower liabilities are perfect substitutes from the point of view of the lender's portfolio, by assumption. Therefore, i_t is also the two-periods nominal rate of return on the privately issued securities. Consequently, $1 / (1 + i_t)$ is the current nominal prices of the government promise and of the borrower's promise to pay one unit worth of goods in period $t+2$. At the beginning of period t , the lender earns $1 \cdot P_t$ units of nominal income and anticipates G_t units of nominal government transfer payments. To solve his problem, he takes $P_t, i_t, P_{t+1}, P_{t+2}, G_t$ as given and chooses $m_t^L(t+1), m_t^L(t+2), B_t$ and L_t .

The solution to the above problem yields the demand functions

for $m_t^l(t+1)$, $m_t^l(t+2)$, B_t and L_t . With the above "regularly shaped" utility function, $C_t^l(t+1) > 0$; then $m_t^l(t+1) > 0$. Thus, the actual competition between money, and bonds and private securities, to enter the individual's portfolio, is related to their purchasing capacity in period $t+2$. For each nominal unit of forgone consumption in period t there are three possible classes of solution. If $i_t < 0$, bonds and private securities will be driven out of the economy. If $i_t = 0$, money, bonds and private securities will be perfect substitutes and undistinguishable from each other. Those situations do not concern us in this paper. If $i_t > 0$, the lender will use only bonds and private securities to provide for his consumption in period $t+2$; then, $m_t^l(t+2) = 0$ and $B_t + L_t > 0$. Let us concentrate our attention on this class of solution. Dropping the term $(t+1)$ from the notation of $m_t^l(t+1)$, the first-order conditions are:

$$\frac{C_t^l(t+1)}{C_t^l(t)} = \frac{P_t}{P_{t+1}} \quad (2a)$$

$$\frac{C_t^l(t+2)}{C_t^l(t)} = (1+i_t) \frac{P_t}{P_{t+2}} \quad (2b)$$

$$C_t^l(t) = 1 + \frac{G_t}{P_t} - \frac{m_t^l}{P_t} - (1+i_t)^{-1} \frac{B_t + L_t}{P_t} \quad (2c)$$

$$C_t^l(t+1) = \frac{m_t^l}{P_{t+1}} \quad (2d)$$

$$C_t^l(t+2) = \frac{B_t + L_t}{P_{t+2}} \quad (2e)$$

For given values of P_t , i_t , P_{t+1} , P_{t+2} and G_t the system above determines the consumption plan

$$[C_t^l(t), C_t^l(t+1), C_t^l(t+2)]$$

and the demands for m_t^l , B_t and L_t . Inserting (2c), (2d) and (2e) into (2a) and (2b), and simplifying, we obtain

$$2m_t^l + (1+i_t)^{-1}(B_t + L_t) = P_t + G_t \quad (3a)$$

$$m_t^L + 2(1+i_t)^{-1}(B_t + L_t) = P_t + G_t \quad (3b)$$

By collapsing them, we find the demand function for m_t^L and the joint demand for B_t and L_t :

$$m_t^L = (P_t + G_t) / 3 \quad (4a)$$

$$(1+i_t)^{-1}(B_t + L_t) = (P_t + G_t) / 3 = m_t^L \quad (4b)$$

By re-writing (4b) we get:

$$1+i_t = \frac{B_t + L_t}{m_t^L} \quad (5)$$

which is entirely analogous to the expression

$$1+i_t = \frac{B_t}{M_t}$$

which we derived in connection with the solution of our original model of interest (Martins 1980, p. 181). The only differences are (i) that now the concept of future liquidity encompasses bonds and private securities instead of bonds alone; and (ii) that now the quantity of money that enters in the expression (5) is less than the total stock of money outstanding in the beginning of period t . The remainder of this stock is carried over period $t+1$ by the borrower.

In the next section we formally state and solve the borrower's problem.

4. The Borrower

The t^{th} borrower maximizes

$$u_t^b = \log C_t^b(t) + \log C_t^b(t+1) + \log C_t^b(t+2)$$

with respect to

$$(m_t^b(t+1), m_t^b(t+2), B_t^b, L_t),$$

subject to

$$P_t C_t^b(t) + m_t^b(t+1) + \frac{B_t^b}{1+i_t} = \frac{L_t}{1+i_t} + W P_t \quad (6a)$$

$$P_{t+1} C_t^b(t+1) + m_t^b(t+2) = m_t^b(t+1) \quad (6b)$$

$$P_{t+2} C_t^b(t+2) = (1-W) P_{t+2} + m_t^b(t+2) + B_t^b - L_t \quad (6c)$$

where the superscript b refers to borrower and where the variables $C_t^b(i)$, $m_t^b(t+1)$, $m_t^b(t+2)$ and B_t^b are completely analogous to their correspondents in the lender's side of the model, in the last section. Let us remember that W and $1-W$, $0 < W < 1$, are the borrower's real income endowments. The remaining variables, P_t , i_t , P_{t+1} , P_{t+2} and L_t , have already been defined.

At the beginning of period t , the borrower earns $W \cdot P_t$ units of nominal income and borrows a net amount of $L_t / (1 + i_t)$ nominal units of income, by selling L_t nominal units of his own liabilities, at a discount price of $1 / (1 + i_t)$. To highlight the theoretical role played by the variable L_t ,

transfer payments from the government. As it is intuitive, this increases his demand function for borrowings. At the beginning of period $t+2$ he earns $(1-W) P_{t+2}$ nominal units of income by selling his last period's endowment, and pays back his debt L_t . solve his problem, he takes P_t , P_{t+1} , P_{t+2} and i_t as given, and chooses $m_t^b(t+1)$, $m_t^b(t+2)$, B_t^b and L_t .

The solution to the above problem yields the demand functions for $m_t^b(t+1)$, $m_t^b(t+2)$, B_t^b , and the supply function of private securities L_t .

class of solution for which $i_t > 0$. With "regularly shaped" utility function $C_t^b(t+1) > 0$; then $m_t^b(t+1) > 0$. With $i_t > 0$ bonds dominate money in period $t+2$; then $m_t^b(t+2) = 0$ and we can drop the term $(t+1)$ to simplify the notation of $m_t^b(t+1)$. Finally, notice that i_t is both the rate that the borrower earns on his bond holdings and the rate that he pays to accumulate these holdings. He is therefore completely indifferent as to accumulating two-periods bonds or not. To deal with this situation, let us assume that the borrower only undertakes action that actually increases his real opportunity set. Then, $B_t^b = 0$. The first order conditions are:

$$\frac{C_t^b(t+1)}{C_t^b(t)} = \frac{P_t}{P_{t+1}} \quad (7a)$$

$$\frac{C_t^b(t+2)}{C_t^b(t)} = (1+i_t) \frac{P_t}{P_{t+2}} \quad (7b)$$

$$C_t^b(t) = W + (1+i_t)^{-1} \frac{L_t}{P_t} - \frac{m_t^b}{P_t} \quad (7c)$$

$$C_t^b(t+1) = \frac{m_t^b}{P_{t+1}} \quad (7d)$$

$$C_t^b(t+2) = (1-W) - \frac{L_t}{P_{t+2}} \quad (7e)$$

For given values of P_t , i_t , P_{t+1} , P_{t+2} and W , the system above determines the consumption plan

$$\{ (C_t^b(t), C_t^b(t+1), C_t^b(t+2)) \},$$

the demand function for m_t^b and the supply function of L_t . Inserting (7c), (7d) and (7e) into (7a) and (7b), and simplifying, we obtain:

$$2m_t^b - (1+i_t)^{-1} L_t = W \cdot P_t \quad (8a)$$

$$-m_t^b + 2(1+i_t)^{-1} L_t = (1+i_t)^{-1} (1-W) P_{t+2} - W P_t \quad (8b)$$

By collapsing them, we find the demand function for m_t^b and the supply function of privately issued securities, L_t :

$$m_t^b = [W \cdot P_t + (1+i_t)^{-1} (1-W) P_{t+2}] / 3 \quad (9a)$$

$$L_t = [2(1-W) P_{t+2} - (1+i_t) W \cdot P_t] / 3 \quad (9b)$$

where we assume that

$$(1+i_t) < 2(1-W) P_{t+2} / W \cdot P_t;$$

then

$$L_t > 0.$$

As can be readily appreciated, the quantity demanded for m_t^b depends only upon the present value of the borrower's nominal income endowments, and not upon the undiscounted values of these endowments, and upon the discount rate, separately.

In the next section we present the market solution.

5. The Market Solution

We wish to find the solutions for the nominal rate of interest

and for the price level as functions of the government policies with respect to money and bonds. To do so, we shall separate very clearly the demand for and the supply of assets' side of the model. Let M_t be the total stock of money at the beginning of period t .

is then given by the horizontal sum of the individual demand functions for m_t^q and m_t^b ,

Now let us remember that the individual demands for government bonds, B_t , and for private securities, L_t , are also the total demands. Therefore, the demand for assets' side of the model can be represented as follows:

$$3 M_t = (1 + W) P_{t+2} + (1 + i_t)^{-1} (1 - W) P_{t+2} + G_t \quad (10)$$

$$3 (1 + i_t)^{-1} (B_t + L_t) = P_t + G_t \quad (4b)$$

The supply of private securities by the borrower is given (9b) and the supplies of money and bonds are both determined by the government budget constraint. At the start of period t , generation $t-2$ wishes to retire B_{t-2} nominal units of bonds and the government wants to transfer G_t nominal units of income to the lender of generation t .

the same instant, government revenue is given by the issuance of money $M_t - M_{t-1}$,

the unitary price of $1 / (1 + i_t)$. This budget constraint and the supply of private securities, (9b),

the model, which can then be represented as follows:

$$G_t + B_{t-2} = M_t - M_{t-1} + (1 + i_t)^{-1} B_t \quad (11)$$

$$3 L_t = 2 (1 - W) P_{t+2} - (1 + i_t) W \cdot P_t \quad (9b)$$

In the above demand and supply system of equations, the values of M_{t-1} and B_{t-2} are given at the start of period t . Therefore, there are seven variables to be dealt with: $P_t, i_t, P_{t+2}, M_t, B_t, G_t$.

market, and to let M_t, B_t and G_t to be established by the government budget financing policy. We first find a provisional solution for i_t . To do that, the value of G_t as given by (11), is:

$$1 + i_t = \frac{(1 - W) P_{t+2} + B_t}{M_t} \quad (12)$$

which already shows that the nominal rate of interest in this model is basically determined by a combination of two clearly separable influences, a Fisherian one, given by the presence of the term P_{t+2}/M_t , and a Gibsonian one, given by the presence of the term B_t/M_t , as stated in the beginning of this paper. Moreover, it can also be hinted that a positive supply of government bonds has the effect of increasing the equilibrium nominal rate of interest above the level which would be determined by Fisherian considerations alone.

Expression (12) is very helpful to solve the sequence of the price level over time. By plugging (11) and (12) into (10), and simplifying, we obtain the following solution for the price level:

$$(1+W) P_t = M_t + M_{t-1} + B_{t-2} \quad (13)$$

This solution depends only upon the history of the stock of unbacked government securities over three periods and it is entirely analogous to the one encountered in our original model (Martins 1980, p. 183). It should be emphasized, however, that this present solution does not depend at all upon the traded stock of the privately issued securities, L_t , which are backed by borrowers' real income endowments. This model is not plagued, therefore, by the issue of whether or not the government should impose legal restrictions on private intermediation in order to achieve a better control of the sequence of the price levels over time (cf. Wallace 1983, p. 1-6). It should not.

The above expression can be readily generalized for any point in time. By plugging the appropriate value of P_{t+2} into (12) and by simplifying we get:

$$1 + i_t = \frac{1-W}{1+W} \cdot \frac{M_{t+2} + M_{t+1}}{M_t} + \frac{2}{1+W} \cdot \frac{B_t}{M_t} \quad (14)$$

which solves the model.

6. Concluding Remarks

The widely accepted standard quantity-theory models present two central implications: "that a given change in the rate of change in the quantity of money induces (i) an equal change in the rate of price inflation; and (ii) an equal change in the nominal rates of interest." These two laws "... possess a combination of theoretical coherence and empirical verification shared by no other propositions in monetary economics." (Lucas 1980, p. 1005). However, and despite this theoretical and empirical strength, the standard monetarist models still fail to explain some very important empirical phenomena. As

one example we mention the occurrence of the Gibson Paradox in Great Britain for a quarter of a millennium (see Shiller and Siegel 1977, p. 891-907); as another, we indicate the patterns of return of U.S. saving bonds and Treasury bills, which are considered as paradoxical by Neil Wallace (see Wallace 1983, p. 1). These paradoxes or rather, these yet-to-be-explained phenomena, warn us that there is both room and need for improving the standard quantity-theory models.

Our original model of interest (Martins 1975 and 1980, p. 174-185) was an attempt to show that the theoretical locus of Fisherian macroeconomics is too narrow for monetary analysis. In particular, we then argued that the issuance of government bonds should affect both the price level and the nominal rate of interest. The core of the argument was the notion that the only theoretically relevant distinction between money and government bonds lies in the difference between their holding periods and hence that they should be modeled in exactly the same theoretical way, as essentially pieces of paper, backed only by the public trust. The result was a theory that did imply a sharply devised influence of government bonds on both prices and interests and shed some light on the working of the Gibson Paradox (see Martins 1980, p. 183-4). Nonetheless, we did not build any concept of capital into that original model; that is, we did not construct any mechanism that could serve as a basis for the working of the Fisher-effect. Consequently, and contrary to the central implications of the quantity theory of money, and of much empirical evidence, its nominal

mined by the bond/money ratio (which in the context of the model represents a proxy for the cost of waiting for the bond maturity date), bearing no close relationship to the rate of expansion of the price level.

This paper tries to bridge this gap by incorporating a private credit market for individual borrowings and lendings in that original framework. The outcome is a clear cut, more general, expanded quantity theory of money, which is subjected to the qualification that the only difference between money and bonds lies in the difference between their holding periods. It is summarized by the price-equation (13) and by the interest-equation (14). According to these the two standard central quantity-theoretic laws will appear as characteristics of steady state equilibria only if the bond/money ratio is kept constant over time. Otherwise there will be deviations from these laws.

Notice that this expanded model could be readily specialized. By setting the supply of government bonds equal to zero we would obtain a pure Fisherian quantity-theory model, together with its two central implications, which would come about without any type of qualification. By eliminating the reasons for borrowing and lending

(i. e., would get our own original model back. In general, however, its solution for the nominal rate of interest depends on a combination of two clearly separable influences. The first is the rate of change in the quantity of money, represented by the quotient $(M_{t+2} + M_{t+1}) / M_t$ in the interest-equation (14), which accounts for Fisherian interest-inflation relationships. The other is the bond/money ratio, B_t/M_t , which opens the possibility for simulating the Gibson Paradox as we have done elsewhere (Martins 1980, p. 183-4). As a result, the interest variable may exhibit examples of a pure Fisher-effect, a pure Gibson Paradox, or a mixture of them, depending upon the way the government chooses the time paths of money and bonds alike. In other words, this expanded model is really a more general framework of analysis, for it encompasses both the standard quantity theory of money and the special type of qualification provided by our own original model, as polar, special cases.

In 1981 Preston Miller tried to show that our original model was nothing more than a standard monetarist model (albeit an obviously more complicated one). He did so by introducing "Capital. . . into the model with a storage technology that matches the payout pattern of bonds: zero gross return after one period and a positive gross return after two periods.", in which neither bonds nor claims on capital can be traded. . . ". As a result he obtained ". . . that many monetary propositions follow. First, the rate of inflation is equal to the growth rate of money (and independent of the growth rate of bonds). Second, an exact Fisher-effect obtains, so that the nominal interest rate on bonds is equal to the real rate of return on capital plus the rate of inflation (and does not depend on the bond/money ratio)." (see Miller, 1981, p. 2-3).

To understand the meaning of Miller's proposal, we firstly notice that the Gibson Paradox could be readily reintroduced in his own set up. In order to achieve this goal it would suffice to expand the model from a three — to a four-periods framework and to assume that his storage technology presents zero gross return after one period and after two periods,

This could be easily done from a purely formal point of view. From a theoretical standpoint it suggests that the understanding of the Gibson Paradox depends on the consideration of the differences between the holding periods of the assets that compete to enter the individual's portfolio. Finally, we recall that the simulation of this Paradox requires a model in which the real rate of interest is allowed to vary in response, for instance, to government deficit financing policies. In other words, a model that imposes a predetermined pattern of real rate of interest from the outset to which all the other rates must adapt by assumption, is condemned to shed no light at all on the working of

that phenomenon. Miller's capital good has no underlying supply function, presents a fixed real rate of return that matches the payout pattern of bonds, and can always be exchanged for consumption goods at the rate of one to one, at maturity, regardless of everything else. By imposing such an amount of restriction at once, Preston Miller has indeed artificially locked our original model to the standard Fisherian monetary theory from which, we hope, we are free again.

References

- LUCAS, Robert E., Jr. Two illustrations of the quantity theory of money. In *The American Economic Review* 70, Dec., 1980.
- MARTINS, Marco A.C. The private and the social optimum quantity of public debt in a pure consumption-loans model (unpublished Ph.D. dissertation). University of Chicago, June, 1975.
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- _____. A nominal theory of the nominal rate of interest and the price level. In *Journal of Political Economy* 88, Feb., 1980.
- MILLER, Preston J. Fiscal policy in a monetarist model. In *Federal Reserve Bank of Minneapolis Research Department Staff Report* 67, Sept., 1981.
- SHILLER, Robert J. & SIEGEL, Jeremy J. The Gibson paradox and historical movements in real interest rates. In *Journal of Political Economy* 85, Oct., 1977.
- SAMUELSON, P. A. An exact consumption-loan model of interest with or without the social contrivance of money. In *Journal of Political Economy* 66, Dec., 1958.
- WALLACE, Neil. A legal restrictions theory of the demand for "money" and the role of monetary policy. In *Federal Reserve Bank of Minneapolis Quarterly Review* 7, Winter, 1983.