INTERTEMPORAL SUBSTITUTION IN CONSUMPTION: AN AMERICAN INVESTIGATION FOR BRAZIL

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Resumo

O artigo examina formulações diferentes da equação de Euler com o objetivo de estimar o impacto da taxa de juros sobre o consumo no Brasil durante 1980. O coeficiente estimado de elasticidade de substituição intertemporal para os consumidores que não sofrem restrições à liquidez é estatisticamente significativo, mas menor do que a unidade. O resultado sugere que um aumento da taxa de juros no Brasil durante este período gerava tanto um efeito riqueza quanto um efeito substituição.

Abstract

This paper examines different formulations of the Euler equation in order to assess their magnitude of the intertemporal substitution effect in Brazil during the 1980. Estimates of the intertemporal elasticity of substitution for those consumers that are not liquidity constrained are statistically significant, but less than one. The result suggest that an increase in interest rates in Brazil during this period had both income and substitution effects.

1. Introduction¹.

Intertemporal substitution in consumption² implies that a higher expected real interest rates make consumers defer consumption, everything else held constant. If consumer can be induced to postpone consumption by increases in interest rates, then movements in interest rates will make consumption decline whenever other components

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²I.e., consumes willingness to substitute consumption tomorrow for consumption today.

of aggregate demand rise. This is particularly relevant in the case of financed government spending, as intertemporal substitution in consumption implies that a rise in the interest rates will automatically induce higher debt holdings. Therefore, the magnitude of this intertemporal substitution effect is a central issue in macroeconomics.

For Brazil, the magnitude of this effect is particularly important given the costs associated with a policy of high real interest rates, where government is the principal debtor in the domestic financial market due to constant budget deficits. On the one hand, high real interest rates reduce the impact of the budget deficit and public debt on aggregate demand. On the hand, it increases the costs of government borrowing in the domestic market and, therefore, the burden of the national debt.

This policy trade-off has received a considerable amount of attention, notably in the aftermath of failed heterodox stabilization programs. These so called heterodox stabilization programs managed to achieve rapid disinflation in the short-run, but success was short-lived. Rapid disinflation was accompanied by bursts of consumption and increased demand for real assets, wich eventually contributed to the collapse of these programs.

The first consumption boom, wich occurred during the cruzado plan, was associated with expansionary monetary policy pursued by the central bank, aimed at reducing the weight of the public debt in the governments budget. The decline in nominal interest rates was believed to be too fast, however, and led to massive withdraw from savings and time deposits. Later programs tried to avoid the shift away from financial assets and into consumers goods, once indexation clauses where eliminated, by targeting high real interest rates after deindexation measures were introduced, and by imposing penalties for early withdraw of deposits. Their efforts met little success, nevertheless.

Alternative explanations for the apparent small intertemporal elasticity of substitution were suggested. Pastore (1990) argued that consumption was unresponsive to interest rate policies because the central bank was selling treasury securities with repurchase agreements and therefore the policy of high real interest rates did not in-

duce more savings, nor impose penalties on early withdraws. Werlang and Neri (1990) and Neri (1990) developed the argument that savings behavior in Brazil was mostly aimed at buffering income against inflation and, therefore it was not the change in interest rates, but disinflation itself, that triggered the massive withdraw of savings deposits during the first heterodox program. Simonsen (1990) claimed that individuals demand indexed assets as hedge against the variance of relative prices, which tends to increase with inflation. Thus, in periods of rapid disinflation they liquidate their portfolios of indexed assets and purchase consumer goods and real assets, as holding indexed assets losses its purpose.

Despite the importance of the topic, the empirical literature reports only two measure the intertemporal elasticity of substitution of consumption for Brazil. Leme and Málaga (1990) found no evidence of intertemporal substitution in consumption for the period 1983-1990. Their exercise however has several weak points. First, they disregard simultaneity problems involved in the estimation of Euler equations and report ordinary least squares (OLS) coefficients, instead of instrumental variable estimates. Second, the fail to recognize that the theoretical framework they are testing implicitly assumes constant real interest rates, a condition hardly met by the Brazilian data. Finally, their test is built on a proxy for quarterly consumption derived from monthly data on employment in the consumption goods industry which is much less smooth, i.e., has a larger standard deviation, than overlapping series derived from National Income Accounts.

Gleizer (1991) used annual data for two periods, 1960-85 and 1971-85, and included a term for variable real interest rates. His results indicated that for the period until 1985 elasticities of substitution were statistically significant but relatively low, suggesting that consumers were reluctant or unable to intertemporally substitute. However, his analysis disregarded issues such as liquidity constraints and nonseparability in the utility function, which might have affected the size of the coefficient estimated. Moreover, he used annual data, that went only until 1985. Therefore, the analysis did not cover the most recent years where (a) most of the attention is centered, (b) one should

dramatic swings in real interest rates and other relevant variables, and (c) there was a shortening of the maturity of the domestic debt that led to the collapse of the term structure of interest rates, which in turn rendered the interest rate policy more effective.

Drawing from a more recent and accurate estimate for quarterly private consumption in Brazil, this paper pursues an empirical investigation of the correlation between the time series of private consumption and real interest rates during the 1980-89 period. More specifically, the paper examines alternative formulations of the Euler equation approach in order to assess the magnitude of the intertemporal substitution effect.

The rest of this paper is organized as follows. Section 2 reviews the literature on intertemporal substitution in consumption and the efforts to reconcile the evidence from the data with the theory. Section 3 presents three models of intertemporal substitution in consumption. Section 4 empirically implements these models. Section 5 discusses the specificities of the Brazilian case. Section 6 examines the welfare effects of liquidity constraints. And section 7 closes the paper with the main conclusions³.

2. Literature Review.

Intertemporal substitution in consumption is the corner stone of the permanent income hypothesis first presented in Friedman (1957), and later recast by Hall (1978) in a intertemporal maximization framework. Consumption was modeled as obeying the first order condition for the optimal choice of a single, fully informed and forward looking consumer; also called the Euler equation approach. Hall's basic proposition was that the marginal utility of consumption should be a trended random walk when interest rates are constant over time. This implied that expected changes in consumption were small and linearly dependent on the ex-ante real interest rate.

The assumption of constant real interest rates was however the main weakness of Hall's framework. Mankiw (1981) improved upon

³In addition, there are two appendices. The first appendix present the statistical diagnostic tests, and the second appendix provides the data sources used in this chapter.

this test of the permanent income hypothesis by allowing the ex-post real interest rate into the simple intertemporal maximization framework, yet failed to find econometric support for the restrictions implied by the Euler equation.

Later empirical work followed the same path. Hansen and Singleton (1982, 1983) estimated a significant and correctly signed intertemporal elasticity of substitution, however the restrictions implied by the orthogonality conditions of the instruments were violated. Hall (1988) studied the US twentieth century consumption data and found no strong evidence that the intertemporal elasticity of substitution was positive for permanent income consumers. Meanwhile, Flavin (1981, 1985) and Campbell and Mankiw (1989, 1990) argued that consumption was too sensitive to present income to be consistent with intertemporal substitutability alone.

The failure of the Euler equation to explain the joint time series of consumption and asset returns produced three lines of research⁴. A first line of research has suggested that the data on aggregate consumption is best viewed as generated not by a single forward looking consumer but by two types of consumers. One type of consumer that is forward looking and substitutes consumption intertemporally in response to interest rates movements. And another type of consumer who is liquidity constrained and follows a "rule of thumb" of consuming his current income (Flavin 1981, 1985; Campbell and Mankiw, 1989, 1991; Hall and Mishkin, 1982; Hayashi, 1987). Aggregate consumption therefore would be a weighted average of current and permanent income.

Campbell and Mankiw (1989, 1991) report estimates of this model for six developed countries⁵. For all of these countries, except Japan, changes in aggregate consumption respond to changes in current income. The fraction of liquidity constrained consumers

⁴A fourth line of research that preceded the Euler equation studies was the Ricardian equivalence hypothesis pioneered by Barro (1974). Barro consolidated private and government accounts and suggested that, given disposable income and wealth (including government debt), intertemporal shifts in the pattern of taxes, with no change in their present value, produced shifts in the pattern of consumption.

⁵Canada, France, Japan, Sweden, United Kingdom and United States.

lies in the range from 20% in Canada to nearly 100% in France, with Sweden, the U.S., and the U.K. falling in between. However, when they account for liquidity constraints, the effect of interest rates on consumption is negligible in most of these countries.

The second line of research focused on possible specification errors. In particular, if variables other than consumption have an impact on its marginal utility. That is, the household utility may be nonseparable between consumption and some other variable.

Four candidates for nonseparability received attention in the literature. Bernanke (1985) examined the relationship between households' nondurables and service expenditure and their stocks of durable goods. Mankiw, Rotemberg and Summers (1985), and Eichenbaum, Hansen and Singleton (1988) looked for evidence of nonseparability between consumption and labor supply. Auschauer (1985) and Bean (1986) checked for nonseparability between consumption and government purchases. And Arrau (1990), Koenig (1990) and Poterba and Rotemberg (1987) investigated the relationship between consumption and money holdings.

Only the two last candidates claimed to have fared well in empirical test. However, the nonseparability between public and private consumption claimed by Auschauer (1985) was later not confirmed by Campbell and Mankiw (1990), while evidence on the money in the utility function approach is mixed.

The results on the money in the utility function approach reported by Poterba and Rotemberg (1987) implied a negative relationship between consumption and liquid assets. Koenig (1990) improved upon their analysis by, among other things, correctly specifying the utility function⁶, and found that households in the US tend to concentrate their consumption expenditures in intervals during which holdings of liquid assets are larger. Arrau (1990) performed a similar test for Chile and Mexico and found that the intertemporal elasticity of substitution for these countries was larger than one

⁶Koenig (1990) allows for two types of drift terms in the household's utility function. The first drift term captures the tendency for consumption to become more or less valued through time. The second drift term allows the valuation of money relative to consumption to rise or fall over the period.

and statistically significant when he included money in the utility function, while the intratemporal elasticity of substitution, although statistically significant, was less than one and relatively small⁷.

A third extension questioned the use of expected utility as a criteria by which consumers would rank different income streams (Epstein and Zin, 1989, 1991; Farmer, 1990; and Weil 1990). Expected utility maximization implies a rigid inverse link between the elasticity of substitution and risk aversion, which should be established empirically rather than imposed by the choice of a utility function. Drawing from a more general theory provided by Kreps and Porteus (1978), who relax the indifference with respect to resolution of uncertainty about consumption streams implied by expected utility maximization, they obtain a more general model in which risk aversion and intertemporal elasticity of substitution are decoupled.

This last approach requires a precise definition of the assets included as private wealth, which makes it difficult to implement empirically for developing countries where there is not enough information on assets privately held abroad. Therefore, we chose to disregard this formulation in our empirical investigation and focus on the three first methods of estimating the intertemporal elasticity of substitution, namely (a) the unconstrained Euler equation, where individuals maximize consumption Intertemporally subject only to the lifetime budget constraint; (b) the Euler equation that allows a fraction of the consumers to be credit constrained; and (c) the money in the utility function approach. In the next section we present the models associated with these three approaches and Section 4 empirically implements the models for Brazil.

3. The Models.

We begin with the permanent income hypothesis model first derived by Hall (1978) and later developed by Mankiw (1981), where the individual's lifetime utility at time s is given by:

⁷One unattractive feature of Arrau's results is that, in the absence of a co-integration theory between the variables of the model, he assumed that the trend in the variables were deterministic and some how compensate each other to yield stationary errors.

$$U_t = \sum_{s=t}^T \delta^s \cdot U(C_s); U' > 0, U'' < 0 \quad (1)$$

$$U(C_s) = \frac{1}{1-\alpha} \cdot C_s^{1-\alpha} \quad (2)$$

where δ is the rate of time preference, C_s is real consumption at time s , and α is the coefficient of relative risk aversion. The form of the utility function is a standard von Neumann-Morgenstern utility function with constant relative risk aversion.

If complete markets exist, the only binding constraint on the consumer's utility maximization problem is the lifetime budget constraint.

$$\sum_{s=t}^T \frac{1}{r_{s+1}} \cdot (Y_s - C_s) \geq 0 \quad (3)$$

where r_{s+1} is the real rate of interest, defined as the ratio between the nominal interest rate and consumer price inflation, both denoted between period s and $s+1$.

The necessary condition for optimality of a household's consumption plan is that the household be indifferent between one unit of consumption at time s and $(1 + r_{s+1})$ units of consumption at time $s+1$.

Formally,

$$\delta \left[\frac{C_{s+1}}{C_s} \right]^{-\alpha} \cdot r_{s+1} = 1 \quad (4)$$

Moving to a stochastic setting this first order condition that defines the Euler equation is reformulated as

$$E_s \left[\delta \left(\frac{C_{s+1}}{C_s} \right)^{-\alpha} \cdot r_{s+1} \right] = 1 \quad (5)$$

where E_s is the expectation operator. The realized value of the equation (5) above must obey:

$$\left[\delta \left(\frac{C_s + 1}{C_s} \right)^{-\alpha} \cdot r_{s+1} \right] = 1 + e_{s+1} \quad (6)$$

Equation (6) provides us with the relationship between consumption and ex-post real interest rates, and we estimate (6) to recover the intertemporal elasticity of consumption $1/\alpha$.

Next, turning to the credit rationing modification of equation (6), consider that one group of consumers follows the intertemporal optimization rule, while another group is constrained to consume at their current level of income. The later part captures the fraction of consumers that are at a corner solution because they are excluded from credit markets. Thus, changes in the log of consumption in period s by optimizing and credit rationed consumers are given, respectively, by:

where the superscripts o and c stand for optimizing individuals and credit constrained individuals,

$$C_{s+1}^o = C_0^s \sqrt[\alpha]{\delta r_{s+1}} \quad (7)$$

$$C_{s+1}^c = Y_{s+1}^c \quad (8)$$

respectively.

Equations (7) and (8) simply state that optimizing individuals plan to consume according to their permanent income, while credit constrained consumers are constrained to consume their current income.

In order to identify the consumption allocated to each group, some simplifying assumptions are needed. Assume that a fraction τ of total income is received by credit rationed consumers, i.e.,

$$Y_{s+1}^c = \tau Y_{s+1} \quad (9)$$

Moreover, assume that a fraction $(1 - \tau)$ of total consumption is made by optimizing consumers, i.e.,

$$C_{s+1}^o = (1 - \tau) C_{s+1} \quad (10)$$

Then the total consumption in period $s + 1$ is given by:

$$C_{s+1} = (1 - \tau)C_s \sqrt[\alpha]{\delta r_{s+1}} + \tau Y_{s+1} \quad (11)$$

Normalizing (11) and re-writing it in a stochastic setting yields:

$$E_s \left[\frac{C_{s+1} - \tau Y_{s+1}}{(1 - \tau)C_s \sqrt[\alpha]{\delta r_{s+1}}} \right] = 1 \quad (12)$$

The realized value of equation (12) must obey:

$$\left[\frac{C_{s+1} - \tau Y_{s+1}}{(1 - \tau)C_s \sqrt[\alpha]{\delta r_{s+1}}} \right] = 1 + e_{s+1} \quad (13)$$

Equation (13) provides us with the relationship between consumption, interest rates and current income, and we estimate (13) to recover the intertemporal elasticity of consumption ($1/\alpha$) and the fraction of consumers that are constrained to consume their current level of income (τ).

Finally, in order to incorporate money holdings in the intertemporal maximization framework we need to re-write the Euler equation to include money in the utility function. We begin with the necessary condition for optimality of a household's consumption plan, i.e., that the household be indifferent between one unit of consumption at time s and $(1 + r_{s+1})$ units of consumption at time $s + 1$, where r_{s+1} is the real rate of interest. Formally,

$$E_s \left[\frac{(1 + r_{s+1}) \cdot V_{s+1}}{(1 + \delta) \cdot V_s} \right] = 1 \quad (14)$$

where E_s is the expectation operator, V_s is the marginal utility of consumption, and δ is the rate of subjective time preference.

The realized value of the equation above must obey:

$$\left[\frac{(1 + r_{s+1}) \cdot V_{s+1}}{(1 + \delta) \cdot V_s} \right] = 1 + e_{s+1} \quad (15)$$

Taking the logarithm of both sides and using the Taylor approximation for $\log(1 + x)$, we can rewrite equation (15) above as:

$$r_s - \delta + \log V_{s+1} - \log V_s = e_{s+1} - \frac{1}{2} \cdot e_{s+1}^2 \quad (16)$$

Specifying the marginal utility function as log-linear in its arguments

$$V_s = \mu_s \cdot [C_s^{-\alpha}] \cdot [M_s^\beta] \quad (17)$$

we can write the $\log V_s$ as

$$\log V_s = \log \mu_s - \alpha \log C_s + \beta \log M_s \quad (18)$$

where C_s and M_s denote consumption and real money balances, respectively; and μ_s , α and β are constants. Concavity requires that α be positive, while $\beta = 0$ implies that the utility function is additively separable between consumption and real money balances.

Replacing (17) in equation (18) and rearranging the terms we obtain

$$\log \Delta C_s = a_0 + a_1 r_s + a_2 \Delta \log M_s + u_s \quad (19)$$

where $a_0 = 1/2 \cdot \sigma^2 - \delta/\alpha$, $a_1 = 1/\alpha$, $a_2 = \beta/\alpha$, and $u_t = 1/\alpha \cdot (1/2 \cdot e_s^2 - 1/2 \cdot \sigma^2 - e_s)$.

Equation (19) provides us with the relation between the log of consumption, the ex-post real interest rate and changes in the log of real money balances. We estimate (19) to recover the intertemporal elasticity of substitution ($1/\alpha$) and the intratemporal elasticity of substitution between consumption and liquidity services (β/α).

Equations (6), (13) and (19) provide the basic framework to estimate (a) the magnitude of the intertemporal substitutability in consumption; (b) the fraction of the consumers that are in a corner solution and therefore are constrained to consume their current level of income; and (c) the intratemporal elasticity of substitution between consumption and liquidity services. In the next section we empirically implement these models.

4. Empirical Implementation.

Before we can estimate these models, three specification issues that arise from the nature of the aggregate time series need to be addressed. The first issue relates to the fact that equation (6), (13) and (19) cannot be estimated via ordinary least squares, as r_t , Y_t and M_t may be correlated with e_t . But since e_t is uncorrelated with any other variable known at $t - 1$, we can rely on instrumental variable estimation⁸, and any vector Z_{t-1} known at time $t - 1$ and correlated with r_t , Y_t and M_t is a valid instrument⁹.

A second data problem arises because consumption and income are measured as quarterly averages rather than at points in time. If the permanent income hypothesis holds in continuous time, then measured consumption is the time average of a random walk. Therefore, the change in consumption will have a first-order serial correlation of 0.25, which would lead us to reject the model even if it was true¹⁰. We deal with this problem by lagging the instruments more than one period in order to obtain a test of the models that is valid for time-averaged data.

The third specification issue concerns the presence of unit roots in the time series of the models. Unit roots in time series imply that they are not stationary and this violates the assumptions of classical regression analysis. Therefore we transformed the variables in order to make the time series stationary by first-differencing their logs.

⁸I.e., we assume that $E(e_{t+1}/I_t) = 0$; $E(e_{t+1}^2/I_t) = \sigma^2$; and therefore e_{t+1} is uncorrelated with any variable known in time t .

We test for the orthogonality of the instruments by regressing the residual from the instrumental variables regression on the instruments, and then comparing T times R^2 from this regression, where T is the sample size, with the χ^2 distribution with $(k - 1)$ degrees of freedom. The test statistic is reported in table 1.

⁹The choice of instruments was done according to the model specification. For the unconstrained Euler equation lagged values of r_t were used. For the permanent income model with liquidity constrained consumers lagged values of r_t and Y_t were selected. And for the Money in the Utility Function approach lagged values of r_t and M_t were chosen.

¹⁰This point is made in Working (1960) and was implemented in a similar setting by Campbell and Mankiw (1989 and 1990).

Table 1, on the next page, reports the results for the three models¹¹. The first row gives the estimated parameters for the unconstrained Euler equation, along with the Ljung-Box Q-statistics¹² and the overidentified restrictions test¹³. The second row presents these estimates for the model where a fraction of the consumers are liquidity constrained, and the third row has the estimates for the model where consumers place money in the utility function.

Table 1.
Instrumental Variable Estimation¹

Equation	$1/\alpha$	τ	β/α	Q(15)	T
Permanent Income Consumers	-0.80 (-1.12)			15.78	0.24
PIH with Liquidity Constrained Consumers	-0.54 (-2.36)	0.32 (2.27)		10.67	3.37
Money in the Utility Function Approach	-0.76 (-1.38)		-0.30 (-1.28)	12.44	1.44

¹ t-statistics in parentheses.

The results indicate that the Euler equation that allows a fraction of consumers to consume their current level of income is the one that best captures the behavior of the time series of consumption over the 1980-89 period. The estimated coefficients for that equation

¹¹ Appendix A2 reports other diagnostic tests for the winning model, i.e., the rule of thumb consumption, and Appendix B2 provides the data sources.

¹²The Ljung-Box Q-statistics is used to test the null hypothesis that the first m autocorrelations are equal to zero. It is distributed as a X^2 with (m) degrees of freedom.

¹³This tests for the orthogonality of the instruments by regressing the residual from the instrumental variables regression on the instruments, and then comparing T times the R^2 from this regression, where T is the sample size, with the X^2 distributions with $(k - 1)$ degrees of freedom.

are statistically significant at the 5% level, there is no evidence of violation of the overidentified restrictions and the Q-statistic is well in the acceptance region. However, the unconstrained Euler equation and the model with Money in the Utility Function fail to provide significant coefficients, despite the fact that they do not violate the overidentified restrictions test and that the test for residual autocorrelation is in the acceptance region.

The coefficient for the liquidity constraint reveals that 32% of the consumers are constrained to consume their current level of income. At first sight, this may seem a very low percentage of the total consumer population. For instance, Campbell and Mankiw (1989) report estimates of around 50% for the US. However, in a highly inflationary economy like Brazil, where real income varies dramatically between the periods of wage adjustment to inflation, most households save during the months following the wage agreement, when wages have just been inflated to their real level, in order to smooth consumption in the months that precede the next wage agreement, when real wages are at their lowest level. Therefore, we would expect to find only the poorest households, those unable to save to buffer against income fluctuations, consuming at the current level of income¹⁴.

This point can be better understood with the help of the Figure 1, which describes wage policy in Brazil during most of the 1980. Under a policy of lagged or imperfect indexation, real earnings had a saw edged time series pattern. The real value of earnings reaches its highest bound just after they have been adjusted to past inflation, and its lowest bound just before its adjustment to past inflation. The straight line at 90 represents average earnings when the rate of inflation is constant and also the ideal path of consumption. Hence, the household would save the amount represented by the area above the line during the first months following the wage agreement, in order to dissave during the months before the readjustment to inflation, where wages were below the line, and therefore smooth consumption.

The most important result is the intertemporal elasticity of substitution. The coefficient is statistically significant only when we ac-

¹⁴A similar argument was developed in Neri (1989) and Neri and Werlang (1990). See section 6 for an analysis of the welfare implications of liquidity constraints.

Figure 1.
Financial assets as percentage of GDP
Brazil - 1970-89

count for the liquidity constrained consumers, and even in this case the intertemporal elasticity of substitution is less than one. This suggests that, for the share of the consumer population that is not liquidity constrained, an increase in the income effect of an increase in real interest rates outweighs the substitution effect. We examine an interpretation for an estimated income effect larger than the substitution effect in the next section.

5. The income and the substitution effects.

In this section we examine one possible reason for an estimated elasticity of substitution smaller than one. The basic argument developed in this section can be made by simply inspecting a Slutsky

equation, that decomposes the change in demand due to an interest rate change into the income and the substitution effects.

Consider a rise in interest rates which will raise the price of consumption today as compared to consumption tomorrow:

$$\frac{\partial c^t}{\partial(1+r)} = \frac{\partial c^s}{\partial(1+r)} + (y-c) \frac{\partial c^y}{\partial y} \quad (20)$$

The first term in the right-hand side denotes the substitution effect and always works opposite the direction of price. Therefore, it has a negative sign. The last term on the right hand side, how consumption changes as income changes, will be positive provided consumption this period is a normal good. The whole expression will depend on the sign of $(y - c)$.

For a borrower, this term is negative and therefore the whole expression is unambiguously negative. An increase in interest rates will lower today's consumption. For a lender, however, the effect is ambiguous. If the income effect outweighs the substitution effect, an increase in interest rates may lead the lender to consume today.

As an estimated elasticity of substitution smaller than one implies that the income effect outweighs the substitution effect, what remains to be shown is how such a large percentage of the Brazilian consumer population (68%) became net lenders. The argument can be made if we understand how the market for government securities works in Brazil.

Primary and secondary markets for Brazilian government securities emerged in the early 1970. These securities provided a convenient source of funds for the government in face of shrinking demand for money due to inflation, as they permitted agents to maintain relatively high yields and liquidity without resorting to nonfinancial and foreign assets. Although the availability of treasury bills and indexed bonds contributed to the falling demand for money, the minimum denomination and transaction costs associated with holding and trading these instruments ensured that most agents would retain funds in zero or low-interest accounts.

In the early 1980s the Brazilian government lost access to the foreign credit market and, in order to continue funding its budget

deficit the government resorted to increase borrowing in the domestic market. It did so, in part, by gradually reducing the minimum denomination for holding and trading in government securities. Furthermore, it allowed financial institutions to use savings and time deposits to back their portfolio of government securities, and did so by issuing repurchase agreements that could be used whenever the investors decided to withdraw the deposits from these institution. The end result was that money balances (currency plus demand deposits) as a percentage of GPD fell from 9.32% in 1980 to 2.21% in 1989.

Figure 2 provides a graphical description of this phenomena. Money holdings (currency plus demand deposits) accentuate their secular decline during the 1980s, except for the period of rapid disinflation of the **cruzado** plan in 1986. Meanwhile, indexed assets (i.e., public debt held outside the central bank, savings deposits, and time deposits) increase their share in the universe of financial assets of the economy. Thus, financial assets other than money holdings were being used by the population as a unit of account and a store of value, typical functions of money.

Being a net lender *vis-à-vis* the government did not imply that these households were accumulating assets. These households dissaved as often as they saved, since they saved during the first months followings the wage agreement, in order to dissave in the months before the wage adjustment to past inflation and, hence, smooth consumption.

However, since government debt was an increasing share of domestic income during all of this period, except for the year of the **cruzado** plan (1986), this implied that at the aggregate level households were becoming net lenders. They did so, in part, by reducing current consumption — the substitution affect — and in part by shifting their money holdings into indexed assets, which allowed them to capture the income effect. As a result of the deepening of this process by the end of the decade the income effect had outweighed the substitution effect.

6. Welfare implications of liquidity constraints.

The works of Cochrane (1989) and Thaler (1990) have argued that empirical studies on aggregate consumption identify deviations

Figure 2.
Inflation and consumption in an inflationary
economy subject to half-yearly wage adjustment

from the permanent income hypothesis that have only second-order effects on utility. This is an interesting issue for Brazil, as liquidity constraints are associated with lower levels of income and access to indexed assets. Therefore, in this section we examine the welfare implications of having a fraction of the consumers consuming their current level of income.

Consider the case where the actual consumption path provides utility U , and the optimal consumption path is U^* . A second-order Taylor expansion of the difference between U and U^* gives us

$$U - U^* = E_t \sum_{i=0}^{\infty} \beta^i u'(C_{*t+i}) \cdot (C_{t+i} - C_{*t+i})$$

$$+(1/2)E_t \sum_{i=0}^{\infty} \beta^i u''(C_{t+i}^*) \cdot (C_{t+i} - C_{t+i}^*)^2 \quad (22)$$

The first sum on the right hand side of (22) is zero for any perturbation from the optimal consumption plan. Utility costs can therefore be approximated by considering the second sum on the right hand side of (22). The expectation of the first element of this sum, divided by u' to convert into dollars and then divided by C_t^* to express the per-period utility loss as a fraction of optimal consumption, is

$$(1/2)E \left[\frac{u''(C_t^*)(C_t - C_t^*)^2}{u'(C_t^*) \cdot C_t^*} \right] \quad (23)$$

$$= (1/2)E \left[\frac{C_t^* u''(C_t^*) \cdot (C_t - C_t^*)^2}{u'(C_t^*) \cdot (C_t^*)^2} \right] \quad (24)$$

$$= (1/2)\gamma \text{Var}(C_t - C_t^*) \quad (26)$$

$$= (1/2)\gamma \text{Var} \left[\frac{(C_t - C_t^*)}{C_t^*} \right] \quad (25)$$

where the equality in the last line assumes that the agent has power utility with relative risk aversion α (see equation 2), and that the proportional difference between actual and optimal consumption can be approximated by the log difference.

The utility costs implied by the constrained Euler equation depends on whether there is a single representative agent who consumes $C_t = \tau Y_t + (1 - \tau)C_t^*$, or two heterogeneous groups consuming Y_t and C_t^* respectively. In the former case, the representative agent's consumption is

$$L_1 = \frac{\delta \tau^2}{2(1 - \tau)^2} \cdot \text{Var}(C_t - Y_t) \quad (27)$$

If there are two groups of agents, the utility loss to the current income consumers is instead

$$L_3 = \frac{\delta}{2(1 - \tau)^2} \cdot \text{Var}(C_t - Y_t) \quad (28)$$

The average utility loss across both groups of consumers, L_2 , is τL_3 . For small values of τ , there can be large differences between the utility loss measures because $L_1 = \tau L_2 = \tau^2 L_3$.

Table 2 reports utility loss measures corresponding to the estimate of liquidity constrained consumers of 32% reported in Table 1. The table expresses utility losses in percentage points, as a fraction of optimal consumption, for different values of relative risk aversion α . The results indicate that, for a range of coefficients of relative risk aversion, the utility loss is small for low degrees of relative risk aversion. This is credited to the fact that (a) the liquidity constraint is not binding for most of the consumer population and (b) the standard deviation (of the difference) between consumption and income is relatively small when compared to other countries¹⁵.

Table 2.
Utility Losses Implied by Liquidity Constraints¹

Loss Measure	L_1	L_2	L_3
$\alpha = 1.00$	0.002	0.008	0.025
$\alpha = 5.00$	0.011	0.040	0.125
$\alpha = 10.00$	0.022	0.075	0.250

¹ Standard Deviation of $(C_t - Y_t) = 0.025$.

Nevertheless, when one takes account of the possible heterogeneity across agents, the utility loss increases dramatically for high coefficients of relative risk aversion. Moreover, for the same coefficient of relative risk aversion for both types of consumers, the value of loss measure L_3 is more than 10 times larger than those of loss measure L_1 . In the limit, when the permanent income consumer is less risk

¹⁵Campbell and Mankiw (1990) report standard deviations between income and consumption for six developed countries that range between 0.024 for the US to 0.094 for Japan.

averse than the current income consumer, the value of loss measure L_3 is 125 times larger than the loss measure L_1 .

Therefore, on average, liquidity constraints do not account for large welfare losses, since the constraint is not binding for most of the consumer population and the standard deviation between consumption and income is relatively small. Yet these costs increase considerably when one takes account of the possible heterogeneity across agents.

7. Conclusions.

The purpose of this paper was to contribute to the understanding of the correlation between the time series of consumption and interest rates in Brazil during the 1980. More specifically, this paper examines several specifications of the Euler equation in order to assess the magnitude of the intertemporal elasticity of substitution. Four findings emerge:

- (a) The Euler equation that allows for a fraction of the consumer population to be credit constrained is the one that best describes the behavior of private consumption in Brazil during the 1980.
- (b) The percentage of the consumer population that is constrained to consume at their current level of income, however, is low (32%) when compared to the US (50%), for instance. This result can be explained by the fact that the existence of an active market for indexed assets allowed consumers in Brazil to spread income over the short term horizon at a very low cost and risk.
- (c) The magnitude of the intertemporal substitution effect is significant only when we account for liquidity constrained consumers, and, even in this case, it is smaller than one. This suggests that there is both a substitution and an income effect associated with higher real interest rates. The income effect is credited to the large holdings of indexed assets by the consumer population which allowed them to earn an income on their buffer stock saving.
- (d) On average, liquidity constraints do not account for large welfare losses, since the constraint is not binding for most of the consumer population and the standard deviation between con-

sumption and income is relatively small. Yet these costs increase considerably when one takes account of the possible heterogeneity across agents.

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Appendix A: Diagnostic Tests

In this appendix we perform a series of diagnostic tests for the model specification given in equation (13). First, we begin with the Lagrange Multiplier test of the 4th order residual autocorrelation, which runs the regression

$$\hat{u}_t = \sum_{i=2}^4 \alpha_i \hat{u}_{t-i} + e_t \quad (29)$$

This test is distributed as a $X(4)^2$ under the null hypothesis of no autocorrelation, i.e., that the errors are white noise.

Second, we test for autocorrelated squared residuals, also called ARCH (AutoRegressive Conditional Heteroscedasticity) test, which test the hypothesis that $\alpha = 0$ in the following regression

$$E(u_t^2 / u_{t-1}, \dots, u_{t-r}) = c_0 + \sum_1^r \alpha_i u_{t-i}^2 \quad (30)$$

where $\alpha = (\alpha_1, \dots, \alpha_{t-r})$. This test is asymptotically distributed as a F-statistics under the null hypothesis.

Third, we implement the Jarque-Bera test for normality of the regression disturbances. This is a X^2 test based on the estimation of skewness and kurtosis of the residuals compared to their counterparts for the normal distribution.

Fourth, we test for heteroscedasticity of the error term. The White (1980) test for heteroscedasticity involves regressing the estimated error term squared on the original regressors and their squares. The null hypothesis is unconditional homoscedasticity, and the alternative hypothesis that the variance of the error term depends on the regressors and their squares.

Finally, the fifth column reports the coefficient of correlation (R^2), which was not reported above for lack of space in Table 1 above.

The results are reported in Table A1, along with the critical values, and show no evidence of dynamic misspecification. The LM

test for 4th order autocorrelation accepts the null of white noise errors, and the Jarque-Bera test for normality accepts the hypothesis of normality of the regression disturbances. There is no evidence of dynamic ARCH effects, and White's test heteroscedasticity accepts the null hypothesis of constant variance.

Table A1.
Diagnostic Tests

4th order Autocorrelation	Autoregressive Conditional Heteroscedasticity	Normality	Heteroscedasticity	R^2
3.01	0.72	0.30	9.05	37.1
Critical Value CHI ² (4)	Critical Value F(4,36)	Critical Value CHI ² (2)	Critical Value CHI ² (5)	
9.49	5.72	5.99	11.0	

Appendix B: Data Sources

This paper uses data from two sources: the IBGE, the Brazilian Institute of Statistics, and the Central Bank of Brazil. From the first source, we took the time series for quarterly GDP measured in 1980 cruzeiros as measure of income, as supplied in publications **Indicadores Conjunturais** (several issues). From this series we estimated residually the time series for quarterly private consumer spending. Both series were then normalized the population in order to obtain per capita figures.

The series for quarterly consumer spending was estimated residually by subtracting (a) exports minus imports of goods and non-factor services, (b) fixed and inventory investment, and (c) public consumption. The series for exports and imports were derived from the series of

export and import **quantum** published in the *Conjuntura Econômica* by the Getúlio Vargas Foundation. For fixed and inventory investment we used the series published by the IPEA/INPES. And the series for public consumption was built by taking the information published by the **Balanco Geral da União** and pro-rating this value quarterly according weights given by the wage bill in each quarter.

From the second source, we obtained the time series for money balances (currency plus demand deposits) and interest rates paid on government securities backed by overnight deposits, as supplied in the **Boletim Mensal do Banco Central** (several issues). Both were deflated to 1980 prices by the general price index — domestic supply (IGP-DI) published in the Getúlio Vargas Foundation's *Conjuntura Econômica*. The series for money deposits was also normalized by the population in order to obtain per capita figures. Quarterly interest rates were obtained by compounding the monthly interest rates and than taking the geometric average in that quarter.