AGENTS, ECONOMETRICIANS AND THE IDENTIFICATION OF RATIONAL EXPECTATIONS SYSTEMS*

Renato G. Flôres Jr.**
Ariane Szafarz***

Abstract

This paper generalises previous results on the identification of rational expectations (r.e.) models, establishing necessary and sufficient conditions on the structural form of static and dynamic models, without specific assumptions on the stochastic processes generating the endogenous and exogenous variables. The approach allows the econometrician to explore the information in predetermined variables not previsible by the agents. Ways of making operational this knowledge are discussed. As a consequence, the set of identification strategies is broadened and a better insight is gained on the cost/benefit of an early selection on the solutions set.

*We are grateful to Marianne Bertrand, to participants in seminars at CORE, Louvain-la-Neuve, and IMPA, Rio de Janeiro, and to two anonymous referees. A. Szafarz acknowledges support from SES — Région Wallonne; R. Flôres benefitted from stays at CAESAR, Université de Paris X (agreement CAPES + COFECUB 093/88) and at CEME, Bruxelles; both authors were also funded by programme ARC from the Communauté Française de Belgique. All errors are our own.

**EPGE/FGV and UFRJ, Rio de Janeiro.
***CEME and ECARE, ULB, Bruxelles.
1. Introduction.

General theorems concerning the identification of the structural form parameters of linear dynamic rational expectations (r.e.) systems were stated in Flôres and Szafarz (1992). These results, obtained without any assumption whatsoever on the processes generating the model variables, show, as previously raised by Salemi (1989) and Broze et alii (1987), that identification can be achieved without concern on the solutions set. Before adding further assumptions to single out a solution — as stationarity or a transversality condition — many models may have their structural parameters already identified. This stood for qualifying as minimal such and approach to identification in a r.e. context. Of course, in practice, estimation issues and the proper fit to the existing data of a particular solution still have to be faced. It is within this last purpose that developments like those in Whiteman (1983), Salemi (1989) himself, Pesaran (1988) and actually most of the related literature should be viewed, and not as general answers to the identification problem.

Broze and Szafarz (1991) and Flôres and Szafarz (1992) also pointed out that, even keeping a priori assumptions to a minimum, identification can change with the information set available to the econometrician. In this paper a complete characterization of this situation is given, while hypotheses on the data generating process are still kept rather weak. The results generalise recent contributions by Turkington and Bowden (1988) and Rayner (1991), which share similar ideas. The use of different information sets for solving r.e. problems is becoming a standard tool, as the volatility test proposed by West (1988) exemplifies.

After presenting the main idea in the next section, a discussion follows on how to characterize the econometrician's knowledge. The basic identification theorem is stated in section 4. Section 5 provides three examples that set the results in a broader view; section 6 concludes.

These two papers were apparently developed independently of our work which started with Flôres and Szafarz (1989).
2. The change in identification.

As in Flôres and Szafarz (1992) we begin with model:

$$Ay_t + B_0y_t^c + B_1y_{t+1} + \ldots + B_ky_{t+k} + Cx_t + Dx_{2t} = u_t$$  \hspace{1cm} (2.1)

where $A, B_0, \ldots, B_k$ are $n \times n$ matrices and $C$ and $D$ are, resp., $n \times m$ and $n \times m_2$ matrices; column vectors $y_t, n \times 1$ and $x_t, m \times 1$ account, resp., for the endogenous and exogenous variables, vector $x_t$ being equal to the piling up of vectors $x_{1t}, m_1 \times 1$ and $x_{2t}, m_2 \times 1$, $m = m_1 + m_2$, the first being previsible with respect to the information set at the beginning of time $t, I_{t-1}$, defined below; $u_t$ accounts for the disturbances, a multivariate process with zero means and (constant) contemporaneous dispersion matrix $\Sigma_u$. The disturbances are an innovation with respect to $I_{t-1}$ and vector $x_t$ is predetermined in (2.1), in the sense that $\text{cov}(x_t, u_{t+j}) = 0$, for all $j \geq 0$, Engle et alii (1983); all r.e. $y_t^c, y_{t+1}^c, \ldots, y_{t+k}^c$ and $x_{2t}^c$ are conditional on the (same) $\sigma$-algebra generated by set $I_{t-1} = \{y_{t-1}, x_{t-1}, y_{t-2}, x_{t-2}, \ldots\}$, as said, being measurable in this algebra.

Different normalizations may be imposed on matrices $A$ or $B_i, 0 \leq i \leq k$, depending on how the model is treated. Matrix $C$ will sometimes be split into its two components $C_1$ and $C_2$, related, resp., to $x_{1t}$ and $x_{2t}$.

The rational expectations in (2.1) are non-observables, in the sense of Goldberger (1972) and Griliches (1974), and for $i = 0, 1, \ldots, k$ one can write:

$$y_{t+1}^c = y_{t+1} - e_{t+1}$$  \hspace{1cm} (2.2)

$e_{t+i}$ being orthogonal to the information set $I_{t-1}$.

Conditioning model (2.1) on any information set $\Omega_{t-1}$ smaller or equal to that of the agents, yields:

$$AE[y_t | \Omega_{t-1}] + B_0E[y_t | \Omega_{t-1}] + B_1E[y_{t+1} | \Omega_{t-1}] + \ldots + B_kE[y_{t+k} | \Omega] + C E[x_t | \Omega_{t-1}] + D E[x_{2t} | \Omega_{t-1}] = 0$$

which, after collecting the common terms, becomes:
Agents, econometricians and the identification

\[(A + B_0)E[y_t|\Omega_{t-1}] + B_1E[y_{t+1}|\Omega] + \ldots + B_kE[y_{t+k}|\Omega_{t-1}] + C_1E[x_1t|\Omega_{t-1}] + (C_2 + D)E[x_2t|\Omega_{t-1}] = 0\]

(2.3)

If, however, expectations are taken with respect to a larger set \(\Omega_{t-1}\), provided that \(E[u_t|\Omega_{t-1}] = 0\), this leads to:

\[AE[y_t|\Omega_{t-1}] + B_0y_t^c + B_1y_{t+1}^c + \ldots + B_ky_{t+k}^c + CE[x_t|\Omega_{t-1}] + Dx_2t^c = 0.\]

(2.4)

Now let \(\Omega_{t-1}\) denote the information set available to the econometrician and \(L\) the set of initial constraints on the structural parameters of (2.1). Identification is defined as:

**DEFINITION:** (Flöres and Szafarz (1992)). A set of admissible values (i.e. satisfying the constraints in \(L\)) \(A^0, B_i^0(i = 0,1,\ldots,k), C^0, D^0\) is \(\Omega\)-identifiable iff for any other admissible values \(A, B_i(i = 0,1,\ldots,k), C, D\), and for every \(j, 0 \leq j \leq k:\)

\[E[y_{t+j}|\Omega_{t-1}] = E^0[y_{t+j}|\Omega_{t-1}] \Rightarrow A = A^0, \quad B_i = B_i^0(i = 0,1,\ldots,k), \quad C = C^0, \quad D = D^0.\]

This means that, beyond the given \(a \text{ priori}\) conditions \(L\), the system is identified if equality\(^2\) of all expectations up to time \(t + k\) entails unicity of the admissible parameters.

The gist of the minimal identification idea lies in that in the case \(\Omega_{t-1} \subseteq I_{t-1}\), the equalities in the Definition, together with (2.3), bear a straight relationship with the corresponding distributed lags model, i.e., the one obtained by substituting the values given from (2.2)

---

\(^2\)The right-hand side of the equality means the conditional expectation when \(A^0, B_i^0(i = 0,1,\ldots,k), C^0, D^0\) are substituted for the parameters, while in the left-hand member the other admissible values \(A, B_i(i = 0,1,\ldots,k), C, D\) are considered.
for the expectations in (2.1) and equating out the new error term. The distributed lags model then provides (a kind of) reduced form parameters that, together with conditions L, allow the establishment of identification.

However, if $\Omega_{t-1} \supset I_{t-1}$ two new things stand out. The first is that, as (2.4) shows, overidentification may be a fact: independently of set L, application of the definition seems to allow for a better separation of the parameters. But equation (2.4), as (2.3), is not estimable, and resorting to the corresponding distributed lags model amounts to arrive at the same rearrangement of coefficients as in the previous case. The point is then how to translate the additional information clearly present into a workable condition.

The above discussion can also be linked to the exogeneity status of $\{x_t\}$. Indeed, equations (2.3) and (2.4) tell that interest in the first-order parameters of the distribution of $\{y_t\}$ requires weak exogeneity of $\{x_t\}$. However, the existence of non-observables and the role played by set L can change the situation, as long as identification is concerned; that is why the whole $\{x_t\}$ process has been assumed only to be predetermined in (2.1). For estimation purposes, it is likely that at least full weak exogeneity will have to be verified.

Now, for all exogenous not $I_{t-1}$ previsible, let $w_t = x_{2t} - x_{2t}^c$ be the error vector orthogonal to all expectations appearing in (2.1). Taking the covariance of both sides of (2.1) with $w_t$ yields the following result:

$$A \text{cov}(y_t, w_t) + C \text{cov}(x_t, w_t) = 0$$

or, after rearrangement and consideration of the partitions of $x_t$:

$$\text{cov}(Ay_t + C_2x_{2t}, w_t) = [A \quad C_2] \begin{bmatrix} \text{cov}(y_t, w_t) \\ \text{cov}(x_{2t}, w_t) \end{bmatrix} = 0$$

(2.5)

Defining $z_t' = (y_t', x_{2t}')$, (2.5) asserts the first-order stationarity of the multivariate process $\{[A \quad C_2]z_t w_t'\}$, as

$$E([A \quad C_2]z_t w_t') = \text{cov}(Ay_t + C_2x_{2t}, x_{2t} - x_{2t}^c) = 0$$

(2.6)
irrespectively of the processes followed by the endogenous and exogeneous.

The above fact suggests that the $\Omega_{t-1}$ of interest are those which differ from $I_{t-1}$ because some of the imperfectly predicted (by the agents) exogenous $x_{2t}$ are measurable in them. The key idea of this paper is to explore this fact as a possible translation of (2.4). Ways of rendering it operational are the subject of the next section.

3. The econometrician’s knowledge.

Let us suppose that the econometrician possesses a further knowledge on some coordinates of process $x_{2t}$; let $x_{2t}^*, p \times 1, p \leq m_2$, be this subvector. Ideally, such a knowledge would enable her to evaluate the corresponding $w_t^* = x_{2t}^* - (x_{2t}^*)^c$ entries in equation (2.5) and then the covariances.

The fact that the latter may not be known in any time point is not necessarily a shortcoming. As will be seen, identification can in principle be checked by merely examining the structure of the matrix of covariances figuring in (2.5). However, if a large sample is available, (2.6) suggests a classical approximation through the weak law of large numbers.

Interesting examples, in which the elements in $w_t^*$ can be expressed by simple formulae, are:

i) $x_{2t}^*$ is a martingale with respect to the filtration $I_{t-1}$;

ii) $x_{2t}^*$ is a VARMA $(p,q)$ process: $\Theta(L)x_{2t}^* = \tau(L)e_t$ where $\Theta(L) = I - \Theta_1 L - \Theta_2 L^2 - \cdots - \Theta_p L^p$, and $\tau(L) = I + \tau_1 + \tau_2 L^2 + \cdots + \tau_q L^q$ are matrices of lag polynomials. This includes a VAR $(p)$ specification and bears relationship with the case in which $x_{2t}^*$ is stationary and admits a Wold (possibly infinite) moving average representation;

iii) $x_{2t}^*$ plus some elements of $I_{t-1}$ form a Gaussian process.

Now, if for a sample of observations $z_t, w_t^*, t = 1, \ldots, T$, one defines $M_T = \sum_t z_t w_t^*$, supposing valid the hypotheses of the weak
law of large numbers and (2.5), it follows that:

$$\lim (1/T)[A C_2] M_T = [A C_2] \lim (1/T)M_T = 0$$

(3.1)

The finite version of (3.1) thus provides a workable identity incorporating the additional information contained in $\Omega_{t-1}$. It is a procedure similar to the one used in the generalised method of moments estimation technique, Hansen (1982).

Carrying the analogy with estimation procedures a bit further, though the discussion was based on a subset of vector $x_{2t}$, the same ideas would be valid if the econometrician had at her disposal a vector process $v_t$, not previsible by $I_{t-1}$, such that $\text{cov}(v_t, u_t) = 0$. This suggests an enlargement of the concept of instruments for the identification case.

4. A basic theorem.

Though the main concern is with the structural form coefficients, we shall include in the a priori restrictions conditions on the (constant) contemporaneous covariance matrix of the innovation process. Set $L$ is then defined through $q$ initial constraints in the form of continuously differentiable functions:

$$\Phi_i(B_k, B_{k-1}; \ldots, B_0, A, C_1, C_2, D, \Sigma_u) = 0, \quad 1 \leq i \leq q, \quad (4.1)$$

which will be stacked as function $\Phi = (\Phi_1, \Phi_2, \ldots, \Phi_q)'$.

Let $\pi$ be the matrix formed by placing sideways the “pseudo-reduced form parameters” of the minimal case, i.e., $\pi = [P_1 P_2 \ldots P_{k+1} P_{k+2}]$ where:

$$P_i = -B_k^{-1}B_{k-i}, \quad 1 \leq i \leq k-1, \quad P_k = -B_k^{-1}C_1$$

$$P_{k+1} = -B_k^{-1}(A + B_0) \quad P_{k+2} = -B_k^{-1}(C_2 + D). \quad (4.2)$$

---

3 See, for instance, Gnedenko (1969), page 211, for a necessary and sufficient condition for the weak law of large numbers for an arbitrarily dependent sequence of random variables.

4 These reduced form parameters are obtained from the distributed lags model associated to equation (2.3) [see Flöres and Szafarz (1992)].
Agents, econometricians and the identification

Matrix \( \pi \) will be viewed as \( \pi = [\pi_1 \pi_2] \), where \( \pi_1 = [P_1 P_2 \ldots P_k] \) and \( \pi_2 = [P_{k+1} P_{k+2}] \).

The two additional equations are:

\[
B_k[\pi_1 \pi_2] + \hat{A} = 0, \tag{4.3}
\]

where

\[
\hat{A} = [B_{k-1} \cdots B_1 \ C_1 \ (A + B_0) \ (C_2 + D)], \tag{4.4}
\]

and, from (4.4):

\[
[A \ C_2]H = 0 \tag{4.5}
\]

where \( H \) is a known \((n + m_2) \times p\) matrix which can be formed either by the actual covariances for a given \( t \) or by a suitable approximation like the \((1/T)M_T\) discussed in the previous section.

Calling \( \beta_k, \sigma \) and \( \alpha \) the vectorizations\(^5\) of \( B_k, \sum_U \) (only the \( n(n+1)/2 \) distinct elements) and \( [B_{k-1} \cdots B_1 \ C_1] \), resp.; and \( (a, c_2) \) and \( (b_0, d) \) those of matrices \([A \ C_2]\) and \([B_0 \ D]\), resp., we have:

**Theorem 1.** Let \( \Theta = (\beta_k, \alpha, (a, c_2), b_0, d, \sigma) \) be a regular point of the parameter space, and suppose the pseudo-reduced form parameters in (4.2) are identified.

A. For locally identifying the \( n^2(k+2) + nm + nm_2 + n(n+1)/2 \) parameters of model (2.1), it is necessary and sufficient that:

i) if the \( \sigma \)-algebra being used is \( \Omega_{t-1} \subseteq I_{t-1} \):

\[
\text{rank} \left[ W^* \left( \frac{\partial \Phi}{\partial (b_0, d)} - \frac{\partial \Phi}{\partial (a, c_2)} \right) \frac{\partial \Phi}{\partial \sigma} \right] = 2n^2 + nm_2 + \frac{n(n+1)}{2}
\]

where \( W^* = \frac{\partial \Phi}{\partial \beta_k} (I_n \bigotimes B_k) + \frac{\partial \Phi}{\partial \alpha^*} (I_n \bigotimes \hat{A}) \)

and \( \alpha^* = (\alpha, (a, c_2)) \)

and the derivatives in the difference are evaluated at corresponding entries in matrices \( A, B_0 \) and \( C_2, D \);

---

\(^5\)All vectorizations are assumed to be row vectors formed by sequentially placing the rows (from the first to the last one) of the corresponding matrices.
ii) if the \( \sigma \)-algebra being used is \( \Omega_{t-1} \supset I_{t-1} \)

\[
\text{rank} \begin{bmatrix}
-I_n \otimes (B_k[B_0 \ D]H)' & -I_n \otimes H' & 0 \\
W^* & \frac{\partial \Phi}{\partial (b_0, d)} - \frac{\partial \Phi}{\partial (a, c_2)} & \frac{\partial \Phi}{\partial \sigma}
\end{bmatrix} =
\]

\[
= 2n^2 + nm_2 + \frac{n(n + 1)}{2}
\]  

(4.7)

where \( W^* \) is as above.

B. If the \( \Phi_i \) are linear functions of the structural parameters, then the system is globally identified iff depending on the \( \sigma \)-algebra being used, fulfillment of the conditions in i) or ii) above takes place.

PROOF: See the Appendix.

Conditions (4.6) and (4.7) reflect how identification changes if the econometrician is able to incorporate extra information of the kind of (2.5). A more suggestive version is given by the Corollary below, which adds two extra hypotheses, not too strong:

**Corollary 2.** Under the hypotheses of the Theorem, and calling \( r \) the rank of \( H, 1 \leq r \leq \min(n + m_2, p) \), if the space generated by the first \( np \) rows of the matrix in (4.7) has a trivial intersection with the one generated by the remaining rows then a necessary and sufficient condition for the local identification of the structural parameters in (2.1) is that:

\[
\text{rank} \begin{bmatrix}
W^* & \frac{\partial \Phi}{\partial (b_0, d)} - \frac{\partial \Phi}{\partial (a, c_2)} & \frac{\partial \Phi}{\partial \alpha}
\end{bmatrix} = 2n^2 + nm_2 + \frac{n(n + 1)}{2} - nr
\]

(4.8)

PROOF: The hypothesis of null intersection between the two spaces implies that the rank of (4.7) will be the sum of the dimensions of the spaces. As the one generated by the first \( np \) rows has dimension \( nr \) the condition follows immediately from the main Theorem.

\[6\text{If only non-previsible exogenous are used as identifying instruments this minimum equals } p.\]
It is interesting to contrast the "gain" from (2.5), translated by the difference in the ranks figuring in (4.6) and (4.7), with the general issue raised by (2.4). If only the $x_{2t}$ are used as instruments, the Corollary says that the most one can get is a reduction in the rank condition as great as the number of elements in matrix $C_2$ (or $D$). This sort of upper limit is a consequence of how the general issue was made operational, as pointed out in section 2.

A final remark should also be made regarding the presence of unit roots in $\{y_t\}$. In the case that some $\{x_t\}$ are lagged endogenous and the remaining ones are stationary processes, Broze et alii (1990) have shown that an Engle-Granger representation can apply, Engle and Granger (1990), and that (2.1)\(^7\) has the same cointegration properties as the corresponding perfect foresight version.

Therefore, the long run equilibria are common to both models, differences lying in the way short-run adjustments take place. Though this may give rise to specific estimation strategies, the identification problem is not affected. Indeed, either there is a priori knowledge about the unit roots — which should then be included in set $L$ — or there is none. In both cases, the definition of identifiability in section 2 holds and the Theorem remains valid.

5. Examples.

In order to better qualify the previous results and illustrate the strategies opened up by the proposed approach, three examples are briefly discussed here.

I. The static model.

Consider model $Ay_t + B_0y_t^e + Cx_t = u_t$. As known, this model allows to solve for the expectation of the endogenous and substitute back, giving way to:

$$Ay_t - B_0(A + B_0)^{-1}Cx_t^e + Cx_t = u_t$$

\(^7\)Actually, their model is more general as it allows for different information sets for computing the expectations.
If $x_t$ is previsible, a true reduced form can be obtained, but $A$ and $B_0$ continue unidentified. If $x_t$ is not previsible, the pseudo-reduced form will coincide with the (previous) true reduced form, but from the error term. The specification by the econometrician of a way to compute $x_t - x_t^e = w_t$ will enable her to use the extra nm conditions $A \text{cov}(y_t, w_t) + C \text{cov}(x_t, w_t) = 0$. Of course, if she chooses a VAR (p) representation for the exogenous process, she will be able to estimate the pseudo-reduced form and check identification along lines proposed since Wallis (1981). The attraction of this case is that incorporation of the econometrician's knowledge will many times allow for a true reduced form, so that a further treatment of the identification is possible. This point is pursued by Turkington and Bowden (1988), who arrive at classical exclusion restrictions for the one-equation case, and is also dealt with in Broze and Szafarz (1991).

However, all this does not preclude the use — if needed — of the nm conditions. Moreover, the econometrician's belief may be attached to a wider class of processes that will violate Turkington and Bowden (1988) assumptions. In these cases, she still can estimate the pseudo-reduced form and proceed to identification according to the lines suggested in the present paper, while the other approaches fail to help her. Even in the case that $x_t$ is previsible, if instruments $z_t^e$ are available, she may accordingly use the corresponding extra equations.

II. A stylised version of Sargent (1976)'s model:

$$p_t = b_{12}y_t + m_t + c_{10}(m_{t-1} - p_{t-1}) + \Sigma c_{1j}x_{jt} + u_{1t}$$
$$y_t = b_{21}(p_t - p_t^e) + \Sigma c_{2j}x_{jt} + u_{2t}$$

with $p_t$ (GNP price deflator) and $y_t$ (real GNP) endogenous, $m_t$ standing for the money supply and the $x_{jt}$ for varied previsible exogenous; all variables are in logs.

This is a classical instance of the identification problem in r.e.; the trouble being with $b_{21}$, the coefficient of the "surprise"

\[8\]In the sense defined in the end of section 3, i.e., $\text{cov}(z_t, u_t) = 0$. 

R. de Econometria 14(1) abril 1994/outubro 1994 81
Agents, econometricians and the identification term (see Pudney (1982)). If \( m_t \) is supposed non-previsible and the econometrician has a good way of estimating \( w_t = m_t - m_t^c \), the second row of the matrix equation (2.5) yields immediately
\[
b_{21} = \frac{\text{cov}(y_t, w_t)}{\text{cov}(p_t, w_t)}. \]
If this is not the case, and no other instruments are available, \( b_{21} \) is not identified as noticed by Wegge and Feldman (1983). The interesting point when \( m_t \) is previsible is that if a true instrument \( z_t \) is at hand, then \( b_{21} = b_{12}^{-1} \). To see this, as the covariance values are not necessarily stationary and (2.5) becomes (with, now, \( w_t = z_t - z_t^c \)):
\[
\begin{align*}
\text{cov}(p_t, w_t) - b_{12} \text{cov}(y_t, w_t) &= 0 \\
-b_{21} \text{cov}(p_t, w_t) + \text{cov}(y_t, w_t) &= 0;
\end{align*}
\]
it must be that:
\[
1 - b_{21} b_{12} = 0.
\]

III. A general current “surprise-effect” model.

The previous example is generalised below, where emphasis is on the surprise terms:
\[
A_1 y_{1t} + A_2 y_{2t} + B_{10} y_{1t}^c + B_{10}^* (y_{1t} - y_{1t}^c) + C_1 x_{1t} \\
+ C_{20} x_{2t} + C_{21} x_{2,t-1} + \ldots + C_{2p} x_{2,t-p} = u_t
\]

By redefining
\[
A = [A_1 + B_{10}^* \quad A_2], \quad B_0 = [B_{10} - B_{10}^* \quad 0] \quad \text{and}
\]
\[
C = [C_1 \quad C_{20} \quad C_{21} \quad \ldots \quad C_{2p}]
\]
it clearly becomes a special version of the static model discussed in the first example. The problem is that \( B_{10}^* \) does not appear in the pseudo-reduced form\(^9\), so that additional information is always needed to identify it — a point long known in the literature. By exploring the characteristics of the above structure, and assuming as the econometrician’s knowledge that \( x_t \) is generated by a \( VAR(p^*) \), \( p^* > p \), Rayner (1991) has obtained several rank conditions which can ultimately be traced as special forms of (4.7).

\(^9\)It obviously disappears when the term \( A + B_0 \) is computed.
6. Conclusions.

This paper has stressed the different role played by agents and econometricians in the identification of r.e. models. It must be said that there is no consensus on the amount of information either can possess. Examples in both directions, i.e., one knowing more than the other, can be easily found in the literature. We allow for both situations but assume that the agents as well as the econometricians know the structure of the true model.

Within this setting, the Theorem in section 4 gives general necessary and sufficient conditions for the identification of structural parameters. Use is made of the pseudo reduced form coefficients and (possibly) of identifying instruments. Conditions (4.6) and (4.7) encompass all previous results on the subject. Moreover, as the latter and the examples in section 5 show, the Theorem may be translated into various specific versions, depending on further simplifications of the basic model or on particular ways of characterizing the econometrician's knowledge.

Since identification precedes estimation, it perspective the scope of the result. In this light, the remarks on weak exogeneity, unit roots and the possible models for the predetermined variables might reveal themselves crucial in subsequent estimation procedures.

Appendix

Proof of the main theorem.

A. Local identification:

i) The matrix to be analysed at a regular point can be written as:

\[
\begin{bmatrix}
I_n \otimes \pi'_1 & I_n^{2(k-1)+nm_1} & 0 & 0 & 0 \\
I_n \otimes \pi'_2 & 0 & I_n^{2+nm_2} & I_n^{2+nm_2} & 0 \\
\frac{\partial \phi}{\partial \beta_e} & \frac{\partial \phi}{\partial \alpha} & \frac{\partial \phi}{\partial (a,c_2)} & \frac{\partial \phi}{\partial (b_0, \sigma)} & \frac{\partial \phi}{\partial \sigma}
\end{bmatrix}
\]  

(A.1)

where the derivatives of the conditions (4.1) have been taken.
considering first all those of the entries in the submatrix corresponding to the block $B_k \pi_1$ and only afterwards the ones related to the remaining $n \times (n^2 + nm^2)$ matrix. The mechanics for passing from this matrix to the one in the condition of the Theorem is similar in cases i) and ii). As the latter is a bit more involved, the proof will be outlined for it.

ii) The additional conditions given by (4.5) supply $np$ extra rows for the matrix to be investigated; so that the new version of (A.1) is:

\[
\begin{bmatrix}
I_n \otimes \pi_1' & I_{n^2(k-1)+nm_1} & 0 & 0 & 0 \\
I_n \otimes \pi_2' & 0 & I_{n^2+nm_2} & I_{n^2+nm_2} & 0 \\
0 & 0 & I_n \otimes H' & 0 & 0 \\
\partial \Phi \\
\partial \beta_k \\
\partial \Phi \\
\partial \alpha \\
\partial \Phi \\
\partial (a,c_2) \\
\partial \Phi \\
\partial (b_0,d) \\
\partial \Phi \\
\partial \sigma 
\end{bmatrix}
\]  

(A.2)

where the new $np$ rows have been inserted between those corresponding to equation (4.3) and those related to the $a$ priori conditions.

Relying on Rothenberg (1971), the necessary and sufficient condition for identification is that the rank of (A.2) be equal to the dimension of the parameter space. By a permutation of the first three columns, (A.2) can be viewed as a four blocks matrix with a $n^2k + nm$ identity matrix at the upper left corner. Simple rank properties then imply that its total rank will be $n^2k + nm$ plus that of the matrix formed by subtracting from its lower right block the product of the

\[10\] The jacobian (A.1), but for a permutation of rows and columns, is equal to the corresponding one in Flores and Szafarz (1992).
remaining two. This product is:

\[
\begin{bmatrix}
0 & I_n \otimes H' \\
\frac{\partial \Phi}{\partial \alpha} & \frac{\partial \Phi}{\partial (a,c_2)}
\end{bmatrix}
\begin{bmatrix}
I_n \otimes \pi'_1 & 0 & 0 \\
I_n \otimes \pi'_2 & I_{n^2+nm_2} & 0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
I_n \otimes H' \pi'_2 & I_n \otimes H' & 0 \\
\frac{\partial \Phi}{\partial \alpha} I_n \otimes \pi'_1 + \frac{\partial \Phi}{\partial (a,c_2)} I_n \otimes \pi'_2 & \frac{\partial \Phi}{\partial (a,c_2)} & 0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
I_n \otimes H' \pi'_2 & I_n \otimes H' & 0 \\
\frac{\partial \Phi}{\partial \alpha} I_n \otimes \pi' & \frac{\partial \Phi}{\partial (a,c_2)} & 0
\end{bmatrix}
\] (A.3)

so that, the matrix to be investigated is:

\[
\begin{bmatrix}
-I_n \otimes ([A + B_0 \quad C_2 + D]H)' & -I_n \otimes H' & 0 \\
\frac{\partial \Phi}{\partial \beta_k} + \frac{\partial \Phi}{\partial \alpha^*} I_n \otimes A' (B_k^{-1})' & \frac{\partial \Phi}{\partial (b_0,d)} - \frac{\partial \Phi}{\partial (a,c_2)} \frac{\partial \Phi}{\partial \alpha}
\end{bmatrix}
\] (A.4)

The result is obtained by right multiplying this matrix by the full rank one:

\[
\begin{bmatrix}
I_n \otimes B'_k & 0 \\
0 & I_{n^2+nm_2+\frac{n(n+1)}{2}}
\end{bmatrix}
\] (A.5)

and by noticing that (4.5) implies that \([A + B_0 \quad C_2 + D]H = [B_0 \quad D]H\).

B. The proof follows from a standard reasoning as \(\sum_u\) does not appear in either (4.2) or (4.5) and the \((A + B_0)\) and \((C_2 + D)\) terms in (4.2) still contribute with constant terms to the various derivatives.

(Received July 1993. Revised June 1994)

References

Agents, econometricians and the identification


Sargent, T.J. 1976. “A classical macroeconometric model for the
Flores & Szafarz
