PROFIT SHARING WITH HETEROGENEOUS ENTREPRENEURIAL PROWESS

Renato Fragelli Cardoso*

Resumo

Este artigo analisa o impacto da adoção compulsória de participação nos lucros sobre os incentivos dos indivíduos para montarem seus negócios próprios. Ele apresenta um modelo de acumulação de capital no qual os indivíduos são igualmente capacitados para serem trabalhadores mas diferem em suas habilidades para gerenciam uma firma. Mostra-se que a imposição obrigatória de participação nos lucros pode inibir iniciativas empresariais, reduzir o número de firmas em operação, o produto agregado e o estoque de capital de longo prazo da economia. Apresenta-se também, um exemplo no qual apesar de suas conseqüências deletérias, a adoção compulsória de participação nos lucros poderia ser introduzida numa sociedade democrática se esta escolha fosse motivo de votação.

Abstract

This paper analyses the impact of the compulsory adoption of profit sharing on the incentives that individuals face to set up their own business. It presents a model of capital accumulation in which individuals are equally skilled to be workers but differ in their abilities to manage a firm. It is shown that the mandatory imposition of profit sharing can inhibit entrepreneurial initiatives, reducing the number of firms in operation, the aggregate output and the economy's long run capital stock. It is also shown an example where, despite its deleterious consequences, mandatory profit sharing could be introduced in a democratic society if this choice were put to vote.

Palavras-Chave: Capital accumulation, profit sharing, income policy

Código JEL: E22, E25, E64

* Escola de Pós-Graduação em Economia. Fundação Getúlio Vargas. 22253-900, Rio de Janeiro, RJ.
1. Introduction.

The usual models on profit sharing study the consequences of sharing contracts freely negotiated between employers and employees in order to appraise their impact on unemployment. In general they commend sharing schemes on its power to reduce unemployment. Moreover, some of these models provide an appealing explanation for the fact that profit sharing be so rare in the West, which leaves room for some coordination role designed to foster this alternate compensation scheme.

The model presented in this paper does not focus on the impact of profit sharing on unemployment, nor on the coordinating issues involving it. It aims at addressing another aspects of sharing schemes, namely: (1) the incentives that an individual faces to set up his own business, thus creating new jobs, instead of being a worker and competing for one of the existing jobs, and; (2) the impact of profit sharing on capital accumulation.

The model of this paper differs from the usual models in four ways. First, in stark contrast with the models that focus on the frictions that generate unemployment, it is assumed that the wage is flexible, which rules out involuntary unemployment. Second, it studies profit sharing schemes that are not freely negotiated between workers and firms, but are imposed from outside by, say, a government decree. Third, it takes into account that the number of firms in operation in an economy is not invariant to the compensation scheme adopted. And fourth, it analyzes the impact of profit sharing on capital accumulation.

The work of Martin Weitzman (1983), (1985) and (1987) kindled interest on profit sharing as an alternative compensation system which could be a useful tool to reduce the main nominal rigidity that causes unemployment. Since it emulates a flexible labor market, i.e., one where the nominal wage adjusts instantaneously in order to provide full unemployment, profit sharing was advocated as a remarkable form of avoiding the dreaded Keynesian unemployment.
In Weitzman's sharing economy, the marginal cost of labor would be below the average cost of labor, creating permanent excess demand for labor, thus eliminating unemployment. Nordhaus (1988) showed that for Weitzman proposition to be valid, two conditions must hold: the supply price of labor must fall very sharply in recessions, and the marginal cost of labor must be very far below the average cost of labor.

Weitzman's explanation for the fact that profit sharing be rare in most western countries was based on strong externality effects. When one wage firm converts to a share contract it will be guaranteeing employment not only to its own (internal) workers, but also serving as the employer of last resort for all other (outside) workers. In bad times internal workers would see their compensation falling in order to rescue outsiders. Since most of the benefits accrue not to its own workers, but to the working class as a whole, internal workers face no incentive to accept profit sharing when the workers of other firms do not accept it as well.

This argument is analogous to Keynes' explanation of why nominal wages are fixed in the short run: no one is willing to be the first to reduce nominal wages. As a result, the economy ends up in an inefficient Nash equilibrium without profit sharing. Weitzman suggests that in order to overcome this perverse coordination failure some sort of government incentive to profit sharing schemes is in order. Brunello (1992) shows that if internal promotion were the only way to climb the rungs of a career within a firm, then internal workers would favor profit schemes since the high rungs would only be attained if outsiders were hired to fill the low ones.

Following Weitzman's emphasis on short run aggregate fluctuations, Cooper (1988) presents a model of monopolistic competition in the presence of multiplier effects in which the introduction of share contracts in one sector changes the response to the adverse shocks and alters the nature of the interaction between the sectors. In his model there is only one very special share contract which Pareto
dominates the fixed-wage system. That is, there is only one very special contract which can balance the gains and losses to the various groups of agents in the economy from the introduction of share contracts. Nothing can assure, however, that the real economy has the arcane power be to pick out that special contract. Moreover, even if the firms were able to single out the special contract, how could it be coordinated in order to create, say, a (stable) Nash equilibrium?

John (1991) casts additional light in the factors that impinge on profit sharing. When a firm’s (marginal or total) revenue are very sensitive to employment, greater employment fluctuation may arise in a share firm. While Cooper’s skepticism about profit sharing was based on general equilibrium effects, John’s was based on fundamentals at the firm’s level.

James Mead (1986) not only agrees with Weitzman’s view that profit sharing provides greater stability of employment than a wage economy, but also believes that a firm in a share economy will be in excess demand for labor. He also suggests that the share system might increase labor effort. By making worker’s income a function of profits, the incentives to shirk are reduced. Moreover, each worker will tend to help with the supervision of fellow workers and might even impose social sanctions to those who shirk. However unobservability of individual effort may undermine this argument when profit sharing is based on collective output. González (1992) discusses this point. Kandel & Lazear (1992) explores how peer pressure may overcome unobservability of individual effort.

The impact of profit sharing on investment was studied by Vadhwani (1987). He concludes that it increases the cost of capital, thus tending to reduce the level of the capital stock. This point is analyzed in a more complete setting in this paper. The conclusions are at one with Vadhwani’s.

The empirical evidence on profit sharing has focused both on productivity as well as on employment. Jones & Svejnar (1985) found evidence that profit sharing had positive productivity effects in Italy.
Fitzroy & Kraft (1987) found strong influence of profit sharing on factor productivity in a sample of medium-sized metalworking firms in Germany. Blanchflower & Oswald (1987) found no evidence that profit sharing influenced employment in the United Kingdom. Cable & Wilson (1989) estimated productivity gains of between 3 and 8% due to profit sharing in the UK engineering industry. Kruse (1992) presents evidence that, although in a small magnitude, profit sharing did indeed increase productivity in the USA. Bell & Newmark (1993) using firm-level data for the union sector of the US economy found evidence that profit sharing reduced labor cost growth at firms that adopted these plans.

In short, the literature provides three main cases for more income sharing: (i) the morale and productivity argument, i.e., profit sharing may incentive workers to increase their effort; (ii) the wage flexibility argument, i.e., profit sharing may reduce the incentives of firms to sack workers in recessions, and; (iii) Weitzman’s macro economic argument, i.e., profit sharing may eliminate unemployment through the creation of permanent excess demand for labor. Argument (i) seems to be a compelling one when profit sharing is adopted in a small firm where observability of individual effort is possible; (ii) also seems to be true, specially in highly cyclical sectors; argument (iii), however, has cogent theoretical, as well as empirical evidence, against it.

The model presented in this paper does not address these three arguments summarized above, but it focuses on two aspects of profit sharing that have been overlooked by the literature.

First, it analyzes the impact of sharing schemes on the individual incentives to set up a firm. Although neoclassical macro economists have been obsessed with micro foundations for their macro models, very often they take for granted who is employing whom. Why should the number of firms operating in an economy be invariant to the compensation scheme adopted? The present paper takes into account this first choice of individuals before turning to the second
choice of how many workers a firm will hire.

Second, the model departs from the short run analysis in order to study the impact of profit sharing on the capital accumulation. Weitzman’s claim that all compensation systems have the same long run equilibrium - proposition 1 in his (1993) paper - does not hold if the micro choices are more carefully studied. Actually the model shows that Weitzman’s claim does not hold even in the short run.

The paper is organized as follows. Section 2 describes the economic environment. In section 3 the model is solved. This section is divided into three subsections in order to simplify the exposition. 3.1 works out the short run equilibrium, 3.2 the long run equilibrium, 3.3 analyses the dynamics following the introduction of profit sharing. Section 4 discusses some political economy issues that might lead short sighted democracies to vote for the adoption of a mandatory profit sharing rule in the economic environment of the model, despite its deleterious effects. Section 5 concludes.

2. The Economic Environment.

The economy is populated by a constant (large) number $\bar{N}$ of individuals dispersed over the interval $[0, 1]$ and characterized by a label $\lambda \in [0, 1]$. A distribution function $F: [0, 1] \to [0, 1]$ with density $f$ defines, for each $\lambda$, the fraction $F(\lambda)$ of individuals whose labels are lower than $\lambda$. $\lambda$ will be seen below to stand for an entrepreneurial prowess parameter.

There is only one good which can be produced, consumed or transformed into capital (i.e., saved). All individuals know the production technology, but some individuals are more talented to be entrepreneurs than others. If an individual $\lambda$ sets up a firm, employing $K_\lambda(t)$ units of capital and $N_\lambda(t)$ workers, his flow of production $Y_\lambda(t)$ at instant $t$ will be given by

$$Y_\lambda(t) = \lambda^c K_\lambda(t)^a N_\lambda(t)^b$$  \hspace{1cm} (1)
where: \(0 < a < 1, 0 < b < 1, 0 < a+b < 1\) and \(c > 0\). The constraint \(0 < a + b < 1\) is necessary to generate pure profits in a competitive market. It is not necessary that the sum \((a + b + c)\) be equal to one.

The capital and labor markets are competitive and are continuously in equilibrium at rental \(r(t)\) and wage \(w(t)\). The gross profit of entrepreneur \(\lambda\) at instant \(t\), \(\pi_\lambda(t)\), is given by

\[
\pi_\lambda(t) = Y_\lambda(t) - r(t)K_\lambda(t) - w(t)N_\lambda(t).
\] (2)

The profit sharing parameter \(\delta\), which is exogenous to the model, defines the net profit of an entrepreneur \(\lambda\) as \((1 - \delta)\pi_\lambda(t)\). Each worker at firm \(\lambda\) earns the wage \(w(t)\) and a profit-sharing income \(P_\lambda(t)\) which is the workers’ share on gross profits equally divided among the \(N_\lambda\) employees

\[
P_\lambda(t) = \delta \frac{\pi_\lambda(t)}{N_\lambda(t)}.
\] (3)

Equation (3) allows for the possibility that the profit-sharing income may vary across firms. As will later be proven, this will not be the case for the Cobb-Douglas technology will assure that \(P_\lambda(t)\) will be the same for all \(\lambda\) - see equation (10).

Individual \(\lambda\)'s goal in life is to maximize his life utility as given by

\[
\int_0^\infty e^{-\rho t} U [C_\lambda(t)] \, dt
\] (4)

where \(C_\lambda(t)\) is his flow of consumption at instant \(t\) and \(\rho > 0\) is the rate of time preference. The function \(U\) is strictly increasing, concave and continuously differentiable. \(\rho\) does not vary across individuals.

Let individual \(\lambda\)'s assets be \(A_\lambda(t)\) and \(\theta > 0\) be the rate of capital depreciation. Since \(r(t)\) is the gross rate of return on his assets, his inter temporal budget constraint is

\[
\dot{A}_\lambda(t) = I_\lambda(t) - C_\lambda(t) + [r(t) - \theta] A_\lambda(t)
\] (5)
where $I_\lambda(t)$ is his non-interest income. The \textit{dotted} variable represents the time derivative of the underlying variable. Since he can freely choose to be an entrepreneur or a worker, he does so seeking to maximize $I_\lambda(t)$, i.e.,

$$I_\lambda(t) = \max \{(1 - \delta)\pi_\lambda(t); \ w(t) + P(t)\}.$$  \hfill (6)

Given equation (1)–(6), each individual facing $r(t)$ and $w(t)$ will, at each instant $t$, maximize his intertemporal utility in the following way:

1. Maximize (2) subject to (1) in order to calculate the profit $\pi_\lambda(t)$ he would receive if he chose to be an employer;
2. Compare $(1 - \delta)\pi_\lambda(t)$ with $P(t) + w(t)$ according to (6);
3. Once he has chosen what kind of agent to be, he will determine his consumption flow $C_\lambda(t)$ in order to maximize (4) subject to (5), where $I_\lambda(t)$ is given by (6).

3. Solving the Model.

Focusing on individual $\lambda$, if he chose to be an entrepreneur, he would first maximize (2) subject to (1). This means that his demand for capital would be

$$K_\lambda^D(t) = \left[ \left( \frac{b}{w(t)} \right)^b \left( \frac{a}{r(t)} \right)^{1-b} \right]^{\frac{1}{1-a-b}} \lambda^{\frac{c}{1-a-b}}$$ \hfill (7)

and his demand for labor would be

$$N_\lambda^D(t) = \left[ \left( \frac{b}{w(t)} \right)^{1-a} \left( \frac{a}{r(t)} \right)^{a} \right]^{\frac{1}{1-a-b}} \lambda^{\frac{c}{1-a-b}}$$ \hfill (8)
substitution of (7) and (8) into (2) yields,

\[
\pi_\lambda(t) = (1 - a - b) \left[ \left( \frac{b}{w(t)} \right)^b \left( \frac{a}{r(t)} \right)^a \right]^{\frac{1}{1 - a - b}} \lambda^{\frac{c}{1 - a - b}}. \tag{9}
\]

Expression (9) shows that the greater the factor prices, the lower will be the profits. Likewise, the efficiency parameter \( \lambda \) increases the profits.

Substituting (8) and (9) into (3) one finds that the profit sharing income \( P_\lambda(t) \) does not vary across \( \lambda \),

\[
P_\lambda(t) = (1 - a - b) \delta \frac{w(t)}{b}.
\]

Since the profit sharing income does not depend on \( \lambda \), one can define labor’s compensation (or earnings) \( E \) as the sum of his wage and profit sharing income

\[
E(t) = w(t) + P(t) = \frac{w(t)}{b} \left[ b + \delta(1 - a - b) \right]. \tag{10}
\]

Examining equations (9) and (10) one concludes that the workers’ earnings do not vary across \( \lambda \) while net profits are an increasing function of \( \lambda \). Thus, given \( w(t) \) and \( r(t) \), those individual for whom \( (1 - \delta)\pi_\lambda(t) \) exceeds \( E(t) \) will be entrepreneurs, whereas those for whom \( E(t) \) exceeds \( (1 - \delta)\pi_\lambda(t) \), will be workers. Therefore there must be a watershed \( \Lambda(t) \) such that individuals with \( \lambda \in [0, \Lambda(t)] \) will be workers and individuals \( \lambda \in [\Lambda(t), 1] \) will be entrepreneurs. The existence of such \( \Lambda(t) \) is assured by the hypothesis of a flexible labor market which adjusts the wage \( w(t) \) so that there are \( \lambda \in [0, 1] \) such that \( E(t) \) exceeds \( (1 - \delta)\pi_\lambda(t) \).

The hypothesis that wages are flexible in the short run was chosen for convenience. Most papers on profit sharing adopt the hypothesis that there is a minimum fixed wage plus the profit sharing part.
of the workers’ compensation. The flexible labor market avoids the need to go into bargaining or implicit contracts models to determine how the fixed part of the compensation is settled.

3.1. Short-run general equilibrium.

The aggregate supply of capital $K(t)$ is a state variable fixed in the short run. At each instant $t$ the variables $\Lambda(t)$, $w(t)$, and $r(t)$ are determined in general equilibrium. The supply of labor $N(t)$ is the number of individuals $\tilde{N}$ times the fraction of them who choose to be workers,

$$N(t) = \tilde{N} F(\Lambda(t)). \quad (11)$$

Recalling that there are $\tilde{N} f(\lambda)$ entrepreneurs of type $\lambda$, the aggregate demand for capital $K^{D}(t)$ is obtained integrating (7) from $\Lambda(t)$ to 1:

$$K^{D}(t) = \int_{\Lambda(t)}^{t} K^{D}_{\lambda}(t) \tilde{N} f(\lambda) d\lambda$$

$$= \left[ \left( \frac{b}{w(t)} \right)^{b} \left( \frac{a}{r(t)} \right)^{1-b} \right]^{1-b} \tilde{N} \int_{\Lambda(t)}^{1} \lambda^{\frac{a}{1-b-\epsilon}} f(\lambda) d\lambda. \quad (12)$$

Similarly, the aggregate demand for labor $N^{D}(t)$ is given by the integration of (8) from $\Lambda(t)$ to 1

$$N^{D}(t) = \left[ \left( \frac{b}{w(t)} \right)^{1-a} \left( \frac{a}{r(t)} \right)^{a} \right]^{1-b-\epsilon} \tilde{N} \int_{\Lambda(t)}^{1} \lambda^{\frac{a}{1-a-\epsilon}} f(\lambda) d\lambda. \quad (13)$$

The individuals $\lambda = \Lambda(t)$ are indifferent between being entrepreneurs or workers. From (9) and (10) this indifference condition
R. Fragelli Cardoso

is given by

\[(1 - \delta)(1 - a - b) \left[ \left( \frac{b}{w(t)} \right)^{b} \left( \frac{a}{r(t)} \right)^{a} \right]^{\frac{1}{1-a-b}} \Lambda(t)^{\frac{\delta}{1-a-b}} = \]

\[= \frac{w(t)}{b} [b + \delta(1 - a - b)]. \quad (14)\]

The capital market equilibrium equation is given by (12) and the aggregate supply of capital \(K(t)\), which will be defined later - in equation (23)

\[K(t) = \left[ \left( \frac{b}{w(t)} \right)^{b} \left( \frac{a}{r(t)} \right)^{1-b} \right]^{\frac{1}{1-a-b}} \bar{N} \int_{\Lambda(t)}^{1} \lambda^{\frac{\delta}{1-a-b}} f(\lambda) d\lambda. \quad (15)\]

The labor market equilibrium equation is obtained from (11) and (13)

\[F(\Lambda(t)) = \left[ \left( \frac{b}{w(t)} \right)^{1-a} \left( \frac{a}{r(t)} \right)^{a} \right]^{\frac{1}{1-a-b}} \int_{\Lambda(t)}^{1} \lambda^{\frac{\delta}{1-a-b}} f(\lambda) d\lambda. \quad (16)\]

The short-run equilibrium variables \(r(t), w(t)\) and \(\Lambda(t)\) are calculated by the system of equation (14), (15) and (16). It must be noted how the model differs from the usual settings in which the number of workers and firms are exogenously are only two equations - the labor and capital markets equilibrium equations - and the corresponding two adjusting variables - the rental and the wage. Here, the number of firms - and hence of workers - is endogenous, so there is a third equation - the indifference condition (14) - and a third variable - \(\Lambda(t)\).

From (14),

\[\left[ \left( \frac{b}{w(t)} \right)^{1-a} \left( \frac{a}{r(t)} \right)^{a} \right]^{\frac{1}{1-a-b}} = \frac{b + \delta(1 - a - b)}{(1 - \delta)(1 - a - b)} \Lambda(t)^{\frac{\delta}{1-a-b}}. \quad (17)\]
Substituting the right hand side of (17) into (16) one gets

\[
\frac{\Lambda \frac{\Gamma}{1-a-b} F(A)}{\int_{A}^{1} \lambda \frac{\Gamma}{1-a-b} f(\lambda) d\lambda} = \frac{b + \delta(1 - a - b)}{(1 - \delta)(1 - a - b)}. \tag{18}
\]

The right hand side of (18) is an increasing function of the profit sharing parameter \(\delta\), while the left hand side is an increasing function of \(A\). Since neither the rental \(r(t)\) nor the wage \(w(t)\) are present in (18), one concludes that the watershed \(A\) does not vary with time. This is why the time variable \(t\) has been dropped from the watershed \(A\) in (18). This fact greatly simplifies the dynamics of the model since, for any given profit sharing parameter, the accumulation/decumulation of capital - and hence the changes in the short run equilibrium wage \(w(t)\) and rental \(r(t)\) - does not change the number of firms in operation. This result is summarized in Proposition 1 below.

**Proposition 1:** The number of entrepreneurs is a decreasing function of the profit sharing parameter \(\delta\). For a given \(\delta\) it does not vary with time.

For a given parameter \(\delta\), and hence of \(A\), the wage \(w(t)\) and rental \(r(t)\) are determined from (15) and (17):

\[
\frac{b}{w(t)} = \frac{\alpha}{r(t)} \frac{\bar{N}}{K(t)} \frac{b + \delta(1 - a - b)}{(1 - \delta)(1 - a - b)} \int_{A}^{1} \lambda^{\frac{\Gamma}{1-a-b}} f(\lambda) d\lambda. \tag{19}
\]

Substitution of (18) into (19) yields

\[
\frac{b}{w(t)} = \frac{\alpha}{r(t)} \frac{\bar{N}}{K(t)} F(A). \tag{20}
\]

Equation (20) shows that, for a given stock of capital, the rental is proportional to the wage. The coefficient of proportionality is a linear
function of $F(\Lambda)$. Therefore, the larger the number of individuals who supply labor - and hence, the smaller the number of firms - then the higher the direct cost of using capital vis-à-vis labor.

The short run rental $r(t)$ is obtained substituting (20) into (15):

$$
\frac{r(t)}{a} = \left(\frac{\bar{N}}{K(t)}\right)^{1-a} [F(\Lambda)]^b \left(\int_\Lambda^{1} \lambda^{\frac{\delta}{1-a-b}} f(\lambda) d\lambda\right)^{1-a-b}. \quad (21)
$$

Equation (21) shows that the rental is a decreasing function of the aggregate capital stock in the short run. When capital is scarce, its cost is high.

The short run wage is given through substitution of (21) into (20):

$$
\frac{w(t)}{b} = \left(\frac{K(t)}{\bar{N}}\right)^a \left(\frac{\int_\Lambda^{1} \lambda^{\frac{\delta}{1-a-b}} f(\lambda) d\lambda}{[F(\Lambda)]^{1-b}}\right)^{1-a-b}. \quad (22)
$$

Equation (22) shows that the wage is an increasing function of the aggregate capital stock in the short run. When capital is abundant, labor is relatively scarce and thus it cost is high.

The stock of capital is given as the sum of all individuals' assets:

$$
K(t) = \int_0^{1} A(\lambda(t)) \cdot \bar{N} f(\lambda) d\lambda. \quad (23)
$$

Since $A(\lambda(t))$ is a state variable, in the short run $K(t)$ is a fixed variable. As a result, in what follows equation (23) will be used only in the characterization of the dynamics of the model, i.e., its path to the long run equilibrium.

The short run comparative statics of the rental $r(t)$ and wage $w(t)$ with respect to the profit sharing parameter $\delta$ is presented in Proposition 2.


Profit sharing

**Proposition 2:** For a given $K(t)$ the rent $r(t)$ and the wage $w(t)$ are decreasing functions of the profit sharing parameter $\delta$.

*Proof:* See Appendix.

In order to write the labor compensation $E$ of (10) as a function of $\Lambda$, the profit sharing parameter must be eliminated from that equation. Let $Z = Z(\Lambda)$ stand for the left hand side of (18). Then (18) yields

$$
\delta = \frac{Z(1 - a - b) - b}{(1 - a - b)(1 + Z)}.
$$

(24)

Substituting the expression above into (10) gives

$$
E(t) = (1 - a) \frac{w(t)}{b} \frac{Z}{1 + Z}.
$$

(25)

Substituting (22) and $Z$ into (25), one gets

$$
E(t) = (1 - a) \left( \frac{K(t)}{N} \right)^{\alpha} \frac{\Lambda^{\frac{\delta - c}{\delta - a - b}} \left[ F(\Lambda) \right]^b \left( \int_{\Lambda}^1 \lambda^{\frac{\delta - c}{\delta - a - b}} f(\lambda) d\lambda \right)^{1 - \frac{\delta}{\delta - a - b}}}{\Lambda^{\frac{\delta - c}{\delta - a - b}} F(\Lambda) + \int_{\Lambda}^1 \lambda^{\frac{\delta - c}{\delta - a - b}} f(\lambda) d\lambda}.
$$

(26)

The short run comparative statics of the labor compensation $E(t)$ with respect to the profit sharing parameter $\delta$ is presented in Proposition 3.

**Proposition 3:** Given $K(t)$, there exists $\delta_{SR}$ such that for any profit compensation $E(t)$ is an increasing function of $\delta$.

*Proof:* See Appendix.

Proposition 3 shows that the reduction of the wage $w(t)$ due to the introduction of profit sharing may be smaller than the profit
sharing income $P(t)$ received by workers, leading to an overall increase in workers’ total compensation $E(t) = w(t) + P(t)$. For this to hold, however the profit sharing parameter must not be too large. If it is too large, then the supply of labor increases so much, while the number of firms demanding their work decreases in the same number, then the net effect is a reduction of $E$.

Output of firm $\lambda$ is obtained from (1), (7) and (8)

$$Y_\lambda(t) = \lambda^{\frac{a}{1-a-b}} \left[ \left( \frac{b}{w(t)} \right)^a \left( \frac{a}{\tau(t)} \right)^b \right]^{\frac{1}{1-a-b}}. \quad (27)$$

Substituting (21) and (22) into (27), the output of firm $\lambda$ can be written as a function of the aggregate capital stock and $\Lambda$

$$Y_\lambda(t) = \left( \frac{K(t)}{N} \right)^a \frac{[F(\Lambda)]^b}{\left( \int_{\Lambda}^{1} \lambda^{\frac{a}{1-a-b}} f(\lambda) d\lambda \right)^{a+b}} \lambda^{\frac{\sigma}{1-a-b}}. \quad (28)$$

Aggregate output is given by integration of (28) from $\Lambda$ to 1

$$Y(t) = \int_{\Lambda}^{1} Y_\lambda(t) \tilde{N} f(\lambda) d\lambda$$

$$= (1 - a - b)K(t)^a \tilde{N}^{1-a} [F(\Lambda)]^b \left( \int_{\Lambda}^{1} \lambda^{\frac{\sigma}{1-a-b}} f(\lambda) d\lambda \right)^{1-a-b}. \quad (29)$$

The expression of aggregate output above shows that, although at the firm level the production is homogeneous of degree $(a + b)$ on labor and capital, at the aggregate level the production is homogeneous of degree one on these two factors. Decreasing returns to scale at the firm level leads to constant returns to scale at the aggregate level, when the employer-employee choice is taken into account. The invisible hand works in order to produce constant returns to scale
Profit sharing

at the aggregate level through the closing of the least efficient firms and their potential owners going to join the labor market.

The short run comparative statics of the output of firm $\lambda$, $Y_\lambda(t)$, and the aggregate output $Y(t)$ with respect to the profit sharing parameter $\delta$ is given in Proposition 4.

**Proposition 4:** Given $K(t)$, firm $\lambda$’s output $Y_\lambda(t)$ is an increasing function of the profit $Y(t)$ is a decreasing function of $\delta$.

**Proof:** See Appendix.

Proposition 4 shows that although profit sharing may increase the production of the firms that keep on operating, it decreases the production of the economy as a whole, since it reduces the number of firms in operation.

Comparing (9) and (27) one finds that the short run net profit of firm $\lambda$, $(1 - \lambda)\pi_\lambda(t)$, and its output $Y_\lambda(t)$ are related as

$$(1 - \delta)\pi_\lambda(t) = (1 - \delta)(1 - a - b)Y_\lambda(t).$$  \hspace{1cm} (30)

From (18) and (24), one gets

$$(1 - \delta)(1 - a - b) = \frac{1 - a}{1 + Z}$$ \hspace{1cm} (31)

where, as defined above, $Z$ stands for the left hand side of (18). From the definition of $Z$ and equations (28), (30) and (31), the short run net profit on firm $\lambda$ is given by

$$(1 - \delta)\pi_\lambda(t) = (1 - a)\left(\frac{K(t)}{N}\right)^a \lambda^{\frac{c}{1 - a - b}} \frac{[F(A)]^b \left(\int_A^1 \lambda^{\frac{c}{1 - a - b}} f(\lambda)d\lambda\right)^{1 - a - b}}{\int_A^1 \lambda^{\frac{c}{1 - a - b}} f(\lambda)d\lambda + A \lambda^{\frac{c}{1 - a - b}} F(A)}.$$  \hspace{1cm} (32)
The short run aggregate net profits \((1 - \delta)\pi(t)\) is given by the integration of (32) from \(\Lambda\) to 1:

\[
(1 - \delta)\pi(t) = \int_{\Lambda}^{1} \pi_\lambda(t) \tilde{N} f(\lambda) d\lambda
\]

\[
= (1 - a)K(t)^a \tilde{N}^{1-a} \left[ F(\Lambda) \right]^b \left( \int_{\Lambda}^{1} \lambda^{\frac{1}{1-a-b}} f(\lambda) d\lambda \right)^{2 - a - b} \frac{1}{\int_{\Lambda}^{1} \lambda^{\frac{1}{1-a-b}} f(\lambda) d\lambda + \Lambda^{\frac{1}{1-a-b}} F(\Lambda)}. (33)
\]

The short run comparative statics of firm \(\lambda\)'s net profit \((1 - \delta)\pi_\lambda(t)\) and the aggregate net profit \((1 - \delta)\pi(t)\) are given in Proposition 5.

**Proposition 5:** Given \(K(t)\), firm \(\lambda\)'s net profit \((1 - \delta)\pi_\lambda(t)\) and the aggregate net profit \((1 - \delta)\pi(t)\) are both decreasing functions of the profit sharing parameter.

**Proof:** See Appendix.

3.2. Long run general equilibrium.

Each individual's consumption decisions are determined by the maximization of (4) subject to a (5). Given the profit sharing parameter \(\delta\), and hence \(\Lambda\), individuals \(\lambda < \Lambda\) will choose to be workers and, therefore, their income \(I_\lambda\) will be given by the right hand side of the max function in (6); individuals \(\lambda \geq \Lambda\) will choose to be entrepreneurs and their income \(I_\lambda\) will be the net profit of the left hand side of the max function in (6). It follows that, for a fixed profit sharing parameter, the max function of (6) will not be a constraint for an individual after he has decided what kind of agent he will be. Therefore maximization of (4) will be made subject only to (5) for each individual.
The solution of this program is determined by the usual Keynes-Ramsey rule:

\[
\frac{\dot{C}_\lambda(t)}{C_\lambda(t)} = \sigma(C_\lambda(t)) \left[ r(t) - (\rho + \theta) \right]
\]

where \(\sigma\) is the elasticity of substituting of consumption

\[
\sigma(C) = - \frac{U'(C)}{CU''(C)}
\]

and the transversality condition

\[
\lim_{t \to \infty} e^{-\rho t} U'[C_\lambda(t)]A_\lambda(t) = 0.
\]

The long run aggregate equilibrium is defined as a steady state equilibrium at which each individual's savings are just enough to replace the depreciation of his assets. The flow of consumption \(C_\lambda(t)\) will be constant for each individual. It follows that the long run rental \(r^*\) will be the sum of the depreciation rate and the rate of time preference:

\[
r^* = \theta + \rho.
\]

The long run capital stock \(K^*\) is given through the substitution of (37) into (21) with \(r(t) = r^*:\)

\[
\left( \frac{K^*}{N} \right)^{1-a} = \frac{\alpha}{\rho + \theta}[F(\Lambda)]^b \left( \int_{\Lambda}^{1} \lambda^{1-a-b} f(\lambda) d\lambda \right)^{1-a-b}.
\]

Equation (38) shows that the long run capital stock is a decreasing function of the rate of time preference and depreciation. If individuals are too anxious to consume or if capital depreciates too fast, then individuals will choose not to accumulate much capital. It is proportional to the size of the population. The relation between the long run capital stock and the profit sharing parameter is given below.
Proposition 6: The long run aggregate capital stock $K^*$ is a decreasing function of the profit sharing parameter $\delta$.

Proof: See Appendix.

Equation (21) helps explain the negative relation between the profit sharing parameter and the long run capital stock. The right hand side of (21) shows that the rental depends (1) on the economy’s relative scarcity of capital vis-à-vis (potential) labor and (2) on the number of individuals who do not supply labor - which is equal to the number of firms. The higher the profit sharing parameter, the lower will be the number of firms demanding capital. In order to keep the long run rental at $\rho + \theta$, the capital stock must become relatively scarce vis-à-vis labor.

The long run wage $w^*$ is obtained through substitution of (38) into (22) with $K(t) = K^*$:

$$
\left( \frac{w^*}{b} \right)^{1-a} = \left( \frac{a}{\rho + \theta} \right)^{a} \left( \frac{\int_{\Lambda}^{1} \lambda^{1-\delta-b} f(\lambda) d\lambda}{F'(\Lambda)} \right)^{1-a-b} \quad (39)
$$

The relation between the long run wage and the profit sharing parameter is given below.

Proposition 7: The long run wage $w^*$ is a decreasing function of the profit sharing parameter.

Proof: See Appendix.

The right hand side of (22) shows that the wage depends (1) on the economy’s relative scarcity of (potential) labor vis-à-vis capital and (2) the number of individuals that actually supply labor. The higher the profit sharing parameter, the higher will be the economy’s
Profit sharing

long run relative abundance of (potential) labor - as proved in Proposition 6 - and the larger the number of individuals who supply labor. Both effects contribute to the reduction of the long run wage.

The long run labor compensation $E^*$ is given though substitution of (38) into (26) with $K(t) = K^*$:

$$E^* = (1 - a) \left( \frac{a}{\rho + \theta} \right)^{\frac{1}{1-a}} \frac{\Lambda^{\frac{1}{1-a}} \left( \int_{A}^{1} \lambda^{\frac{1}{1-a}} f(\lambda) d\lambda \right)^{\frac{1-a-b}{1-a}}}{\Lambda^{\frac{1}{1-a-b}} F(A) + \int_{A}^{1} \lambda^{\frac{1}{1-a-b}} f(\lambda) d\lambda}. \quad (40)$$

The long run relation between the profit sharing parameter and the worker's compensation is given below.

**Proposition 8:** There exists $\delta_{LR}$ such that, for any profit sharing parameter $\delta$ such that $\delta < \delta_{LR}$, the long run labor compensation $E^*$ is an increasing function of $\delta$.

**Proof:** See Appendix.

Proposition 8 could have been proven using equation (26) and the results of Proposition 3 and 4. Equation (26) shows that the impact of profit sharing on workers’ compensation $E$ is twofold. First through $\Lambda$, second through the capital stock $K$. The effect on $K$ is not present in the short run. Proposition 3 showed that the increase in $\Lambda$ due the introduction of profit sharing reduces the worker’s compensation $E$ for a given $K$. Proposition 6 assures that the long run capital stock is reduced by profit sharing. From (26), it follows that both effects contribute to reduce $E$ in the long run.

The long run output of firm $\lambda$ is obtained through the substitution of (38) into (28) with $K(t) = K^*$

$$Y_\lambda^* = \left( \frac{a}{\rho + \theta} \right)^{\frac{1}{1-a}} \lambda^{\frac{1}{1-a-b}} \left[ \frac{F(\Lambda)}{\int_{A}^{1} \lambda^{\frac{1}{1-a-b}} f(\lambda) d\lambda} \right]^{\frac{b}{1-a}}. \quad (41)$$
The long run aggregate output $Y^*$ is determined through the substitution of (38) into (29) with $K(t) = K^*$, or through the integration of (41) from $\Lambda$ to 1

$$Y^* = \tilde{N} \left\{ \left( \frac{a}{\rho + \theta} \right)^a [F(\Lambda)]^b \left( \int_{\Lambda}^{1} \lambda^{1-\frac{\varepsilon}{1-a-\delta}} f(\lambda) d\lambda \right)^{1-a-b} \right\}^{\frac{1}{1-a}}. \quad (42)$$

From (41) and (42) one finds the following proposition.

**Proposition 9:** The long run output of firm $\lambda$ is an increasing function of the profit sharing parameter, while the long run aggregate output is a decreasing function of $\delta$.

*Proof:* See Appendix.

Proposition 9 could have been proven using equations (1), (7) and (8) together with Proposition 7. Since in the long run the rental is constant, those equations assure that the output per firm is a decreasing function of the wage. Proposition 7 assures that the wage falls in the long run. So the output per firm increases in the long run.

The long run net profit of firm $\lambda \pi^*_\lambda$ is obtained through the substitution of (38) into (32) with $K(t) = K^*$:

$$\begin{align*}
(1 - \delta)\pi^*_\lambda &= (1 - a)\lambda^{\frac{\varepsilon}{1-a-\delta}} \\
&\quad \left\{ \left( \frac{a}{\rho + \theta} \right)^a [F(\Lambda)]^b \left( \int_{\Lambda}^{1} \lambda^{1-\frac{\varepsilon}{1-a-\delta}} f(\lambda) d\lambda \right)^{1-a-b} \right\}^{\frac{1}{1-a}} \\
&\quad \int_{\Lambda}^{1} \lambda^{1-\frac{\varepsilon}{1-a-\delta}} f(\lambda) d\lambda + \Lambda^{1-\frac{\varepsilon}{1-a-\delta}} F(\Lambda) \quad (43)\end{align*}$$

Finally, the long run aggregate net profit $(1 - \delta)\pi^*$ is given through the substitution of (38) into (33) with $K(t) = K^*$ or through

Revista de Econometria, 15(2) Novembro/Março 1996
the integration of (43) from $\Lambda$ to 1

\[
(1 - \delta)\pi^* = (1 - a)\tilde{N} \frac{\left[\left(\frac{a}{\rho + \theta}\right)^b \left(\int_1^\Lambda \lambda^{1 - \frac{\epsilon}{a - \beta}} f(\lambda) d\lambda\right)^{\frac{1 - a - b}{1 - a}}\right]}{1 + \left[\Lambda^{1 - \frac{\epsilon}{a - \beta}} F(\Lambda)/\int_1^\Lambda \lambda^{1 - \frac{\epsilon}{a - \beta}} f(\lambda) d\lambda\right]}.
\]

(44)

Equations (43) and (44) yield the following proposition.

**Proposition 10:** The long run net profit of firm $\lambda$ and the long run aggregate net profit are decreasing functions of the profit sharing parameter $\delta$.

**Proof:** See Appendix.

Proposition 10 is a consequence of equations (32) and (33), and propositions 5 and 6. Net profit per firm as well as aggregate net profits are increasing functions of the aggregate capital stock - according to (32) and (33) - and decreasing functions of $\Lambda$ - according to the Proposition 5. Since profit sharing causes the aggregate capital stock to decrease in the long run, it follows that net profits must decrease in the long run.

3.3. The dynamics.

Starting from a profit sharing parameter equal to zero for instants $t < 0$, how would the economy react to an increase of that parameter to a (small) positive value from instant $t = 0$ onwards? The dynamics is depicted in chart 1.

Initially the economy is in a long run equilibrium with rental equal to the rate of time preference plus depreciation. The increase in the profit sharing parameter reduces the (net) profit of the least
efficient entrepreneurs to a level below their employees' compensation. These entrepreneurs, therefore, no longer want to keep their firms operating.

Immediately after $t = 0$ some companies close down, and its owners become workers. These individuals were the least efficient entrepreneurs in the equilibrium without profit sharing. The creation of profit sharing increases the worker's (new) earning $E(0+)$ to a level above the (new) net profits $\pi_L(0+)$ they would receive if they remained entrepreneurs instead of becoming workers.

The short run stock of capital is constant but the supply of labor increases. The number of firms demanding capital and labor is smaller so the prices of renting capital $r(t)$ and employing labor $w(t)$ are reduced in general equilibrium.

Since the direct cost of capital and labor fail, the firms that keep on operating increase their demand for these factors, thus increasing each company's output. The fall of the aggregate output shows that the reduction in the number of firms is proportionally larger than the increase in the (average) production of each firm.

The reduction of the (gross) return on assets to a level below the sum of the rates of time preference plus depreciation, $r(t) < \rho + \theta$, makes each individual's flow of consumption increase. Savings become lower than capital depreciation, leading to a gradual reduction of the capital stock.

As the aggregate capital stock is gradually reduced, the rental starts to increase. As capital becomes scarcer, labor becomes more abundant, thus the wage falls continually. Each firm's output is reduced for the use of capital is less intensive, so aggregate output gradually falls. Labor compensation also falls continually since gross profits and wages fall.

The economy eventually reaches a new long run equilibrium when the (gross) rate of return on capital has risen to the sum of the rates of time preference plus depreciation. At this new long run equilibrium the economy as a whole is poorer, for aggregate output
Profit sharing

is much lower than it was with a zero profit sharing parameter.

In short, profit sharing has two main effects. In the short run it inhibits entrepreneurial initiatives, leading some entrepreneurs to close down their business and become workers. Later it promotes a spending binge which depletes the economy’s stock of capital.

If the profit sharing parameter is sufficiently low, workers will be better off after its introduction, while entrepreneurs will be worse off. In the real world, the fact that workers can be better off even in the long run might induce populist governments to enact mandatory profit sharing schemes.

4. The Middle Voter Choice for Profit Sharing.

It was shown that for low values of the sharing parameter, the introduction of profit sharing increases workers’ total compensation in the short as well as in the long run. It also reduces the rental in the short run and net profits in the short and in the long run. Given these facts, one can describe a situation in which a majority vote can lead to the economy-wide mandatory adoption of profit sharing despite its impoverishing aggregate consequences.

In fact, consider the following situation. The economy is in a long run equilibrium: (1) the distribution of entrepreneurial prowess is such that when there is no profit sharing, workers outnumber entrepreneurs, and; 2) the past history has determined a distribution of assets across individuals such that each worker’s accumulated assets are negligibly low when compared to the wage he receives. This implies that the rentiers do not include workers, but only entrepreneurs.

Under these conditions, in the short run the economy-wide adoption of profit sharing would increase the income of workers and reduce the income of entrepreneurs. This is so because in the short run total compensation of workers would increase (Proposition 4) and the rental and net profits would fall (Proposition 2 and 5). In the long run, the income of workers would still be higher than its level before
the adoption of profit sharing (Proposition 8), whilst the income of entrepreneurs would be lower (Proposition 9).

This implies that if the adoption of profit sharing were put to vote, the yes vote would win. This is so because those who loose out are fewer than those who win out. This means that the democratic society would freely prefer to reduce the aggregate output because the burden of the loss would fall on the minority of rentiers and entrepreneurs.

5. Conclusion and Extensions.

The model presented above is an example that mandatory profit sharing schemes may harm the economic incentives that lead individuals to set up their own business. Moreover, since it can increase worker’s gross compensation at the expense of their employer’s pure profits, populist governments may feel tempted to meddle with labor-firms relations, introducing compulsory profit sharing systems. Weitzman’s case of public policy measures taken to induce firms to adopt share-type compensation schemes might be used as the populist’s theoretical underpinnings to go on bullying the work of the invisible hand.

The model does not deny that profit sharing schemes freely negotiated between firms and workers may indeed reduce risk bearing by firms of very cyclical sectors, thus leading to higher rates of employment. Nor does it deny that the impact of sharing schemes on workers’ effort may also be of importance in some industries. In short, my view on profit sharing is that it can be of great help in some industries and of great harm in others. Profit sharing is certainly of help in very cyclical industries. The same holds for industries where the individual effort plays a key role in the final result of the production process, and where this effort is observable.

While the micro conditions in one sector of the economy may recommend sharing schemes, they may not recommend it in other sectors. Aggregate arguments such as Weitzman’s cannot justify any
public intervention to foster profit sharing. The choice of whether to adopt sharing schemes or not must be left to the agents directly involved: workers and firms.

\[ F(\Delta(t)) \]

Number of Workers

\[ r(t) \]

Rental

\[ w(t) \]

Wage

\[ E(t) \]

Worker's Compensation

\[ (1-\delta) \tau_\lambda(t) \]

Net Profit per Firm

\[ K(t) \]

Aggregate Capital Stock

\[ Y(t) \]

Aggregate Output
Appendix

Proof of Proposition 2

Since Proposition 1 assures that $\Lambda$ is an increasing function of $\delta$ it must be proven that the right hand sides of (21) and (22) are decreasing functions of $\Lambda$. This is immediate for (22). For (21) define the real valued differentiable function $G$ as

$$G(\Lambda) = [F(\Lambda)]^b \left( \int_\Lambda^1 \lambda^{\frac{\delta}{1-\delta-b}} f(\lambda) d\lambda \right)^{1-a-b}.$$ 

Taking logarithms and differentiating one gets:

$$\frac{d \ln G(\Lambda)}{d\Lambda} = \frac{G'(\Lambda)}{G(\Lambda)} = \frac{f(\Lambda)}{F(\Lambda)} \left[ b - (1-a-b)\Lambda^{\frac{\delta}{1-\delta}} \frac{F(\Lambda)}{\int_\Lambda^1 \lambda^{\frac{\delta}{1-\delta-b}} f(\lambda) d\lambda} \right]. \quad (A1)$$

Substituting (18) into the expression above one gets:

$$\frac{G'(\Lambda)}{G(\Lambda)} = \frac{f(\Lambda)}{F(\Lambda)} \left[ b - \frac{b+\delta(1-a-b)}{1-\delta} \right] = \frac{-f(\Lambda)}{f(\Lambda)} (1-a) \frac{\delta}{1-\delta} < 0. \quad (A2)$$

Proof of Proposition 3

Since Proposition 1 assures that $\Lambda$ is an increasing function of $\delta$, it must be proven that there exists $\delta_{SR}$ such that, for any $\delta < \delta_{SR}$, the right hand side of (26) is a decreasing function of $\Lambda$. In order to do so, define the real valued differentiable function $H$ as

$$H(\Lambda) = \Lambda^{\frac{\delta}{1-\delta-b}} F(\Lambda) + \int_\Lambda^1 \lambda^{\frac{\delta}{1-\delta-b}} f(\lambda) d\lambda. \quad (A3)$$
Profit sharing

From (A1) it follows that (26) can be written as

\[ E(t) = (1 - a) \left( \frac{K(t)}{N} \right)^a \frac{\Lambda^{\frac{c}{1-a-b}} G(\Lambda)}{H(\Lambda)}. \]  

(A4)

Straightforward differential calculus and some algebra show that

\[
\frac{d}{d\Lambda} \ln \left[ \frac{\Lambda^{\frac{c}{1-a-b}}}{H(\Lambda)} \right] = \frac{c}{1-a-b} \frac{\int_{\Lambda}^{1} \lambda^{\frac{c}{1-a-b}} f(\lambda) d\lambda}{\Lambda H(\Lambda)}
\]

\[ = \frac{c}{(1-a-b)\Lambda} \left[ 1 + \frac{\int_{\Lambda}^{1} \lambda^{\frac{c}{1-a-b}} F(\Lambda) d\lambda}{\int_{\Lambda}^{1} \lambda^{\frac{c}{1-a-b}} f(\lambda) d\lambda} \right]^{-1}. \]

Substituting the right hand side of (18) into the expression above, one gets:

\[
\frac{d}{d\Lambda} \ln \left[ \frac{\Lambda^{\frac{c}{1-a-b}}}{H(\Lambda)} \right] = \frac{c}{(1-a-b)\Lambda} \left[ 1 + \frac{b + \delta(1-a-b)}{(1-\delta)(1-a-b)} \right]^{-1}
\]

\[ = \frac{c(1-\delta)}{(1-a)\Lambda}. \]  

(A5)

From (A2), (A4) and (A5), it follows that

\[
\frac{d}{d\Lambda} \ln E(t) = \frac{c(1-\delta)}{(1-a)\Lambda} - \frac{f(\Lambda)}{F(\Lambda)} \frac{1}{1-\delta}. \]  

(A6)

From (24), the profit sharing parameter \( \delta \) can be expressed as a function of \( Z \) - recall that \( Z \) is the left hand side of (18) - so (A6) can be written as

\[
\frac{d}{d\Lambda} \ln E(t) = \frac{c}{1-a-b} \frac{1}{(1+Z)\Lambda} - \frac{f(\Lambda)}{F(\Lambda)} [Z(1-a-b) - b]. \]  

(A6')
From (18), it can be seen that $Z$ is a real valued continuous function of $\Lambda$, and hence of $\delta$, with the following properties:

$$\frac{dZ}{d\delta} > 0, \quad \lim_{\delta \to 0} Z(\delta) = \frac{b}{1 - a - b}, \quad \lim_{\delta \to 1} Z(\delta) = +\infty.$$ 

Since $\Lambda$ is also an increasing function of $\delta$, let $\Lambda_0$ be its value at $\delta = 0$ and from (18) $\lim_{\delta \to 1} \Lambda(\delta) = 1$.

From the properties of the functions $Z$ and $\Lambda$ one has:

$$\lim_{\delta \to 0} \frac{d}{d\Lambda} Ln E(t) = \frac{c}{(1 - a)\Lambda_0} > 0$$

$$\lim_{\delta \to 1} \frac{d}{d\Lambda} Ln E(t) = -\infty < 0.$$ 

Continuity assures the existence of $\delta_{SR} \in (0, 1)$ such that for any $\delta \in [0, \delta_{SR})$, $\frac{d}{d\delta} E(t) < 0$.

**Proof of Proposition 4**

Since Proposition 1 assures that $\Lambda$ is an increasing function of $\delta$, it follows from (28) that firm $\lambda$’s output is also an increasing function of $\delta$.

From (29) and (A1) the aggregate output can be written as

$$Y(t) = (1 - a - b)K(t)^a \tilde{N}^{1-a} G(\Lambda)$$

where $G$ is a decreasing function of $\Lambda$ and hence of $\delta$, as shown in the Proof of Proposition 2.

**Proof of Proposition 5**

Applying the same notation used in the Proof of Proposition 3, (32) can written as

$$(1 - \delta)\pi(1) = (1 - a) \left( \frac{K(t)}{N} \right)^a \lambda^{\frac{1-\delta}{1-\gamma}} \frac{G(\Lambda)}{H(\Lambda)}$$
where $G$ is decreasing and $H$ is increasing. Expression (33) can be written as

$$(1 - \delta)\pi(t) = (1 - a)K(t)^a \tilde{N}^{1-a} \frac{G(\Lambda)}{H(\Lambda)} \int_{\Lambda}^{1} \lambda^{\frac{\delta}{1-a-b}} f(\lambda) d\lambda$$

which is also decreasing in $\Lambda$ since $G$ and the integral are both decreasing and $H$ is increasing.

**Proof of Proposition 6**

The same Proof of Proposition 2 applies.

**Proof of Proposition 7**

The right hand side of (39) is a decreasing function of $\Lambda$ and hence of $\delta$.

**Proof of Proposition 8**

Recalling the definition of functions $G$ and $H$ used in the Proof of Proposition 5, comparing (26) and (40), it follows that

$$\frac{d}{d\Lambda} \ln E^* = \frac{d}{d\Lambda} \left[ \frac{\Lambda^{\frac{\delta}{1-a-b}}}{H(\Lambda)} \right] - \frac{1}{1-a} \frac{d}{d\Lambda} \ln G(\Lambda). \quad (A7)$$

From (A2), (A5) and (A7):

$$\frac{d}{d\Lambda} \ln E^* = \frac{c(1-\delta)}{(1-a)\Lambda} - \frac{f(\Lambda)}{F(\Lambda)} \frac{\delta}{1-\delta}. \quad (A8)$$

Expression (A8) is essentially equal to (A6), so the Proof of Proposition 5 applies.
Proof of Proposition 9

The right hand side of (41) is an increasing function of $\Lambda$ and hence of $\delta$. The right hand side of (42) is a decreasing function of $\Lambda$ as was shown in the Proof of Proposition 2.

Proof of Proposition 10

The right hand side of (43) is a quotient function of $\Lambda$ whose numerator is decreasing - see definition of function $G$ in Proposition 2 - and the denominator is increasing. The same holds for the right hand side of (44).


References


Profit sharing


