A KEYNESIAN MODEL OF NOMINAL WAGE RIGIDITY*

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Abstract

We present a Model that reflects Keynes' intuition concerning nominal wage rigidity: workers like to keep their relative status on society. Several results are presented, including the analysis of the influence of risk aversion. The appendix has two new results on symmetric games.

1. Introduction.

Nominal wage rigidity is the fundamental mechanism in the traditional macroeconomic model of Keynes. It is through this that the government can adjust the economy to the full employment level. Yet no good model to explain this phenomenon exists. Models of price rigidity, like Stiglitz and Weiss (1981), and Shapiro and Stiglitz (1982), or similar ones, consider only real prices. The one I shall present here is consistent only with nominal rigidities. The existing formal models of nominal price rigidity are of the type of the "kinked demand curve" as in Henderson and Quandt (1958). As is well known this is a completely "ad-hoc" procedure. The explanation that is most popular, however, is "money illusion". This argument basically says that the employees do not pay attention, at least immediately, to the real wage. They take some time to realize that they gained or lost purchasing power, by the decrease or increase of the general price level. In other words,

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the workers are deluded by nominal figures. Although this idea is intuitively appealing, it would mean a kind of irrational behaviour on the part of the employees. After all it is not necessary to be a genius, or even to make calculations, to understand that other prices are going up, but not your wage.

Another argument in favour of money wages rigidity is the empirical evidence. In fact this has been noticed in various countries. Most striking evidence is the stickiness of money wages during the great depression, and the recent Chilean experience, where the government tried to control inflation and unemployment through a monetary approach to the balance of payments, but never succeeded to control the latter, because of the rigidity of the nominal wages. However the economist should try to explain every component of his model, even when the hypothesis is empirically observed.

How then could we analyse the problem? The answer, as Tobin (1972) notices, is given by Keynes (1936) in his General Theory, Ch. 2.III. Let us quote the passage, to avoid misinterpretation:

Though the struggle over money-wages between individuals and groups is often believed to determine the general level of real wages, it is, in fact, concerned with a different object. Since there is imperfect mobility of labour, and wages do not tend to an exact equality of net advantage in different occupations, any individual or group of individuals, who consent to a reduction of money-wages relatively to the others, will suffer a relative reduction in real wages, which is sufficient justification for them to resist it. On the other hand it would be impracticable to resist every reduction of real wages, due to a change in the purchasing-power of money which affects all workers alike; and in fact reductions of real wages arising in this way are not, as a rule, resisted unless they proceed to an extreme degree.

As we see this idea is much more plausible: workers are concerned about their relative position in the society. But somebody has to start the process of cutting wages. So the employees of that firm will resist the wage cut, in order to avoid a worsening relative to the others. The same does not occur when the cut in the wages occurs through a loss of purchasing-power, because every worker is affected in the same way.
The aim of this article is to set up a formal model for the above argument.

2. Firms.

In this model firms will be considered as usually profit maximizers, taking wages and prices as given. If we suppose a production function $y_i = F_i(N)$ with $F'_i > 0$ and $F''_i < 0$ for a particular firm $i$, then $N_i$ (the demand for labour of firm $i$) is given by $F'_i(N_i) = \frac{W}{P}$. Now an external shock occurs and alters the derivative of $F_i$. Assume it is a negative supply shock ($F'_i(N_i) > F_i''(N_i) \forall N_i$). Under the assumption that firms always operate on the demand curve (this may be contradicted sometimes), we have as a consequence that either $N_i$ or $\frac{W}{P}$ decreases. Again let us suppose the price $P$ cannot be affected at all, but there are frictions on the labour market. Then the employers may offer a wage cut to the employees in order to avoid massive firing. We will analyse here the extreme case in which the firms offer a cut in wages or fire workers. It is clear that a more complete analysis would require a compromise between wages and employment. This discussion justifies the following behaviour of firms, taken as the first hypothesis:

(a) Firm $i$ proposes to its employees the following scheme: either you accept an once and for all reduction in noninal wages by $\theta_i \%$, or we will have to fire at random a proportion $p_i \%$ of you.

Observe that, in the way the cut in wages is modeled here, one does not allow a dependence on the unknown price level, otherwise the model could give as a result rigidity in real wages. The main question is whether or not the reduction offered by the firms is nominal or real. The viewpoint of this paper is that the firms offer a nominal cut.

3. Workers.

They are the key agents of the model. To formalize Keynes idea, we have to assume that workers have a utility function depending upon relative wages, on the expected utility hypothesis of Von Neumann-Morgenstern. This can be seen as a consumption externality on the usual neoclassical model, where preferences
over bundles are also influenced by prices in this particular way. This can be justified by means of envy, or status desire.

We will ignore here effects other than the externality itself. This gives us the second hypothesis:

(b) The employees are concerned only with their relative position in society, represented only by their wages relative to the rest of society.

It is obvious that this is not entirely realistic, even in the simple context we will be dealing with. In a more general model one could also think that the employees worry about unemployment.

Another hypothesis will be that workers of the same firm act together, in the sense that they all together accept or refuse to have their wages altered. This can be viewed as a kind of union acting only in each firm. Also we will suppose that the answer they will give to the employers is either “yes” or “no”. They could also:

(i) go on strike; (ii) counter offer a cut less than \( \theta \% \) (they could go even further: ask for increases in their nominal wages); (iii) quit the job to search for a better wage; (iv) take into account that they will, very probably, face a similar situation again. Most of these points have been noticed by Simonsen (1982). However the idea is to keep the model as simple as possible to get the results we will, without departing from the Keynesian viewpoint. It should be clear that it would be interesting to complicate the model to include all these effects. This discussion can be summarized as follows:

(c) The employees answer to (a) independently per firm, and only accepting or refusing the wage cut.

Finally, as a simplification we will assume:

(d) The unemployment wage is zero.

4. The model.

As Simonsen (1982) noticed, the Keynesian view of the subject is a game theoretic one.\(^1\) He also understood that the simple argument in the General Theory would admit as the only equilibrium point the downward rigidity of nominal wages. As it is well known this does not happen in practice: in the U.S.A. money wages fell between 1929 and 1933. What is empirically observed is the resistance to a decrease, and not absolute rigidity. The

\(^1\) Fraga (1981) also noticed this point, independently.
model that will be developed in this section will take this point into account.

Two other assumptions (in addition to (a) – (d)) are needed. The first one is just a consistency one, to make explicit that the behaviour of the labour market cannot be a competitive one:

(e) The labour market has imperfect mobility.

The second one avoids unnecessary complications:

(f) The firms employ the same kind of worker with the same utility function and the same initial wage.

With hypotheses (a) to (f), and, for the moment, assuming that there are only two firms, we have the following game:

(i) two players, each one representing the workers of one firm;
(ii) each of them answering "yes" or "no" to the proposal of its firm. If "yes" he keeps the job, but his wage decreases. If "no" he has a chance of being fired, and face unemployment. They will be supposed to act non-cooperatively.

Let the utility of player \( i \) be \( u(x) \) where \( x \) is the wage of employees of firm \( i \) divided by the average wage of employees of firm \( j, j \neq i \). With two firms \( u_1(w_1, w_2) = u(\frac{w_1}{w_2}) \), and \( u_2(\bar{w}_1, w_2) = u(\frac{w_2}{\bar{w}_1}) \). With \( n \)-firms:

\[
\begin{align*}
    u_i(\bar{w}_1, \ldots, w_i, \ldots, \bar{w}_n) &= u\left(\frac{1}{n-1} \sum_{j \neq i} w_j\right),
\end{align*}
\]

being \( \bar{w}_j \) the average nominal wage for the workers of firm \( j \), and \( w_i \) the wage of the persons of firm \( i \). Let \( w \) be the initial money wage, and \( \theta_i \) and \( p_i \) be given by (a).

We have four cases;

(i) Both players accept the cut: in this case we have that no employee is fired and both wages are cut:

\[
\begin{align*}
    w_1 = \bar{w}_1 = w(1 - \theta_1) \\
    w_2 = \bar{w}_2 = w(1 - \theta_2)
\end{align*}
\]

\[
\Rightarrow \quad \begin{cases} 
    u_1 = u(\frac{1 - \theta_1}{1 - \theta_2}) \\
    u_2 = u(\frac{1 - \theta_2}{1 - \theta_1})
\end{cases}
\]

(ii) Player one accepts the cut and player two does not: in this case
\[ w_1 = \bar{w}_1 = w(1 - \theta_1) \text{ and} \]
\[ \bar{w}_2 = p_2 \cdot 0 + (1 - p_2) \cdot w = w(1 - p_2) \Rightarrow \]
\[ u_1 = u\left(\frac{1 - \theta_1}{1 - p_2}\right) \quad w_2 = \begin{cases} 
0 & \text{if unemployed} \\
 w & \text{if employed} 
\end{cases} \Rightarrow \]
\[ \Rightarrow u_2 = p_2 \cdot u(0) + (1 - p_2) \cdot u\left(\frac{1}{1 - \theta_1}\right) \]

(iii) Player two accepts the cut and player one does not: this is symmetric to (ii).

\[ u_1 = p_1 \cdot u(0) + (1 - p_1) \cdot u\left(\frac{1}{1 - \theta_2}\right) \]
\[ u_2 = u\left(\frac{1 - \theta_2}{1 - p_1}\right) \]

(iv) Both players refuse to accept the cut:

\[ \bar{w}_1 = p_1 \cdot 0 + (1 - p_1)w = w(1 - p_1) \]
\[ \bar{w}_2 = p_2 \cdot 0 + (1 - p_2)w = w(1 - p_2) \Rightarrow \]
\[ w_1 = w_2 = \begin{cases} 
0 & \text{if unemployed} \\
 w & \text{if employed} 
\end{cases} \]
\[ u_1 = p_1u(0) + (1 - p_1) \cdot u\left(\frac{1}{1 - p_2}\right) \]
\[ u_2 = p_2u(0) + (1 - p_2) \cdot u\left(\frac{1}{1 - p_1}\right) \]

This gives us the following payoff configuration:

As it will be seen in an example in a following section anything can occur in such a situation: any of the four configurations can be achieved as Nash equilibrium for a convenient choice of the parameters \( \theta_1, \theta_2, p_1, p_2 \).
5. The symmetric case.

The first thing to be noticed is existence of equilibrium (Nash) in this context, by Theorem 1 in the appendix, since our general model of the last section is a symmetric game in the case $p_1 = p_2 = p$, and $\theta_1 = \theta_2 = \theta$, that is treated here.

Symmetry allows us to say something about the behaviour of the workers.

**Proposition 1.** Given any continuous utility function then $\forall \theta > 0 \exists p^*(\theta) > 0$ such that $p < p^*(\theta) \Rightarrow (\text{No}, \text{No})$ is Nash Equilibrium.
Furthermore \( p^*(\theta) \) is strictly increasing in \( \theta \) in case \( p^*(\theta) \neq 1 \) for all \( \theta \).

PROOF: By (b) \( u \) is a strictly increasing function. Without loss of generality we may suppose \( u(0) = 0 \). Therefore (No, No) is a N.E. if and only if: \( u_1(\text{No, No}) \geq u_1(\text{Yes, No}) \) and \( u_2(\text{No, No}) \geq u_2(\text{No, Yes}) \). This is the same as:

\[
(1 - p)u\left(\frac{1}{1 - p}\right) \geq u\left(\frac{1 - \theta}{1 - p}\right)
\]  

(1)

Let \( F(\theta, p) = u\left(\frac{1 - \theta}{1 - p}\right) - (1 - p)u\left(\frac{1}{1 - p}\right) \). Then (No, No) is N.E. if and only if \( F(\theta, p) \leq 0 \). By definition \( F(\theta, 0) = u(1 - \theta) - u(1) < 0 \). Define \( N(\theta) = \{ p; F(\theta, p) < 0 \} \). It follows from the continuity of \( u \) that \( N(\theta) \) contains an interval of the type \([0, \bar{\theta}], \bar{\theta} > 0 \). Let \( p^*(\theta) = \sup\{ \bar{\theta}; [0, \bar{\theta}] \subset N(\theta) \} \). This satisfies the desired properties. We will show now that \( \theta > \theta' \Rightarrow p^*(\theta) > p^*(\theta') \) (if \( p^*(\theta) \neq 1 \forall \theta > 0 \), it is sufficient to notice that \( \theta > \theta' \Rightarrow F(\theta, p) < F(\theta', p) \Rightarrow N(\theta) \subset N(\theta') \Rightarrow p^*(\theta) \geq p^*(\theta') \).

But, again by the fact \( F \) is strictly decreasing in \( \theta \) for every given \( p \), it is clear that \( p^*(\theta) = p^*(\theta') \) cannot happen. This completes the proof.

The intuition behind this result is very clear: if the probability of being fired is low enough (where “enough” depends on the cut in nominal wages in an increasing way) then it is a rational attitude for the workers to refuse the wage cut.

It is important to notice that if the employees are risk lovers, then it may be optimal to refuse the wage cut always, no matter how low is the probability of being fired. This will be seen in the next section, in example 4. Nonetheless this kind of behaviour can never be verified in risk averse or risk neutral individuals. In fact:

**Proposition 2.** If \( u \) is risk neutral or risk averse, and continuous in \([0, +\infty[\), then: (i) \( p^*(\theta) < 1 \), and (ii) \( p > p^*(\theta) \Rightarrow (\text{No, No}) \) is not a Nash equilibrium.

PROOF: Let us assume again \( u(0) = 0 \), and \( F(\theta, p) \) is defined as in the proof of the first proposition. We will show that there exists one and only one \( p^*(\theta) \in [0,1[ \) such that \( F(\theta, p^*(\theta)) = 0 \), and as \( F(\theta, 0) < 0 \), this will show that the theorem is true. In fact,
\[ F(\theta, p) = 0 \iff u\left( \frac{1 - \theta}{1 - p} \right) = (1 - p)u\left( \frac{1}{1 - p} \right) \iff (\text{as } u \text{ is strictly increasing}) \theta = 1 - (1 - p)u^{-1}\left( (1 - p)u\left( \frac{1}{1 - p} \right) \right). \]

Define \( H(p) = 1 - (1 - p)u^{-1}\left( (1 - p)u\left( \frac{1}{1 - p} \right) \right) \). Then \( H(0) = 0 \). To calculate \( \lim_{p \to 1} H(p) \) we prove that \( (1 - p)u\left( \frac{1}{1 - p} \right) \) is bounded, \( \forall p \in [0, 1] \).

Notice that risk neutral individuals also have concave utility functions. Then \( \forall p \in [0, 1] \): by concavity \( pu\left( \frac{1}{p} \right) + (1 - p)u\left( \frac{1}{1 - p} \right) \leq u(2) \Rightarrow (1 - p)u\left( \frac{1}{1 - p} \right) \leq u(2) - pu\left( \frac{1}{p} \right) \leq u(2) \). It follows then that \( \lim_{p \to 1} H(p) = 1 \). Now we show that \( H(p) \) is strictly increasing in \( p \), and this proves the result, being it sufficient to take \( p^*(\theta) = H^{-1}(\theta) \). To prove this observe that it is enough to show that \( (1 - p)u\left( \frac{1}{1 - p} \right) \) is nonincreasing in \( p \), which follows from the fact that \( u \) concave and \( u(0) = 0 \Rightarrow \frac{u(z)}{z} \) is nonincreasing in \( z \in [0, +\infty[ \), fact 0 < \( z_1 < z_2 \Rightarrow 0 < \frac{z_1}{z_2} < 1 \) and \( z_1 = \frac{z_1}{z_2}z_2 + (1 - \frac{z_1}{z_2})0 \Rightarrow u(z_1) \geq \frac{z_1}{z_2} u(z_2) + (1 - \frac{z_1}{z_2})u(0) = \frac{z_1}{z_2} u(z_2) \Rightarrow \frac{u(z_1)}{z_1} \geq \frac{u(z_2)}{z_2} \), and the result is shown. It is obvious that \( p^*(\theta) \) thus defined agrees with the definition of Proposition 1.

This proposition shows that risk aversion is an important feature to avoid triviality of results: if employees are risk averse (or risk neutral) and the probability of being fired is high enough (again, as in Theorem 1, "enough" depends in an increasing way on the intensity of the cut in nominal wages), then it is not rational for them to refuse the wage cut.

The next proposition characterizes the rationality of the acceptance of the wage cut.

**Proposition 3.** Given a continuous utility function in \([0, +\infty[\), strictly increasing, then \( \forall \theta > 0 \) there exists \( 0 < p^**(\theta) < 1 \), strictly increasing with \( \theta \), such that \( p \geq p^**(\theta) \Rightarrow (\text{Yes, Yes}) \) is a Nash Equilibrium. Furthermore \( p < p^**(\theta) \Rightarrow (\text{Yes, Yes}) \) is not a N.E.

**Proof:** (Yes, Yes) is N.E. if and only if \( u_1(\text{Yes, Yes}) \geq u_1(\text{No, Yes}) \).
Yes) and $u_2(\text{Yes, Yes}) \geq u_2(\text{Yes, No})$. This is the same as (being $u(0) = 0$) $u(1) \geq (1 - p)u(\frac{1}{1 - \theta})$. Take $p^*(\theta) = 1 - \frac{u(1)}{u(\frac{1}{1 - \theta})}$.

The rest follows easily. □

The intuition of the result above is obviously exactly the opposite of Propositions 1 and 2. If the chance of being fired is high enough then the workers of both firms accept the cut. If it is low enough then it is not optimal to accept the cut.

The corollaries that follow just show when the behaviour is straightforward.

**Corollary 1.** If $p < \min\{p^*(\theta), p^{**}(\theta)\}$ then (No, No) is the unique N.E.

**Proof:** Trivial from the fact the number of players is two. □

**Corollary 2.** In case of risk neutrality or risk aversion, we have that $p > \max\{p^*(\theta), p^{**}(\theta)\}$ implies (Yes, Yes) is the unique N.E.

**Proof:** The same as above. □

These corollaries show one of the main points: we can have sticky wages, but when the probability of being fired becomes high enough, like in a great depression, then the rational action is to accept the reduction in wages.

By means of Theorem 2 of the appendix, one sees that the actions that result from Corollaries 1 and 2 are strictly dominant strategies. This could induce one to think that no game interaction is present in the model. However two points should be noted. First, this is a general property of symmetric games which, by no chance, is particular to the model. Second, there are lots of situations where non-trivial behaviour can occur. As later examples will show, even in the symmetric case. A particularly interesting kind of nontrivial behaviour is an example where all equilibria are asymmetric. This happens, for the following utility function:

$$u(x) = \begin{cases} \frac{2x}{x + 1}, & 0 \leq x \leq 1 \\ \frac{1}{2} + \frac{x}{2}, & x \geq 1 \end{cases}$$

If one computes $p^*(\theta)$ and $p^{**}(\theta)$ for $\theta = 0.99$, one gets $p^*(0.99) < p^{**}(0.99)$, and $\forall p \in \{0.99, 0.99\}$ and for $\theta = 0.99$...
I have asymmetric behaviour (this means (No, Yes) and (Yes, No) are the only equilibria). The example could easily be rebuilt, in such a way that the values of the parameters would be more credible.

To end this section we will make some remarks on the case of \( n \) firms. Again (considering \( u(0) = 0 \) ) (No, \( \cdots \), No) gives the payoff: \( u_i = (1 - p)u(\frac{1}{1 - p}) \). For the configuration (No, \( \cdots \), No, Yes, No, \( \cdots \), Yes, No, Yes, \( \cdots \), Yes) in \( u_i = (1 - p)u(\frac{1}{1 - p}) \). Similarly (Yes, \( \cdots \), Yes) results in the payoff \( u_i = u(1) \) and (Yes, \( \cdots \), Yes, No, Yes, \( \cdots \), Yes) in \( u_i = (1 - p)u(\frac{1}{1 - p}) \). Therefore it follows that propositions analogous to 1, 2 and 3 are still valid. Notice that we cannot say the same with respect to corollaries 1 and 2, and a more detailed analysis should be done. However we do not think these corollaries are central results, and for this reason we will not deepen in these details. The model is then robust with argumenting the number of firms.


The risk aversion coefficient, as defined in Pratt (1964) plays a fundamental role on the results. As one would expect, the more risk averse the individual, the less propense he is to submit himself to unemployment. Therefore as a result one would like to get that, given the wage cut \( \theta \), the more risk averse he is, the more likely he is of accepting the cut and, the less likely he is of refusing it. The next proposition shows that this is the case.

**Proposition 4.** Let \( u_1 \) and \( u_2 \) be continuous in \([0, +\infty[\), concave, in creasing. Suppose \( u_1 \) is at least as risk averse as \( u_2 \). Then \( p_1^*(\theta) \leq p_2^*(\theta) \) and \( p_1^{**}(\theta) \leq p_2^{**}(\theta) \), \( \forall \theta \epsilon [0, 1[. \)

**Proof:** Notice that by a result of Pratt (1964, Theorem 1), \( u_1 \) is at least as risk averse as \( u_2 \) if and only if there exists \( k \) concave (a nd increasing) such that \( u_1 = k \circ u_2 \). Assuming again \( u_1(0) =
\[ u_2(0) = 0 \] we have \( k(0) = 0 \). Let us start with \( p^{**}(\theta) \):

\[
p^{**}_1(\theta) = 1 - \frac{k(u(1))}{k(u(\frac{1}{1-\theta}))}. \text{ But } \frac{k(u(1))}{u(1)} \geq \frac{k(u(\frac{1}{1-\theta}))}{u(\frac{1}{1-\theta})} \Rightarrow \\
\Rightarrow 1 - \frac{k(u(1))}{k(u(\frac{1}{1-\theta}))} \leq 1 - \frac{u(1)}{u(\frac{1}{1-\theta})} = p^{**}_2(\theta).
\]

Then \( p^{**}_1(\theta) \leq p^{**}_2(\theta), \forall \theta \in [0,1]. \)

To prove the same result for \( p^*(\theta) \), let us recall that \( p^*(\theta) \) is the solution to \( \theta = H(p) \), where \( H(p) = 1 - (1 - p)^{-1}u_1^{-1}(1 - p)u(\frac{1}{1-p}) \), as in Proposition 1. Let \( H_1(p) \) and \( H_2(p) \) be the corresponding functions for \( u_1 \) and \( u_2 \). We want to show that \( p^*_1(\theta) \leq p^*_2(\theta) \), what ends up being the same as proving \( H_1(p) \geq H_2(p), \forall p \in [0,1]. \) But:

\[
H_1(p) - H_2(p) = (1 - p)[u_2^{-1}((1 - p)u_2(\frac{1}{1-p})) - u_1^{-1}((1 - p)u_1(\frac{1}{1-p}))]. \text{ Without loss of generality we can assume } u_1(1) = u_2(1) = 1 \Rightarrow k(1) = 1. \text{ Then: } x = (1 - p)u_2(\frac{1}{1-p}) \leq 1. \text{ However } u_2(\frac{1}{1-p}) > 1 \Rightarrow \frac{k(u_2(\frac{1}{1-p}))}{u_2(\frac{1}{1-p})} \leq 1 \Rightarrow (1 - p)u_1(\frac{1}{1-p}) = (1 - p)k(u_2(\frac{1}{1-p})) \leq (1 - p)u_2(\frac{1}{1-p}) = x.
\]

Therefore \( H_1(p) - H_2(p) \geq (1 - p)[u_2^{-1}(x) - u_2^{-1}(k^{-1}(x))] \). As \( x \leq 1 \), we have \( k^{-1}(x) \leq 1 \), and \( \frac{k(k^{-1}(x))}{k^{-1}(x)} \geq 1 \Rightarrow x \geq k^{-1}(x) \Rightarrow u_2^{-1}(x) - u_2^{-1}(k^{-1}(x)) \geq 0 \Rightarrow H_1(p) \geq H_2(p), \forall p \in [0,1]. \)

The proposition above says that, for given \( \theta \), the region where (No, No) is equilibrium \( (p \leq p^*(\theta)) \) shrinks with the increase of risk aversion; whereas the region where (Yes, Yes) is equilibrium \( (p \geq p^{**}(\theta)) \) enlarges with it.

In the next section some examples are shown.
Diagram 2.
Region 1: (No, No) Unique Nash Equilibria
Region 2: (Yes, Yes) Unique Nash Equilibria

7. Examples.

This section is a collection of examples. They are intended to show the richness of behaviour allowed by the model, and to stress particular points that were not taken into account explicitly.

Example 1: The Influence of Risk Aversion in a Simple Behavior

This example will be done by using the function \( u(x) = x^{1-\alpha} \).

It has the property of constant relative risk aversion \( \frac{\alpha u''(x)}{u'(x)} \), that equals to \( \alpha \). In here we will consider \( \alpha < 1 \). The case of constant relative risk aversion \( \alpha \geq 1 \) is not consistent with \( u(0) = 0 \).

After a simple calculation \( p^*(\theta) = p^{**}(\theta) = 1 - (1 - \theta)^{1-\alpha} \). The figure, for \( 0 < \alpha < 1 \), would be:

No w, if \( \alpha \) is varying, we have:

As the figure shows, as risk aversion increases it grows the area where the reduction in wages is accepted, and the opposite when it decreases. This will be the general influence of risk aver-
ension in the model: People are more afraid of being fired with its increase, what makes them behave more carefully, accepting more frequently the wage cut.

**Example 2:** The presence of non-trivial behavior

Take \( u(x) = \frac{x}{x+1} \). Then \( p^*(\theta) = 1 - (1 - \theta)^{1/2} \), and \( p^{**} = \theta/2 \). The graph is as follows:

**Example 3:** The Influence of Risk Aversion in a Complex Model

Let \( u(x) = 1 - e^{-\alpha x} \). This is a constant absolute risk aversion utility function, where risk aversion (absolute) is \( \alpha \).

The graph of \( p^*(\theta) \) and \( p^{**}(\theta) \) is as in the later example. The influence of risk aversion may be easily calculated, and by Proposition 4 both curves decrease when risk aversion increases.

**Example 4:** The behavior of risk lover individuals

Now we will consider \( u(x) = e^{\alpha x} - 1 \) (constant absolute risk aversion = −1). In this case we have that \( p^*(\theta) = 1 \) for \( \theta > .35 \) (indeed a number a bit lower), basically because the function \( H(p) \) of Proposition 2 is not strictly increasing. It is possible to calculate numerically the behaviour:

**Example 5:** The Presence of Asymmetries
Diagram 4.

Diagram 5.
Let \( u(x) = x \), a very simple utility function, and let us consider that firms do not act symmetrically. Then we have the following game:

We have nine cases. Only four of them will be shown, since the other five are just degenerate cases of these four:

(i) \( p_1 < \theta_1 \) and \( p_2 < \theta_2 \) ⇒ (No, No) unique Nash Equilibrium.
(ii) \( p_1 < \theta_1 \) and \( p_2 > \theta_2 \) ⇒ (No, Yes) unique Nash Equilibrium.
(iii) \( p_1 > \theta_1 \) and \( p_2 < \theta_2 \) ⇒ (Yes, No) unique Nash Equilibrium.
(iv) \( p_1 > \theta_1 \) and \( p_2 > \theta_2 \) ⇒ (Yes, Yes) unique Nash Equilibrium.

These results show that if the behaviour of the firms is not the same, then it may be optimal for its workers to act differently. Moreover they act in a quite intuitive way: if both chances of being fired are small, then none accept the wage cut. On the other hand, if one is high and the other is low, the former will accept and the latter will not. The other cases have similar intuition.

8. Increases in nominal wages.

In this situation the apparently innocuous hypothesis (c) is crucial. In fact, let us suppose it prevails an unemployment rate of \( \delta \% \) uniformly in all the unions of the firms. Also assume the
Table 2.

<table>
<thead>
<tr>
<th>Employees - Firm 1</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>(\delta u(0) + (1-\delta)u(\frac{1+\theta}{1-\delta+p}))</td>
<td>(\delta u(0) + (1-\delta)u(\frac{1+\theta}{1-\delta+p}))</td>
</tr>
<tr>
<td>No</td>
<td>(\delta u(0) + (1-\delta)u(\frac{1+\theta}{1-\delta+p}))</td>
<td>(\delta u(0) + (1-\delta)u(\frac{1+\theta}{1-\delta+p}))</td>
</tr>
</tbody>
</table>

Workers are offered the following scheme (this scheme is compatible with the behaviour of the firms, as described in paragraph 2):

\((g)\) We will increase your wage in \(\theta\)% or reconvert \(p\)% of the unemployed of your union (in the model it is not possible \(p > \delta\)).

Now, if a worker decide only in a selfish way, it always pays him to accept the wage increase. This is obvious from the fact that his relative position in society can never be worsened in case his wage increase and the number of unemployed people remain the same.

Nevertheless, this will not hold if the employees care about his companions. To make the calculations, let us compute the average wages, under the assumption that the same \(p's\) and \(\theta's\) prevail.
If the increase is accepted: \( w = \begin{cases} 0 \text{ unemployed (δ\%)} \\ w(1 + \theta) \text{ employed (1 − δ\%)} \end{cases} \)

and \( \bar{w} = (1 − δ)w(1 + \theta) \).

If it is refused: \( w = \begin{cases} 0 \text{ unemployed ((δ − p)\%)} \\ w \text{ employed ((1 − δ + p)\%)} \end{cases} \)

and \( \bar{w} = (1 − δ + p)w \).

This gives us, in the two firm example, the payoff matrix:

For these calculations we supposed that all the workers decide about the cut. That is why the utilities are averaged. Otherwise the utility of the players would differ. From this we have our last result:

**Proposition 5.** If \( u \) is' continuous in \([0, +\infty[ \) and \( δ > 1 - \frac{u(1) - u(0)}{u(1 + \theta) - u(0)} \), then there exists \( p \) such that the workers will refuse the wage increase.

**Proof:** Take \( u^*(x) = u(x) - u(0) \). Then we can make every calculation with \( u(0) = 0 \). But (No, No) is N.E. if \((1 − δ + p)u(\frac{1}{1 − δ + p}) \geq (1 − δ)u(\frac{1 + \theta}{1 − δ + p}) \). Define \( F(\theta, δ, p) = (1 − δ + p)u(\frac{1}{1 − δ + p}) - (1 − δ)u(\frac{1 + \theta}{1 − δ + p}) \). By the hypothesis \( F(\theta, δ, δ) = u(1) - (1 − δ)u(1 + \theta) > 0 \), and by the continuity of \( F \) (that comes from \( u \)) it is possible to choose \( 0 < p < δ \), near to \( δ \), such that \( F(\theta, δ, p) > 0 \). This implies (No, No) is N.E. for these combinations of \( \theta \), \( δ \) and \( p \).

The economic intuition behind this result is clear: if the unemployment rate is high enough, then the union will take care of its unemployed. In this case the workers are altruistic. On the other hand if \( δ \) is low enough, then every increase in wages is accepted. Also we have that if the unemployment cut is low, then the wage increase is accepted.

9. **Conclusion.**

A model which sheds some light on the resistance of workers to reduce their nominal wage is presented. There is no “money illusion”. Instead the behaviour is obviously utility maximizing. It is provided a rationale for a “kinked” like supply of labour. The
model also allows that the wage goes down in some situations, that are clearly the ones in which the probability of being fired is high enough. Moreover, the conclusions are robust to the number of firms. The influence of risk aversion is observed: it increases the region where the cut in wages is accepted, and decreases the region where it is refused. We include also a section in which wage increases are discussed within the framework of the model. In this case wage increases are refused only in case of high prevailing unemployment rate and also high cut of unemployed employees by the firms.

One point that one could observe also, is the fact that wages, as we measure, are only referred to the employed part of the population. If one sees employment as a lottery, as this model suggests, it could be argued that the level of nominal wages should be corrected to incorporate the value of this lottery: in our simple case this would mean only multiplying it by the probability of not being fired.

Another message we wanted to make clear is never to believe in nominal wage flexibility (downward) to the adjustment of the economy: either it must happen through the price level or should be done in a centralized way, unilaterally (this because the situation that everybody is with the wage cut is Pareto optimal in the context of the model in its symmetric form).

For further developments we would think upon closing the model, by including unemployment in the utility of employees and the production side together. Also one could try to relax the hypotheses, especially the one concerning strikes.

References


APPENDIX

Symmetric n-person two-strategies games

In this appendix two results concerning symmetric games, with two strategies only for each player, are presented. This has a direct application to the main text, because the model in its symmetric form, on section 5, is a particular case of it.

The first result is the existence of Nash equilibrium for these games (in pure strategies). The second result helps one to understand a little bit more about the features of the model when the Nash equilibrium is unique: in this case it is also a strictly dominant equilibrium.

**DEFINITION 1:** An n-person game $\Gamma = (S_1, \ldots, S_n, \ldots, u_1, \ldots, u_n)$, where $S_i$ is a set $\forall i$, and $u_i : \prod_{j=1}^{n} S_j \rightarrow \mathbb{R}$ is a payoff function of player $i \forall i$, is said to be symmetric if:

(i) $S_1 = \cdots = S_n$;

(ii) $\forall \sigma : \{1, \ldots, n\} \rightarrow \{1, \ldots, n\}$ a permutation of the set $\{1, \ldots, n\}$, we have $u_i(s_1, \ldots, s_n) = u_{\sigma(i)}(s_{\sigma(1)}, \ldots, s_{\sigma(n)})$.

**EXAMPLE 1:** In the two-person case it just means that the game has the structure:

**DEFINITION 2:** Given a game $\Gamma$, and a strategy $\bar{s}_i \in S_i$, we say that $\bar{s}_i$ is a (strictly) dominant strategy for player $i$ if: $\forall s_{-i} \in \prod_{j \neq i} S_j$, $u_i(\bar{s}_i, s_i) > u_i(s_i, s_{-i}), \forall s_i \in S_i, s_i \neq \bar{s}_i$.

**DEFINITION 3:** Given a game $\Gamma$, and an n-tuple of strategies $(\bar{s}_1, \ldots, \bar{s}_n) \in \prod_{i=1}^{n} S_i$, we say that $(\bar{s}_1, \ldots, \bar{s}_n)$ is a (strictly) dominant strategy equilibrium if: $\forall i, \bar{s}_i$ is a (strictly) dominant strategy for player $i$. 

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Example 2: The prisoner's dilemma is an example of a game where the usual equilibrium (non-cooperative) is a strictly dominant one.

Theorem 1. Let $\Gamma$ be a symmetric $n$-person game, where $\#S_i = 2 \forall i$. Then there exists a Nash equilibrium in pure strategies.

Proof: The proof is done by induction. For $n = 1$ the result is trivial. The induction step will make use only of $n - 1 \geq 1$, so that we can go directly to it, without having to bother with $n = 2$. Therefore, suppose it is true for $n - 1$. I will prove it is true for $n$. In fact, let us fix the action of player $n$. Suppose also $S_1 = \cdots = S_n = \{a, b\}$. Without loss of generality we can assume we fixed his action as $a$. We now have a game with the $n - 1$ first players that is also symmetric. Thus, by the induction hypothesis there is an $n$-tuple $(s_1, \cdots, s_{n-1})$ that is an equilibrium of this game (with the $n^{th}$ player restricted to play $a$). To check if $(s_1, \cdots, s_{n-1}, a)$ is a Nash equilibrium of $\Gamma$, we have only to check for player $n$. Let us suppose that this is not Nash. Then I will construct another $n$-tuple that is a Nash equilibrium, and this will conclude the proof. If $(s_1, \cdots, s_{n-1}, a)$ is not Nash, then it must be the case that $u_n(s_1, \cdots, s_{n-1}, b) > u_n(s_2, \cdots, s_{n-1}, a)$. Suppose $i$ such that we have $s_i = a$. It follows by the symmetry hypothesis (taking $\sigma$ to be the permutation that exchange $i$ with $n$ and leave the rest fixed), in this case, that $u_n(s_1, \cdots, s_{n-1}, a) = u_i(s_1, \cdots, s_{n-1}, a) \geq (by\ the\ induc-
tion hypothesis) \( u_i(\bar{s}_1, \ldots, b, \ldots, \bar{s}_{n-s}, a) = (\text{by symmetry}) \)
\( u_n(\bar{s}_1, \ldots, a, \ldots, \bar{s}_{n-1}, b) = (\text{as } \bar{s}_i = a) = u_n(\bar{s}_1, \ldots, \bar{s}_{n-1}, b) \).
But this would contradict \( u_n(\bar{s}_1, \ldots, \bar{s}_{n-1}, b) > u_n(\bar{s}_1, \ldots, \bar{s}_{n-1}, a) \)
\( \Rightarrow \bar{s}_1 \neq a \forall i \neq n. \)

Then \( \bar{s}_i = b \forall i. \) I claim that, in this case, the \( n \)-tuple \((b, \ldots, b)\)

is a Nash equilibrium. To see this, by symmetry, it is enough to check the Nash inequality for any one given player. Let us check it for player \( n. \) We have \( u_n(b, \ldots, b) > u_n(b, \ldots, b, a) \).

This proves the inequality for player \( n, \) and therefore the theorem follows. \( \Box \)

**Theorem 2.** Let \( \Gamma \) be a symmetric \( n \)-person game, where \( \#S_i = 2, \forall i. \) Suppose there is a unique Nash equilibrium. Then it is a strictly dominant strategy equilibrium.

**Proof:** The proof is again by induction. For \( n = 1 \) the result is trivial. Let the result be valid for \( n - 1, n \geq 2. \) Call the equilibrium \( n \)-tuple \((\bar{s}_1, \ldots, \bar{s}_n), \bar{s}_i \in S_i. \) Suppose for some \( i \) and \( j \) we have \( \bar{s}_i \neq \bar{s}_j. \) Let it be \( \bar{s}_i = a \) and \( \bar{s}_j = b. \) As a short hand, call \( \bar{s}_{-i,j} \) to \( \bar{s}_{i,j} \)

an element of \( \prod_{k \neq i, j} \bar{s}_k. \) Then: (1) \( u_i(\bar{s}_{-i,j}, a, b) = u_i(\bar{s}_{-i,j}, b, b), \)
(2) \( u_j(\bar{s}_{-i,j}, a, b) = u_j(\bar{s}_{-i,j}, a, a). \) But applying, as in the previous theorem, the permutation \((ij)\) (change the places of \( i \) and \( j, \) letting the rest unchanged), we have: \( u_i(\bar{s}_{-i,j}, a, b) = u_j(\bar{s}_{-i,j}, b, a), \)
\( u_j(\bar{s}_{-i,j}, a, b) = u_j(\bar{s}_{-i,j}, b, a), \) \( u_j(\bar{s}_{-i,j}, a, a) = u_i(\bar{s}_{-i,j}, a, a) \) and hence by (1) and (2) the Nash inequalities are verified for \((\bar{s}_{-i,j}, b, a)\)

for players \( i \) and \( j. \) If \( k \neq i, j, \) by symmetry \( u_k(s_{-i,j}, s_i, s_j) = u_k(s_{-i,j}, s_j, s_i) \forall (s_1, \ldots, s_n). \) This implies that Nash inequalities are verified also in this case \( \Rightarrow (\bar{s}_{-i,j}, b, a) \) is another Nash equilibrium. Therefore if \((\bar{s}_1, \ldots, \bar{s}_n)\) is the unique one, it must be the case that \( \bar{s}_1 = \cdots = \bar{s}_n. \) Without loss of generality we may assume \((a, \ldots, a)\) is this point. Consider now the game \( \Gamma; \) as being \( \Gamma \) when we restrict player \( i \) to play \( a. \) This is a \((n - 1)\)-person game. I claim that \((a, \ldots, a)\) is the unique Nash equilibrium of this game, \( \forall i. \) Suppose it is not. Let \((\bar{s}_{-i})\)
be it. Then \( u_j(\hat{s}_{-i}, a) \geq u_j(\hat{s}_{-i}, j, a) \) \( \forall s_i \in S_i \). Suppose there exists \( j \neq i \) such that \( \hat{s}_j = a \). Then we could easily verify that the Nash inequality would hold for \( i \) in the point \((\hat{s}_{-i}, a)\) (by symmetry). This would be a contradiction with the fact \((a, \cdots, a)\) is the unique Nash for \( \Gamma \). Therefore it has to be the \( n \)-times case \( \hat{s}_j = b \ \forall j \neq i \). We may have \( u_i(\hat{s}_{-i}, a) \geq u_i(\hat{s}_{-i}, b) \). In this case \((\hat{s}_{-i}, a)\) is Nash and is different from \((a, \cdots, a)\) \( \Rightarrow \) contradiction. On the other hand we could have \( u_i(\hat{s}_{-i}, b) > u_i(\hat{s}_{-i}, a) \), being \( \hat{s}_{-i} = (b, \cdots, b) \) \( \Rightarrow \) \((b, \cdots, b)\) is a Nash equilibrium also \( n \)-times different from the original one. This is also a contradiction, and therefore we have that \((a, \cdots, a)\) is the unique Nash for \( \Gamma_i \), given \( (n-1) \)-times any \( i \). By the induction hypothesis it is a strictly dominant strategy equilibrium for \( \Gamma_i \). By symmetry we want to check only that \( a \) is strictly dominant strategy for player 1. Let us look at \( \Gamma_1 \). Then \( \forall \vec{s} \) and \( \forall s_{-1,i} \), we have:

\[
\text{1st ith} \quad u_1(a, a, s_{-1,i}) = u_i(a, a, s_{-1,i}) > u_i(a, b, s_{-1,i}) = u_i(b, a, s_{-1,i}).
\]

Therefore if there exists \( i \neq 1 \) such that \( s_i = a \), it follows \( u_1(a, s_{-1}) > u_1(b, s_{-1}) \). We have only to check what happens when \( s_{-1} = (b, \cdots, b) \). But if \( u_1(a, b, \cdots, b) \leq u_1(b, \cdots, b) \), then \( (n-1) \)-times \((b, \cdots, b)\) would be Nash of \( \Gamma \) \( \Rightarrow u_1(a, s_{-1}) > u_1(b, s_{-1}) \forall s_{-1} \). This \( (n-1) \)-times proves the theorem.

The hypotheses of the theorem are strictly necessary. It is easily seen that the symmetry is indispensable. The following example shows that if the number of strategies increase the result will not hold.

**Example 3:** Let \( \Gamma \) be as below:

\((a, a)\) is the unique Nash equilibrium. However \( u_1(a, c) = 2 < 5 = u_1(b, c) \).
Table 4.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>(1,1)</td>
<td>(3,0)</td>
<td>(2,0)</td>
</tr>
<tr>
<td>b</td>
<td>(0,3)</td>
<td>(0,0)</td>
<td>(5,-1)</td>
</tr>
<tr>
<td>c</td>
<td>(0,2)</td>
<td>(-1,5)</td>
<td>(4,4)</td>
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