TESTING EXPORTS UNDERINVOICING UNDER A DUAL EXCHANGE RATE REGIME: EVIDENCE FOR BRAZILIAN EXPORTS*

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Resumo

O presente trabalho utiliza um modelo de utilidade esperada para explicar o fenômeno de subfaturamento de exportações sob um regime de câmbio dual (mercado oficial e paralelo). Desenvolve-se, em seguida, uma nova metodologia que possibilita testar as hipóteses de subfaturamento via preço e via quantidade utilizando regressões lineares. As regressões estimadas são Modelos de Correção de Erros (MCE), dada co-integração entre as séries temporais usadas. Os resultados dos testes confirmam a existência de subfaturamento via preço, mas não via quantidade. A fuga de capitais associada a esse subfaturamento foi estimada em no mínimo US\$ 13,9 bilhões entre 1972-88, um montante suficiente para recompor uma parte significativa da dívida externa brasileira a preços de mercado.

Abstract

The paper uses an expected utility model to explain exports underinvoicing under a dual exchange rate regime. Econometric estimation is done using an Error Correction Model (ECM). The hypotheses of export price and quantity underinvoicing are tested and the model confirms the former but not the latter. Using the estimated ECM, consistent estimates of capital flight due to price underinvoicing are calculated. The sample lower bound of the hard currency "loss" due to the existence of a dual exchange rate regime is US\$ 13.9 billion, sufficient to repurchase a large portion of the Brazilian external debt at today's market value.

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1. Introduction.

Dual exchange rate regimes arise when agents in a given country are not freely allowed to buy and sell foreign exchange. The most common dual exchange regime is the coexistence of official and black market exchange rates. Since black market rates are usually higher than official rates,¹ an exporter may be tempted to underinvoice a given export transaction to increase its total return. There are two ways in which this can be accomplished: the first is by smuggling tradable goods and then selling the obtained hard currency in the black market. The second is a more subtle one, in which the exporter conceals from its central bank a portion of the hard currency obtained in a legal export transaction, latter selling it at the black market rate. They are labelled respectively underinvoicing via quantity and via price for obvious reasons.

The usual way in which central banks investigate these phenomena is by comparing trade accounts across countries. This methodology is very costly, since each country has several trading partners. Moreover, it will not detect underinvoicing via quantity, because smuggling is usually registered in *neither* trading partners' accounts. The methodology proposed here is more economical in the sense that only trade figures for the country suspected of underinvoicing need to be collected. Furthermore, it allows the distinction between price and quantity underinvoicing.

The starting point of our methodology is a trade model, where export price and quantity are jointly determined assuming they do not depend on the exchange rate gap (black market vs. official market rates). The next step is to model optimal price underinvoicing. At this stage, an expected utility model is used to show that an increase in the exchange rate gap increases optimal price underinvoicing. This last result, regression based tests for both price and quantity underinvoicing hypotheses. Consistent estimates of capital flight are finally obtained using regressions results.

¹For the Brazilian economy this inequality holds. Exceptionally, for some countries, it may be reversed for a brief period of time. See Pechman (1984) pp.34-36.

The empirical results for Brazilian exports figures indicate that exports underinvoicing have been occurring in this country for the last twenty years. They also suggest that underinvoicing via price was the only type practiced by exporters. This result seems intuitive: on one hand, smuggling goods outside a country involves physically moving merchandise across borders, which is usually a very risky (illegal) operation. On the other hand, export price underinvoicing is an illegal financial operation, which by its own nature is of very low risk of detection. Therefore, it is not surprising that when faced with the choice between underinvoicing via price or via quantity exporters select the former.²

Section 2 discusses the methodology used. In Section 3, estimated trade equations are presented. Section 4 includes a lower bound estimate of the hard currency capital flight in exports. Section 5 concludes.

2. Methodology.

The issue of capital flight via trade have had some space in the economic literature: Pitt (1984) and Macedo (1987) both deal with optimal smuggling. The last paper presents a theoretical model which integrates two illegal markets: smuggled goods and black market for exchange rates. Although it seems that the likelihood of price underinvoicing is at least as high as that of smuggling, little research was done on the former. An exception is Barbosa *et al.* (1988), a pioneer work on that area. There, price underinvoicing is modelled using the theory of choice under risk. In the present paper, I borrow from Barbosa *et al.* the same basic theoretical model, which is modified and augmented. The changes allow for testing the two distinct forms of underinvoicing (price and quantity) in a very convenient way.

I start by assuming that the exporter determines the optimal underinvoicing amount by using a two stage model. In the first stage, based on standard trade theory, exports quantum and prices are determined independently from the exchange rate gap (black market vs. official market rates). In the second stage, the exporter decides how

² Notice that risk of detection alone will not be the sole determinant of the exporter's choice, but it enters the analysis via an expected loss function.

much hard currency per unit of exports will be sold to the central bank, based on the exchange gap. This latter stage uses an expected utility model. The safe attitude is to sell to the central bank 100% of the hard currency obtained per unit of exports, whereas the risky attitude is to sell some of it in the black market for a higher bid. Whenever the exporter does the latter, he or she bears the risk of being caught doing such an illegal transaction.

It is important to note that by proceeding this way one critical assumption is made:

ASSUMPTION 1: Exporters actually behave in that two stage fashion, where the exchange rate gap does not affect the equilibrium price and quantity of exports. As a consequence, smuggling of exports are ruled out. Since this assumption imposes too much structure in the exporter's decision problem, a test of its validity was developed and is presented below.

In contrast with assumption 1, Macedo's (1987) model only delivers no smuggling at a corner solution. His model however omits any possibility of price underinvoicing, being in that sense incomplete. Preferably, one should model jointly optimal decisions on export price and quantity underinvoicing. However, accomplishing this task is very costly: there is no evidence they both occur simultaneously and the mathematical model needed to accommodate both gets very complex. Because quantity underinvoicing is intrinsically a more risky operation than price underinvoicing, it seems natural to start modelling these phenomena by assuming the non-existence of the former (*i.e.*, assumption 1). Since that assumption can be tested, this approach still answers relevant empirical questions while keeping simple the mathematical theoretical model.

The trade model used follows Goldstein and Khan (1985), Rios (1987) and most of the trade literature. Demand and supply of exports are:

$$\log X_t^d = \alpha_0 + \alpha_1 \log \left(\frac{PX_t^d}{PW_t} \right) + \alpha_2 \log YW_t + u_t^d \quad (1)$$

$$\log X_t^s = \beta_0 + \beta_1 \log \left(\frac{PX_t^s \cdot E_t \cdot (1+S_t)}{P_t} \right) + \beta_2 \log U_t + u_t^s \quad (2)$$

$$\log X_t^* \equiv \log X_t^d = \log X_t^s \quad (3)$$

$$\log PX_t^* \equiv \log PX_t^d = \log PX_t^s \quad (4)$$

where X^d is the exports quantum demand, X^s is the exports quantum supply, PX^d is the dollar demand price of Brazilian exports, PX^s is the dollar supply price of Brazilian exports, PW is the price index of world exports, YW is the income of the rest of the world, E is the nominal (official) exchange rate for the Brazilian currency³ (CR\$/US\$), S is the exports subsidy rate, P is the domestic price index for the Brazilian economy, U is the capacity utilization of the Brazilian economy and the u 's are random shocks.

As is clear from (1)-(4), the system above determines jointly $\log PX^*$ and $\log X^*$, the equilibrium price and quantity of exports. The expected signs of the coefficients of equation (1) are: α_1 is negative and α_2 is positive. The first captures the substitution effect and the second the income effect if Brazilian exports are normal goods. The expected signs of the coefficients of equation (2) are: β_1 is positive; β_2 is negative if exporters treat the domestic market as a customer's market or if they regard exporting to be riskier than selling to the domestic market see Goldstein and Khan (1985) p.1061.

The reduced form for $\log X^*$ using the system (1)-(4) is:

$$\log X_t^* = \gamma_0 + \gamma_1 \log \left(\frac{PW_t \cdot E_t \cdot (1+S_t)}{P_t} \right) + \gamma_2 \log U_t + \gamma_3 \log YW_t + \xi_t \quad (5)$$

where:

$$\gamma_0 \equiv \frac{\alpha_0 \beta_1 - \beta_0 \alpha_1}{\beta_1 - \alpha_1}; \gamma_1 \equiv -\frac{\alpha_1 \beta_1}{\beta_1 - \alpha_1} > 0; \gamma_2 \equiv -\frac{\alpha_1 \beta_2}{\beta_1 - \alpha_1} < 0;$$

$$\gamma_3 \equiv \frac{\alpha_2 \beta_1}{\beta_1 - \alpha_1} > 0 \text{ and } \xi_t \equiv \frac{\beta_1 u_t^d - \alpha_1 u_t^s}{\beta_1 - \alpha_1}.$$

With $\log PX^*$ and $\log X^*$ determined jointly by (1)-(4), the exporter has to now decide how much hard currency *per unit of exports* will be sold to the central bank.⁴ Denote r^0 and r^b as the official and black market exchange rates respectively, where $r^b \geq r^0$ holds. Assume these two prices are known with certainty before the exporter's

³All legal export revenues are converted at the official rate.

⁴Conditioned on assumption 1, this is equivalent to optimally decide on how much hard currency is sold to the central bank.

decision takes place. It is natural to regard r^0 as the safe asset and r^b as the risky asset, the only uncertainty being if the exporter will be caught selling hard currency in the black market. If the exporter is caught he/she pays a penalty $\alpha(\varepsilon)$, which depends on the proportion ε of PX^* sold to the central bank. The probability of the exporter being caught is assumed fixed and equal to p .

In what follows it is assumed that the exporter is risk averse with Von Neumann-Morgenstern utility function $U(\cdot)$ ($U'(\cdot) > 0$, $U''(\cdot) < 0$), that $r^b - \alpha(\varepsilon) \leq r^0 \leq r^b \forall \varepsilon$, $\alpha'(\varepsilon) < 0$ and $\alpha''(\varepsilon) > 0$. Therefore, the exporter's problem is to choose ε ; $0 \leq \varepsilon \leq 1$; as to solve:

Max

$$p \cdot U[PX^*(\varepsilon r^0 + (1 - \varepsilon)(r^b - \alpha(\varepsilon)))] + (1 - p) \cdot U[PX^*(\varepsilon r^0 + (1 - \varepsilon)r^b)]$$

The (interior) first order condition to this problem is:

$$\begin{aligned} & p \cdot U'[PX^*(\varepsilon^* r^0 + (1 - \varepsilon^*)(r^b - \alpha(\varepsilon^*)))]PX^* \\ & \times (r^0 - (r^b - \alpha(\varepsilon^*)) - \alpha'(\varepsilon^*)(1 - \varepsilon^*)) + \\ & + (1 - p) \cdot U'[PX^*(\varepsilon^* r^0 + (1 - \varepsilon^*)r^b)]PX^*(r^0 - r^b) = 0 \end{aligned}$$

Where $PX^* \cdot \varepsilon^*$ is the optimal choice of registered exports per unit of exported good. Therefore, $PX^* \cdot (1 - \varepsilon^*)$ is the optimal exports underinvoicing per unit of exported good.

Proposition 1. Define $G = r^b/r^0$ to be gap between the exchange rates and $Ra(y) = -U''(y)/U'(y)$ to be the exporter's absolute risk aversion coefficient. If the exporter has a decreasing absolute risk aversion coefficient, i.e., if $Ra'(y) < 0$, then:

$$\frac{\partial \varepsilon^*}{\partial G} < 0$$

PROOF: See appendix.

Proposition 1 delivers a very intuitive result: it says that, at the individual level, the higher the gap of the exchange rates the lower will be ε^* . Define registered exports price as $PX = PX^* \cdot \varepsilon^*$. Under assumption 1 and proposition 1, there is a negative relationship between PX and G . This implies, still under assumption 1, a negative

relationship between G and the registered value of exports, $PX \cdot X^*$. If these negative relationships hold at the aggregate level, the registered (observable) exports price is PX instead of PX^* and the registered (observable) exports value is $PX \cdot X^*$ instead of $PX^* \cdot X^*$. Notice that X^* can still be found if there is a constructed price series for registered exports (PX). To get X^* , one only has to deflate the registered value of exports ($PX \cdot X^*$) using PX . Notice also that this constructed quantum of exports does not depend on G under assumption 1, but it does if assumption 1 is false. Even though PX can be obtained using trade figures, PX^* cannot be calculated from data on the value of exports alone, since it is measured with an asymmetric error. These ideas are concisely discussed in proposition 2.

Proposition 2. *Define $X = PX \cdot X^* / PX^*$ to be a “transformed quantum” of exports. If $\log \varepsilon^*$ depends linearly on $\log G$, then, $\log X$ depends on $\log G$. Moreover, if assumption 1 is correct, $\log X^*$ does not depend on $\log G$.*

PROOF: If there is data on PX , we can find X^* by dividing the value of exports by PX . Conditioned on assumption 1 above, the last part of proposition 2 is true by definition. For the first part of proposition 2, rewrite $\log X$ as:

$$\log X_t = \log X_t^* + \log \left(\frac{PX}{PX^*} \right)_t = \log X_t^* + \log \varepsilon_t^*$$

Assuming a contemporaneous relationship between G and ε^* as follows:

$$\log \varepsilon_t^* = \gamma_4 + \gamma_5 \log G_t$$

And combining this expression with (5) we get:

$$\begin{aligned} \log X_t = & (\gamma_0 + \gamma_4) + \gamma_1 \log \left(\frac{PW_t \cdot E_t \cdot (1 + S_t)}{P_t} \right) + \gamma_2 \log U_t \\ & + \gamma_3 \log YW_t + \gamma_5 \log G_t + \xi_t \end{aligned} \quad (6)$$

By proposition 1, $\gamma_5 < 0$, thus the result follows.

Proposition 2 offers a test on whether there is price or quantity underinvoicing. If we run a regression of $\log X^*$ on the set of regressors in (5) and on $\log G$, we should get an insignificant coefficient

for the latter if assumption 1 is correct. Thus, finding the opposite refutes the no smuggling assumption. Moreover, if we estimate (6), we should get a (negative) significant coefficient for $\log G$ under proposition 1. Finding the opposite refutes price underinvoicing. If we are able to find consistent estimates for γ_i ; $i = 1, \dots, 5$ in (6), we can estimate how much of hard currency is "lost" per period due to capital flight. This latter issue is discussed in section 4 below.

3. Empirical results.

Before performing the underinvoicing tests, some discussion about the data is necessary. First, it seems justifiable to apply these tests to manufactured exports, because it is much harder to illegally price underinvoice agricultural exports than manufactured exports. Agricultural commodities are traded daily in world markets, their prices are common knowledge and product homogeneity predominates. Hence, it is very easy for central banks to detect price underinvoicing by using a list of minimum export prices. Unlike agricultural commodities, price clearing of manufactured exports is done privately and product homogeneity is uncommon. Thus, even if central banks can compile a price list, exporters can still get around them by doing cosmetic product differentiation together with registered price reduction.

The data is available quarterly from 1971.I to 1988.IV for most series. Even though, in general, data quality is good, in some cases proxies were used and in others substitutions were made. They were:

- (i) PX^* is unobservable and a proxy had to be used in order to obtain the series for X . The proxy chosen was the price index of manufactured goods for the U.S. economy. Since the bulk of manufactured Brazilian exports come to the U.S., this seems a natural choice, because these two series must have the same long run pattern.

Given assumption 1 and proposition 1, the registered value of exports is $PX \cdot X^*$ and it depends on G through PX . Clearly, the only way to eliminate this dependence is by deflating the value of exports by PX . In principle, using any other deflator would yield a "quantum" which depends on G . Denote PX' as a proxy to PX^* .

The advantage of using a proxy like PX' to deflate $PX \cdot X^*$ is that we can write:

$$\begin{aligned}\log \left(\frac{PX \cdot X^*}{PX'} \right) &= \log X^* + \log \left(\frac{PX}{PX'} \right) \approx \log X^* + \log \left(\frac{PX}{PX^*} \right) \\ &= \log X^* + \log \varepsilon^*\end{aligned}$$

Therefore, we can model this transformed quantum by using equation (5) and $\log G$ in a way similar to equation (6).

- (ii) The subsidy series is only computed annually, therefore, in order to fulfill the quarterly data requirement, the strong assumption that it did not change during the year had to be made. Even though this is a very unrealistic assumption, the variable $PW \cdot E(1+S)/P$ performed better than its counterpart which omitted the subsidy rate.
- (iii) The series used for X^* is not the registered quantum of *manufactured* exports, but the registered quantum of *total* exports. The latter was used despite the existence of a registered manufactured exports series provided by Fundação Getúlio Vargas (FGV). The discussion in Pinheiro and Motta (1990) should convince any final user of data to avoid using FGV's export price series in empirical work. This issue is discussed in greater length in section 3.2 below.

The data series used, with sources in parentheses, are listed below:

- (i) X was found by dividing the value of manufactured exports (Macrodados database) by the price index of manufactured goods for the U.S. economy (Citibase).
- (ii) X^* used the quantum of total exports calculated by Pinheiro and Motta (1990).
- (iii) The income of the rest of the world used total real world exports (IFS database).
- (iv) The reduced form real exchange rate used the nominal exchange rate Cruzeiro/Dollar (Macrodados database); the wholesale price index for the Brazilian economy (Macrodados database); the price of world exports (IFS database); the subsidy rate calculated by Bauman (1989).

- (v) The capacity utilization of the Brazilian economy was extracted from the quarterly research gathered by Fundação Getúlio Vargas (FGV database).
- (vi) The gap of the exchange rates (Cruzeiro/US\$) was calculated using official and black market rates (Macrodados database).

The first step before estimating (5) and (6) is to infer the order of integration of the several time-series to be used in estimation. The unit-root test applied to the data series was the "Augmented Dickey-Fuller (ADF) test". Since the test result sometimes depend on the regression run — number of lags of the regressand and the use of a trend or a constant — several specifications were considered: lags zero up to lag 5 for the regressand in each "trend", "no trend", and "no constant" regression.⁵ For each time series, there was a total of 32 regressions run, and the final order of integration of each series was determined after a careful scrutiny of those. Below are presented the *t*-statistics of the ADF test, using the most probable lag structure with white noise errors. Three seasonal dummies were used since none of the series were desazonalized.

A test for two unit roots was also performed, however none of the series indicated any tendency to be integrated of order higher than 1. This result, combined with those in Table 1, indicates that all variables except $\log G$ are $I(1)$. Although it seems that $\log G$ is $I(0)$, it is a borderline case: its test statistic using the "no trend" specification has a unit root at 5%, but not at 10%. To investigate this issue further, a cointegration test between $\log r^b$ and $\log r^0$ was performed.⁶ An $I(0)$ series can be obtained with $[1, -1.02]$ as the estimated cointegrating vector. This result is practically one of long run proportionality between these two variables.⁷ Because of it, in estimating equations (5) and (6) below, $\log G$ will be treated as $I(0)$ and will be calculated as $\log r^b - 1.02 \log r^0$ instead of $\log r^b - \log r^0$.

⁵See Engle and Yoo (1987).

⁶Both series after 1st. differencing accepted the null of one unit root, thus being $I(2)$ processes.

⁷This implies that $\log G$ is $I(0)$. Since both series $\log r^0$ and $\log r^b$ are $I(2)$, long run proportionality implies they are jointly $CI(2, 2)$ in Engle and Granger (1987) terminology.

Table 1.
Augmented Dickey-Fuller test

Variable	ADF(K)	Trend	No trend
X	5	-2.51	
X^*	5	-2.06	
$PW \cdot E(1 + S)/P$	5		-2.74
YW	5	-2.65	
G	4	-4.35	
U	5		-1.35

NOTES: All series in logs. K is the maximum number of lags of the series in the right hand side. Sample size is 65 for most series. Critical values for a sample size of 65 are the following:⁸ at 1%, Trend = -4.104, No Trend = -3.533. At 5%, Trend = -3.479, No Trend = -2.906 extracted from McKinnon (1990).

Cointegration tests were performed prior to estimation in order to determine the $I(0)$ representation of equations (5) and (6). Since we are dealing with four $I(1)$ variables in each of these equations, an appropriate method of testing for cointegration must be applied. Notice that the usual Engle-Granger two step method will not estimate the whole cointegrating space if there is more than one cointegrating vector. Based on the discussion in Gonzalo (1990), Johansen's (1988) methodology was used. The first set of cointegrating test results, which includes $\log X$, $\log[PW \cdot E(1 + S)/P]$, $\log YW$ and $\log U$, is presented in Tables A.1, A.2 and A.3 in the appendix.

Table A.1 presents results of the test for the number of cointegrating vectors (conversely, common trends). There are no strict rejections of any of the null hypotheses neither at 5% nor at 10%. However, at 20%, the test statistic rejects the null that there is at most zero cointegrating vectors. Since the last acceptance was that of the existence of one cointegrating vector, we conclude the latter (at 20%).⁹ Table A.2 presents the estimate of this cointegrating vector.

⁸Critical values changes is very small for sample sizes near 65. Therefore this critical values were used for all series.

⁹Although this is not the usual significance level used for hypothesis testing, the error term will be included in estimation due to its theoretical importance.

Since we are estimating a reduced form, there is very little hope of getting any economic interpretation from it. Table A.3 provides Error Correction (EC) model coefficient estimates for the unique error term. Interestingly enough, the coefficients of Z_1 in the equations of $\log YW$, $\log[PW \cdot E(1+S)/P]$ and $\log U$ are all relatively small, a necessary condition for weak exogeneity of these variables in the $\log X$ equation.¹⁰

The second set of cointegrating test results, including $\log X^*$, $\log[PW \cdot E(1+S)/P]$, $\log YW$ and $\log U$, is presented below in Tables A.4, A.5 and A.6 in the appendix. From Table A.4, we conclude that there are no significant cointegrating vectors at 10%. Even in this case, we may want to use the most significant cointegrating vector in the estimation of equation (5). Table A.5 presents the estimate of this cointegrating vector. Table A.6 provides EC model coefficients for this unique error term. Again, as in the case of $\log X$, the coefficients of the error term for equations other than $\log X^*$ are relatively small when compared to the former. This suggests that we can treat $\log YW$, $\log[PW \cdot E(1+S)/P]$ and $\log U$ as weakly exogenous in the $\log X^*$ equation.

At this point, a qualification is appropriate about the functional form of equations (5) and (6) above. They coincide with the so called "static regression model", and therefore are nested in a general type of Autoregressive-Distributed lag $[AD(\cdot)]$ model exposed by Hendry *et al.* (1984). The simplest version of the $AD(\cdot)$ family $\{AD(1,1)\}$, with two time series and one lag of each variable is:

$$y_t = \beta_1 z_t + \beta_2 z_{t-1} + \beta_3 y_{t-1} + \mu_t \quad (7)$$

where the error term μ_t is *i.i.d.* $N(0, \sigma^2)$ and $|\beta_3| < 1$.

From (7), the long run relationship is $y = K \cdot z$, where $K = (\beta_1 + \beta_2)/(1 - \beta_3)$.

Transform (7) as follows:

$$\begin{aligned} \Delta y_t &= \beta_1 z_t + \beta_2 z_{t-1} + (\beta_3 - 1)y_{t-1} + \mu_t \\ &= \beta_1 \Delta z_t + (\beta_1 + \beta_2)z_{t-1} + (\beta_3 - 1)y_{t-1} + \mu_t \\ &= \beta_1 \Delta z_t + (\beta_3 - 1)(y - K \cdot z)_{t-1} + \mu_t \end{aligned} \quad (8)$$

¹⁰See Engle and Granger (1987).

where (8) is an EC model.

Clearly, there is an isomorphism between (7) and (8). If y and z are both $I(1)$ and cointegrated, we can estimate an EC model as in (8). This will be equivalent to estimating the level equation in (7). The advantage is an efficiency gain in estimation, since we are reducing uncertainty of the estimated model by imposing the knowledge of the long run relationship contained in the cointegrating vector $(1, -K)'$.

Estimating the static models (5) and (6) may not capture the dynamic structure of $\log X$ and of $\log X^*$, since there is no way to know whether the constraints they impose on their nesting $AD(\cdot)$ counterparts are correct or not. Usually, attempts of using static regressions in modelling economic time series have been unproductive. In general, static regression errors are not innovations and residual autocorrelation plagues those models. If we allow for a more general formulation of the $AD(\cdot)$ type, there is no loss of information if these static models are correct. However, if the constraints they impose on their $AD(\cdot)$ counterparts are inconsistent with the data, using an $AD(\cdot)$ model will help in capturing the "correct" dynamic structure of these equations. Therefore, a "General to Simple" approach will be used here, starting with an unconstrained $AD(\cdot)$ model with a high number of lags, leading to the estimation of a parsimonious model. The search for the "correct" dynamic specification is preferably done using the EC model, due to the referred efficiency gains. Thus, the lag search will be performed during the EC model estimation. This strategy is only acceptable because there is an isomorphism between the $AD(\cdot)$ class and the class of EC models, as the above exercise on (7) illustrates.

3.1. Estimating equation (6).

The estimation method used for equation (6) was Instrumental Variables (IV). OLS cannot be used for two basic reasons: first, since the exporter sells some of the hard currency proceeds in the black market, the aggregate supply of hard currency in this market is a function of $\log X$, even though the individual exporter may treat the black market exchange rate parametrically. The fact that $\log G$ is

a function of $\log X$ creates a simultaneity problem if OLS is used. Second, it is realistic to assume that the exporter does not know with certainty both r^b and r^0 for the time when the proceeds of the export transaction will be received. Thus, he/she could not possibly condition his/her decision of optimal underinvoicing on $\log G$. In that case, he/she must forecast $\log G$, and this is exactly what instrumental variables do. Because $\log G$ is autocorrelated, a good candidate for instrument is $\log G$ lagged. Lagged values of the other variables in (6) can be used as instruments as well.

The IV estimation of the (parsimonious) Error Correction model using equation (6) is presented in Table 2 below. The sample period covers 1972.II to 1988.III with 66 observations. Three seasonal dummies ($Q1, Q2, Q3$) were included in the regression since the data have no seasonal adjustment. Two other dummies were also included: DNR (impulse dummy 1985.I) captures the uncertainty effect of the transmission of power from a 20 year old military regime to civilians. DCRU (impulse dummy 1986.IV) captures the effects the pre-announced end of a price freeze. In these two occasions $\log X$ plunged sharply in an unusual fashion relative to other observations in the same respective quarter, resulting in two large outliers. If these dummies are not included, the residuals display 7th order autocorrelation, most probably due to the fact that these two outliers are exactly seven quarters apart.

The results of Table 2 above indicate a good degree of adequacy of the proposed model. The significance of $\log G$ is satisfactory, confirming previous suspicions that this variable negatively affects $\log X$. Other variables with high explanatory power are: the capacity utilization; the income from the rest of the world and the EC term (Z_1). The first indicates that the recent exports boom may be related to the continued recession that the Brazilian economy has been experiencing since the beginning of the 1980's. The second indicates that, as a developing country, Brazil is able to supply manufactured exports at an increasing rate. The reduced form exchange rate is important in the long run (through Z_1) but not in the short run, since its coefficient is insignificant.¹¹ This probably reflects little room for short

¹¹In the empirical evidence abroad, the fact that real exchange rates are relatively

Table 2.
Instrumental variables estimation
dependent var.: $\Delta \log X$

Expl. var.	Est. coeff.	T-stat.
$\log G$	-0.276	-3.13
DNR	-0.292	-4.60
$DCRU$	-0.300	-4.38
$\Delta \log X_{-2}$	-0.192	-2.72
$\Delta \log X_{-4}$	0.190	2.70
$\Delta \log YW$	1.689	4.48
$\Delta \log U$	-1.354	-3.92
$Z1_{-1}$	-0.344	-4.64
Constant	2.660	4.50
$Q1$	0.037	0.58
$Q2$	0.003	0.08
$Q3$	0.122	2.20

List of Instruments: $Z1_{-2}$, $\Delta \log YW_{-1}$, $\Delta \log U_{-1}$, $\log G_{-1}$

Diagnostic Tests:

$\sigma = .0647$; $\chi^2(14)/14$ $\beta = 0$: 25.72

Durbin-Watson = 2.202; AR 1-4 $\chi^2(4)/4$: 0.80

AR 1-8 $\chi^2(8)/8$: 1.09; Normality (Jarque-Bera) $\chi^2(2) = 2.381$

White's Heteroskedasticity: Prob > $F = 0.97$

term price adjustments in trade contracts.

The results obtained so far largely support the hypothesis that the exchange rate gap negatively influences the registered value of exports. With regard to the boom experienced recently by Brazil-

unimportant in the short run for trade figures is well documented. Goldstein and Khan's (1985) survey of econometric results suggest that long run price elasticity of exports is roughly twice as big as the short run elasticity.

ian manufactured exports, it seems to be a combination of domestic recession and increasing world trade.

3.2. Estimating equation (5).

We now turn to the estimation of equation (5). If $\log G$ is significant in this regression, this is a sign that the registered exports quantum and the actual exports quantum differ, *i.e.*, smuggling may be occurring. Instrumental Variables estimation was again used by the same argument applied to the estimation of equation (6).

The IV estimation of the (parsimonious) Error Correction version of equation (5) is presented below. The sample period covers 1977.IV to 1988.III with 44 observations.

The results of Table 3 above show a good fit for equation (5). All estimated coefficients have the expected signs. The strategy of using Z_1 in the EC model estimation proved right since its estimated coefficient is significant at the 1% level. Remarkably, this is true despite the fact that the corresponding cointegrating vector failed Johansen's test. The coefficient for $\log G$ is insignificant, indicating that there is no smuggling occurring for Pinheiro and Motta's (1990) *total exports* series. This implies that there is no smuggling for *manufactured exports* as well, unless there is overinvoicing for *non-manufactured exports*. Since this event is very unlikely, the no smuggling assumption taken first should be accepted. There is one caveat to this result. If we do the same test using the quantum resulting by dividing the value of manufactured exports by the price of manufactured exports provided by Fundação Getúlio Vargas (FGV), the no smuggling hypothesis is rejected. Results of this test are presented in Table A.7 in the appendix.

Despite the adverse result using FGV's price series, I think that the no smuggling result should be accepted. Pinheiro and Motta (1990) compiled a much better total exports series than the one provided by FGV.¹² Comparing their methodology with FGV's they concluded that price series for the latter have one major flaw: a relatively low flexibility of commodity composition in a very dynamic export portfolio. This may lead to poor "quantum figures" for value

¹²FGV calculates exports series for several exports categories.

Table 3.
Instrumental variables estimation
dependent var.: $\Delta \log X^*$

Expl. var.	Est. coeff.	<i>T</i> -stat.
$\log G$	0.0381	0.32
$\Delta \log YW$	1.0284	1.74
DNR	-0.1977	-2.67
$DCRU$	-0.3714	-4.69
$\Delta \log X^*_{-1}$	0.2132	1.70
$\Delta \log U$	-1.4655	-3.26
$Z1_{-1}$	0.7065	2.50
Constant	6.7327	2.46
$Q1$	-0.0453	-0.46
$Q2$	0.2040	4.38
$Q3$	0.0891	1.03

List of Instruments: $Z1_{-2}$, $\log G_{-1}$

Diagnostic Tests:

$\sigma = .07514$; $\chi^2(11)/11$ $\beta = 0$: 17.58

Durbin Watson = 2.012; $\text{Ar } 1-5$ $\chi^2(5)/5$: 2.09

Normality (Jarque-Bera) $\chi^2(2) = 1.075$; Arch-Test: $\chi^2(5) = 4.28$

data using FGV's price series as a deflator. As a consequence, Table A.7 may be displaying a spurious association between $\log G$ and FGV's $\log X^*$. Supporting this spurious association hypothesis are the estimated coefficients for the $\Delta \log YW$'s and the $\Delta \log U$'s in Table A.7. They mostly have reverse signs with significant *t*-statistics. This is a very odd behaviour vis-a-vis the results of Tables 2 and 3 above.

Using all the evidence at our disposal, it seems that there is underinvoicing being practiced by Brazilian exporters, but it only occurs via price.

4. Estimating exports underinvoicing.

To estimate manufactured exports underinvoicing, equation (6) was re-estimated using $\log G = \log r^b - \log r^0$. In this case, if we set G to 1 (and $\log G$ to zero), theory would predict that no underinvoicing of any sort would take place.¹³ Hence, if the estimated coefficient of $\log G$ is consistent, the model's prediction of the growth rate of the actual value of exports¹⁴ ($\Delta \log(\hat{P}X^* \cdot X^*)$) can be obtained by using the model's sample forecast under the assumption that $\log G$ is zero.

The result of this exercise is presented in Table A.8 in the appendix. To construct this table, ($\Delta \log(\hat{P}X^* \cdot X^*)$) was obtained by inflating regression forecasts using the price index of manufactured goods for the U.S. economy. At this point, an identification problem emerged: to get ($\log(\hat{P}X^* \cdot X^*)$), an observation on ($\log(PX^* \cdot X^*)$) is needed. Since there are none,¹⁵ a lower bound for ($\log(\hat{P}X^* \cdot X^*)$) was calculated using period $t-1$'s registered exports value ($\log(PX \cdot X^*)$) as a base to calculate period t 's ($\log(\hat{P}X^* \cdot X^*)$). This is a lower bound since theory would predict that in *no* event $\log(PX^* \cdot X^*) < \log(PX \cdot X^*)$, because it makes no economic sense to overinvoice exports.

The estimated export value lower bound seems reasonable, having the welcomed property that exports' underinvoicing is higher towards the end of the sample, when the level of the exchange gap is the highest. A plot of the total manufactured exports underinvoicing (in current US\$) and $\log G$ is presented in Graph 1 in the appendix. The picture clearly shows a positive relationship between these two variables, as expected.

The results in Table A.8 indicate a significant amount of capital flight due to the existence of a dual exchange rate regime. For the whole sample period, registered exports underestimated actual exports by *at least* 4.5%, and, for the latest 8 years of the sample period by *at least* 6%. The lower bound for the total "loss" of hard

¹³There are no gains of exports underinvoicing since black market and official exchange rates coincide and, there is always some positive probability that the exporter would be caught trying to do an illegal transaction.

¹⁴In what follows, I denote the forecasted value of the product AB by $(\hat{A}\hat{B})$.

¹⁵At least one of ($\log PX^*$, $\log X^*$) is non-observable.

currency in the sample period is US\$ 13.9 billion (in 1988 updated value using the "Libor" rate). This is sufficient to repurchase a substantial portion of the outstanding Brazilian external debt at today's market value.

5. Conclusions.

Initial suspicions that the existence of a dual exchange rate regime had an impact on trade figures were confirmed. It seems that the evidence supporting this fact is overwhelming for the case of Brazilian manufactured exports: there is exports underinvoicing being done primarily via price. The collective results do not indicate any significant smuggling occurring in either manufactured exports or total exports.

In economic terms, it seems that maintaining a dual exchange rate regime is unjustified. At the theoretical level, it may be argued that such regime is welfare reducing and inefficient, since economic agents are not free to legally buy or sell foreign exchange at will. At the pragmatic level, even with the government controlling the official exchange rate, a market mechanism has a definite influence in concealing from the central bank significant amounts of hard currency. Although the primary goal of exchange centralization is the central bank control over the flow of hard currency, it seems that the government fails to achieve this goal due to the existence of a dual exchange system. The remedy is not to abolish the free rate (black market) but the official rate. A freely convertible currency would make surface these concealed trade figures, Brazilian exports will become more competitive abroad and resource allocation will be done more efficiently.

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Appendix

PROOF OF PROPOSITION 1: The proof will be constructive. If some continuity and differentiability assumptions are made on $U(\cdot)$, the optimal response function $\varepsilon^*(r^b, r^0)$ will be continuous and differentiable. Let's first perform a change of variables in $\varepsilon^*(r^b, r^0)$ to express it as a function of G . Since $G = r^b/r^0$, define $\theta_1 = G \cdot r^0$ and $\theta_2 = r^b/G$. Notice that $\theta_1 = r^b$ and $\theta_2 = r^0$. Thus, we can write:

$$\varepsilon^*(r^b, r^0) = \varepsilon^*(\theta_1, \theta_2)$$

Since both θ_1 and θ_2 are functions of G , we can totally differentiate $\varepsilon^*(\cdot)$ w.r.t. G to get:

$$\frac{\partial \varepsilon^*}{\partial G} = \frac{\partial \varepsilon^*}{\partial \theta_1} \cdot \frac{\partial \theta_1}{\partial G} + \frac{\partial \varepsilon^*}{\partial \theta_2} \cdot \frac{\partial \theta_2}{\partial G} = \frac{\partial \varepsilon^*}{\partial \theta_1} \cdot r^0 - \frac{\partial \varepsilon^*}{\partial \theta_2} \cdot \frac{r^b}{G^2}$$

Thus, to get $\partial \varepsilon^*/\partial G < 0$ it suffices to show that $\partial \varepsilon^*/\partial \theta_1 < 0$ and $\partial \varepsilon^*/\partial \theta_2 > 0$.

Unfortunately, without assuming a functional form for $U(\cdot)$ we cannot get a closed form solution for $\varepsilon^*(r^b, r^0)$. Thus, to get the signs of both $\partial \varepsilon^*/\partial \theta_1$ and $\partial \varepsilon^*/\partial \theta_2$ we have to resort to the implicit function theorem.

Define the following expression for the (interior) first order condition to the exporter's problem:

$$\begin{aligned} FOC[\varepsilon^*(r^b, r^0); r^b, r^0] \equiv & p \cdot U'[PX^*(\varepsilon^*r^0 + (1 - \varepsilon^*)(r^b - \alpha(\varepsilon^*)))] \times \\ & \times PX^*(r^0 - (r^b - \alpha(\varepsilon^*)) - \alpha'(\varepsilon^*)(1 - \varepsilon^*)) \\ & + (1 - p) \cdot U'[PX^*(\varepsilon^*r^0 + (1 - \varepsilon^*)r^b)] \\ & \times PX^*(r^0 - r^b) = 0 \end{aligned}$$

Use now the definition of θ_1 and θ_2 into the above expression to get:

$$\begin{aligned} FOC[\varepsilon^*(\theta_1, \theta_2); \theta_1, \theta_2] \equiv & p \cdot U'[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*)))] \times \\ & \times PX^*(\theta_2 - (\theta_1 - \alpha(\varepsilon^*)) - \alpha'(\varepsilon^*)(1 - \varepsilon^*)) \\ & + (1 - p) \cdot U'[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)\theta_1)] \\ & \times PX^*(\theta_2 - \theta_1) = 0 \end{aligned}$$

Apply now the implicit function theorem to this expression to get:

$$\frac{\partial \varepsilon^*}{\partial \theta_1} = - \frac{\partial FOC / \partial \theta_1}{\partial FOC / \partial \varepsilon^*}$$

And

$$\frac{\partial \varepsilon^*}{\partial \theta_2} = - \frac{\partial FOC / \partial \theta_2}{\partial FOC / \partial \varepsilon^*}$$

Due to the assumptions on $\alpha'(\varepsilon)$ and $\alpha''(\varepsilon)$, the sufficient second order condition to this problem holds. Thus:

$$\text{sign } \frac{\partial \varepsilon^*}{\partial \theta_1} = \text{sign } \frac{\partial FOC}{\partial \theta_1}$$

And

$$\text{sign } \frac{\partial \varepsilon^*}{\partial \theta_2} = \text{sign } \frac{\partial FOC}{\partial \theta_2}$$

Therefore, to show that $\partial FOC / \partial \theta_1 < 0$ and $\partial FOC / \partial \theta_2 > 0$.

To show the first differentiate the first order condition *w.r.t.* θ_1 to get:

$$\begin{aligned} \frac{\partial FOC}{\partial \theta_1} = & p \cdot U''[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*)))](PX^*)^2 \\ & \times (\theta_2 - (\theta_1 - \alpha(\varepsilon^*)) - \alpha'(\varepsilon^*)(1 - \varepsilon^*))(1 - \varepsilon^*) - \\ & - p \cdot U'[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*)))]PX^* + \\ & + (1 - p) \cdot U''[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)\theta_1)](PX^*)^2(\theta_2 - \theta_1) \times \\ & \times (1 - \varepsilon^*) - (1 - p) \cdot U'[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)\theta_1)]PX^* \end{aligned}$$

Use now the definition of $Ra(\cdot)$ to get:

$$\begin{aligned} \frac{\partial FOC}{\partial \theta_1} = & -p \cdot Ra[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*)))] \times \\ & \times U'[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*)))](PX^*)^2 \times \\ & \times (\theta_2 - (\theta_1 - \alpha(\varepsilon^*)) - \alpha'(\varepsilon^*)(1 - \varepsilon^*))(1 - \varepsilon^*) - \\ & - p \cdot U'[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*)))]PX^* - \\ & - (1 - p) \cdot Ra[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)\theta_1)] \times \\ & \times U'[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)\theta_1)](PX^*)^2(\theta_2 - \theta_1)(1 - \varepsilon^*) - \\ & - (1 - p) \cdot U'[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)\theta_1)]PX^* \end{aligned}$$

Table A.1.
Cointegrating results using Johansen's (1988)
technique (uses $\log X$)

Eigenvalues (μ_i)	$-T \sum_{j \leq i} \ln(1 - \mu_j)$	Critical value at 5% (10%)	Null hypothesis
0.0145	0.9829	9.243 (7.525)	$\exists$ at most 3 coint. vectors
0.0923	7.4694	19.964 (17.852)	$\exists$ at most 2 coint. vectors
0.2388	25.7540	34.910 (32.003)	$\exists$ at most 1 coint. vectors
0.2861	48.3364	53.116 (49.648)	$\exists$ at most 0 coint. vectors

Multiply the first order condition by:

$$Ra[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*)))]PX^*(1 - \varepsilon^*)$$

And add the result (zero) to $\partial FOC / \partial \theta_1$. After rearranging the result is:

$$\begin{aligned} \frac{\partial FOC}{\partial \theta_1} = & -p \cdot U'[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*)))]PX^* - \\ & - (1 - p) \cdot U'[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)\theta_1)]PX^* - \\ & - (1 - p) \cdot U'[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)\theta_1)](PX^*)^2(\theta_1 - \theta_2) \times \\ & \times (1 - \varepsilon^*) \cdot \{Ra[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*)))] - \\ & - Ra[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)\theta_1)]\} \end{aligned}$$

Table A.2.
Standardized β'
est. cointegrating vector

$\log X$	$\log YW$	$\log(PW \cdot HSE)/P$	$\log U$
1.00	-2.33	-0.87	1.75

Since we assumed $Ra'(\cdot) < 0$ and:

$$(\varepsilon^* \theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*))) < (\varepsilon^* \theta_2 + (1 - \varepsilon^*) \theta_1)$$

We get:

$$Ra[PX^*(\varepsilon^* \theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*))) - Ra[PX^*(\varepsilon^* \theta_2 + (1 - \varepsilon^*) \theta_1)] > 0$$

Therefore, all the terms in the right hand side of $\partial FOC / \partial \theta_1$ are negative and $\partial FOC / \partial \theta_1 < 0$ is proved.

Now to show that $\partial FOC / \partial \theta_2 > 0$. We will use an argument analogous to the one used to prove that $\partial FOC / \partial \theta_1 < 0$. First differentiate the first order condition *w.r.t.* θ_2 to get:

$$\begin{aligned} \frac{\partial FOC}{\partial \theta_2} = & p \cdot U''[PX^*(\varepsilon^* \theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*)))](PX^*)^2 \times \\ & \times (\theta_2 - (\theta_1 - \alpha(\varepsilon^*)) - \alpha'(\varepsilon^*)(1 - \varepsilon^*))\varepsilon^* + \\ & + p \cdot U'[PX^*(\varepsilon^* \theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*)))]PX^* + \\ & + (1 - p) \cdot U''[PX^*(\varepsilon^* \theta_2 + (1 - \varepsilon^*) \theta_1)](PX^*)^2(\theta_2 - \theta_1) \\ & \varepsilon^* + (1 - p) \cdot U'[PX^*(\varepsilon^* \theta_2 + (1 - \varepsilon^*) \theta_1)]PX^* \end{aligned}$$

Apply now the definition of $Ra(\cdot)$ to the expression above and then add the result to the first order condition multiplied by:¹⁶

$$Ra[PX^*(\varepsilon^* \theta_2 + (1 - \varepsilon^*) \theta_1)] \cdot PX^* \cdot \varepsilon^*$$

After cancelling terms and rearranging, the result is:

$$\begin{aligned} \frac{\partial FOC}{\partial \theta_2} = & p \cdot U'[PX^*(\varepsilon^* \theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*)))]PX^* + \\ & + (1 - p) \cdot U'[PX^*(\varepsilon^* \theta_2 + (1 - \varepsilon^*) \theta_1)]PX^* + \\ & + p \cdot U'[PX^*(\varepsilon^* \theta_2 + (1 - \varepsilon^*) \theta_1)](PX^*)^2 \times \\ & \times \varepsilon^* \cdot (\theta_2 - (\theta_1 - \alpha(\varepsilon^*)) - \alpha'(\varepsilon^*)) \times \\ & \times \{Ra[PX^*(\varepsilon^* \theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*))) - \\ & - Ra[PX^*(\varepsilon^* \theta_2 + (1 - \varepsilon^*) \theta_1)]\} \end{aligned}$$

¹⁶ Again, this is equivalent to adding zero to the previous expression.

Table A.3.
Standardized α EC coeff.
in each equation

Dep. var.	Z_1
$\log X$	-0.69
$\log YW$	-0.06
$\log(PW \cdot HSE)/P$	0.07
$\log U$	0.05

Since $r^b - \alpha(\varepsilon) \leq r^0 \leq r^b \forall \varepsilon$ and $\alpha'(\cdot) < 0$, $(\theta_2 - (\theta_1 - \alpha(\varepsilon^*)) - \alpha'(\varepsilon^*)) > 0$. Using this result in conjunction to the assumption that $Ra'(\cdot) < 0$ yields:

$$(\theta_2 - (\theta_1 - \alpha(\varepsilon^*)) - \alpha'(\varepsilon^*)) \cdot \{Ra[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)(\theta_1 - \alpha(\varepsilon^*)))] - Ra[PX^*(\varepsilon^*\theta_2 + (1 - \varepsilon^*)\theta_1)]\} > 0$$

Therefore all the terms in the right hand side of the above expression are positive, thus, $\partial FOC / \partial \theta_2 > 0$ and the result follows.

Table A.4.
Cointegrating results using Johansen's (1988)
technique (uses $\log X^*$)

Eigenvalues (μ_i)	$-T \sum_{j \leq i} \ln(1 - \mu_j)$	Critical value at 5% (10%)	Null hypothesis
0.0051	0.2180	9.243 (7.525)	$\exists$ at most 3 coint. vectors
0.0749	3.5664	19.964 (17.852)	$\exists$ at most 2 coint. vectors
0.2782	17.5867	34.910 (32.003)	$\exists$ at most 1 coint. vectors
0.3757	37.8447	53.116 (49.648)	$\exists$ at most 0 coint. vectors

Table A.5.
Standardized β' est. cointegrating vector

$\log YW$	$\log(PW \cdot HSE)/P$	$\log U$	$\log X^*$
1.00	-0.42	-1.30	-0.44

Table A.6.
Standardized α EC coeff. in each equation

Dep. var.	Z_1
$\log X^*$	0.94
$\log YW$	-0.27
$\log(PW \cdot HSE)/P$	0.38
$\log U$	0.25

Table A.7.

Instrumental variables estimation
 dep. var.: $\Delta \log X^*$ (Uses FGV's
 price series as a deflator)

Expl. var.	Est.	T-stat.
$\log G$	-.21828	-2.03
$\Delta \log X^*_{-4}$	.23223	2.45
$\Delta \log YW$	1.61780	3.18
$\Delta \log YW_{-2}$	-2.34784	-4.55
$\Delta \log YW_{-3}$	-1.83664	-3.27
$\Delta \log U_{-3}$	2.09349	3.13
$\Delta \log U_{-4}$	.80527	1.69
$\Delta \log \left(\frac{PW \cdot E(1+S)}{P} \right)_{-1}$	.29474	1.57
$Z1_{-1}$	-.38496	-5.11
Constant	-.01342	-.22
$Q1$	.06087	.61
$Q2$	.15638	3.54
$Q3$	.23853	2.26
DNR	-.35864	-4.53
$DCRU$	-.44387	-4.40

List of Instruments: $\Delta \log X^*_{-1}$, $\Delta \log YW_{-1}$, $\Delta \log U_{-1}$, $\log G_{-1}$, $Z1_{-2}$

Diagnostic Tests:

$\sigma = .0804361$

$\chi^2(15)/15 \quad \beta = 0: 12.46$

Durbin Watson = 2.208

AR 1-7 $\chi^2(7) = 3.996$

Normality (Jarque-Bera) $\chi^2(2) = .657$

Heteroskedasticity (white) $PR > F[24,19] = .9979$

Testing exports underinvoicing

Graph 1.

Manufactured Exports Undervoicing and US\$ gap (Ranges are matched)

Table A.8.

Consistent lower bound estimate of manufactured
exports underinvoicing current US\$ and
end of sample US\$

QTR	Current US\$	In 1988.4 US\$	QTR	Current US\$	In 1988.4 US\$
71.1			80.2	143765	336995
71.2			80.3	119591	272118
71.3			80.4	28860	63951
71.4	2497	11865	81.1	10425	22262
72.1	38631	180623	81.2	125617	258166
72.2	2383	10989	81.3	69261	136797
72.3	4884	22213	81.4	121773	230556
72.4	11428	51193	82.1	479649	877187
73.1	4944	21815	82.2	505238	891471
73.2	4861	21059	82.3	135741	231170
73.3	2024	8586	82.4	404033	666707
73.4	82332	340233	83.1	153026	246430
74.1	3414	13776	83.2	115044	181135
74.2	9589	37860	83.3	460558	708746
74.3	36578	140559	83.4	593626	891029
74.4	38804	144469	84.1	55260	80977
75.1	5649	20506	84.2	61726	88249
75.2	16919	60303	84.3	58959	81949
75.3	53858	188511	84.4	52703	71158
75.4	80093	274812	85.1	245837	323950
76.1	55864	188168	85.2	386506	497878
76.2	43767	145175	85.3	694718	876693
76.3	385952	1259832	85.4	14051	17381
76.4	194453	625103	86.1	275651	334322
77.1	13281	42110	86.2	69991	83289
77.2	52085	162881	86.3	329838	385920
77.3	85272	262801	86.4	1775584	2046093
77.4	82672	250783	87.1	1221905	1387382
78.1	34688	103341	87.2	180740	202074
78.2	55875	163385	87.3	226831	249141
78.3	63417	181780	87.4	263564	284309
78.4	37626	105513	88.1	133331	141076
79.1	106502	290671	88.2	127118	132209
79.2	171416	455370	88.3	477092	487117
79.3	209056	541239	88.4		
79.4	128892	324576			